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# Essays on Optimal Mix of Taxes, Spatiality and Persistence under Tax Evasion

Mohammad Yunus

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# ESSAYS ON OPTIMAL MIX OF TAXES, SPATIALITY AND PERSISTENCE UNDER TAX EVASION

BY

MOHAMMAD YUNUS

A Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree
of
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ACCEPTANCE

This dissertation was prepared under the direction of the candidate's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics in the Andrew Young School of Policy Studies of Georgia State University.

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Despite the numerous comments and suggestions I received, errors and omissions are my own responsibility.

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ABSTRACT

ESSAYS ON OPTIMAL MIX OF TAXES, SPATIALITY AND PERSISTENCE UNDER TAX EVASION

BY

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This dissertation analyzes the optimal mix of direct and indirect taxes in an economy with multiple tax collecting authorities when both the taxes are subject to evasion and to what extent the tax compliance behavior of individuals in the United States are persistent and spatially dependent.

Essay I derives and provides an intuitive interpretation of: (i) impact of the changes in the government instruments on tax evasion by firms, the expected prices they charge, and the expected tax rates they face; (ii) a generalized version of Ramsey rule for optimal commodity taxation which accounts for income tax evasion from either or both the tax authorities; (iii) generalized formulae for the optimal income tax rate for each of the tax authorities; and (iv) the tradeoff between optimal tax rates and audit probabilities for each of the tax authorities. It also re-examines controversies surrounding the uniform income taxes and the differentiated commodity taxes, and investigates how income tax evasion affects the progressivity of the income tax rates. It concludes that whether or not

tax evasion calls for reductions in the optimal income tax rates hinges on how tax evasion and the associated concealment costs vary across individual taxpayers.

Essay II introduces the twin issues of spatiality and persistence in the individual income tax evasion. While the issue of persistence arises through accumulated learning over time, spatiality arises for several reasons. Some these include the exchange of information between taxpayers; the social norm of tax compliance: an individual would comply if everybody in the society complies and vice versa; individuals faced with dynamic stochastic decision problems that pose immense computational challenges may simply look to others to infer satisfactory policies and interpersonal dependence works through learning by imitating rather than learning by doing. State-level annual per return evasion of individual income tax and related data were used to examine the above hypotheses and found supports for both of them in the individual income tax evasion in the United States.

# ESSAY I: TAX EVASION AND OPTIMAL MIX OF TAXES

## *INTRODUCTION*

Depending on the feasibility of choices, policymakers in both developed and developing economies face the challenging task of striking a delicate balance between the direct and indirect taxes in the face of differential tax compliance. Insofar as taxes cannot be collected without costs, different proposals for tax instruments and tax structures should be judged on the basis of their administrative advantages, differential compliance, equity, efficiency, adequacy, and induced concealment costs. The World Bank and the International Monetary Fund usually prescribe that developing countries rely more on indirect taxation in some form of uniform commodity taxes, especially the value added taxes because indirect taxes are more difficult to evade. The justification for this prescription is that poorly developed institutional structures of the tax collecting agencies in these countries are not conducive to collecting direct taxes in the face of ubiquitous nature of tax noncompliance and venal tax officials. The critics of this prescription usually point at the regressive nature of indirect taxation and question the presumption that evasion of commodity taxes is innocuous.

The optimal taxation theory offers hardly any guidance to resolve this debate, for this literature is primarily concerned with characterizing the tax structure that minimizes inefficiencies when the tax base is observable at no costs while giving due cognizance to equity concerns. Feasibility constraints in tax collection had remained outside the usual purview of optimal taxation literature until tax evasion literature made inroads into public finance. How do differences in evasion characteristics of taxes affect the choice and design of optimal tax systems? Why do the direct and the indirect taxes coexist even in presence of tax evasion in most countries? If both the direct and the indirect taxes are to be collected, how should one strike a balance in designing an optimal mix? The existing literature does not provide much substantive ground to resolve these issues.

This essay is an exploration of the role of tax evasion in the design of tax systems. Our particular interest lies in how tax evasion affects the optimal mix of direct and indirect taxes in the presence of multiple tax collecting authorities. It attempts to build on Boadway, Marchand, and Pestieau (1994). These authors provide a justification for an optimal tax solution in a model with linear commodity taxes and non-linear income taxes when *only the latter* can be evaded at private costs. Despite offering an important insight into how evasion of both direct and indirect taxes affects the optimal tax structures, their work fails to account for some realistic features of actual tax systems. For instance, evaders in their model are never caught in their cheating. Thus, these are models of tax avoidance with differential compliance costs, and therefore lack a crucial feature of tax evasion—the uncertainty it introduces into the agents' decision making due to the presence of random audits. This is of critical relevance scores of studies have shown that uncertainty brings significant changes to the fundamental rules suggested by certainty models of optimal taxation. Further, it is also unrealistic to assume that commodity taxes are not subject to evasion.

We consider a model with linear income and commodity taxes both of which can be evaded at private costs in a world with two tax collecting authorities. One may think of the two tax authorities as the federal and state revenue authorities as in the U.S. or tax authorities at the national and local government levels in other countries. In practice, there may be more than two levels of tax authorities in some countries such as in the U.S. Thus, the assumption of two tax authorities would serve as a prototype of the real world complexities, especially for the large western capitalist economies. This is equally true for many developing economies with large territorial boundaries. The sheer size of these economies makes it hard for the tax administration to control everything from the center. Besides, with federal constitutional system allows the local governments in many of the developing countries to impose and collect taxes in their own jurisdictions apart from the taxes imposed by the federal government.

While firms pay commodity taxes to a single tax authority, income earners have to pay taxes to two different tax authorities. Our analysis shows how the standard results in the optimal taxation literature are affected by evasion of direct and/or indirect taxes by individuals and firms while accounting for the uncertainty arising from tax audits by at least one of the tax authorities and the costs associated to the concealment of true tax liability. By including the audit probabilities on both income and commodities in the set of feasible instruments, our model is one of optimal taxation and optimal enforcement of the tax laws. From this perspective, our model is closer to Slemrod (1994). With countries gradually heading towards uniform tax rates, the model provides a justification for the relevance of the analysis and a logically consistent framework to study important policy questions related to the optimal mix of taxes, the design of optimal tax structures, and the optimal enforcement of the tax base.

The optimal tax system in our model involves a mix of linear income taxes and differentiated commodity taxes. We derive and provide an intuitive interpretation of: (i) the impact of the changes in the government instruments on tax evasion by firms, the expected prices they charge, and the expected tax rates they face; (ii) a generalized version of Ramsey rule for optimal commodity taxation which accounts for income tax evasion from either or both the tax authorities; (iii) generalized formulae for the optimal income tax rates for each of the tax authorities; and (iv) the tradeoff between optimal tax rates and audit probabilities for each of the tax authorities. We also re-examine controversies surrounding the uniform income taxes and the differentiated commodity taxes, and how income tax evasion affect the progressivity of the income tax rates. We conclude that whether or not tax evasion calls for reductions in the optimal income tax rates (and thus lower progressivity) hinges on how tax evasion and the associated concealment costs vary across individual taxpayers.

Section 2 provides a brief review of the literature on the optimal taxation and tax evasion. Section 3 develops a model and discusses the behaviors of firms and individuals together with some key comparative static results. Sections 4 and 5 restate the optimal tax problem, and derive and discuss the expressions that characterize the optimal commodity tax structure and the optimal income tax rates for the two tax authorities, compare them with their well-known analogues in standard optimal tax models, and discuss conditions under which the two coincide while highlighting the new features of our model. Section 6 investigates the issue of the progressivity of the optimal income tax rates. Section 7 characterizes the tradeoff between optimal commodity tax rates and audit probabilities and the optimal enforcement of income taxes. Finally, Section 8 concludes with a summary of results and the avenue of future research.

## *A BRIEF REVIEW OF TAX EVASION AND OPTIMAL TAXATION*

The essay touches two strings of issues in public finance—optimal taxation and tax evasion. The optimal tax literature has been primarily concerned with characterizing the tax structure that minimizes the distortions that second best taxation entails to individual choices of work and consumption while achieving certain equity goals. Personal commodity consumption is not observable (or very costly to observe) in Ramsey's (1927) optimal linear commodity tax problem. The tax authority observes aggregate demand or output of each commodity, and uses this observation as a screening device based on *a priori* information about the differences in consumption patterns between individuals with different ability. On efficiency grounds, the Ramsey rule suggests higher tax rates on commodities with lower elasticity of demand. But it does not specify the conditions that make uniform commodity tax rate optimal. In the optimal income taxation models following Mirrlees (1971), the tax authority observes income of the individuals but cannot observe their ability and/or labor supply independently. Taxes are then levied on income that is taken as a proxy for an individual's productivity. A fundamental normative question of tax policy is then determining the optimal degree of income tax progressivity.

Following Ramsey (1927) and Mirrlees (1971), Atkinson and Stiglitz (1976) show that tax structure does not involve a mix of taxes when preferences between consumption and leisure is weakly separable; a general income tax suffices to achieve the revenue and redistributive goals. When the only set of instruments available to the government are a linear income tax and linear commodity taxes, Deaton (1979, 1981) shows that uniform commodity taxation is optimal if preferences are separable between goods and leisure and the Engel curves are linear. The linear income tax suffices, as commodity taxes do not serve any additional redistributive goal. Auerbach (1985), Stern (1987), and Stiglitz (1988) provide thorough surveys on the optimal taxation literature. Atkinson and Stiglitz (1980) give a comprehensive treatment of the subject.

The basic tenet of tax evasion models is that neither individual income nor consumption (nor sales of firms) are observable to tax authorities without costs. Tax assessments are largely based on taxpayers' reports. Occasionally tax authorities conduct audits on taxpayers to determine their true tax liability. Since the availability of audits expands the tax authorities' set of instruments, it brings forth new tradeoffs in the design of optimal tax policy. Further, the efficiency costs of the tax system are no longer limited to the usual distortions of individual labor supply and consumption choices. The costs of administering and enforcing the tax system and the compliance costs of taxpayers impose an additional deadweight loss to the society. These include audit costs, filing costs along with the costs incurred by taxpayers to conceal their true tax liability. As a result, tax authorities' policies are no longer preoccupied with choosing the optimal tax base and structure of taxation as the same revenue objective can now be achieved through a combination of several policy instruments.

Allingham and Sandmo (1972), Kolm (1973), and Srinivasan (1973) are the seminal papers that first studied the tax evasion problem. These authors treat tax evasion as a problem of individual choice under uncertainty to which standard portfolio allocation theory could be applied. Sandmo (1981) and Kaplow (1990) were the first to look into the problem of optimal income and commodity taxation respectively. They derive modified rules for optimal income and commodity tax rates that account for tax evasion and characterize the factors that determine the choice between higher taxation and tighter enforcement of tax laws. Cremer and Gahvari (1993) introduce tax evasion into the Ramsey commodity taxation problem and derive a modified version of Ramsey rule for optimal commodity taxes, characterize the tradeoff between optimal tax rates and audit probabilities and discuss how tax evasion and its concealment affect some standard results such as the optimality of uniform commodity taxes. With the exception of Sandmo (1981), the above works focus on a representative consumer economy and hence do not examine fully the issue of tax progressivity. Slemrod (1994) studies the impact of tax avoidance on optimal income tax progressivity in a linear income tax model with a costly tax enforcement mechanism, and concludes that increased tax enforcement is an alternative to higher tax rate for ensuring income tax progressivity. Cremer and Gahvari (1994) also focus on evasion in the linear tax problem but study the role of the income concealment technology, particularly how the possibility to influence the probability of being caught evading and the costs of concealment affect the progressivity of optimal linear income tax. Cremer and Gahvari (1996) study the effect of evasion and concealment costs in the optimum general income tax framework. Cowell (1990), Andreoni, Erard and Feinstein (1998), Alm (1999), and Slemrod and Yitzhaki (2002) are classic surveys on tax evasion literature.

The above results on optimal taxation and tax evasion do not provide any satisfactory ground to justify the common coexistence of direct and indirect taxation. Boadway, Marchand, and Pestieau (1994) are the first to use differences in tax evasion as a rationale for justifying a mix of direct and indirect taxes as optimal. They consider the choice between a general income tax and linear commodity tax in a two-class economy where taxes have different evasion characteristics, including concealment costs and show that, in general, the optimal tax solution involves a mix of taxes—a result in sharp contrast to Atkinson and Stiglitz (1976). Under the premise that indirect taxes are harder to evade, their model allows for evasion of only the direct taxes.

Boadway, Marchand, and Pestieau (1994) lack an enforcement mechanism to audit income reports—a crucial issue that comes with tax evasion as it introduces uncertainty into the agents' decision making. This is of critical relevance since works by Weiss (1976), Eaton and Rosen (1980a, 1980b), Varian (1980), Stiglitz (1982), Hamilton (1987), and Cremer and Gahvari (1995) show that optimal taxation rules are substantially modified in the presence of uncertainty. Moreover, enforcement parameters in these studies are excluded from the set of feasible tax revenue instruments of the tax authorities. But Slemrod (1994) stresses that optimal tax problem should consider both the choice of tax rules and their enforcements.

Our model combines *two* linear income taxes and a linear commodity tax subject to evasion at private costs. We extend Cremer and Gahvari's (1993) commodity tax evasion problem to an economy with risk averse firms and juxtapose two linear income taxes for two different tax authorities that are also subject to evasion. In contrast to Cremer and Gahvari (1994), we relax the assumption of quasi-linear preferences in analyzing the income taxes, and show that this is crucial to determining the extent to which tax evasion affects optimal tax rules. By expanding the set of feasible instruments to include audit probabilities, we investigate the tradeoffs that arise between tax rates and audit probabilities as an alternative means to raise tax revenues. Since the difficulties in observing personal consumption patterns and higher costs of administering and enforcing a system of multiple taxes dictate a heavy reliance on a few tax rates, our model bears relevance to most real world tax systems.

### *A MODEL OF TAX EVASION AND MIX OF TAXES*

Consider a competitive economy with *n* industries each with "many" identical firms producing a homogenous commodity. Individuals have identical preferences over the set of *n* commodities and leisure but differ in their abilities. There are two tax authorities—A and B; tax authority A collects a fixed amount of revenue by imposing taxes on wage income while B collects revenue by imposing taxes on both wage income and commodity sales.

Like other tax evasion models none of the tax authorities observe true incomes of individuals or sales of firms (by tax authority B). Therefore, tax assessments are based on individuals' income and firms' sales reports. Tax authorities carry out costless cursory examinations that reveal tax-dodgers unless the latter spend resources to conceal incomes and output. Tax authorities also carry random independent audits of taxpayers that reveal true incomes and sales proceeds.

Tax authority B collects a fixed amount of revenue $M_B$ by choosing the $n \times 1$ vector of commodity tax rates $\underline{t}$, with $t_i \in [0,1]$ and industry specific audit probabilities, $\underline{\sigma}$, with $\sigma_i \in (0,1)$, the parameters of linear income tax function $t_B$, the lump sum grant $\alpha_B$ and the probability of randomly audited incomes $\sigma_B$. Similarly, tax authority A collects a fixed amount of revenue $M_A$ by choosing the parameters of linear income tax function $t_A$, the lump sum grant $\alpha_A$ and the probability of randomly audited incomes $\sigma_A$. Finally, there is a 'super government' that maximizes a strictly concave Bergson-

Samuelson social welfare function that represents the social preferences for redistribution subject to the revenue constraints of the two tax authorities.

Firm's Sales Report

Firms in our model are characterized by constant returns to scale technology. The marginal cost of production, $c_i$, in industry *i* is constant for all firms, but it differs across industries. Industry *i* sells its output, $X_i$, in a competitive market at price $p_i$ and is subject to a commodity tax rate $t_i$. Tax authority B assesses the due taxes of a typical firm in the industry based on its reported sales. A firm can evade taxes by reporting only a fraction $0 \leq \delta_i \leq 1$ of its sales. The tax authority B carries out a cursory scrutiny of the firm's report that reveals true sales proceeds unless the latter spends resources to conceal its unreported sales. Each firm incurs a cost of concealment $G_i(1-\delta_i)$ per unit of output $z_i$. These costs are proportional to unreported output. Let us assume that $G_i(1-\delta_i)$ is strictly convex with $G_i^{'}(0) \to 0$ and $G_i^{'}(1) \to \infty$. Define $g_i(1-\delta_i) = (1-\delta_i)G_i(1-\delta_i)$. This implies that $g_i^{'}(0) \to 0$ and $g_i^{'}(1) \to \infty$.

Firms in each industry face a probability, $\sigma_i$, of audit that is independent of their sales reports. When caught cheating, firms have to pay the true taxes due plus a penalty (θ -1) proportional to the amount of taxes evaded. The penalty rate is assumed to be exogenous and equal in all industries. Firms take prices and enforcement parameters set by the tax authority B as given, and maximize expected profits by choosing output and the proportion of sales reported for tax assessments. Thus, firm *i*'s sales report is

$\Gamma_{NC_i} = p_i - g_i - \delta_i t_i$ if it is not caught cheating and $\Gamma_{C_i} = p_i - g_i - t_i - (\theta - 1)(1 - \delta_i)t_i$ if it is caught cheating.

We will derive the expected profit assuming that firms are risk-averse.[1] The competitive assumption together with the constant marginal costs and proportional concealment costs imply that output will be determined endogenously in the case of risk-averse firms. This assumption is at variance with Cremer and Gahvari (1993) who adopt separability due to assumption of risk neutrality. This separability no longer exists when firms are risk-averse as is typically the case for small-scale owner-managed businesses. Under these circumstances, the problem faced by firms in this economy can be analyzed by focusing on a representative firm. Hence, firm *i* in one of the industries solves:

$$\max_{\delta_i} E(\pi_i) = (1 - \sigma_i)\pi_i[(p_i - g_i - \delta_i t_i)z_i] + \sigma_i \pi_i[\{p_i - g_i - t_i - (\theta - 1)(1 - \delta_i)t_i\}z_i] - c_i z_i \quad (3.1)$$

The assumptions of risk aversion of firms and the convexity of the concealment technology imply that the above expected profits function is strictly concave. Thus, the first and the second order conditions for this problem are:

$$(1 - \sigma_i)\pi_i'(\Gamma_{NC_i})(g_i' - t_i)z_i + \sigma_i \pi_i'(\Gamma_{C_i})(g_i' + t_i(\theta - 1))z_i = 0 \quad (3.2)$$

$$(1 - \sigma_i)\left[\pi_i''(\Gamma_{NC_i})(g_i' - t_i)^2 z_i^2 - \pi_i'(\Gamma_{NC_i})g_i'' z_i\right] + \sigma_i\left[\pi_i''(\Gamma_{C_i})(g_i' + t_i(\theta - 1))^2 z_i^2 - \pi_i'(\Gamma_{C_i})g_i'' z_i\right] < 0 \quad (3.3)$$

[1] It is a polemic issue if firms are risk-averse or risk neutral. To us risk aversion is a tenable assumption for the small-scale firms while risk neutrality assumption is necessary for firms with corporate environment. Although large firms with corporate culture individually pays more tax revenue to tax authority, tax payments of the small firms as a group far exceed their counterpart of the large firms. Thus modeling firms as risk averse comes closer to reality. Further, the relevant propositions, theorems etc. for risk neutral firms are special case of the risk neutral firms.

The first order condition can be rewritten as:

$$g_i'(1-\delta_i) = \frac{t_i\left[(1-\sigma_i)\pi_i'\left(\Gamma_{NCi}\right) - \sigma_i\pi_i'\left(\Gamma_{C_i}\right)(\theta-1)\right]}{(1-\sigma_i)\pi_i'\left(\Gamma_{NC_i}\right) + \sigma_i\pi_i'\left(\Gamma_{C_i}\right)} \tag{3.2'}$$

Since by assumption $g_i'(1-\delta_i) > 0$, a necessary condition for $\delta_i > 0$ is that returns to evasion be positive or equivalently $(1-\sigma_i)\pi_i'\left(\Gamma_{NCi}\right) > \sigma_i\pi_i'\left(\Gamma_{Ci}\right)(\theta-1)$. Since we are interested in interior solution, we assume that this condition is satisfied. Note that the equivalent condition derived by Cremer and Gahvari (1993) is a special case of (3. 2′) under assumption of risk neutrality of firms.

Let us define

$$t_i^e = t_i\{\delta_i + (1-\delta_i)\sigma_i\theta\} \tag{3.4}$$

to be the expected tax rate on the *i*-th commodity. Given a large number of firms the pricing condition in industry *i* then becomes:

$$p_i = c_i + g_i(1-\delta_i) + t_i^e \tag{3.5}$$

where $g_i(1-\delta_i)$ and $t_i^e$ are evaluated at $\delta_i^*$ that solves (3.2). Proposition 1 addresses how the equilibrium is affected if one of the tax rates or the audit probabilities changes:

*Proposition 1: The effect of a change in the statutory tax rate would have an ambiguous impact on the sales reports and the effective tax rate but would have a positive effect on the prices charged by firms. The effect of a change in the probability of industry-specific audit would be positive on the sales reports, the effective tax rate, and the prices charged by firms.*

$$\frac{\partial \delta_i}{\partial t_i} = \frac{(1-\sigma_i)\left[\pi_i''(\Gamma_{NC})\left(g_i' - t_i\right)\delta_i z_i + \pi_i' z_i\right] + \sigma_i\left[\pi_i''(\Gamma_C)\left(g_i' + t_i(\theta-1)\right)\left(1 + (\theta-1)(1-\delta_i)\right)z_i - \pi_i'(\theta-1)z_i\right]}{|D|} \gtrless 0 \qquad (3.6)$$

$$\frac{\partial t_i^e}{\partial t_i} = \delta_i + (1-\delta_i)\sigma_i\theta + t_i\frac{\partial \delta_i}{\partial t_i}\left(1-\sigma_i\theta\right) \gtrless 0 \qquad (3.7)$$

$$\frac{\partial p_i}{\partial t_i} = -g_i'\frac{\partial \delta_i}{\partial t_i} + (1-\delta_i)\sigma_i\theta \gtrless 0 \qquad (3.8)$$

$$\frac{\partial \delta_i}{\partial \sigma_i} = \frac{(1-\sigma_i)\pi_i'\left(\Gamma_{NC_i}\right)\left(g_i' - t_i\right) - \sigma_i\pi_i'\left(\Gamma_{C_i}\right)\left(g_i' + t_i(\theta-1)\right)}{|D|} > 0 \qquad (3.9)$$

$$\frac{\partial t_i^e}{\partial \sigma_i} = t_i\left[(1-\delta_i)\theta + \frac{\partial \delta_i}{\partial \sigma_i}\left(1-\sigma_i\theta\right)\right] > 0 \qquad (3.10)$$

$$\frac{\partial p_i}{\partial \sigma_i} = (1-\delta_i)\sigma_i t_i\theta > 0 \qquad (3.11)$$

where $|D|$ is negative as defined in (3.3).

*Proof:* Differentiate (3.2), (3.4) and (3.5) with respect to the instruments at the disposal of the tax authority B. □

It is not surprising that most of the results of Cremer and Gahvari (1993) can be derived from the above conditions when risk neutrality of firm is invoked. Condition

(3.6) implies that an increase in the statutory commodity tax rate would induce an increase or decrease in tax evasion depending upon the degree of absolute risk aversion. This result is at variance with Cremer and Gahvari (1993) who report a positive relationship between statutory tax rate and evasion in the case of risk-neutral firms. When firms are risk-averse their proposition no longer holds since both the income and substitution effects are now at work. However, condition (3.7) shows that the impact of a change in the statutory tax rate transcends from the evasion decision to the effective tax rate and hence the effect is ambiguous. Since we assume constant cost industry, conditions (3.8) and (3.11) are not unexpected. Anything that increases the cost per unit would also increase price proportionately.

Condition (3.9) implies that a higher industry-specific audit probability would lead to higher tax compliance. This implies that higher audit probability would increase industry-specific effective tax rate. These results are expected since higher audit rate would have a salutary effect in protecting both the tax base and the statutory tax rate. These results simply reflect the natural responses of a risk-averse firm to policies that affect the expected return to tax evasion.

*Lemma 1: Changes in* $t_k$ *and* $\sigma_k$ *do not affect the values of* $p_i$ *and* $\delta_i$.

$$\frac{\partial \delta_i}{\partial t_k} = \frac{\partial \delta_i}{\partial \sigma_k} = \frac{\partial p_i}{\partial t_k} = \frac{\partial p_i}{\partial \sigma_k} = 0 \qquad \forall \mathrm{i} \neq \mathrm{k} \tag{3.12}$$

Proof: It follows from industry-specific tax rate and audit probability assumptions. □

Individual's Problem

Consider a continuum of individuals with different abilities distributed according to a continuous distribution function *F(w)* with support on $[w^l, w^h]$ – a closed interval on $R_+$. Each individual is endowed with 1 unit of time to allocate between leisure and labor $\ell$. The value of *w* gives the relative efficiency of labor supplied per unit of time. Given the assumption of linearly homogeneous technology, this represents the marginal productivity of labor for a worker of ability *w*. The total productivity of a worker will then be equal to her wage income $y \equiv w\ell$. Let $X^n \subset \Re^n_+$ be the commodity space and $\underline{x} \equiv \{x_i\}_{i=1}^{i=n}$ be a vector of commodities. Individuals have identical preferences over $X^n$ and leisure $1-\ell$ represented by a well-behaved utility function $U(\underline{x}, 1-\ell)$.

Individuals face linear income tax systems from tax authorities A and B with constant marginal tax rates $t_A$ and $t_B$ and receive uniform lump sum transfers $\alpha_A$ and $\alpha_B$ respectively from them as guaranteed income. They evade taxes by reporting proportions $\delta_A$ and $\delta_B$ of their pre-tax wage income *y*. Tax authorities assess due taxes based on reported incomes while carrying out cursory examinations that reveal cheating unless the taxpayer spends resources to conceal true income. Concealment costs per \$1 are given by $K_i(1-\delta_A-\delta_B+\delta_A\delta_B)$. These costs are assumed to be proportional to undeclared income and $0 \le \delta_A \le 1$ also $0 \le \delta_B \le 1$. Let us assume that K(.) is strictly a quasi-convex function with $K_i(0) = K_i'(0) \to 0$ and $K_i'(1) \to \infty$. Let us also assume as a simple case that $k_i(1-\delta_A-\delta_B+\delta_A\delta_B) = (1-\delta_A-\delta_B+\delta_A\delta_B)K_i(1-\delta_A-\delta_B+\delta_A\delta_B)$. Then the above restrictions on $K_i(1-\delta_A-\delta_B+\delta_A\delta_B)$ imply that $k_i(0) = k_i'(0) \to 0$ and $k_i'(1) \to \infty$.

Individuals face random audits with probabilities $\sigma_A$ and $\sigma_B$ from tax authorities A and B, which are independent of the declared incomes to the tax authorities. Audit by a tax authority would reveal the true income to that tax authority only. Once evasion is established, individuals have to pay exogenous fines ($\theta_A - 1$) and ($\theta_B - 1$) proportional to evaded taxes in addition to their true tax liabilities following the amendments Yitzhaki (1974) made to the Allingham-Sandmo (1972) model.

An individual might find herself in four different possible states depending on whether she is audited by tax authority A or B, or by both, or not audited at all. Assume that she makes her labor supply and income report decisions at the beginning of the reference period, prior to knowledge of audit lotteries. These decisions determine her wage income net of concealment costs. Depending on the outcome of the audit lotteries her post-tax wage rate in four possible contingencies becomes:

$$\left.\begin{aligned}
\underline{w}_1 &= w\{1 - t_A - t_B - k(1-\delta_A-\delta_B+\delta_A\delta_{BB}) - (\theta_A - 1)(1-\delta_A)t_A - (\theta_B - 1)(1-\delta_B)t_B\} \\
\underline{w}_2 &= w\{1 - t_A - \delta_B t_B - k(1-\delta_A-\delta_B+\delta_A\delta_B) - (\theta_A - 1)(1-\delta_A)t_A\} \\
\underline{w}_3 &= w\{1 - \delta_A t_A - t_B - k(1-\delta_A-\delta_B+\delta_A\delta_B) - (\theta_B - 1)(1-\delta_B)t_B\} \\
\underline{w}_4 &= w\{1 - \delta_A t_A - \delta_B t_B - k(1-\delta_A-\delta_B+\delta_A\delta_B)\}
\end{aligned}\right\} \tag{3.12}$$

with the associated probabilities of occurrence as:

$$\left.\begin{aligned}
S_1 &= \sigma_A\sigma_B \\
S_2 &= \sigma_A(1-\sigma_B) \\
S_3 &= \sigma_B(1-\sigma_A) \\
S_4 &= (1-\sigma_A)(1-\sigma_B)
\end{aligned}\right\} \tag{3.13}$$

Hence, the expected net return to her labor supply becomes $\sum_{j=1}^{4} S_j \underline{w}_j$. At the end of the period, her post-audit tax treatment determines her after-tax income $y_j \equiv \underline{w}_j \ell$ available to allocate over consumption of the $n$ commodities in the state. No savings are allowed. Hence, she has to devise a contingent consumption plan that meets the virtual budget constraint in all four states. Let $\underline{p} = (p_1, p_2, \ldots, p_n)$ be the vector of consumer prices. The virtual budget constraint for the individual can then be written as:

$$y_j = \underline{w}_j \ell + \alpha = \underline{p}\underline{x}_j \text{ ; j = 1, 2,…, 4 and } \alpha \equiv \alpha_A + \alpha_B \qquad (3.14)$$

Assume that the individual maximizes the expected utility. That means she takes the vector of prices $\underline{p}$, the income tax parameters ($t_A$, $t_B$, $\alpha_A$, and $\alpha_B$), and chooses the state contingent vector of commodities $\underline{x}_j$, labor supply $\ell$ and the proportions of income to report ($\delta_A$ and $\delta_B$) to solve:

$$Max \;\; \Psi = \sum_{j=1}^{4} S_j U_j\left(\underline{x}_j, 1-\ell\right) \qquad \text{s.t.} \quad (3.14) \qquad (3.15)$$

where $\Psi$ is assumed to be a twice continuously differentiable and well-behaved function. She simultaneously chooses her labor supply, and allocates the resulting pre-tax labor income to a lottery with sure return (true tax liability report) and a random return (income concealed from either tax authority or both) at a cost that is increasing and proportional to the share of the risky asset on her portfolio. She buys off the opportunity to affect her effective income tax rates through her choice of the proportions of income reported. As a result, both the *ex-ante* and *ex-post* effective marginal tax rates to tax authorities A and B are no longer constant across individuals. Since uncertainty is not resolved until the end of the period, when the individual learns whether she is audited or not by the tax authorities A or B, her choice of consumption bundle becomes state contingent. Of course, when choosing $\delta_A$ $\delta_B$ and $\ell$, the individual has to take into account her optimal choice of $\underline{x}_j$. The Lagrangian and the associated first order conditions for this problem are given in Appendix A.

The first order conditions for $\underline{x}_j$ and $\delta_m$ (m = A, B) can be used to derive necessary conditions for interior solutions for $\delta_m$. Rewriting (A.7) and (A.8) in Appendix A gives:

$$k'_A(1-\delta_A-\delta_B+\delta_A\delta_B)=\frac{t_A[(\lambda_3+\lambda_4)-(\theta_A-1)(\lambda_1+\lambda_2)]}{\sum_{j=1}^{4}\lambda_j} \tag{3.16}$$

$$k'_B(1-\delta_A-\delta_B+\delta_A\delta_B)=\frac{t_B[(\lambda_2+\lambda_4)-(\theta_B-1)(\lambda_1+\lambda_3)]}{\sum_{j=1}^{4}\lambda_j} \tag{3.17}$$

Since $k'_m(1-\delta_A-\delta_B+\delta_A\delta_B) > 0$ in an interior solution, (3.16) and (3.17) imply:

$$\frac{(\lambda_3+\lambda_4)}{(\lambda_1+\lambda_2)}\equiv\frac{S_3U'_{x_{i3}}+S_4U'_{x_{i4}}}{S_1U'_{x_{i1}}+S_2U'_{x_{i2}}}>(\theta_A-1) \tag{3.18}$$

$$\frac{(\lambda_2+\lambda_4)}{(\lambda_1+\lambda_3)}\equiv\frac{S_2U'_{x_{i2}}+S_4U'_{x_{i4}}}{S_1U'_{x_{i1}}+S_3U'_{x_{i3}}}>(\theta_B-1) \tag{3.19}$$

i.e., the marginal rate of substitution of consumption of each commodity between “good” and “bad” states must be greater than the penalties for income tax evasion imposed by the respective tax authorities. Thus, she would evade taxes from either of the tax authorities as long as the above inequalities hold. The strict concavity of utility function *U* is both a necessary and a sufficient condition to guarantee that the solutions for $\delta_m$ are indeed global maximum.

**Income Tax Evasion and Optimal Decision Rules**

Note that the individual’s optimal choice involves substitution across three margins: (i) for a given after-tax income from the tax authorities, the composition of her consumption bundle on each state of nature; (ii) the usual, now disaggregated, leisure-consumption tradeoff; and (iii) the tradeoff between the benefits of evasion through enhanced consumption possibilities and its concomitant costs (risky consumption and concealment costs). Given the general specification of consumer preferences, these tradeoffs are intertwined in a non-trivial way. For instance, optimal labor supply will, in general, depend on the choice of income reports and the resulting concealment costs since these affect the net return to work. Similarly, tax evasion behavior will, in general, be influenced by commodity prices and non-wage income.

In order to characterize these tradeoffs, the first order conditions (A.2) through (A.8) in Appendix A can be combined and simplified to get:

$$\frac{U'_{x_{ij}}\left(\underline{x}_j, 1-\ell\right)}{U'_{x_{kj}}\left(\underline{x}_j, 1-\ell\right)} = \frac{p_i}{p_k}; \quad j = 1, 2, 3, 4; \quad i = 1, 2, \ldots, \text{n and } i \neq k \tag{3.20}$$

$$-\frac{S_3 U'_{x_{i3}} + S_4 U'_{x_{i4}}}{S_1 U'_{x_{i1}} + S_2 U'_{x_{i2}}} = \frac{k'_A + t_A(\theta_A - 1)}{k'_A - t_A} \tag{3.21}$$

$$-\frac{S_2 U'_{x_{i2}} + S_4 U'_{x_{i4}}}{S_1 U'_{x_{i1}} + S_3 U'_{x_{i3}}} = \frac{k'_B + t_B(\theta_B - 1)}{k'_B - t_B} \tag{3.22}$$

$$-\frac{\sum_{j=1}^{4} S_j U'_{\ell j}\left(\underline{x}_j, 1-\ell\right)}{\sum_{j=1}^{4} S_j U'_{x_{ij}}\left(\underline{x}_j, 1-\ell\right)} = \frac{w}{p_i}\left[1 - t_A - t_B - k + (1-\delta_A)k'_A + (1-\delta_B)k'_B\right] \tag{3.23}$$

Condition (3.20) implies that, conditional on an audit state, the structure of the optimal consumption bundle is determined by equating the marginal rate of substitution between two commodities and their respective price ratio. Thus, if the consumer knew the outcome of audit lotteries before they are resolved, the substitution at the margin between any two commodities would be governed by their market terms of trade. With the uncertainty arising from tax evasion, the optimal consumption bundle will also be affected by the terms of trading risks across audit states.

Conditions (3.21) and (3.22) which characterize optimal income tax evasion implicitly, illustrate this result from the optimal choices of $\delta_A$ and $\delta_B$. While the right hand sides of these conditions show the rate of substitution between good and bad states, the left hand sides are the slopes of the respective boundary of the budget set defined by all feasible pairs $[\{(\boldsymbol{x}_{i3}, \boldsymbol{x}_{i4}), (\boldsymbol{x}_{i1}, \boldsymbol{x}_{i2})\}$ and $\{(\boldsymbol{x}_{i2}, \boldsymbol{x}_{i4}), (\boldsymbol{x}_{i1}, \boldsymbol{x}_{i3})\}]$ given the individual's labor supply and the tax enforcement parameters of the tax authorities A and B. These slopes are given by the ratios of net marginal benefits of tax evasion from tax authorities A and B in each possible state. Thus, conditions (3.21) and (3.22) imply that the willingness to transfer consumption of any commodity from the ‘good’ to the ‘bad’ state must be equal to the implicit “relative price” of trading risk (income concealment from either tax authority) and no risk (income reported truly to both the tax authorities).

Finally, condition (3.23) describes the modified optimal tradeoff between leisure and consumption of any commodity. The ratio of the marginal utilities to leisure for consumption of each commodity must equal the post-tax relative wage after concealment costs are taken into account. With no evasion of taxes owed to tax authorities A and B, (3.23) gives the usual optimality condition. With income tax evasion, the tradeoff is affected by the marginal tax rates, the choices of $\delta_A$ and $\delta_B$, and the concealment technology, all of which affect the return to labor supply. With $\delta_A$ and $\delta_B$ at their optimal levels, a higher per capita cost, $k_i(1-\delta_A-\delta_B+\delta_A\delta_B)$, reduces the net return to work effort. In contrast, higher income concealment (i.e., lower values of $\delta_A$ and $\delta_B$) reduces the expected income tax rates and thus increases the return to labor supply. It is, thus, clear that labor-leisure tradeoff critically hinges on the tax evasion decisions of the individual.

Given the general specification of individual preferences in our model, the interdependence among tax evasion from the tax authorities, consumption and labor supply determines $\delta_A$ and $\delta_B$, which, in turn, determine how much consumption is put at stake in audit lotteries. The choice of state contingent consumption bundle provides numerous ways to diversify the risks of tax evasion over the *n* commodities. Differences in income elasticity both across commodities and individual types intertwine the realized consumption pattern to the tax evasion behavior. Existing complementarities between consumption and leisure interlock an individual’s labor supply and tax evasion decisions.

In order to illustrate these issues, consider the case of quasi-linear preferences in consumption and leisure, so that individuals are risk neutral and income effects of audits are immaterial. In that case (3.21) and (3.22) reduce to:

$$k'_A = t_A\left[1-(S_1+S_2)\theta_A\right] \tag{3.21′}$$

$$k'_B = t_B\left[1-(S_1+S_3)\theta_B\right] \tag{3.22′}$$

These conditions imply that $\delta_A$ ($\delta_B$) equates the marginal cost of income concealment from tax authority A (B) to income tax evasion. Note that both of the evasion decisions are independent of lump sum grants, pre-tax wage rate, and prices, and hence commodity tax parameters. Since per capita concealment costs $K_i(.)$ are assumed to be the same for all individual types, everyone evades same fraction of their income and hence faces the same effective tax rates:

$$t_A^e = t_A\left[\delta_A + \theta_A(S_1+S_2)(1-\delta_A)\right] \tag{3.24}$$

$$t_B^e = t_B\left[\delta_B + \theta_B(S_1+S_3)(1-\delta_B)\right] \tag{3.25}$$

Substituting (3.21′) and (3.22′) into (3.23) and simplifying yields:

$$-U'_\ell(1-\ell) = \frac{w}{q_i}\left[1-t_A^e-t_B^e-k\right] \tag{3.26}$$

where $t_A^e$ and $t_B^e$ are evaluated at the optimal values of $\delta_A$ and $\delta_B$. This condition implies that the optimal labor supply is determined by equating the marginal utility of leisure to the net expected return to work effort. Since the partial derivatives of (3.26) with respect to $\delta_A$ and $\delta_B$ result in (3.21′) and (3.22′), labor supply is independent of tax evasion decisions, conditional on the optimal values of $\delta_A$ and $\delta_B$.

Note that (3.21′) and (3.22′) imply that income tax evasion will unambiguously increase with a higher marginal income tax rate and/or a lower probability of audit by either or both the tax authorities. These results are the two tax authority generalization of Cremer and Gahvari (1994) and Boadway, Marchand, and Pestieau (1994). However, these results and that $\delta_A$ and $\delta_B$ are constant across individuals dependent crucially on the assumption of risk aversion and are yet to be supported by empirical evidence. In general, the optimal values of $\delta_A$ and $\delta_B$ differ from the ones derived through benefit-cost analysis, and are affected by income effects arising from changes in lump sum transfers, commodity tax rate, income tax rates, and audit probabilities. Since the intensities of these income effects will vary along the distribution of abilities, the propensities to evade taxes will vary across individuals and hence will affect their consumption-leisure tradeoff in a non-trivial way. As will be detailed later, such income effects along with the uncertainties arising from tax evasion play a crucial role in shaping the impact of tax evasion on optimal tax structures.

**Compensated Demands, Labor Supply, and Comparative Statics**

Conditional on the exogenous penalty rates, the first order conditions (A.2) through (A.12) in Appendix A can be used to derive the corresponding commodity demand functions $x_{ij}$, labor supply $\ell$ and the optimal proportions of income reported, $\delta_A$ and $\delta_B$, for an individual as:

$$x_{ij}\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right);\ \mathrm{i} = 1, 2,\ldots, \mathrm{n};\ \mathrm{j} = 1, 2,\ldots, 4. \qquad (3.27) – (3.30)$$

$$\ell\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right) \qquad (3.31)$$

$$\delta_A\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right) \qquad (3.32)$$

$$\delta_B\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right) \qquad (3.33)$$

Note that while $x_i$'s are random variables before the realization of income tax audit lotteries, $\ell$, $\delta_A$ and $\delta_B$ are not. Commodity tax evasion parameters affect these optimal choices through their affect on market prices. To characterize the response of individual to changes in the policy parameters of the two tax authorities, obtain the indirect utility function by substituting the vector of commodity demands and labor supply functions into (3.15) as:

$$V\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right)=\sum_{j=1}^{4}S_jU_j\left(\underline{x}_j(.),1-\ell(.)\right) \qquad (3.15')$$

Given the optimal choices, the following envelope results can be derived by partially differentiating the above indirect utility function:

$$V_{p_k}=-E\left[\lambda_j x_{kj}\right]\equiv-\sum_{j=1}^{4}\lambda_j x_{kj}<0;\quad k = 1, 2, 3, \ldots, \mathrm{n}. \qquad (3.34)$$

$$V_{t_A}=-E\left[\frac{\partial y_j}{\partial t_A}\lambda_j\right]\equiv-w\ell\left[\underbrace{(\lambda_3+\lambda_4)\delta_A+(\lambda_1+\lambda_2)(1-\theta_A(1-\delta_A))}_{\equiv\lambda_{t_A}}\right]<0 \qquad (3.35)$$

$$V_{t_B} = -E\left[\frac{\partial y_j}{\partial t_B}\lambda_j\right] \equiv -w\ell\left[\underbrace{(\lambda_2+\lambda_4)\delta_B + (\lambda_1+\lambda_3)(1-\theta_B(1-\delta_B))}_{\equiv \lambda_{t_B}}\right] < 0 \tag{3.36}$$

$$V_\alpha\left(= V_{\alpha_A} \equiv V_{\alpha_B}\right) = E[\lambda_j] \equiv \sum_{j=1}^{4}\lambda_j > 0 \tag{3.37}$$

$$V_{\sigma_m} = \sum_{j=1}^{4}\frac{\partial S_j}{\partial \sigma_m}U_j\left(\underline{x}_j(.), 1-\ell(.)\right) < 0 \qquad ; \text{m = A, B} \tag{3.38}$$

where $\lambda_{t_A}$ ( $\lambda_{t_B}$ ) denotes the change in the expected marginal utility of income due to a change in the statutory income tax rate by tax authority A (B), $V_\alpha\left(= V_{\alpha_A} \equiv V_{\alpha_B}\right)$ is the expected utility of certain income, and $V_{\sigma_m}$ shows the change in the expected marginal utility of income due a change in the probability of detection by the tax authorities. As expected, an increase in the price of the *k*-th good (for instance, through a higher commodity tax and/or audit probability), or an increase in the income tax rates and penalty rates by tax authorities A and/or B, all reduces the expected utility. However, the impact of audit probability is ambiguous in view of the fact that higher audit probability by one tax authority can be more than compensated for by the lower audit probability by the other tax authority. Since with tax evasion income and thus consumption are random variables, these expressions are just analogous to the usual expressions obtained in the absence of uncertainty, and readily highlight the difference made by the uncertainty that ensues with audit lotteries.

Let us derive the compensated demand and labor supply functions. Given the fact that the choice of income reports to tax authorities A and B are not the direct sources of utility, we may consider the tax compensated dual problem. Since the lump sum income (the sum of grants $\alpha = \alpha^A + \alpha^A$) is the same regardless of the outcome of audit lotteries, the dual problem becomes:

$$\min \alpha = \underline{p}\underline{x}_j - \ell S_j \underline{w}_j \qquad s.t. \qquad \sum_{j=1}^{4} S_j U_j\left(\underline{x}_j, 1-\ell\right) \geq V^* \tag{3.39}$$

Although this problem differs from the usual one in duality theory with certainty, it has an almost similar intuitive interpretation. The optimal solution consists of the corresponding state contingent commodity bundles, labor supply, and the proportions of income reported to tax authorities A and B, so that the compensation required to provide the individual with the level of utility $V^*$ be minimum. Since the individual commits to a level of labor supply and proportions of income reports to tax authorities A and B before the realizations of audit lotteries, it is easier to think of the solution in two stages. In the first stage, for a fixed vector of $\underline{x}_j$, one chooses $\delta_A$, $\delta_B$ and $\ell$ based on the expected return to labor supply $\underline{w}_j$ and these determine the pre-tax labor income net of concealment costs. In the second stage, given the optimal choices of $\delta_A$, $\delta_B$ and $\ell$, the individual chooses the consumption vector, $\underline{x}_j$, to minimize $\alpha$. The associated Lagrangian and the first order conditions are presented in Appendix B.

Not surprisingly, holding utility constant the individual behaves as a risk neutral agent. Hence, the optimal income reports are determined qualitatively the same way as in the absence of income effects (i.e., (3.21′) and (3.22′) for quasi-linear preferences). Therefore, $\delta_A^c$ and $\delta_B^c$ are independent of $w$ and consumer prices. Solving the system of equations (B.2) through (B.9) in Appendix B yields the compensated demand functions, $x_{ij}^c$, labor supply function, $\ell^c$, income reports, $\delta_A^c$ and $\delta_B^c$, and the expenditure function, $\alpha\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, V^*\right)$. We can then define the following equivalences using the fact that the lump sum grant $\alpha \equiv \alpha_A + \alpha_B$ is just sufficient to attain the specified level of indirect utility:

$$x_{ij}\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, \alpha(.)\right) \equiv x_{ij}^c\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, V^*\right) \tag{3.40}$$

$$\ell\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, \alpha(.)\right) \equiv \ell^c\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, V^*\right) \tag{3.41}$$

$$\delta_A\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, \alpha(.)\right) \equiv \delta_A^c\left(t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, V^*\right) \tag{3.42}$$

$$\delta_B\left(w, p, t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, \alpha(.)\right) \equiv \delta_B^c\left(t_A, t_B, \theta_A, \theta_B, \sigma_A, \sigma_B, V^*\right) \tag{3.43}$$

By the usual duality properties, we can then find the derivatives of the expenditure function as the marginal rate of substitution of the indirect utility function. Hence, from (3.34) through (3.38) one can derive:

$$\frac{\partial \alpha}{\partial p_k} = -\frac{V_{p_k}}{V_\alpha} = \frac{\sum_{j=1}^{4} \lambda_j x_{kj}}{\sum_{j=1}^{4} \lambda_j} > 0 \tag{3.44}$$

$$\frac{\partial \alpha}{\partial t_A} = -\frac{V_{t_A}}{V_\alpha} = \frac{w \ell \lambda_{t_A}}{\sum_{j=1}^{4} \lambda_j} > 0 \tag{3.45}$$

$$\frac{\partial \alpha}{\partial t_B} = -\frac{V_{t_B}}{V_\alpha} = \frac{w \ell \lambda_{t_B}}{\sum_{j=1}^{4} \lambda_j} > 0 \tag{3.46}$$

$$\frac{\partial \alpha}{\partial \sigma_m} = -\frac{V_{\sigma_m}}{V_\alpha} = -\frac{\sum_{j=1}^{4} \frac{\partial S_j}{\partial \sigma_m} U_j\left(\underline{x}_j(.), 1-\ell(.)\right)}{\sum_{j=1}^{4} \lambda_j} > 0 \qquad \text{m = A, B} \tag{3.47}$$

Expressions (3.44) through (3.47) show the required change in the lump sum transfers needed to compensate an individual due to marginal changes in tax instruments by the two tax authorities so that she can still attain the level of utility $V^*$.

*Proposition 2: The optimal responses of an individual of a particular type to changes in the tax authorities' parameters that affect prices, the return to labor supply, and income tax evasion are ambiguous.*

*Proof:* See Appendix C. □

As expected, unless further restrictions on preferences are imposed, it is not possible to determine the sign of these expressions due to conflicting income and substitution effects. Insofar as the individual can substitute across several margins such as, across commodities, risk vis-à-vis no risk, and labor/leisure, in response to the policies by either tax authority A and B or both, the expected utility maximization is consistent with a variety of individual behavior. However, it may be noted that if the proportion of declared income decreases with lump sum grant, a tighter enforcement of the income tax laws by both the tax authorities (i.e., higher $\sigma_A$ and/or $\sigma_B$), or a policy by tax authority B that increases consumer prices (i.e., higher $t_i$ and/or $\sigma_i$) will induce taxpayers to unambiguously report higher proportions of their true income to both the tax authorities. This follows from the fact that $\frac{\partial \delta_A^c}{\partial \sigma_m}$>0 and $\frac{\partial \delta_B^c}{\partial \sigma_m}$>0 (from differentiation of (B.8) and (B.9) respectively) and that an increase in the price of any goods affects evasion behaviors purely through an income effect that makes the individual relatively poorer. In general, the signs of (C.1) through (C.16) crucially hinge on the nature of relationship between goods, between goods and leisure, and the degree of absolute and relative risk aversion of the individual.

### *THE OPTIMAL LEVEL OF TAXES TO THE TAX AUTHORITIES*

Multi-agency tax evasion adds new ingredients to the standard optimal taxation problem of balancing equity and efficiency. In such a case the efficiency costs of the tax system will not be limited to the usual excess burden of taxation. Since incomes of individuals and sales of firms cannot be observed without costs by tax authority B, concealment and audit costs impose an additional deadweight loss to the society. Similar inefficiency also arises as a result of tax authority A's inability to observe incomes of the individuals without additional costs. Further, lack of coordination between the tax authorities may increase the inefficiency costs that can actually be avoided. The more acute the tax evasion by individuals and firms, the more the tax authorities may have to divert to the enforcement of the tax laws. In addition, the efficiency costs in commodity taxation, in terms of the distortion in the relative prices, will now depend on the evasion behavior of firms across industries, through the differences in the technology affecting the concealment costs. Further, multi-agency tax evasion is likely to limit the redistributive role of the tax system since audit costs and costs that may arise due to lack of coordination reduce the levels of transfers to individuals and thus hamper redistribution. Differential commodity taxation to favor the poor needs to be balanced against the above mentioned price distortions caused by commodity taxation. Insofar as different taxes have different evasion characteristics, the equivalence between the observed individual incomes and consumption or even between aggregate reported incomes and observed aggregate commodity consumption, no longer holds. Observed individual incomes and reported sales of firms, which now constitute the total tax base, are less reliable measures of individual welfare.

New tradeoffs emerge in the design of optimal tax policy for tax authority B. Due to the uncertainty that comes with tax audits, the choice of taxes becomes an issue of the extent to which revenue collection should rely more heavily on commodity taxes collected from firms or on income tax collected from individuals. There may now be additional social welfare gains from reducing the risk of tax evasion. Moreover, tax policies to both tax authorities A and B are no longer exclusively concerned with choosing optimal tax bases and the structures of taxation; as the set of instruments now include audit probabilities, tighter enforcement or a combination of tighter enforcement and higher tax rates or increased coordination between tax authorities A and B appears as an alternative. Welfare gains might be obtained by targeting certain goals with particular instruments so that the optimal policy may more likely involve a mix of taxes.

Under this backdrop let us now analyze the problems encountered by tax authorities A and B. In doing so, we make the assumption that the penalty rates, ($\theta_A$-1), ($\theta_B$-1) and ($\theta$-1), on firms and individuals cannot be set to eradicate tax evasion fully. The costs of audits are given by functions $c_A(\sigma_A, \sigma_B)$ and $c_B(\sigma_A, \sigma_B)$ for the individuals and $c(\underline{\sigma})$ for the firms. While function $c$ is strictly quasi-convex in all of its arguments, function $c_B$ is strictly increasing in $\sigma_B$ and non-increasing in $\sigma_A$. Similarly, function $c_A$ is strictly increasing in $\sigma_A$ and non-increasing in $\sigma_B$. Further, we also assume that

$c_A(0) = c_B(0) = c(0) = 0$;

$$\frac{\partial c_A(\sigma_A,\sigma_B)}{\partial \sigma_A}\left(=\frac{\partial c_B(\sigma_A,\sigma_B)}{\partial \sigma_B}\right) \to \infty \text{ as } \sigma_{A(=\sigma_B)} \text{ approaches unity.}$$

$$\frac{\partial c_A(\sigma_A,\sigma_B)}{\partial \sigma_B}\left(=\frac{\partial c_B(\sigma_A,\sigma_B)}{\partial \sigma_A}\right) \to 0 \text{ as } \sigma_{A(=\sigma_B)} \text{ approaches unity.}$$

In this setting, $\underline{t}^e$, $t_A^e$ and $t_B^e$ are of primary interest in the computation of tax revenues accrued to tax authorities A and B. Owing to a continuum of individuals, the realized means of the random variables are assumed to be equal to their expected values so that $S_1$ is the fraction of individuals actually caught cheating by both the tax authorities, $S_2$ and $S_3$ are the fractions of individuals caught cheating only by tax authority A and B respectively, and $S_4$ is the fraction of individuals never caught by either of the tax authorities. Similarly, $\sigma_i$ is the fraction of firms in industry *i* caught cheating by the tax authority B. We can then define:

$$x_i = E\left[x_{ij}\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right)\right] = \sum_{j=1}^{4} S_j x_{ij}(.) \tag{4.1}$$

$$X_i = \int_{w^l}^{w^h} x_i dF_w \tag{4.2}$$

$$R_A = R\left(w,\underline{p},t_A,t_B,\theta_A,\theta_B,\alpha_A,\alpha_B,\sigma_A,\sigma_B\right) = t_A^e w\ell - \alpha_A \tag{4.3}$$

$$R_B = R\left(w, \underline{p}, t_A, t_B, \underline{\tau}, \theta_A, \theta_B, \theta, \alpha_A, \alpha_B, \sigma_A, \sigma_B, \underline{\sigma}\right) = \sum_{i=1}^{n} t_i^e x_i + t_B^e w\ell - \alpha_B \tag{4.4}$$

where $R_A$ is the expected taxes net of transfers collected from an individual by tax authorities A, and $R_B$ is the expected taxes net of transfers collected from an individual and firm by tax authority B. Note that $x_{ij}$ and $\ell$ are given by the optimizing individual demand and labor supply functions defined earlier and that tax collections depend on the tax evasion behaviors of individuals and firms and the respective effective tax rates. The 'super government' chooses to maximize a strictly quasi-concave (indirect) social welfare function $H\left(V\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \alpha_A, \alpha_B, \sigma_A, \sigma_B\right)\right)$ subject to the respective revenue constraints for the tax authorities. Assuming the per capita revenue requirements for tax authorities A and B, the problem of the 'super government' can be stated as:

$$\max \int_{w^l}^{w^h} \left[H\left(V\left(w, \underline{p}, t_A, t_B, \theta_A, \theta_B, \alpha_A, \alpha_B, \sigma_A, \sigma_B\right)\right)\right] dF_w \tag{4.5}$$

$$\text{s.t.} \int_{w^l}^{w^h} \left[R_A\left(t_A, t_B, \theta_A, \theta_B, \alpha_A, \alpha_B, \sigma_A, \sigma_B\right)\right] dF_w - c_A\left(\sigma_A, \sigma_B\right) = M_A \tag{4.6}$$

$$\text{s.t.} \int_{w^l}^{w^h} \left[R_B\left(w, \underline{p}, t_A, t_B, \underline{t}, \theta_A, \theta_B, \theta, \alpha_A, \alpha_B, \sigma_A, \sigma_B, \underline{\sigma}\right)\right] dF_w - c(\underline{\sigma}) - c_B\left(\sigma_A, \sigma_B\right) = M_B \tag{4.7}$$

The Lagrangian and the associated first order conditions are presented in Appendix D. Combining (3.37), (D.3) and (D.4), one can derive the expressions for the Lagrangian multipliers of tax authority A's and B's revenue constraints as:

$$\gamma_A = -\frac{\int_{w^l}^{w^h}\left[H'(.)\sum_{j=1}^{4}\lambda_j\right]dF_w\left\{\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_B}\right]dF_w - \int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_A}\right]dF_w\right\}}{\int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_A}\right]dF_w\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_B}\right]dF_w - \int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_B}\right]dF_w\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_A}\right]dF_w} \tag{4.8}$$

$$\gamma_B = -\frac{\int_{w^l}^{w^h}\left[H'(.)\sum_{j=1}^{4}\lambda_j\right]dF_w\left\{\int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_A}\right]dF_w - \int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_B}\right]dF_w\right\}}{\int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_A}\right]dF_w\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_B}\right]dF_w - \int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_B}\right]dF_w\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_A}\right]dF_w} \tag{4.9}$$

These can be interpreted as the average social marginal utilities of certain income using the revenues of tax authorities A and B as numeraire. As $\int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_A}\right]dF_w < 0$ and $\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_B}\right]dF_w < 0$, if we assume that the product of own effects dominates that of cross effects, and $\int_{w^l}^{w^h}\left[\frac{\partial R_j}{\partial \alpha_i}\right]dF_w \geq 0$; $i \neq j$, then $\gamma_A$ and $\gamma_B$ are both positive as required for an interior solution. We assume that theses conditions are satisfied. As a result, the first order conditions imply that:

$$\int_{w^l}^{w^h}\left[\frac{\partial R_A(.)}{\partial t_A}\right]dF_w > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_A(.)}{\partial t_B}\right]dF_w > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_B(.)}{\partial t_A}\right]dF_w > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_B(.)}{\partial t_B}\right]dF_w > 0;$$

$$\int_{w^l}^{w^h}\left[\frac{\partial R_B(.)}{\partial t_k}\right]dF_w > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_A(.)}{\partial \sigma_A}\right]dF_w > \frac{\partial c_A(.)}{\partial \sigma_A} > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_B(.)}{\partial \sigma_B}\right]dF_w > \frac{\partial c_B(.)}{\partial \sigma_B} > 0;$$

$$\int_{w^l}^{w^h}\left[\frac{\partial R_B(.)}{\partial \sigma_k}\right]dF_w > \frac{\partial c(.)}{\partial \sigma_k} > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_A(.)}{\partial \sigma_B}\right]dF_w - \frac{\partial c_A(.)}{\partial \sigma_B} > 0;\ \int_{w^l}^{w^h}\left[\frac{\partial R_B(.)}{\partial \sigma_A}\right]dF_w - \frac{\partial c_B(.)}{\partial \sigma_A} > 0$$

That is, in an interior optimum there should be positive net marginal revenue from increasing $t_A$ and $\sigma_A$ by tax authority A and by increasing $t_k$, $t_B$, $\sigma_B$, and $\sigma_k$ by tax authority B. Further, tax authorities can jointly minimize their income tax audit costs through coordination of their audit information. Otherwise, since the instruments are distortionary, aggregate welfare could be increased by simply setting the instruments to zero by either or both the tax authorities so as to eliminate distortion.

$$\frac{\partial R_A}{\partial t_k} = w\frac{\partial p_k}{\partial t_k}\left( t_A^e \frac{\partial \ell}{\partial p_k} + t_A \ell \left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial p_k}\right) \tag{4.10}$$

$$\frac{\partial R_A}{\partial t_A} = w\left( t_A^e \frac{\partial \ell}{\partial t_A} + \ell\left(\frac{\partial t_A^e}{\partial t_A} + t_A\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial t_A}\right)\right) \tag{4.11}$$

$$\frac{\partial R_A}{\partial t_B} = w\left( t_A^e \frac{\partial \ell}{\partial t_B} + \ell t_A\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial t_B}\right) \tag{4.12}$$

$$\frac{\partial R_A}{\partial \alpha_A} = w\left( t_A^e \frac{\partial \ell}{\partial \alpha_A} + t_A \ell\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial \alpha_A}\right) - 1 \tag{4.13}$$

$$\frac{\partial R_A}{\partial \alpha_B} = w\left( t_A^e \frac{\partial \ell}{\partial \alpha_B} + t_A \ell\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial \alpha_B}\right) \tag{4.14}$$

$$\frac{\partial R_A}{\partial \sigma_A} = w\left( t_A^e \frac{\partial \ell}{\partial \sigma_A} + t_A \ell\left( \theta_A\left(\frac{\partial S_1}{\partial \sigma_A} + \frac{\partial S_2}{\partial \sigma_A}\right)\left(1-\delta_A\right) + \left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial \sigma_A}\right)\right) \tag{4.15}$$

$$\frac{\partial R_A}{\partial \sigma_B} = w\left( t_A^e \frac{\partial \ell}{\partial \sigma_B} + t_A \ell\left( \theta_A\left(\frac{\partial S_1}{\partial \sigma_B} + \frac{\partial S_2}{\partial \sigma_B}\right)\left(1-\delta_A\right) + \left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial \sigma_B}\right)\right) \tag{4.16}$$

$$\frac{\partial R_A}{\partial \sigma_k} = w\frac{\partial p_k}{\partial \sigma_k}\left[ t_A^e \frac{\partial \ell}{\partial p_k} + t_A \ell\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial p_k}\right] \tag{4.17}$$

$$\frac{\partial R_B}{\partial t_k} = \frac{\partial p_k}{\partial t_k}\left[\sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial p_k} + w\left(t_B^e \frac{\partial \ell}{\partial p_k} + t_B \ell \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial p_k}\right)\right] + x_k \frac{\partial t_k^e}{\partial t_k} \tag{4.18}$$

$$\frac{\partial R_B}{\partial t_A} = \sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial t_A} + w\left(t_B^e \frac{\partial \ell}{\partial t_A} + \ell t_B \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial t_A}\right) \tag{4.19}$$

$$\frac{\partial R_B}{\partial t_B} = \sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial t_B} + w\left(t_B^e \frac{\partial \ell}{\partial t_B} + \ell\left(\frac{\partial t_B^e}{\partial t_B} + t_B \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial t_B}\right)\right) \tag{4.20}$$

$$\frac{\partial R_B}{\partial \alpha_A} = \sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial \alpha_A} + w\left(t_B^e \frac{\partial \ell}{\partial \alpha_A} + t_B \ell \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial \alpha_A}\right) \tag{4.21}$$

$$\frac{\partial R_B}{\partial \alpha_B} = \sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial \alpha_B} + w\left(t_B^e \frac{\partial \ell}{\partial \alpha_B} + t_B \ell \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial \alpha_B}\right) - 1 \tag{4.22}$$

$$\frac{\partial R_B}{\partial \sigma_A} = \sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial \sigma_A} + w\left(t_B^e \frac{\partial \ell}{\partial \sigma_A} + t_B \ell \left(\theta_B\left(\frac{\partial S_1}{\partial \sigma_A} + \frac{\partial S_3}{\partial \sigma_A}\right)\left(1-\delta_B\right) + \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial \sigma_A}\right)\right) \tag{4.23}$$

$$\frac{\partial R_B}{\partial \sigma_B} = \sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial \sigma_B} + w\left(t_B^e \frac{\partial \ell}{\partial \sigma_B} + t_B \ell \left(\theta_B\left(\frac{\partial S_1}{\partial \sigma_B} + \frac{\partial S_3}{\partial \sigma_B}\right)\left(1-\delta_B\right) + \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial \sigma_B}\right)\right) \tag{4.24}$$

$$\frac{\partial R_B}{\partial \sigma_k} = \frac{\partial p_k}{\partial \sigma_k}\left[\sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial p_k} + w\left(t_B^e \frac{\partial \ell}{\partial p_k} + t_B \ell \left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial p_k}\right)\right] + x_k \frac{\partial t_k^e}{\partial \sigma_k} \tag{4.25}$$

These expressions and the first order conditions in (D.1) through (D.10) can be used to characterize the level and structure of optimal taxes and audit probabilities in our prototype economy. Since the model is non-linear the closed form solution for these key parameters cannot be derived at this level of generalization. To what extent these restrictions are satisfied has to be verified empirically based on data from the real world. Nevertheless, our analytical approach is useful in illustrating the main qualitative relationships within a logically consistent framework.

## *OPTIMAL TAX RULES UNDER TAX EVASION*

Insofar as both the direct and the indirect taxes can be evaded and there are tax authorities to enforce their tax rules, optimality involves both types of taxes to both the tax authorities. So, we will derive expressions for optimal commodity and income taxes, and compare them with the expressions derived for optimal commodity taxation by Ramsey (1927) and optimal income tax rate derived by Dixit and Sandmo (1977).

### Optimal Commodity Tax Rates

For the purpose of optimal commodity taxation, let us treat $\underline{\sigma}$, $t_A$, $t_B$, $\sigma_A$, and $\sigma_B$ as fixed and substitute (3.34), (C.1) through (C.4), (4.10), and (4.18) into (D.7) to get:

$$-\int_{w^l}^{w^h}\sum_{j=1}^{4}\lambda_j x_{ij}\left[\frac{H'}{\gamma_B}+\frac{1}{\sum_{j=1}^{4}\lambda_j}\left\{\begin{array}{l}\sum_{i=1}^{n}t_i^e\frac{\partial x_i}{\partial\alpha}+w\left(\gamma t_A^e+t_B^e\right)\frac{\partial\ell}{\partial\alpha}\\ +w\ell\left(\gamma t_A\left(1-\theta_A\left(S_1+S_2\right)\right)+t_B\left(1-\theta_B\left(S_1+S_3\right)\right)\right)\frac{\partial\delta_A}{\partial\alpha}\end{array}\right\}\right]dF_w$$
$$+\sum_{i=1}^{n}t_i^e\frac{\partial X_i^c}{\partial p_k}+\int_{w^l}^{w^h}\left[w\left(\gamma t_A^e+t_B^e\right)\frac{\partial\ell^c}{\partial p_k}\right]dF_w+X_k\frac{\partial t_i^e}{\partial t_k}/\frac{\partial p_k}{\partial t_k}=0 \qquad (5.1)$$

where $\gamma=\dfrac{\gamma_A}{\gamma_B}$, the ratio of the marginal utilities of income to tax authorities A and B modifies the usual Ramsey rule. The modified rule now takes into account not only the type of commodities but also who are the main consumers of these goods. Further, the modified rule clearly takes into consideration both the equity and efficiency aspects of optimal taxation. In our setting the efficiency aspect includes the change in the expected tax resulting from the income effect of the lump sum transfer on income reported to the tax authorities. Given the concavity of $H$ and $U$, $b^w$ tends to be biased in favor of the low income individuals.

*Proposition 3: In an economy with linear income and commodity taxes that are subject to random audits by multiple tax authorities, optimal commodity tax rates satisfy the following relationship,* $\forall k = 1,2,...,n$*:*

$$\frac{\sum_{i=1}^{n} t_i^e \frac{\partial X_k^c}{\partial p_k}}{X_k} = \frac{1}{X_k}\int_{w^l}^{w^h}\left[\left(\frac{\sum_{j=1}^{4}\lambda_j x_{ij}}{\sum_{j=1}^{4}\lambda_j}\right)b^w\right]dF_w - \frac{\int_{w^l}^{w^h}\left[w\left(\gamma t_A^e + t_B^e\right)\left(\frac{\partial \ell^c}{\partial p_k}\right)\right]dF_w}{X_k} - \frac{\partial t_i^e}{\partial t_k} / \frac{\partial p_k}{\partial t_k} \qquad (5.2)$$

Proof: Use (4.13) and (4.22), the symmetry of the matrix of compensated demand functions and divide (5.1) by $X_k$ □

The term $b^w \equiv \frac{H'\sum_{j=1}^{4}\lambda_j}{\gamma_B} + \gamma\left(\frac{\partial R_A}{\partial \alpha_A} + 1\right) + \left(\frac{\partial R_B}{\partial \alpha_B} + 1\right)$ in the above modified Ramsey rule can be termed as the net social marginal utility of certain income after taking into account the net effect of transfers from tax authorities A and B on the expected taxes collected from an individual. Note that $b^w$ is analogous to $\gamma^h$ in Diamond (1975), which plays an important role in the optimal taxation literature. It captures the weight of individual's ability in the social welfare function depending on how aggregate welfare and the tax payment by the individual change with the lump sum transfer.

The left hand side of (5.2) gives the percentage reduction in the compensated demand for commodity $x_k$ caused by the change in the tax rate on it and is usually referred to as "index of discouragement" à la Mirrlees (1976). The first term in the right hand side measures the extent to which taxes are levied on goods that are consumed by individuals with a high or low net social marginal utility of income i.e., the rich or the poor. Recall that expression (3.44) gives the change in the lump sum transfer that is required to compensate an individual for a price change induced by the change in the statutory tax rates. With no income tax evasion from either tax authority, this is just $x_{kj}$. This compensation varies in accordance with the differences in consumption patterns. For instance, low-income individuals should be compensated more when necessities are taxed heavily. The more the *k*-th commodity is consumed by individuals with a high social marginal utility of income or income elastic tax payments (i.e., high income elasticity estimates for goods or low income elasticity estimates of labor supply and tax evasion), the smaller the reduction in the compensated demand for $x_k$, and the lower tends to be its expected tax rate. Insofar as this correlation is expected to become pronounced for necessities, (5.2) suggests that the commodities consumed by the poor should be subject to a lower expected tax rate.

The second term reflects the marginal income tax revenue accrued to tax authorities A and B due to the substitution effect on labor supply of a change in $t_k$. With no income tax evasion, such as in the usual Ramsey formula, this term is usually subsumed in the left hand side summation. Given that commodity taxes and income taxes levied by tax authorities A and B have different evasion characteristics, this is no longer the case. Its sign depends on the relationship between good being taxed and labor (which depends on the income tax rates by tax authorities A and B). If the good being taxed and labor supply are net complements, an efficiency loss would result due to the induced reduction in the work effort. As a result, goods that are net complements to leisure i.e., net substitute for labor, should be taxed more heavily on both equity and efficiency grounds.

The last term is a correction factor first discussed by Cremer and Gahvari (1993) and reflects the distortions that commodity tax evasion creates in the price of the good per unit of tax revenue collected. This distortion reflects the increase in concealment costs in the *k*-th industry, and accounts for the difference between increases in $t_k^e$ and $p_k$. The higher this term, the smaller the increase in concealment costs and less social resources are wasted. The extended Ramsey now calls for a proportionate reduction in compensated demand to be smaller for goods that entail the smaller evasion distortions.

## Optimal Income Tax Rates

In presence of multiple tax collecting authorities, the optimal income tax rates set by tax authorities A and B are affected by evasion in incomes and commodity taxes. It may be noted that these tax rates are not close form solutions as there are obvious simultaneity between the two rates due to their mutual interdependence. To see this let us treat $\underline{\sigma}$, $\underline{t}$, $\sigma_A$, and $\sigma_B$ as fixed and use (C.5) through (C.12), (D.1), (D.2), (4.11), (4.12), (4.19), and (4.20) and manipulate to get:

$$
\begin{aligned}
&\int_{w^l}^{w^h} - w\ell\lambda_{t_A}\left[\frac{H'}{\gamma_B} + \frac{1}{\sum_{j=1}^{4}\lambda_j}\left\{\begin{aligned}&\sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial \alpha} + w\left(\gamma t_A^e + t_B^e\right)\frac{\partial \ell}{\partial \alpha} + \gamma w t_A \ell\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial \alpha}\\ &+ w t_B \ell\left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial \alpha}\end{aligned}\right\}\right] dF_w\\
&+\int_{w^l}^{w^h}\left[w\left(\gamma t_A^e + t_B^e\right)\frac{\partial \ell^c}{\partial t_A}\right]dF_w + \int_{w^l}^{w^h}\left[\gamma w\ell\left(\delta_A + \theta_A\left(S_1+S_2\right)\left(1-\delta_A\right)\right)\right]dF_w + \sum_{i=1}^{n} t_i^e \frac{\partial X_i^c}{\partial t_A} \qquad (5.3)\\
&+\int_{w^l}^{w^h}\left[w\ell\left\{\gamma t_A\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A^c}{\partial t_A} + t_B\left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B^c}{\partial t_A}\right\}\right]dF_w
\end{aligned}
$$

$$
\begin{aligned}
&\int_{w^l}^{w^h} - w\ell\lambda_{t_B}\left[\frac{H'}{\gamma_B} + \frac{1}{\sum_{j=1}^{4}\lambda_j}\left\{\begin{aligned}&\sum_{i=1}^{n} t_i^e \frac{\partial x_i}{\partial \alpha} + w\left(\gamma t_A^e + t_B^e\right)\frac{\partial \ell}{\partial \alpha} + \gamma w t_A \ell\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A}{\partial \alpha}\\ &+ w t_B \ell\left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B}{\partial \alpha}\end{aligned}\right\}\right] dF_w\\
&+\int_{w^l}^{w^h}\left[w\left(\gamma t_A^e + t_B^e\right)\frac{\partial \ell^c}{\partial t_B}\right]dF_w + \int_{w^l}^{w^h}\left[w\ell\left(\delta_B + \theta_B\left(S_1+S_3\right)\left(1-\delta_B\right)\right)\right]dF_w + \sum_{i=1}^{n} t_i^e \frac{\partial X_i^c}{\partial t_B} \qquad (5.4)\\
&+\int_{w^l}^{w^h}\left[w\ell\left\{\gamma t_A\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A^c}{\partial t_B} + t_B\left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B^c}{\partial t_B}\right\}\right]dF_w
\end{aligned}
$$

*Proposition 4: With tax evasion by individuals from multiple tax authorities, the optimal income tax rates by the tax authorities A and B are characterized implicitly by the following formulae:*

$$
t_A = \frac{\int_{w^l}^{w^h}\left[\frac{w\ell\lambda_{t_A}}{\sum_{j=1}^{4}\lambda_j} b^w - \gamma w\ell\left(\delta_A + \theta_A\left(S_1+S_2\right)\left(1-\delta_A\right)\right)\right]dF_w - \sum_{i=1}^{n} t_i^e \frac{\partial X_i^c}{\partial t_A}}{\int_{w^l}^{w^h}\left[\frac{w\ell}{t_A}\left\{\left(\gamma t_A^e + t_B^e\right)\frac{\partial \ell^c}{\partial t_A}\frac{1}{\ell} + \gamma\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A^c}{\partial t_A} + \frac{t_B}{t_A}\left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B^c}{\partial t_A}\right\}\right]dF_w} \qquad (5.5)
$$

$$t_B = \frac{\int_{w^l}^{w^h}\left[\frac{w\ell\lambda_{t_B}}{\sum_{j=1}^{4}\lambda_j} b^w - w\ell\left(\delta_B + \theta_B\left(S_1 + S_3\right)\left(1-\delta_B\right)\right)\right] dF_w - \sum_{i=1}^{n} t_i^e \frac{\partial X_i^c}{\partial t_B}}{\int_{w^l}^{w^h}\left[\frac{w\ell}{t_A}\left\{\left(\gamma t_A^e + t_B^e\right)\frac{\partial \ell^c}{\partial t_B}\frac{1}{\ell} + \gamma\left(1-\theta_A\left(S_1+S_2\right)\right)\frac{\partial \delta_A^c}{\partial t_B} + \frac{t_B}{t_A}\left(1-\theta_B\left(S_1+S_3\right)\right)\frac{\partial \delta_B^c}{\partial t_B}\right\}\right] dF_w} \tag{5.6}$$

Proof: Use (4.13) and (4.21) and the definition of $b^w$ in (5.2). □

The two equations in the above proposition show the joint dependence of the income tax rates by the both tax authorities. Even though they do not give closed form solutions, they contain the main elements to characterize the optimal income tax policies in this prototype economy. In order to interpret these expressions, consider the income tax rate in the absence of income and commodity tax evasion provided by Dixit and Sandmo (1977):

$$t = -\frac{\int_{w^l}^{w^h}\left[w\ell\left(\bar{b}^w - 1\right)\right] dF_w}{\int_{w^l}^{w^h}\left[w\left(-\frac{\partial \ell^c}{\partial t}\right)\right] dF_w} = -\frac{Cov(y, \bar{b}^w)}{\int_{w^l}^{w^h}\left[w\left(-\frac{\partial \ell^c}{\partial t}\right)\right] dF_w} \tag{5.7}$$

where $\bar{b}^w$ is the corresponding analog of $b^w$ defined above and Cov(,.,) denotes the covariance computed with respect to the ability distribution. It is easy to verify that (5.5) and (5.6) reduce to expressions similar to (5.7) when there are no income or commodity tax evasion, but they are not exactly the same due to inherent simultaneity in income tax rates between tax authorities A and B. In the standard interpretation, the covariance term in (5.7) reflects the social valuation of raising additional revenues by increasing taxes and distributing the proceeds uniformly across the population. In other words, it shows the social value of a more progressive income tax. The denominator captures the welfare loss of the tax due to the negative substitution effects on labor supply. With the concavity of the social welfare function, the covariance term is negative so that an optimal tax solution involves a positive income tax rate. The counterpart to (5.7) with income tax evasion derived by Cremer and Gahvari (1994) under quasi-linear preferences is:

$$t = -\frac{Cov(y, \tilde{b}^w) - \frac{\partial m}{\partial t}}{\int_{w^l}^{w^h} \left[ w\left( -\frac{\partial \ell^c}{\partial t} \right) \right] dF_w} \qquad \text{where } \tilde{b}^w = \frac{H'}{E[H']} \tag{5.8}$$

where m is the amount of income concealment analogous to $k(1 - \delta_A - \delta_B + \delta_A \delta_B)$ in our prototype economy. Thus, the basic formula is corrected to include the resource cost of income concealment. A comparison among (5.5) through (5.8) illustrates how the presence of income and commodity tax evasion and the simultaneity of income tax rates between tax authorities A and B affect the optimal income tax formulae.

The first two terms in the numerators of (5.5) and (5.6) are analogous to the covariance terms in (5.7) and (5.8), accounting for the different effects that changes in $t_A$ and $t_B$ have on the effective income tax rates $t_A^e$ and $t_B^e$ across individuals. These measure whether after accounting for differences in evasion behavior, income redistributions benefit individuals with a low or high social marginal utility of income. Note that the first terms in the numerators in (5.5) and (5.6) give the required compensation for the loss in the utility caused by a higher tax rate by the concerned tax authority. In (5.7), this is simply given by the additional income taxes paid. With multi-agency tax evasion at work, this is no longer a reliable measure of welfare loss of an individual since $t_A^e$ and $t_B^e$ change with $\delta_A$ and $\delta_B$ and the outcome of audit lotteries. However, this is not apparent in (5.8) because with quasi-linear preferences, the proportion of income reported and hence $t_A^e$ and $t_B^e$ are constant across individuals. This is the key difference between the results of this essay and those derived by Cremer and Gahvari (1994). The second terms are the average marginal tax collections, also modified to account for differences in the taxes paid by individuals due to multi-agency income tax evasion. The third terms reflect the differences made by the presence of commodity taxes. These terms capture the effect of changes in the marginal tax rate by the respective tax authority on the marginal excess burden of existing commodity taxes as measured by the induced change in indirect tax revenue. Due to the negative semi-definiteness of the Slutsky substitution matrix, these terms are positive, so that the presence of commodity taxes tend to make optimal income tax rate by either authorities higher in order to compensate for the loss in commodity tax revenue to tax authority B.

The terms in the denominators measure the welfare losses of income taxation due to substitution effects, which now involve distortions in labor supply and the additional costs of income concealment from the tax authorities. Other things remaining the same, the higher the compensated labor supply and evasion responses to changes in $t_A$ and $t_B$, the higher the efficiency costs of redistribution. Higher concealment costs tend to call for lower optimal tax rates relative to the case of no income tax evasion whatsoever.

Given that the denominators in (5.5) and (5.6) are negative and H(.) is strictly concave, a positive marginal tax rate requires that the numerators are negative. This will be the case as long as there are net positive distributional gains from linear income taxation. This, in turn, requires that an increase in $t_A$ ($t_B$) causes $t_A^e$ ($t_B^e$) to increase more for higher ability individuals and/or $\frac{\partial \delta_A^c}{\partial t_B}$ ($\frac{\partial \delta_B^c}{\partial t_A}$) is ‘large’ and positive. This would be the case if $\delta_A$ ($\delta_B$) strictly decreases with w, i.e., if the high ability individuals conceal a lower proportion of their income.

## Tax Mix and Uniform Taxation: Some Specific Results

Expressions (5.2), (5.5), and (5.6) make it clear that the presence of evasion in commodity and the income taxes set by tax authorities affects the structure of direct and indirect taxation. In order to grasp the difference that each of the three components of evasion makes, let us consider some special cases in which the above formulae reduce to familiar expressions in literature. Of particular interest for the issues of uniform taxation and the optimal tax mix is the result of Deaton (1979). He shows that when preferences are weakly separable between leisure and goods with linear Engel curves, optimal tax system involves either (i) a linear income tax or (ii) equivalently uniform commodity taxes accompanied by a lump sum transfer. We will examine its robustness in the presence of multi-agency tax evasion.

Consider first the case that firms do not evade commodity taxes but individuals do evade their income taxes owed to authorities A and B. We thus have $t_i^e = t_i$, $\forall$i and the last term in (5.2) is equal to 1. Then, it might seem that optimality calls for a zero marginal income tax rate by tax authority B and the use of commodity taxes along with the lump sum transfer for redistribution. By adjusting the vector of commodity tax rates so as to raise the same revenue as with the income tax in place, social welfare could be potentially improved in two ways: (i) eliminating the wasteful costs of income concealment and the costs of audits, and (ii) eliminating the uncertainty that ensues with income tax audit lotteries. This would allow a higher lump sum transfer or lower commodity taxation, and would eliminate the utility costs that audit lotteries impose on risk-averse individuals.

Nevertheless, at this level of generalization, this result could not be immediately established. With multi-agency income tax evasion, $t_A^e$ and $t_B^e$ vary across individuals so that the features of the multi-agency income tax system cannot be trivially reproduced by a uniform commodity tax that resembles a wage tax. If low ability individuals evade a higher proportion of their income than their high ability counterparts, reported income will still provide somewhat reliable signals of the rankings of true income. Since $t_A^e$ and $t_B^e$ will then be lower for low ability individuals, taxes would carry some information about the ability of individual types. Income tax evasion could then be served as passive "screening" devices to sift the low ability individuals from their high ability counterparts relaxing the information constraints faced by the tax authorities. With tax evasion the income tax systems operate as a non-linear tax schedule. It is then possible that evasion lowers both marginal and the average tax rates faced by low-income individuals and thus increase the progressivity of the income taxes. If these redistributive benefits outweigh the above welfare costs, then the optimal tax structure could still involve a mix of income and commodity taxes.

Obviously, if high ability individuals tend to have high $\delta_A$ and $\delta_B$, the optimal policy involves sole reliance on commodity taxation. In this case, as indicated before, the numerators of (5.5) and (5.6) would be positive and $t_A = t_B = 0$ is the optimal tax solution. Therefore, if high ability individuals evade proportionately more and evasion of commodity taxes is unimportant, then increasing reliance on commodity taxation approximates the "optimal" policy prescription set forth by the World Bank and the International Monetary Fund for the developing countries.

In contrast, as long as $t_A > 0$ and/or $t_B > 0$, even without commodity tax evasion, it cannot be immediately presumed from (5.2) that commodity tax rates should be uniform. This again follows from the aforementioned non-equivalence between a wage tax and uniform commodity tax in this prototype economy. Since $\delta_A$ and $\delta_B$ and hence $t_A^e$ and $t_B^e$ vary with w, tax evasion introduces non-linear features into the income tax, which can be favorable to redistribution and are not trivially reproduced by commodity taxation. Moreover, individuals pay different proportions of their income in taxes regardless of their consumption patterns. If we allow, as Cremer and Gahvari (1994) did, individuals to pre-commit a given level of consumption of certain commodities regardless of the outcome of the tax audits, the optimal tax system would then involve a mix of differentiated commodity taxes together with an income tax for tax authority B. Hence, the advocacy of tax practitioners in developing countries for a heavier reliance on uniform commodity taxation cannot be justified on the grounds of differences in the evasion characteristics of taxes that favor commodity taxation.

Consider now the opposite situation where only commodity taxes are subject to evasion. Let us re-examine two specific results in the literature that make a case for uniform commodity taxation: (i) exogenous (or untaxed) labor income and (ii) independent commodity demands. Using dots to refer to the value of the variables without income tax evasion, we can rewrite (5.2) as:

$$\frac{\sum_{i=1}^{n} t_i^e \frac{\partial \ddot{X}_i^c}{\partial p_k}}{\ddot{X}_k} = Cov\left(\frac{\ddot{x}_k}{\ddot{X}_k}, \ddot{b}^w\right) - \frac{\int_{w^l}^{w^h} \left[ w(\gamma t_A + t_B)\left(\frac{\partial \ddot{\ell}^c}{\partial p_k}\right)\right] dF_w}{\ddot{X}_k} - \frac{\partial t_i^e}{\partial t_k} / \frac{\partial p_k}{\partial t_k} \qquad (5.2')$$

This is analogous to the optimal commodity tax rule obtained by Cremer and Gahvari (1993) for a representative consumer in a prototype economy of heterogeneous taxpayers with linear income taxes by A and B are in place. As we will see, most of their results generalize to the case where equity concerns are incorporated and a linear income tax is among the policy instruments for tax authority B. The left hand term—the index of discouragement—reflects the Ramsey efficiency arguments for taxing inelastic goods more heavily. The covariance term is negative if $x_k$ is a normal good and lower in absolute value for necessities; it reflects the equity concerns embodied in the social welfare function. On efficiency grounds the demand for the necessities should then be discouraged less by increased taxation. The last term is the distortion caused by commodity tax evasion in industry *k*.

When income is endogenous, we are back to a second best situation. As (5.2′) reveals, there is no *a priori* presumption that an optimal solution involves zero commodity tax rates, i.e., sole reliance on the linear income tax for tax authority B. More often then not differences in evasion behavior across industries tend to make optimal commodity tax rates non-uniform; instead an optimal mix of differentiated taxes emerges (Cremer and Gahvari 1993).

When labor income is untaxed by the authority B, Deaton's (1979) result – with weak separability between leisure and consumption along with identical linear Engel curves, income tax suffices – applies. In this case, the consumption pattern is the same for all commodities across the individuals, i.e., $\frac{\ddot{x}_k}{\ddot{X}_k}$ is the same for $\forall k$, so that commodity sales carry no information about individuals' ability and differentiated commodity taxation can serve no redistributive role. In this case commodity tax evasion compounds the deadweight loss by adding the concealment and audit costs by tax authority B. In the general case, even though commodity taxes may still play a redistributive role, any such redistributive benefits have to outweigh the efficiency losses caused by concealment and audit costs by B.

It may be recalled that in the absence of commodity tax evasion and when commodity demands depend only on own price so that $\frac{\partial x_i}{\partial p_k} = 0,\ \forall \mathrm{i} \neq \mathrm{k}$, optimal commodity tax follows the so-called "inverse elasticity rule" when there is no commodity tax evasion and when commodity demands depend only on own price. Letting $\varepsilon_{kk}$ be the absolute value of the average compensated own price elasticity of the *k*-th good, from (5.2′) we obtain:

$$\frac{t_k^e}{p_k} = \frac{1}{\varepsilon_{kk}}\left[ Cov\left(\frac{\ddot{x}_k}{\ddot{X}_k}, \ddot{b}^w\right) - \frac{\int_{w^l}^{w^h}\left[ w(\gamma t_A + t_B)\left(\frac{\partial \ddot{\ell}^c}{\partial p_k}\right)\right] dF_w}{\ddot{X}_k} - \frac{\partial t_i^e}{\partial t_k} / \frac{\partial p_k}{\partial t_k} \right] \tag{5.9}$$

In the absence of commodity tax evasion, this is simply the ratio of the "social luxury index" of $x_k$ (which calls for a lower price on necessities) and its compensated own price elasticity (which calls for a lower tax on such commodities). Deaton (1979) argues that since under additive preferences luxuries tend to be price elastic and necessities price inelastic, redistributive forces calling for a lower tax on necessities shown by the covariance term in (5.9) are offset by the efficiency pressure for a higher tax suggesting an approximate uniform tax solution. With commodity tax evasion, the result necessarily refers to the expected tax rate and setting the statutory tax rates and audit probabilities to be equal would not suffice to guarantee the result. This would now depend on whether the distortions of commodity tax evasion are greater in industries producing necessities or in industries producing luxuries. This, in turn, depends on the differences of the concealment cost technology. Since there is no *a priori* presumption of correlation between concealment costs and the elasticity of demand, the optimality of uniform commodity taxation is highly unlikely.

Finally, once redistributive concerns are introduced, it is not possible to conclude that with equal price responsiveness tax evasion increases the expected commodity tax of the good subject to evasion as in Cremer and Gahvari (1993). Fulfillment of the condition would now require the equality of the covariance term for both goods.

***TAX EVASION AND PROGRESSIVITY OF INCOME TAXES***

The foregoing analysis suggests that income tax evasion calls for lower marginal income tax rates set by tax authorities A and B and thus make linear income tax systems set by both the tax authorities less progressive. However, formulae in (5.5) and (5.6) give only implicit solutions. At this level of generalization it is impossible to make any specific conclusion. To derive specific conclusions we follow Cremer and Gahvari (1994): comparing the behavior of social welfare as $t_A$ and/or $t_B$ change with and without tax evasion. To this end we use quasi-linear preferences $U(c, 1-\ell) = c + f(1-\ell)$ where c is a composite commodity with price normalized to unity. Using the results derived earlier, one can get the following simplified results:

$$\sum_{j=1}^{4}\lambda_j = 1; \qquad \frac{\partial \ell}{\partial t_A} = \frac{\partial \ell^c}{\partial t_A}; \qquad \frac{\partial \ell}{\partial t_B} = \frac{\partial \ell^c}{\partial t_B}; \qquad \frac{\partial R_A}{\partial \alpha} = \frac{\partial R_B}{\partial \alpha} = -1;$$

$$k'_A\left(1-\delta_A-\delta_B+\delta_A\delta_B\right) = t_A\left[\left(\lambda_3+\lambda_4\right)-\left(\theta_A-1\right)\left(\lambda_1+\lambda_2\right)\right];$$

$$k'_B\left(1-\delta_A-\delta_B+\delta_A\delta_B\right) = t_B\left[\left(\lambda_2+\lambda_4\right)-\left(\theta_B-1\right)\left(\lambda_1+\lambda_3\right)\right];$$

$$\gamma = \frac{\int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_B}\right]dF_w - \int_{w^l}^{w^h}\left[\frac{\partial R_B}{\partial \alpha_A}\right]dF_w}{\int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_A}\right]dF_w - \int_{w^l}^{w^h}\left[\frac{\partial R_A}{\partial \alpha_B}\right]dF_w}; \text{ and } \tilde{b}^w = \frac{H'}{\gamma_B}.$$

With these results the expressions in (5.5) and (5.6) is simplified to:

$$t_A = \frac{\int_{w^l}^{w^h} \left[ w\ell\lambda_{t_A}\tilde{b}^w - \gamma w\ell(\delta_A + \theta_A(S_1+S_2)(1-\delta_A)) \right] dF_w}{\int_{w^l}^{w^h} \left[ \frac{w\ell}{t_A} \left\{ (\gamma t_A^e + t_B^e)\frac{\partial \ell}{\partial t_A}\frac{1}{\ell} + \gamma(1-\theta_A(S_1+S_2))\frac{\partial \delta_A}{\partial t_A} + \frac{t_B}{t_A}(1-\theta_B(S_1+S_3))\frac{\partial \delta_B}{\partial t_A} \right\} \right] dF_w} \quad (5.5')$$

$$t_B = \frac{\int_{w^l}^{w^h} \left[ w\ell\lambda_{t_B}\tilde{b}^w - w\ell(\delta_B + \theta_B(S_1+S_3)(1-\delta_B)) \right] dF_w}{\int_{w^l}^{w^h} \left[ \frac{w\ell}{t_A} \left\{ (\gamma t_A^e + t_B^e)\frac{\partial \ell}{\partial t_B}\frac{1}{\ell} + \gamma(1-\theta_A(S_1+S_2))\frac{\partial \delta_A}{\partial t_B} + \frac{t_B}{t_A}(1-\theta_B(S_1+S_3))\frac{\partial \delta_B}{\partial t_B} \right\} \right] dF_w} \quad (5.6')$$

Since individuals are now risk-neutral, the marginal utility of income is constant. Everybody chooses the same amount of income to report ($\delta_A$ and $\delta_B$) so that $t_A^e$ and $t_B^e$ are again constant. It is then evident that labor supply increases unambiguously with the earning potential of individuals, so does the pre-tax income. Income reports become a reliable indicator of individual's well-being so that the numerators are unambiguously negative. The linear income taxes by tax authorities A and B will then be clearly progressive in that average tax rates increase with pre-tax income, exhibit properties closer to those in the absence of tax evasion. Nevertheless, one cannot infer whether $t_A$ and $t_B$ should be smaller than their counterparts in the absence of tax evasion. While marginal concealment costs still add to the social deadweight loss, the effect of tax evasion on the marginal excess burdens are ambiguous as both the level and elasticity of labor supply are affected by tax evasion. Since income differs with and without tax evasion, even in the most favorable conditions considered so far, a restriction on the shape of the social welfare function will not suffice to conclude that tax evasion lowers the progressivity of the income taxes.

The difference between these findings and those of Cremer and Gahvari (1994) may be attributed to the specification of the concealment technology. Cremer and Gahvari (1994) allowed the probability of being caught to depend on the amount and proportion of income concealed as well as the amount spent in concealment. Nevertheless, their results on progressivity are derived under the special case when probability is not affected by the proportion of income evaded. With this restriction, labor supply is not affected by tax evasion. Since evasion behavior and concealment technology are independent of pre-tax income, everybody chooses to evade the same amount and spend the same in concealment. The probability of being caught is then effectively the same for everyone. Labor supply is unambiguously higher for high-ability individuals. High-income individuals evade a lower proportion of their income than their low-income counterparts and thus face higher effective marginal tax rates. Given that individuals evade the same amount and labor supply is not affected by evasion, concealment and audit costs make everyone worse off. Under these circumstances, it is not surprising that a lesser concern for redistribution embodied in the social welfare function leads to a less progressive tax system.

In our prototype economy, concealment technology consists of constant probability of being caught by tax authorities A or B or both but the concealment costs are proportional to the amount evaded. Individuals pick the same proportion of income to evade rather than the absolute amount; this then affects the leisure-consumption trade offs. In contrast to Cremer and Gahvari (1994), the proportions of income concealed turn out to be the same for everyone as well as the expected tax rates, but the high-income individuals evade more in absolute terms and pay higher concealment costs. Further, the lower effective tax rates cause optimal labor supply to be higher in the presence of tax evasion. Additional assumptions are then required to resolve the ambiguities regarding the differences in the levels of lump sum transfers and marginal social welfare. Still, in the most favorable scenario, even though $\delta_A$ and $\delta_B$ are the same for everyone so that reported income still increases with pre-tax income, there remains the question of whether tax evasion makes the tax base a less reliable measure of well-being.

The forgoing analysis implies that it is not possible to come up with clear-cut statements regarding the effect of tax evasion on the progressivity of linear income tax system. Such statements are even more elusive under more general structures of individual preferences where income effects are important. The analysis does, however, point out some of the important elements involved. It does not follow that the social welfare loss imposed by concealment and audit costs necessarily makes the society worse off. It is possible that for tax evasion to have a positive impact on welfare through redistributions in the excess burden of the existing taxes. The impact on aggregate welfare would depend on the shape of the distribution of skills and how tax evasion and the concomitant concealment costs vary across the population.

## *THE OPTIMAL ENFORCEMENT OF THE TAX BASE*

An important aspect of our explicit modeling of the uncertainty that entails tax evasion is that the set of tax policy instruments now includes the audit probabilities, which provide an alternative way to raise revenues for the tax authorities. For instance, by increasing the audit probabilities and thus reducing the expected return to commodity tax evasion, tax authority B can induce firms to report a higher proportion of their sales and can therefore increase revenue collections with lower tax rates. Since enforcement is costly and also causes distortions, it is important to characterize the trade offs involved in choosing these various alternatives.

## The Optimum Enforcement of Commodity Taxes

Using (D.8), (4.17) and (4.25) and undertaking similar simplifications that led to (5.2), we obtain an equivalent expression for the Ramsey rule in terms of audit probabilities:

$$\frac{\sum_{i=1}^{n} t_i^e \frac{\partial X_k^c}{\partial p_k}}{X_k} = \frac{1}{X_k}\int_{w^l}^{w^h}\left[\left(\frac{\sum_{j=1}^{4}\lambda_j x_{ij}}{\sum_{j=1}^{4}\lambda_j}\right)b^w\right]dF_w - \frac{\int_{w^l}^{w^h}\left[w\left(\gamma t_A^e + t_B^e\right)\left(\frac{\partial \ell^c}{\partial p_k}\right)\right]dF_w}{X_k} - \left(\frac{\partial t_k^e}{\partial \sigma_k} - \frac{\partial c(.)}{\partial \sigma_k}\right) / \frac{\partial p_k}{\partial \sigma_k} \quad (5.10)$$

The close resemblance of this expression to (5.2) comes without surprise. An increase in audit probability in the *k*-th industry raises the expected tax rate on $x_k$ at the cost of distorting the price of the good and at an additional audit cost which adds to social deadweight loss. The third term of this expression is thus analogous to the tax evasion distortion term discussed earlier. It captures the distortion that tighter enforcement creates in the price of $x_k$ per unit of tax revenue collected net of audit costs. It is evident that $t_k$ and $\sigma_k$ represent closely substitute instruments for tax authority B to raise commodity tax revenue.

*Proposition 5: The optimal trade off between higher commodity tax rates and tighter enforcement of commodity tax laws is characterized by the following relationship*

$$\frac{\partial t_k^e}{\partial t_k} / \frac{\partial p_k}{\partial t_k} = \left( \frac{\partial t_k^e}{\partial \sigma_k} - \frac{\partial c(.)}{\partial \sigma_k} \right) / \frac{\partial p_k}{\partial \sigma_k} \tag{5.11}$$

*Proof:* Use (5.2) and (5.10) above. □

This is exactly the result derived by Cremer and Gahvari (1993) in a representative consumer economy with no linear income tax. It simply says that, for optimality, the changes in commodity tax rates and audit probabilities that result in an increase of $1 in prices should yield the expected per unit increase in net tax revenue. The fact that the rule remains intact in this considerably more general setting means that the trade offs between higher tax rates and tighter enforcement of sales tax laws are not affected by equity considerations or tax evasion of income. The optimal policy is guided only by efficiency considerations in terms of how tax rates and audit probabilities affect the costs of output concealment and thus consumer prices.

## The Optimum Enforcement of Income Taxes

Slemrod (1994) argues that the issue of the optimal progressivity of the income tax and the optimal enforcement of the tax system is intertwined. Earlier we found that, *ceteris paribus*, optimal income tax rates would tend to be lower the higher the costs of income concealments and the more elastic the response of $\delta_A$ and $\delta_B$ to changes in $t_A$ and $t_B$. These two parameters are clearly a function of the audit probabilities $\sigma_A$ and $\sigma_B$ and therefore of the resources spent on audits. Audit policy offers an alternative means to increase expected tax revenue collections by discouraging income tax evasion. Under random audits, this is achieved by setting the constant probabilities of audit. In contrast to the income tax rates, audit policy has an associated direct cost in addition to the indirect costs of any distortionary effects on individual and firm behavior. On the other hand, because of risk-aversion, there are social benefits from a higher audit probability resulting from the reduction of the risk of tax lotteries. Setting the optimal audit probability involves balancing these costs and benefits for different type of taxpayers. In order to evaluate this question closely, one can characterize optimal audit probabilities by holding $\underline{\sigma}$, $\underline{t}$, $t_A$, and $t_B$ constant. Using (D.5), (D.6), (4.15), (4.16), (4.23), and (4.24), one can implicitly solve for $\sigma_A$ and $\sigma_B$ as function of $t_A$, and $t_B$ along with other parameters and see how tax rates are affected as the audit probabilities change. However, at this level of generalization, it is not possible to give closed form solutions and hence any definitive conclusion again begs for empirical verification.

## *CONCLUSIONS*

In this essay we have extended the recent literature on the role of tax evasion in the design of an optimal tax system. We examined how the optimal mix and the characteristics of linear income and differentiated commodity taxes are affected in the presence of evasion by competitive firms and individuals (from tax authorities A and B) under uncertainty of random audits. Several important lessons were learned from the analysis.

First, the introduction of multi-agency tax evasion sensibly affects the orthodox prescriptions of the optimal tax literature. The standard optimal tax formulae should be modified in such a way as to account for the impact of multi-agency tax evasion and audits. Further, the usual restrictions on individual preferences generally do not suffice to render either form of taxation as useless. In contrast to the current paradigm, we find that the presence of both income and commodity tax evasion makes a strong case for differentiated commodity taxation. Hence, we find little justification for the prescription based on the "Washington Consensus" that developing countries should rely heavily on uniform commodity taxation on grounds of differences in tax compliance. Insofar as differential tax rates can help minimize the differential price distortions created by commodity tax evasion, uniform commodity taxes are sub-optimal when concealment of sales varies across industries. Eventually, the optimal tax system involves a mix of linear income and differentiated commodity taxes.

Second, the expansion of the range of tax authorities' instruments through inclusion of independent audits and sharing of audit information bring important changes to optimal tax policy. In essence, as a direct mechanism to control tax evasion, audit probabilities together with sharing information offer alternative ways to raise revenues so that some optimal trade offs with tax rates emerge. By directly affecting tax evasion and the concomitant concealment, audit policies and information sharing affect the responses of taxpayers to changes in tax policies. Any level of revenues by tax authorities A and B can now be achieved through a combination of taxes, audits and information sharing. Further, in the case of income tax evasion, independent audits and sharing of audit information offer a direct welfare enhancing mechanism by reducing individuals' exposure to audit risks.

Third, we find little justification for the prevailing orthodoxy that tax evasion calls for lower optimal marginal tax rates and reduce the progressivity of income taxes. However, these results seem to critically hinge on modeling evasion of income taxes and the associated concealments. Our formulation allows evasion to affect directly the terms of labor-leisure trade off. Further, whether tax evasion affects negatively the distributional properties of the income taxes depends on the shape of the distributional skills of individuals and how the proportions of income concealed varies across the taxpayers.

The issues dealt with in this essay can be expanded to several directions. *First*, it would be theoretically interesting to analyze the issues using a game-theoretic approach. As several studies found, tax authorities do not pre-commit the audit rules but base such decisions depending on the nature and extent of returns filed by the taxpayers. *Second*, our model is based on competitive market structure. While this assumption is analytically tractable it does not fit well with real world data especially from the developing countries. It would be interesting to explore the commodity tax evasion under various non-competitive market structures. *Third*, it would be interesting to see if and how our conclusions change once expected utility maximization is replaced by generalized non-expected utility or prospect theory. *Fourth*, it would be of primary policy interest to conduct simulations of the model in order to quantify the parameters highlighted in the essay.

# ESSAY II: SPATIALITY AND PERSISTENCE IN THE U.S. INDIVIDUAL INCOME TAX COMPLIANCE

## *INTRODUCTION*

The income tax system in the United States operates on a self-assessment basis. Individuals annually determine their tax liability and pay whatever they deem due. In the course of time taxpayers learn more about the tax system, especially about the loopholes of the tax code. Individuals also can and possibly do communicate with other filers in filing their own returns. Insofar as the taxpayers may recall their own past filing experience and may communicate with other taxpayers, these issues provide a rough basis to shape the nature and extent of this year's reporting amount. Thus, two interesting phenomena emerge from past reporting experience and exchange of information and experience with other filers: one of them relates to the dynamic behavior of the taxpayer; the other shapes the nature and extent of spatial dependence.

If the individual got away with tax evasion in the past, she may tend to evade more this year. If she was audited and caught cheating by the IRS in the past, she may tend to evade less this year. In either case this year's tax evasion decision is shaped by the past evasion experience. There is, thus, an element of persistence in individual income tax evasion in the United States. Dubin (2004) gives another explanation for the possible persistence in tax evasion. He argues, "…taxpayers may adjust their reported taxes based on a mixture of taxes reported in the previous year and the optimal level of taxes due based on existing or current conditions." He cites delayed audit completion cycle as a possible reason for the presence of persistence.

There are several possible explanations of the second phenomenon. Given that taxpayers usually exchange their experience with each other, they influence and are influenced by the tax evasion behavior of the other taxpayers. However, the precise nature of interaction between the taxpayers with regard to evasion decision hinges on the nature of enforcement by the IRS; if the enforcement of tax codes is strict in the sense that they have a target revenue collection in mind, then higher evasion by one taxpayer should be followed by lower evasion by the other, thereby resulting in a negative relationship. If, instead the enforcement of the tax codes were lax, then a higher evasion by one taxpayer would follow non-negative evasion by the other taxpayer. In view of the declining audit rates by the IRS over the years, the latter scenario seems more plausible than the former. Hence, we expect a positive relationship between the reporting behaviors of two taxpayers.

There is a possible alternative explanation for the above interdependence phenomenon. Suppose the IRS has a fixed amount of revenue in mind. In that case, if one of the taxpayers successfully evaded more of her tax liability it increases the probability of being audited of the other taxpayer. In this case the dependence comes through the probability function and leads to a negative relationship in the reporting behaviors of two taxpayers.

Alm, McClelland, and Schulze (1999) provide another explanation of the interdependence of tax compliance. They argue that "… an individual will comply as long as he or she believes that compliance is the social norm. Conversely, if noncompliance becomes pervasive then the social norm of compliance disappears." They test this paradigm with experimental data and find substantive evidence. Their argument and the experimental findings also imply that there is a positive relationship between the reporting behaviors of two taxpayers.

There is a third explanation based on experimental evidence by Manski (1991). He argues that individuals faced with dynamic stochastic decision problems that pose immense computational challenges may simply look to others to infer satisfactory policies. McFadden (2006) argues that interpersonal dependence "… works through … learning by imitating rather than learning by doing. … (P)rimary information … come(s) from others,  through observation, advice, and association. … In addition to providing information, social networks may discipline the behavior of members through consensus on social norms, accountability of choices, and sanctions for behavior that violates norms."

These twin issues of persistence and spatial dependence, albeit with their importance, are never raised in the theoretical models nor tested in the empirical analyses of income tax evasion. This essay will, thus, succinctly extend the original Allingham-Sandmo-Yitzhaki model of income tax evasion by incorporating the above twin issues. It will also test the empirical validity of these issues in the context of the U.S. federal individual income tax evasion. As will be reviewed later in this essay, several empirical studies have been conducted to assess the determinants of income tax evasion in the U.S. However, if either of the issues has had any role in shaping the magnitude and growth of income tax evasion, then those results were biased and/or inconsistent. Hence, any policy prescription based of those results will be misleading.

The "tax gap" attributable to individual income tax has grown from $29 billion in 1973 to $81 billion in 1981. It was reduced to $70 billion in 1985 before jumping to $95 billion in 1992. According to various IRS sources, the annual tax gap is estimated to be $345 billion, or about 10 percent of what is collected each year from individuals and businesses. The Internal Revenue Service (IRS) estimates that three-quarters of this tax gap is attributable to individual taxpayers. At that rate, individuals currently represent $260 billion of the tax gap, which is close to triple the level estimated in 1985. The rising loss in tax revenues due to non-compliance has drawn attention of the policymakers. The successive U.S. governments have taken several measures to reduce this ever-yawning gap. In the light of apparent magnitude and growth of the tax gap, the time seems right to reassess the determinants of individual income tax evasion. These issues will also warrant some modifications of the original Allingham-Sandmo-Yitzhaki model of income tax evasion.

The essay is organized as follows. In the next section we briefly review the past studies conducted on individual income tax compliance. Section 3 extends the original Allingham-Sandmo-Yitzhaki model by incorporating persistence and spatiality. Section 4 deals with data and related methodological issues. Section 5 discusses the methods employed in the empirical work and analytical issues related to the construction of variables. Section 6 presents the descriptive statistics and the estimation results where the spatiality and persistence issues are examined separately. The final section summarizes the findings, discusses their implications, and suggests areas for further research.

## *LITERATURE REVIEW*

After the seminal works by Allingham and Sandmo (1972), Srinivasan (1973), and Yitzhaki (1974), theoretical work on tax evasion has progressed in leaps and bounds.

In contrast, the empirical literature has been thwarted primarily due to lack of data on tax evasion. We focus on those studies that directly link to our point of departure set forth in the Introduction. Further, we confine our analysis to the empirical studies. In doing so, we divide the review into two sections: cross-sectional studies and time-series studies. Cowell (1990), Andreoni, Erard, and Feinstein(1998), Alm (1999), and Slemrod and Yitzhaki (2002) provide summaries of literature on tax evasion from both the theoretical and empirical fronts. Also, we confine ourselves within the purview of individual income tax evasion as opposed to corporate income tax or sales tax evasion.

## Cross-section Studies

The empirical dimension of tax evasion literature started with Clotfelter (1983) based on the 1969 TCMP data. He divides the income tax returns into three broad groups: non-business returns, non-farm business returns, and farm returns and estimates a tobit model, explaining, for each group, noncompliance as a function of the combined federal and state marginal income tax, after-tax auditor-adjusted gross income, and set of demographic variables available on tax returns. The most striking conclusions are: (i) net income and marginal tax rates positively affect evasion; (ii) wages as proportion of adjusted gross income (AGI) and interests and dividends also as proportions of AGI negatively affect how an individual underreports. Note that his finding on the effects of marginal tax rates is inconsistent with the Allingham and Sandmo (1972) model. Married taxpayers were found to underreport more than the single taxpayers.

Witte and Woodbury (1985) aggregate the 1969 TCMP data at the three-digit zip code level. They discuss how direct audit, audit of the other classes, multiple penalty and progressive tax structures together with community characteristics affect individual income tax evasion. Using SURE estimation method they find that taxpayers evade less as lagged direct audit rate increases. They argue that increase in any of the three measures of penalty discourages evasion if the taxpayers are risk neutral or risk averse. They also find that tax evasion is positively related to "opportunities" for evasion and related to income in a non-linear way, with non-compliance at its highest at both very low and very high levels of income. Their finding of positive association between information reporting and taxpayer compliance provides a strong empirical support to the TEFRA 1982.

Slemrod (1985) studies the issue of primary and secondary tax evasion based on one-fourth of the data from stratified random sample of the U.S. Treasury File for 1977. He uses an index of the presence of evasion to position taxable income with the fifty-dollar bracket and regresses dummies for fungible items, age, and marital status together with adjusted gross income and marginal tax rate. He finds that the tendency to evade taxes is associated with higher marginal tax rates, the presence of fungible items, being less than 65 year of age and being married. If income is added to the list of regressors, the coefficient of the marginal tax rate switches sign from positive to negative. However, none of the coefficient estimates are precise. His estimates, thus, fail to distinguish the tax effect from the income effect. Besides, his comparison of models' coefficients with and without income variable is inconsistent with basic econometrics in that if income is a valid regressor then estimates in column 1 of his Table 3 biased and estimates in column 2 are correct. But if income is not relevant, then coefficient estimates of column 2 are not efficient.

Poterba (1987) uses one observation from each of the 1965, 1969, 1973, 1976, 1979, 1982 TCMP data sets. He discusses how the marginal tax rate affects capital gains tax evasion. He finds that a decrease in the marginal income tax rate discourages evasion, even though the coefficient estimate is not precise. However, with six observations and three parameters to be estimated, any inference is highly unreliable.

Beron, Tauchen, and Witte (1988) use five of the seven groups of the 1969 TCMP data aggregated at the three-digit zip code level. Using 2SLS, they estimate three separate equations for reported AGI, reported tax liability, and the log odds of an audit. They find that the deterrence effect of audit is small and it is more effective for detecting income subtractions than the report of income itself. One notable omission of their work is the marginal tax rate. Further, their use of 2SLS to deal with the simultaneity problem is not convincing; more could be gained by utilizing the system GMM estimation method. Tauchen, Beron, and Witte (1989) apply the same technique on the 1979 TCMP individual data and find similar results. Additionally, they find that the "ripple effect" of audit is many times higher than the revenue yields from the direct audit.

Dubin and Wilde (1988) divide the 1969 TCMP data set into seven audit classes. They discuss whether the IRS audit rate is endogenous and, if so, how it affects evasion among these audit classes. They use the IRS budget as an instrument variable to estimate taxpayers' compliance behavior equation. They find that the audit rate is an endogenous variable in four of the seven audit classes. Moreover, the effect of the IRS auditing strategies outweighs the deterrent effect in three of the four cases in which audit rates are endogenous.

Alm, Bahl, and Murray (1990) develop a model to discuss how public policies affect taxpayers' evasion and avoidance behavior. Policy variables include marginal tax rates, payroll tax contributions and benefits, probability of auditing, and penalty rules. They use 1983 Jamaican micro level data to estimate the share equations for the tax bases that rise with higher benefits for payroll tax collections and falls with higher marginal tax rates. Further, tax bases also fall with more severe penalties and a higher audit probability as individuals substitute avoidance for evasion.

Alm, Bahl, and Murray (1991) use 1983 Jamaican micro level data to discuss how the self-employed people evade income tax in response to policy changes. They find that a lower marginal income tax rate deters evasion. Moreover, fraction of income declared by the self-employed people increases less than that of their actual income.

Feinstein (1991) adopts a "fractional detection model," which captures the fact that IRS examiners can detect some (but perhaps not all) of income tax evasion. He uses a small portion of the individual-level data from the 1982 and 1985 TCMP data sets. He discusses how income, marginal tax rate, and various socioeconomic characteristics of the filers affect tax evasion and finds that the both the likelihood and magnitude of evasion increases with taxpayers' income and the marginal tax rate when he uses the two TCMP data sets separately.

Kamdar (1995) examines the importance of information reporting on the tax compliance of individuals using data from the 1971 TCMP. He estimates separate compliance equations for incomes subject to differential reporting requirements and find that third-party information reporting is an important deterrent to noncompliance.

Martinez-Vazquez and Rider (2005) examine the theoretical and empirical implications of accounting for multiple modes of tax evasion we use the 1985 TCMP to estimate an empirical model with two modes of evasion and find that increased enforcement effort has a positive effect on compliance in the targeted mode, a negative effect in the untargeted mode, and a positive overall effect on tax compliance.

### Time-series Studies

Dubin, Graetz, and Wilde (1987) use 2SLS method to estimate the determinants of tax evasion based the state-level data in the *Annual Report* 1977-85. They use the IRS budget per return and the percentage of income tax returns filed to total tax returns filed as instruments for the audit rate. Further, they use "percentage of individual income tax returns per audit" (i.e., "the amount of penalty" over the "total collections from individual income tax") as a measure of taxpayers' noncompliance. Their explanatory variables are as follows: the lagged audit rate, the lagged socioeconomic variables such as the unemployment rate, the percentage of adult population with a high school education, per capita income, per capita income squared, the percentage of population over 45, the percentage of population employed in the manufacturing, and time trend. They find that the following variables positively affect taxpayers' noncompliance (at the 1 percent significance level): the percentage of adult population with a high school education, per capita income, time trend, and the predicted lagged audit rate. The last one implies that the IRS audit rate is endogenously determined. In contrast, "the unemployment rate" and the "per capita income squared" negatively affect "the percentage return per audit" at the 1 percent significance level. Finally, the actual lagged audit rate negatively affects (but only at the 20 percent significance level) "the percentage return per audit." This implies that an increase in the audit rate deters evasion. However, the deterrent effect is dominated by the effect of the IRS effort on auditing.

There are some notable problems in Dubin, Graetz, and Wilde (1987). First, they use eight years' state-level data, which is perhaps not enough to capture the interdependent actions of the taxpayers and the IRS. Second, they should not include employment taxes in calculating total collections of individual income tax. They made this mistake because the *Annual Report* does not separate the employment tax from the individual income tax at the state-level. Third, more importantly, their dependent variable, "the percentage return per audit" is inappropriate to reflect how much a taxpayer evades. It should be replaced by "penalty per return examined", which conforms to the theoretical model of Allingham-Sandmo as amended by Yitzhaki. Finally, they neglected the information on the sources of individual income presented in the *Individual Income Tax Returns* (later in the *Statistics of Income Bulletin*), which is more relevant than some of their states' characteristics variables. Finally, their model is incomplete in that it does not consider the effect of marginal tax rate on evasion.

Dubin, Graetz, and Wilde (1990) also use 2SLS method to discuss the same issue raised in Dubin, Graetz, and Wilde (1987): the overall role of audit rates in the Federal revenue collections process. Their data sources are the *Annual Report* 1977-1986 and the *Individual Income Tax Returns* (later in the *Statistics of Income Bulletin*). They consider that audits may have "spillover" effects. The "spillover" effects mean that people report more taxes when they believe audits are more likely, no matter whether they are actually audited or not. They use "the IRS budget per return" and "number of information returns other than W-2 form filed divided by total number of returns" as instruments for the audit rate equation. They use "the reported taxes per returns filed" in the *Individual Income Tax Returns* as a measure of taxpayers' compliance. The explanatory variables include: the current audit rate, socioeconomic variables and a time dummy. The socioeconomic variables can be decomposed into three types, the first of which is only related to the tax base. It includes: the percentage of population over 65, total number of households on welfare divided by the total number of households, and total number of households divided by the number of population. The second one is only related to taxpayers' compliance behavior. It includes: the percentage of adult population with a school education, the percentage of the workforce employed in manufacturing, total number of farms divided by total population, the percentage of labor force in a service industry. The third one is related to both the tax base and taxpayers' compliance behavior. It includes: per capita income, the unemployment rate, and the average state income tax rate. Their findings are as follows: per capita income positively affects the reported taxes per return filed (at the 1 percent significance level). In contrast, the following variables negatively affect the reported taxes per return filed (at the 10 percent significance level): total number of farms divided by total population, the percentage of the work force employed in service industry, the state income tax rate, and the unemployment rate. Finally, the audit rate is an endogenous variable and is positively related to reported taxes per return filed.

Dubin, Graetz, and Wilde (1990) also suffer from some notable shortcomings. First, they still neglect information on the sources of taxable income in the *Individual Income Tax Returns* (later in the *Statistics of Income Bulletin*). Second, they may bias their results when they include “per capita income as an explanatory variable. We never observe the “actual income” in the aggregate-level data. When we use “per capita income” as a proxy for “actual income”, the effects of other variables on “the reported taxes per return filed” may be masked. This is because “per capita income” and “the reported taxes per return filed” are highly positively correlated but may not be causally linked. Finally, it is impossible to separate the tax base effect from the compliance effect on “the reported taxes per return filed” for the third type of socioeconomic variables.

Plumley (1996) extends the analysis in Dubin, Graetz, and Wilde (1990) using the state-level data from 1982 to 1991. He modifies some of the compliance equations by incorporating the income and offsets equations and the ratio of tax returns filing to expected filings. He is the first to show that criminal investigation enforcement activities are significant and positively related to compliance.

Dubin (2004) uses the state-level data between 1988 and 2001 to discuss the effects of criminal investigation enforcement activities on taxpayers’ compliance behavior. This is perhaps the only study that tries to explore one of the issues we will be investigating later in our empirical analysis. Using the dynamic panel estimation method, he concludes that criminal investigation activities have a measurable and significant effect on voluntary tax compliance. While incarceration and probation have the most significant effect on compliance, sentenced cases and media attention do not seem to have any significant influence.

The past studies, even without the above two crucial issues, present conflicting evidence as to the relative importance of sanctions, audit rates, and marginal tax rates on tax compliance. In general, sanctions are negatively related to evasion in the theoretical models of tax compliance (Allingham and Sandmo 1972), but are often statistically insignificant in empirical studies (Witte and Woodbury 1985). Audit rates are significant for some, but not all audit classes (Dubin and Wilde 1988), and the relationship between the marginal tax rates and the level of compliance is still a polemic issue in the empirical studies (Yitzhaki 1974; Clotfelter 1983; Slemrod 1985; Dubin, Graetz, and Wilde 1987, 1990). Besides, none of the studies reviewed above addresses the twin issues raised in the Introduction. Because empirical models without addressing the issues of spatiality and persistence will be misspecified, the ignorance of the above issues may well render their results biased and inconsistent and the policy prescription based on any of those studies will be misleading.

### *A MODIFIED MODEL OF INDIVIDUAL INCOME TAX COMPLIANCE*

Before we make any effort to incorporate the interdependence and the persistence a caveat is in order. That is, no single theoretical model can address all three different explanations outlined in the Introduction simultaneously. Therefore, our approach is both eclectic and demonstrative. Let us start with the original Allingham-Sandmo-Yitzhaki model of income tax evasion. Consider an individual *i* whose true income at time *t,* $y_{it}$, is known only to him but not to the tax authority. Tax is levied at constant rate, $\tau_{it}$, on the declared income. However, with some probability $p_{it}$ taxpayer *i* will be subject to investigation by the tax authority and if found under reported will be subject to a penalty rate, $\theta_{it}$, on the evaded income, $E_{it}$. Note that $\theta_{it}$ is higher than $\tau_{it}$. As discussed earlier, taxpayer *i* in evading the amount $E_{it}$ from the tax authority takes into consideration the amount successfully evaded in the previous period, $E_{it-1}$, by himself as well as the amount, $E_{jt}$, contemporaneously evaded by another taxpayer, *j*. Suppose the IRS audit rule is as follows: whether or not individual taxpayer *i* will be audited this year depends on how much that individual evaded last year as well as how much individual *j* evades this year.

Thus, individual *i*'s after-tax income is state dependent and given as

$$W_1 = y_{it} - \tau_{it}(y_{it} - E_{it}) \quad \text{if she is not caught cheating, and} \tag{3.1}$$

$$W_2 = y_{it} - \tau_{it} y_{it} - \theta_{it}\tau_{it}E_{it} \quad \text{if she is caught cheating} \tag{3.2}$$

Suppose that the individual maximizes Neumann-Morgenstern utility. Taxpayer, *i*, will then choose $E_{it}$ so as to maximize

$$E(U_{it}) = (1 - p_{it}(E_{it-1}E_{jt}))U_{it}(y_{it} - \tau_{it}(y_{it} - E_{it})) + p_{it}(E_{it-1}E_{jt})U_{it}(y_{it} - \tau_{it}y_{it} - \theta_{it}\tau_{it}E_{it}) \tag{3.3}$$

where E is the expectation operator, and $p_{it}(E_{it-1}E_{jt})$ implies that evasion of income by individual *i* at time *t* is conditional on his evasion in the previous period together with the contemporaneous evasion by another individual *j*.

Now if conditioning of $p_{it}$ on $E_{it-1}$, $E_{jt}$ is ignored, the familiar Allingham-Sandmo-Yitzhaki first order condition can be derived as

$$(1 - p_{it}(.))U'_{it}(W_1) = p_{it}(.)\theta_{it}U'_{it}(W_2) \tag{3.4}$$

where single prime on shows the first partial derivative of $U_{it}$ with respect to $E_{it}$, $W_1$ and $W_2$ are after-tax-income in the two states of nature as defined above. In this case all of the basic results of the Allingham-Sandmo-Yitzhaki model follow. However, if the probability of being audited is conditional on $E_{it-1}$, $E_{jt}$, it greatly complicates the analysis of the optimal choice of $E_{it}$, since both $E_{it-1}$, $E_{jt}$ are now arguments on both sides of the first order condition (3.4). The influence of $E_{it-1}$ or $E_{jt}$ on $E_{it}$ can be obtained by totally differentiating (3.4) treating all other parameters constant for the sake of brevity:

$$\frac{dE_{it}}{dE_{jt}} = \frac{U_{it}^{'}\left(W_1\right) + \theta_{it} U_{it}^{'}\left(W_2\right)}{\left(1 - p_{it}\left(.\right)\right)U_{it}^{''}\left(W_1\right) + p_{it}\left(.\right)\theta_{it}^{2} U_{it}^{''}\left(W_2\right)} \frac{dp_{it}\left(.\right)}{dE_{jt}} \tag{3.5}$$

$$\frac{dE_{it}}{dE_{it-1}} = \frac{U_{it}^{'}\left(W_1\right) + \theta_{it} U_{it}^{'}\left(W_2\right)}{\left(1 - p_{it}\left(.\right)\right)U_{it}^{''}\left(W_1\right) + p_{it}\left(.\right)\theta_{it}^{2} U_{it}^{''}\left(W_2\right)} \frac{dp_{it}\left(.\right)}{dE_{it-1}} \tag{3.6}$$

where the double prime on $U_{it}$ shows the second partial derivatives with respect to $E_{it}$. From the second order condition characterizing the optimal choice of $E_{it}$, the first term in the denominator of the right hand sides of (3.5) and (3.6) must be negative. Also, the first term in the numerator of the right hand sides of (3.5) and (3.6) must be positive. Hence the impact of a change in $E_{jt}$ or $E_{it-1}$ on the level of $E_{it}$ depends on the sign of the second term in each of the cases. Insofar as the sign of $\frac{dp_{it}\left(.\right)}{dE_{jt}}$ or $\frac{dp_{it}\left(.\right)}{dE_{it-1}}$ cannot be determined *a priori*, the sign of the impact of either $E_{jt}$ or $E_{it-1}$ or both must be determined empirically.

The solution of the taxpayer's utility maximization results in the following general functional form:

$$E_{it} = f\left(E_{jt}, E_{it-1}, X_{it}\right) \tag{3.7}$$

where $X_{it}$ is set of characteristics that influences the evasion behavior of the individual, and $E_{it}$, $E_{it-1}$, and $E_{jt}$ are as defined above. This brings us to the issue of testing dynamic and/or spatial effects. This is done after we describe the data used in the model and some methodological issues.

### *DATA AND METHODOLOGICAL ISSUES*

The empirical analysis of this essay is based primarily on the state, district, and regional level data collected from the Annual *Report of the Commissioner of the Internal Revenue* as well as the *IRS Data Book* for 1979-1997. Appendix E presents the list of the IRS districts and regions. These publications contain state-level information on the number of individual income tax returns filed, the number of returns examined and the amounts of additional taxes and penalties recommended by the IRS offices at the district and regional levels. The IRS also records data on these variables against the *service center(s)* of the IRS regions. For data reported against service centers, treatments are given as follows: the number of returns examined that are recorded against the IRS service center(s) in a region are prorated among the constituting states in proportion to the number of returns filed; the amount of additional taxes and penalties recorded against the service center(s) in a region are prorated among the constituting states in proportion to the number of returns examined.

The recent IRS Reform Act reorganized the entire district system and required many district offices to be responsible for the tax returns filed by multiple states. As a result, most of the district-level statistics in 1997 included services provided to multiple states. Since only state-level data are used in the analysis, we take the 1996 allocations of examinations, additional taxes for each state among all states in the newly defined districts and extrapolate the annual figures for 1997 based on the 1996 proportions. For states with multiple districts, the district-level data are aggregated to the state level. Further, the data on Adjusted Gross Income (AGI), the number of returns with wages and salaries, with itemized deductions, and the total number of exemptions are obtained from the *Statistics of Income Bulletin* of the IRS.

These are augmented by data on 'retail trade employment', 'proprietors' employment', 'service sector employment', and 'total employment' from the *Bureau of Economic Analysis*. The data on total, non-white, and population over 65 years of age, and the Gini coefficient were obtained from the *Statistical Abstracts of the United States* from the *Bureau of the Census*. Finally, the unemployment data are obtained from the *Handbook of U.S. Labor Statistics*. Along with the variables dictated by the tax evasion model, these additional variables were used in many of the previous studies.

From the IRS data, additional taxes and penalties recommended were divided by the number of individual income tax returns filed in a state to get a proxy for the individual income tax evasion ($E_{it}$). We, however, know that additional taxes and penalties recommended differ from the additional taxes and penalties assessed due to subsequent bilateral and legal settlement between the IRS and the individual corporation. But, data on the additional taxes and penalties assessed are not available from the IRS published documents.

Even though the probability of detection is related to a myriad of factors, audit rates have traditionally been regarded as the most important. We thus use the number of individual income tax returns examined divided by the number of individual income tax returns filed times 100 to get a proxy for individual income tax audit rate. Audit rates have been the focus of much attention in the tax evasion literature, and the IRS believes that audits are one of the most effective deterrent tools. It may, however, be noted that the central focus on audit is changing in the IRS. Since the 1980s the IRS has been intensifying the use of other deterrence and enforcement tools to supplement the declining role of audit. Unfortunately, data on these tools by state and year are not publicly available.

We form four new variables from data available in the *Statistics of Income Bulletin* and the *IRS Data Book*: (i) per return adjusted gross income: adjusted gross income divided by the number of individual income tax returns filed; (ii) percent of returns filed with wages and salaries: total number of returns filed with wages and salaries divided by total number of returns filed; (iii) percent of returns filed with itemized deductions: total number of returns filed with itemized deductions divided by total number of returns filed, and (iv) per return exemptions: total number of exemptions claimed divided by total number of returns filed.

From the BEA employment data, we created three series: (i) percent of proprietors' employment: total number of proprietors divided by total number of people employed; (ii) percent of retail trade employment: total number of people employed in retail trade divided by total number of people employed; and (iii) percent of service sector employment: total number of people employed in the service sector divided by total number of people employed.

Non-compliance in our model is not independent of the marginal tax rates. In order to test the relationship between tax rates and the tax evasion of the individual we need a measure of individual tax rate. We resolve this issue by using the dollar weighted marginal tax rates available at from the National Bureau of Economic Analysis.[2] These rates are calculated by the NBER TAXSIM model from micro data for a sample of the U.S. taxpayers. The figures are generated by first calculating the tax liability of each eligible return, and then increasing all income types by 1 percent and recalculating the tax liability under the assumption that itemized deductions are constant. The difference in aggregate tax divided by the difference in aggregate income is the marginal tax rate on the average dollar of income.

The rates take account of most features of the tax code including the maximum tax, minimum tax, alternative taxes, partial inclusion of social security, earned income tax credit, phase outs of the standard deduction and lowest bracket rate, etc. Because state of residence for taxpayers with AGI>$200,000 is not given in the data, high income taxpayers are assigned randomly to states in proportion to the number of high income taxpayers listed in the *Statistics of Income* annual volumes of the Internal Revenue Service. Thiss caveat should be borne in mind while interpreting the coefficient of the variable.

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[2] See at http://www.nber.org/~taxsim/ally/ally.csv for details; Internet, accessed in May, 2006.

Data on the marital status were also obtained from the NBER.[3] Similar to data on marginal tax rates, these data are also are available by state and by year for returns with AGI≤ \$200,000. Returns with AGI > \$200,000 are, therefore, prorated based on the distribution of income in the state in that particular year. This caveat should be borne in mind while interpreting the coefficient of the variable.

## *ECONOMETRIC MODELS*

Following our discussion of the possible presence of the spatial effect and dynamic effect, we posit a general model in which an individual *i*'s income tax evasion ($E_{it}$) depends on individual *j*'s income tax evasion behavior, on that individual's past income tax evasion, and on a set of local socioeconomic variables:

$$E_{it} = \rho \sum_{j \neq i}^{N} \omega_{ij} E_{jt} + \gamma E_{it-1} + \mathrm{x}'_{it}\beta + \eta_i + u_{it}; \qquad i = 1, \ldots, N; j = 1, \ldots, N; \text{ and } t = 1, \ldots, T \qquad (5.1)$$

$$u_{it} = \lambda \sum_{j \neq i}^{N} \omega_{ij} u_{jt} + \varepsilon_{\mathrm{it}}; \qquad i = 1, \ldots, N; j = 1, \ldots, N; \; t = 1, \ldots, T \qquad (5.2)$$

Since we use a cross-section of states over time, subscripts *i* and *t* represent an average individual in the state and time periods, respectively; $\rho$ is a scalar parameter measuring the slope of the reaction function; $\omega_{ij}$ are spatial weights used to compute the effect of the

[3] The author is grateful to Dr. Daniel Feenberg, NBER, for making data on marital status available.

individual income tax evasion of states relevant to state $i$; $\omega_{ij} \neq 0$ if individuals in states $i$ and $j$ interact strategically, and, by convention, $\omega ii$=0; $\mathbf{x}_{it}$ is ($k\times 1$) is a vector of individual $i$'s socioeconomic conditions, $\gamma$ the coefficient of persistence in evasion, and $\beta$ is the corresponding vector of coefficients on the other conditioning variables. The first element of $\mathbf{x}_{it}$ is unity to allow for the intercept. We assume that the parameters $\rho$, $\gamma$ and $\beta$ are constant across time and space. The spatial and the dynamic analyses are special cases of the above general model. We will discuss the estimation technique in one type of analysis assuming that the other effect is absent for the sake of brevity. Further, we omit discussion of the features of the model when neither spatial effects through the dependent variable or the error term nor the dynamic effects are present, for it then becomes a typical panel model whose features and estimation techniques are outlined in any standard text on panel econometrics. However, it may be noted that even simple fixed or random effects model will produce biased results in this case since the audit rate is endogenous.

## Estimation of Panel Model with Endogenous Audit Rate

Since the audit is endogenous, we need to find proper instruments to address it. Some of the past studies used the IRS cost per return as an instrument for this purpose. However, for us this is a bad instrument in that both the costs per return and the audit rate are jointly determined. Besides, the cost per return directly affects the level of tax evasion thus violating the fundamental assumption of being a valid instrument. In order to substantiate our claim, we checked the validity of the instrument for audit rate. When cost per return alone is used as an instrument, it makes the audit coefficient statistically insignificant; when used with other instruments the test of over-identifying restriction is rejected. Thus, one needs to look for instruments that have no direct effect on the level of tax evasion but influence the audit selection. These instruments are the political affiliation of the President of the United States, the composition of both chambers of the U.S. Congress, and the party affiliation of the state Governor. The choice of these instruments is guided by Schulz and Wood (1998), who argue that audit enforcement is by and large governed by the responsive of the IRS to elected officials. Our assumption is that the audit rate of individual income tax would be lower if the party affiliation of the President is Democratic, the Senate and the House each have a Democratic majority and the state governor is Democratic. This negative association can be expected not because the Democrats are lenient to tax evaders but because they are proclaimed to be pro general public as opposed to pro corporation and rich. We checked the validity of instruments using Sargan (1958) test of over-identifying restrictions and found that our instruments as valid. This will be detailed while discussing the fixed and random effects instrumental variable estimates.

Estimation of Spatial Panel Dependence with Spatial Error Correlation

Note that the usual panel model, spatial panel model and dynamic panel model are all special cases of the above general model. For, instance, let us assume that the dynamic effect is absent. Then forming vectors of observations in *t*, the model becomes

$$E_t = \rho W_t E_t + \beta X_t + I_N \eta + u_t; \qquad t = 1, \ldots, T \tag{5.3}$$

where $\boldsymbol{E_t}$=$(E_{1t},\ldots,E_{Nt})'$ is the ($N$×1) vector of individual income tax evasion for the cross-section of $N$ states at time $t$, $W_t$ an ($N$×$N$) matrix of spatial weights, and $\boldsymbol{X_t}$ is an $N$×$K$ matrix with rows given by the set of vectors $x'_{it}$; η is a ($N$×1) vector of the unobserved heterogeneity, and $\boldsymbol{u_t}$ is the corresponding ($N$×1) error term vector.

Under this structure, tax evasion is determined endogenously in equilibrium because the spatial lag term, $W_tE_t$, is correlated with the error term, $\mathbf{u_t}$, and ordinary least squares (OLS) yields inconsistent estimates of the parameters. It is therefore commonly estimated by using maximum likelihood techniques (Anselin 1988; Anselin and Hudak 1992). Removing the simultaneity in model (5.3), estimation can be carried out with alternative methods under the assumption that errors are *iid*. The reduced form equation is

$$E_t = (I_N - \rho W_t)^{-1}\beta X_t + (I_N - \rho W_t)^{-1}(I_N\eta + u_t) \qquad ; t = 1, \ldots, T \qquad (5.4)$$

where $I_N$ denotes an identity matrix of size $N$. Equation (5.4) can be estimated by maximum likelihood method under normality assumptions. Case, Rosen, and Hines (1993) were, perhaps, the first to use this method in the case of fiscal policy interdependence.

Even in the absence of spatial autocorrelation (ρ=0), the estimation of model (5.4) can lead us to conclude erroneously that there is interaction if the error term itself is subject to spatial autocorrelation, for example, in the form of (5.2) written in matrix form:

$$u_t = \lambda W_t u_t + \varepsilon_t\,; \qquad t = 1, \ldots, T \qquad (5.5)$$

where $\varepsilon_t$ is distributed with mean zero and covariance matrix $\sigma_\varepsilon^2 I_N$. In this case, spatial dependence in the error—for example, resulting from similar geographical conditions — can induce correlation in tax evasion even though individuals may have no interactions. Uncorrected spatial correlation in the error term would not affect the consistency of the estimated parameter β, but it would reduce its efficiency. If there is strategic interaction ($\rho \neq 0$), ignoring the spatial lag term $W_t E_t$ in the estimation is more serious, since it yields inconsistent estimates of β. It is important therefore to test for both kinds of spatial dependence (in the dependent variable and in the error term).

Maximum likelihood estimation is complicated when we account for spatial correlation in the error term by possible identification problems (Anselin 1988). We follow the instrumental variables approach because it avoids this issue, it is computationally easier to implement, and it does not require distributional assumptions on the error term ε. W assume that the parameters $(\beta', \rho)'$ and $(\sigma_\varepsilon^2, \lambda)$ are time-invariant. This allows us to estimate the model pooling the panel of observations, stacking them over the time index as:

$$\mathrm{E} = \rho \mathrm{WE} + \beta \mathrm{X} + I_N \otimes i_T \eta + \mathrm{u} \; ; \qquad t = 1, \ldots, T \tag{5.6}$$

with

$$\mathrm{u} = \lambda \mathrm{Wu} + \varepsilon \; ; \qquad t = 1, \ldots, T \tag{5.7}$$

where $E = (E_1',\ldots, E_T')'$ and $\varepsilon = (\varepsilon_1',\ldots, \varepsilon_T')'$ are $(NT \times 1)$ vectors, with $T$ equal to the total number of periods. $\boldsymbol{X}$ is the $(NT \times K)$ matrix of stacked exogenous variables, and $\eta$ is a $(NT\times 1)$ vector of the unobserved heterogeneity. W is an $(NT \times NT)$ block-diagonal matrix of spatial weights, with $T$ copies of $W$ along the diagonal, in the case of time-invariant weights, and with matrices $(W_1,\ldots, W_T)$ in the case of time-variant weights. $I_N$ is an $(N\times N)$ identity matrix and $i_T$ is a $(T\times 1)$ vector of unity. Finally, we assume that the covariance matrix of $\varepsilon$ is given by $\sigma_\varepsilon^2 I_{NT}$.

When the true model is given by the (5.6) and (5.7), an application of Kelejian and Prucha's(1998) generalized two stage least squares (G2SLS) procedures performed in three steps is outlined as follows. In the first step the (demeaned) regression model in (5.6) is estimated by 2SLS procedure using the instruments $H \subset \left[X, WX, W^2X\right]$ that are linearly independent columns. In the second step the spatial error correlation parameter $\lambda$ is estimated in terms of the residuals obtained via the first step and the generalized moments procedure suggested by Kelejian and Prucha (1999). Finally, in the third step, the original (demeaned) regression model in (5.6) is re-estimated by 2SLS after transforming it via a Cochrane-Orcutt type transformation to account for the spatial error correlation.

## Estimation of Dynamic Panel Model

Instead if we assume that the spatial dependence and the spatial error effects are absent, the model involving (5.1) and (5.2) becomes a dynamic panel model. In the context of a persistent dependent variable and endogenous regressors, neither fixed effects nor the random effects nor even the spatial panel estimators outlined above gives consistent estimates. As argued earlier, one of the right-hand-side variables in (5.1) is correlated with the random error term. This contention is confirmed by Hausman (1978) test in the next section. This makes both the fixed effects and the random effects estimates inconsistent. For this reason, in addition to the usual panel and spatial panel estimators, we estimate the instrumental variable estimation following Anderson and Hsiao (1982), Arellano and Bond (1991), and Blundell and Bond (1998) to obtain consistent estimates of tax evasion equation in the presence of dynamics and endogenous explanatory variables. Essentially, the approach involves writing the model in (5.1) and (5.2) without the first terms, and first difference (5.1) to get rid of the individual heterogeneity as:

$$\mathrm{E}_{\mathrm{it}} - \mathrm{E}_{\mathrm{it-1}} = \alpha(\mathrm{E}_{\mathrm{it-1}} - E_{\mathrm{it-2}}) + (\mathrm{x}_{\mathrm{it-1}} - \mathrm{x}_{\mathrm{it-2}})' \beta + (\mathrm{u}_{\mathrm{it}} - \mathrm{u}_{\mathrm{it-1}}) \tag{5.8}$$

By construction, the differenced lag of the tax evasion equation $(E_{it-1} - E_{it-2})$ in the above equation is endogenous. Further, as noted earlier, $\mathbf{x}_{\mathrm{it}}$ contains endogenous variables, such as the audit rates. Therefore, we need instruments to consistently estimate the above equation. The differenced right-hand-side variables are instrumented with appropriately lagged levels. On the assumption that the error term in (5.2) are serially uncorrelated, i.e., $E(u_{it}u_{is}) = 0$, the following moment condition yield the appropriate instruments for the differenced lagged dependent variable and the endogenous explanatory variables.

$$E\left(E_{it-s}\Delta u_{it}\right)=0 \text{ for } t = 3,\ \ldots,\ T \text{ and } s \geq 2 \tag{5.9}$$

$$E\left(\mathbf{x}_{it-s}\Delta u_{it}\right)=0 \text{ for } t = 3,\ \ldots,\ T \text{ and } s \geq 2 \tag{5.10}$$

When the moment conditions (5.9) and (5.10) hold, one can use the lagged levels variables as instruments for the first differenced variables. However, when the lagged levels are weakly correlated with subsequent first differences, the Arellano and Bond (1991) differenced GMM estimator suffers from small sample bias (Blundell and Bond 1998). To deal with the potential problem with the differenced GMM estimates, Arellano and Bover (1995) proposed an estimator that makes use of additional information in levels. This relatively new estimator is referred to as the system GMM estimator. This approach combines two sets of equations—one set in the first differences and another in levels—into a system of equations. This introduces additional T-2 moment restrictions given by:

$$E\left[\left(\eta_i + u_{it}\right)\Delta E_{it-1}\right]=0 \tag{5.11}$$

$$E\left[\left(\eta_i + u_{it}\right)\Delta \mathbf{x}_{it-1}\right]=0 \tag{5.12}$$

The system GMM estimator uses the moment conditions in equations (5.9) through (5.12) to consistently estimate the parameters of interest in equation (5.1) under the assumption made at the beginning of this sub-section.

It should be noted that valid instruments should be correlated with the included endogenous explanatory variable(s) and, at the same time, orthogonal to the error term. To ensure the validity of the instruments, we conduct the Sargan (1958) test of over identifying restrictions to jointly test the appropriateness of the instruments.[4] The null for the test is that the instruments are valid in that they are not correlated with the errors. Under the null, the test statistic is distributed as $\chi^2_{(L-k)}$, where $L$ is the number of instruments and $k$ is the number of parameters in the model.

Further, as noted earlier, the consistency of the GMM estimation depends on whether errors in the levels equation are white noise. If the errors are serially correlated the GMM will lose its consistency. We thus, test for the second order autocorrelation in the differenced equation. The test statistic developed by Arellano and Bond (1991) is detailed in their appendix along with its distributional characteristics. By construction, we expect first order serial correlation in first differenced equation but not second or higher order autocorrelation.

## *EMPIRICAL ANALYSIS*

This section covers the empirical results obtained from estimating the various models developed in the previous section. After the descriptive statistics of the data, we start off with the most restricted estimates—the simple panel IV estimates—as the benchmark. We then relax the assumption of spatiality and persistence in sequence to see what happens to the estimates. We find that the simple estimates are biased since these estimates ignore the twin issues of spatiality and persistence. Along the same line of

[4] Also see Hansen (1982) for details.

argument, the spatial and dynamic panel estimates in isolation are also biased since either of the estimates ignores the other effects. This bias of the dynamic panel estimate is confirmed when we checked the Moran I and LM statistics of the error term.

## Descriptive Statistics

The definition and the descriptive statistics of the variables used in the study are presented in Table 1. The average real per return evasion of individual income taxes is about $4 thousand over the sample period. The mean individual income tax audit rate is 1.4 percent. The average marginal income tax rate (state and federal combined) is 30 percent. However, it varies between 22 and 43 percent across states and years. The mean real adjusted gross income is about $30 thousand. About 85 percent of the returns are filed by earners of wages and salaries which contain information of income and tax withheld by the third-party, the employers.

About one-third of the returns filed present itemized deductions and more than two exemptions for dependants are claimed. However, there are wide variations in both filing with itemized deductions and claimants of dependants across years and states. Around half of the returns are filed jointly by married couples.

The IRS spends around $28 for running the administration of which enforcement is a major component. Of the total per return costs, it incurs $22 as compensation to the personnel at the national office and the field staff in the IRS districts and regions. Of the rest, a sizeable amount is spent for legal pursuit of the cases against the evaders. It is puzzling why the IRS does not expand its audit coverage when one compares the costs and returns of the tax base enforcement.

Table 1. Descriptive Statistics of the Variables Used: 1979-97

| Variables | Mean | Std. | Min | Max |
|---|---|---|---|---|
| Per Return Evasion ($) | 3774.152 | 1905.116 | 1045.989 | 20858.460 |
| Audit Rate (%) | 1.418 | 0.633 | 0.407 | 4.827 |
| State & Federal Combined Tax Rate (%) | 29.897 | 3.323 | 22.440 | 42.910 |
| Real AGI ($ 000) | 29.532 | 4.068 | 20.150 | 49.737 |
| Returns with Wages & Salaries (%) | 85.296 | 3.673 | 61.017 | 94.131 |
| Returns with Itemized Deduction (%) | 30.580 | 7.294 | 13.432 | 50.292 |
| Per Return Exemption (no.) | 2.285 | 0.188 | 1.505 | 2.953 |
| Joint Returns (%) | 46.237 | 5.554 | 26.980 | 65.220 |
| Proprietors in Total Employment (%) | 16.786 | 3.688 | 9.314 | 27.179 |
| Retail Trade in Total Employment (%) | 16.524 | 1.164 | 11.646 | 19.777 |
| Services in Total Employment (%) | 25.472 | 4.735 | 15.804 | 43.670 |
| Elderly Population (%) | 12.370 | 2.414 | 2.740 | 21.186 |
| Non-white Population (%) | 18.738 | 13.594 | 0.683 | 71.441 |
| Gini Coefficient | 0.377 | 0.026 | 0.330 | 0.446 |
| Unemployment Rate (%) | 6.425 | 2.129 | 2.225 | 18.017 |
| Party Affiliation of President* | 0.368 | 0.483 | 0.000 | 1.000 |
| Senate Democrat Ratio | 51.632 | 4.936 | 45.000 | 58.000 |
| House Democrat Ratio | 57.871 | 5.112 | 46.897 | 63.678 |
| Party Affiliation of State Governor* | 0.566 | 0.492 | 0.000 | 1.000 |
| Real Personnel Costs Per Return ($) | 21.90 | 5.76 | 12.76 | 47.89 |
| Real Capital Costs per Return ($) | 0.75 | 0.60 | 0.08 | 6.23 |
| Real Other Costs Per Return ($) | 6.31 | 2.13 | 1.72 | 19.20 |

Note: *Party affiliation of the President = 1 if he is Democrat and 0 otherwise. Party affiliation of the State Governor is similarly defined.

It must be remembered that compliances by the individuals are influenced by many other factors not included in the above list. Appendix F gives correlation matrix between the variables involved. There is no high correlation between the explanatory variables and their partial correlation with dependent variable implies that the variables explain part of the variation in the income tax evasion.

There is no definite time trend in the data except per return evasion and the audit rate. While per return shows an upward trend, the audit rate shows the downward trend. The mean per return evasion was less than $3000 between 1979 and 1985 but jumped to around $4000 after 1986 and stayed around at that level. Apparently this is a contradictory finding in that the TRA was introduced in 1986 and was in effect during this period.

In contrast, the audit rate was around 1.7 percent between 1979 and 1885. But it dropped to barely above 1 percent after the TRA was introduced. When plummeting audit rate is juxtaposed with soaring tax evasion, it resolves the apparent ineffectiveness of the TRA. That is, the IRS was not given enough resources to bring the evaders to book. The resource constraint for tax base enforcement was equally a common phenomenon when the Republican and the Democratic presidents held power.

## Estimation Results

Table 2 reports the results of the simple fixed and random effects models with treatment given to the endogeneity of audit rate as discussed before but ignoring both the persistence and the spatial effects. Even though these are intermediate results, we will make some passing remarks and compare them to the findings of the previous studies along this line. Note that these fixed effects results assume that all effects are in long-run equilibrium. However, even if we ignore the spatial effects discussed later, taxpayers are assumed to change their behavior and modify their reported taxes due based on their past experience. We conducted test for individual heterogeneity by performing an F-test.[5] The null is that there is no individual heterogeneity. The test statistic reported at the bottom of Table 2 rejects the null at 1 percent significant level implying evidence of individual effects in the data. We also conducted the Hausman (1978) test in order to decide which set of results are valid. The estimated test statistic reported at the bottom of Table 2 implies the explanatory variables are correlated with the individual heterogeneity and hence the fixed effects estimation is the appropriate procedure.

Note that most of the explanatory variables included in model affect the tax evasion at the 10 percent level with expected signs. Based on these results one arrives at conclusions similar to Jou (1992) but not Dubin, Graetz, and Wilde ((1990) because these latter authors used different dependent variables in their analyses. However, these results albeit corrected for endogeneity of the audit rate are still biased because both the dynamic and spatial effects discussed earlier are ignored.

Since we have only one endogenous variable and we used three instruments for it, the model is over-identified. We thus conducted the test of over-identifying restrictions following Sargan (1958) and Hansen (1982) that are used in IV estimation. The test statistic reported at the bottom of Table 2 shows that the instruments are valid. However, the strength of these instruments should be judged in the light of conditions outlined in Bound, Jaeger, and Baker (1995). The main condition is that use of weak instruments can lead to inconsistent IV estimates. Further, these biases are in the same direction as the OLS estimates in finite samples.

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[5] This test procedure is detailed in Baltagi (2001).

Table 2. Fixed and Random Effects IV Estimates

| Variables | FE Estimates | RE Estimates |
|---|---|---|
| Audit Rate | -1,692.694*** | -1,487.922*** |
| | (359.013) | (398.446) |
| State & Federal Combined Marginal Tax Rate | -118.311*** | -105.664** |
| | (40.264) | (42.572) |
| Adjusted Gross Income | 28.414 | 79.381*** |
| | (33.419) | (27.596) |
| Percent of Returns with Wages and Salaries | -19.313 | -86.811*** |
| | (40.846) | (33.589) |
| Percent of Returns with Itemized Deduction | 59.787*** | 47.932** |
| | (19.936) | (19.161) |
| Number of Exemptions per Return | 2,516.503*** | 2,584.621*** |
| | (936.129) | (744.535) |
| Percent of Joint Returns Filed | 6.103 | -10.448 |
| | (21.854) | (19.603) |
| Percent of Proprietors in Total Employment | 3.226 | 21.012 |
| | (74.968) | (42.873) |
| Percent of Retail Trade in Total Employment | 92.051 | -36.763 |
| | (120.289) | (87.968) |
| Percent of Service Sector in Total Employment | 219.119*** | 147.435*** |
| | (54.112) | (30.570) |
| Percent of Elderly Population | -237.947*** | -186.450*** |
| | (89.122) | (61.228) |
| Percent of Non-white Population | 41.041** | 26.014*** |
| | (16.832) | (9.463) |
| Gini Coefficient | -12,427.455** | -4,893.577 |
| | (5,081.210) | (4,433.670) |
| Unemployment Rate | -90.748** | -33.285 |
| | (42.234) | (36.187) |
| Constant | 2,944.570 | 7,591.937** |
| | (4,498.785) | (3,517.772) |
| Observations | 950 | 950 |
| Individual Effects [F(49,886)] | 6.81***[0.000] | |
| Sargan-Hansen Test [$\chi^2_{(3)}$] | 8.711**[0.0334] | |
| Hausman Test [$\chi^2_{(14)}$] | 41.84*** [0.0001] | |

Note: Standard errors in parentheses. One asterisk implies significant at 10 percent; two asterisks imply significant at 5 percent; and three asterisks imply significant at 1 percent. Figures in the brackets are p-values.

The fixed effects IV estimator is less efficient if the audit rate is in fact not endogenous. Therefore, it is useful to test for endogeneity of the audit rate to see whether the IV estimation method is at all necessary. In order to do this, we do our panel regression analysis in two stages: in the first stage, we regress the audit rate on the party affiliation of the U.S. President, the ratio of Democrats to the Republicans in both the chambers of the U.S. Congress, and the party affiliation of the State Governors, and all other exogenous variables in the original model. We then substitute the predicted value of the audit rate for the actual audit rate and estimate the model by panel estimation method.

Table 3. Reduced Form of Audit Rate

| Variables | Dependent Variable: Audit Rate |
|---|---|
| Party Affiliation of President | -0.104** (0.047) |
| Senate Democratic Ratio | 0.023*** (0.007) |
| House Democratic Ratio | -0.055*** (0.007) |
| Party of Affiliation State Governor | -0.003 (0.031) |

Note: Standard errors in parentheses. One asterisk implies significant at 10 percent; two asterisks imply significant at 5 percent; and three asterisks imply significant at 1 percent. We do not report all other coefficients except those associated with the instruments.

Table 4. Test of Endogeneity of Audit Rate

| Variables | Dependent Variable: Evasion |
|---|---|
| Audit Rate | -1,692.694*** (354.153) |
| Residual of Audit Rate | 611.027* (374.269) |

Note: Standard errors in parentheses. One asterisk implies significant at 10 percent; two asterisks imply significant at 5 percent; and three asterisks imply significant at 1 percent. We do not report all other coefficients except those associated with the actual audit rate and the residual from the first stage regression.

The results of the first stage regression are summarized in Table 3. Audit enforcement becomes lax with a Democratic President and majority Democrats in the House in power. This may imply that the Democrats try to serve the interest of the common people as opposed to the Republicans who are alleged to serve corporate interests. If this claim is valid then sign of coefficient of the ratio of Democrats in the

Senate is "wrong." Finally, as expected, state Governors have little influence over the federal audit strategy in their states. We do not discuss this equation as it is a reduced form equation as opposed to structural form of the audit equation.

Finally, the test in Table 4 supports the hypothesis that audit rate is endogenously determined. This is because the estimate for residual of the audit rate from the first stage regression is different from zero at the 10 percent significance level. However, the marginal significance cast doubt about the strength of these instruments and reminds us of the observations made by Bound, Jaeger, and Baker (1995).

Estimation of the tax evasion model specified in (5.6) and (5.7) requires specification of proximity of states. According to the theory outlined in the earlier section individual *i*'s tax evasion behavior is dependent on *j*'s behavior. Given the per capita state data on evasion some metric of proximity must be used. However, the weight matrix must be exogenous to the regressors. Anselin (2002) pointed out that construction of the distance metric based on any of the regressors makes the model highly non-linear with endogeneity problem that must be instrumented out. As a result of this constraint, use of weight matrix based on income or population is ruled out. At the same time the weight must be meaningful enough to represent dependence in the dependent variable or the error term.

In view of these we selected three alternative metrics: (i) neighbors belong to the same Division in the U.S. Bureau of Census, (ii) neighbors belong to the same Region in the Bureau of Economic Analysis, and (iii) neighbors belong to the same Region in Internal Revenue Service. Note that clustering of states based on the above are not ad hoc; rather they are based on the co-movement of several socioeconomic factors. As such these matrices contain both time variant information such as per capita income differential and time invariant such as their geographic proximity.

As indicated earlier, the efficiency and properties of estimators as well as the properties of other statistics will in general depend upon whether or not a model's disturbance terms are indeed spatially correlated and whether the models have spatially lagged dependent variable. As a result, it is important to estimate spatial correlation both in the dependent variable as well as the error term and check if there are strategic dependence both in the dependent and the error term. Any evidence in either of these evidence will render the panel estimates discussed earlier biased and/or inconsistent.

Table 5 reports the results of the general spatial model. It, however, still ignores the dynamic persistence. Note that the spatial lag coefficient is not precise when weight matrix is defined following BEA Regions or Census Divisions; it is only marginally significant when the weight matrix following the IRS regions. The last results indicate that there is strategic interaction among neighboring states in the determination of individual income tax evasion. Furthermore, this interaction suggests a positively sloped reaction function in tax evasion, as expected. In terms of elasticity, a unit less measure, a 10 percent increase in the individual income tax evasion in the neighboring states results in an increase of 3.3 percent in a state's individual income tax evasion. It may be noted that the coefficient of the spatial error switches sign when the spatial weight is based on the IRS divisions. Most of the explanatory variables are statistically significant. Since the estimate of spatial error correlation is significant, it is evident that there are substantial spatial error effects.

Table 5. General (Kelejian–Prucha) Spatial Estimates

| Variables | Weight Matrix | | |
|---|---|---|---|
| | BEA Regions | Census Divisions | IRS Regions |
| Spatial Dependence (λ) | 0.052 | 0.111 | 0.333*** |
| | (0.124) | (0.102) | (0.125) |
| Audit Rate | -1333.700*** | -1536.540*** | -1017.580*** |
| | (277.869) | (295.184) | (253.037) |
| State & Federal Combined | -61.031** | -46.316 | -85.478*** |
| Marginal Tax Rate | (29.621) | (30.772) | (27.550) |
| Adjusted Gross Income | 133.242*** | 135.441*** | 122.209*** |
| | (21.035) | (21.230) | (20.496) |
| Percent of Returns with | -141.376*** | -133.916*** | -146.127*** |
| Wages and Salaries | (23.082) | (23.539) | (22.572) |
| Percent of Returns with | 14.786 | 7.675 | 22.437* |
| Itemized Deduction | (14.055) | (14.522) | (13.475) |
| Number of Exemptions per Return | 3105.910*** | 3168.220*** | 2896.470*** |
| | (560.109) | (564.977) | (542.387) |
| Percent of Joint Returns Filed | -35.175** | -34.882** | -36.255** |
| | (16.773) | (16.893) | (16.342) |
| Percent of Proprietors | 28.865 | 38.752 | 12.280 |
| In Total Employment | (29.133) | (29.761) | (27.822) |
| Percent of Retail Trades | -105.693* | -122.069** | -78.602 |
| In Total Employment | (60.517) | (61.265) | (58.775) |
| Percent of Service Sector | 122.904*** | 132.239*** | 101.762*** |
| In Total Employment | (21.488) | (22.087) | (19.818) |
| Percent of Elderly Population | -181.809*** | -197.601*** | -142.734*** |
| | (39.991) | (41.305) | (37.569) |
| Percent of Non-white Population | 12.486** | 13.160** | 9.594* |
| | (6.042) | (6.111) | (5.860) |
| Gini Coefficient | 3226.750 | 2781.480 | 4543.490 |
| | (3390.990) | (3423.020) | (3262.750) |
| Unemployment Rate | 57.165* | 66.212** | 49.506* |
| | (30.809) | (31.376) | (30.038) |
| Constant | 8057.850*** | 7202.390** | 7799.750*** |
| | (2863.510) | (2914.450) | (2772.660) |
| Spatial Error (ρ ) | 0.087*** | 0.073*** | -0.009*** |
| | (0.001) | (0.020) | (0.001) |

Note: Standard errors in parentheses. One asterisk implies significant at 10 percent; two asterisks imply significant at 5 percent; and three asterisks imply significant at 1 percent.

Since we have identified that there is strategic interaction (mainly through the error), the effects of the exogenous variables will have an impact on the entire configuration of equilibrium level of individual income tax evasion. This is true even if they take place in only one of the interacting states; through their repercussions on the state's tax evasion they may exert significant influence on the tax evasion behavior of the other states. As will be detailed out later, the so-called Moran I and LM statistics vindicate our claim. It may, however, be noted that the spatial estimation results, albeit elegant, are both biased and inconsistent as the estimation ignores the persistence in tax evasion.

The results of Arellano-Bond Dynamic GMM estimation are presented in Table 6. These results are nice and elegant with most of the coefficients vary in the expected way. However, the estimates show the combined effects of direct and spatial influences. However, if we look at the spatial diagnostic tests, which indicate serious spatial correlations, it is evident the results are potentially misleading due to model misspecification. To that end, we conducted spatial diagnostic tests detailed in Appendix G. The results of Moran I and the LM statistics presented in Table 7 imply that there are indeed spatial effects no matter what form of the contiguity weight matrix is used. Thus the results of the Arellano-Bond GMM estimates should be interpreted with caution.

In order to check the validity of the instruments used in the Arellano-Bond GMM estimates we conducted the test of over-identifying restrictions. The Hansen (1982) statistic implies that the lagged values used as instruments satisfy the moment conditions discussed before. The Arellano-Bond estimators introduce first order serial correlation in the data, but if there are higher order serial correlations in the data then use of this estimator is inappropriate.

Table 6. Arellano-Bond GMM Dynamic Panel Estimates

| Variables | Coefficient Estimates |
|---|---|
| Lagged Per Return Evasion | 0.400*** |
| | (0.023) |
| Audit Rate | -1,215.226*** |
| | (136.653) |
| State & Federal Combined Marginal Tax Rate | -47.026** |
| | (18.282) |
| Adjusted Gross Income | 147.171*** |
| | (29.990) |
| Percent of Returns with Wages and Salaries | -103.292*** |
| | (24.733) |
| Percent of Returns with Itemized Deduction | 74.758*** |
| | (11.192) |
| Number of Exemptions per Return | 2,963.688*** |
| | (638.264) |
| Percent of Joint Returns Filed | -28.945 |
| | (31.068) |
| Percent of Proprietors in Total Employment | 116.383** |
| | (48.130) |
| Percent of Retail Trade in Total Employment | 101.651 |
| | (178.937) |
| Percent of Service Sector in Total Employment | 67.320** |
| | (28.987) |
| Percent of Elderly Population | -99.903** |
| | (48.214) |
| Percent of Non-white Population | 6.035 |
| | (10.390) |
| Gini Coefficient | 10,183.174*** |
| | (2,339.135) |
| Unemployment Rate | 170.505*** |
| | (39.437) |
| Constant | -7,149.161*** |
| | (2,527.553) |
| Observations | 900 |
| Hansen Test[$\chi^2_{(34)}$] | 37.00 [0.332] |
| AR(1) | -3.36 [0.001] |
| AR(2) | -0.89 [0.375] |

Note: Standard errors in parentheses. One asterisk implies significant at 10 percent; two asterisks imply significant at 5 percent; and three asterisks imply significant at 1 percent. Figures in the brackets are p-values.

Table 7. Spatial Diagnostics of Errors

| Weight Matrix | Moran I Statistic | LM Statistic |
|---|---|---|
| BEA Regions | 0.049***(2.630) | 5.944[0.015] |
| Census Divisions | 0.078***(3.785) | 12.894[0.000] |
| IRS Regions | 0.060***(4.871) | 19.829[0.000] |

Note: Figures in the parentheses are z- statistics, those in the brackets are p-values

We also test for first and second order serial correlation following Arellano and Bond (1991). The results reported in Table 6 show evidence of first order serial correlation but absence of any higher order serial correlation.

We find that audit rate constrains the level of evasion irrespective of the type of weight matrix used. These results are similar to previous findings based on cross-section and time-series data (Tauchen, Witte, and Beron 1989; Dubin, Graetz, and Wilde 1990). Our results suggest that raising the audit rate by one percentage point would have decreased the level of evasion by more than one third of a percent from the mean level. This is equal to a reduction of more than $1200 of evasion in the short run. Given the estimated persistence of around one quarter, the reduction is more than $2000 per individual return. These results offer strong support for the deterrence effect of audit. Our results also suggest that the decline in the audit rate over the last three decades may be partly responsible for the decline in voluntary tax compliance.

One of the novelties of our results is the use of combined actual average marginal tax rates for the federal and state governments. Most of the past studies had to use the average state tax rate for this purpose. Contrary to the popular orthodoxy, our results suggest that evasion decreases as the marginal tax rate increases, a result reminiscent of Yitzhaki (1974) who argued that if penalty is proportional to the taxes evaded, then an increase in the marginal tax would unambiguously decrease the level of evasion if there is decreasing absolute risk aversion of income. Our results suggest that an increase in the marginal tax rate by 10 percentage points would decrease the level of evasion by 0.2 percentage point in the short run. Given the level of persistence this implies that a 10 percent increase in the audit rate would decrease evasion by about one third of a percent from the mean level. This also dispels the misconception that the federal income tax rate in the U.S. has reached the maximum of the so-called "Laffer Curve."

The coefficient of adjusted gross income (AGI) is positive and significant. These results are consistent with the hypothesis of increasing absolute risk aversion, commonly accepted as a reasonable assumption in models of individual choice under uncertainty. Our results suggest that a one percent increase in the AGI would lead to more than one percent increase in evasion in the short run and more two percent in the long run.

The percent of returns filed with wages and salaries was associated – as expected – with better compliance. Clotfelter (1983) cited two possible factors for such a strong positive relationship. First, as wages and salaries are held by a third party and report to the IRS, the taxpayers may well be convinced that detection of evasion is more likely with these types of income. In addition, these incomes are relatively simple to report in the tax form. Our results suggest that a one percent increase in the returns filed with wages and salaries would be associated with more than two percent decline in the level of evasion in the short run and about four percent in the long run. In contrast itemized deductions and the number of exemptions claimed are not subject to any verification unless the return is audited. As such these variables were found to be positively associated with the level of evasion. While a one percent increase in the returns with itemized deductions increases evasion by about $74, successful claim of an additional exemption per return increases evasion by about $3000.

There are tendency for the married couple to evade less than the other groups but the coefficient estimates is imprecise. These results corroborates finding of some of the previous studies (Tauchen, Witte, and Beron 1989) but at variance with others (Kamdar

1995). However, in the absence of any solid theoretical foundations, the above results should be interpreted with caution. Moreover, the parameter estimates are also imprecise. Hence any policy prescription based on these estimate would be fraught with danger.

Several past studies and anecdotal observations point to certain groups for tax noncompliance. Some of these include income from sole proprietorship, retail trade, and service sector employments. To examine these propositions, we used the percent share of proprietors, retail trade, and service sector employment to total employment, with an expected positive relationship between evasion and each of them. The results corroborate our claims in that we found strong positive relationship between evasion and percent share of proprietorship and service sector and imprecise but positive with the retail trade.

The elderly population appears to have tendency to evade less than their younger counterparts. Dubin, Graetz, and Wild (1990) found similar relationship in their estimation. One explanation for this relationship could be that most of the elderly live at the subsistence income level and hence do not have any supernumerary income to hide from the tax authorities or even a fewer of them exceed the threshold income level to file a tax return.

Contrary to popular myth, the percent of population that is nonwhite is not significantly related to evasion although there is an imprecise but positive relationship between their fraction in the population and the level of evasion. Since most of the nonwhite population are at lower income stratum and the evasion is used in this essay is an average measurement, this may mask the true relationship, because Tauchen, Witte, and Beron (1989) find a positive relationship between the nonwhite population and the level of evasion for the low income audit classes.

It is usually postulated that social norms break down in societies with high income inequality. Since tax payments is strongly shaped by social norms (Alm, McClelland, and Schulze 1999) we expected a positive relationship between the level of tax evasion and the extent of income inequality measured by the Gini coefficient. Our results corroborate the above proposition.

The positive relationship between the rate of unemployment and the level of evasion suggests that evasion becomes higher during economic recession. These findings are similar to most of the previous studies (Dubin, Graetz, and Wilde 1990; Jou 1992). One of the reasons for this relationship could be that unemployed people work in the underground economy for cash payments and do not report their income. Our results suggest that a three percent reduction in unemployment rate would lead to about one percent decline in the level of evasion in the short run and more than one percent in the long run.

Following Dubin, Graetz, and Wilde (1990), we also make an experiment about the intertemporal effects of declining audit rate. Based on the estimates based on the unfiltered and filtered data reported in Table 6, we calculated the predicted values of the level of evasion that would have declined had the audit rate remained at the 1981 level. By 1996 we estimate that maintaining the audit rate at the 1981 level would have reduced total evasion by $48.5 billion. As noted above the estimates based on these estimates are contaminated due to the ignorance of spatial effects. In view of the positive spatial effects both through the dependent variable and the error term this amount is certain to rise. However, in the absence of any estimates that deals with both the issues simultaneously the precise magnitude cannot be measured.

## *CONCLUSIONS*

The identification of persistence and strategic interaction in models of individual income tax evasion has important implications for the equilibrium configuration of tax evasion. In this essay we attempted to modify the basic Allingham-Sandmo-Yitzhaki model of individual income tax evasion and tested it with state level data of individual income tax evasion. We found evidence of both persistence and strategic interaction among individuals in the cluster of states in the determination of tax evasion behavior. Interaction among states that belongs to a particular group or region based on the BEA, Census or the IRS criteria, appeared to results in dependence, weakly through the level of evasion but strongly through common but unidentified shocks.

We initiated an important econometric problem: how to account for spatial effects in the dynamic panel data model. Since individuals interact and their tax evasion and influence and are influenced by others either directly or through some common unobservable shocks, one expects spatial dependence either directly or indirectly. In addition individuals learn by doing. As no unified approach exists to address this twin issue, we left the two loose ends untied.

Our results are encouraging, however, because once the presence of persistence and strategic interactions is identified, the natural extension is to try to account for specific models of tax evasion to analyze the normative implications of tax evasion behavior we discussed in the Introduction.

In our analysis we only used weight matrices that are time invariant for obvious reason of filtering limitation. It is well established that the results of spatial analysis are very sensitive to the choice of the weight matrix. Thus, in order to check the robustness of our results one needs to use a variety of weight matrices such as the so-called Mahalanobis distance etc.

Further, we did not model explicitly the source of strategic interaction between individuals. This is a limitation of the analysis in the sense that the source of interaction is not identified. Also, the empirical analysis is based on the U.S. data. It would be interesting to see if this conclusions hold for other developed and developing countries.

It would have been ideal to estimate the spatial and the dynamic effects simultaneously. However, such an estimation method is not available at the moment. Hence, it would be interesting to devise an estimator that can tackle these issues and check its robustness using Monte Carlo experiment.

### *APPENDIX A: THE PRIMAL PROBLEM OF THE CONSUMER*

Given the preference function of the taxpayer defined in (3.15) the Lagrangian is:

$$Max \;\; \Im = \sum_{j=1}^{4} S_j U_j\left(\underline{x}_j, 1-\ell\right) - \sum_{j=1}^{4} \lambda_j \left[y_j - \underline{p}\underline{x}_j\right] \tag{A.1}$$

The first order conditions for interior solutions can be written as:

$$\frac{\partial \Im}{\partial x_{ij}} = S_j U'_{ij}\left(\underline{x}_{ij}, 1-\ell\right) - \lambda_j p_j = 0 \text{ ; i = 1, 2,…, n, j = 1, 2, …, 4} \qquad \text{(A.2 – A.5)}$$

$$\frac{\partial \Im}{\partial \ell} = -\sum_{j=1}^{4} S_j U'_{\ell j}\left(\underline{x}_{ij}, 1-\ell\right) + \sum_{j=1}^{4} \lambda_j \underline{w}_j = 0 \tag{A.6}$$

$$\frac{\partial \Im}{\partial \delta_A} = (\lambda_1 + \lambda_2)\left[t_A(\theta_A - 1) + k'_A(1 - \delta_A - \delta_B + \delta_A \delta_B)\right] + (\lambda_3 + \lambda_4)\left[-t_A + k'_A(1 - \delta_A - \delta_B)\right] = 0 \tag{A.7}$$

$$\frac{\partial \Im}{\partial \delta_B} = (\lambda_1 + \lambda_3)\left[t_B(\theta_B - 1) + k'_B(1 - \delta_A - \delta_B + \delta_A \delta_B)\right] + (\lambda_2 + \lambda_4)\left[-t_B + k'_B(1 - \delta_A - \delta_B)\right] = 0 \tag{A.8}$$

$$\frac{\partial \Im}{\partial \lambda_j} = 0 \text{ ; j = 1, 2, …, 4.} \qquad \text{(A.9 – A.12)}$$

where $-\sum_{j=1}^{4} S_j U'_{\ell j}\left(\underline{x}_{ij}, 1-\ell\right)$ is the 'weighted' average of marginal utilities of leisure and $\lambda_j$ are the usual Lagrange multipliers.

### *APPENDIX B: THE DUAL PROBLEM OF THE CONSUMER*

Given the dual problem of the taxpayer defined in (3.39) the Lagrangian and the first order conditions for an interior solution are:

$$\Gamma = \underline{\mathrm{p}}\underline{x}_j - \ell S_j \underline{w}_j + \nu\left[V^* - \sum_{j=1}^{4} S_j U_j\left(\underline{x}_j, 1-\ell\right)\right] \tag{B.1}$$

$$\frac{\partial \Gamma}{\partial x_{ij}} = p_i - \nu \sum_{j=1}^{4} S_j U'_{ij}\left(\underline{x}_{ij}, 1-\ell\right) = 0 \text{ ; i = 1, 2,…, n, j = 1, 2, …, 4} \tag{B.2 – B.5}$$

$$\frac{\partial \Gamma}{\partial \ell} = -\sum_{j=1}^{4} S_j \underline{w}_j + \nu \sum_{j=1}^{4} S_j U'_{\ell j}\left(\underline{x}_{ij}, 1-\ell\right) = 0 \tag{B.6}$$

$$\frac{\partial \Gamma}{\partial \delta_A} = k'_A - t_A\left[1 - \left(S_1 + S_2\right)\theta_A\right] = 0 \tag{B.7}$$

$$\frac{\partial \Gamma}{\partial \delta_B} = k'_B - t_B\left[1 - \left(S_1 + S_3\right)\theta_B\right] = 0 \tag{B.8}$$

$$\frac{\partial \Gamma}{\partial \nu} = V^* - \sum_{j=1}^{4} S_j U_j\left(\underline{x}_j, 1-\ell\right) = 0 \tag{B.9}$$

where $\nu$ is the usual Lagrange multiplier.

## *APPENDIX C: SOME COMPARATIVE STATIC RESULTS*

The following relations are derived by differentiating the identities (3.40) – (3.43) with respect to $p_k$, $t_A$, $t_B$, $\theta_A$, $\theta_B$, $\sigma_k$ and assuming that both leisure and at least one of the goods are normal.

$$\frac{\partial x_{ij}}{\partial p_k} = -\frac{\partial x_{ij}}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\lambda_j x_{kj}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial x_{ij}^c}{\partial p_k} \gtrless 0 \tag{C.1}$$

$$\frac{\partial \ell}{\partial p_k} = -\frac{\partial \ell}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\lambda_j x_{kj}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \ell^c}{\partial p_k} \gtrless 0 \tag{C.2}$$

$$\frac{\partial \delta_A}{\partial p_k} = -\frac{\partial \delta_A}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\lambda_j x_{kj}}{\sum_{j=1}^{4}\lambda_j}\right) \gtrless 0 \tag{C.3}$$

$$\frac{\partial \delta_B}{\partial p_k} = -\frac{\partial \delta_B}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\lambda_j x_{kj}}{\sum_{j=1}^{4}\lambda_j}\right) \gtrless 0 \tag{C.4}$$

$$\frac{\partial x_{ij}}{\partial t_A} = -\frac{\partial x_{ij}}{\partial \alpha}\left(\frac{w\ell\lambda_{t_A}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial x_{ij}^c}{\partial t_A} \gtrless 0 \tag{C.5}$$

$$\frac{\partial \ell}{\partial t_A} = -\frac{\partial \ell}{\partial \alpha}\left(\frac{w\ell\lambda_{t_A}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \ell^c}{\partial t_A} \gtrless 0 \tag{C.6}$$

$$\frac{\partial \delta_A}{\partial t_A} = -\frac{\partial \delta_A}{\partial \alpha}\left(\frac{w\ell\lambda_{t_A}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \delta_A^c}{\partial t_A} \underset{<}{>} 0 \tag{C.7}$$

$$\frac{\partial \delta_B}{\partial t_A} = -\frac{\partial \delta_B}{\partial \alpha}\left(\frac{w\ell\lambda_{t_A}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \delta_B^c}{\partial t_A} \underset{<}{>} 0 \tag{C.8}$$

$$\frac{\partial x_{ij}}{\partial t_B} = -\frac{\partial x_{ij}}{\partial \alpha}\left(\frac{w\ell\lambda_{t_B}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial x_{ij}^c}{\partial t_B} \underset{<}{>} 0 \tag{C.9}$$

$$\frac{\partial \ell}{\partial t_B} = -\frac{\partial \ell}{\partial \alpha}\left(\frac{w\ell\lambda_{t_B}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \ell^c}{\partial t_B} \underset{<}{>} 0 \tag{C.10}$$

$$\frac{\partial \delta_A}{\partial t_B} = -\frac{\partial \delta_A}{\partial \alpha}\left(\frac{w\ell\lambda_{t_B}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \delta_A^c}{\partial t_B} \underset{<}{>} 0 \tag{C.11}$$

$$\frac{\partial \delta_B}{\partial t_B} = -\frac{\partial \delta_B}{\partial \alpha}\left(\frac{w\ell\lambda_{t_B}}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \delta_B^c}{\partial t_B} \underset{<}{>} 0 \tag{C.12}$$

$$\frac{\partial x_{ij}}{\partial \sigma_m} = \frac{\partial x_{ij}}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\frac{\partial S_j}{\partial \sigma_m}U_j\left(\underline{x}_j(.),1-\ell(.)\right)}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial x_{ij}^c}{\partial \sigma_m} \underset{<}{>} 0 \qquad m = A, B \tag{C.13}$$

$$\frac{\partial \ell}{\partial \sigma_m} = \frac{\partial \ell}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\frac{\partial S_j}{\partial \sigma_m}U_j\left(\underline{x}_j(.),1-\ell(.)\right)}{\sum_{j=1}^{4}\lambda_j}\right) + \frac{\partial \ell^c}{\partial \sigma_m} \underset{<}{>} 0 \qquad m = A, B \tag{C.14}$$

$$\frac{\partial \delta_A}{\partial \sigma_m} = \frac{\partial \delta_A}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\frac{\partial S_j}{\partial \sigma_m}U_j\left(\underline{x}_j(.),1-\ell(.)\right)}{\sum_{j=1}^{4}\lambda_j}\right)+\frac{\partial \delta_A^c}{\partial \sigma_m}\begin{matrix}>\\<\end{matrix}0 \qquad \text{m = A, B} \qquad \text{(C.15)}$$

$$\frac{\partial \delta_B}{\partial \sigma_m} = \frac{\partial \delta_B}{\partial \alpha}\left(\frac{\sum_{j=1}^{4}\frac{\partial S_j}{\partial \sigma_m}U_j\left(\underline{x}_j(.),1-\ell(.)\right)}{\sum_{j=1}^{4}\lambda_j}\right)+\frac{\partial \delta_B^c}{\partial \sigma_m}\begin{matrix}>\\<\end{matrix}0 \qquad \text{m = A, B} \qquad \text{(C.16)}$$

## *APPENDIX D: SOCIAL WELFARE MAXIMIZATION OF THE 'SUPER GOVERNMENT'*

Let W be the Lagrangian function related to the optimization problem of the 'super government' defined in (4.5) – (4.7). One can then derive the following first order conditions for the 'super government' as:

$$\frac{\partial W}{\partial t_A} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial t_A} + \gamma_A \frac{\partial R_A(.)}{\partial t_A} + \gamma_B \frac{\partial R_B(.)}{\partial t_A} \right] dF_w = 0 \qquad \text{(D.1)}$$

$$\frac{\partial W}{\partial t_B} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial t_B} + \gamma_A \frac{\partial R_A(.)}{\partial t_B} + \gamma_B \frac{\partial R_B(.)}{\partial t_B} \right] dF_w = 0 \qquad \text{(D.2)}$$

$$\frac{\partial W}{\partial \alpha_A} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial \alpha_A} + \gamma_A \frac{\partial R_A(.)}{\partial \alpha_A} + \gamma_B \frac{\partial R_B(.)}{\partial \alpha_A} \right] dF_w = 0 \qquad \text{(D.3)}$$

$$\frac{\partial W}{\partial \alpha_B} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial \alpha_B} + \gamma_A \frac{\partial R_A(.)}{\partial \alpha_B} + \gamma_B \frac{\partial R_B(.)}{\partial \alpha_B} \right] dF_w = 0 \qquad \text{(D.4)}$$

$$\frac{\partial W}{\partial \sigma_A} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial \sigma_A} + \gamma_A \frac{\partial R_A(.)}{\partial \sigma_A} + \gamma_B \frac{\partial R_B(.)}{\partial \sigma_A} \right] dF_w - \gamma_A \frac{\partial c_A(.)}{\partial \sigma_A} - \gamma_B \frac{\partial c_B(.)}{\partial \sigma_A} = 0 \qquad \text{(D.5)}$$

$$\frac{\partial W}{\partial \sigma_B} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial \sigma_B} + \gamma_A \frac{\partial R_A(.)}{\partial \sigma_B} + \gamma_B \frac{\partial R_B(.)}{\partial \sigma_B} \right] dF_w - \gamma_A \frac{\partial c_A(.)}{\partial \sigma_B} - \gamma_B \frac{\partial c_B(.)}{\partial \sigma_B} = 0 \qquad \text{(D.6)}$$

$$\frac{\partial W}{\partial t_k} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial p_k}\frac{\partial p_k}{\partial t_k} + \gamma_A \frac{\partial R_A(.)}{\partial t_k} + \gamma_B \frac{\partial R_B(.)}{\partial t_k} \right] dF_w = 0 \qquad \text{(D.7)}$$

$$\frac{\partial W}{\partial \sigma_k} = \int_{w^l}^{w^h} \left[ H'(.)\frac{\partial V(.)}{\partial p_k}\frac{\partial p_k}{\partial \sigma_k} + \gamma_A \frac{\partial R_A(.)}{\partial \sigma_k} + \gamma_B \frac{\partial R_B(.)}{\partial \sigma_k} \right] dF_w - \gamma_B \frac{\partial c(.)}{\partial \sigma_k} = 0 \qquad \text{(D.8)}$$

$$\frac{\partial W}{\partial \gamma_m} = 0 \qquad ;\ \text{m = A, B} \qquad \text{(D.9) – (D.10)}$$

## *APPENDIX E: IRS DISTRICTS AND REGIONS BY STATES*

| STATE | DISTRICT | REGION | STATE | DISTRICT | REGION |
|---|---|---|---|---|---|
| Alabama | Gulf Coast | Southeast | Montana | Rocky Mountain | Western |
| Alaska | Pacific North West | Western | Nebraska | Midwest | Midstates |
| Arizona | Southwest | Western | Nevada | Southwest | Western |
| Arkansas | Arkansas-Oklahoma | Midstates | New Hampshire | New England | Northeast |
| California | Central California | Western | New Jersey | New Jersey | Northeast |
| | Los Angeles | | New Mexico | Southwest | Western |
| | Northern California | | New York | Brooklyn | Northeast |
| | Southern California | | | Manhattan | |
| Colorado | Rocky Mountain | Western | | Upstate New York | |
| Connecticut | Connecticut-Rhode Island | Northeast | North Carolina | North-South Carolina | Southeast |
| Delaware | Delaware-Maryland | Southeast | North Dakota | North Central | Midstates |
| Florida | North Florida | Southeast | Ohio | Ohio | Northeast |
| | South Florida | | Oklahoma | Arkansas-Oklahoma | Midstates |
| Georgia | Georgia | Southeast | Oregon | Pacific North West | Western |
| Hawaii | Pacific North West | Western | Pennsylvania | Pennsylvania | Northeast |
| Idaho | Rocky Mountain | Western | Rhode Island | Connecticut-Rhode Island | Northeast |
| Illinois | Illinois | Midstates | South Carolina | North-South Carolina | Southeast |
| Indiana | Indiana | Southeast | South Dakota | North Central | Midstates |
| Iowa | Midwest | Midstates | Tennessee | Kentucky-Tennessee | Southeast |
| Kansas | Kansas-Missouri | Midstates | Texas | North Texas | Midstates |
| Kentucky | Kentucky-Tennessee | Southeast | | South Texas | |
| Louisiana | Gulf Coast | Southeast | Utah | Rocky Mountain | Western |
| Maine | New England | Northeast | Vermont | New England | Northeast |
| Maryland | Delaware-Maryland | Southeast | Virginia | Virginia-West Virginia | Southeast |
| Massachusetts | New England | Northeast | Washington | Pacific North West | Western |
| Michigan | Michigan | Northeast | West Virginia | Virginia-West Virginia | Southeast |
| Minnesota | North Central | Midstates | Wisconsin | Midwest | Midstates |
| Mississippi | Gulf Coast | Southeast | Wyoming | Rocky Mountain | Western |

Source: The Internal Revenue Service

## *APPENDIX F: CORRELATION MATRIX OF THE VARIABLES*

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 1.00 | | | | | | | | | | | | | | |
| B | -0.23 | 1.00 | | | | | | | | | | | | | |
| C | -0.19 | 0.29 | 1.00 | | | | | | | | | | | | |
| D | 0.31 | -0.02 | 0.05 | 1.00 | | | | | | | | | | | |
| E | -0.24 | 0.11 | 0.23 | 0.11 | 1.00 | | | | | | | | | | |
| F | 0.17 | 0.05 | 0.52 | 0.35 | 0.13 | 1.00 | | | | | | | | | |
| G | -0.22 | 0.19 | 0.34 | -0.54 | 0.29 | 0.15 | 1.00 | | | | | | | | |
| H | -0.26 | 0.08 | 0.18 | -0.54 | 0.08 | 0.07 | 0.74 | 1.00 | | | | | | | |
| I | -0.04 | 0.05 | -0.24 | -0.43 | -0.60 | -0.29 | 0.15 | 0.34 | 1.00 | | | | | | |
| J | 0.00 | -0.29 | -0.25 | -0.13 | -0.23 | -0.20 | -0.18 | 0.02 | 0.24 | 1.00 | | | | | |
| K | 0.34 | -0.03 | -0.15 | 0.54 | -0.21 | 0.12 | -0.59 | -0.53 | -0.16 | 0.18 | 1.00 | | | | |
| L | 0.00 | -0.38 | -0.09 | 0.03 | -0.21 | -0.23 | -0.20 | -0.06 | 0.09 | 0.42 | 0.32 | 1.00 | | | |
| M | 0.16 | 0.12 | 0.03 | 0.19 | 0.16 | 0.05 | -0.12 | -0.41 | -0.37 | -0.15 | 0.16 | -0.17 | 1.00 | | |
| N | 0.16 | -0.10 | -0.36 | 0.08 | -0.08 | -0.43 | -0.29 | -0.38 | -0.01 | -0.01 | 0.26 | 0.26 | 0.51 | 1.00 | |
| O | -0.01 | 0.13 | 0.16 | -0.28 | 0.06 | 0.15 | 0.40 | 0.27 | -0.13 | -0.23 | -0.23 | -0.23 | 0.09 | -0.03 | 1.00 |

Legends: A= Per return Evasion, B = Audit Rate, C= State & Federal Combined Marginal Tax Rate,
D = Real Adjusted Gross Income, E = Percent of Returns with Wages and Salaries,
F = Percent of Returns with Itemized Deductions, G = Number of Exemptions per Return, H = Percent of Joint Returns,
I = Percent of Proprietors in Total Employment, J = Percent of Retail Trade in Total Employment,
K = Percent of Service Sector in Total Employment, L = Percent of Elderly Population,
M = Percent of Non-white Population, N = Gini Coefficient, and O = Unemployment Rate.

## *APPENDIX G: SPATIAL DIAGNOSTIC TESTS*

There are two approaches to test for spatial error dependence based on the results of the within estimation. The null is expressed as $H_0$: $\lambda = 0$ in both the approaches. One approach is based on the extension of Moran's I test for spatial autocorrelation, the other on the Lagrange multiplier principle. However, as Kelejian and Prucha (2001) showed that Moran I statistic defined is not robust to misspecification. To give credence to our claim and to check the robustness of the Moran I, we also computed the LM statistic following Anselin (1988).

*Moran's I Statistic*

Moran's (1950) I statistic is a well known test for spatial autocorrelation. It is a weighted correlation coefficient used to detect departures from spatial randomness. It is produced by standardizing the spatial autocovariance by the variance of the error and depends on a spatial structural specification such as a spatial weights matrix or a distance related decline function. The Moran's I statistic is defined as

$$I = \frac{N \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} e_i e_j}{S_0 \sum_{i=1}^{N} e_i^2} \quad \Longleftrightarrow \quad \text{Or in matrix form } I = \frac{Ne'We}{S_0 e'e} \qquad \text{(G.1)}$$

where N equals the number of spatial units, $w_{ij}$ is a weight denoting the strength of the connection between spatial units $i$ and $j$, $e_i$ is the regression residual and $S_o$ is the sum of the weights defined as

$$S_0 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} \text{ , } i \neq j \qquad \text{(G.2)}$$

The expectation of $I$ under the null hypothesis is:

$$E(I) = -\frac{N \times tr\left|(e'e)^{-1} e'We\right|}{2S_0(N-K)} \qquad \text{(G.3)}$$

The variance of $I$ is determined normality assumption. The assumption of normality is useful when we have good reason to believe the errors follow a normal distribution. The variance of Moran I is defined as:

$$Var(I) = \frac{N^2 \left\{ 4S_0 + 2tr\left( \left[ (e'e)^{-1} e'We \right]^2 \right) - tr\left[ (e'e)^{-1} e'(W + W')^2 e \right] - 2\left( tr\left[ (e'e)^{-1} e'We \right] \right)^2 / (N - K) \right\}}{(2S_0)^2 (N - K)(N - K + 2)} \tag{G.4}$$

*The Lagrange Multiplier Statistic*

The approach towards testing for spatial error dependence that is based on the language Multiplier principle is outlined in Anselin (1988). In formal terms, the statistic is very similar to the Moran's I except for the use of a different scaling constant. Its properties are asymptotic. The statistic is:

$$LM_{err} = \frac{(e'We / S_0^2)^2}{tr(WW + W'W)} \tag{G.5}$$

where the notations are as above. It is asymptotically distributed as a $\chi^2$ variate with one degree of freedom. A high value of the statistic (and a low value of the probability) implies rejection of the null hypothesis of no spatial association.

## *REFERENCES*


Allingham, Michael G. and Agnar Sandmo. 1972. Income tax evasion: a theoretical analysis. *Journal of Public Economics* 1, no. 3-4: 323-338.

Alm, James R. 1999. Tax Compliance and Administration. In *Handbook on Taxation*, ed. W. Bartley Hildreth and James A. Richardson:741-768. New York: Marcel Dekker, Inc.

Alm, James R., Roy W. Bahl, Jr., and Matthew N. Murray. 1990. Tax Structure and Tax Compliance. *Review of Economics and Statistics* 72, no. 4: 603-13.

________. 1991. An Evaluation of the Structure of the Jamaican Individual Income Tax. In *The Jamaican tax reform*:87-152. Cambridge, Mass.

Alm, James R., Gary H. McClelland, and William D. Schulze. 1999. Changing the Social Norm of Tax Compliance by Voting. *Kyklos* 52, no. 2: 141-71.

Anderson, T. W. and Cheng Hsiao. 1982. Formulation and Estimation of Dynamic Models Using Panel Data. *Journal of Econometrics* 18, no. 1: 47-82.

Andreoni, James, Brian Erard, and Jonathan Feinstein. 1998. Tax Compliance. *Journal of Economic Literature* 36, no. 2: 818-60.

Anselin, Luc. 1988. *Spatial Econometrics: Methods and Models*. Norwell, Mass.; London and Dordrecht: Kluwer Academic.

________. 2002. Under the Hood: Issues in the Specification and Interpretation of Spatial Regression Models. *Agricultural Economics* 27, no. 3: 247-67.

Anselin, Luc and Sheri Hudak. 1992. Spatial Econometrics in Practice: A Review of Software Options. *Regional Science and Urban Economics* 22, no. 3: 509-36.

Arellano, Manuel and Stephen Bond. 1991. Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. *Review of Economic Studies* 58, no. 2: 277-97.

Arellano, Manuel and Olympia Bover. 1995. Another Look at the Instrumental Variable Estimation of Error-Components Models. *Journal of Econometrics* 68, no. 1: 29-51.

Atkinson, Anthony B. and Joseph E. Stiglitz. 1976. The Design of Tax Structure: Direct versus Indirect Taxation. *Journal of Public Economics* 6, no. 1-2: 55-75.

________. 1980. *Lectures on Public Economics*. New York: McGraw-Hill Companies.

Auerbach, Alan J. 1985. The Theory of Excess Burden and Optimal Taxation. In *Handbook of Public Economics. Volume 1*:61-127. New York and Oxford.

Baltagi, Badi H. 2001. *Econometric Analysis of Panel Data*. Chichester and New York: John Wiley and Sons, Ltd.

Beron, Kurt J., Helen V. Tauchen, and Ann D. Witte. 1988. A Structural Equation Model for Tax Compliance and Auditing: National Bureau of Economic Research, Inc, NBER Working Papers: 2556.

Blundell, Richard and Stephen Bond. 1998. Initial Conditions and Moment Restrictions in Dynamic Panel Data Models. *Journal of Econometrics* 87, no. 1: 115-43.

Boadway, Robin, Maurice Marchand, and Pierre Pestieau. 1994. Towards a theory of the direct-indirect tax mix. *Journal of Public Economics* 55, no. 1: 71-88.

Bound, John, David A. Jaeger, and Regina M. Baker. 1995. Problems with Instrumental Variables Estimation When the Correlation between the Instruments and the Endogenous Explanatory Variable Is Weak. *Journal of the American Statistical Association* 90, no. 430: 443-50.

Case, Anne C., Harvey S. Rosen, and James R. Hines, Jr. 1993. Budget Spillovers and Fiscal Policy Interdependence: Evidence from the States. *Journal of Public Economics* 52, no. 3: 285-307.

Clotfelter, Charles T. 1983. Tax Evasion and Tax Rates: An Analysis of Individual Returns. *Review of Economics and Statistics* 65, no. 3: 363-73.

Cowell, Frank A. 1990. *Cheating the government: The economics of evasion*. Cambridge, Mass. and London: MIT Press.

Cremer, Helmuth and Firouz Gahvari. 1993. Tax evasion and optimal commodity taxation. *Journal of Public Economics* 50, no. 2: 261-275.

________. 1994. Tax Evasion, Concealment and the Optimal Linear Income Tax. *Scandinavian Journal of Economics* 96, no. 2: 219-39.

________. 1995. Uncertainty and Optimal Taxation: In Defense of Commodity Taxes. *Journal of Public Economics* 56, no. 2: 291-310.

________. 1996. Tax evasion and the optimum general income tax. *Journal of Public Economics* 60, no. 2: 235-249.

Deaton, Angus. 1979. Optimally uniform commodity taxes. *Economics Letters* 2, no. 4: 357-361.

________. 1981. Optimal Taxes and the Structure of Preferences. *Econometrica* 49, no. 5: 1245-60.

Diamond, Peter A. 1975. A Many-Person Ramsey Tax Rule. *Journal of Public Economics* 4, no. 4: 335-42.

Dixit, Avinash K. and Angar Sandmo. 1977. Some Simplified Formulae for Optimal Income Taxation. *Scandinavian Journal of Economics* 79, no. 4: 417-23.

Dubin, Jeffrey A. 2004. Criminal Investigation Enforcement Activities and Taxpayer Noncompliance: California Institute of Technology, Division of the Humanities and Social Sciences, Working Papers: 1200.

Dubin, Jeffrey A., Michael J. Graetz, and Louis L. Wilde. 1987. Are We a Nation of Tax Cheaters? New Econometric Evidence on Tax Compliance. *American Economic Review* 77, no. 2: 240-45.

________. 1990. The Effect of Audit Rates on the Federal Individual Income Tax, 1977-1986. *National Tax Journal* 43, no. 4: 395-409.

Dubin, Jeffrey A. and Louis L. Wilde. 1988. An Empirical Analysis of Federal Income Tax Auditing and Compliance. *National Tax Journal* 41, no. 1: 61-74.

Eaton, Jonathan and Harvey S. Rosen. 1980a. Labor Supply, Uncertainty, and Efficient Taxation. *Journal of Public Economics* 14, no. 3: 365-74.

________. 1980b. Optimal Redistributive Taxation and Uncertainty. *Quarterly Journal of Economics* 95, no. 2: 357-64.

Feinstein, Jonathan S. 1991. An Econometric Analysis of Income Tax Evasion and Its Detection. *RAND Journal of Economics* 22, no. 1: 14-35.

Hamilton, Jonathan H. 1987. Optimal Wage and Income Taxation with Wage Uncertainty. *International Economic Review* 28, no. 2: 373-88.

Hansen, Lars Peter. 1982. Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica* 50, no. 4: 1029-54.

Hausman, Jerry A. 1978. Specification Tests in Econometrics. *Econometrica* 46, no. 6: 1251-71.

Jou, Jyh-Bang. 1992. Income Tax Evasion: Theory and Empirical Evidence from the U.S. State-Level Data, Tax Year 1976-1989. Ph.D. Dissertation, University of Chicago.

Kamdar, Nipoli. 1995. Information Reporting and Tax Compliance: An Investigation Using Individual TCMP Data. *Atlantic Economic Journal* 23, no. 4: 278-92.

Kaplow, Louis. 1990. Optimal Taxation with Costly Enforcement and Evasion. *Journal of Public Economics* 43, no. 2: 221-36.

Kelejian, Harry H. and Ingmar R. Prucha. 1998. A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances. *Journal of Real Estate Finance and Economics* 17, no. 1: 99-121.

________. 1999. A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model. *International Economic Review* 40, no. 2: 509-33.

________. 2001. On the Asymptotic Distribution of the Moran I Test Statistic with Applications. *Journal of Econometrics* 104, no. 2: 219-57.

Kolm, Serge-Christophe. 1973. A Note on Optimum Tax Evasion. *Journal of Public Economics* 2, no. 3: 265-270.

Manski, Charles F. 1991. Nonparametric Estimation of Expectations in the Analysis of Discrete Choice under Uncertainty. In *Nonparametric and semiparametric methods in econometrics and statistics: Proceedings of the Fifth International Symposium in Economic Theory and Econometrics*:259-75. Cambridge; New York and Melbourne.

Martinez-Vazquez, Jorge and Mark Rider. 2005. Multiple Modes of Tax Evasion: Theory and Evidence. *National Tax Journal* 58, no. 1: 51-76.

McFadden, Daniel. 2006. Free Markets and Fettered Consumers. *American Economic Review* 96, no. 1: 5-29.

Mirrlees, James A. 1971. An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies* 38, no. 114: 175.

________. 1976. Optimal Tax Theory: A Synthesis. *Journal of Public Economics* 6, no. 4: 327-58.

Moran, P.A.P. 1950. Notes on Continuous Stochastic Phenomena. *Biometrika* 37, no. 1/2: 17-23.

Plumley, Alan H. 1996. *The Determinants of Individual Income Tax Compliance: Estimating the Impacts of Tax Policy, Enforcement, and IRS Responsiveness*. Internal Revenue Service, Publication 1916 (Rev. 11-96).

Poterba, James M. 1987. Tax Evasion and Capital Gains Taxation. *American Economic Review* 77, no. 2: 234-39.

Ramsey, Frank P. 1927. A Contribution to the Theory of Taxation. *Economic Journal* 37, no. 145: 47-61.

Sandmo, Agnar. 1981. Income tax evasion, labour supply, and the equity--efficiency tradeoff. *Journal of Public Economics* 16, no. 3: 265-288.

Sargan, J. D. 1958. The Estimation of Economic Relationships using Instrumental Variables. *Econometrica* 26, no. 3: 393-415.

Scholz, John T. and Dan B. Wood. 1998. Controlling the IRS: Principals, Principles, and Public Administration. *American Journal of Political Science* 42, no. 1: 141-162.

Slemrod, Joel B. 1985. An Empirical Test for Tax Evasion. *Review of Economics and Statistics* 67, no. 2: 232-38.

________. 1994. Fixing the Leak in Okun's Bucket: Optimal Tax Progressivity When Avoidance Can Be Controlled. *Journal of Public Economics* 55, no. 1: 41-51.

Slemrod, Joel B. and Shlomo Yitzhaki. 2002. Tax Avoidance, Evasion, and Administration. In *Handbook of Public Economics. Volume 3*:1423-70. Amsterdam; London and New York: Elsevier Science, North-Holland.

Srinivasan, T. N. 1973. Tax evasion: A model. *Journal of Public Economics* 2, no. 4: 339-346.

Stern, Nicholas H. 1987. Theory of Optimal Commodity and Income Taxation. In *The Theory of Taxation for Developing Countries*, ed. David M. Newbery and Nicholas H. Stern. New York: Oxford University Press.

Stiglitz, Joseph E. 1982. Utilitarianism and Horizontal Equity: The Case for Random Taxation. *Journal of Public Economics* 18, no. 1: 1-33.

________. 1988. Pareto Efficient and Optimal Taxation and the New New Welfare Economics: National Bureau of Economic Research, Inc, NBER Working Papers: 2189.

Tauchen, Helen V., Ann Dryden Witte, and Kurt J. Beron. 1989. Tax Compliance: An Investigation Using Individual TCMP Data: National Bureau of Economic Research, Inc, NBER Working Papers: 3078.

Varian, Hal R. 1980. Redistributive Taxation as Social Insurance. *Journal of Public Economics* 14, no. 1: 49-68.

Weiss, Laurence. 1976. The Desirability of Cheating Incentives and Randomness in the Optimal Income Tax. *Journal of Political Economy* 84, no. 6: 1343-52.

Witte, Ann D. and Diane F. Woodbury. 1985. The Effect of Tax Laws and Tax Administration on Tax Compliance: The Case of the U.S. Individual Income Tax. *National Tax Journal* 38, no. 1: 1-13.

Yitzhaki, Shlomo. 1974. Income tax evasion: A theoretical analysis. *Journal of Public Economics* 3, no. 2: 201-202.

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