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Student Participation in Mathematics Discourse in a Standards-based Middle Grades Classroom

Brian S. Lack
Georgia State University

ACCEPTANCE

This dissertation, STUDENT PARTICIPATION IN MATHEMATICS DISCOURSE IN A STANDARDS-BASED MIDDLE GRADES CLASSROOM, by BRIAN LACK, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chair, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty. The Dean of the College of Education concurs.

____
Barbara Meyers, Ed. D.
Committee Chair

____
Joel Meyers, Ph. D.
Committee Member

____
Lynn Hart, Ph. D.
Committee Member

____
Susan Swars, Ph. D.
Committee Member

____
Date

____
Barbara Meyers, Ed. D.
Chair, Department of Early Childhood Education

____
R. W. Kamphaus, Ph. D.
Dean and Distinguished Research Professor
College of Education

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Brian Lack
4250 Brumby Lane
Cumming, GA 30041

The director of this dissertation is:

Dr. Barbara Meyers
Department of Early Childhood Education
College of Education
Georgia State University
Atlanta, GA 30303-3077

CURRICULUM VITAE

Brian Lack

ADDRESS: 4250 Brumby Lane
Cumming, GA 30041

EDUCATION:

Ph.D.	2010	Georgia State University Early Childhood Education
M.Ed.	2000	University of Georgia Early Childhood Education
B.S.	1999	University of Georgia Early Childhood Education

PROFESSIONAL EXPERIENCE:

2008-Present	Middle School Math Teacher Forsyth County Schools, Cumming, GA
2007-2008	Elementary School Teacher Atlanta Public Schools, Atlanta, GA
2004-2007	Doctoral Teaching and Research Fellow Georgia State University, Atlanta, GA
2000-2004	Elementary School Teacher Clarke County and Gwinnett County, GA

PUBLICATIONS:

Lack, B. (2011). Anti-democratic militaristic education: An overview and critical analysis of KIPP Schools. In R. Ahlquist, T. Montano, and P. Gorski (Eds.), *Assault on working-class kids: Hyper-accountability, corporatization, Ruby Payne, and deficit theories.* New York: Peter Lang.

Lack, B. (2010). Education funding. In C. Clauss-Ehlers (Ed.), *Encyclopedia of cross-cultural school psychology* (pp. 414-415). New York: Springer.

Lack, B. (2009). No excuses: A critique of the Knowledge is Power Program (KIPP) within charter schools in the USA. *Journal for Critical Education Policy Studies, 7*(2), 126-153.

Breault, R. A., & Lack, B. (2009). Equity and empowerment in PDS work: A review of literature (1999 to 2006). *Equity and Excellence in Education, 42*(2), 152-168.

Swars, S. L., Meyers, B., Mays, L. C., & Lack, B. (2009). A two-dimensional model of teacher retention and mobility: Classroom teachers and university partners take a closer look at a vexing problem. *Journal of Teacher Education, 60*(2), 168-183.

PRESENTATIONS:

Breault, R. A., & Lack, B. (2008, March). *Equity, empowerment and student learning in PDS: A review of literature.* Paper presented at the annual meeting of the American Educational Research Association, New York.

Lack, B. (2007, April). *No excuses: A democratic critique of the Knowledge is Power Program.* Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Lack, B., Mays, L. C., Swars, S. L., & Meyers, B. (2007, April). *Examining teacher retention and mobility in a Professional Development School through collaborative inquiry.* Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Lack, B., and Meyers, B. (2006, November). *The many faces of mobility: A school-university partnership study of school climate and culture.* Presented at the annual meeting of the Southeastern Regional Association of Teacher Educators, Towson, MD.

Lack, B., & Meyers, L. (2006, October). *Integrating social studies, technology, and children's literature to teach about the American Civil Rights movement.* Presented at the annual meeting of the Georgia Council for the Social Studies, Athens, GA.

Truscott, D., Dangel, J. R., Meyers, B., Lack, B., Swars, S. L., Jordan, L., et al. (2006, February). *Partnership work in high-needs schools: Reflection on practice.* Presented at the annual meeting of the Association of Teacher Educators, Atlanta, GA.

PROFESSIONAL SOCIETIES AND ORGANIZATIONS:

2006-Present	American Educational Research Association
2008-Present	National Council for Teachers of Mathematics

PROFESSIONAL SERVICE:

2010	Mentor, Forsyth County Schools Mentoring Program
2009	Assisted peer review for Journal for Research in Mathematics Education
2007	McNair Program research mentor to Julie Owens, ECE undergraduate student, Georgia State University
2006-2007	Guest speaker for Ed.S. course on teacher development
2006	Teaching apprentice to Dr. Laura Meyers, social studies methods undergraduate course in ECE
2005-2007	Interviewer for admissions to undergraduate program in ECE
2005-2006	Volunteer, academic tutoring and percussion instructor, KIPPWAYS, Atlanta, GA
2004-2006	ECE student representative for the Student Activities Fees Oversight Committee

ABSTRACT

STUDENT PARTICIPATION IN MATHEMATICS DISCOURSE IN A STANDARDS-BASED MIDDLE GRADES CLASSROOM
by
Brian Lack

The vision of K-12 standards-based mathematics reform embraces a greater emphasis on students' ability to communicate their understandings of mathematics by utilizing adaptive reasoning (i.e., reflection, explanation, and justification of thinking) through mathematics discourse. However, recent studies suggest that many students lack the socio-cognitive capacity needed to succeed in learner-centered, discussion-intensive mathematics classrooms. A multiple case study design was used to examine the nature of participation in mathematics discourse among two low- and two high-performing sixth grade female students while solving rational number tasks in a standards-based classroom. Data collected through classroom observations, student interviews, and student work samples were analyzed via a multiple-cycle coding process that yielded several important within-case and cross-case findings. Within-case analyses revealed that (a) students' access to participation was mediated by the degree of space they were afforded and how they attempted to utilize that space, as well as the meaning they were able to construct through providing and listening to explanations; and (b) participation was greatly influenced by peer interactional tendencies that either promoted or impeded productive contributions, as well as teacher interactions that helped to offset some of the problems related to unequal access to participation. Cross-case findings suggested that

(a) students' willingness to contribute to task discussions was related to their goal orientations as well as the degree of social risk perceived with providing incorrect solutions before their peers; and (b) differences between the kinds of peer and teacher interactions that low- and high-performers engaged in were directly related to the types of challenges they faced during discussion of these tasks. An important implication of this study's findings is that the provision of space and meaning for students to participate equitably in rich mathematics discourse depends greatly on teacher interaction, especially in small-group instructional settings where unequal peer status often leads to unequal peer interactions. Research and practice should continue to focus on addressing ways in which students can learn how to help provide adequate space and meaning in small-group mathematics discussion contexts so that all students involved are allowed access to an optimally rich learning experience.

STUDENT PARTICIPATION IN MATHEMATICS DISCOURSE IN A STANDARDS-BASED MIDDLE GRADES CLASSROOM

by

Brian Lack

A Dissertation

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ABBREVIATIONS

NRC	National Research Council
NCLB	No Child Left Behind
NCTM	National Council of Teachers of Mathematics
NMAP	National Mathematics Advisory Panel
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1

INTRODUCTION

Just over twenty years ago, in response to widely publicized indictments of American education including *A Nation at Risk* (1983) and *A Nation Prepared: Teachers for the 21st Century* (1986), the National Council of Teachers of Mathematics (NCTM) issued its radical reconceptualization of mathematics education in its *Curriculum and Evaluation Standards for School Mathematics* (1989). Developers of the *Standards* envisaged reformed mathematics classrooms as "discourse communities where conjectures are made, arguments presented, strategies discussed," (Romberg, 1993, p.37) or, in short, classrooms that promote genuine understanding of mathematics. As a result, current K-12 standards-based reform embraces a greater emphasis on child-centered instructional practices that prioritize mathematical *processes* in addition to the traditional focus on mathematical *content* alone (NCTM, 2000). This generally means that students of today are expected to take a more active and meaningful role in the learning process, which often translates to classroom behaviors such as verbal or written reasoning, discussion, debate, and inquiry. Instead of responding succinctly to questions that have predetermined answers, students in today's standards-based classrooms are expected to address richer, more complex questions that draw on their ability to monitor, reflect on, and communicate their thinking processes. Put differently, questions such as "What is the answer?" have been supplemented (if not supplanted) by questions of the following nature: What do you mean? How do you know? How does what you said compare or contrast with what someone else said? Why do you agree or disagree? Will that always be the case? Can you think of a counterexample? Can you explain your reasoning to another student? How can you be certain?

Because the vision of mathematics education reform has placed greater emphasis on students' ability to effectively communicate their mathematical understandings, the nature of *classroom discourse* (i.e., student-to-student, student-to-teacher) has become an important instructional element of standards-based elementary and middle school mathematics classrooms (Silver & Smith, 1996; Walshaw & Anthony, 2008). Discourse, however, is an ambiguous term that has been defined and studied in very different ways, all the while masquerading under one generic label (Gee, Michaels, & O'Connor, 1992). NCTM (1991) has operationalized its conception of *discourse* as such:

> *Discourse* refers to the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in . . . tasks. The discourse embeds fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity . . . Teachers, through the ways in which they orchestrate discourse, convey messages about whose knowledge and ways of thinking and knowing are valued, who is considered able to contribute and who has status in the group. (¶ 4)

Mathematics classrooms that espouse rich discourse are widely viewed as a remedy to the traditional forms of instruction that have rendered students as passive learners and often left them with only superficial understanding of mathematics (Cobb, Wood, & Yackel, 1990). However, endorsement of discourse-rich mathematics instruction for the mere sake of increasing the ratio of student-initiated talk to teacher-talk underestimates the complexity of discourse as an effective instructional intervention for all students (Nathan & Knuth, 2003; Truxaw & DeFranco, 2008). While a stable body of research on classroom mathematics discourse has emerged over the last two decades, only a handful of researchers have looked specifically at low-achieving children and their participation in discussion-oriented classrooms (Baxter, Woodward, & Olson, 2001; Empson, 2003; Lubienski, 2000a; 2000b) and of these studies, only Empson's (2003) focuses on children's understanding of fractions and other rational number constructs. A perusal of the relevant literature has led me to reflect on my own experiences as a middle grades mathematics teacher (one who embraces discourse as a potentially useful instructional tool), resulting in several complex and discomforting questions. For instance, *who* benefits from mathematical discourse communities, and to what extent? What factors might impede the efficacy of classroom discourse as an effective intervention for all students? How can *all* students profit from participation in discourse communities? In today's milieu of standards-based reform, these questions bear a critical degree of import, especially for advocates of discussion-intensive mathematics instruction.

Proponents of discourse-based mathematics instruction call for the development of classroom communities that empower students to engage in the processes of "doing mathematics" (NCTM, 1989, p. 7), which entails "conjecturing, scrutinizing, and defending one's ideas, as well as learning about it" (Nathan & Knuth, 2003, p. 176). The teacher, no longer the sole arbiter of truth in the classroom, plays a less dominant but nevertheless critical role in facilitating students' knowledge construction. Cazden (2001) argues that teaching and learning roles within such communities are necessarily distributed more fluidly and democratically among all participants and suggests that the quality of discourse communities depends not only on teacher expertise but also on students' academic contributions and social relationships. Cazden's juxtaposition of traditional and non-traditional mathematics classrooms is thorough, yet succinct, and therefore worth quoting at length:

> In more traditional classrooms, social relationships are extracurricular, potential noise in the instructional system and "interference" with real school work. What counts are relationships between the teacher and each student as an individual, both in whole-class lessons and in individual seatwork assignments. In non-traditional classrooms, the situation has fundamentally changed. Now each student becomes a significant part of the official learning environment for all the others, and teachers depend on students' contributions to other students' learning, both in discussions and for the diffusion of individual expertise through the class. (p. 131)

Given the increasing emphasis on non-traditional, discussion-oriented approaches to mathematics pedagogy, coupled with the well-documented history of difficulties in understanding fractions and other rational number concepts, empirical attention to children's engagement in classroom discourse about rational number learning is of critical value.

Study Rationale

Lack of Emphasis on Mathematical Processes

Since the late 1990s, international studies of mathematics achievement have consistently revealed somewhat disappointing comparisons of American students' performance relative to students of other industrialized nations. A look at the Trends in International Mathematics and Science Study (TIMSS) of 2007 shows that U.S. fourth-graders ranked 11th in overall mathematics achievement and only 13th when compared to other nations in the domain of *cognitive reasoning* (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008), which assesses the ability to draw inferences, make generalizations and justifications, and solve novel tasks—the very actions that discourse communities aim to enculturate in students. In light of these data, further investigation of students' understanding of basic rational number concepts in addition to the communicative processes that best enable genuine mathematical learning of such concepts is in great demand.

In 2001, members of the National Research Council (NRC) promulgated perhaps the most balanced and comprehensive notion of *mathematical knowledge* to date, identifying five distinct but related attributes that contribute to the development of individual students' overall "mathematical proficiency" (Kilpatrick, Swafford, & Findell, 2001, p. 116). The five elements of mathematical proficiency are listed in Table 1, along with a corresponding definition of each element. Although each of these notions are interconnected in complex ways, this study placed particular emphasis on children's use of *adaptive reasoning,* which currently is in need of empirical attention in the discourse literature on rational number learning.

Table 1

Characteristics of Mathematical Proficiency

Element	Definition
Conceptual understanding	Comprehension of mathematical concepts, operations, and relations
Procedural fluency	Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
Strategic competence	Ability to formulate, represent and solve mathematical problems
Adaptive reasoning	Capacity for logical thought, reflection, explanation, and justification
Productive disposition	Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Classrooms in the United States have given only superficial attention to the development of adaptive reasoning as evidenced by a lack of instructional emphasis on student reasoning and justification of mathematical concepts and procedures (Hiebert, Stigler, Jacobs, Givvin, Garnier, Smith, et al., 2005). However, recent research has proven that encouraging the use of rich forms of discourse, like explaining one's reasoning and comparing multiple problem-solving strategies, leads to deeper levels of understanding (Chi, De Leeuw, Chiu, & LaVancher, 1994; Rittle-Johnson, Siegler, & Alibali, 2001; Star, 2005).

Like the NRC, NCTM (2000) has also called for greater efforts on the part of teachers to integrate explicit mathematical processes, like reflection, reasoning, and explanation, into daily classroom instruction. In 2006, the state in which the current study was conducted responded to NCTM's call for greater emphasis on these processes by publishing a revised edition of the state's K-12 public education standards, which currently includes a set of process standards. In two years of teaching middle grades mathematics, the author has observed a disproportionate amount of emphasis placed on content standards and only superficial attention dedicated to process standards. While deemphasizing these critical process standards, students and teachers remain limited to traditional forms of teacher-centered instruction buttressed by a pedagogical culture that promotes passivity, isolation, and complicity on the part of students, along with the prioritization of procedures over concepts in determining what type of knowledge is of optimal value (NCTM, 2000). Research and practice devoted to students' learning of mathematical content is clearly a critical area of interest for educators and policymakers, but can not be divorced from research and practice pertaining to the processes by which such content is made accessible to students.

Equitable Opportunities to Learn

Scholarly analyses of student learning and participation that claim to address issues of *equity* often identify one or more particular underrepresented cultural groups, such as African-Americans, females, or students from low-income families, as a focal population. Generally, the intended aim of such work is to generate raised consciousness about unequal distribution of benefits or resources among such cultural groups (Breault & Lack, 2009). This study does not address a particular cultural group, but rather, students based on their measured performance in mathematics, which includes both high- and low-performing students. This study design was constructed from the primary interest of creating equitable opportunities to learn mathematics (Esmonde, 2009). In short, this study examines *who* has access to opportunities to learn mathematics and also aims to explore *how* and perhaps *why*.

Notwithstanding contention, a fundamental assumption of standards-based reform, and specifically inscribed into the language of the equity principles, is that *all* children can learn mathematics to a degree of functional proficiency (usually measured by performance on standardized assessments). As one might reasonably expect, much attention has been devoted to tailoring mathematical content to students of varying ability in order to effectively reach diverse students, but nuanced mathematical processes have been overlooked. For instance, textbook companies have responded to public demands related to equity concerns by providing differentiated tasks, such as supplemental readiness and enrichment activities, that address state content standards for students of varying proficiency levels. But minimal emphasis has been placed on differentiation of process standards, like students' communication of, reasoning about, and representations of mathematics. The present study embraces Cazden's (2001) assertion that "educational purpose and equitable opportunities to learn remain the most important (instructional) design principles. Both teachers and researchers need to monitor who participates and how, and who doesn't and why" (p. 81). The implications of this study's findings may help elevate awareness of the need to place a greater priority on mathematical processes that facilitate the learning of mathematical content. In sum, by focusing on both low and high-performing students' access to participation in discourse, this study addresses issues of equity and access to learning opportunities.

Rational Number Learning

The introduction of rational number concepts represents a critical developmental shift in children's mathematical thinking (Lamon, 1996; 2006). Fraction or part-whole concepts are typically the first form of rational number a child encounters in his or her formal education experience (Booker, 1996; Pothier & Sawada, 1983) and often in making this developmental transition, children experience substantial interference from their crystallized knowledge of whole number properties (Behr, Wachsmuth, & Post, 1985; Behr, Wachsmuth, Post, & Lesh, 1984; Johanning, 2008). Between grades 4 and 8, rational number concepts are featured in national and state mathematics standards because they help build the necessary developmental skills one needs to master more advanced forms of mathematics (Lamon, 2006). Recently, a body of research has revealed that student mastery of fraction skills and concepts is among the strongest predictors of subsequent success in higher mathematics (Brown & Quinn, 2007a; Lamon, 2007; National Mathematics Advisory Panel, 2008; Wu, 2001). However, researchers have also long articulated the struggle that American students have endured in attempts to understand fractions (Kieren, 1976; Mack, 1990; Moss & Case, 1999). Further compounding this phenomenon is the well-documented fact that many adults, including a significant amount of teachers, hold misconceptions regarding fractions and fraction operations (Ball, 1993a; Hanson & Hogan, 2000; Newton, 2008; Post, Harel, Behr, & Lesh, 1991).

Fraction and rational number proficiency is strongly associated with success in later mathematics learning, and particularly algebra. Brown and Quinn (2007b) noted a significant correlation between students who performed poorly in a first-year high school algebra course and their performance on a fraction test. Wu (2001) contended that the "proper study of fractions provides a ramp that leads students gently from arithmetic up to algebra" (p. 1) and underscored the critical role that fraction proficiency plays in the ability to manipulate linear functions in advanced algebra. Just recently, the National Mathematics Advisory Panel (NMAP) issued a call for increased scholarly focus and greater instructional emphasis on children's learning about fractions. The authors of the report clearly asserted that future success in algebra hinges on substantive learning about fractions, decimals, and percents in the K-8 curriculum and reported that a nationally representative sample of high school algebra teachers described their incoming students as having very poor preparation in rational number concepts, particularly those associated with operations involving fractions and decimals (NMAP, 2008). Although critics lamented the NMAP's narrow selection of "scientifically-rigorous" studies from which it based its analysis and subsequent recommendations, there is very little contention over the belief that early learning of fractions and other rational numbers leads to subsequent success in mathematics. Moreover, few question the fact that early rational number learning continues to be problematic for many students.

Study Significance

The majority of mathematics discourse studies have focused on very young children's experiences with whole-number arithmetic. Given the observation that the introduction of rational number concepts poses a critical developmental shift in mathematical thinking, in addition to the assertion by some researchers that equitable participation in classroom discourse among older students is more difficult to achieve (Cazden, 2001; NCTM, 2000), discourse studies of middle school students engaged in learning about rational numbers is of considerable import. It is also worth pointing out that much of the work on mathematics classroom discourse has been at the primary-age level (kindergarten through grade 5), where the ramifications of social and emotional development may play a distinct role in the nature of children's participation in discourse. NCTM (2000), for example, acknowledges the reality that, during adolescence, "students are often reluctant to do anything that causes them to stand out from the group, and many middle-grades students are self-conscious and hesitant to expose their thinking to others. Peer pressure is powerful, and a desire to fit in is paramount" (p. 268). By examining the nature of older students' participation in mathematical talk about rational number concepts, particularly those who have failed to master basic fraction concepts like equivalence and order, this study makes a contribution to the extant literature on rational number understanding within a mathematics discourse community.

While much of the literature on mathematics discourse focuses on the role of the teacher in facilitating students' participation in making contributions, few studies of mathematics classroom discourse have examined peer interactions in small-group instructional settings. Notwithstanding the critical role a teacher plays in orchestrating rich discussion of mathematical content, small-group peer interactions without the assistance of the teacher represent a fundamental characteristic of standards-based mathematics classrooms. In short, students assume the critical role of facilitating discourse when the teacher is not present to do so. No studies that addressed small-group peer-led discussions among the body of work dedicated to mathematics discourse communities were found. This study therefore aims to help fill this critical gap.

This study also aims to address a methodological gap in the recent classroom discourse literature by employing video-recording as a method of eliciting student awareness and reflection on the nature of participation in mathematics discourse. Few classroom discourse studies have sought to address students' own perspectives regarding their roles in classroom participation, which ignores the possibility that students may actually have complex reasons for assuming various roles or for choosing to overtly participate or not participate in classroom discourse. For example, Baxter et al. (2001) characterized low-performing students as passive, quiet, and disengaged in the context of whole-group discussion, but failed to address why these students appeared to behave in such ways. Instead, the researchers relied on deficit assumptions to explain students' lack of capacity for participating in whole-class mathematics discussion. To the contrary, researchers who have used video monitoring of students' interactions during collaborative instructional tasks suggest that some students may not necessarily be aware of their behavior in a group discussion setting and, specifically, how their behavior relates to the shared norms and expectations of the group itself. For example, Nastasi and Young (1994) found that after viewing and discussing audiovisual recordings of collaborative group work with students who were struggling to work effectively together, students reported greater degrees of consciousness about their collaborative interactions. Allen (1992) also shared video-recordings of classroom discussions with her students to illuminate inequitable participation trends and subsequently involved students in offering ideas for improvement. Hatch (2002) contends that the use of video playback is a powerful stimulus for getting students to share their perspectives or interpret their own behavior during classroom events. Therefore, this study may reveal important insights about students' perspectives of their roles in mathematics discourse communities.

Finally, this study addresses Empson's (2003) call for research to examine differences between the interactions and participation of low- and high-performing students while talking about mathematics. In fact, like Empson's research, this study applied participant frameworks as a lens to help analyze students' interactions and contributions made during discussion of mathematics tasks.

Research Question

This study was guided by the following research question:

What is the nature of low- and high-achieving students' participation in classroom discourse about rational number tasks in a standards-based sixth grade classroom?

CHAPTER 2

REVIEW OF THE LITERATURE

In order to investigate the nature of low- and high-achieving students' participation in classroom discourse about rational number tasks in a standards-based sixth grade classroom, a broad review of the existing literature is provided that addresses some of the most salient concepts germane to the research question, including mathematics discourse communities, cooperative learning, and social and emotional dimensions of participating in group task discussions. Before examining the empirical research related to these topics, it is important to lay out the theoretical assumptions that guided this study from its inception.

The first part of this chapter is therefore devoted to describing how the major theoretical assumptions related to this study. Since sociocultural learning theory and pedagogical constructivism were the major lenses through which the research design was filtered, these theories of learning are identified and described first. This is followed by an explanation of participant frameworks, which is a tool used for analyzing students' interactions, specifically focusing on how individuals animate one another into certain intellectual roles by their interactions. The final part of this chapter is dedicated to reviewing the empirical literature related to the research question.

Theoretical Framework

Sociocultural Theory

According to Cobb (2007), theoretical contributions in the field of mathematics education have come primarily from four traditions: experimental psychology, cognitive psychology, distributed cognition, and sociocultural theory. This study draws primarily on sociocultural theories of learning because of its explicit emphasis on theoretical assumptions regarding social and cognitive development that hinges on participation in cultural practices (e.g., language socialization through participation in classroom discourse, or understanding of mathematics from informal, out-of-classroom experiences, etc.). Moreover, data collection and analysis focused on the processes by which students became participants in various roles and to various extents in mathematics discourse related to rational number tasks.

Sociocultural theories of mathematics learning are generally associated with the seminal work of Vygotsky (1978) and prioritize the socially and culturally situated nature of mathematical activity over individual sensory-motor functions (Cobb, 1994). Vygotsky (1978) identified three general themes fundamental to his theory of development: (a) Higher mental human processes can be best understood by focusing on how and when they occur; (b) higher mental processes, such as memory, concepts, and reasoning, originate between people on the social plane before appearing in the individual on the psychological plane; and (c) higher mental processes are mediated by cultural tools and signs such as language, writing, and symbols. Vygotsky claimed that all higher mental activity originates through a process of internalization, or what some scholars refer to as “appropriation” (Cazden, 2001), which he described as the process by which individuals engage in cultural practices on the *intermental plane* (i.e., through social interaction) before gradually performing these practices independently on the *intramental plane* (i.e., through internalization). The transformation between the social and psychological planes occurs within a zone of proximal development – the space between an individual's independent capabilities and his or her immediate mental potential. In other words, the zone of proximal development is determined by both the child's level of development and the quality of instruction provided to the child (Wertsch, 1985). It is in this space that social interaction between a novice and more knowledgeable others can lead to internalization of higher mental functions. Vygotskian learning theory, and in particular, his contributions regarding the zone of proximal development, essentially paved the foundation for cooperative learning as a viable instructional approach in modern classroom settings (Schunk, 1996).

Other researchers have extrapolated Vygotsky's work into theories that rely on an apprenticeship metaphor (e.g., Lave & Wenger, 1991; Rogoff, 1990), specifically stating that learning occurs in social interaction between novices and more-skilled others through increasingly greater degrees of legitimate participation (Lave & Wenger, 1991). In other words, learning is defined, in part, as a positive change in participation in a set of cultural practices. For example, while co-participating in mathematics discourse communities, teachers or more-able peers initially take a major role in sharing their reasoning aloud. Over time, novice students evolve from relatively marginal or peripheral roles to more autonomous roles through successively greater degrees of participation in the community of practice. Wertsch (1985) draws an important distinction between apprenticeship and school-like instruction. Apprenticeship learning, which derives from labor activity settings, intentionally organizes interaction so that the expert assumes a majority of the responsibility for executing tasks in the earliest of interactions. Therefore, initial interactions of this type might be informed by the assumption that efficient error-free execution is of the highest priority, rendering the novice capable of only executing the easiest steps involved in successfully mastering a task. On the other hand, school-like learning, which derives from instructional activity settings, might intentionally structure interaction so that novices can learn for the sake of understanding by participating freely in all aspects of the task. The important distinction between apprenticeship and school-like settings is that since learning is prioritized in the school-like setting, errors and mistakes are viewed as necessary steps toward true mastery of a task.

Philosophical and Pedagogical Constructivism

Around 600 B.C.E., a Western philosopher named Xenophanes boldly asserted that the state of knowing “truth” (i.e., reality) is impossible because, in order to do so, "we would need access to such a world that does not involve our experiencing it" (von Glasersfeld, 1990, p. 20). More than two centuries later, 17th century Italian philosopher Giambattista Vico famously noted “verum esse ipsum factum” (“truth itself is made”). Put differently, Vico argued that the human mind can only know what the human mind has constructed, thereby delivering one of the earliest references to the epistemology of constructivism (von Glasersfeld, 1990). As a theory of knowledge, constructivism is consonant with the Vygotskian principle that “knowing” is constructed primarily through an individual’s adaptive cognition of the experiential world, and rejects many of the tenets of classical behaviorism (e.g., mental processes insulated from social context; causal relations between environment and an individual’s cognitive processes)

(Noddings, 1990; von Glasersfeld, 1990). Advocates of constructivism can crudely be distinguished by the degree of emphasis they place on individual psychological (i.e., radical constructivists) and social processes (i.e., social constructivists) (Cobb, 1994).

From an individual or psychological perspective, the teacher's primary goal is to provide situations and objects that build on existing constructions of knowledge by either partially restructuring children's thinking (Booker, 1996) or triggering cognitive conflict, known as *disequilibrium* (Piaget, 1969). Through a process of equilibration, which presupposes the conditions of engagement and comprehension, children assimilate new knowledge into previously existing knowledge or accommodate new knowledge by reorganizing cognitive structures (Ginsburg & Opper, 1988). Constructivism is typically reified in classrooms by the use of inquiry- or discussion-based learning (Cobb et al., 1990; Ernest, 1996; Silver & Smith, 1996), and the instructional focus is on development of both mathematical concepts and procedures (Goldin & Shteingold, 2001) through the use of challenging tasks (Silver & Smith, 1996) and manipulatives (i.e., hands-on learning objects) (Noddings, 1990). Social constructivists place additional pedagogical emphasis on collaborative group work and peer interactions (Ernest, 1996).

As a theory of learning, constructivism stresses the role that prior knowledge plays in development of new knowledge. This prior knowledge can be either formal (i.e., that which is taught in schools) or informal (i.e., intuitive or experiential knowledge) (Baroody & Ginsburg, 1990). Informal knowledge is particularly useful in a child's formal educative experiences because it tends to be more meaningful, practical, and interesting than formal knowledge alone (Mack, 1990). Constructivists therefore advocate bridging the cognitive gaps between formal and informal knowledge as a component of effective mathematics instruction. One way to accomplish this, as constructivists assert, is to pay careful attention to and productively respond to children's thinking (Jacobs, Lamb, & Phillip, 2010). Careful consideration of the strategies that students report while doing mathematics allows teachers and researchers to embrace diverse or idiosyncratic forms of reasoning, which sometimes lead students down "unexpected paths to correct answers" (Callingham & Watson, 2004, p. 83). The author asserts that such consideration forms the *sine qua non* of constructivist-guided, student-centered pedagogy.

Many theorists have cautioned that constructivism, as an epistemological and ontological orientation, cannot generate a set of prescriptive teaching practices (Noddings, 1990; von Glasersfeld, 1990; Ernest, 1996). It does, however, inform a pedagogical approach that rejects student passivity and transmission as dominant modes of instruction. This approach is best known as pedagogical constructivism (Richardson, 2003). In mathematics classrooms that adopt constructivist pedagogy, teachers encourage students to become more engaged in reasoning and thinking that leads to solutions rather than solutions as ends in themselves (Yackel & Cobb, 1996). Said differently, as Lampert (1990) contends, in mathematics discourse communities, "the problem is not the question and the answer is not the solution" (p.40). There are some researchers who firmly believe that mathematics, by its essence, lends itself to (if not necessitates) a constructivist pedagogical approach. For instance, Carpenter, Franke, and Levi (2003) argue that the nature of mathematics presupposes that one cannot truly learn mathematics without actively engaging in meaningful discussion and argumentation. In short, a constructivist conceptualization of the mathematics classroom is categorically different from those of the past which were founded primarily upon principles of behaviorism.

Students' Goal Orientations

Given the assumption of situated cognition, constructivist learning theories support the idea that motivation depends on cognitive processes that interact with social and cultural dimensions. According to Schunk (1996), beliefs about one's own ability to learn are also constructed within a social context, and have a strong influence on one's goal-setting schema in school. Put differently, constructivists posit that motivation, which is an indisputably fundamental aspect of the learning process, is highly sensitive to environmental influences and plays a significant role in one's implicit theories of factors that contribute to understanding of content as well as performance in school. Goal orientations, or the degree to which individuals and institutions place emphasis on intrinsic and extrinsic indicators of success, are believed to be play a significant role in influencing students' academic behaviors in classrooms (Dweck, 1986).

According to this theory, classrooms typically emphasize *learning goals* (also known as task-involved goals or mastery goals) and *performance goals* (also known as ability- or ego-involved goals). Learning goals stress the seeking of challenges, true understanding of content, and mastery of tasks. Performance goals, on the other hand, emphasize the demonstration of high ability and avoidance of displays of incompetence (Ames & Archer, 1988; Dweck, 2000). Students with learning goals are governed by intrinsic interest and natural curiosity in solving tasks, as well as developing competence in a specific content area. Students with performance goals instead tend to depend heavily on others for help, avoid challenge and difficulty and instead prefer easy assignments, and are more concerned with grades than developing conceptual understanding (Newman, 1998). Across an array of classrooms (i.e., elementary, middle grades, high school), research has also shown that students with performance-orientations tend to seek shallow or imitative forms of learning, such as rote learning, and often prioritize mere answers over explanations as their primary goal is speedy completion of tasks, not necessarily gaining deep understanding (Elliot & Dweck, 1988; Meece, Blumenfield, & Hoyle, 1988; Meyer, Turner, & Spencer, 1997; Nelson-LeGall & Jones, 1990).

Goal theorists contend that instructional tasks that are optimal for learning typically involve a high degree of challenge and beg if not necessitate risk-taking acts such as conjecturing and a willingness to expose and capitalize on one's errors. On the other hand, performance-oriented instructional tasks are best for avoiding challenge or the embarrassment that may come with failing to solve them successfully, and are generally tasks that students are well-rehearsed at and are marked by a high degree of predictability, but devoid of ripe opportunities to learn something new (Ames, 1992). Given this conception of these competing orientations, one could reasonably argue that learning-oriented tasks are aimed at understanding while performance-oriented tasks are mostly successful at only imitating understanding.

Participant Frameworks

According to Empson (2003), understanding classroom discourse and how it "structures students' participation requires a fine-grained analysis of teachers' and students' interactive talk" (p. 306). Participant frameworks (Goffman, 1974, 1981) can be used to explain how discourse organizes social interaction, or specifically how student and teacher talk animates individuals into certain intellectual roles or identities, such as answer-supplier, evaluators, claim-makers, listeners, solution-reporters, questioners, etc. According to O'Connor and Michaels (1996), the teacher in a mathematics discourse community facilitates language socialization and role-taking by orchestrating interaction among the group, which aims to get students to "identify themselves as people who solve problems, construct arguments, justify claims, generate conjectures, and communicate with others formally and informally about their mathematical thoughts" (Empson, 2003). All members within a learning community position themselves and others as participants in myriad ways, but primarily through markers such as verbal and non-verbal language. For example, when a student asks a peer, "But why did you divide by one-half when there were two people sharing the cake?" the student is positioning his or her peer as a defender of and clarifier of a mathematical claim. In other words, the specific language used by one participant prompts another participant to assume a special role in discourse, in this case a justifier and clarifier. In short, a participation framework at any particular moment in classroom discourse is "the amalgam of all members' participation statuses relative to the current utterance" (O'Connor & Michaels, 1996).

According to Goffman (1974), the traditional dyadic categorization of speaker and hearer grossly misconstrues "the range of ways that humans use talk to create alliances and oppositions and to connect utterance acts with various participants" (O'Connor & Michaels, 1996, p. 69). Rather than thinking of a single speaker, Goffman contends that most utterances involve a *principal* (the person who is the source of the original content whose position has been established), *author* (the person scripting the lines), and *animator* (the person who renders another as a figure) and goes on to suggest that rather than thinking of a single listener in each case to instead view the audience as a group of both addressed and unaddressed recipients (Forman & Ansell, 2002; O'Connor & Michaels, 1996). This is not to suggest that each of these entities represent mutually exclusive social roles for separate individuals – of course, at times during conversation, it is possible for a single speaker to assume each of these three roles simultaneously (Goffman, 1981).

Empson (2003) argues that lower-performing students' success in discourse communities depends on the teacher's ability to provide space and meaning for students' contributions. Many researchers explain the struggles of lower-performing students' ability to participate in discussion-intensive instructional settings as a function of sociocognitive traits, such as a child's limited capacity for listening and responding to others' high-level explanations (e.g., Baxter et al., 2001; Lubienski, 2000a, 2000b; Mulryan, 1995). However, Cohen and Lotan (1995) suggest that even low-performing or low-status students' degree and quality of participation in discourse can increase when teachers orchestrate their interactions skillfully, (e.g., praising a student's contributions during task work, using effective scaffolding practices, etc.). Thus, the teacher's role in facilitating discourse is paramount, but that is not to say that students themselves play a marginal role in producing quality verbal interactions. The few studies to date that have employed participant frameworks as a method of analyzing discourse have focused almost exclusively on teacher-to-student interactions, giving priority to the teacher's role in facilitating discussion among students; this study used participant frameworks as a lens to analyze discourse that emerged not only between the teacher and students, but primarily among the students themselves in both whole-class and small-group instructional contexts. Studies that have moved beyond pigeonholing low-performing students as incapable of engaging in productive discourse with teachers and peers underscore the usefulness of adopting analytical lenses like participation frameworks within the context of mathematics discourse communities.

Conducting the Literature Review

A majority of the research related to this study was reviewed before collecting and analyzing data. To find this literature, the researcher used keywords identified in several of the most relevant previous studies related to mathematics discourse communities (e.g., Baxter et al., 2001; Empson, 2003; Nathan and Knuth, 2003) and also obtained relevant primary sources that were cited in these flagship studies.

Additional literature was reviewed as findings emerged from data analysis. As codes and categories emerged, the *Thesaurus of ERIC Descriptors* was used to generate relevant descriptors, which were then applied in a variety of combinations using Boolean operators to refine and narrow the search according to the contextual features of this study. Examples of ERIC descriptors used were: *adolescents, classroom communication, competition, cooperative learning, discourse analysis, discussion (teaching technique), elementary education, fractions, group discussion, group dynamics, grouping (instructional purposes), mathematics education, rational numbers, student participation,* and *small group instruction*.

I also obtained recently-published secondary source documents from *Review of Educational Research;* one addressed the teacher's role in classroom discourse (Walshaw & Anthony, 2008) while the other provided a review of the scholarship on equitable opportunities to learn in cooperative mathematics instructional settings (Esmonde, 2009).

These two reviews provided relevant and up-to-date references related to the significant themes that emerged from this study's findings.

The remainder of this chapter is dedicated to critically reviewing and synthesizing the extant empirical literature related to the research question, beginning with a review of empirical studies that capture the complexity of learning about rational numbers. Next, an overview of classroom discourse communities is provided by comparing traditional forms of discourse to a reform-based model, followed by an analysis of the literature related to challenges to building and maintaining successful discourse communities. The next section addresses student participation in discourse communities, with an emphasis on the nature of students' explanations. Finally, research on peer interactions in small-group settings is reviewed, as well as important social and emotional developmental processes that mediate such interactions.

The Complexity of Rational Number Learning

Several theories have been advanced to explain why student understanding of fractions and related rational number concepts has been so elusive. Cognitive psychologists and mathematics educators have identified several interdependent theoretical subconstructs of rational numbers (Kieren, 1976), each playing a critical role in the development of overall understanding. However, upper-elementary mathematics instructors have traditionally spent little time developing students' substantive understanding of the meaning of fractions beyond the narrow focus on part-whole shading tasks (Cramer, Post, & delMas, 2002), thus addressing only one of several critical rational number subconstructs (Lamon, 2007). Other researchers ascribe blame to the way fractions have been taught in school, specifically focusing on different types of knowledge development. In a review of studies, Moss and Case (1999) posited that student deficiencies associated with fraction learning stem primarily from a traditional instructional emphasis on syntactic knowledge over semantic knowledge. Put differently, teachers have long prioritized memorization of rules and procedures (or computation) over construction of meaning and concepts (or relationships) (Hiebert & Wearne, 1985; Klein, 2003; Schoenfeld, 1988).

Empirical studies have highlighted specific characteristics of fractions that contribute to students' difficulty in understanding them. Many scholars cite the complex notational system (Ball, 1993a; Leinhardt, 1988; Mack, 1990; Moss & Case, 1999; Post et al., 1985; Streefland, 1993) that differs greatly from children's intuitive and formal understandings of the whole number notational system. Other researchers underscore the significance of rudimentary fraction concepts, like order and equivalence, which often elude student understanding and account for later developmental obstacles (Behr et al., 1984; Cramer et al., 2002; Post et al., 1985; Smith, 1995). Studies on understanding of fraction order and equivalence point to the necessity of two prerequisite abilities: partitioning (Pothier & Sawada, 1983) and concept of units (Lamon, 1996). The ability to perform partitions on geometric figures through strategies such as fair sharing and splitting are critical concepts to grasp in the early stages of fraction learning (Pothier & Sawada, 1990; Streefland, 1993). Equivalence has traditionally been taught as a mechanical procedure disconnected from semantic representation (Cramer et al., 2002), and the role of unit has often been implicitly taught, if not overlooked altogether (Yoshida & Sawano, 2002). This body of literature clearly suggests that in order to attain a proficient level of understanding, students must fully develop a concept of the magnitude of rational numbers: As Cramer et al. (2002) contend, “without this conceptual foundation, they cannot operate on fractions in a meaningful way" (p. 129).

Many students tend to confound whole number properties with rational number properties, which typically “involves the comparison and construction of ratios based on absolute rather than on relative difference” (Streefland, 1993). Rather than applying multiplicative or proportional reasoning, students often incorrectly apply additive thinking to rational numbers. Several researchers have analyzed these misconceptions, which can manifest in various forms, such as: treating fractions as whole numbers; referring to fractional parts as whole number quantities; and using additive rather than multiplicative or proportional reasoning when operating on fractions (Behr et al., 1984; Lamon, 2006; Mack, 1990; Mix, Levine, & Huttenlocher, 1999; Moss & Case, 1999; Post et al., 1985). An experimental curriculum carried out by Moss and Case (1999) demonstrated the resounding observation that although many students learn to manipulate rational numbers successfully through traditional instructional methods, they lack a deep conceptual understanding of these numbers and tend to struggle in novel instructional contexts where some degree of flexible or adaptive knowledge application is necessary. Similarly, Behr et al. (1984) found that even when students were successfully taught to overcome their tendencies to confuse fraction properties with the properties of whole numbers, they still struggled when asked to apply their fraction knowledge to problem solving contexts. These studies collectively infer that children’s firmly rooted arithmetic conceptions of whole numbers often pose a great challenge to their learning of fractions and other rational numbers, especially when instructional focus is on syntactic knowledge.

A Brief Overview of Classroom Discourse

Traditional classroom dialogue, which generally consists of teacher initiation, student response, and teacher evaluation (i.e., IRE sequence), limits students' participation statuses to mere responders to "display questions" to which the teacher already knows the answer (Cazden, 2001, p.41). In discourse communities, on the other hand, all participants *animate* each other into specific roles and identities by responding to each other's utterances in diverse ways. One might argue that traditional patterns of classroom discourse, such as IRE, allow students to participate in discourse, however one of the most salient distinctions between IRE and inquiry-and-discussion-intensive discourse is that IRE tends to follow a linear, predictable direction of verbal exchange (i.e., teacher to student), whereas richer forms of discourse often assume a multilateral trajectory (i.e., student 1 to teacher to student 2). Truxaw and DeFranco (2008) classify the latter as *dialogic discourse* because it is better characterized by a give-and-take flow of information that uses dialogue as opposed to lecture or transmission as a central process for thinking.

According to Hatano and Inagaki (1991), information flow can also be described by the relative hierarchical status of the participants involved in an exchange. *Horizontal* interaction occurs among peers at a comparable level of ability, while *vertical* interaction occurs between a novice and an expert. While vertical interaction seems consonant with the idea of apprenticeship learning, Hatano and Inagaki assert that productive classroom discourse generally involves a balance of the two. Because of the complex nature of interactions that can occur in mathematics discourse communities, students can be called upon to assume diverse roles (e.g., evaluator, dissenter, justifier, explainer, questioner, etc.) while co-constructing mathematical knowledge. Rich and diverse verbal interactions between members of discourse community are typically characterized by an amalgam of horizontal and vertical interactions along with dialogic discourse trajectories.

Mathematics Discourse Communities

Norm Setting

Mathematics discourse, particularly the practice of students reasoning aloud or explaining their thinking, is often a new experience for students who enter learner-centered classrooms that emphasize communication of mathematical thinking; socialization into this type of learning environment often takes considerable time for some students as they adjust to revised classroom norms (Hufferd-Ackles, Fuson, & Sherin, 2004; Yackel & Cobb, 1996). Yackel and Cobb underscore the importance of establishing norms that are amenable to rich discourse; in doing so, they distinguish between *social* and *sociomathematical* norms: "The understanding that students are expected to explain their solutions and their ways of thinking is a *social norm*, whereas the understanding of what counts as an acceptable mathematical explanation is a *sociomathematical norm*" (1996, p. 461). For example, Yackel and Cobb describe situations in mathematics classrooms where students acquiesce to social cues when asked a question instead of assertively supporting their answers with mathematical explanations or justifications. During one classroom episode, a child supplied an answer before hearing other students begin to dissent. The child subsequently wavered between two different answers without providing an explanation of her reasoning. The teacher then skillfully reasoned to the child that answers cannot be rationalized by social pressure, but only by mathematical logic. Yackel and Cobb pointed out examples like this to argue that sociomathematical norms should be emphasized in addition to social norms.

Classroom culture, which has significant ramifications on the nature of discourse, is not something that a teacher necessarily creates alone, but rather is co-constructed and negotiated by the teacher's interactions with all of the students in a particular classroom (Empson, 2003; O'Connor & Michaels, 1996; Sliver & Smith, 1996). Therefore, teachers in mathematics discourse communities must remain sensitive to the unique abilities, interests, and social and emotional characteristics of the students in the class. Much of the literature on mathematics discourse communities emphasizes what teachers can do to alter the instructional environment so that it is conducive to productive discursive practices between students and teachers.

Finally, the significance of an individual child's biographical experiences with learning and doing mathematics cannot be exaggerated. By the time most students reach middle school, they have been exposed to thousands of hours of classroom socialization. Lampert (1990) argues that children's early exposure to traditional mathematics instruction often results in the formation of rigid cultural assumptions about the act of doing and knowing mathematics:

> Commonly, mathematics is associated with certainty: knowing it, with being able to get the right answer, quickly. These cultural assumptions are shaped by school experience, in which *doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher. Beliefs about how to do mathematics and what is means to know it in school are acquired through years of watching, listening, and practicing. (p. 32)

Lampert's words have continued to resonate in scores of empirical studies that have since analyzed the development of classroom discourse. This body of work has conveyed several distinct challenges involved in facilitating a discursive community of learners.

Challenges to Building and Sustaining Discourse Communities

Upon entering a reform-based discourse community, many students struggle to identify with new and often unfamiliar roles of engagement in learning because of their learning histories in traditional settings. Hufferd-Ackles et al. (2004) found that even when third-grade students felt confident that they knew the correct answers to problems presented in whole-class discussions, many were nervous about sharing thinking because they lacked experience with doing so in front of peers. Several studies of mathematics discourse communities have shown that because of inexperience, students often do not know how to explain their mathematical reasoning (Lampert, 1990; Nathan & Knuth, 2003; Pape, Bell, & Yetkin, 2003). Lampert's (1990) reflection on her work with fifth-grade students is illustrative of many of the problems demonstrated when teachers attempt to build discourse communities among a group of students with limited prior experience in discussion-intensive classrooms. Her study revealed that students' uncertainty about discourse expectations impeded their participation in classroom dialogue about mathematics. For example, when the teacher questioned a child's thinking (i.e., asked the student to explain), Lampert observed that other students automatically inferred that the child's proposed solution was incorrect. Lampert explained this tendency as the product of a social norm—similar to Yackel's and Cobb's (1996) example of acquiescing to social cues rather than relying on mathematical content—that is culturally constructed through years of prior classroom experience.

Often times, students appear to be unaware of the necessary metacognitive skills needed to explain their reasoning aloud. Lampert's (1990) students often resorted to explanations such as "I just know," "That's the way my teacher taught me to do it," or "I don't know how I figured it out" (1990, p. 56). Sometimes, they agreed or disagreed with certain students based on whether they identified those students as smart or incompetent. Lampert also found that some students succumbed to intimidation tactics when their ideas were challenged by peers; even when the teacher made it clear to students that such behavior would not be tolerated in whole-group discussions, many students continued to address their peers negatively in small-group problem-solving settings. Lampert concluded that classroom culture is among the most important dimensions of discourse communities. "When classroom culture is taken into consideration," she argues, "it becomes clear that teaching is not only about teaching what is conventionally called *content* [italics added]. It is also teaching students what a lesson is and how to participate in it" (p. 34). In sum, many of the challenges described in Lampert's study underscore the importance of sociomathematical norms (Yackel & Cobb, 1996).

Another significant challenge experienced in mathematics discourse communities stems from the constraint of time. Rich, meaningful discourse, which requires teachers to listen carefully to student's responses, encourages students to evaluate each other's claims and questions, and elicit reflection and interaction among students simply takes more time than pure transmission approaches. For example, a study conducted by Hufferd-Ackles et al. (2004) demonstrated how a teacher struggled to keep pace with the mathematics curriculum pacing schedule after she began to give students greater responsibility in the teaching and learning process. During the study, which lasted one complete school year, the teacher began the year teaching third-grade students while implementing a didactic, teacher-centered approach. However, the teacher expressed a desire to overhaul her approach to teaching, and subsequently began giving students more talking and teaching roles, such as asking students to explain their thinking rather than soliciting answers in isolation. Although the researchers reported great success in transitioning from a traditional- to a reform-based discourse community, they also noted that instructional lessons that previously took a day to finish began to take multiple days once the students took a greater role in facilitating classroom discourse. One might argue that the tradeoff made by the teacher in this case was that she sacrificed quantity-of-learning for depth-of-learning, an argument that has surfaced from time to time in support of the current standards-based reform movement (Stein, Smith, Henningsen, & Silver, 2000).

Time constraints also pose an issue for conceptual knowledge development, which typically demands more time than traditional instruction on procedural knowledge alone (Post, Wachsmuth, Lesh, & Behr, 1985). Moreover, in reform-based mathematics classrooms, students are encouraged to use multiple representations to construct knowledge rather than relying exclusively on symbolic or procedural algorithms that stress speedy production of answers (Cuoco & Curcio, 2001). One result of encouraging students to represent their thinking in diverse ways is the additional consumption of time and effort required to do so. Historically, it is important to point out that early 20th century advocates of social efficiency or Taylorism in schools and classrooms, such as E. L. Thorndike and Franklin Bobbitt, espoused transmission approaches like IRE because of the large quantities of information that could be processed within relatively short periods of time (Tyack, 1974; Urban, 2004).

Other studies have touched upon the notion of unequal participation in mathematics classroom discourse. Baxter et al. (2001) conducted a qualitative study of 16 low-achieving third grade students' participation in classroom discussion. The researchers observed 34 whole-class lessons of teachers implementing reform-based mathematics instruction from the *Everyday Mathematics* program (Bell, Bell, & Hartfield, 1993). Throughout all 34 observations, only on three occasions did low achievers even volunteer to speak; when they did volunteer, they "offered one-word answers or remained silent while a peer spoke" (p. 536). Even when teachers attempted to induce their involvement in class discussions, these students tended to offer simple one or two word responses, or teachers resorted to oversimplifying the questions they asked of these students. Low-performing students were often off-task and disengaged during whole-class discussions, as classroom discussions were dominated by the most articulate and high-performing students. On 24 of 28 observed instances while working in pairs or small groups, lower attaining students relegated themselves to menial roles, merely copying their partner's work or volunteering to organize materials rather than being responsible for providing reasoning and solutions. In short, their roles "tended to be supportive rather than substantive" (Baxter et al., 2001, p. 540).

Studies by King (1993) and Mulryan (1995) revealed similar findings. King (1993) examined the interactions among two low- and two high-achieving third grade students as they work collaboratively towards solving four separate mathematical tasks. Task discussions were dominated by high-achievers, who were largely responsible for initiating ideas and providing most of the answers. Low-achieving students reported being confused for the majority of the task discussions. When they did make contributions, low-achievers focused exclusively on the procedural aspects of the task and reported not being able to comprehend the contributions of their peers. Low-achievers also explained that they wanted to make important contributions to the group, but they felt rushed to do so and consequently "left out", as high-achievers often outpaced them when "crunching" numbers or deciding how to solve a problem. All in all, across four separate tasks, only ten requests for help were issued, all by low-achievers. When low-achieving students sought help from their peers, they often received vague explanations, if any at all.

Mulryan (1995) studied the participation of 48 fifth and sixth grade students during small group mathematics task work in three different schools. Specifically, she investigated time-on-task using a three-point rubric to analyze students' involvement in discussion, of which her a priori categories were a) on-task: engrossed, b) on-task: involved but not engrossed, and c) minimally on-task. After seven weeks of observing students in both whole-class and small-group settings, Mulryan concluded that all students spent a significantly greater percentage of time on-task during small-group discussions than they did during whole class-discussions, but that low-achieving students spent significantly less time on-task than high-achievers in both settings. Like King (1993), Mulryan (1995) observed that low-achievers asked significantly more questions than high-achievers. Finally, students reported that speed of task completion was a criterion for success in small group task settings, which Mulryan conjectured may have had complex psychological and social ramifications for low-achieving students especially.

Lubienski (2000a, 2000b) studied the participation of 8 seventh grade girls of various SES and academic backgrounds in a researcher-taught mathematics class. Lubienski interviewed these students at the beginning, middle, and end of the school year about their perceptions of participation in a reform-based mathematics instructional setting. She also tape-recorded 14 lessons and analyzed them by coding student contributions across 20 categories developed from analysis of interview data. Lubienski found that lower-SES students were not comfortable with the roles they were asked to assume in her classroom. She also found that lower-SES pupils were often timid during class discussions and later reported that whole class discussions tended to confuse rather than inform these students. Lubienski (2000a) acknowledged that her analysis was primarily filtered through cultural deficit theories, but she did not clearly elaborate on whether or how she addressed the development of classroom norms that had the potential to promote safe exchange of ideas, risk-taking, and mutual respect for participants. The reader is ultimately left to wonder how this might have affected participation by lower-attaining students. Several cases of students making cruel gestures and remarks to other students, directly and indirectly, are mentioned throughout the study, but no analysis of social norms is proffered.

Lubienski concluded that discourse-based pedagogy might privilege students of higher ability or from home cultures that nurture argumentative discourse as a means toward intellectual development. Other studies support this assertion, particularly to the extent that access to participation in mathematics discourse communities is a function of the disconnect between informal home discourse and formal school discourse (Gee & Clinton, 2000; Walkerdine, 1988). According to such an explanation, parents of high-SES or high-achieving children tend to use hints or scaffolds that emphasize self-questioning and questioning of others rather than simply giving their children answers. Children gradually internalize this discursive practice and as a result, are better able to engage in rich forms of talk (what Truxaw and DeFranco (2008) would likely deem as "dialogic discourse"), such as elaborating and questioning rather than providing simple statements or ideas (Wood, 1989). Ridlon's (2001) case study of a seventh grade student who resisted non-traditional problem-based pedagogy because of his cultural beliefs about teacher authority and traditional student roles lends additional credibility to theories that focus on home-and-family influences on interactive talk within a classroom context.

Although these studies effectively problematize the notion of participation in discourse communities, they are not clear on if and how they used multiple representations (e.g., graphical, verbal, pictorial, etc.) to support students' participation in classroom discussion. These representations serve not only as a vital instructional scaffold for students, but primarily, they serve as additional tools and objects that mediate the process of meaning-making. Oral discussion without the support of rich and meaningful representations of students' thinking fails to provide a key instructional scaffold for students who need them in order to communicate their mathematical thinking effectively (Goldin & Shteingold, 2001). It is also important to reiterate that NCTM's (2000) definition of discourse includes representations as a critical medium of communication. A transparent focus on multiple representations is an absolute necessity if one of the goals of classroom discourse is to provide equitable access to all students.

Lastly, the majority of the aforementioned studies were conducted by outside researchers. While there are distinct advantages to this scenario, it is possible that students' behavior was significantly altered by the presence of strangers in the classroom. For instance, some children may have been more timid about speaking in front of the researchers. Also, as Mulryan (1995) points out, her limited rapport with and background knowledge about the students she studied may have also caused her to overlook potentially significant factors that affected students' participation. Of course, the design of this study features a teacher who is also the researcher. However, unlike several of the studies that involved researchers doubling up as the classroom teacher for a few weeks, this study was orchestrated by a researcher who was a permanent full-time teacher.

The Teacher's Role in Mathematics Discourse Communities

The literature on discourse communities prioritizes the role of the teacher in creating and maintaining an instructional environment that invites rich and productive forms of mathematics discourse. The literature is therefore replete with narratives of teachers' diverse experiences with facilitating discourse communities, much of which includes in-depth analyses of the degree to which teachers should be involved.

One resounding theme in the literature deals with the pretense that is created by merely increasing the percentage of student talk. For example, a study conducted by McClain and Cobb (2001) described a teacher who used strategy-sharing in whole class discussions to elicit a greater degree of student talk. However, the teacher made no substantive attempt to synthesize students' ideas, or compare and contrast them with one another. In many cases, he failed to revoice student contributions to the class, which suggested that he assumed a) that other students had no problem comprehending these contributions, and moreover, b) that *his* interpretations of students' contributions were consistent with *their* own interpretations. O'Connor and Michaels (1996) contend that these two specific assumptions often undermine the quality of discourse in a classroom that prioritizes student talk. Furthermore, Jacobs et al. (2010) argue that negligent pedagogical practices, like failing to carefully listen to, interpret, and effectively utilize students' contributions, generally inhibit the development of students' mathematical thinking. A study conducted by White (2003) further suggests that successful interactive discourse depends largely on the teacher's willingness to place all students at the forefront of instruction. Her observational data led to the emergence of four key themes that promote rich classroom discourse for all students: (a) valuing students' ideas, (b) exploring students' answers, (c) incorporating students' background knowledge, and (d) encouraging student-to-student communication. These studies collectively suggest that an affective element of care and desire to listen to and empower students is central to a teacher's ability to successfully promote rich discourse for all students.

Because students naturally do talk more in reform-based classrooms, listening carefully to student contributions is therefore yet another integral facet of mathematics discourse communities (Ball, 1993b; Jacobs et al., 2010; O'Connor & Michaels, 1996; Yackel & Cobb, 1996). Ball (1993b) refers to this as the "teacher's capacity to hear children" and respect their thinking, even in cases where a student appears to be applying reasoning that is partially correct and incorrect at the same time. Jacobs et al. (2010) stress the significance of three important teacher behaviors: "attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172). Ball (1993b) and other researchers (e.g., Cazden, 2001; Lampert, 1990) often attribute teachers' ability to effectively enact such behaviors as their *pedagogical content knowledge* (Shulman, 1986), that is, their ability to effectively synthesize their knowledge of mathematical content with their knowledge of high-quality pedagogy. Pedagogical content knowledge, according to Shulman (1986), is what separates merely intelligent teachers from effective teachers.

One reason that listening carefully to students' contributions is so important in mathematics discourse communities is because teachers are often called upon to mediate students' contributions. That is, the teacher necessarily assumes a large portion of the responsibility in assisting listeners with comprehension of individual speakers' utterances. Several researchers have studied teachers' attempts to make individual student contributions accessible to all students in the class, which is primarily done through the act of revoicing (Empson, 2003; Forman & Ansell, 2001; O'Connor & Michaels, 1996), where a teacher rebroadcasts a student's contribution back to the student as well as the entire group. Revoicing is essential for several reasons: First, it allows a teacher to capture the gist of student contributions but also re-express the novice's thoughts in terms that are more clear, coherent, and succinct. Because students typically struggle to articulate their thinking by using highly fluent expression during exploratory talk (i.e., talk about content that is relatively new to students or within their zone of proximal development), revoicing is often necessary to provide clarity for all students. Second, it allows teachers to position themselves as inference-makers, rather than the sole validating authority of student contributions, allowing individual children to maintain ownership of original ideas while setting a context that enables the group to reflect on each other's contributions (O'Connor & Michaels, 1996). A study conducted by Forman and Ansell (2001) revealed that students were more involved in providing explanations of their reasoning in classrooms where revoicing was strategically implemented by teachers. However, in a different study, the practice of revoicing appeared to often interrupt the flow of discussion (Hufferd-Ackles et al., 2004), suggesting that the nature of revoicing as an effective instructional strategy is complex.

Many studies have analyzed the importance of teacher decision-making related to "stepping in and stepping out" of the classroom discussion in order to optimally facilitate student engagement in learning (Empson, 2003; Goos, 2004; Hufferd-Ackles et al., 2004; Fraivillig, Murphy, & Fuson, 1999; Nathan & Knuth, 2003; O'Connor, 2001; Turner, Meyer, Midgley, & Patrick, 2003; Williams & Baxter, 1996), which embodies sociocultural notions of both scaffolded instruction and legitimate peripheral participation (Lave & Wenger, 1991). For example, just how involved should a teacher be in a mathematics discourse community? And are students capable of engaging in productive discursive practices without the direct involvement of the teacher? A study by Nathan and Knuth (2003) of a teacher's attempt to increase the level and depth of student-to-student interactions in a mathematics discourse community illustrates the complexity of such a goal. The researchers conducted a two-year study of a middle school math teacher and her academically diverse sixth grade classroom in which they videotaped weekly lessons and conducted regular interviews with the teacher about student interactions. During the first year of using discourse-based teaching practices, students speaking accounted for 28% of the speech acts, however students only addressed each other 1.2%

of the time. The teacher was the "hub of these whole class conversations" (p. 191), often mediating all student utterances. Her scaffolding was twice as likely to focus on mathematical content (i.e., *analytic scaffolding*) rather than norms of social participation in class (i.e., *social scaffolding*). The following year, the teacher sought to increase student-to-student interactions by monitoring and reducing her own role as a participant in classroom discussion. Student-to-student interactions increased from 1.2% to 33% of all whole classroom talk, but the teacher's analytic scaffolding dropped precipitously. Because this teacher "essentially removed herself from the analytic aspects of the classroom discourse and gave her attention primarily to the social aspects, there was no clear authority for students to turn to in the face of their uncertainty" (p. 198). The researchers concluded that this teacher may have thus compromised student learning for the sake of increasing student-to-student talk. O'Connor (2001) also reflects on the complexity of teacher moves in discourse settings, such as decisions to refrain from correcting a student's imprecise or inaccurate language use, or perhaps being faced with the difficulty of choosing to emphasize a student's miscalculation in spite of a well-formed conceptual explanation:

> In exploratory talk, students are maximally unclear because they themselves are under the greatest processing demands: they are trying to figure out new ideas and present them in public in coherent fashion. The teacher needs to understand them, to keep track of the sequence of contributions, and to monitor what other students are understanding, as well as to plan her own responses in a conversationally appropriate two or three seconds. The ideas the students are proposing are tenuously stated and tenuously conceived. A superb insight might be couched within a contribution that contains a hideously incorrect computation. What to focus on, and when? And how to decide? (p. 175)

O'Connor contends that, depending on the instructional context (e.g., episodes of exploratory talk versus summative review), a teacher must adjust her criteria for what counts as an acceptable explanation from students. Put differently, but similar to Wertsch's (1985) description of apprenticeship versus school-like instruction, O'Connor (2001) believes that teachers should push students to strive for correctness and accuracy when talking about relatively simple concepts or review material, while perhaps allowing greater flexibility during discussion involving relatively unfamiliar or perhaps difficult material.

Several studies provide portraits of pedagogical approaches that extend discussion beyond superficial purposes (Fraivillig et al., 1999; Franke & Kazemi, 2001; Pape et al., 2003; Stein, Grover, & Henningsen, 1996). An analysis of exemplary teachers who used classroom discourse as a central feature of mathematics instruction yielded three major pedagogical elements: eliciting, supporting, and extending (Fraivillig et al., 1999). When teachers modeled these behaviors and encouraged students to gradually co-participate in these practices on a consistent basis, they helped enrich classroom discourse and develop mathematical agency in students. Pape et al. (2003) observed a teacher that used an explicit self-regulated learning approach to guide her discursive interactions with students. Informed by three distinct phases of self-regulation (i.e., forethought, volition control, and self-reflection), the teacher was successful in eliciting richer levels of talk about mathematics for most students, especially those within the average achievement range. The researchers did note that low-achievers and high-achievers found some aspects of the self-regulated approach to be "bothersome" or "unnecessary" (p. 194), but they also were very clear in describing how the use of multiple mathematical representations were used to scaffold children's understanding. Finally, a teacher in O'Connor's study (2001) enriched classroom discourse by challenging students' assertions with the use of counterexamples to encourage critical thinking about rational numbers. These counterexamples initiated shifts in discourse, in which children were afforded the opportunity to move the discussion of mathematical content into spheres of greater abstraction. Cobb, Boufi, McClain, and Whitenack (1997) refer to such shifts as *mathematizing discourse,* because it naturally places students into the act of "doing mathematics," which often provides fertile ground for enriching students' understanding of mathematical relationships.

Overall, the research on mathematics discourse communities provides an optimistic view of meaningful mathematics teaching and learning. One criticism, however, of the body of literature in general is that analytical emphasis is placed exclusively on classroom culture and the teacher's role in orchestrating productive discourse and, interestingly, a majority of the research on mathematics discourse communities is published in mathematics education journals. More importantly, little attention is devoted to students' individual emotional and social characteristics that seem to play a critical role in influencing participation in the discourse community itself. Unsurprisingly, much of the literature on children's active involvement in talk about mathematics comes from educational psychology, as the analytical focus rarely extends to classrooms and teachers and is instead placed on students' social cognition as well as critical interactions between self and others. What follows is a review of some of the most relevant literature pertaining to students' participation in classroom-based discursive practices.

Student Participation in Discourse

Peer Interactions in Small-group Settings

Most empirical studies that have applied a micro-level analysis of student participation in interactive classroom talk are found within the cooperative learning literature. Cooperative learning, also known as small-group learning, has been researched extensively over the last three decades (Esmonde, 2009). In small group settings, students are afforded more opportunities to speak than in whole-class settings, and therefore can play a greater role in contributing to their own learning and to the learning of other students as well. Since learning is peer-directed in most small-group settings, interactions that occur among students are among the most critical variables that affect learning outcomes. Webb (1991) asserts that the outcomes of small-group learning cannot be fully explained without systematic study of group processes, and more specifically, analysis of task-related verbal interactions that occur in small-group settings. The following studies examined peer interactions in small-group mathematics instructional settings.

Individual and group characteristics are often associated with the quality of peer interactions during mathematics task-related discussions. A meta-analysis conducted by Webb (1991) summarized the most salient predictors of peer interactions as: (a) the nature of the questions students asked one another, (b) ability of the individual, (c) composition of the group based on ability differences, (d) personality characteristics of the individual, (e) gender of the student and (f) gender composition of the group.

One would be safe in assuming that ability is correlated with group interactions, for many empirical studies have confirmed this finding. Studies have repeatedly shown that higher-performing students tend to provide most of the explanations during small-group task discussions (King, 1993; Mulryan, 1995; Webb & Mastergeorge, 2003). There is even some evidence that suggests individual ability is related to whether or not a student receives adequate and relevant assistance from peers when requested. Although some studies have found no relationship between ability and not receiving help, Webb (1984) found that low-performing students received help less frequently than their higher-performing peers when they requested it.

Some empirical work has examined the relationship between the composition of small groups and the interactions that occur within, specifically with respect to ability differences. Although these studies have yielded contradictory results, the evidence is reasonably conclusive that all students are significantly more involved in providing explanations and asking questions when the range of differences in ability among group members is not extreme (Webb, 1991). Specifically, studies have shown that a greater degree of equal participation exists in groups with a moderate range of ability (e.g., medium to high, medium to low, not high to low) (Webb, 1982, 1984; Webb & Cullian, 1983; Webb & Kenderski, 1984).

Research shows that receiving help from peers can be positively correlated with mathematics achievement, but this depends in part on the quality of the content-related help provided (i.e., whether the help provided is the mere statement of an answer to a problem or if it is a detailed explanation of the content). Webb (1991) conducted a meta-analysis of studies that examined mathematics task-related peer interactions in small-groups and found that while receiving detailed explanations is only sometimes positively related to achievement, there is overwhelming evidence that "receiving less elaboration than is needed, such as asking for an explanation and being told only the correct answer, is negatively related to achievement" (p. 376), underscoring the importance of productive interactions among students in mathematics discourse communities. Webb's meta-analysis also examined studies on effective peer interactions aimed at help-seeking. The majority of studies reviewed by Webb suggested that whether or not students receive high-quality, responsive help significantly depends on the nature of the request made for help. For instance, specific requests for help, such as, "Why did you multiply 2 by 1/2?" are much more successful in eliciting elaborate and appropriate explanations than general requests, such as "I'm confused" or "I don't get it."

There has been much public debate about the impact of small-group learning for higher-achieving students. To what extent is participation in cooperative learning beneficial for the students that are the highest achieving? Webb's (1991) meta-analysis addressed this question, as well, and found that when higher-performing students gave content-related explanations to lower-performing students in small-group settings, their mathematics achievement increased. Webb conjectured that the cognitive restructuring needed to provide an accurate and detailed explanation of mathematical content helped contribute to the increase in achievement.

In order for student interactions to be optimally effective for help-seekers, the student seeking help must use the explanation provided by the peer helper to solve similar problems or execute similar tasks independently. Studies have found this to be one of the most conclusive variables that influence effectiveness of peer interactions in small groups. For instance, Webb and Mastergeorge (2003) examined the behavior of seventh grade students who prompted peers for help during small-group mathematics task work as well as the interactive processes that enabled or obstructed their ability to solve multi-step problems involving the use of decimals. Specifically, the researchers wanted to investigate why some students demonstrated success after receiving detailed explanations from peers whereas other students who also received well-articulated explanations did not. After analyzing the interactions of 48 students that sought help during small-group task work, the researchers found that students' overall success varied according to observed behavioral differences in three domains. First, students who asked for specific explanations for why certain numbers or procedures were used were much more successful than students who sought only answers or calculations through general requests for help, such as "I don't get it," or "I am confused." Second, successful students were much more persistent in seeking help by asking questions iteratively, modifying them as necessary, until they fully understood the explanations provided by their peers. Unsuccessful students, on the other hand, often accepted others' answers or suggestions without asking for clarification of their explanations. Finally, before and after asking for help, successful students often attempted to solve the problems independently, whereas unsuccessful students often did not attempt to solve problems independently before asking for help, and made few attempts to solve similar problems independently after receiving explanations.

Social and Emotional Dimensions of Peer Interactions

Developmentally, adolescence is a period in which children begin to develop a greater awareness of self-concept, while becoming more attuned to social comparisons. An elevated need for peer acceptance is accompanied by a greater need for autonomy (Berk, 2005). The net effect of these opposing changes sometimes manifests as passivity in the classroom (Eccles & Midgley, 1989). As students begin to demonstrate greater awareness of social comparison among peers, particularly with regard to academic ability and social competence, they naturally develop perceptions regarding status hierarchies that in turn play a significant role in students' quality and frequency of classroom participation. Research suggests that students in classrooms and small groups develop status orders based on perceived differences in ability and social attractiveness (Cohen & Lotan, 1995; Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997; Nelson-LeGall & Glor-Schieb, 1986; Newman, 2000). Cohen and Lotan (1995), for example, claim that higher-status students tend to interact and participate more frequently than lower-status students, and that perhaps more significantly, these differences in participation can lead to unequal learning outcomes.

Developmental research has conclusively shown that as students age, they become increasingly more adept at self-regulating their learning (Berk, 2005), which includes (but is not limited to) seeking help from peers and adults through verbal interactions. But while some students consistently engage in productive interactions, like adaptive help-seeking (e.g., asking for clarification aimed at specific rather than general aspects of a vague explanation), many instead assume a passive role in the learning process, for example by infrequently seeking academic assistance when needed (van der Meij, 1988) or excessive help-seeking attempts (e.g., asking only for an answer rather than an explanation) (Nelson-LeGall & Glor-Schieb, 1986). There are several complex social and emotional dimensions that influence peer interactions in the classroom.

Much of the help-seeking literature from educational psychologists acknowledges the fundamental role of academic goal structures in classrooms and personal goals of individual students (Ames & Archer, 1988; Butler, 1993; Elliot & Dweck, 1988; Newman, 1998). A study by Butler (1993) revealed that students with learning goals were more likely to correct errors, resolve disputes or impasses, and move the group toward mastery of the task. Students with performance goals, however, demonstrated excessive forms of help-seeking, such as asking peers for answers to problems without first trying to solve it themselves. Newman (1998) conducted an examination of the influence of achievement goals (i.e., task vs. performance goals) on 78 fourth and fifth graders' help-seeking behaviors during small-group mathematical task work. The researcher observed students interacting across eight separate classrooms (all with varying achievement goal-structures), and measured each student's personal goal affiliation. Newman concluded that while learning-goal oriented students placed in learning-goal oriented classrooms demonstrated the most adaptive forms of help-seeking (e.g., asking for specific explanations by using specific questions), findings revealed that students with strong performance goals engaged in more adaptive forms of help-seeking in classrooms that prioritized learning goals than those who were placed in classrooms that emphasized performance goals. In other words, Newman (1998) argued that congruence between personal and classroom goals, namely learning-oriented goals, results in the greater likelihood that children who need help will seek it out in adaptive ways.

Students' reluctance to ask for help when confused can be explained by several logical reasons. Anxiety stemming from perceptions of dumbness associated with help-seeking manifest early in children and can become more intense in middle school. Graham and Barker (1990) found that as early as age 7, children begin to equate the act of receiving help from peers or teachers as a social cue for low-ability. Newman and Schwager (1993) found that third graders were more likely to identify students who asked the most questions as the "dumb kids" rather than the "smart kids." In a study conducted by Newman and Goldin (1990), second grade students expressed fears that teachers might think they were "dumb" as a result of asking questions. These researchers interviewed 65 students in second, fourth, and sixth grade regarding their perceptions about asking questions in class and found that across all grade levels, students perceived teachers as more helpful than peers in being able to effectively respond to their questions. Students also expressed far greater concern about the possibility of receiving negative reactions (i.e., being perceived as "dumb") from peers than teachers as a result of asking questions aimed at seeking help with mathematical content. In other words, there was a greater degree of social risk involved in asking questions of peers. Similar to Webb's (1991) findings, Newman and Goldin (1990) found a positive correlation between the frequency of question-asking and achievement.

Ryan, Gheen, and Midgley (1998) investigated help-avoidant behaviors of 516 sixth-grade students and found that students with lower levels of self-efficacy regarding their mathematics ability reported significantly higher frequencies of help-avoidance. The researchers also measured students' individual goal structures (i.e., learning oriented vs. performance oriented goals) and found that students with intrinsic, task-oriented learning goals reported fewer incidents of help-avoidance, whereas students with extrinsic, performance-oriented goals engaged in higher levels of help avoidance. In other words, the worse they felt about their ability to do well in math, the more they tended to eschew potentially helpful interactions. Collectively, these studies on help-seeking avoidance suggest that students are often burdened with the task of attempting to negotiate competing social and academic goals.

Some research has addressed how social status characteristics of individual students, such as perceived popularity or ability, influence group interactions. The findings of these studies build on expectation-states theory (Berger, Cohen, & Zelditch, 1972) ,which explains how social status characteristics structure social interaction in small groups. Berger et al. (1972) performed laboratory experiments in which people of equal ability and status performed group tasks together but were informed by researchers that certain individual members of the group possessed higher status or educational attainment. Each participant was presented with a task, which was to be solved independently at first, followed by a group discussion of the task, and then a final phase in which participants could revise their original solutions. The researchers found that people with lower status tended to be influenced greatly by those with higher status, and that higher status individuals were rarely influenced by their lower-status counterparts. In sum, the theory contends that individuals form specific performance expectations about other individuals based on high and low states of a status characteristic.

Since Berger et al. (1972) generated this theory, educational researchers have attempted to extrapolate and test this theory in actual classrooms. Cohen and Lotan (1995) designed a classroom experiment in which teachers across 13 different classrooms (grades 2 through 6) implemented two interventions ("multiple ability treatment" and "assigning competence to low-status students"), both of which were aimed at increasing the participation rates and academic influence of low-status students. The researchers used sociometric measures to measure two salient social status characteristics germane to classroom interaction research: popularity (as measured by the number of friends one has), along with perceived ability (which was measured by the number of students who identify one as being smart) in order to quantify status levels of each student involved in the study. Cohen and Lotan hypothesized that low-status students would participate more during small-group interactions with higher-status peers because teachers a) emphasized the importance of multiple intelligences in being able to resolve complex tasks and b) gave students instructional feedback that was public, very specific, and positive in nature. The researchers found that greater rates at which teachers used status treatment interventions resulted in significant increases in the participation of low-status students, while demonstrating no negative effect on the participation rates of high-status students. Several other studies have confirmed these findings in general (Bianchini, 1999; Cohen, Lotan, Scarloss, & Arellano, 1999; Dembo & McAullife, 1987; Lotan, 2003).

There is also evidence that social status is related to the type of help that low-status students seek from their higher status peers. Using sociometric measures of peer status and academic competence, Nelson-LeGall and Glor-Schieb (1986) recorded extensive observations of third and fifth graders in small-group mathematics cooperative learning settings. Interestingly, the findings revealed that students who were not well-liked by their peers and students who were perceived to be weak at mathematics tended to solicit excessive forms of help-seeking interactions (i.e., asked peers for answers only, rather than explanations or demonstrations of the content). By contrast, the researchers found virtually no correlation between how well-liked a student was perceived to be by peers and the tendency of that student to solicit instrumental help-seeking interactions (i.e., asking a peer for an explanation or demonstration, rather than a mere answer).

These findings also suggested that the relationship between peer status and the nature of peer interactions is highly sophisticated, and may vary based on the type of help requested from peers. Nelson-LeGall and Glor-Schieb (1986) additionally speculated that children who seek excessive help from more-able peers may actually be perceived by their peers as possessing low social status because of assumptions related to free-rider effects (i.e., more-able peers may develop resentment for less-able peers because they are suspected of free-riding).

Finally, one is likely to assume that friendship may be related to the interactions. Several studies have examined the effects of friendship and peer interactions, however the results are inconclusive. A few studies suggest that most students prefer to interact with close friends instead of peers whom they barely know or rarely associate with (Azmitia & Montgomery, 1993; Strough, Berg, & Meegan, 2001; Zajac & Hartup, 1997). These studies also found that students are more likely to dissent ideas and work productively through disagreements when grouped with peers they consider to be friends. On the other hand, research has also shown that students sometimes prefer to work with strangers or peers whom they do not consider close friends and that this preference is not subsequently related to the quality of interactions (Mitchell, Rosemary, Bramwell, Solnosky, & Lilly, 2004; Walker, 2006).

Summary of Empirical Research

Collectively, the literature on peer interactions during small group discussion of mathematics tasks suggests that participation in discourse at the middle school level is complex due to the interaction of various social and emotional dimensions. The literature on the teacher's role in facilitating mathematics discourse has optimistically overlooked these important developmental aspects, especially within the context of small-group learning, where the teacher is often not present to facilitate discourse. Because rational number learning also presents significant cognitive challenges for many middle grades students, the nature of middle grades students' participation in mathematics discourse related to rational numbers clearly warrants further empirical investigation.

CHAPTER 3

METHODOLOGY

The goal of this study was to describe the nature of low- and high-performing students' participation in discourse about rational number tasks in a standards-based sixth grade classroom. I used a multiple case-study design to examine the interactions among students as well as the contributions they made during discussion of challenging mathematical tasks. Merriam (1998) concludes that case-study methodology is an appropriate design choice when a single, delimited object of study (i.e., a case) warrants intensive and holistic analysis. Yin (2003) adds that when investigation of a phenomenon occurs within its real-life context, and analytic generalization (i.e., theory development) is a goal of scholarly inquiry, case study design is often a sound choice.

One defining characteristic of qualitative research is its fluid nature. As such, qualitative inquiry cannot be employed in a mechanistic or prescriptive format (Ernest, 1996; LeCompte & Schensul, 1999a). However, most researchers agree on a certain degree of consistency with regard to effective methods of data collection, analysis, and report writing (Creswell, 2003). In keeping with such recommendations, this chapter begins with a description of the context in which the study was conducted, including a description of the research setting and the focal participants. Included in this description of the study context are aspects of the mathematics classroom in which data collection took place are detailed, including ways in which cultural norms regarding participation were introduced and reinforced. The two major small-group tasks are then described in order to provide the reader with a context of the instructional content that students encountered during the study. Finally, the methods used to collect and analyze data while maintaining trustworthiness are explained, with particular emphasis on describing how findings were generated from analysis of raw data. This chapter concludes with a summary of the overall study design.

Context

Setting Description

The setting in which this study took place was a suburban middle school (grades 6-8) in a large metropolitan area located in the Southeastern U.S. Pryor Middle School (a pseudonym) is located is one of the fastest-growing and wealthiest counties in the nation, and was one of five new schools opened during the 2009-2010 school year. The district itself serves over 30,000 students across more than 30 schools.

The school district's commitment to cutting-edge instructional technology is evident in that all schools own several portable notebook stations and every classroom from grades K through 12 features at least 4 networked desktop computers for student use, as well as an interactive whiteboard and LCD projector. Each year, the county budget allows for substantial expenditures toward the purchase of site-licenses and subscriptions to educational web or software-based programs such as explorelearning.com GIZMOS, BrainPOP, SAFARI Montage, United Streaming, HoughtonMifflin's Skills Tutor, and educational data management and communication programs such as Edusoft, InfiniteCampus, and AngelWEB. While boasting one of the lowest millage rates in the metropolitan area, the school district allocated nearly $200 million toward instructional purposes for the 2010 fiscal year, despite recent state- and county-wide budget cuts.

Pryor Middle School is fairly homogenous with regard to its demographic makeup. 83% are White, 8.3% Latino, 5.3% Asian or Pacific Islander, 1.7% African-American, and 1.7% Multi-racial; approximately 4% of the students qualified for the federally-funded free- or reduced-lunch program in 2009. Students at Pryor Middle fall mostly in the average-to-high performing range on standardized tests of achievement. For instance, over 90% of all students passed the annual state criterion-referenced exam in mathematics in both 2009 and 2010. During the 2009-2010 school year, 98.4% of the focal teacher's 127 students met expectations[1] on the state test, while two-thirds exceeded expectations.

Pryor Middle School, like all public schools in the state, is dedicated to implementing and evaluating standards-based education. The principal at Pryor encourages teachers to make the content standards visible and understandable to all students. Many teachers at Pryor Middle post the actual wording of the standards on their walls for student viewing and report using the language of the standards while teaching them to students. Each quarter, the students are assessed on mastery of the standards outlined by the county's curriculum pacing guide, and teachers subsequently engage in "data digging" by identifying standards of concern related to these quarterly testing outcomes. County-wide professional development activity over the last two years has been explicitly driven by improvement of standardized testing results related to the content standards by encouraging teachers to collaboratively reflect on and generate a

[1] "Met Expectations" is defined as a specific raw percentage of correct items, which turned out to be 49% on this exam. Therefore it is important to recognize that "meeting expectations" is not necessarily tantamount to "proficiency" or what schools traditionally consider "passing" (i.e., 70% or above).

variety of ideas for realizing higher achievement outcomes. The county also recently adopted a state reform initiative which would allow individual schools greater flexibility and local control in return for increased accountability regarding achievement outcomes consistent with the No Child Left Behind Act. The principal at Pryor Middle has openly subscribed to this initiative.

The students who participated in this study were those enrolled in the researcher's very own co-taught sixth grade inclusion math class. This group was composed of several lower-performing students as defined primarily by their recent performance in mathematics on the state standardized assessment, but also of average- and high-performing students determined by the same criteria, as well as students who received various special education services.

The Teacher-Researcher

The author of this dissertation served as both the classroom teacher and the lone researcher in this study. In order to avoid confusion for the reader, the teacher will be identified by the use of a pseudonym (Mr. Yorke).

Norm-setting and Community Building

Data collection for this study began approximately seven weeks after the school year started. Given important considerations related to the establishment of classroom norms discussed in Chapter 2, the teacher attempted to create an atmosphere conducive to eliciting rich, dialogic discourse. Table 2 demonstrates interventions aimed at developing a well-connected classroom community.

Table 2

Norms and Corresponding Classroom Interventions

Norm	Intervention
Social	Personal introduction of teacher
	Partnered personal interviews and introductions
	Mini-vignettes and discussion about conditions needed to foster safe and open exchange of ideas
	Construction of "listening" and "when you need help" norms
	Continued reference to list of class social norms
Sociomathematical	Lesson: "What is an acceptable mathematical explanation?"
	Whole-class evaluation of student writing samples
	Dissemination of explanation (speaking) norms
	Continued reference to list of class sociomathematical norms

In order to begin building community among students on the first day of school, the teacher introduced himself to the students by sharing photographs of his family as well as photos and videos of him engaging in various hobbies, such as music and sports. The teacher expressed hope that this would help his students to perceive him as affable and approachable.

Because most students who entered Pryor Middle School in 2009 came from one of three "feeder" elementary schools, many students were familiar with each other at the beginning of the school year. In fact, the two low-performing participants in this study previously attended the same elementary school, as did both high-performing students (but each pair of students attended separate schools). Because some children appeared to gravitate to pre-established social cliques on the first day of school while others seemed socially isolated, the teacher decided to "shake up" the class so that they might make new acquaintances and friendships. Therefore, students were randomly assigned to pairs to interview each other about their background and interests before concluding with a partner-initiated introduction of one another to the class.

On the second day of school, the teacher facilitated a class discussion about class participation and communication of mathematical ideas. Students were assigned to groups and asked to think about conditions necessary for allowing open and safe exchange of ideas. This activity was followed by a discussion in which the teacher recorded a list of important ideas generated by the students. Finally, the teacher presented three mini-vignettes aimed at encouraging the students to think about desirable behaviors needed to sustain a healthy community of learners. Students responded to these vignettes and discussed their responses in small groups. The lesson concluded with a whole-class discussion of the students' reflections, and the teacher subsequently integrated the most salient ideas into a list that became the official class norms (see Appendix A).

Over the next few weeks, the teacher reviewed these class norms whenever a relevant situation occurred. For instance, during a warm-up discussion a few weeks into the year, one student was mocked by a few of his peers after sharing an answer that they apparently deemed ridiculous. The teacher intervened immediately by referring all students to the class norms and facilitating a brief discussion on the importance of respecting everyone's contributions. The students who laughed at their classmate apologized and the lesson moved on. The teacher was consistent with reinforcing classroom norms by following this same general procedure when necessary. He did not solely reference class norms in punitive contexts; he also referenced the norms when students exemplified them on a regular basis.

In order to help facilitate students' ability to explain their thinking and meet the criteria of what the teacher considered an adequate and appropriate mathematical explanation (i.e., sociomathematical norms), he first implemented an instructional lesson in which he presented a multi-step word problem and provided five different sample responses that were all considered by the teacher to be "unacceptable" mathematical explanations. Students were asked to critique each response before the teacher presented them with an elaborate "acceptable" mathematical explanation. The students were finally asked to describe specific characteristics of the acceptable sample explanation that provided clarity for the audience.

Over the next several weeks, the teacher presented the students with several writing opportunities. While the students were writing, he would walk around the room and help them when they demonstrated difficulty with aspects of the content and writing processes. He would then allow student volunteers to share their writing with the class, followed by an opportunity to receive constructive feedback. The teacher also scanned copies of anonymous writing samples from students in other classes to share with the class. This helped the students to identify specific qualities of good mathematical explanations.

Instructional Tasks

One of the defining features of a standards-based classroom is the implementation of high-quality instructional tasks, which provide a great deal of potential for engaging a community of students in rich mathematical discourse (Cohen, 1994; Lampert, 1990; Silver & Smith, 1996). Empson (2003) characterized tasks as "semantically rich problems that afford a variety of strategies … which can provide a basis for productive interactions between teacher and students" (p. 337). Because complex or ill-structured tasks typically cannot be solved by the mere application of a single procedural algorithm, and because they include misleading or unfamiliar features (i.e., those not explicitly taught through textbook instructional lessons), they require flexible and adaptive thinking that also provides a fertile backdrop for cognitive dissonance and rich, dialogic discussion (Cohen, 1994; Stein et al., 2000). Piaget argued that such features are integral to demonstrating true understanding rather than the ability to merely recall information in the same context in which it was explicitly taught (Ginsburg & Opper, 1988).

Despite the widespread championing of mathematical tasks as a robust instructional tool by advocates of standards-based reform, several researchers caution that the level of cognitive functioning demanded by specific tasks is most often significantly modified by teachers' interactions with students (Empson, 2003; Hiebert, Carpenter, Fennema, Fuson, Human, Murray, et al., 1996; Stein et al., 2000). In other words, low-level cognitive tasks, such as basic computation problems can be transformed into higher-level tasks depending on the types of questions the teacher poses about the problem. Likewise, a teacher's over-scaffolding of higher-level tasks can easily diminish the level of cognitive demand needed to solve such problems, effectively reducing the richness of the task. The nature of interactions during discussion of mathematical tasks is thus a key domain of interest for research and practice.

Standards-based reform has explicitly endorsed the use of challenging tasks as critical learning tools. However, standards-based advocates have also relied greatly on appraising the success of standards-based teaching not on content but primarily on the outcomes of standardized testing. One of the greatest risks involved in advocating the implementation of standards-based learning is that teachers who prioritize narrowly-defined outcomes, such as test scores, are sometimes susceptible to teaching merely to the test. The teacher in this study described himself as one who held concern for testing outcomes, but was primarily driven intrinsically by what he considered to be “high-quality learning contexts,” which featured rich mathematical tasks.

Tasks that were amenable to high levels of cognitive demand were intentionally selected to be administered for both whole- and small-group work. These tasks originated from an amalgam of sources: (a) *Everyday Mathematics* (Bell, Bell, Bretzlauf, Dillard, & Flanders, 2007), a reform-based approach developed by the University of Chicago School Mathematics Project; (b) the Rational Number Project (Cramer, Behr, Post, & Lesh, 2009; Cramer, Wyberg, & Leavitt, 2009), a 20-plus-year-research project funded by the National Science Foundation; (c) NCTM’s *Navigating through Number and Operations in Grades 6-8* (Rachlin, Cramer, Finseth, Foreman, Geary, Leavitt, et al., 2006); (d) an activity book published by the AIMS Organization (Wiebe, 1998), which is closely aligned with NCTM Standards and reform-based mathematics pedagogy; (e) *Holt, Rinehart, and Winston Mathematics Course 1, Grade 6* (Bennett, Burger, Chard, Jackson, Kennedy, Renfro, et al., 2007), a mostly-traditional mathematics textbook, and (f) the teacher’s original ideas and those that had been passed on over the years by thoughtful colleagues. Before making a final decision to implement each task, two sixth-grade math colleagues at Pryor Middle were asked to review them and co-appraise the level of cognitive demand to ensure that each task was appropriate and capable of inviting rich levels of discussion among students. The task analysis guide (Stein et al., 2000, p.16) was used to guide the analysis of each task’s level of cognitive demand.

Because the two small-group task discussions provided most of the data related to participation in this study, a brief description of both tasks is provided below. For a description of tasks featured during whole-class discussions, see Appendix B.

Small-group Task: "Fraction Maze" (Bennett et al., 2007)

About three weeks into the rational number unit, students completed an assignment in which they had to move across and down a grid filled with fractions, improper fractions, and mixed numbers—each time going from a smaller number to a larger number. Because the numbers on the grid were presented in a variety of representational forms, the task offered a level of ambiguity that was inviting of rich discussion. The task, in its entirety, consisted of 22 separate moves, which the students were expected to discuss and debate when they deemed it necessary. The students completed and discussed this task simultaneously (i.e., they did not attempt to solve it independently before discussing it). Because of the cognitive demand of this task, which was, for the most part, at the students' independent cognitive level, the teacher was minimally involved in facilitating this discussion. It took the students approximately 25 minutes to complete this task discussion. A copy of the assignment is included (see Appendix C).

Small-group Task: "Science Fair" (Rachlin et al., 2006)

At the end of the 9-week unit on rational numbers, the students were given the following problem:

> Three middle schools are going to have a science fair in an auditorium. The amount of space given to each school is based on the number of students participating. Bret Harte Middle School has 1000 participants, Malcolm X Middle School has 600 participants, and Kennedy Middle School has 400 participants. Respond to the following questions: a) What fraction of the space should each school get based on number of participants? Show how you know; b)

> If the schools share the cost of the science fair based on the number of students, what percent of the cost should each school pay? Show how you figured these percentages; and c) If the cost of the science fair is $300.00, how much should each school pay based on the number of students? Explain how you know.

The students were given one whole class period to solve this task independently. The following day, the teacher facilitated a small-group discussion based on the three questions provided above. Because of the cognitive demand of the task, which was, for the most part, at the instructional cognitive level of the students, the teacher played a significant role in facilitating the discussion. This facilitation included calling on individual students at times to share their ideas, as well as scaffolding their ideas to assist them towards successful completion of the task. Discussion of this task took approximately 45 minutes to complete. A copy of this task assignment is included (see Appendix D).

Participants

Sampling Procedures

Because one of the researcher's aims was to compare interactions among and contributions by students of varying achievement levels, the researcher used dichotomous case selection (LeCompte &Schensul, 1999a). The focal participants were 4 female students in the researcher's fifth-period sixth-grade math class. The peers of these focal students were also participants of this study, but only to the extent to which they engaged in important interactions with focal participants during whole-class discussion. Because potentially important between-gender factors are outside the scope of this study, male students were not selected as focal participants. Although this class consisted of four students with specific learning disabilities, none of the focal participants were receiving special education services, nor had they been identified for psychological testing related to a potential special education service referral. The following criteria were devised to aid in selecting the four focal students.

According to LeCompte and Schensul (1999a), selection criteria should take logistical and definitional considerations into account. Logistical criteria have to do with economy and what is readily available. The students in the researcher's fifth-period sixth-grade math class (n = 25 students) constituted a convenience sample. Purposeful sampling (n = 4 students) was used to select focal students from this group for the multiple case study.

Definitional criteria determine exactly who the participants will be and how they are to be identified. In keeping with the ideals of standards-based classrooms, where all students are expected to succeed academically as measured by standardized assessments, a variety of formal assessment data were used to determine the best fit for focal students. The researcher examined the previous two years (2008, 2009) of the state standardized test data, 2009 norm-referenced test results [i.e., composite/categorical National Percentile Rankings (NPR)], and classroom work samples, such as quizzes and tests and subsequently chose two low-performing students based on the following criteria: (a) at or near-failing results on state standardized math assessments (i.e., a score of 820 or below), (b) below 34^{th} percentile NPR on state norm-referenced math test, (c) failing grades (below 70 percent) on class quizzes and tests, and (d) daily warm-ups (i.e., review of recently taught content) that are incomplete or error-ridden. Two high-performing students were chosen based on the following criteria: (a) exceeds expectations on state standardized math assessment (i.e., a score of 850 or higher), (b) above 60^{th} percentile NPR on state norm-referenced math test, (c) exceptional grades (above 90 percent) on class quizzes and tests, and (d) daily warm-ups that are complete and accurate. Reading comprehension and vocabulary performance data from the norm-referenced state test were also referenced as a proxy measure for each participant's level of written language proficiency, although these data were not used to select participants. In addition to assessment data, student attendance records were considered as a final criterion, because the researcher wanted to know with a reasonable degree of confidence that the focal students would be in attendance on a regular basis for data collection purposes. In fact, during the entire nine-week span of the study, the focal students missed only one, two, four, and six days respectively[2].

Description of the Participants

Rachel

Rachel (a pseudonym), a white, middle-class, low-performing female student from a two-parent family, was approximately 11.5 years old at the time of study. Usually quiet and lacking confidence, Rachel expressed that math, of all subjects, posed the greatest degree of challenge and difficulty for her, and this sentiment was corroborated by her performance in math, as she often failed weekly quizzes and summative unit assessments. The previous year, as a fifth-grader, she was identified by teachers and administrators as "at-risk." Consequently, she received supplemental instructional services for mathematics that entire year. Motivation, however, was not wanting, as she usually completed daily homework assignments and opted to re-take nearly every summative unit exam that she had failed. She also regularly and voluntarily attended

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[2] Although 4-6 days may seem significant in the context of 9 weeks, it is important to point out that data were not collected each day during the 9 weeks. In fact, across all nine videorecorded whole-group task discussions, all students were present for eight of these lessons, and the only participant to be absent during one of the whole-group discussions was Rachel. Most importantly, all students were present for both small-group task discussions.

weekly after-school tutorials. Despite her struggles in math, she maintained B's and C's in her other core subjects. At a parent conference in late October, Rachel's language arts instructor characterized her as "comfortable" and "very funny," two traits rarely observed in math class. A comparison of her math and language performance on norm-referenced standardized tests helps to explain why this may have been the case. For example, her performance in the areas of vocabulary and reading comprehension was average to slightly below-average in 2009, while her math percentile scores were consistently in the bottom quartile. It seemed that her sense of self-efficacy with regard to mathematics performance was extremely low, and this may have directly affected her inclination to participate in discussions about mathematics. Rachel missed six days of school during the course of this study. However, arrangements were made to re-teach the information she missed when new concepts were introduced.

Heidi

Heidi (a pseudonym), a white, middle-class, low-performing female student from a two-parent family, was also approximately 11.5 years old at the time of the study. Although Heidi appeared relatively motivated about participating in discussions about mathematics, and showed intermittent flashes of creativity and sharp mental math ability, her performance on quizzes and tests in math was surprisingly and consistently low. Heidi could sometimes talk astutely about mathematics concepts, but often struggled to demonstrate her ability on pencil-and-paper assessments. Heidi was also identified as a candidate for supplemental mathematics instruction in fifth-grade as part of an early intervention program. As a C-student in her other classes, who also demonstrated average performance in vocabulary and reading comprehension domains on the state's norm-referenced test, Heidi had more important issues on her mind outside of school: all year long, she coped with a undisclosed family trauma. Although she sometimes expressed the desire to attend weekly tutorial sessions, she was only able to attend one during the entire school year. She missed only four days of school during the 9-week period of this study.

Marie

Marie (a pseudonym), a white, middle-class, high-performing female student from a single-parent family, had just turned 11 years old at the onset of this study. She too dealt with a considerable degree of family trauma during the study. In spite of this, she almost always appeared passionately motivated and, at times, bubbly about participating in talk about mathematics. In 2008, shortly after transferring from a school on the west coast, she earned admission into the district's gifted education program. Although she often lamented her struggles to maintain pace with her peers in her other core classes, she expressed a high sense of self-efficacy about mathematics and often stated that math was not only her best subject, but her favorite as well. She performed highly on norm-referenced measures of vocabulary (84^{th} percentile) and reading comprehension (70^{th} percentile). Marie volunteered to share her mathematical contributions significantly more than any other student in the class, and at times struggled to contain her excitement, often interrupting others or blurting out responses due to her intense degree of enthusiasm during discussions about mathematics. During the course of the study, Marie only missed two days of school.

Patty

Patty (a pseudonym), a white, middle-class, high-performing female student from a two-parent family, was approximately 11.5 years old at the time of the study. She qualified and entered the county's gifted-education program when only in third-grade. Although she expressed a greater affinity for reading over mathematics, she often cited her father, who holds a degree in physics, as an inspiration for her divergent sense of thinking. Patty preferred to represent her mathematical thinking pictorially, and often gravitated toward the use of manipulatives (e.g., fraction pattern blocks, fraction circle pieces) when engaging in problem solving and discussion. Like Marie, she scored exceptionally high on norm-referenced measures of vocabulary (93rd percentile) and reading comprehension (80th percentile). She also demonstrated eagerness to share her thinking with the whole class, as she sometimes took non-conventional approaches to problem-solving. Patty described herself as highly competitive and motivated. Over the entire school year, she never once failed to turn in a homework assignment on time and missed only one day of school during the study.

Data Collection

Data were collected in a variety of ways in order to bolster the overall design of this study. Data collection instruments included fieldnotes of video-recorded observations, interviews with focal participants, and students' written work samples.

Instruments

Vide-recorded Task Discussions: Whole-class

Video-recording of instructional lessons occurred nine times over the course of a nine-week unit on rational numbers. Each lesson was recorded in its entirety (i.e., approximately 45 minutes) to minimize the risk that potentially valuable data would be lost. Purposeful video sampling of entire lessons was performed in order to extract relevant data. In accord with a recommendation made by Shensul, LeCompte, Nastasi, and Borgatti (1999), the researcher selected only relevant segments of video data that addressed the study research question (e.g., cooperative interactions between focal participants) and simply summarized the non-relevant portions in order to capture the larger context of events within the research setting. When typing up the fieldnotes of these nine instructional episodes, italic font was used to indicate summarized portions that did not relate directly to the research question.

During whole-class instruction, the video camera was placed on a tripod in the corner of the classroom in order to obscure its presence while at the same time capturing a wide-angle view of all participants. Students who did not wish to participate in the study were seated in a location outside the view of the camera lens. Since video-recording endured from the beginning of class until the end, it was not necessary to operate the video camera during the course of instruction.

Video-recorded Task Discussions: Small-group

Video-recording of the four focal participants engaged in collaborative discussion of rational number tasks was conducted twice during the unit: once at the end of the third week and once at the end of the ninth week. The teacher was present in the room during both of these tasks, and had initially planned to remain relatively uninvolved in the discussion of the mathematical content related to the task, or social scaffolding of the group as they worked on completing the task together. After observing the first task discussion and subsequently analyzing the relevant data, the researcher wavered over the degree to which he thought the teacher should be involved in the discussion of the final task (for reasons that will be explained in greater detail in the next chapter) and eventually decided to play a more substantial role in facilitating the discussion of the final task (i.e., what many researchers in the extant literature refer to as "stepping in.")

The researcher initially concluded that it would be most authentic and meaningful to capture and examine focal participants' interactions *in situ* – that is, in the context of a real classroom. However, it was determined prior to the data collection period that ambient classroom noise (i.e., other groups of students talking out loud together simultaneously) inevitably compromised the ability to accurately hear and interpret student dialogue when analyzing the video-recorded data of the task discussions, so the focal participants were relocated to a separate room for the small-group task discussions. Therefore, this should be considered a limitation of the overall study design.

Although the video camera was set on a tripod and placed approximately three feet from the table at which the students were seated, the researcher supplemented the video recording with a hand-held digital voice recorder, as a precautionary measure to reduce the possibility that some parts of the dialogue could not be heard clearly during transcription.

In the first assignment, participants were asked to work together to solve a task, in which they were given 30 minutes to complete in an empty classroom (with the teacher-researcher present). During this time, non-focal participants were also working in small groups to complete the same task in the teacher's classroom under the direction of a co-teacher. The teacher began by describing the task directions and expectations in detail, providing examples, and answering students' questions about the directions. Students were then explicitly instructed to: 1) contribute to solving the task by thinking, showing work, and responding to and asking questions of one another, 2) attempt to interact with all of the group members, and 3) take time collectively at each step to explain their reasoning for the choices they made. The actual solving and discussion of this task took approximately 20 minutes and was transcribed verbatim.

In the second and final assignment, focal participants were asked to work individually to solve a task, which they were given 50 minutes to complete. The following day, focal participants were relocated to an empty classroom where the teacher began with a brief review of the task instructions, followed by an in-depth discussion of the task questions, which was facilitated by the teacher. This discussion lasted for 45 minutes and was transcribed verbatim.

The video-recordings of both small-group task discussions were subsequently edited for the purpose of follow-up interviews with each participant. The editing of each video is described in greater detail in the section on semi-structured interviews.

Fieldnotes of Video-recorded Observations

Teacher-researchers who study their own classrooms face unique methodological challenges concerning observation. The responsibility of attending to the diverse needs of more than twenty students each day precluded the option of sitting passively in a corner of the classroom while making fieldnotes of observations. Other researchers who have designed studies of their own classrooms (e.g., Canterbury, 2006) have relied on mental notes and reflections of classroom events based on short-term memory. Since analysis of student talk was a major component of this study, the prospect of recalling events from memory immediately after the conclusion of each instructional lesson was inadequate. Therefore, nine whole-class instructional lessons and two small-group task discussions were video-recorded in order to enable comprehensive and accurate transcription of spoken dialogue and observation of key nonverbal gestures. This afforded the researcher the benefit of seeing and hearing classroom interactions that otherwise might have been overlooked *in situ* or perhaps forgotten from short-term memory.

Same-day viewing of audiovisual data occurred each day data were collected. During these viewings, fieldnotes of classroom observations were made, as if the researcher were observing the classroom in real-time. When recording fieldnotes, the researcher looked specifically for spoken contributions made by focal participants during discussion of mathematics tasks. All spoken dialogue involving focal participants was transcribed verbatim. When this interactional dialogue involved non-focal participants, the dialogue of non-focal participants was transcribed as well.

To organize fieldnotes, the verbal utterances of each participant were transcribed in an integrated, chronological fashion. Everything the participants said, in addition to important nonverbal gestures (e.g., communicating covertly with a peer while another student was sharing her thinking, raising hand to speak, facial expressions implying confusion), were recorded in fieldnotes. Nonverbal communication and other relevant observations were distinguished from verbal utterances by the use of parentheses during transcription of fieldnotes. Important background information, like the context of the discussion or perhaps a description of the problem being discussed, was also included in fieldnotes and was generally distinguished from other data by the use of italics. Several researchers recommend separating interpretations and reflections from observed factual data in order to mitigate the threat of conflating observations with interpretations of observations (Merriam, 1998; Stake, 1995; Yin, 2003). Doing so helps the researcher to maintain fidelity while describing behavioral observations without attributing meaning to or drawing inferences from observations (Schensul, Schensul, & LeCompte, 1999). Therefore, reflections on the data were generally made manually in the margins of the fieldnotes printouts; observational data other than spoken contributions (e.g., non-verbal gestures) were electronically recorded in-line and separated by parentheses.

During initial informal classroom observations and daily viewing of the audiovisual data, a number of issues were addressed, such as: frequency and quality of student contributions and interactions; body language or other non-verbal communication; and the types of language students used to make claims, ask questions, invite or exclude other participants, and so forth. Initial coding of each set of fieldnotes subsequently led to the development of new questions, hunches, and a formative list of phenomena to explore in greater detail as the study progressed.

To compliment observational data from fieldnotes, Miles and Huberman (1984) recommend the use of researcher memos, which entail regular summary and reflection of field notes in order to frame and reframe the focus of qualitative inquiry as it evolves. These memos were recorded electronically each day, with the date and time of the memo listed at the onset of each entry. This was a more practical way of reflecting on the day's events, because these memos were typically recorded immediately after class each day. Each day that fieldnotes of video-recorded observations were made, and even on some days when informal data were collected (e.g., observations made on instructional days that were not video-recorded), the researcher read through the data and recorded reflections in the researcher memo. These reflections focused specifically on how students participated in discourse during task discussions, the kind of roles they assumed and language they used during discussions, musings on how the data related to the research question and the relevant literature reviewed prior to the data collection phase, discussions of emerging patterns, codes, categories, themes and concepts, and even the researcher's reservations and anxieties that developed as the study carried on.

Interviews

According to Hatch (2002), when capturing participants' perspectives is a goal of research design, interviewing is often an essential empirical method. Two types of interviews were implemented in this study: informal interviews and formal semi-structured interviews.

Informal interviews. Informal interviews of focal participants took place sparingly, usually when the researcher deemed that relevant information would best be elicited in an informal context. For instance, when it appeared that one of the focal participants was not interacting with her peers during a group task, the teacher asked her, "Why did you raise your hand for me to help you? Why didn't you first discuss it with your partners?" On a different occasion, one participant shared her feelings of aversion for another focal participant with the teacher in confidence. Additional data were elicited from conversations with one participant who attended after-school tutorial sessions. Data gleaned through informal interviews were recorded and reflected upon in the daily researcher memo as soon as class concluded.

In most cases, important questions that emerged from ongoing analysis of observed video-recorded data were better suited to be asked of focal participants during formal, semi-structured interviews. This was because the context of a sit-down face-to-face semi-structured interview was perceived as more conducive to eliciting focused and substantive responses from the students, but also because it enabled the researcher to record and accurately transcribe participant's responses.

Semi-structured interviews. Semi-structured interviews were conducted with each focal participant individually, following the video-recording of their small group task work together. All interviews were audio-recorded and transcribed verbatim. The researcher chose to interview each student individually (as opposed to together) to reduce the likelihood that they would hold back authentic feelings and reactions to questions. These interviews were conducted in two separate phases: one interview was administered *immediately* following the task discussion on the day it occurred, and a subsequent interview was conducted one to three days later immediately *after* they watched an edited video clip of the task discussion. The researcher anticipated that interviewing students in these two separate phases might result in heightened self-awareness of their contributions and interactions, therefore helping to elicit additional substantive data related to the research question. Moreover, questions would certainly be more relevant and comprehensible to students immediately after viewing video excerpts of their participation as opposed to simply asking them to answer questions about the task discussions based on short-term memory without being able to refer to a concrete situation.

Since focal participants were observed engaging in small-group discussion twice during this study, each student was interviewed four times (once before viewing video playback for both tasks and once after viewing video playback for both tasks). These interviews lasted between 5 and 15 minutes for each participant and occurred in the teacher's classroom during non-instructional time. The researcher gained permission from connections teachers (e.g., drama, P.E., art, music, etc.) to pull students from their classes in order to conduct the interviews.

Immediately after the small-group task discussion concluded, the researcher questioned each focal participant about her perceptions of participation and interactions among the group. That same afternoon or evening, the researcher viewed and analyzed the video of the small-group interactions and subsequently put together a 5 to 8 minute reel of edited video data (for the purposes of efficiency) to play back simultaneously during follow-up interview sessions, which took place within three days after the task discussions took place.

In order to provide an effective and representative video sample for participants to view and subsequently respond to, several criteria were devised beforehand to help determine what might constitute a representative sample of the small-group task discussions. First, the researcher sought to capture meaningful interactions among the students. Meaningful interactions occurred when decisions were discussed collaboratively, which sometimes involved consensus, dissent, and even one student dominating the discussion. For instance, during the Fraction Maze task, some of the most important speech acts were those that represented the initiation of a move along the grid from one rational number to a greater value. During the Science Fair task, some of the most important speech acts included those that represented major shifts in the central idea being discussed by the group. For instance, when the group shifted focus from partitioning Kennedy's and Malcolm X's share of the right-half of the auditorium from

1/6 and 2/6 to 1/8 and 3/8. These meaningful interactions were taken from video segments that ranged from roughly 45 sec to 1 min 30 sec.

Additionally, the amount of speech acts made by each participant in the video sample was intended to be generally proportional to the total amount of speech acts made by each participant throughout each entire small-group task discussion. To accomplish this, the researcher viewed the video of the small-group task discussion in its entirety and then generated estimated percentages of time spent talking for each participant. After cutting and pasting relevant video segments into a sample video clip, the researcher viewed the sample video clip in its entirety to check for relative proportionality among each individual's time spent talking.

After viewing the video playback of the small-group interactions, the researcher posed questions aimed at getting interviewees to elaborate on contributions made during the task discussion as well as their perspectives on the experience of participating in the group (see Appendix E for sample questions). Although the researcher posed some of the same questions to each participant, some questions were based on the specific nature of interactions that occurred in the small-group setting or perhaps between two individuals, and therefore were personalized to each individual participant. For example, at one point during the Science Fair task discussion, while Heidi was posing a question to the group, Patty got out of her seat and relocated so that she could explain her reasoning to Marie, apparently ignoring Heidi's contribution to the group. It seemed that Patty ignored Heidi and instead chose to privately share her thinking with Marie. In that particular case, after watching the video clip of the scenario, the researcher asked Heidi, "What happened here? Did you feel like you were being listened to by the group and why or why not?"

and to Patty, "What happened here? Were you listening to Heidi's contribution? Why or why not?" Furthermore, in several cases, focal participants' responses to semi-structured interview questions generated additional ideas for unforeseen follow-up questions that were also asked of students in these interviews.

Students' Written Work Samples

Although discourse is most often associated with verbal communication of mathematical ideas, it also includes written communication. In order to gain a fair and robust portrait of students' participation in discourse (especially those who preferred to remain relatively silent during discussions), the teacher had all students (n = 25) record written reflections (n = 8) of their thinking at various points during the 9-week rational number unit. Toward the end of class on the days that lessons were video-recorded, the teacher allowed time for students to reflect on their understanding of the instructional content.[3] In these written reflections, students were expected to summarize their learning by responding to structured prompts that were related to the specific content being taught, such as "Write a letter to a student who is having trouble with subtracting mixed numbers with regrouping and explain how this is similar to and different than subtracting whole numbers with regrouping." Several of the writing assignments were designed to require students to analyze an error or mistake made by a fictitious peer and to subsequently write a letter to this peer explaining the error, why it was not a reasonable solution, and the correct solution. Sometimes this was posed simply as, "Do you agree with Bob? Why or why not?"

[3] In some cases, when time ran out, I was forced to allow the students to write the following day. If a student was absent when a writing assignment was collected, I arranged to meet with her before or after school to review the lesson content and allow her to record a written reflection.

The teacher modeled this process the first 2 months of school by sharing student- and teacher-created responses of varying quality in order to get students involved in evaluating the quality of these reflections. In some cases, students required additional scaffolding during these assignments, even near the end of the school year. Analysis of these written data helped to illuminate students' ability to communicate their mathematical thinking and the quality of their written communication. See Appendix F for details regarding the individual writing topic for each written work sample.

It also helped to provide a more accurate portrait of students' potential to participate in productive classroom discourse—especially for Rachel, who was often silent during task discussions. For instance, Empson (2003) demonstrated how she was able to triangulate data from an initial clinical interview of a child who previously made an ambiguous contribution during small-group discussion. Using the interview data, the researcher made an informed conjecture about the student's intentional meaning despite sharing an unclear explanation of his reasoning during classroom discussion. See Appendix G for copies of illustrative writing samples generated by each participant.

Data Management

In order to prepare the body of collected data for analysis, the researcher maintained a 2-inch binder with labeled dividers denoting each distinct form of data. Schensul and LeCompte (1999) recommended creating data instrument logs as a method for organizing and managing data. To do this, an electronic spreadsheet was used to help keep track of the type of each data source (e.g., whole-class fieldnotes, small-group fieldnotes, interviews, written work samples, researcher memos), date recorded, length of audio or video clip (if recorded audio-visually), and a brief description of the contents.

All student work samples were photocopied and scanned electronically for the purpose of publication. Within each section of the binder, all data were arranged in chronological order to capture the authentic progression in which observed data unfolded in the classroom. Chronological organization also helped to conjure up a realistic rehashing of the events that took place during this study, which helped to streamline the process of reading through and reviewing the data.

During the intensified data analysis phases, units of data were physically sorted into emergent categories, delineated by sticky-notes with category labels written at the top. The categorical data were then sorted into groups of interrelated themes and ideas by physically repositioning the sticky notes on large sheets of chart paper while simultaneously engaging in analytic memo writing, which helped to identify possible relationships between emergent categories and themes. This method of organization helped make Second Cycle coding processes (e.g., synthesizing, interpreting, and modeling the data) optimally efficient.

Data Analysis

First Cycle Coding

The constant comparative method (Corbin & Strauss, 2008) was used to compare information within one data source and then used again to compare data across multiple sources in order to reduce data into salient categories and themes. This general method of comparison was applied to fieldnotes of video-recorded observations, student interviews, and student work samples.

One of the first goals of qualitative analysis is to reduce data into manageable but substantive parts in preparation for further synthesis and interpretation (LeCompte &

Schensul, 1999b). The act of identifying and interpreting meaning from qualitative data typically begins with coding. Saldana (2009) points out that coding "is not a precise science; it's primarily an interpretive act" (p. 4). Because the act of coding data is inevitably filtered through the researcher's particular analytical lenses, it is critical that the report of data analysis is clear and transparent with respect to the processes by which codes are generated (Yin, 2003). In this section, examples are provided at each phase of the coding process to illuminate the otherwise tacit processes by which data were analyzed.

For this study, data were coded in three distinct phases: holistic pre-coding, First Cycle coding (i.e., open-coding), and Second Cycle coding. Saldana (2009) draws a distinction between the goals of First and Second Cycle coding methods by characterizing First Cycle methods as a primary tool of data reduction, while Second Cycle methods represent greater levels of data abstraction, such as "classifying, prioritizing, integrating, synthesizing, abstracting, conceptualizing, and theory building" (p. 45). To be clear, holistic pre-coding and open coding occurred at different times but both were considered First Cycle methods because they were both aimed at reducing raw data.

Holistic Coding

Because data were collected over a period of nine weeks and, to some degree, the researcher wished to analyze the data as a teacher engaged in classroom inquiry might, the first phase of coding was holistic (Dey, 1993). Holistic Coding is the process of attempting to "grasp basic themes or issues in the data by absorbing them as a whole rather than by analyzing them line by line" (Dey, 1993, p. 104). Instead of coding line by line each day as transcripts of interviews and fieldnotes were created and student work samples were collected, the researcher read the entire body of data collected for a particular day and analyzed it in meaningful and contextualized “chunks.” For example, rather than seeking to apply a code to a segment of a participant’s utterance during whole class discussion, the researcher recorded meaningful descriptive phrases in the margins, such as “student’s explanation included only numbers” or “student providing answer tentatively, in question form.” Also in these margins, the researcher recorded reflections in the form of summary, questioning, and relevant narratives. For instance, in a memo dated October 23, 2009, the researcher wrote the following in response to a series of vague explanations proffered by Marie during small-group task discussion. “By pointing to her paper as she talks, she seems to be relying on her representations to do the explaining. The other students seem lost.” This instance represented the first time the researcher identified VAGUE REFERENT as an emergent code that was later found to be central to the nature of students’ participation in discourse. Such reflections on the data also helped to generate follow-up questions for semi-structured interviews with participants, as well as general hunches to be explored in greater detail as data collection progressed. Taking time to reflect on the data via memo-writing is an essential part of the process of data analysis (Corbin & Strauss, 2008).

Open Coding

As soon as the data collection period officially concluded, the researcher engaged in a more rigorous form of analysis: open coding, which was used in order to generate as many concepts related to the research question as possible, without overlooking potentially important nascent meaning within the data. The research question intentionally featured the phrase "nature of participation" because the researcher held only a vague preconception of what was to be looked for in the data (only knowing that whatever it was, it would loosely be related to the idea of participation and discourse). Therefore, in order to establish and maintain consistency, the researcher followed the recommendation of Auerbach and Silverstein (2003), who advised researchers to keep a copy of the salient components of the research proposal, such as the research question, theoretical framework, and goals of the study, nearby at all times in order to maintain focused coding decisions. The researcher created a condensed list of these study design aspects and referred to it regularly as data were coded.

Open coding of the data consisted of several interrelated steps. First, the researcher wrote shorthand summarizing phrases in the margins of fieldnotes, interview transcripts, and student work samples. Phrases (sometimes In Vivo phrases), rather than single words, were intentionally used in an attempt to avoid over-abstracting the data. Several methodologists caution that making high-level inferences, especially at early phases of data analysis, can threaten the authenticity and trustworthiness of study findings (Bogdan & Biklen, 1998; Corbin & Strauss, 2008; LeCompte & Schensul, 1999a). After physically labeling the data, the researcher entered each code phrase into an electronic coding manual. Consistent with Saldana's (2009) recommendation, for each coded unit of data, the researcher included the code phrase, the location (i.e., date, file, line number), and an example or description of the data (which often involved copying and pasting the specific data in order to provide a concrete, contextualized excerpt of the actual data). This method of organization played an instrumental role in helping to maintain consistency among interpretations of the data. For example, there were times when the researcher experienced difficulty in deciding which code to use for a specific piece of data; when this happened, the researcher would simply look back at previous coding decisions and make semantic comparisons in order to resolve ambiguity. Finally, the researcher included a column for analytic memos, which provided the opportunity to reflect on emerging categories and hunches regarding salient themes.

Some of the salient formative codes and categories that emerged after First Cycle analysis of each participant's data are listed in Table 3. The codes, listed in all-capital letters, correspond with the participant noted at the top of each column, are grouped by a list of related categories on the far left hand column.

Table 3

Codes and Categories Generated from First Cycle Coding

	Low-performing students		High-performing students	
Categories	Rachel	Heidi	Marie	Patty
Independent Contributions	SOCIAL ARBITRATION DOING IT ON PAPER	NARRATING OPTIONS REPORTING A SOLUTION	INITIATING MOVE DIRECTING THE GROUP	USING MANIPULATIVES INITIATING MOVE
Dependent Contributions	SO DO THIS? INCOMPLETE CHALLENGE	SO DO THIS? BUT WHY?	CATCHING ERRORS AGREEING	CHALLENGING CLAIMS ARGUING
Access to Participation	WAIVING SPACE CONFUSED TAKING ANSWERS	HANG ON! I'M CONFUSED TAKING ANSWERS	BLURTING DYING TO SHARE VAGUE REFERENT	I, I, I COMPETITIVE IN THE TUNNEL VAGUE EXPLANATION

Second Cycle Coding

Codes that resulted from First Cycle analysis were subsequently grouped under more abstract terms (i.e., categories) through a process of Second Cycle coding. Categories were constantly refined through ongoing and recursive analysis, which was achieved primarily by constantly referring back to the original data sources for contextualized interpretation and subsequently making conjectures about the relationship between the individual units of data and the broader emergent categories.

To facilitate rearranging codes into various conceptual categories, spreadsheet printouts from the coding manual were cut into pieces by coded units of data. These individual units of data were then grouped by adhering them to sticky notes with the conceptual title labeled at the top. All sticky notes were placed on large, laminated sheets of chart paper so that lines could be drawn to connect interrelated categories and codes with an erasable marker. When an apparent theme had emerged, the researcher went back through the data to look not only for corroborating instances but disconfirming ones as well (Stake, 1995). In some cases, disconfirming evidence prompted reorganization of the emergent categories. For example, after noticing a repetitive degree of tentativeness in Heidi's solution reporting, the researcher began to attribute this to low self-efficacy. However, interview data appeared to disconfirm this initial hunch:

> Int: It seemed to me like when you were offering your ideas on which fraction you should go to next, you tended to ask the group rather than tell the group what to do. Like, instead of saying "Let's go to 1 3/4," you'd almost ask, "Should we go to 1 3/4?" Why ask instead of tell?
>
> Heidi: Because you told us that we should ask questions and interact together; not be like, "This is what we're going to do," and stuff. (Interview 2)

Because Heidi attributed what was initially perceived as "tentativeness due to lack of confidence" to her desire to follow the teacher's expectations regarding peer interactions, her use of tentative language while reporting solutions could not validly be ascribed to a social or emotional dimension, such as low self-efficacy or social comparison (at least, not by this instance of data).

Reconceptualizing the data through category sorting and codeweaving were two major strategies used during Second Cycle coding processes. The cardinal rule of Second Cycle coding is perhaps best articulated by Glaser (1978): "Data should not be forced or selected to fit pre-conceived or pre-existent categories or discarded in favor of keeping an extant theory intact" (p. 4). Not forcing a particular conceptualization of the data sometimes called for collapsing of separate categories into more appropriate or streamlined groupings. Sometimes, this collapsing was done when the researcher observed specific codes that included very few units of data. For example, after open coding of the data was performed and preliminary categories began to develop, two separate categories, CITING OTHERS' IDEAS and MIMICKING OTHERS' STRATEGIES, seemed to better represent a single code (at the time, very few instances were listed under MIMICKING OTHERS' STRATEGIES and disproportionately more under CITING OTHERS' IDEAS). Consequently, the two codes were combined together under the label USING OTHERS' IDEAS). Physical representation of the data on large chart paper helped facilitate the trial-and-error process of reconceptualizing and collapsing data.

The researcher also drew connections between emergent categories via a process of "codeweaving" (Saldana, 2009). Saldana used the term codeweaving to describe the process of using actual codes and key words in integrated narrative statements as a practical method of insuring that the researcher is constantly thinking about possible networks of meaning among the data. For example, after noticing several instances in which Heidi responded to questions before anyone in the group had the opportunity to attempt to think strategically about them, the researcher jotted the following memo (codes that emerged from analysis of raw data are listed in capital letters): “THINKING ALOUD seems to be a practical way of CLAIMING SPACE in discussion, for Heidi (a low-performing student), to help offset the undesirable effects of differences in GROUP PACING.” This narrative helped to illuminate a potential explanation for Heidi’s use of space during task discussions, which was later confirmed through a subsequent interview.

While attempting to collapse First Cycle codes into categories and related themes, the researcher began to notice patterns that emerged from analysis of each student’s contributions during whole-class and small-group task discussions. Namely, four distinct classifications became salient: the type of explanations students provided (whether relational or computational in nature); the quality of explanations students provided (which ranged from complete and correct to ambiguous or incorrect); the types of contributions students made (dependent vs. independent); and the texture of student communication (whether they expressed ideas declaratively or tentatively). Because these classifications emerged in part through First Cycle coding attempts, each were subsequently identified as individual a priori codes by which all student contributions could be analyzed. Thus, the researcher combed through the data in its entirety again, but this time using only each of the four classifications described above to analyze all student contributions made during whole-class and small-group discussions. Also during this phase of data analysis, the researcher coded student work samples based on whether written explanations were complete or ambiguous, as well as computational or relational. Analysis of these data provided further grounding for findings related to the nature of students' contributions during discussion of rational number tasks.

Cross-case Analysis

Qualitative researchers have recommended various similar strategies to assist with cross-case synthesis of findings. Miles and Huberman (1984), for example, proposed the use of meta-matrices, or tabular displays of condensed data across cases or key variables as a format for making comparisons. Yin (2003) recommended the use of word tables to display data from individual cases according to a conceptual framework. Eisenhardt (1989) described a tactic in which the researcher organizes the data around specific themes in order to mine the data across cases or dimensions for intergroup similarities and differences. "The result of these forced comparisons can be new categories and concepts which the investigators did not anticipate" (Eisenhardt, 1989, p. 541).

After completing within-case analyses, the researcher met with a faculty advisor to present the salient themes that emerged from First and Second Cycle coding processes. During this meeting, additional comparison and reconceptualization of the data took place until the resulting categorical concepts were consistent across the individual cases. This reconceptualization entailed close examination of the properties and dimensions of the previously identified themes. For example, USE OF SPACE was identified as a salient overarching concept for all four cases. However, the specific ways in which students used space varied across the cases. After reconceptualizing the thematic labels, the data was read again in its entirety to test the fit between coded data and emergent themes.

Finally, the researcher constructed a tabular display of the condensed findings of each individual case on a large poster board, separating the data by overarching themes that emerged from within-case analyses (i.e., contributions, use of space, meaning-making, peer and teacher interactions). While looking across categories for commonalities, two overarching concepts emerged as significant. The researcher immediately began writing analytic memos about the relationships among these concepts across each case and concluded the process by looking back at the raw data for supporting evidence as well as disconfirming instances. Finally, the cross-case findings were refined based on constant comparison of the entire set of data.

In addition to coding and reducing data, several steps were taken to bolster trustworthiness and authenticity of the study findings. The following sections explicate some of the measures taken to accomplish this.

Trustworthiness

Trustworthiness is a term unique to qualitative research that represents the validity and reliability of the study design (Lincoln & Guba, 1985). In short, trustworthiness is used to judge the quality of qualitative inquiry. Guba and Lincoln (1994) delineate four criteria for determining trustworthiness (which they juxtapose with benchmarks of design rigor from the positivist and postpositivist traditions of inquiry). The four criteria of quality are: credibility (which parallels internal validity), transferability (which parallels external validity), dependability (which parallels reliability), and confirmability (which parallels objectivity). Creswell (2003) and

Merriam (1998) note that trustworthiness has often been used interchangeably with terms such as “authenticity” and “credibility”, and recommend some of the following strategies, which adequately address Guba and Lincoln’s (1994) four criteria of quality: data triangulation, member-checking, thick description, negative or discrepant case analysis, prolonged engagement, peer debriefing, external auditing, persistent observation, and clarification of researcher bias. What follows is an explanation of how some of these strategies were used in this study.

Data Triangulation

Data triangulation entails cross-checks of multiple data sources in order to ensure valid results (Schensul et al., 1999). Triangulation of the data partially addresses the problem of construct validity because the multiple sources of evidence provide diverse measures of the same phenomenon (Yin, 2003). Yin cautions that triangulation is not achieved by the mere inclusion of multiple data sources, but rather when “the events or facts of the case study have been supported by more than a single source of evidence” (p. 99). As codes and categories were constructed during data analysis, all data sources (i.e., fieldnotes, interviews, student work samples) were probed with the aim of corroborating or rethinking emergent interpretations of the data. As mentioned before, the researcher looked carefully for signs of both agreement and disagreement across data sources. An additional strength of the study design is that the researcher was able to compare his interpretations of student participation in mathematics discourse (via observations, fieldnotes, and work samples) with those of the students (through interview data).

Thick Description

Although generalizability is not an intended objective of most qualitative studies, a thick, rich contextual description of the research setting, participants, and the nature of classroom teaching and learning provides the reader with some sense of transferability, that is, the potential that findings might be applicable to other similar settings. Verbatim accounts of classroom dialogue have been included in the study findings to allow the reader transparency with respect to the types of interactions and contributions that students made while learning about rational number concepts.

Additionally, the researcher has accounted for a potential source of bias that stems from the nature of studying his own classroom. Thick, rich description of classroom interactions and tasks bolster treatment integrity, that is, the extent to which the teacher taught the way the researcher claims.

Negative Case Analysis

Because social science research very rarely results in findings that are immune to contradictions or exceptions, it is important to use negative case analysis when reporting the results of a study. Creswell and Miller (2000) observe the following:

> In practice, the search for disconfirming evidence is a difficult process because researchers have the proclivity to find confirming rather than disconfirming evidence. Further, the disconfirming evidence should not outweigh the confirming evidence. As evidence for the validity of a narrative account, however, this search for disconfirming evidence provides further support of the account's credibility because reality, according to constructivists, is multiple and complex. (p. 128)

Incidents that run counter to suggested empirical themes are reported within the findings of the study. This not only helps in the process of refining or challenging initial categorization of the data, but also adds to the reader's perception of the researcher's authenticity and credibility (Creswell, 2003). For instance, as previously mentioned, the researcher noted that Heidi demonstrated a tendency at times to share ideas tentatively rather than assertively. The researcher began to ascribe this tentativeness to a low self-efficacy and Heidi's admission that she tended to struggle in mathematics, but Heidi explicitly disconfirmed this hunch during a subsequent interview.

Additionally, since the researcher did not employ scientifically reliable sociometric or psychological tests to measure social status, goal orientation or motivation, and instead relied solely on the triangulation of observational and interview data, discrepant instances were noted in researcher memos as across all phases of data analysis. For instance, although cross-case analysis of students' participation in mathematics discourse revealed salient themes related to each student's goal orientation and the degree of concern they each expressed with "being wrong" during discussions, for some participants, like Patty, data sometimes showed that she demonstrated behaviors consonant with both intrinsic and extrinsic forms of motivation. Rather than describing Patty as a student driven exclusively by an extrinsic goal orientation, the researcher reports instances which contradict patterns associated with extrinsic motivation, resulting in the observation that Patty was driven by both intrinsic and extrinsic goals, but primarily by the latter.

Prolonged Engagement

One of the benefits of playing the dual role of teacher and researcher is that access to the research setting before, during, and after the implementation of this study was relatively unencumbered. Before the study began, the teacher-researcher was able to spend a considerable amount of time building trust and rapport with all participants. Unlike many discourse studies which provide mere snapshots of student participation in dialogue, this study allowed the researcher to monitor development of and changes in students' participation in mathematics discourse over the course of five weeks. After the formal data collection period concluded (which lasted nine weeks), the teacher-researcher spent the remainder of the year (i.e., five months) with the participants in the continued role as their classroom teacher, which allowed further insight into ongoing developments with regard to their participation in mathematics discourse.

Persistent Observation

Persistent observation allows researchers to elicit data through a recursive, cyclical data collection and analysis process. The study design allowed for many opportunities to collect rich data in varied contexts. Also, by conducting observations of whole class discourse nine times, in addition to two lengthy observations of small group interaction and subsequent follow-up interviews (all over the course of nine weeks) data saturation was easily attained.

Audit Trail

Creswell (2003) recommends the use of systematic procedures for collecting and analyzing qualitative data in a way that will provide optimal clarity and organization. Merriam (1998) asserts that in order to confirm that the results of the study are actually consistent with the data collected, the investigator must explain "how the data were collected, how categories were derived, and how decisions were made throughout the inquiry" (p. 207). Yin (2003) adds that a clear chain of evidence (i.e., audit trail) helps bolster the reliability (or dependability) of the researcher's claims. This was achieved by clearly linking study findings to specific sources of data in the case study database. Each time raw data are cited in Chapter 4 (the study findings), a reference is noted at the end of each citation identifying the exact data collection instrument from which the data were taken.

The Role of the Researcher

Kilbourn (2006) argues that researchers should briefly comment on their own biographical experiences as they relate to the researcher's understanding of and commitment to the specific nature of the proposed topic of inquiry. Merriam (1998) contends that this is a necessary step toward addressing researcher bias in qualitative research design. By acknowledging these experiences, I lay out my assumptions about mathematics teaching and learning and how these assumptions filter my perspective of reality.

As a student who attended K-12 public schools and one who excelled in mathematics, and now a teacher of middle grades mathematics, I draw from a wide range of experiences that allow me to reflect deeply on my own understanding of mathematics. As a child, I never experienced the kind of student-centered, constructivist-oriented, hands-on, discussion-intensive pedagogy that is now subscribed to, at least rhetorically, in standards-based classrooms of today. Yet I still excelled in mathematics and even developed a solid conceptual understanding of mathematical relationships and meanings in spite of the teacher-and-textbook-centered quality of instruction that I received. In short, this transmission-style of instruction did not leave me with an impoverished understanding of mathematics. For whatever reasons, I was immune to the sources of confusion, misunderstanding, and failure that many of my peers (who received the same quality of instruction) inevitably fell susceptible to. By this, I realize that some students are resilient and can develop rich understandings of mathematics autonomously in spite of transmission-based instruction – however, my experience also tells me that such students are few and far between.

It was during my student teaching semester that I began to realize how I could make a difference for the majority of students who struggle to understand mathematics. The school in which I was assigned had recently adopted a large-scale reform, known then as the National Numeracy Project. Under this reform approach, I witnessed students being exposed to multiple, unconventional methods of problem solving and computation, which I would later see again in *Everyday Mathematics* as an elementary school math teacher. These reform-based approaches appealed to me because they reconstructed mathematics as a discipline that can be taught in multiple ways that draw on students' informal or intuitive knowledge as opposed to the near-exclusive use of predetermined abstract algorithms. I now wholeheartedly believe that this reconstructed approach to teaching mathematics is the key to reaching a greater share of students.

Before giving further wholesale endorsement to my vision of a mathematics discourse community, I now attempt to respond to the research question raised in Chapter 1. In light of the questions and points of contention I have raised thus far in this dissertation, I now explain the study findings, knowing that some questions have been answered while many have merely led to the generation of further complicated questions.

CHAPTER 4

FINDINGS

A multiple case study design was used to examine the following research question: "*What is the nature of low- and high-performing students' participation in discourse about rational number tasks in a standards-based sixth grade classroom?*" The four cases included two low-performing students, Rachel and Heidi, and two high-performing students, Marie and Patty. The findings are presented separately for each case, followed by cross-case comparisons. Both within- and cross-case analyses are organized around salient aggregated themes.

The Nature of Participation in Discourse: An Overview

For the purpose of this study, the researcher defined *participation in discourse* as engagement in communication of ideas through thinking, speaking, writing, and listening in an educational setting, and *student contributions* as speech acts related to mathematical problem solving. Participation in discourse generally manifested in response to prompts (both explicit and implicit[4]) within the context of steps taken to solve mathematical tasks, students' spoken contributions, or scaffold questions posed by the teacher. Students' contributions can be classified into two major types depending on the point of reference:[5] (a) *independent contributions* (i.e., reporting a solution to a problem, making or initiating

[4] An example of an explicit prompt is a written direction, instruction, or question within the context of a problem, such as "Determine which value is greater and explain how you know." An example of an implicit prompt is a question such as "How much space should each school be awarded?" that entails dividing or partitioning, but does not explicitly instruct students to do so.

[5] This classification of student contributions emerged from data analysis, not a priori definitions.

a claim, or offering a potentially useful idea), which generally were not made in direct reference to others' contributions, or (b) *dependent contributions,* which typically were made in reference to previous contributions put forth by others (i.e., evaluating others' claims—which took the form of either aligning oneself with others' claims or challenging others' claims—or not completely evaluating others' claims and instead settling for "taking" or "accepting" an idea without demonstrating the necessary thinking). Further, students' contributions varied in quality based on type (i.e., computational vs. relational) as well as clarity and precision (i.e., complete and correct, ambiguous and correct, or incorrect). Interpretation of the quality of students' contributions relied on a priori codes. For each case, the participant's *contributions* made while solving mathematical tasks are described with emphasis devoted to each of the subthemes defined in Figure 1.

Further, the results regarding the nature of student participation in discourse about rational number topics will be organized around salient empirical themes related to notions of *space* and *meaning*. *Peer and teacher interactions* also emerged from the data as a key domain of interest. Together, these three domains represented contextual elements that mediated students' *access to participation* in mathematics discourse. Consequently, the domains *use of space*, *meaning-making,* and *peer and teacher interactions* are applied as a framework for reporting the study findings related to *access to participation*. Figure 1 presents the relationship among the major themes that emerged through data analysis.

Use of space refers to the ways in which space was used (or not used) by students during discussion to express their participation. For instance, students employed a

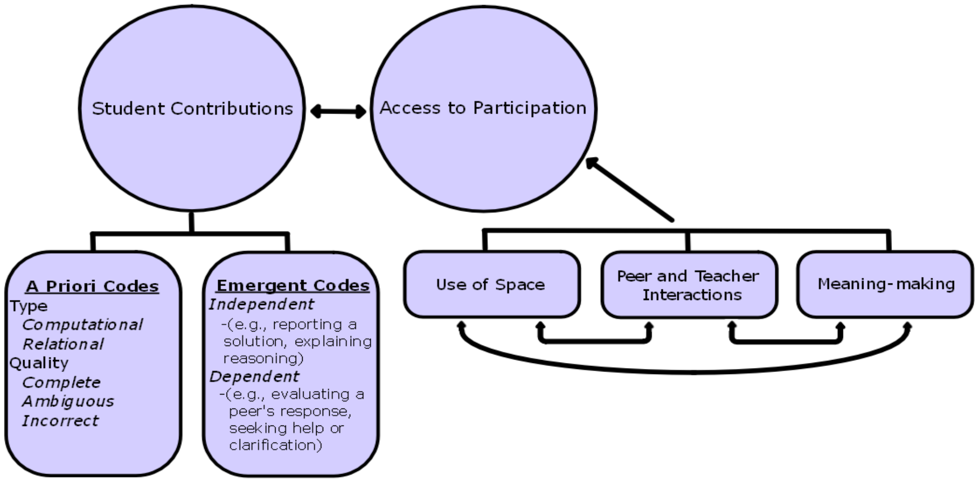

Figure 1. A model of salient themes related to the nature of students' participation in mathematics discourse.

variety of behaviors in their attempts to claim or waive space in discussion, such as thinking aloud, seeking clarification, blurting out ideas, and interrupting one another. Differences in ability with respect to pacing (i.e., speed of calculating solutions) also played a role in determining the distribution of space among students. *Peer and teacher interactions* refer to the observed tendencies regarding students' interactions with specific peers and the teacher and the nature of these interactions. In other words, an examination of whom each participant interacted with is provided, as well as how they interacted with one another and perhaps why. Finally, *meaning-making* represents how and why meaning was shared (or not shared) among students. For example, an ambiguous mathematical explanation can be understood by students who accurately comprehend its implicit or intended meaning, but confusing to other students who do not follow its intended meaning. Constructivists refer to this as the notion of "intersubjectivity" (e.g., Lerman, 1996)—that is, the extent to which participants developed a mutual sense of meaning about the content they discussed. Meaning-making played a significant role in influencing who gained access to participation and who did not. Figure 1 illustrates each of the major themes and subthemes related to the nature of students' participation in mathematics discourse. The themes and subthemes featured in the model are interrelated in complex ways, reiterating the strength of the qualitative research design used in this study.

In the following sections, findings related to the research question are examined in the following order: First, descriptive statistics regarding student contributions to discourse are presented before individual cases in order to provide a clear context of each student's relative degree and quality of participation in mathematics discourse. Then, each individual case is presented by describing the types of contributions students made, as well as factors that affected their access to participation, while supplementing this description with illustrative data (e.g., quotes, vignettes, etc.) and interpretive commentary. This chapter concludes with a cross-case summary that highlights important similarities and differences between low- and high-performing students' participation in mathematics discourse related to rational numbers.

Cross Case Frequencies

The quantity of participation is inextricably linked to the quality of participation in mathematics discourse. The relative frequency of attempts by students to claim space in discussion by making contributions arguably represents the starting point to further appraising quality of participation.

The following table shows the number of utterances, defined simply as a speaking turn, made during each observational setting over the course of the study. Speaking turns ranged from single-word statements or questions to elaborated explanations.

Table 4

Total Number of Utterances during the Study

	Low-performing students		High-performing students	
	Rachel	Heidi	Marie	Patty
Small group	46	192	214	149
Fraction Maze	22	75	115	99
Science Fair	24	117	99	50
Whole class	39	43	54	53
Total	85	235	268	202

Note. Utterances made during whole class settings were often initiated by the teacher (the student may or may not have volunteered to speak, but in most cases they did). Relatively low variation among the frequencies of utterances made during whole class discussion is most likely due to classroom norms associated with parity. In other words, during whole class discussion, the teacher tried to maintain a relative balance among not only the four focal participants, but the additional 21 students in the classroom, as well.

The difference between Heidi and Rachel (both low-performing students) regarding the frequency of utterances made during small group task discussion is underscored in Table 4. Interestingly, Heidi made more utterances than anyone during the Science Fair discussion. However, as expected, high-performing students accounted for a larger percentage of speaking turns during both instructional contexts. In spite of these differences, it is important to note that the total number of utterances alone does not reveal substantial details regarding the quality of spoken contributions.

The data were examined to identify the frequency and quality of each student's contributions, specifically with regard to the number of explanations they shared, and whether these explanations were *computational* or *relational* in nature. Skemp (1978) defined a computational explanation as simply stating what to do in order to solve a problem. In contrast, a relational explanation is characterized not only by knowing what to do in order to solve a problem, but also explaining why. Table 5 shows the total

Table 5

Frequency of Contributions Made during Whole Class Discussion and Type of Accompanying Explanation

	Low-performing students		High-performing students	
	Rachel	Heidi	Marie	Patty
Total contributions	10	8	11	10
No. of explanations	4	5	8	8
relational	0	3	3	4
computational	4	2	5	4

Note. The number of contributions made during whole class discussion of rational number tasks occurred over the course of 9 video-recorded instructional lessons. The difference between the number of contributions and the number of explanations represents the number of contributions that were given but not accompanied by an explanation. Generally, students were always asked to accompany solution reporting with a corresponding explanation.

number of contributions made during whole-class discussion, as well as the type of explanation (i.e., relational or computational). Although the frequency of contributions made by all 4 participants during whole class discussion of rational number tasks was relatively equal, high-performing students tended to accompany a greater percentage of their contributions with explanations. Also, all students' explanations were characterized by a balance between relational and computational explanations except for Rachel, whose explanations were all computational in nature.

Additionally, to determine the quality of students' explanations, all explanations offered during whole class discussions were coded as *complete*, *ambiguous*, or *incorrect* (Franke, Webb, Chan, Ing, Freund, & Battey, 2008). An example of each type is provided in Table 6. The quality of students' explanations was analyzed by examining each instance in which a participant provided an explanation while making a contribution during whole-class discussion. The results are shown in the Table 7.

Table 6

Quality of Explanation Rubric

Sample Problem: Is 7/12 closest to 0, 1/2, or 1 whole? And why?		
Quality Type	Example 1	Example 2
Complete	7/12 is closest to 1/2 because 7 out of 12 is a little bit more than half but still really far away from being 1 whole.	7/12 is closest to 1/2 because 6/12 equals 1/2, which means that 7/12 is just 1/12 more than 1/2 but 5/12 less than 1 whole.
Ambiguous	7/12 is closest to 1/2 because it's 1 away from it.	7/12 is closest to 1/2 because 7 is closer to 6.
Incorrect	7/12 is closest to 1 because 7 and 12 are big numbers.	7/12 is closest to 0 because it's less than half.

Table 7

Frequency and Quality of Explanations during Whole Class Discussion

	Low-performing students		High-performing students	
	Rachel	Heidi	Marie	Patty
No. of contributions with explanation	4	5	8	8
Complete	0	3	3	2
Ambiguous	2	1	4	6
Incorrect	2	1	1	0

Results show that all students struggled to express their mathematical thinking clearly and coherently. As anticipated, low-performing students provided incorrect reasoning at a greater rate than high-performing students. It is interesting to note that Heidi, a low-performer, produced the highest rate (3 out of 5) of complete explanations while her low-performing counterpart, Rachel, demonstrated incorrect reasoning in half of her attempts to explain her solutions. Also notable is the significant rate (10 out of 16) at which high-performing students provided unclear explanations of their thinking.

Additionally, students' contributions were coded when they were responsible for initiating a move during the Fraction Maze small-group task discussion (initiating a move was a critical role in solving this particular task). Each contribution was identified by the type of explanation as well as its quality (in some cases, no explanation was provided, which explains the difference between the total number of initiated moves and the sum of relational and computational explanations offered). The results of the first task (Fraction Maze) are presented in Table 8.

Table 8

Frequency of Initiated Moves and Type and Quality of Explanations during the Fraction Maze Task Discussion

	Low-performing students		High-performing students	
	Rachel	Heidi	Marie	Patty
No. of initiated moves	0	1.5[a]	13	7.5[a]
No. of accompanying explanations	0	0	9	6
Type				
Relational	-	-	7	5
Computational	-	-	2	1
Quality				
Complete	-	-	1	2
Ambiguous	-	-	8	4
Incorrect	-	-	-	-

Note: An initiated move was a central aspect of participation during the Fraction Maze task discussion because of the nature of the task, which entailed 22 necessary "moves" between the start and finish lines of the puzzle. In other words, a minimum of 22 decisions were required in order to solve this task. The initiator was the student who made an explicit claim to the group regarding the move that needed to be made.

[a]Heidi and Patty were both responsible for initiating 1 of the 22 moves involved; hence, they were each credited for one-half of an initiated claim.

Data included in Table 8 reflect the finding that high-performing students were predominantly responsible for initiating moves during the Fraction Maze task. Although many important mathematical contributions were made during this discussion that did not necessarily involve initiation of a move through the maze grid, the results from Table 8 indicate that high-performing students took on leadership roles through directing and telling the others what to do. As the data will reveal in subsequent sections, low-performing students found it difficult to maintain pace (and therefore find or create space to make contributions) with their higher-performing peers during this task discussion.

Finally, the same type of analysis was applied to the Science Fair task discussion. The results are presented in Table 9.

Table 9

Frequency and Quality of Explanations during the Science Fair Task Discussion

	Low-performing students		High-performing students	
	Rachel	Heidi	Marie	Patty
No. of contributions accompanied by explanation	0	8	9	10
Type				
Relational	-	8	4	7
Computational	-	-	5	3
Quality				
Complete	-	2	2	-
Ambiguous	-	3	5	7
Incorrect	-	3	2	3

The fact that Heidi, a low-performing student, relied exclusively on relational explanations, even though only one-fourth of her attempts to explain her thinking were coded as complete and correct, is surprising when noting the relative balance between relational and computational explanations provided by both high-performing students. Also noteworthy is the significant rate of ambiguous explanations provided during this task discussion: over half (15 of 27) of all explanations were coded as ambiguous. Unlike the Fraction Maze task discussion results, which rendered both low-performing students as assuming passive roles, Table 9 demonstrates the sharp contrast between the two low-performing students' participation during the Science Fair task discussion, as Rachel failed to provide a single explanation of her thinking, while Heidi often did.

Within-case Findings

The within-case analysis of data resulted in important findings regarding the nature of each student's contributions to discourse about rational number tasks as well as several important features related to each student's access to participation in discourse. For each participant, the types of contributions made are described, followed by three salient dimensions of access to participation: a) use of space during discussion, b) meaning-making, and c) peer and teacher interactions.

Low-performing Student: Rachel

As demonstrated in Table 4, Rachel made very few contributions in small-group settings, and even though she made almost as many contributions as each of the other participants during whole-class discussions, most of these were solicited by the teacher. When she did make contributions, she rarely provided elaboration of her thinking and often articulated her thoughts incompletely and tentatively. When she provided explanations of her reasoning, they were always computational in nature, as shown in Table 5. Her attempts to interact with high-performing peers were generally characterized by excessive help-seeking gestures (i.e., asking for answers, not explanations) and were often unsuccessful. She reported being confused much of the time during task discussions, and ascribed her reticence to her lack of understanding.

Types of Contributions

Of the four participants, Rachel was by far the least involved in sharing contributions publicly. Because she rarely raised her hand to volunteer to share her thinking, the teacher attempted to elicit contributions from her by calling on her (i.e., animating her as a solution reporter) in order to position her as a successful participant in discussion—especially at times when the teacher sensed that she had some degree of understanding that would allow her to successfully engage in substantive discourse or at least to a provide reasonable explanation. This practice is consonant with that of other researchers who have either used participant frameworks or expectation-states theory as a lens for analyzing student participation in discourse (e.g., Cohen & Lotan, 1995; Empson, 2003; O'Connor & Michaels, 1996). Despite this attempt, Rachel almost never engaged in independent speech acts, such as initiating a claim or reporting a solution.

When confused, Rachel tended to be somewhat reticent. Very rarely did she engage in dependent contributions. For example, although she reported in interviews that she was often confused during small-group task discussions, she rarely prompted others for clarification, and on the occasion that she did seek clarification, she tended to express the source of her confusion vaguely or incompletely. Generally, when she wanted clarification, she would ask solely for an answer instead of an explanation. Moreover, only on three occasions during the entire study did she overtly attempt to challenge another student's idea, but when she did, she did not fully explicate her reasons for dissenting. For example, while playing a game called "Fraction Capture", she contested a move initiated by Marie (a move that was actually legal), but stopped before articulating a complete reason. Just after Marie had captured a square on the game board that was filled two-thirds of the way, the following interaction took place:

1	Rachel:	"No, cause that has to be (long pause)"—
2	Marie:	"No, it's, I filled in more than one half!"
3	Rachel:	(staring at the paper) "Oh." (Fieldnotes, October 21, 2009)

On a separate occasion, during the Fraction Maze task discussion, Rachel again made it appear as if she was beginning to challenge another student's claim but stopped in mid-sentence before completing it with an explanation:

1-2	Heidi:	All I'm saying is that 25 over 5 is 5 so I don't think we can use that.
3-4	Patty:	Oh! I thought you said "we should go there!" (smiling) I'm like, "Nooooo!"
5-6	Marie:	(moving the group along as they are giggling) Alright, let's do 5 1/3 cause that's like the only thing you can do.
7	Rachel:	(quietly) but it's (pause) — ooooooh.

In line 7 above, it is not apparent why Rachel attempted to challenge a previous claim because she did not cite the idea that she was dissenting, nor did she follow through with an explanation of why. Weeks later, during the Science Fair task discussion, Rachel demonstrated this tendency again while she mused over whether or not the fraction three-tenths could be written as a decimal: "But three-tenths (pause)—oh yeah, it does!" When Rachel experienced cognitive dissonance while thinking aloud, she never clearly articulated what caused her to change her thinking or even the specific details of her revised thinking.

Although Rachel almost never publicly challenged others' claims, she sometimes did engage in using others' ideas, although this too was often done privately (usually in the form of scratching out calculations on paper). On occasion, she would also make dependent contributions, usually by simply agreeing (e.g., "Yeah, that's what I got" or "Me too") with another student's previous contribution. When asked during interviews about checking others' claims for herself, she admitted that she rarely did so.

Rachel most frequently engaged in incomplete evaluation of others' contributions, or made no successful evaluation at all. In fact, the most frequent form of public interaction that Rachel engaged in was taking or accepting others' ideas without evaluating them, and seeking clarification by asking questions that were relatively shallow in nature. For example, during the Fraction Maze task discussion, while the group was trying to decide whether one-sixteenth is greater than one-eighth, Patty commanded the attention of the group by using fraction circle pieces to demonstrate the relationship between these two fractions.

1	Heidi:	4 3/16? That would beeeee (long pause)
2	Patty:	I'm doing—
3	Heidi:	(interrupting) Or what about 4 1/16?
4-6	Patty:	(pause, looking annoyed that Heidi just interrupted her) I'm doing the one below it (referring to 4 3/16) because 1/16 (reaching for fraction circle pieces, pausing), think of a twelfth, and then an—
7	Heidi:	(interrupting) Do we circle 4 1/16?
8	Patty:	an eighth.
9	Marie:	We don't have eighths (fraction pieces).
10-17	Patty:	This is just a comparison though. Like this one's teeny (pointing to her drawing of 1/12) and this one's big (pointing to her drawing of 1/8). (laughing) No, what's smaller than a twelfth? Nothing. (picking up a third) Okay, a third; let's try that. (juxtaposing a third with a twelfth) See the comparison? It's smaller! (pointing out that the twelfth is smaller than the third, so presumably attempting to argue that a sixteenth would therefore be smaller than an eighth).

18 Marie: No, it looks bigger (smirking). I'm just kidding!
19 Heidi: (to Patty) I'm confused! (ignored by Patty)
20 Rachel: (looking at Patty) Soooo, 4 1/16???
21 Patty: (looking at her puzzle paper) 4 3/16. That's what I put.
22-23 Marie: Yeah, that's what I put, too. (Rachel then circles 4 3/16 on her paper and moves to the next problem.)

Rather than seeking clarification of Patty's lengthy and somewhat elusive explanation, Rachel merely asked for confirmation of Patty's answer and subsequently accepted it without contest. Moments later, a similar interaction followed:

1 Heidi: Okay, and then 4 1/3???
2 Patty: I'm going to that, and then obviously 6 2/7 (looking ahead)
3 Rachel: (her eyebrows expressing confusion) Wait, so circle 4 1/3???
4-7 Patty: (looking over to help Rachel) Yeah, that's what I put. (Rachel then circles 4 1/3, which moments later, would be successfully challenged by Marie, who was privately evaluating this claim at the time.)

Nearly all of Rachel's contributions posed in the format of a question followed this same basic structure (i.e., ask a question, receive a response, and accept the response with no follow-up). In fact, most of her questions were prompts to repeat an answer or idea; she never asked higher-level evaluative or interpretive questions. For instance, not once did she follow such a simple question with a prompt for justification or explication by asking something to the effect of "Why?" or "How do you know that?"

A majority of Rachel's contributions to group discussion were expressed in a tentative manner. Rather than offering ideas assertively or in a declarative way, she often expressed her ideas as questions, or qualified them with words like "maybe" and phrases like "I guess…." During small-group task discussion, she was observed mouthing or mumbling short responses, but she was usually inaudible, or someone else spoke over her or would interrupt her. Several times, in small-group discussion, she would timidly ask questions several times before finally being heard and responded to (often, the teacher would have to step in and project her question to the group or ask her to speak up as in lines 10-11 below).

1-2	Rachel:	5 2/9?!?! (as if she cannot find it on the puzzle, presumably because it is written as 47/9 on the puzzle).
3	Marie:	And then I think—
4	Patty:	5 1/3 (no one attends to Rachel's confusion)
5	Heidi:	5 1/3???
6	Rachel:	Where's 5 2/9???
7	Patty:	(to Heidi) yeah (as Marie simultaneously says "No!")
8	Marie:	Oh yeah, because you simplify!
9	Patty:	We just debated that! We just debated that!
10-11	Teacher:	(noticing that no one has responded to Rachel's question) Where is 5 2/9?
12	Heidi:	Cause 25 over 5 would equal 5 as a whole number.
13	Rachel:	Where's 5 2/9??? (looking at Patty's paper now)
14	Marie:	Oh, uh, 47 over 9 is 5 2/9
15	Rachel:	Ooooooh, okay.

Triangulated data analysis revealed that Rachel preferred to engage in private forms of participation, most frequently manifesting in the form of "working it out" on paper. Rachel rarely ever engaged in mental math operations, as she clearly preferred to write her thinking out. She admitted during interviews that she was not "good" at mental math and she felt like she could generate answers quicker and more reliably when solving on paper.

If Rachel was not working problems out on paper, she was usually listening to others' ideas. Although observing the act of listening is elusive from an empirical standpoint, Rachel rarely showed physical signs of disengagement or off-task behavior. In almost all cases when she was not working out problems, she was usually making eye contact with the speaker and demonstrating facial expressions that suggested she was hearing and attempting to make sense of others' contributions (this was most often evidenced by a look of confusion).

Access to Participation: Use of Space

Interview and observational data confirmed that while she wanted to "talk more" and felt pressure to speak up, she did not do so, mostly because she was either confused or lagging behind the others in her thinking processes. During an interview after the Science Fair task discussion, she explained, "I was confused at some parts, and I didn't really know what to say and so I just kind of listened to them" (Interview 3). After completing the Fraction Maze task, Rachel admitted that she did not speak much at the beginning of the discussion because she was confused by the directions and had to catch on first before feeling comfortable enough to speak up. Other students perceived Rachel as participating "in the background" or "off to the side," which meant that they assumed that she was listening and trying her best to follow along by engaging in working out problems on paper, despite the fact that she rarely ever spoke. When the researcher asked the other participants about ways to get Rachel more involved, they thought "telling" or "showing" her how to solve problems would help, as well as asking her questions like, "What do you think?" In spite of these ideas being offered by the students themselves, these actions were rarely ever observed throughout the unit.

Rachel rarely volunteered to share her thinking in front of the group. In fact, when she did volunteer, it was typically in response to a concrete, low-level question or a question that could be answered in very brief words. The only times Rachel took the opportunity to explain her thinking was when she was explicitly asked to do so by the teacher. When the researcher asked Rachel during an interview about the notion of taking risks (i.e., offering to share ideas even if the student thinks she may be wrong, in hopes of learning from mistakes), she referred to Heidi, and specifically her tendency to take risks during discussion:

> Rachel: Um, even if Heidi has it wrong, she'll still be like, well, this is my answer and I'm going to go with it.
> Int: Uh huh, she puts it out there even if she like … even if she's wrong, she takes risks, as you might say. Do you think that's good?
> Rachel: Mmm, hmm.
> Int: Well … if there's a bad thing about that. What might that be? What would make that hard to do?
> Rachel: People would be like, "Nooo!!!" (in a mocking tone).
> Int: And how would that make you feel?
> Rachel: Like you're stupid. (Interview 3, December 17, 2009)

Interview responses from both Heidi and Patty also lent credence to Rachel's perceptions of "feeling stupid" or experiencing diminished self-concept as a result of being wrong in front of others. Said Heidi, "I guess because she just doesn't really, like, I don't wanna say she doesn't wanna feel stupid, but I just wanna say she's, I guess she's kinda like, like I guess she doesn't really wanna butt in, because she doesn't wanna feel sad or mad about herself that she didn't know that, so I guess that's why" (Interview 3). Patty simply attested, "She [Rachel] doesn't wanna seem dumb. That's how everyone feels" (Interview 3).

Interestingly, Rachel showed a greater degree of confidence and willingness to speak up during weekly after-school tutorial sessions. The tutorial groups typically consisted of the same children each week (mostly low-performers in math class). In a memo dated November 12, 2009, just moments after the conclusion of a weekly tutorial session in which Rachel's participation stood out, the researcher noted the following:

> She seemed much less timid about asking questions and sharing her thinking aloud in this small group tutorial today. So afterwards, I asked her "Why the change?" and she responded that she felt "more comfortable." When I asked her why, she just shrugged her shoulders and said, "I don't know why." Perhaps it's because of the more intimate

> environment, or it could have something to do with parity—in other words, she probably knows that the other two students present at tutorial are in the same "academic ballpark" as her; therefore she is not afraid of being perceived as "stupid" or "dumb."

Unfortunately, outside of the weekly tutorials, Rachel did not exemplify the same spirit of confidence and courage to share her thinking and ask questions when confused.

Access to Participation: Meaning-making

Although Rachel commended Heidi's apparent immunity to the "fear of being wrong," she never really adopted the same sense of fearlessness during discussion. In fact, Rachel admitted that the discussion-based context was not her favorite way to learn, partly because she did not "really want to sit there and talk" about math because she just gets "confused." When asked during an interview about her interactions with Marie during the Science Fair task, Rachel explained that it was difficult to interact with others because she could not follow or comprehend what they were attempting to say, particularly Marie:

> Int: Did you interact with Marie?
> Rachel: Not really, because what she was saying, I got really confused. That's why I was like (showing a confused look on her face). (Interview 4, December 18, 2009)

When asked to be more specific, Rachel added: "Cause she wasn't um, like, saying what she was talking about" (Interview 4). The researcher then read an excerpt of one of Marie's utterances at a critical point during the discussion of the Science Fair Task and asked Rachel what, specifically, was so confusing, and she responded, "Well, she didn't use like 'this school' or 'this class,' she was like, 'that' and 'this'" (Interview 4). The use of vague pronoun referents, particularly by high-performing students, played a salient role in group discussion (which will be discussed in greater detail later). Interestingly, Rachel confessed to merely accepting Marie's thinking during the Fraction Maze task because she thought that Marie was "in charge" and that her thinking was plausible because of this. When asked why she did not evaluate others' ideas, Rachel explained that she was confused and constantly lagging behind the group and that the group moved on without her understanding "like 70% of the time" (Interview 2). In other words, Rachel felt that the degree to which she was involved in discussion was mediated by the degree to which other students' contributions were clear and easy to follow. In short, her participation depended on the quality of others' contributions.

Access to Participation: Peer and Teacher Interactions

During interactions with her peers, Rachel was never observed taking a leadership role with respect to mathematical content. The only time Rachel ever directed the group was in a primarily social context. During the Fraction Maze task discussion, for example, Patty and Heidi argued over whether or not Heidi stated a mixed number properly, to which Rachel stepped in and pleaded, "Okay! It doesn't matter. Let's just move on!" Her directing was a form of social arbitration or procedural management; it was not related to the mathematical content of the task. As noted in Table 8, she was the only participant to not record a single initiated move during this task discussion.

Rachel tended to interact with Heidi, another low-performing student, more frequently than Patty or Marie (both high-performing students) while working in small-group settings. A few times, Rachel was observed unsuccessfully attempting to seek interaction with Marie or Patty. The following excerpt is from the researcher's fieldnotes of a lesson on estimating fractions to the nearest half-unit (e.g., zero, one-half, or one whole):

1	Heidi:	(to Rachel) Okay, 7/12?
2	Rachel:	(pause) That would beeee –

3	Heidi:	It's either half or zero.
4	Rachel:	One-half. I think it's a half.
5	Heidi:	You wanna ask Mr. Yorke?
6-10	Rachel:	(turns around to find the teacher but can't get his attention because he is working with another group of students nearby, so she turns around and looks at Heidi) I think it's a half. (Now she looks at Marie and Patty) What do you guys think? (No immediate response from the group. About 3 seconds elapse.)
11-14	Heidi:	(to Patty and Marie who are working together) What would 7/12 be? Would it be equal to a half? (No response; teacher happens to be passing by now, Heidi directs the question toward him now.) Would 7/12 be equal to a half or zero?
15	Teacher:	7/12?
16-17	Patty:	No, it's closer to a half. Closer! Closer to half! (Fieldnotes, October 13, 2009)

Why it took so long for Rachel and Heidi to finally receive a response to their prompts for confirmation and why Patty finally responded as soon as the teacher approached the group is not known. It is interesting, however, to point out that Heidi and Rachel offered nothing beyond a mere guess at the answer, not even an attempt at explaining their reasoning, despite clearly indicating that they were capable of such reasoning through various empirical observations.

The majority of Rachel's interactions with peers during the Fraction Maze task were characterized by excessive forms of help-seeking. She was confused for most of the task and had trouble keeping up with the group for the most part. All of the questions she posed during the discussion were aimed at getting answers from others without concern for relevant explanations. For example, often, when the group made a move, Rachel would ask for confirmation of the correct fraction or mixed number to circle by asking the group, "So, circle this?"

During the Science Fair task discussion, most of Rachel's contributions were elicited by the teacher's prompts. On several occasions, the teacher credited Rachel with ownership of an idea, albeit an idea elicited by teacher request and one that she never attempted to elaborate on. For instance, moments after Rachel was asked to share her fractions for the divvying of the auditorium (her conjecture was a 3/8 and 1/8 split between Malcolm X and Kennedy Middle Schools), the teacher cited an idea originally shared by Rachel to help the group resolve confusion at a later point in the discussion: "Okay, Rachel just said that Malcolm X should pay more than Kennedy." Despite several direct references by the teacher to position Rachel as an originator of a useful idea or as a solution reporter, there were no successful attempts at eliciting follow-up explanations from her during the Science Fair task discussion (as demonstrated in Table 9).

Low-performing Student: Heidi

Heidi made many significant contributions during discussion in both whole- and small-group instructional settings. Although she too reported being confused much of the time, she engaged in a mixture of adaptive and excessive forms of help-seeking and frequently prompted others for clarification of their reasoning. In small-group discussions, Heidi desperately sought to claim space in discussion, but found it elusive due to pacing disparities and a tendency to be interrupted or overlooked by high-performing peers. Heidi showed a willingness to get involved in discourse and thus risk making mistakes because she reported learning best under such conditions.

Types of Contributions

Despite performing poorly on formal measures of mathematics achievement, Heidi engaged in a wide variety of roles when making contributions during group discussion activities. She volunteered to share her thinking as frequently as any student in the class, in spite of her tendency to confuse ideas, offer incorrect reasoning, and require additional teacher scaffolding. During the Fraction Maze task discussion, Heidi made several independent contributions, mostly via attempts at initiating claims by offering a fraction or mixed number as a possible solution. She almost always did this as soon as the group acknowledged consensus on the previous solution. Many times, she would initiate a move by narrating the possible options to the group, followed by an attempt to solve the problem by thinking out loud, usually elongating her speech, as in lines 2-3 and 9 in the interaction below.

1	Marie:	Uh, 7 and, 7 and, no! 7 over 3!
2-3	Heidi:	Yeah, 7/3, that would be about (looking up to the ceiling) 2 and 1—
4	Patty:	No.
5	Heidi/Marie:	(in unison) third!
6	Heidi:	2 1/3.
7	Marie:	Yeah, that's good. (everyone circles this)
8	Marie:	And then I think we go to two and seven-eighths—
9	Heidi:	That would beeeeee—
10	Marie:	Because that's the only thing we can do.
11-12	Patty:	Yeah, or else, we'd just make a box if we go to one-sixteenth.

During the Fraction Maze task discussion, Heidi frequently prompted others for clarification of their contributions by admitting, "I'm confused," or "I just don't get that at all!" A few times, she went beyond merely acknowledging her state of confusion by specifically citing the part of another student's contribution that was nebulous to her, as in the excerpt below, taken from the Science Fair task discussion:

1-9	Patty:	Um, these work because I did 600 over 2000 and you keep dividing that down, and you, divide it by 2, and it's 300 over 1000, then you divide that and it's 150 over 500, and then you divide that and it's 30 over 100, and I know what that means, uh, you divide that down too or 3/10, so I got that. And then, for Kennedy Middle School, it's kind of obvious (*Rachel looks up with a blank stare on her face*), cause you add 50 plus, uh, 5/10

		plus 3/10, which is 8/10, and then you have 2/10 left over, so (*Rachel still looking confused*).
10	Marie:	That's what I got.
11	Heidi:	But there's stuff left over, so what would you do with that?
12-13	Marie:	No, you don't have anything left over cause if you add all those together, you get 10/10.
15	Heidi:	But she said there's 2/10 left over and then I asked her after that.
16-17	Patty:	(to Heidi) So since you have 2/10 left over, that's Kennedy Middle School's kids.
18	Heidi:	Oh!

In line 11, Heidi evaluated Patty's explanation as incomplete because Patty concluded her reasoning with "and then you have two-tenths left over …" which Marie confirmed. Heidi's question in line 11 animated Patty as a clarifier of her previous claim. Rather than ignoring Patty's vague explanation, Heidi sought clarification for understanding, and consequently Patty was challenged to use greater precision in explaining her reasoning.

At times, Heidi's prompts for clarification were ignored by others, and when the teacher observed this happening, he sometimes challenged, if not reminded, the others to respond to these prompts. The vignette taken from the small-group discussion of the Science Fair task below is illustrative of this type of contribution. After agreeing that 50% of $300 is exactly $150, two members of the group then went on to claim that 30% of $300 is $90. After Marie claimed that 30% of $300 would equal $90, Heidi interjected:

1-2	Heidi:	Wouldn't you do it [30%] out of 150 because there's only 150 dollars left?
3	Teacher:	Oh, that's a good question.
4-6	Marie:	Because you'd go like this (motioning for me to look at her paper as she writes some numbers with her pencil—until the lead breaks accidentally).
7-8	Heidi:	Cause 150 has already been paid by Bret Harte and so there's 150 left.
9	Teacher:	Right.

10	Heidi:	So you should have done 30% of 150.
11-12	Teacher:	What do y'all think about that? (Everyone is writing; *are they listening???*)
13-14	Marie:	I'm just gonna stay with my answer, um, because you do 30% of 2, of 300, which would be—
15	Teacher:	But did y'all hear what Heidi said though? (no responses)
16-17	Heidi:	(to Marie) 150 of that has already been paid, so there's 150 left, soooo—
18-19	Patty:	But still you have to think of 30% of 300 so if you do 30% of 150, it'd be a totally different number.
20-22	Teacher:	What would it be? (Patty takes pencil and begins to explore this idea.) It's a great question because now that you've asked that, I'm thinking too, why not 30% of what's left?

In the example above, Heidi challenged a claim made by Marie and then implicitly animated others as evaluators of her idea (the teacher made this explicit by asking the group what they thought). Initially, Marie avoided this animation by reiterating her strategy, but the teacher quickly re-positioned her and the others by repeating the question. The group went on to explore Heidi's conjecture, which was initially offered in the form of a question for clarification, but ended up serving as a justification of why taking 30% of the residual amount (i.e., $150) would not constitute a fair, proportionate amount for Malcolm X Middle School to pay for the science fair. Heidi did not always provide a detailed explanation of her alternate thinking when prompting others for clarification, but, in this case, the specific question she posed provided a rich context for deeper examination of key mathematical content. Moreover, Heidi often expressed during interviews that she valued mistakes because discussing her errors was the best way for her to avoid being confused in math:

> Sometimes, whenever I get an answer, like say there's a math question and I say the answer is 30 and then Patty says it's like 56 and then I say like, "How did you get that?" and she tells me and then I say "Well, what if you do like this instead of that?" and then she kind of just would tell me why and so that's how I learn better cause I learn from my mistakes. (Interview 3, December 17, 2009)

Not only did Heidi often prompt others for clarification during discussion, but she also explained her reasoning in a way that was often clear and accessible to others, as in the following contribution:

> Um, I put it so that, um, the whole space had, um, Bret Harte had the most space because they had the most kids. And, then, um, so they got half of the space. And then I put that, um, Malcolm X and Kennedy Middle School had 25% because there's half of the auditorium left and if you split those in, if you split half in half, that's 25%, so I gave both of them 25%, and that's how I did it. (Science Fair task fieldnotes, December 17, 2009)

When the group discussed challenging parts of a task, Heidi's contributions often took the form of evaluating her thinking out loud, which sometimes would break down:

> And then you'd probably figure out how much space there is. Because where I got the 200 was there's [sic] 200 more students in Kennedy, I mean, in Malcolm X Middle School than there are in Kennedy. So, like, and there's 1000 kids left, and so 200 out of those 1000—Wait, that wouldn't work. (Science Fair task fieldnotes, December 17, 2009)

Often, during episodes where the group appeared to be at an impasse, Heidi would offer a potentially useful idea, although not always followed by explication or justification for it. Interestingly, most of the time, Heidi's contributions were presented in question format. "Why don't we split it into sixths? Like, instead of fourths, into sixths?" was an idea she posed later on during the discussion of the Science Fair task. In analyzing Heidi's contributions, she most often offered ideas either in question format or by using the words *maybe* and *probably* as qualifiers, rather than stating her ideas and opinions declaratively. The excerpt below demonstrates a typical example of this:

1	Heidi:	Wouldn't it be uh, an eighth and a sixth? Could we try that?
2	Marie:	Or 2/8 and a fourth!
3	Heidi:	Like instead of a fourth, it would be a sixth.
4-5	Teacher:	Well, 1/6 is interesting, I don't think anyone said that, although you (pointing to Marie) originally you did.
6	Marie:	I did.
7-8	Heidi:	But, like, wouldn't it be, okay, there's half and Rachel said 1/8 and 1/4, couldn't we either change 1/8 to a sixth?

When asked during an interview about her tendency to couch ideas as questions rather than declarations, Heidi cited the teacher's instructions to "ask each other questions and interact" instead of directing others, "This is what we're going to do," suggesting that she was merely following the teacher's instructions rather than demonstrating a lack of confidence in her ideas (Interview 2, October 26, 2009).

Although many of her ideas seemed to be presented tentatively, she demonstrated a tendency to state summative claims at apparent impasses in problem solving, perhaps as a metacognitive strategy for clearing confusion by taking stock of conditions known to be true up to that particular point in time. At an impasse during the Science Fair task, she declared, "All we know right now is that Bret Harte is good. That's how many, how much space they get…. So I guess we have to like add more space on to, uh, what is it? Malcolm X." This statement not only evaluated the group's cumulative progress toward solving the problem, but it also re-framed or reiterated the intermediate goal at that point in time, which was the need to add more space to Malcolm X Middle School's share of the auditorium (i.e., more than her original conjecture of 25%).

Heidi also engaged in private participation, as she too scratched out calculations and algorithms related to the intermediate steps of task problem solving. But she made it clear during an interview that she did not want to take a passive role in discussion, despite recognizing the disparity in performance-levels between the group members.

> I feel like I struggle, and, um, like I said … see I just kinda wanna get more involved so that I get it more, instead of just kinda being off in my little corner and kinda just listening. I just wanna like, get involved, so I *understand it* [emphasis hers]. (Interview 1, October 23, 2009)

Heidi also displayed an affinity for mental math, despite making a high percentage of mistakes. Near the end of the Science Fair task discussion, Heidi put her mental math acuity on display. When asked to mentally divide 2000 by 100, she used a combination of mental math and proportional reasoning: “Uh, it goes in, let’s see, 100 goes into 1000 ten times, so that would be 20 times.” Heidi often made it apparent when she was engaging in mental math by either looking up at the ceiling or thinking aloud. On several occasions, she began to engage in mental math only to have the answer blurted out by either Marie or Patty before she could finish.

Access to Participation: Use of Space

During task discussion, Heidi often expressed frustration with being outpaced by Marie and Patty when working out calculations on paper or mentally. As students worked feverishly to rename improper fractions and mixed numbers for the sake of comparison during the Fraction Maze task, Heidi would often seek to initiate each move by calling out the possible options to the group. Many times, she would start to claim space by thinking aloud, only to be interrupted by Marie or Patty, who would typically rename the rational numbers more quickly than Heidi would. At one point during the discussion, Heidi desperately pleaded, “Hang on!!!”

1-3	Patty:	5 into 17? It would be 3 and something (smiling, now working on paper; all girls now are working it on paper, except Marie who is scribbling with her fingers).
4	Marie:	Let’s do it on paper.
5	Teacher:	Good call!
6	Patty:	That’s what I’m doing, see???
7	Marie:	It’d be 2 and ssss—
8	Heidi:	Hang on!!! (in a whiny tone)
9	Patty:	3 2/5! (Heidi looks miffed)

Just moments later, near the end of the discussion, Heidi desperately pleaded for additional time and space to participate, but ultimately failed to be granted such, as demonstrated in lines 6 and 7 below:

1-2 Heidi: Okay, and then 37/4??? That would beeeeee—that would go into—
3 Marie: That'd be 8 and something
4 Heidi: No, that would beeeee—
5 Marie: 8 5/4!!!
6 Heidi: That would be nine, wait, hang on, I'll show you, I'll show you…
7 Patty: It's bigger (to Marie). Yeah, circle it because it's bigger. (Fraction Maze task, October 23, 2009)

Heidi characterized this disparity in pacing as a source of both distraction and confusion during the Fraction Maze task discussion. When asked during an interview what more she could have done to improve her participation, Heidi said she wished that she could have "pitched in more." When asked why she did not pitch in as much as she would have liked, she explained:

> Cause they were like all off and then I was like confused cause I'd be in one spot and they'd be like all the way in another spot and then I'd have to like catch up, because I get like distracted and I'd be working on one problem and they'd be on like three more problems. (Interview 1, October 23, 2009)

She added that because of the pacing disparity, she found it nearly impossible to check or evaluate Patty's and Marie's claims, perhaps further limiting her ability to make substantive contributions to the discussion.

Although it is reasonable to expect interruptions to occur often during small-group discussion, Patty and Marie showed a noticeable tendency to interrupt Heidi. During a lesson early in the rational number unit, the teacher asked the students to write a letter to a fictional student who mistakenly showed through a drawing that six-eighths is greater than three-fourths. After a few minutes, he allowed students to share their writing in small groups. Note how Heidi encounters difficulty in finding space to share:

> Heidi opens her mouth and says, "I put"—but is immediately cut off by Patty who says, "I wrote this" and Heidi immediately responds – "Well, then!" in an irate tone (but maybe she is being playful). Patty reads, "Dear person, you are wrong. Your picture is not lined up correctly…" As soon

> as Patty finishes reading, both Marie and Heidi attempt to read their responses. Heidi, again, after getting interrupted says, “Well, then!” but Marie stops reading and politely tells her to go ahead. (Fieldnotes, October 5, 2009)

A few weeks later, during the Fraction Maze small-group task discussion, Heidi again showed an inclination to be interrupted before she could fully express her contribution. In the excerpt below, it takes several turns for her to articulate that she wanted to discount 25 over 5 as a possible option:

1	Heidi:	5 1/3? …. Cause 25 over 5 would equal 5 as a whole number.
2	Rachel:	Where’s 5 2/9??? (looking at Patty’s paper now)
3	Marie:	Oh, uh, 47 over 9 is 5 2/9
4	Rachel:	Ooooooh, okay.
5	Patty:	(to Heidi) Yeah, but a ninth, a third is bigger than a ninth
6-7	Heidi:	(to Patty) I know, but I’m saying, I’m saying, if we, like, from 47/9, there, we have the option of 25/5—
8	Marie:	Which is exactly 5!
9	Patty:	Yeah, but—
10	Heidi:	(finishing her statement) 5/4 or 5 1/3. And I’m saying—
11	Marie:	I think it’d be 5 1/3.
12	Heidi:	Cause I’m saying, all I’m saying is that—
13	Patty:	That’s exact, that’s exactly 5.
14	Marie:	Cause 25 over 5 would be exactly 5.
15	Heidi:	Cause all I’m—
16	Patty:	And then 47 over 9 is, um,
17	Marie/Patty:	(in unison): 5 2/9!
18	Patty:	And then, 25 over 5 is exactly 5.
19-20	Heidi:	All I’m saying is that 25 over 5 is 5, so I don’t think we can use that.
21-22	Patty:	Oh! I thought you said “we should go there!” (smiling) I’m like, “Nooooo!” (Fraction Maze fieldnotes, October 22, 2009)

Although it appears that Heidi was just stating all of the options and attempting to consider each one in turn, Marie and Patty may have misinterpreted Heidi’s intentions when she said, “We have the option of twenty-five-fifths.” Heidi subsequently tried to re-claim her space in lines 10, 12, and 15, in hopes of explaining why she singled out 25/5, but was not able to do so until lines 19-20.

Heidi's engagement in task discussion was not as strong or focused during whole-class settings. Attention issues (i.e., concentration, off-task behavior) sometimes undermined her ability to contribute meaningfully to whole-group discussion. For instance, there were several instances in which Heidi was observed engaging in off-task behaviors such as organizing her notebook, playing with her pencil, and staring at and communicating covertly with other students in the classroom while students were sharing their reasoning about related mathematical content. Sometimes, when Heidi was called on by the teacher to make a contribution, she expressed disorientation with the question or the context of the discussion due to inattentiveness.

Access to Participation: Meaning-making

Although Heidi did a nice job of seeking clarification during small-group task discussion by prompting others to express their thoughts in clearer words, her attempts were not always successful. Particularly challenging for Heidi was following along and making sense of the contributions made by high-performing students, particularly Marie. Whether it was because Marie "talks real fast" or because she "thinks that everybody already has it and so whenever she explains it, she doesn't have to be specific," Heidi made it clear during an interview that the process of interacting with Marie was usually fraught with difficulty.

> Sometimes like she does it all in her head or on the paper or something and whenever she's like explaining it, she thinks that you already know what it means and so she goes "like this and that," and then I'm just like, "Whoa! What do you mean?" . . . Cause she thinks that everybody can do the same thing as her so whenever she explains it, everybody's already looking at their paper and they're just like, "Oh, yeah, I get that," but she needs to be like more specific with what she says. (Interview 4, December 18, 2009)

During the Fraction Maze task, when the teacher was playing a virtually passive role, Heidi's attempts to seek clarification were often unsuccessful and at times she clearly avoided seeking clarification because of apparent pacing disparities between individuals in the group. However, when the teacher "stepped in" during the discussion of the Science Fair task by revoicing students' contributions to the group, as well as scaffolding their reasoning, she posed significantly more clarification questions, some of which resulted in very rich discussion. An illustrative example of this is provided below:

1	Teacher:	Who should pay more: Malcolm X or Kennedy?
2	K and E:	Malcolm X.
3	Teacher:	Malcolm X, they have more kids.
4	Heidi:	Yeah, 6, I mean 200 more.
5-6	Teacher:	But how much more? We still have $150 more to pay *(Patty now raising her hand)*. Half has already paid by Bret Harte.
7-8	Heidi:	So there's 150 more and so I guessed, uh, if you split 100 that's 50; if you split, uh, 50 that's 25 sooooooo 75 dollars?
9	Marie/Patty:	(*shaking their heads in disagreement):* No.
10	Heidi:	Wait (*looking at her paper)* … That's what I said.
11-12	Teacher:	Whoa, wait a second. Yeah, yeah, yeah, you said 75 dollars, they should both pay 75 dollars, right?
13	Heidi:	Right.
14	Teacher:	And that adds up to 150.
15	Heidi:	Right, sooo…
16-17	Teacher:	And Rachel just said that Malcolm X should pay more than Kennedy.
18-20	Heidi:	Yeah, but that's what I had because I had that 25 percent to Malcolm X and 25% to Kennedy. *(Marie is raising her hand emphatically)*
21-24	Teacher:	Yeah, back when you said 1/4 to Malcolm X and 1/4 to Kennedy, that would make sense, but Malcolm X should pay more than Kennedy. So how do we figure out how much each one should pay?
25	Marie:	I wanna say something!!!
26	Teacher:	Marie.
27	Marie:	Um, 30%, for Malcolm X you do 30% into 300—
28	Teacher:	30% of (emphasis) 300.
29	Marie:	Right. Which would be 90 dollars.
30-31	Heidi:	Wouldn't you do it out of 150 because there's only 150 dollars left?

Access to Participation: Peer and Teacher Interactions

Although Heidi's personality could be characterized as friendly and affable, Patty developed an aversion toward Heidi throughout the duration of this study. Patty attributed the disliking to Heidi's distractibility and tendency to talk during inappropriate times. Although Heidi remained attentive and focused during both video-recorded small-group task discussions, she was sometimes observed engaging in off-task behavior during whole-class discussion. The tension between Heidi and Patty was palpable, especially in behavior and actions demonstrated by Patty. This tension manifested in several ways, which will be described in greater detail in the section on Patty's participation. The tension notwithstanding, Heidi often reached out to Patty for assistance. Heidi clearly realized the value of learning in a social context and expressed this during an interview:

> Heidi: Cause you, the like, one person in your group might know it better than you, so whenever they start like getting into it, they'll like kinda help you with what you don't know and then, so, yeah.
> Int: So you can benefit from someone else's knowledge if they're pretty good at it, or vice versa—they might be able to benefit from you if you solidly understand one of the concepts.
> Heidi: Yeah, right, like Patty knows a few things that I don't know. So if I, like, ask her for help, she'll tell me and then I'll get it, so like, yeah. (Interview 2, October 26, 2009)

Heidi demonstrated an example of observing Patty and consequently mimicking her strategy during an ambiguous problem in the Fraction Maze task. After noticing that Patty had successfully employed visual representations of fractions several times during the discussion, Heidi announced to the group that she had decided to follow suit:

1	Patty:	Do we have ninths (referring to fraction circle pieces)?
2	Heidi:	No.
3-4	Patty:	I'm doing circles again! (Patty is now drawing circles on paper again, juxtaposing them.)
5	Heidi:	I know, that's what I'm gonna do (watching Patty).

Although Heidi attempted to interact with Patty and Marie, her attempts were often unsuccessful, as Patty and Marie demonstrated a natural preference to interact solely with one another, leaving Heidi to interact only with Rachel. When asked about this during an interview, Heidi explained, "They're kind of like the faster learners and I have to like go through it like Rachel does …" (Interview 4, December 18, 2009). After noticing that many of Heidi's attempts to interact with Patty and Marie were unsuccessful, particularly in during the Fraction Maze task discussion, Heidi was asked why she thought this happened and she explained,

> I guess they weren't really listening and they were like off on their own and I would like get the answer right and then they'd say like "No it's not!"; then they'd do it, and they'd be like, "This is this answer," and I'd be like, "Well, I just said that. (Interview 1, October 23, 2009)

Toward the end of the study, and perhaps during the height of the spat between Heidi and Patty, the researcher noticed that Heidi began to raise her hand when she needed help during small-group work, rather than seek interaction with Patty or Marie. However, when asked about this, Heidi reported that she sought help from the teacher because she knew he could "explain it better" (Interview 4). Interestingly, Heidi never once acknowledged the tension between her and Patty.

Heidi sometimes offered reasonable claims and explanations that were not publicly evaluated by others, and when this happened the teacher often stepped in to revoice her claims and position the other students as evaluators of her ideas. For instance, during the Science Fair task discussion, just after Heidi delivered an elaborate explanation (although incorrect) of why she chose to allocate an equal amount of space to Malcolm X and Kennedy Middle Schools, the teacher asked the other students what they thought about her conjecture. Marie raised her hand to respond, but immediately began explaining her own solution and completely ignoring Heidi's explanation before almost immediately being interrupted by the teacher: "Whoa! Wait a second! Do you agree with Heidi and why or why not?" Later on, during the same task discussion, the teacher had to explicitly ask the group twice to respond to a critical question posed by Heidi.

Heidi was not only ignored during small-group discussions; she sometimes appeared isolated during group seat-work in whole-class instructional settings. When asked during an interview why she tended to isolate herself, and seek help from the teacher rather than her peers (especially during the latter part of the study), she replied:

> I just think that since (the teacher is) more, like, educated with it, that (he'll) be able to like explain it better, because sometimes like if I ask Marie something, she'll like either speak really fast or do something really fast, and I'll be like, "Where did you go?" (Interview 4, December 18, 2009)

High-performing Student: Marie

Marie was perhaps the most involved in participating during discussion of tasks by offering an array of both dependent and independent contributions. Viewed as the group's leader by each of the other participants, Marie was most active in directing the group and challenging claims and catching the group's mistakes. She interacted almost exclusively with Patty and reported having difficulty with explaining her reasoning clearly enough for low-performing students to comprehend her ideas. Marie found space during discussion quite easily because of how quickly she solved problems and she even appropriated space at times by interrupting others, blurting out answers, and thinking aloud.

Types of Contributions

Marie initiated significantly more contributions than anyone in the group. Very fast at mental calculating and processing, she was often the first student to complete assignments in class as well as the first to raise her hand to share her contributions. She almost always volunteered to share her thinking, regardless of the instructional context. Her contributions were also very diverse in nature.

Interview data revealed that a majority of the students perceived Marie as the group's de facto leader. Anecdotal evidence from task discussions corroborated this sentiment. During the small-group discussion of the Fraction Maze task, Marie was the first student to offer a contribution as well as the last, when she acknowledged to the group that the task had finally been solved. When Heidi would pose a question for clarification or Rachel would seek affirmation of correct answers, Marie would almost always be the first to respond. She often uttered simple evaluative comments to the group, such as "Yeah, let's do that," or a prompt to command the group's attention, like "Okay, guys! Guys, listen!" At several times during small-group task discussions, she would solicit the entire group's attention as she guided them through a specific procedure or explained a pictorial representation of her thinking.

When Marie agreed or aligned herself with another student's contribution, she would often voice her agreement, sometimes by simply saying, "Yeah, that's what I got," or "Yeah, me too." Sometimes, she would qualify her agreement with an explanation or brief justification, as shown below in lines 3 and 15-16 in this excerpt from the Fraction Maze task discussion:

1	Patty:	You can do 14 over 2, because that equals 7.
2	Heidi:	That's 7.
3	Marie:	Yeah, 7 and that's more.
4	Patty:	And then do 8 5/8 cause that's bigger!!!
5-6	Heidi:	Okay, and then 37 fourths??? That would beeeeee—that would go into—
7	Marie:	That'd be 8 and something.
8	Heidi:	No, that would be—

9	Marie:	8 5/4!!!
10	Heidi:	That would be nine, wait, hang on, I'll show you, I'll show you…
11	Patty:	It's bigger (to Marie). Yeah, circle it because it's bigger.
12	Heidi:	(thinking aloud) 4, 9, 36, 1. It would be 9 1/4!
13	Marie:	Yeah.
14	Patty:	It would be 8 5/4 (to Heidi)
15-16	Marie:	(to Patty) I know, which is an improper fraction, so you'd have to go to 9…

Sometimes, she would extend her affirmation of someone else's contribution with additional justification, as in the following excerpt taken from the Science Fair task discussion. After a series of unsuccessful conjectures regarding the exact fraction of the auditorium that each school should be allotted, the group experienced a breakthrough, although not without confusion and prompts for clarification:

1	Marie:	That's what I got.
2	Heidi:	But there's stuff left over, so what would you do with that?
3-4	Marie:	No, you don't have anything left over cause if you add all those together, you get 10/10.
5-6	Heidi:	But she said there's 2/10 left over and then I asked her after that.
7-8	Patty:	(to Heidi) So since you have 2/10 left over, that's Kennedy Middle School's kids.
9	Heidi:	Oh!
10	Marie:	Yeah, and then if you add all those together it'll equal 10/10.
11	Teacher:	Wait, did Kennedy get less than Malcolm X?
12	Patty/Marie:	(in unison): Yeah!
13	Marie:	A lot less.
14	Teacher:	A lot less? How much less?
15	Marie:	Well, no! Only 1/10!

In lines 1 and 12, Marie expressed agreement, but without explanation. In line 10, and then 13 and 15, Marie supplemented her agreement with Patty's contribution in lines 7-8 by explaining to Heidi that the individual parts of space sum to entire space of the auditorium. The teacher then posed a question (line 14) that prompted the students to ensure that the smaller school was given the smallest fraction of space. Marie moved beyond simple affirmation and made a claim: "A lot less." After the teacher prompted her to clarify, she revised her claim after determining the difference was relatively small.

Even though Marie frequently affirmed the contributions of others in diverse ways, her most significant contributions to group discussion were dependent ideas categorized as "challenging claims" or "catching mistakes" ("catching mistakes" was actually an In Vivo code taken from interview data). In the example below, Marie exemplifies both patience and resolve in evaluating the group's hasty decision to move from 9/2 to 4 1/3 (which was originally prompted by Patty in line 2).

1	Heidi:	Okay, and then 4 1/3???
2	Patty:	I'm going to that, obviously 6 2/7 (looking ahead)
3	Rachel:	(looking confused) Wait, so circle 4 1/3???
4-5	Patty:	Hey! 9 over 2 (looking over to help Rachel) Yeah, that's what I put. Look, there's another 9 over 2.
6	Rachel:	Where?
7	Marie:	1…2…3… and this one's bigger (to herself).
8	Heidi:	(to Patty) but that's diagonal (thinking diagonal to 4 1/3)
9	Patty:	No, it's right next to it (Patty is right, if 4 1/3 is the referent)
10	Rachel:	Oh, right next to 4 1/3.
11	Heidi:	Oh, so—
12-13	Marie:	Um, it's not 4 1/3 because I just checked and 4 and a half is bigger than 4 1/3.
14-15	Patty:	(looking at Marie, musing, then slaps her hand on the table as she begins to check into this claim.)
16	Teacher:	(intervening) Is it?
17	Patty:	No, it's not!
18	Marie:	Yeah!
19-22	Patty:	Think of a — Look! (grabbing for fraction circle pieces) A half (pause) and a third (picks up a third to juxtapose the two, then realizes she is mistaken, followed by a pause). Yes, it is (in a humbled, almost humiliated tone).

Eight utterances after Patty directed the group to move to 4 1/3, Marie challenged the claim. After a bit of back-and-forth contention between the two, Patty evaluated Marie's claim by attempting to show the relative magnitude of 1/2 and 1/3 before conceding that she had made an error in assuming that 1/3 is greater than 1/2.

Just moments earlier during the Fraction Maze task discussion, Marie again demonstrated the persistence needed to go against the group's decision in managing to overturn another hastily-made incorrect move. When asked during an interview about her persistence in checking others' claims, Marie explained that the desire for certainty was just part of her personality and that she just did not like accepting others' ideas at face value: "I always kind of do my own thing. I don't know. Um, I like checking answers, not just hearing one and putting it down" (Interview 2). Doing her "own thing" was a key factor that allowed her to check and evaluate others' ideas as prolifically as she did, because in most cases, Marie would check while someone else was speaking or after the group had decided to move ahead.

All of Marie's contributions were made assertively. She did not waffle over competing ideas as she spoke, nor did she use qualifying terms like "maybe," "probably," or "it might be." She expressed her thinking as declarative statements rather than tentative questions.

Access to Participation: Use of Space

Many times during both task discussions, the researcher observed Marie writing feverishly while others were engaged in discourse. It was common for Marie to have her hand raised while someone else was sharing their reasoning, and on several occasions, she struggled to contain her enthusiasm, as she would make inappropriate sounds or gestures while others were talking. Often, in an attempt to get permission to share her reasoning, she would blurt out phrases uncontrollably, such as "Oh, can I say it now?!", "I got it!!! Okay! (Raising her hand)", and "I wanna say something!!!" It was tempting for the teacher to call on her every time she volunteered to speak, but he was intent on providing equal space for all students to share their contributions during whole class discussions and also during the Science Fair task discussion. After viewing the video of this discussion, Marie remorsefully admitted that she was "hyper" because of her affinity for solving challenging math problems.

Marie often demonstrated a tendency to think aloud during task discussions. When Marie went through her thinking out loud, it was usually very abbreviated, decontextualized, and disconnected (i.e., probably not meant to be followed by others). For example, during the Fraction Maze task, while others were representing their thinking on paper, Marie droned, "Sevennnn, wait, sevennnn, twenty-eight, no you can't even do seven!" Sometimes, her thinking aloud appeared to distract others as they were thinking silently to themselves. Interview data corroborated this finding, as Heidi reflected, "When I'm working like with a problem and it's all quiet and then somebody blurts out something, I lose my train of thought…." (Interview 4). Patty added that it sometimes caused a "mind trap" for her, but also argued that at other times, it helped her to think differently about her problem-solving approach. "Sometimes it's good. I think of it as a positive perspective because sometimes she puts me, um, gets my mind sharp where I'm going with the problem and puts me in a new direction that leads to the right answer sometimes," she explained (Interview 4).

Marie usually found space in discussion quite easily, especially during the Fraction Maze task (when the teacher was virtually uninvolved in facilitating the discussion). One advantage that Marie had over the others was the speed at which she could calculate—both mentally and on paper—as well as her quickness at thinking and making decisions. Marie acknowledged the speed in which she could determine solutions as a personal strength. However, interview data revealed that she also recognized the negative impact or duress it may have caused for the other students in the group, as she thought they might have "tried to rush" to keep up with her, consequently leading them to "mess up" or concede altogether and merely "ask for the answers."

Access to Participation: Meaning-making

Marie rarely asked others for clarification of their ideas. In fact, she only prompted a peer for clarification three times during the study, and each time her request for clarification was directed to Patty because of ideas that were expressed vaguely. Not once did she ask Heidi or Rachel to clarify their contributions.

In interviews, Marie described both small-group tasks as "fun" and "challenging." She expressed an affinity for discussion-based learning for several reasons, but mostly because she enjoyed working with friends and she believed that discussion provides the best way to prove that her "answers are right." To Marie, discussion also served as an educational tool to help everyone understand math better. She enjoyed the challenge of debating ideas publicly and even though she acknowledged that she did not like being wrong, she was "okay with it" because she felt like there was much less risk in being wrong in math versus being wrong in her other classes:

Marie:	In some classes, I don't want to be wrong more than others. And in this one, I'm kind of loose with, which is fine.
Int:	You're kind of what?
Marie:	Loose with, like, I'm fine with being wrong in this class. I don't know why.
Int:	I wonder why. Maybe it depends on the subject? Or the people in the class? Or the teacher?
Marie:	I don't know. (Interview 1, October 23, 2009)

During an interview, Marie admitted that she often took risks during math discussion. She committed a few mistakes in her reasoning throughout the course of her interactions and contributions during discussion, but she also caught many mistakes made by herself and others because of her well-developed habit of checking claims.

One of the most interesting findings of all was that Marie struggled to express her thinking clearly when she shared contributions with others. In fact, during the entire study, 17 of the 22 explanations she provided during task discussions were coded as ambiguous. Although she often cited numbers and quantities in her explanations, she typically truncated the meaning by omitting key referents, like nouns or the specific objects that the numbers represented. A typical example comes from her solution for the Science Fair task, where she explains, “You divide like that into these and you can know that it’s almost half,” and just moments later, “You can add up these (pointing to her paper) and then do that (pointing to her paper again).” At another point in the same discussion, she attempted to share a computational explanation with the group that was almost impossible to follow: “I know, I know! You can add up these (pointing to 1000, 600, and 400) and then do that. You can add 600 to 400, and then do what I did to that.” Patty commented later in an interview that it was as if Marie “was talking to the paper, like, she was trying to explain to the paper what was going on” (Interview 4). Heidi suggested that Marie may have suffered from a lack of audience awareness when attempting to explain her thinking: “… whenever she’s like explaining ‘it’, she thinks that you already know what ‘it’ means and so she goes ‘like this’ and ‘that’, and then I’m just like, ‘Whoa! What do you mean?’” (Interview 4). After viewing the video, Marie admitted that explaining her reasoning was “hard” for her:

Marie: I just don’t know how to say it out loud.
Int: So what, what’s, why is it so hard to make it clear when you explain it?

> Marie: Cause I use pronouns. I do…. I just realized now that I was doing like, “Okay this goes there and this like,” I could’ve said like, “Malcolm X get this,” not “this gets this.” (Interview 4, December 18, 2009)

Marie later suggested during an interview that even though explanations provided by her or Patty were not always crystal clear to all students, the ability to deductively infer the intended meaning of a vague explanation is relatively effortless for some students, but elusive to others. The following excerpt illustrates this point, in addition to revealing Marie’s preference to interact with Patty during task discussions because of their propensity to “get” or understand each other.

> Marie: Yeah, cause sometimes they don’t get what we’re thinking. Like with the 2/10 left over (an explanation provided by Patty that Heidi did not originally understand), I think I would’ve probably like got that. With the Kennedy—
>
> Int: Okay, so when we’re having like there’s a tough problem that you’re working on and you’re not sure what the answer is or you want to check it or you want to ask a question, you’re saying it’s probably easiest to ask Patty cause chances are she’s probably gonna know what you’re thinking too.
>
> Marie: Yeah. Yeah. Yeah. (Interview 4, December 18, 2009)

It is important to point out that other researchers have addressed this problem in communication among students (see O’Connor & Michaels, 1996) and that no single participant was immune to the tendency to provide vague explanations, but because of the significant number of contributions that Marie made in which she attempted to explain her reasoning, in addition to the fact that the other three students identified her as the group’s de facto leader, Marie’s struggle to make meaning that was comprehensible to others became a salient theme regarding the nature of participation among low- and high-performing students. Put differently, Marie’s difficulty in articulating her reasoning was significant because she accounted for a large percentage of mathematical contributions.

Access to Participation: Peer and Teacher Interactions

Throughout the study, Marie demonstrated an inclination to interact with Patty a majority of the time. Whenever the teacher directed the students to collaborate on assignments in groups, she would tend to naturally gravitate toward Patty more often than not. During the Science Fair task discussion, Marie and Patty sometimes conversed privately as Heidi was sharing her thinking with the group. During one particular partner activity, the teacher decided to contrive the groups (which he normally did not do): He paired Marie with Rachel while they played a game of Fraction Capture. Many times during the game, the researcher observed Marie discussing her moves with Patty, while Rachel sat idle and confused. At one point, Rachel attempted to get Marie's attention three separate times before Marie responded. The researcher wondered why Marie preferred to interact almost exclusively with Patty, even during tasks in which they were not paired with one another:

> Marie: Because we like both, I guess (long pause) like know a lot. And we like both agree with each other.
> Int: Okay, is it easier to talk to her about what you're thinking than it is to, uh, Heidi or Rachel?
> Marie: Yeah, cause sometimes they don't get what we're thinking. Like with the 2/10 left over (referencing an exchange from the Science Fair task discussion), I think I would've probably like got that. (Interview 3, December 17, 2009)

To Marie, it seemed as if Patty spoke the same language, whereas Heidi and Rachel did not. Marie and Patty also shared several similarities outside of math class: they were both enrolled in the gifted education program, and they shared two classes and homeroom with one another.

Teacher interactions with Marie were predominantly characterized by revoicing of her vague explanations so that the others in the group could follow her reasoning better, as in the following excerpt from the Science Fair task discussion:

1	Patty:	(to Marie) You lost me … when you said "like that."
2-4	Teacher:	Well, she's saying (referring to Marie) that because Malcolm X gets more than Kennedy and together they have one-half of the space—
5	Marie:	It's gonna be more than half.
6-8	Teacher:	And Malcolm X is going to get more than half of a half—as Heidi said, a fourth—Malcolm X should get more than 1/4 and Kennedy should get less than 1/4.

Sometimes, attempts to elicit clarification from Marie did not manifest as revoicing of her contributions, but rather as questions that encouraged her to be more specific. During a whole-class discussion, the following exchange took place:

1-3	Fieldnotes:	*Later, after a student finds 35% of 80ml by finding 5% first and then multiplying it by 7 (to get 28) Marie raises her hand to share an observation.*
4-6	Marie:	Um, what you're doing, practically, is if you put it into fractions, you're dividing, like 50 over 100 and like, 80 over 100.
7	Teacher:	For which one?
8	Marie:	For the first one, and then you can keep doing that.
9-10	Teacher:	Okay, explain a little bit more *(The teacher struggles to follow her reasoning).*
11	Marie:	Like 50 over 100 and then 80 over 100, you're dividing.
12-13	Teacher:	Okay, so you're saying 50 over 100 is one half, right? And you're saying 80 over 100 is (pause) divide 80 by—
14	Marie:	If you did 80 over 100 divided by 50 over 100…
15	Teacher:	80 over 100 divided by 50 over 100, let me see.
16	Marie:	Does that work?
17-19	Teacher:	Well, 80 over 100 divided by 50 over 100 would be … (drawing a symbolic representation of the problem on the board) 8/5 and that would give us—
20	Marie:	Oh, no.
21-23	Teacher:	I'm not sure. I like how you're thinking, though. There is a connection with multiplication and division, keep on thinking about that as we do these last two problems. (Fieldnotes, December 2, 2009)

In lines 7 and 9-10, the teacher animated Marie as a clarifier and elaborator of her conjecture and then in lines 12-13 revoiced her contribution to check for agreement with his interpretation of her contribution. Even though the interaction between student and teacher did not result in clearly expressing or proving her conjecture to be correct, it was successful to the degree that it elicited further clarification of an originally vague idea. Finally, in lines 21-23, the teacher affirmed Marie's creativity and initiative and positively challenged her to continue thinking and refining her idea.

High-performing Student: Patty

Patty made a significant number of contributions during group discussion in both whole- and small-group instructional settings. Although she did not initiate as many claims or moves as Marie, she played a unique and significant role in evaluating and challenging others' claims, mostly by attempting to construct conceptual explanations or sharing pictorial representations of her thinking, although her explanations were not always coherent. However, at times, she also demonstrated a tendency to participate privately and sometimes even appeared unconcerned with seeking others' input. She also demonstrated a distinct preference to interact exclusively with Marie in small-group settings.

Types of Contributions

As demonstrated in Table 8, Patty initiated roughly one-third of the moves during the Fraction Maze task discussion. For a majority of her claims, she used fraction manipulatives or drawings to compare the relative magnitudes of the fractions under consideration. Nearly every time she made these comparisons, she easily captured the group's attention as they all seemed curious about her contributions. One of the first challenging dilemmas that arose during this task discussion was where the students had to decide if 4 1/8 was greater or less than 4 3/16. Instead of simply applying the traditional common denominator procedure for comparing mixed numbers, Patty drew two large circles and partitioned them accordingly. As she drew, she explained why she chose to represent her thinking visually (because she had just realized that several mixed numbers and improper fractions with a denominator of either 8 or 16 were clustered together in the same general area on the maze). The excerpt below also shows how she used her sense of humor in discussion, which she did often in both whole- and small-group settings.

1-2	Patty:	I'm trying to draw 16^{ths} and 8^{ths} cause they all have 16^{ths} and 8^{ths} here. All we have is a pretty circle and picture of a box.
3	Heidi:	Or pieces.
4	Patty:	Yeah.
5	Heidi:	Pieces of pie.
6	Rachel:	That's a good idea.
7-9	Patty:	Oh my gosh! (Everyone looks at Patty.) Okay, I've never drawn a better eighths-circle than that! Never! I'm a bad circle drawer! (laughing)
10	Marie:	Me too.
11	Rachel:	Me too.
12	Marie:	Okay, so…now you need like 16ths? So divide them all into 2 (looking at Patty's eighths circle drawing).

Patty subsequently directed the others toward her representations of sixteenths and eighths to help compare the next group of rational numbers. At the end of this task discussion, her scratch paper was covered with such visual representations. Because Patty preferred the use of pictorial representations instead of symbolic representations, it is not surprising that the majority of her explanations were coded as relational, rather than computational. Of the 24 explanations she offered throughout the study, 16 were coded as relational.

Patty also showed the greatest tendency to offer conceptual arguments or narratives as explanations as opposed to mere computational explanations. Although she sometimes struggled to articulate these thoughts, she often attempted to move “beyond the numbers” by drawing on her informal knowledge and placing her explanations in an intuitive context. For example, near the end of the Science Fair task discussion, Heidi asked Patty why she allotted 30% of the total cost of the science fair to Malcolm X Middle School instead of assessing 30% of the residual cost (after determining that Bret Harte Middle School would cover half of the costs). Patty first attempted to frame a justification primarily around the numbers and procedures but struggled to clearly debunk Heidi’s misconception:

> Yeah, it’s not to the point where, it’s like you can’t do Bret Harte Middle School: 300; then Malcolm X: 150; and then Kennedy, like 30 bucks because they all have, they all have to pay, it has to build up to 300 (dollars). It can’t be like 300, 150—

Moments later, the teacher posed the following question to help clarify Patty’s attempt at justification: “So what would your response be to the critic who says ‘Wait a second, there are three schools here. Each of them should pay one-third of the cost, so in other words, each of them should pay 100 bucks!’? Patty responded,

> It’s basically like giving a huge discount to Bret Harte (Middle School) because they have more kids (1000) than them (Malcolm X Middle School; 600 students) so since they should all pay an equal amount because they had paid for their share of the auditorium [sic].

Although the latter portion of her justification above is confusing, she meant to suggest that by paying one-third of the cost instead of paying for half of it, Bret Harte Middle School would have received a substantial discount. This is essentially the concept of proportionality, which was just being formally introduced at the time of this task discussion (which may help explain why Patty struggled to express this concept clearly). She built on her previous attempt at explaining—which was primarily based on explaining procedural properties—by offering a revised justification based on her intuitive knowledge of proportionality, evidenced by the word "discount."

Thinking visually came naturally to Patty, as her explanations often indicated. During a whole class discussion, the teacher asked the students to decide if 11/20 is closer to one whole or zero and explain. Patty raised her hand and said, "Um, closer to one because 10/20 is half and 11/20 is one slice bigger than half because ten-halves (pause), ten over twenty is half" (Fieldnotes, October 13, 2009). She often combined her visual thinking ability with her intuitive knowledge of rational numbers to make conjectures. During a whole-class discussion, for example, the teacher asked the students to think of a fraction that comes between four-fifths and one whole. While the class struggled to solve this problem, Patty's raised her hand emphatically: "Four-and-a-half-fifths," she said (which was indeed correct reasoning, but violates the rules of fraction notation) (Fieldnotes, October 15, 2009). By visually representing Patty's conjecture on the board, the teacher helped guide the class to see that four-and-a-half-fifths is equivalent to nine-tenths, which is indeed greater than four-fifths but less than one whole.

Although Patty preferred to share conceptual explanations and visual representations of her thinking, she also offered procedural or computational explanations from time to time. Like Marie, she struggled to provide clarity in her procedural explanations, as she often used vague referents that made it difficult for others to follow her thinking. For example, during a lesson on subtracting mixed numbers by regrouping, she noticed a computational pattern that she enthusiastically shared with the whole class, but it took revoicing (lines 5-8; 14-17 below) from the teacher to make it accessible to everyone:

1-4	Patty:	I realized that if you put the first number into the denominator, the numerator into the denominator, add it, and you, and that you don't have to, that equals 9/7, so you don't have to do the long way, like adding it all out.
5-9	Teacher:	So correct me if I'm wrong, but what you're saying is that like with 9 2/7, you slash out the 9, you take one whole away, you know that becomes eight. Then you're saying add the numerator to the denominator and that becomes your new numerator.
10	Patty:	Instead of writing it all out.
11-13	Teacher:	Right. Instead of writing it all out, this is exactly what you can do because the denominator tells you how many of those pieces it takes to make a whole.
14	Patty:	Mmm, hmm.
15-18	Teacher:	So maybe a shortcut that'll make it easier for some of you to understand is what she just said. Just take the numerator and denominator and add them. That becomes your new numerator and then you keep your denominator….
19-20	Marie:	Will that always work? (Marie comments that she doesn't think so.)
21-22	Teacher:	"Will that always work?" is a great question to ask in math. We'll call this one the Patty Method for short . . . (Fieldnotes, October 22, 2009)

Although Patty shared a significant number of contributions throughout the study, she did not show a tendency to engage in thinking out loud. Rather, she preferred to "do it on paper" most of the time, as she would employ a mixture of symbolic problem-solving strategies as well as visual and conceptual representations. Interestingly, she was much more vocal in the Fraction Maze discussion than she was during the Science Fair task discussion. For a majority of the Science Fair task discussion, she engaged in what the researcher coded as IN THE TUNNEL participation, in which she was preoccupied with scratching out calculations and privately attempting to solve the problem. During these times, she made minimal (if any) eye contact with others as they spoke, and posed no evaluative comments or questions, suggesting that she was primarily engrossed in an isolated state with one goal in mind—solving the problem on her own. In follow-up interviews to the Science Fair task, when the researcher asked the others if they felt like their contributions were being listened to by others, they each cited Marie, Heidi, and Rachel as paying attention but questioned Patty's role as a listener. Rachel observed, for example, that she thought Patty was listening, "except when she was like writing…. I think she was working it out in her own way" (Interview 4). Marie, too, thought that Patty's primary objective "was trying to get her own answer" (Interview 4).

Access to Participation: Use of Space

Patty expressed a deeply-rooted sense of competitiveness, both implicitly and explicitly, throughout the study. At one point, during the Fraction Maze task, she became involved in a spirited debate with Marie over whether 4 1/3 is greater or less than 4 1/2, in which she slapped the table and emphatically reached for the fraction circle pieces to debunk Marie's claim.

1-2	Marie:	Um, it's not 4 1/3 because I just checked and 4 1/2 is bigger than 4 1/3.
3-4	Patty:	(looking at Marie, musing, then slaps her hand on the table forcefully as she begins to check into this claim.)
5	Teacher:	Is it?
6	Patty:	No, it's not!
7	Marie:	Yeah!
8-11	Patty:	Think of a — Look! (grabbing for fraction circle pieces) A half (pause) and a third (picks up a third to juxtapose the two, then realizes she is mistaken, followed by a pause). Yes, it is (in a humbled, almost humiliated tone).

Patty also reported that Heidi "frustrated" her, igniting her competitive spirit. Patty also ascribed her instinct to argue as a manifestation of her competitiveness. When asked in an interview what triggered her to openly engage in debate with another student, Patty had the following to say:

Patty:	I guess it's just my nature, I guess. My opinion just kicks in. Instinct, I guess.
Int:	It's a good trait to have – you're one of the few students who I can tell—
Patty:	I'm really competitive. (Interview 1, October 23, 2009)

Patty explained that she did not like being wrong in math because of her competitiveness, as she described herself as "obsessed with checking" her work (Interview 2). Observational data confirmed this sentiment, as Patty often worked problems out more than once to ensure accuracy. She rarely accepted someone else's solutions at face value and even characterized the act of blindly believing in someone else's thinking as "annoying." In an interview, she alluded to her instinctive sense of competitiveness, in addition to the importance of certainty and confidence in her thinking as factors that helped her decide whether or not to challenge someone's ideas: "When I've worked a problem out over and over, time and time again, and I absolutely know it, or um, just the instinct kicks in and I get competitive" (Interview 1).

Access to Participation: Meaning-making

Patty rarely prompted others for clarification of their contributions. Only on two occasions throughout the study did she do so, and both times her prompts were directed to Marie to clarify a vague utterance. Although she evaluated and challenged far more ideas than Heidi and Rachel, Patty's evaluations were not aimed at seeking clarification of another student's ideas.

When Patty offered explanations, they sometimes came across as unclear. Particularly when sharing computational explanations, she showed a tendency to use numbers and procedures in isolation, as when she finally determined the correct allotment of the auditorium in the Science Fair task discussion and attempted to explain. The analytic memo is included in italics below to illustrate the researcher's reflection about the contextual ramifications of Patty's vague but correct explanation:

Patty: Because 3/10 is 30% and 2/10 is 20% and so 50 and then 30 plus 20 is 50 and then 50 plus 50 is 100%. *(Rachel looks on, but shows confusion on her* *face – I wonder if Patty's vague language, i.e. lack of referents, makes it harder for Rachel and others to follow her reasoning).*

Although Patty offered thoughtful contributions to group discussion—ones that often led to extension and further exploration of the problem than what was actually called for—she demonstrated a tendency to give decontextualized procedural explanations. For instance, near the conclusion of the Science Fair task discussion, Patty argued that Bret Harte Middle School would be receiving a "huge discount" if they were only assessed one-third of the cost associated with the Science Fair. The teacher then decided to probe this idea in hopes of making the concept more concrete for all of the students. But when he asked for an explanation, Patty offered one that the teacher understood, but was likely nebulous to the others:

1-2 Teacher: Well how much would Malcolm X and Kennedy Middle School be getting ripped off by if they had to pay 100 dollars?
3-4 Marie: This one (pointing to Malcolm X) 70 bucks and this one (pointing to Kennedy) 80 bucks.
5 Patty: No! 40 (pointing to Malcolm X) and 10 (pointing to Kennedy).
6 Marie: How?
7 Teacher: How did you figure that out (to Patty)?
8 Marie: I did 100 minus—
9 Patty: Because you do 60 plus 40 is 100. 90 plus 10 is 100.

In line 9, Patty refers only to numbers, not the ideas that the numbers represent, therefore making the explanation potentially difficult for others to follow. This was again apparent during a whole class discussion on finding percent of a number using mental math. When the teacher asked Patty to explain how she found 60% of 60, she reasoned,

1-6 Patty: Well, what I did, I put it, I made it smaller, the percent, so that I could build up, so I did half of 60 is 30 and then since I don't know 30 yet, I did 15 and then I took 5 away which is like (pause) 5 up from 10 percent, so it's like 20 (pause) so it's, the answer's 20 for 15, then for 30 you have to add 15 and that's 35 and then 60 is 50.
7 Marie: What???
8 Student: I didn't understand that at all!

She later admitted in a follow-up interview that explaining her thinking in a way that is comprehensible to others was difficult for her. She also pointed out that "knowing it in your head" and "explaining it in words" are fundamentally different abilities, suggesting that the role of metacognition (e.g., audience awareness) explains, at least in part, the elusive difference between the two (Interview 4).

Access to Participation: Peer and Teacher Interactions

Even though Patty often demonstrated that she had fun discussing mathematics with her peers, her sense of playfulness was tempered by the degree to which she was individualistically oriented and embraced her competitive spirit. For example, much of Patty's language during the Fraction Maze task discussion implied that she had made decisions independent of the group's input and was willing to move ahead without consensus or debate. Listed below are several utterances from the discussion where Patty conveyed to the group that she was prepared to make moves without seeking their opinions, nor offering explanations of her own:

> Patty: So we can go down to 3/16; *I'm* doing that. (everyone else copies this move)
> Heidi: Okay, and then 4 1/3?
> Patty: *I'm* going to that, obviously 6 2/7 (looking ahead)
> Rachel: (looking confused) Wait, so circle 4 1/3?....
>
> Patty: *I* was right!....
>
> Patty: Yeah, it's . . . 5 5/6.
> Marie: Yeah, so—
> Patty: *I'm* going there (everyone now circles this) . . . *I'm* gonna sorta go to 5 5/9 (an invalid move that was later contested by Marie)….
>
> Patty: *I'm* gonna do 7 6/7.
> Heidi: Huh?
> Marie: Where are you?

Heidi later commented that Patty "kind of just like goes off on her own and does it" (Interview 4). In fact, in three distinct episodes during the Fraction Maze task discussion, Patty made a hasty, incorrect decision (and moved on to the next problem) without the consent of the group before being successfully challenged and overturned by Marie each of the three times.

Patty showed a natural tendency to interact almost exclusively with Marie during small-group task work. During the Fraction Maze task discussion, she typically channeled her attention solely to Marie when the group was stuck in a vexing dilemma, like in the excerpt below. This single problem represented the most debated and perhaps challenging problem of the entire task, as they were faced with having to decide where to go from 4 3/16 (the options were 31/8, 4 1/8, and 9/2):

1-2	Patty:	We're trying to debate whether a 16th is better than, an 8th is better than—
3	Marie:	Wait a minute, I think I got it, 2 into 9 is—
4	Heidi:	Cause 31 eighths—
5	Patty:	I'm trying to debate whether an eighth is bigger than 3/16
6	Marie:	I got 4 and a half.
7	Heidi:	Cause if you do, um, 8, or I mean, 31 divided by 8, it's 3—
8-9	Patty:	(showing her picture to Marie, and ignoring Heidi). Which one looks bigger to you, Marie?
10	Heidi:	Soooo—

In lines 8-9, Patty animated Marie as the sole evaluator of her claim, effectively excluding Heidi and Rachel from the conversation.

During the discussion of the Science Fair task, Patty had an opportunity to respond to Heidi's confusion regarding the solution to partitioning the auditorium proportionately, but instead opted to privately move ahead to a subsequent part of the task. She then overtly removed herself from participating in the group as a whole by physically getting out of her seat and moving to the other side of the table to confer exclusively with Marie about the problems on the next page.

1	Teacher:	Okay, so 3/10.
2	Heidi:	So half and 3/10, then wouldn't you multiply that?
3	Teacher:	Write 3/10 for Malcolm X.
4	Marie:	I got it, okay! (raising her hand to tell)
5-9	Teacher:	Okay, hang on! Let's let them (Rachel and Heidi) have the same opportunity you guys had. (Patty is now on the next page). Now, erase Kennedy Middle School or scratch it out, if Malcolm X gets 3/10, what should Kennedy get? It's gotta add up to half of the space, right?
10-11	Heidi:	Okay, so 3/10 toooooo, wait! How would we do that? (Rachel looks equally perplexed). I'm confused!
12-16	Teacher:	Remember, Malcolm X and Kennedy get one-half of the space (Patty moves over next to Marie, bumps into me accidentally, apologizes, then converses with Marie privately about the next two problems for about two minutes straight).

The researcher later asked Patty in an interview about her tendency to interact solely with Marie, to which she responded, "Well, I kind of want to see her perspective on things, like I said, she's really on the ball" (a phrase she often used to characterize Marie, Interview 4). When asked to clarify what she meant by "on the ball," Patty explained: "She's really good at math…. She was just like ahead of everyone" (Interview 4). Multiple data sources corroborated the observation that Patty clearly valued interacting with Marie over Rachel and Heidi because she perceived a superior degree of quality in her interactions with Marie. Moreover, when asked how she went about deciding whether or not to challenge someone's ideas during discussion, she indicated that it was perhaps easier to believe in Marie's reasoning than others' by saying,

> If it seems like they wouldn't think of it right, then I would argue, but if they, you know, were really good at, like Marie, she can explain stuff like right after that, I would believe her, you know, I wouldn't argue, I would just ask her, "How did you do it?" and prove it to me. (Interview 3, December 17, 2009)

Patty may have avoided interacting with Heidi and Rachel because of several perceived drawbacks. When asked to talk about her apparent reservations about interacting with or helping Heidi or Rachel, Patty suggested that her interactions with them (which she classified as usually having to explain a concept or walk them through a procedure) were often fruitless, if not counterproductive. Among her most compelling reasons was the perception that these interactions often ultimately confused her.

Int: And she (Heidi) was like, “Wait, it’s still left over, so what do you do?”
Patty: Yeah, so it seems confusing, so I just like going on my own track instead of explaining things to people, because once I, I have this thing where if I explain it to someone where I know it really wear, [sic] well, sometimes it drops out of my head and then I don’t understand it anymore.
Int: I gotcha. So you, you understand it like mentally well, it’s in your head, but—
Patty: But then when I explain it, it comes out wrong and then I confuse myself. (Interview 4, December 18, 2009)

Interestingly, Patty stated in a previous interview that she liked to “hear others’ opinions first” and “see others’ perspectives” (Interview 2), although this was observed during discussion of tasks only when she reached out to Marie. She also identified herself as possessing some sort of exclusive alliance with Marie, and voiced concerns that interactions with Heidi and Rachel had the potential to confuse not only her, but Marie as well through secondary interactions:

Int: But I didn’t see so much (interaction) between like Heidi and Rachel and the group. Why do you think, why do you think that’s the case?
Patty: Because Marie is more on the ball, and (pause) I’m not trying to say that about Rachel and Heidi, it’s just that they seem more confused, and my whole thing about talking and then confusing myself and then I’ll end up, if I probably interact with them, and then with Marie, I’ll probably tell Marie and then get that mindset into Marie’s head and then she would be confused too, so it was (pause)—
Int: I see what you’re saying. So you’re kind of saying like talking about your mathematical thinking with Marie (pause)—
Patty: And then after I explain something which I’m confused then I put it into her mindset, which messes us up, so—

Int: Okay I see, alright so you're saying that it's easier to kind of get your words out and like articulate what it is you're trying to say when you discuss it with Marie, whereas if you discuss it with Heidi or Rachel, you feel like the chances are greater that you might just confuse yourself *and Marie* because they're confused.

Patty: Yeah. (Interview 4, December 18, 2009)

Like Marie, Patty admitted that explaining a concept to others well enough so they understand the explanation is difficult for her. "It's hard for me to explain to other people, when I sort of have it in my head, it's like (she makes a jumbled, nonsensical sound like the teacher in the Charlie Brown cartoons)" (Interview 3). She even sometimes characterized the act of helping others as burdensome and anxiety-inducing. With respect to explaining a concept or procedure to someone who is confused, she admitted "When I see people's brow fruffle [sic] when I do that, I just go, 'Oh God, now I have to help them' (in a reluctant tone)" (Interview 4, December 18, 2009)

Although it seemed that everyone in the group got along well together, Patty approached the teacher in confidence just two weeks into the study with a request to be moved away from Heidi in the classroom. When the teacher asked Patty to explain the problem, she responded that Heidi was always talking at inappropriate times during class, which distracted her. Although the teacher waited until the second semester to relocate the students within the classroom, additional evidence of Patty's opposition toward Heidi was revealed in interview data. For example, after the Fraction Maze task discussion, the researcher asked Patty who she thought was least involved in the discussion and why, to which she responded, "Probably Heidi. She was sorta staring off into space and stuff. And, yeah, um, a little bit off task at times." This was surprising to hear since Heidi offered a disproportionately greater number of contributions than Rachel during the

Fraction Maze task discussion. Several weeks later, the following exchange occurred in an interview just after the students completed the Science Fair task discussion:

> Int: Remember when Heidi had that really good question, that—
> Patty: (interrupting) She kept talking over me, it got really annoying! (Interview 3, December 17, 2009)

Although the students worked well together during this discussion, Patty appeared to be "in the tunnel" for much of the time while Heidi was making contributions. Patty also expressed frustration with the perception that Heidi could not adequately follow or comprehend her explanations, particularly during the Science Fair task discussion:

> Patty: . . . with Heidi, I was getting competitive cause I was kind of getting a little bit frustrated that she didn't get it, but—
> Int: But she was asking questions, though, too.
> Patty: Yeah, that's what I, I don't know why I just get frustrated with stuff like that easily. I'm trying to work on that. (Interview 4, December 18, 2009)

In spite of Patty's stated distaste for Heidi, interactions between the two still occurred, but it was almost always initiated by Heidi, often in the form of a prompt for clarification.

Most of the teacher's interactions with Patty involved eliciting clarification of vague explanations either through revoicing attempts or by positioning her as a claim-justifier through simple follow-up questions, such as "Why?" or "How did you come up with 1/4?" Many times, during both small-group and whole-class discussions, Patty would tend to supply only an answer despite being asked for a reason. It usually required a repeated and explicit attempt to get her to explain her thinking. For example, the following exchange took place during the Science Fair task discussion:

1-2	Teacher:	Okay, let's have a discussion to see where we're at now. What do you guys think?
3	Marie:	I got it!!!
4-5	Teacher:	Patty, why don't you start off by explaining your solution and then (to Marie) we'll come back to you.
6-9	Patty:	Okay, for Bret Harte Middle School, obviously a half. And then, for Malcolm X Middle School I got 3/10, and then for

Kennedy Middle School, 2/10 or 1/5 and then for the monies—

10-13 Teacher: Okay, but you can't just speed past it like that, you have to justify your answer. Tell us why 2/10 and 3/10 works, the other ones that we came up with didn't work. Why did these work?

Patty made several attempts during the Science Fair task discussion to share her solutions privately with the teacher while the group was engaged in discussion of the task. On one of these occasions, the teacher explicitly reminded Patty that the task was to be solved as a group and that any contributions or conjectures she had should be put forth to the group and not the teacher alone. Based on the context of the discussion and observed nonverbal gestures, it appeared that Patty sought private confirmation of her solution from the teacher.

Cross-case Findings

The following section synthesizes the within-case findings across cases. First, a summary of the within-case findings is presented for each pair of low- and high-performing students, followed by the examination of salient themes that emerged from cross-case synthesis of findings.

Summary of Within-case Findings for Low-Performing Students

Although Rachel and Heidi were both classified as "low-performing" in mathematics based on an array of formal assessment data, the nature of their participation in mathematics discourse varied greatly in terms of the contributions they made during small-group task discussion. Rachel rarely ever spoke and expressed obvious self-concept and self-efficacy issues that limited the frequency and quality of interactions. She allowed others to control space during discussion. Heidi, on the other hand, demonstrated an unabashed willingness to learn from mistakes, as well as a natural desire to seek clarification, which manifested in the form of seeking out space to participate in discourse. In contrast to her behavior during small-group task discussions, however, attention issues (i.e., concentration, on-task behavior) sometimes undermined her ability to contribute meaningfully to whole-group discussion.

Although both students demonstrated engagement with task discussions and struggled to varying degrees in meeting the cognitive demand of the tasks, the way they dealt with these struggles was fundamentally different. When Rachel was confused, she almost never admitted it by prompting others for clarification; confusion appeared to silence her. Confusion for Heidi, on the other hand, motivated her to participate so that she could understand her misconceptions, and she was not at all shy at all about pleading to the group, “Wait! I’m confused,” or “But, why?”

When Rachel spoke to the group, she expressed ideas tersely and rarely in context. On the other hand, Heidi was often responsible for initiating a detailed discussion or debate. Unlike Rachel, who felt pressure to “talk more” because everyone else was talking a lot, Heidi characterized her motivation to be more involved in discussion as a means to “learning better,” “understanding more,” and so that she could “get it” on her own. Concerns about diminished social status appeared to play a role in Rachel’s timidity and reticence with regard to speaking in front of the group, whereas Heidi downplayed any status concerns by taking risks during discussions, not being afraid to make mistakes, and repeatedly referring to her conviction that understanding her mistakes was an integral part of the way she preferred to learn. This distinction is perhaps unsurprising given Rachel’s reported discomfort with discussion-based contexts.

Outside of both being classified as low-performing students, Rachel and Heidi did share some similarities. They both reported being frequently confused or not being able to follow the explanations of the high-performing students. They often shared incorrect solutions during task discussions. They also both tended to express ideas and explanations tentatively more frequently than declaratively, which sometimes were expressed in the format of a question. Rarely did they express their thinking assertively or direct others during small-group task discussions. Finally, Rachel's and Heidi's interactions with the teacher during task discussions were mostly characterized by the teacher providing scaffolding in order to move them from incorrect or incomplete reasoning toward correct or complete explanations.

Summary of Within-case Findings for High-Performing Students

Patty and Marie shared many similarities with regard to participation in mathematics discourse. Both offered a significant number of dependent and independent contributions, which were diverse in nature. Their evaluations of others' thinking were characterized by a significant number of challenged claims. They both provided a range of relational and computational explanations of their thinking, and although they rarely demonstrated incorrect reasoning, they did struggle to articulate their thinking in a way that was comprehensible to others, despite reporting that they often understood each other. Both students fell susceptible to the use of vague pronoun referents when explaining their thinking. Neither Marie nor Patty spoke tentatively when making contributions; even when they demonstrated incorrect reasoning, the language they used to communicate their thinking was assertive and declarative (though not always clear). They preferred to interact exclusively with one another, and the few times that they prompted for clarification during group discussion, the prompts were always directed toward one another. They did not ask evaluative questions of Rachel or Heidi. They both reported preferring discussion over direct forms of instruction, mostly because they like to prove that their answers were correct, in addition to the perception that discussion provides a higher-quality learning experience. Finally, high-performing students' interactions with the teacher during task discussions mostly involved the teacher prompting them for clarification of correct but ambiguous ideas through the use of revoicing or additional questioning.

The major difference between Marie and Patty was that Marie demonstrated more pro-collaborative learning behaviors, whereas Patty clearly displayed competitive and self-centered behaviors such as her inclination to work in isolation to solve a problem while others talked through it together, or her tendency to make decisions without the consent of the group. Moreover, Patty tended to engage more frequently in private forms of participation than Marie, who preferred to think out loud. Patty expressed a strong desire to always be correct, which explains why she worked privately for such a significant portion of the Science Fair task discussion. Marie, on the other hand, felt a lesser degree of risk associated with being wrong during task discussions, which helped to explain why she was so willing to think and often improvise out loud. Finally, although they both employed a wide range of explanations, Patty favored relational and visual explanations, whereas Marie employed as many computational explanations as she did relational.

The Nature of Participation in Discourse across the Cases

Cross-case data analysis revealed two salient findings regarding how low- and high-performing students participated in discourse during rational number tasks. First, observed behaviors and interview data suggested that each participant's willingness to participate in discourse was related to their goal orientations, specifically whether they expressed consonance with intrinsic or extrinsic learning goals, how they perceived the risk of "being wrong" during discussion, and resulting concerns about relative peer status. Second, low- and high-performing students faced fundamentally different challenges while participating in discourse about rational number tasks; differences between the kinds of interactions that low- and high-performers engaged in were directly related to the types of challenges they faced during discussion of these tasks. The following sections expound upon these two findings.

Willingness to Make Contributions

One very important finding across cases was that each participant's willingness to share her contributions appeared to be mediated by each individual participant's perceptions of risk associated with sharing incorrect answers or reasoning in front of the group. Students' goal orientations, specifically the degree to which these goals were intrinsic or extrinsic, help to explain their willingness to make contributions to the group. For instance, students that expressed intrinsic goals, that is, those who primarily expressed behaviors consonant with the desire to understand the mathematical content regardless of resulting peer status concerns, were less restricted by fears associated with providing wrong answers or incorrect reasoning, such as "looking dumb" or "not looking smart." Researchers have found that students who place greater emphasis on intrinsic learning goals naturally are less concerned with perceptions of diminished peer status because their primary goal is to understand the learning content while students who place greater emphasis on extrinsic learning goals are optimally concerned with providing correct answers or solutions so that their status is either enhanced or at the very least not diminished (Ames, 1992). Students' contributions and their access to participation in discourse (the two major themes used to frame within-case findings) are reflected in the following analysis.

Triangulated data analysis revealed that Heidi (low-performer) and Marie (high-performer) demonstrated behaviors consistent with intrinsic learning goals, while Rachel (low-performer) and Patty (high-performer) expressed fixation with extrinsic goals, such as concerns about fear associated with being wrong, as well as related status issues. For Patty, competition was important not just for the sake of engaging in spirited debates, but for protecting her status among the group. Making sure that she was not outdone by Heidi, a low-performing peer who often tried to claim space during discussion, and wanting to maintain her status relative to Marie as a high-performing student were particularly high on her list of social priorities. Patty was rather transparent about her desire to always be correct and her resulting fear of "screwing up the problem" in front of her peers. This fear of being wrong led to her self-described obsession with checking her work for accuracy before claiming space in discussions by making contributions to the group. Regarding the importance of always being correct when sharing thinking publicly, she admitted to conceding to social pressure from her high-performing or gifted peers: "If you got like a bunch of stuff wrong," she reported of her peers, "they'll look at you weird the next day, and they'll be like backing away and stuff" (Interview 3).

Finally, she stated that she needed to be sure of her answers or confident in her thinking before volunteering to speak before the group, which explained why she was so quiet during the majority of the Science Fair task discussion as she worked diligently to resolve her initial misconceptions about the task. Despite rarely demonstrating the ability to craft clear and coherent explanations of her reasoning, Patty did not appear to allow this to hinder her willingness to make contributions to the group. In short, she was more concerned with having and sharing the correct answer than the precise explanation for the correct answer. Thus, meaning-making did not play a limiting role in her willingness to make contributions or her access to participating in discussion.

For Rachel, who already possessed low status and self-efficacy, staying quiet and opting to not provide explanations protected her from the likelihood that she would say something that would consequently threaten or diminish her status even further. Rachel often expressed that she felt pressured to "talk more" but rarely felt that she understood the mathematical content well enough to make contributions. Concerns about "feeling stupid" due to sharing incorrect answers or flawed reasoning before the group played a significant role in her decision to waive space and stay out of task discussions. "I would've talked more if I understood" (Interviews 2 and 3), she said on more than one occasion, implying that meaning-making and the lack of shared meaning, particularly, were responsible for her lack of willingness to make contributions publicly. Moreover, Rachel did not have the same fearlessness that Heidi demonstrated during task discussions. She expressed admiration for Heidi in this regard because, even as a low-performing peer, Heidi claimed space in discussions despite knowing that she might be incorrect: Said Rachel, "Even if Heidi has it wrong, she'll still be like, 'This is my answer, and I'm going to go with it'" (Interview 3).

For Heidi, who rarely held back in seeking help or clarification of others' ideas and stated several times that she embraced mistakes as a necessary component of her favored learning process, status concerns did not appear to significantly affect her willingness to participate in discourse. Heidi admitted that she struggled to understand most mathematical content, and felt the need to "get involved" in group discussions because of this. "Instead of being off in my little corner and just kinda listening, I just wanna like get involved so I understand it. Instead of just like, 'Okay, I kinda get this'" (Interview 1). Heidi provided additional corroborating evidence of this finding when she later reported that she felt that getting involved in discussion about mathematical content was especially important "with the harder stuff" and not as critical for the content she deemed easy. When asked if she perceived any negative drawbacks related to sharing incorrect answers or flawed reasoning, she emphasized her desire to be able to do the harder mathematics independently at the expense of any such social status concerns. Her primary goal of learning for understanding helped her to downplay fears associated with "being wrong" during discussion of rational number tasks.

For Marie, who was unanimously perceived as the group's leader and most competent mathematician, status concerns were minimized by the fact that she didn't mind being wrong in math class, and consequently felt like she could take greater risks during discussions without the fear of losing status among her peers. This was evident in part by her unrestricted tendency to emphatically volunteer to speak almost every time a question was posed or a mistake was made by one of her peers. Marie also constantly expressed her thinking out loud, even before she arrived at final answers or explanations. This was in sharp contrast to her high-performing counterpart, Patty, who preferred to reach a degree a certainty and confidence in her solutions before speaking up in front of the group. Marie explained her unrestricted willingness to share her thinking with others as a function of her enthusiasm for mathematics, which on several occasions she claimed was by far her favorite subject.

In sum, Rachel and Patty, whose primary goal orientation could be characterized as extrinsic, held back at times during task discussions due to concerns related to sharing incorrect answers and explanations and the resulting possibility of diminished peer status. Conversely, Heidi and Marie primarily demonstrated an intrinsic goal orientation, which resulted in the perception of minimal risk related to providing wrong answers or explanations because they were concerned more with understanding the mathematical content than protecting their status among the group. Table 10 illustrates the relationship between learning goals and the degree of risk concern as well as peer status concerns associated with making mistakes while discussing rational number tasks.

Table 10

Primary Goal Orientation and Perception of Risk Associated with "Being Wrong"

Primary goal orientation	*Perception of "risk" associated with being wrong*	*Degree of concern with diminished peer status*
Intrinsic	Low	Low
Extrinsic	High	High

Differences in Interactions during Task Discussions

Correct answers and solutions did not come to low-performing students as easily as they did to high-performers. Low- and high-performing students each faced challenges over the course of task discussions, but their challenges resided in fundamentally different contexts. Lows' negotiation of meaning during task discussions often involved the daunting task of attempting to resolve confusion with mathematical content whereas highs were primarily challenged not by providing correct solutions but by providing rich, coherent explanations of their solutions. The interactions initiated by low-performing students in this study were mostly related to help-seeking due to content they found difficult to understand. However, the two low-performing students differed categorically in how they responded to confusion: for Heidi, being confused often led to adaptive forms of help-seeking such as prompting others for clarification or elaboration of their solutions (although she did, at times, seek only answers from peers rather than explanations); confusion for Rachel most often led to reticence or excessive forms of help-seeking, such as accepting others' answers and solutions without demanding elaboration.

For high-performers, whose most significant challenge was not understanding the mathematical content but instead articulating coherent explanations of their reasoning, their explanations appeared to be driven by highly-individualistic, in-group orientations that resulted in being able to explain a solution only to the extent that a correct answer could be somewhat supported by the explanation or so that the other high-performing student in the group could understand it. Their explanations were not crafted so that low-performing students could understand them (although explaining their thinking vaguely was most likely not a conscious act of discrimination against low-performing peers). Explaining for the sake of resolving low-performing students' confusion was not an explicit or implicit objective for high-performers, unless it was forced by subsequent teacher interaction.

Finally, the discourse literature has clearly emphasized the role of the teacher in facilitating productive forms of mathematical talk among students. Although this study focused on students' interactions with one another in small groups, an important finding regarding how students tended to interact with the teacher during discussion of rational number tasks emerged through cross-case analysis. For high-performers, interactions with the teacher mostly involved revoicing, or prompts issued by the teacher demanding further clarification of students' ideas in order to make the explanation more precise and clear so that those listening could understand better. For low-performers, interactions with the teacher also included revoicing, but were mostly characterized by attempts to scaffold their contributions in order to move them from incomplete or incorrect responses and explanations to correct ones.

CHAPTER 5

DISCUSSION

Summary of Findings

The research question that guided this study was "*What is the nature of low- and high-performing students' participation in classroom discourse about rational number tasks in a standards-based sixth grade classroom?*" Within-case analyses revealed that students' access to participation and the roles they assumed during task discussion were mediated by the degree of space they were afforded and how they attempted to utilize that space, as well as the meaning they were able to construct through providing and listening to explanations. Participation was also greatly influenced by peer interactional tendencies that either promoted or impeded productive contributions, as well as teacher interactions that helped to offset some of the problems related to unequal access to participation. Because all students struggled to various degrees to clearly explain their reasoning to others, and low-performing students found it difficult to keep pace with and comprehend the important contributions of high-performing students, the teacher played a significant role in facilitating equitable interactions and helping to neutralize some of the problems related to unequal access to participation in mathematics discourse.

Cross-case findings revealed the salience of social and emotional dimensions, such as interactional tendencies based on perception of others' ability or willingness to help, as well as fears associated with sharing incorrect reasoning or solutions and the consequential effects on peer status. Triangulated data analysis across the cases revealed that each participant's willingness to participate in mathematics discourse was related to her goal orientations, specifically whether she expressed consonance with intrinsic or extrinsic learning goals, how she perceived the risk of "being wrong" during discussion, and resulting concerns about relative peer status. Also, low- and high-performing students faced separate challenges while participating in discourse about rational number tasks; ways in which they interacted with their peers and the teacher were related to these challenges, which significantly affected the nature of their participation in mathematics discourse.

In this chapter, several conclusions are presented based on the findings of this study and how they relate to the extant literature, followed by a discussion of the implications this study's findings hold for teachers who wish to facilitate rich and effective mathematics discourse in their classrooms. The discussion of implications focuses primarily on how participation in discourse can be improved, as well as debate over seemingly incompatible notions related to teaching mathematics in a standards-based classroom. Finally, several limitations of this study design are acknowledged, followed by various recommendations for future research.

Conclusions

Differences in Contributions

The findings of this study resonate with results from previous studies with regard to the tendency for higher-performing students to contribute to mathematics task discussions in ways typically expected of students with the highest ability. For example, high-performers provided more explanations during mathematics task-related discussions (King, 1993; Mulryan, 1995). With regard to dependent contributions, high-performing students checked and challenged ideas much more frequently and made significantly more evaluative claims and explanations than their low-performing peers (Webb 1991; Webb & Mastergeorge, 2003). Highs almost never merely accepted others' ideas without checking the claims themselves first, whereas low-performing students often took answers without demanding explanations (Webb 1991; Webb & Mastergeorge, 2003). High-performing students often challenged incorrect claims by offering counter-explanations and interacting with one another, while low-performing students often failed to explain why they were confused or how they got an answer that was incorrect. Consistent with the findings of Baxter et al. (2001) and Mulryan (1995), high-performing students engaged significantly more in directing the group, whereas low-performing students initiated far fewer ideas and asked more questions (aimed at seeking help) than high-performers. However, contrary to results of the Baxter et al. (2001) study, low-performers were most often engaged (i.e., on-task, attentive) during task discussions even though they did not dominate the dialogue.

In spite of the fact that Rachel, a low-performing student, made significantly fewer contributions than her peers, low- and high-performing students in this study unanimously agreed that all students should participate equally in discussion. Students reported that getting everyone involved in discussion is important, not just for the sake of parity, but for meaningful learning to occur among the group. Although Marie, Patty, and Heidi shared various practical ideas for helping Rachel assume a greater role in participation, they did little in reality to help accomplish this, as they almost never asked Rachel questions or checked to see if she understood the content of the discussion. According to Cohen (1994), this finding is not surprising, as equal interactions rarely occur in mixed-ability peer groups unless there is some structure in place that ensures parity (more will be said about this in a later section of this chapter). As one might imagine, debate over whether cooperative groups should be structured or contrived in ways that alter students' natural interactional tendencies does exist and continues to be a relevant issue today (Cohen, 1994). It is important to point out that the teacher in this study did not assign specific roles to students (nor did he implement group rewards) as the empirical emphasis of this investigation was placed on what naturally occurs in unstructured small-group settings.

Meaning-making during Task Discussion

In this study, all students found it easier to find solutions to the rational number tasks than to explain them in ways that could be easily understood by their peers. In fact, one of the most important findings of this multiple case study is that both low- and high-performing students, when left to their own devices, tended to favor abbreviated, decontextualized, computational explanations aimed almost exclusively at revealing answers rather than otherwise tacit thinking processes. For the most part, it appeared that the students' willingness and ability to provide rich explanations or justification of their thinking, in general, was heavily dependent upon the teacher's degree of involvement in facilitating the discussion, and specifically based on whether the teacher explicitly asked for elaboration. Students did not naturally discuss or explain their thinking, particularly when there was agreement about an answer among some of the group members. Instead, students were fixated on finding answers and immediately moving on to the next problem, especially when the teacher was not involved in facilitating the discussion.

These findings are corroborated by many similar studies (Hufferd-Ackles et al., 2004; Lampert, 1990; Nathan & Knuth, 2003; Pape et al., 2003; Yackel & Cobb, 1996).

While discussing the rational number tasks during this study, rarely did any of the students articulate complete, contextualized, and coherent explanations of their reasoning. When they did attempt to explain their solutions, they tended to prioritize computational or numerical aspects while deemphasizing contextual meaning by employing vague pronoun referents at a high frequency. Patty's attempt to justify exactly how and why she partitioned the auditorium during the Science Fair task discussion provides an illustrative example of this: "These work because I did 600 over 2000 and you keep dividing that down, and you, divide it by 2, and it's 300 over 1000, then you divide that and it's 150 over 500, and then you divide that and it's 30 over 100, and I know what that means, uh, you divide that down too or 3/10, so I got that." Put differently, by relying heavily on numbers and procedures when attempting to explain their thinking—and not explicitly addressing what the numbers represented or why specific computational operations were used—their explanations were difficult to follow, especially for students who expressed confusion over the specific content being discussed. Because the teacher was not always present during small-group discussions, the exchange of vague explanations appeared to have undermined others' access to and opportunity to meaningfully interact and participate in discourse, especially in situations where a request for help was either overlooked or not issued. These results resonate with the findings of Webb and Mastergeorge (2003), who observed that help-seekers benefited significantly more when help-givers provided explanations with verbally labeled numbers rather than explanations with mere numerical procedures. In this study, when teacher assistance was not provided in the form of follow-up questioning and revoicing of student utterances, these ambiguous explanations often went unchallenged or non-clarified in small-group settings, particularly by low-performing students.

This study's findings question the presumption that higher-performing students often provide more detailed and easier-to-understand explanations of their mathematical thinking than their lower-performing peers (Fraivillig et al., 1999; Lubienski, 2000a). Although one low-performer assumed a largely passive role in discussion, the other revealed that she was capable of matching the quantity (and at times, quality) of participation demonstrated by her higher-performing peers, especially when the teacher stepped in to facilitate discourse. The findings of this study also challenge Noddings' (1985) assumption that because of compatible language use, children may find it easier to understand the explanations of their peers than the explanations provided by their teacher. While this may have been true to some extent for high-performers, both low-performers stated that they preferred to seek explanations from the teacher rather than from their high-performing peers because of their perception that the teacher knows "how to explain better." It is likely that the complexity of rational numbers, as discussed in Chapter 2, provides a partial explanation as to why low-performers reported that explanations crafted by the teacher were easier to follow than those given by their peers. Perhaps low-performers might have avoided seeking explanations from their high-performing peers due to the perceived degree of social risk in asking questions that could have been construed by others as dumb or stupid. Nevertheless, an important concept to take into consideration here is the degree of shared understanding of one another's contributions to discussion, or what Wertsch (1985) called *intersubjectivity.* As some of the participants insightfully pointed out, a weak sense of audience awareness may limit ability to explain clearly, just as it would undermine a writer's ability to express ideas clearly to a reader.

Additionally, getting students to evaluate, or in particular, to explicitly cite others' thinking when offering explanations proved to be difficult in this study (Hufferd-Ackles et al., 2004; Lampert & Blunk, 1998). For example, even when the teacher explicitly asked students to evaluate others' ideas, they often ignored this request and instead offered their own independent solutions or ideas, which were aimed primarily at uncovering their own answers and not necessarily discussing, explaining, or justifying their reasoning (or the reasoning of others, for that matter). Individualism (i.e., egocentrism) among students, or more specifically, the tendency to show concern only for one's own contributions to mathematics discourse, might help explain why individual students tend to be solely driven by offering their own answers and explanations with little or no regard for evaluating others'. If this individualistic behavior is perpetuated by various socialization agents (e.g., schools, teachers, parents, peers), then it only seems reasonable that teachers can and should play a significant role in countering this tendency by finding ways to empower students as evaluators of each other's contributions. In order to achieve this, a teacher must tip the balance of analytic and social scaffolding she provides, which may require the teacher to "step out" of the discourse so that students can take on greater roles as evaluators of their peers' contributions (Lampert & Blunk, 1998). As demonstrated by the findings of Nathan and Knuth (2003), however, assigning students the primary responsibility for analytic scaffolding may compromise students' learning if the teacher does not skillfully interact with students (e.g., revoicing, questioning, etc.) as they engage in analytic evaluations of each other's contributions.

Who Benefits? Unequal Use of Space during Discussion

In Chapter 1, I raised the question of who benefits from classroom instruction characterized by rich mathematics discourse. Previous research (e.g., Lubienski, 2000a; Baxter et al., 2001) suggests that high-achievers profit significantly more than low-achievers because rich discourse, characterized by higher-order thinking and questioning, reflects their sociocognitive strengths. While the findings of this study do not clearly dispute the findings of previous research, they do suggest there are no simple answers to this question. Specifically, however, opportunities to participate in discussion about rational number tasks manifested in diverse ways, and the majority of these opportunities were indeed seized by high-performing students. Consistent with findings from previous studies (e.g., Baxter et al., 2001; King 1993; Lubienski, 2000a; Mulryan, 1995), low-performing students did not appear to have nearly as many opportunities to make productive contributions to small groups, especially when the teacher was not present to facilitate the discussion of mathematical content. Unequal access to participation was related to unregulated space and meaning, that is, when the teacher was not involved in facilitating discussion. Just based on empirical observations of each student's participation in discourse during small-group settings (especially the Fraction Maze task discussion), it is safe to presume that the two low-performers would have easily been drowned out of the small-group task discussions had the teacher not stepped in to revoice students' contributions, and position individual students as evaluators, questioners, clarifiers, and solution reporters. Therefore, the findings of this study corroborate the importance of the teacher's role in facilitating discourse, especially with regard to the opportunities for low-performers to contribute effectively (Empson, 2003; Walshaw & Anthony, 2008).

Similar to findings by Baxter et al. (2001), Fraivillig et al. (1999), King (1993), Lubienski (2000a, 2000b), and Mulryan (1995), low-performing students found it difficult to keep pace with high-performing students in problem solving and what they called "working it out." They reported sometimes being lost and overwhelmed with anxiety when they realized that they were significantly behind their higher-performing peers in generating correct solutions. This was more so the case during the Fraction Maze task when the teacher was relatively uninvolved in scaffolding or facilitating the discussion. An important finding of this study then is that being outpaced by high-performing students posed limiting effects on the ability of low-performing students to participate in explaining, justifying, and evaluating each others' thinking. However, consonant with the findings of King (1993), even though lows were outpaced by their high-performing peers, they expressed that they wanted to be more involved and they wanted to take on more important roles in the discussion, but they felt rushed and left out by their more-able peers, who seemed to dominate the discussions. Unlike the findings of Baxter et al. (2001) and Lubienski (2000a), however, which portrayed lower-performing students as disinterested and incapable of playing a significant role in discussions about challenging mathematics content, despite being outpaced by their higher-performing counterparts, one of the low-performers in this study demonstrated substantial effort and motivation when talking about the tasks, while the other was attentive but overwhelmed by confusion and intimidation. Significant differences in the speed at which high- and low-performing students could solve problems appeared to cause distress for low-performing students as they rushed to maintain pace with their higher-performing peers. Interestingly, students in Mulryan's (1995) study cited "speed of task completion" as one of the most important criteria of a "good" cooperative learning group for mathematics problem solving. Given this finding, it is not surprising why a student would feel significant pressure to keep up with the fastest students in the group. While Rachel almost always allowed Marie or Patty to race ahead of her, Heidi was often overt in her attempts to seek clarification or to beg another student to "wait" or "hang on." This distinction was perhaps the most significant difference between the two low-performers because it was reflective of their use of space during task discussions. In short, Heidi sought to claim space while Rachel waived space.

The fact that one low-performer in this study (i.e., Rachel) did not take an active role in speaking and sharing contributions is problematic and thus worthy of further discussion. This pattern was stable throughout the entire year, which suggests that passiveness may be a trait deeply rooted within her mathematical self-identity. Studies have suggested that students can benefit by passively observing and listening to others' contributions without taking an active role in speaking (e.g., Olivera & Straus, 2004; Peterson & Swing, 1985), but triangulated data from this study suggest that Rachel was silent because she did not understand the content being discussed. Just as Mulryan (1992) found with low-ability students, Rachel explained her passivity during group discussions in part as a function of her perception of the task's difficulty level. Also consistent with Mulryan's study findings, common perceptions of other students as to why Rachel remained reticent during discussions included "she was confused," "she is shy," and "she is afraid to say the wrong answer." In addition, Mulryan (1992) similarly reported that even though students who were active participants during discussions identified others as passive participants, they most often did nothing to elicit the involvement of passive students in group work and sometimes ignored passive students when they did attempt to make contributions. Unfortunately, the same was true in this study.

The stark contrast between Rachel's participation in mathematics discourse during after-school tutorials and in-class task discussions raises important questions related to the context of participation, especially for students with a profile similar to Rachel's (i.e., low-performing, low-efficacy, quiet, passive). After-school tutorials were generally attended by three to five low-performing students on a weekly basis, whereas in-class task discussions were facilitated in groups as large as 25 students, with a wide range of mathematics abilities. Rachel's sudden willingness to actively participate in mathematics discourse during after-school tutorial sessions raised important questions about the social and emotional context of participation in mathematics discourse, particularly for passive students. Although she claimed she did not know why she engaged more frequently and productively in interactive talk during the after-school help sessions, it is likely that social and emotional factors played a major role in the difference. Her help-seeking behaviors were clearly less restricted during tutorials, which may have been a response to the perception that her social status was not relatively as low among the group of peers in attendance during tutoring sessions, therefore mitigating the heightened sense of social comparison she may have experienced during interactions in math class. Her increase in question-asking during tutoring is consistent with the findings of Graesser and Person (1994), who observed that seventh-grade students felt more comfortable asking questions during one-to-one tutoring sessions than they did in classroom settings. Moreover, the researchers found no correlation between achievement and the frequency of questions asked, suggesting that intimate tutorial settings may provide a significantly more fertile platform for all students to engage in adaptive forms of help-seeking discourse.

Peer Status and Interactions

Perhaps unsurprisingly, the two high-performing students tended to interact exclusively with each other most often during task discussions, especially when they encountered ambiguous or particularly challenging mathematical content. When given the choice, they always chose to work exclusively with each other in partnered settings. Even though the two low-performing students sought help from their higher-performing peers from time to time, they were often ignored or overlooked, which may help to explain why low-performing students relied more on the teacher for help, whereas high-performing students tended to seek help from each other. High-performing students reported a greater level of ease and comfort in communicating with each other and described the act of explaining mathematical content to low-performing students as burdensome and challenging, a finding also reported by Webb and Mastergeorge (2003). The fact that high-performing students preferred to interact exclusively with one another is not surprising, for example, given the conclusive finding from the literature on cooperative learning that children prefer to interact with peers of equal academic status (Rubin, Bukowski, & Parker, 1998).

Similar to the results of Lampert's (1990) study, peer status perceptions played an important role in influencing the interactional tendencies of students in this study. High-performing students in this study were perceived as highly competent and more able to participate effectively in mathematics task discussions by their low-performing peers, raising the question of what role, if any, self-fulfilling prophecies played in influencing peer interactions. Cohen's (1994) review of research concluded that status differences based on academic ability often lead to the development of status generalizations by all group members in which high status students are expected to play a greater role in solving tasks because of their perceived superior level of competence. As a result, low status students are often cut off from access to participating in substantive task interactions. Similarly, Hatano and Inagaki (1991) argue that vertical interaction, which occurs between novices and experts, most often leads to unequal participation because the novice is often less motivated to exert effort toward the construction of knowledge in part due to the belief that the more able member can easily construct that knowledge herself. Consistent with expectation states theory (Berger et al., 1972) and the findings of Cohen (1994) and Lampert (1990), Rachel, a low-performer with low-efficacy, often expressed agreement with higher-performing students most likely because of status expectations she held about them. While findings showed that Rachel did little to overcome status differences, Heidi clearly sought to offset status differences by attempting to claim space in discussion, which she attributed to her self-reported need to understand the rational number concepts that she struggled to comprehend.

An interesting finding not revealed in the review of the literature is that high-performing students articulated their ideas declaratively while low-performing students tended to express their ideas tentatively, often wavering in their thinking or posing thoughts as questions and conditional statements (signaled by the use of modifying words like “maybe” and “probably” as well as the use of “I think . . .” when they initiated contributions). When low-performers submitted their ideas in question form, it seemed as if they were seeking confirmation from higher-performing peers. High-performing students, on the other hand, almost never used tentative language when making contributions, even when their explanations or solutions were incorrect. Although observed patterns such as these were not discovered in the literature reviewed, they are consistent with findings from the literature on self-efficacy (Kerr, 1994), which show that students with higher levels of self-efficacy make contributions more confidently than students with low levels of self-efficacy. As in King’s (1993) study, help-seeking interactions were almost always initiated by low-performers, and manifested in various forms ranging from excessive to adaptive. Although Heidi was not well-liked by Patty, it did not appear that Heidi engaged primarily in excessive help-seeking interactions with Patty. To the contrary, Heidi was often observed soliciting more elaborate explanations from Patty and challenging the accuracy of her claims (although she did sometimes seek help excessively). Moreover, Heidi reported that she did not like merely asking for or accepting others’ answers without truly understanding the content. This was not consistent with the findings of Nelson-LeGall and Glor-Schieb (1986), who found a significant negative correlation between social attractiveness (i.e., peer status) and excessive help-seeking behavior (i.e., asking for answers without explanations or demonstrations) as well as perceived academic competence and excessive help-seeking behavior. However, for Rachel, it was just the opposite—not very popular and instead rather quiet and shy, Rachel did tend to ask for answers and rarely asked for accompanying explanations. Although sociometric measures of peer status were not employed, specific behaviors were observed throughout the year that helped to determine how well each child was liked by peers, as well as social cliques that existed in and beyond the classroom.

Implications

The findings of this study hold several implications for researchers and practitioners interested in the facilitation of mathematics discourse communities. In this section, various practical recommendations, based on this study's findings, for improving the nature of classroom discourse are discussed, and questions are raised regarding some of the contradictory notions related to the facilitation of mathematics discourse in standards-based classrooms.

Ways to Improve the Nature of Discourse

The findings of this study imply that improving the nature of discourse in mathematics classrooms will entail encouraging students to reflect on their participation and interactions during discussion of mathematics topics. Lampert's (1990) claim that focusing solely on the development of students' mathematical content knowledge is not enough to prepare them to engage in mathematics discourse is indeed a powerful sentiment. Teachers therefore must find ways to begin to get students to reflect on how they are participating (or not participating) in mathematics discourse. To this end, this study reiterates previous research findings that video-recording provides educators with a powerful tool for student reflection (Allen, 1992; Hatch, 2002; Nastasi & Young, 1994) and provides a concrete method for teaching children about the social aspects of participation in group-learning settings. While viewing video playback of one's own participation in mathematics discourse, a student could focus her analysis on a variety of topics, including help-seeking and help-giving behaviors (e.g., to what extent do help-seekers make specific requests for help, and to what extent do help-givers provide hints and scaffolds rather than low-level assistance, like answers without explanations) or another student might reflect on the quality and clarity of explanations given (e.g., to what extent do students use computational or relational explanations, as well as verbal labels for numbers and procedures?). A teacher might even choose to begin the reflective process with something simpler to analyze, perhaps non-verbal cues of attentiveness and engagement while students are sharing contributions before the group. In short, the quality of participation can not change significantly without substantial reflection on the part of both teachers and their students.

Based on the challenges that lows and highs faced during task discussions in this study, it is important that researchers and practitioners continue to find ways to help students overcome these difficulties. Low-performing students must be explicitly taught how to demand specific, clear explanations from peers while high-performers should be taught how to craft clear and precise explanations in order to allow all group members greater access to participation in mathematics discourse. While such goals are unquestionably optimistic and likely difficult to attain, existing research offers multiple suggestions for helping improve the quality of task-related peer interactions, such as (a) providing students with explicit instruction on how to give conceptual rather than computational explanations (Fuchs et al., 1997); (b) teaching students how to distinguish between high- and low-level questions, (King, 1999); and (c) the use of metacognitive prompts and self-regulated learning techniques to allow for comprehension monitoring during group discussions (Mevarech & Kramarski, 1997). Integrating these innovative interventions into practice is no simple task, however, due in part to systemic constraints related to covering state-mandated content standards within a finite time frame. More will be said about this in the following section.

Although there is an abundance of literature on the effects of homogenous and heterogeneous student grouping with regard to ability or performance, the findings of this study illuminate some of the problems with heterogeneous groups. Perhaps the relative ability differences between the two low- and two high-performers in this study was too significant to enable relatively equal and productive contributions from each participant (Webb, 1991). With the interest of low-performing students at hand, the findings beg the following question: How should teachers group students in order to enable more effective verbal task interactions? For instance, should the range of differences in ability (and perhaps personality) be relatively small so as to afford all participants equitable opportunities to participate in mathematics discourse and learning? Although Webb's (1991) meta-analysis clearly suggests that small group interactions are most productive and equitable when the range of ability among individuals is narrow (not exceeding low-to-medium or medium-to-high ability), Cohen and Lotan (1995) caution that there are no silver bullets or prescriptive solutions to this historical problem. They eloquently capture the dilemma of grouping that has generally troubled educators for decades and specifically those in settings where tracking and ability grouping practices have been eradicated:

> Educators (who have adopted mixed-ability grouping) have already discovered that they have exchanged severe problems of status differences between tracks and ability groups for equally severe problems of status differences within classrooms. Many perceptive teachers have also found that cooperative learning techniques so widely recommended for this setting do not solve these status problems. (p. 118)

Although the tone of Cohen's and Lotan's narrative may come across as discouraging, it is helpful to the extent that it problematizes the notion of the mere mixing of diverse students as a quick fix to the problems associated with tracking and ability grouping that have long vexed teachers in modern school settings. Cohen and Lotan recommend the explicit implementation of status treatments, and specifically assigning competence to low-status children in order to positively influence the expectations held by others regarding low-status students' ability and value to the peer group—a recommendation that is reiterated by several other researchers who have studied participation in discourse or task interactions (e.g., Empson, 2003; O'Connor & Michaels, 1996; Webb & Mastergeorge, 2003). In sum, although the practice of mixed-ability grouping is inherently motivated by equity concerns, mere implementation of heterogeneous groups does not guarantee equitable interactions among students.

Revised Classroom Norms

Similar to Lampert's findings, (1990) the teacher in this study also struggled with teaching students to consistently use mathematical content, especially relational content, as a basis for forming their explanations. When students were confused and needed help, they often engaged in excessive forms of help-seeking or even avoidance of help-seeking, like many studies of peer interactions during mathematics problem-solving have demonstrated (Butler, 1993; Nelson-LeGall & Glor-Schieb, 1986; Newman, 1998; Ryan et al., 1998). In light of these observations, a post-hoc analysis of these findings and the relevant research literature on peer interactions suggest that the classroom norms set at the beginning of the study suffered from two glaring omissions. In hopes of providing conditions that may better enable more productive interactions among students in the future, two additional classroom norms are recommended, which are listed in Table 11.

The Paradox of Competition and Collaboration

This study's findings question the extent to which competition and collaboration can co-exist in mathematics discourse communities. Most apparent was Patty's competitive, individualistic behavior, which may have resulted in limiting effects on the quality of discursive exchanges made by group members, especially during the Science Fair task discussion as she worked privately for a majority of the time. She openly characterized herself as highly competitive, which is not surprising given the finding that students enrolled in gifted education programs tend to demonstrate personality traits consistent with perfectionism and competitiveness (Clark, 2001). Patty demonstrated such traits in many ways: she often made hasty decisions without the consent of the

Table 11

Additional Classroom Norms

Norm	Inadequate Example	Adequate Example
Always connect words to numbers when explaining your thinking. Do not use numbers only; always clearly express what the numbers represent.	"To make 1 1/2 times, I added 1/3 to 2/3 and got 1."	"To make 1 1/2 times the recipe, I added 1/3 cup of sugar to the original 2/3 cup of sugar to make a whole cup of sugar."
"I'm confused" or *"I don't get it"* are unacceptable prompts for help. Always tell someone exactly what is confusing you!	*"I'm confused"* or *"I don't get it"*	*"Because you're dividing the cake up for two people to share, why did you divide by 1/2 instead of 2?"*

group, only to be subsequently challenged by either Marie or Heidi; she always spoke assertively when making contributions, hated to be wrong, and rarely invited interactions from low-performing peers; and during task discussions, she failed to make eye contact with others as they shared contributions and instead was often observed writing feverishly so that she could find a solution before anyone else did. Together, these personality characteristics seemed to undermine the potential for productive interactions between Patty and her low-performing peers, although they did not appear to have a significant negative impact on her interactions with Marie, her high-performing counterpart.

While competitive behaviors may be healthy among students of equal ability status, it may be counterproductive when students are grouped heterogeneously and expected to collaborate and help each other learn as is typical in most small-group instructional settings (Cohen & Lotan, 1995). Some researchers have suggested the use of task and group reward structures to help offset the effects of individualism or inter-group competition (Johnson & Johnson, 1998; Slavin, 1995), but this proposition is not without contention, as ideological debate abounds over whether students should be motivated to learn cooperatively by virtue of their own intrinsic interest rather than extrinsic rewards. Notwithstanding contention, there is conclusive evidence that individual students who have the ability to solve tasks by themselves lack the motivation to interact with or help others who are struggling (Slavin, 1995). Simply put, an individual student’s primary goal in the classroom is not to ensure the learning of others but rather her own learning. Moreover, several other researchers who have used participant frameworks as an analytical tool to study students’ interactions have acknowledged that students do not animate one another into diverse roles in the way that a teacher does (Forman & Ansell, 2001; O'Connor & Michaels, 1996). However, this does not necessarily mean that students are incapable of learning how to animate one another into diverse roles. Since teacher modeling of these behaviors alone is not likely to result in their internalization in students, teachers must find ways to explicitly teach children how to assume an array of roles in mathematics discourse, and this may entail experimentation with task and reward structures—especially because of the prominence of small-group non-teacher-facilitated learning in reform-based mathematics classrooms. Teacher education programs would do well to stress the importance of task-related peer interactions, and more specifically, what teachers can do to effectively facilitate them.

If Empson (2003) is correct in contending that the teacher's ability to provide space and meaning for students to engage in productive discourse is the most critical factor affecting the quality of discourse for all students, then the same must be expected of students in small-group settings when the teacher is not present to facilitate the discussion between the students. While the collaborative construction of classroom norms certainly helped to alleviate such problems, these norms were by no means a panacea to the problems. It is therefore critical to continue to address problems associated with group interaction and motivation, and perhaps how an individual's personality characteristics might interact with other individual or group characteristics and how these interactions might influence participation in discourse. Similarly, peer status issues must be addressed in order to establish the kind of instructional setting and productive discourse spoken of so idealistically in the literature on mathematics discourse communities. These responsibilities surely begin with the teacher, but are invariably affected by the unique characteristics of the students in a particular classroom, as well.

The Rhetoric and Reality of Standards-based Reform

So what might the ideal standards-based mathematics discourse community look like? Webb (1991) describes ideal mathematics classroom discourse conditions as those in which students “freely admit what they do and do not understand, consistently give each other detailed explanations about how to solve the problems, and give each other opportunities to demonstrate their level of understanding” (p. 386). Romberg (1993) envisions “discourse communities where conjectures are made, arguments presented, strategies discussed” (p. 37). NCTM strongly advises teachers to use multiple representations to allow students to construct knowledge rather than relying exclusively on didactic instructional delivery approaches that prioritize symbolic algorithms and speedy production of answers (Cuoco & Curcio, 2001).

The rhetoric of standards-based teaching is arguably compatible with these ideals. This language certainly places greater emphasis on students’ conceptual understanding and ability to solve complex mathematical tasks, as well as their capacity to engage in adaptive forms of reasoning, as mentioned in Chapter 1. However, the reality of standards-based teaching, where system-level pacing guides, prescriptive content-based educational standards, and repeated routine standardized assessments of students’ mastery of the standards dictate what, how, and when teachers teach, poses a paradoxical dilemma for teachers who strive to incorporate the ideals of standards-based mathematics education in a mathematics discourse community.

Although the teacher in this study subscribed to standards-based teaching and rich forms of mathematics discourse, he found these ideals difficult to realize. For instance, he had intended to spend a considerable amount of time addressing the writing process in mathematics with his students, as well as the social norms of participating in a discourse community, but he found it difficult to make time for the integration of these ideals because of system-level constraints associated with teaching the content of mathematics. Consistent with existing research on problems encountered in attempting to facilitate classroom discourse, the teacher in this study felt pressured to cover content quickly (Hufferd-Ackles et al., 2004), and feared that students' conceptual knowledge development regarding rational numbers would be compromised under such constraints (Post et al., 1985). For example, the county pacing guide allowed only five weeks for the unit on rational numbers, but in order to teach the concepts included in the state curriculum documents to this particular group of students using reform-based teaching approaches (i.e., multiple representations, hands-on materials, etc.), it took nearly twice as much time. As mentioned in the first two chapters, teaching mathematics for understanding generally requires more time than traditional didactic pedagogy, because much of the learning is hands-on, some is discovery-based, and almost all of the instruction integrates the use of multiple representations of knowledge. Moreover, many of the standards in the rational number unit depend on the students' understanding of basic fraction concepts like order and equivalence, yet only two days are allotted for lessons on fraction equivalence and order (although the teacher in this study used an entire week to teach these concepts.) Given the research that links students' overall struggles with fractions to their failure to understand the basic concepts of equivalence and order, it is no surprise that standards-based instruction, when implemented as prescribed (or perhaps mandated) fails to work well for all students, instead privileging only those who possess the prior knowledge and skills to maintain pace with standardized content pacing guides. Simply put, there is great irony in the fact that educational leaders acknowledge that all children are different, yet they are all expected to master a standardized set of knowledge.

Finally, the contradiction between assessment and learning in the standards-based era of public education can not be overlooked. The ways in which schools measure learning have changed very little over time, which is ironic considering that standards-based reform assumes that the predominant curriculum and pedagogy of the past are fundamentally inadequate. Given the great emphasis that educators continue to place on standardized testing as the primary tool for measuring the quality of student learning, which is most often characterized by multiple-choice, answer-driven questions, it is not surprising that the students in this study tended to resist providing explanations of their reasoning in favor of merely providing concise answers to questions. The message conveyed to children through standardized tests of achievement is that explanations and other forms of adaptive reasoning do not matter nearly as much as knowing the answer to a question. Therefore, in order to improve students' ability to communicate their mathematical reasoning, we must first question the value we place on these standardized forms of assessment, while continuing to advocate for change in the ways we assess learning. How difficult could it really be to include open-ended questions on these tests, where students are asked to explain concepts? The same system that is used for evaluating responses to writing prompts on state examinations of writing proficiency could be used to score students' adaptive reasoning abilities on standardized tests of mathematics proficiency. As long as students only have to choose a predetermined answer from 4 or 5 choices on these tests, it will be difficult to motivate teachers to move beyond traditional instructional methods and toward building mathematics discourse communities. Simply put, the old ways of testing can not co-exist with the new visions of teaching and learning.

Study Limitations and Recommendations for Future Research

Although this study was designed to investigate students' participation in mathematics discourse in both whole-class and small-group settings, data generated from whole-class discussion was limited in comparison with data that were collected from small-group settings. Given the reality that the whole-class research setting included 25 students, the researcher expressed ethical concerns at the onset of the study related to soliciting responses from focal participants disproportionately more than non-focal participants during whole-class discussions of rational number tasks. Even though nine whole-class lessons were video-recorded in hopes of mitigating the likelihood of collecting an inadequate amount of data from whole-class settings, additional data from whole-class settings would have allowed for more-detailed analyses and conclusions regarding students' participation in discourse during whole-class discussions.

Although data saturation was obtained through analysis of the two small-group task discussions, the use of additional tasks in a future study might possibly reveal additional outcomes or themes related to these students' participation in discourse. Additional small-group task discussions would also help to determine whether the nature of students' participation in discourse changed over time or with different types of content or tasks. More cases might also help to uncover additional findings, as well, especially considering the stark differences observed between the two low-performing students in this study.

As is the case with many studies that include observation of participants in a research setting, the potential for observer effects to influence the findings is expected and therefore must be acknowledged. For instance, in this study, the extent to which students' behaviors were altered by their knowledge of serving as participants in a study, or that they were being video-recorded, is unknown. Moreover, how might their behaviors have been affected by the presence of a researcher who was not also their classroom teacher? As a teacher-researcher, I had the distinct advantage of being able to observe each student's behaviors for the entire school year before, during, and after the study and can honestly say that the students' behaviors did not seem significantly different when data were not being collected.

Although students were observed participating in mathematics discourse in the regular classroom setting during whole-class discussions, they were relocated to an isolated classroom for the purposes of small-group data collection. One can only speculate how their participation might have been affected had they been observed in the regular classroom along with the other 21 students. In order to overcome this limitation, future studies that employ a similar design would likely need to use high-tech audio equipment, such as lapel microphones that effectively filter out ambient classroom noise.

Finally, minimal information was known about the distinct mathematics education histories of each the participants of this study. There is no question that each student's particular biographical experience as students of mathematics in elementary school settings played a significant role in influencing their current participation habits. It would certainly be interesting to know more about these past experiences and whether or not specific behaviors have changed or remained stable over time. Future research could track students' diverse experiences with participation in mathematics discourse over several years in different classrooms to explore this proposition.

Future research is needed to help illuminate the characteristics of effective teachers in mathematics discourse communities. Comparisons of teachers' experiences with implementing discourse-rich mathematics instruction may help to elicit these characteristics. Future studies should also focus on examining discursive interactions in small-group learning settings, particularly with the aim of improving participation in mathematics discourse for all students. It may even be interesting to analyze students' participation in discourse as they interact with different peers in flexible grouping settings, where the composition of the peer group changes over time. For instance, how does a child's participation in mathematics discourse vary as her peer group changes? And what factors appear to be related to the changes in participation? Such research may help reveal additional ways in which educators can maximize the potential for all students to gain access to participation in mathematics discourse.

Summary

In examining the differences between the types of contributions these students made during task discussions, it is evident that limitations related to the provision of adequate space and meaning created problems for low-performers that proved difficult to overcome. Without effective teacher intervention, it is highly likely that these differences are naturally sustained if not exacerbated over time. On the same token, the teacher's interactions with high-performing students was also critical, as the clarity of their explanations was improved through the teacher's revoicing and clarification prompts. This study therefore reiterated the important role that teachers play in encouraging high-quality, equitable participation in mathematics discourse.

Finally, this study's findings assert that a student's ability to provide more than an answer to a math problem is not merely a function of cognitive dimensions. This study's findings underscore the importance of social and emotional dimensions that affect students' willingness and ability to participate in rich and diverse forms of mathematics discourse, such as classroom socialization, peer status, motivation and goal orientation, participant frameworks, and above all, the situated context in which learning to participate in mathematics discourse occurs.

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APPENDIXES

APPENDIX A

Class-constructed Norms for Participation in Mathematics Discourse

Listening Norms

1. Do not talk while someone else is sharing their thinking; listen to them instead.
2. Do not make anyone feel bad for not being able to explain their solutions.
3. Do not make anyone feel bad for having the wrong answer.
4. When evaluating someone’s explanation, say something positive before offering critique.

Explanation (Speaking) Norms

1. Explain your thinking by using the language of mathematics.
2. Be precise, clear, and coherent.

“When you need help…” Norms

1. Always be willing to ask your peers for help if you are confused; don’t be withdrawn.
2. Always try to solve the problem first BEFORE asking a peer for help.
3. Always ask for hints only; do not ask for answers.

APPENDIX B

Description of Whole-class Tasks

Exploring Equivalent Fractions (Bell et al., 2007)

Students explored the relationship between physical representations of equivalent fractions and the multiplication and division algorithm used to generate equivalent fractions.

Comparing Fractions to 0, 1/2, and 1

Students used fraction pattern blocks and number lines to help identify the relative proximity to 0, 1/2, and 1.

Fraction Capture (Bell et al., 2007)

Students played a competitive game related to fraction number sense, equivalent fractions, and splitting and adding fractions.

Estimating Fraction Sums to One Whole (Behr et al., 1985)

Students were given six numbers, which were to be used to create as many fraction sums that were close to, but not exactly, one whole.

Subtracting Fractions with Regrouping (Bennett et al., 2007)

Students used fraction pattern blocks to model subtraction of mixed numbers with regrouping.

Multiplying Mixed Numbers with Area Models and Circle Drawings (Wiebe, 1998)

Students modeled fraction and mixed number multiplication expressions by drawing corresponding area models and circle drawings.

Dividing Fractions with Fraction Circles Pieces (Sharp, 1998)

Students modeled fraction division problems with fraction circle pieces and then attempted to discover the common-denominator algorithm from accumulated examples.

"Mental Percents" with Repeated Halfing (Moss & Case, 1999)

Students identified 50%, 25%, 10%, 5% of a number by drawing "water beakers." They were then asked to find as many different parts of the whole as they could by combining, subtracting, multiplying or dividing the original percents (i.e., 50, 25, 10, 5).

Finding Percent of a Whole and Checking with Estimation (50% benchmark)

Students used 50% as a comparative benchmark for estimating the percent of a whole number in order to evaluate the reasonableness of their answers.

APPENDIX C

Fraction Maze Task

Directions: You must ALWAYS move from a smaller to a larger number. You may move left, right, up, and down only (no diagonal moves are allowed).

Send

$\frac{1}{8}$ $\frac{3}{16}$	$\frac{1}{16}$ $1\frac{1}{2}$	$\frac{7}{3}$ $1\frac{3}{5}$	$2\frac{7}{8}$ $1\frac{1}{8}$	$\frac{17}{5}$ $3\frac{3}{4}$	$2\frac{15}{16}$ $\frac{25}{8}$
$\frac{1}{8}$	$\frac{5}{4}$	$1\frac{1}{2}$	$\frac{16}{4}$	$\frac{33}{8}$	$4\frac{1}{16}$
$\frac{25}{5}$ $5\frac{1}{6}$	$\frac{47}{9}$ $5\frac{1}{3}$	$4\frac{5}{6}$ $\frac{9}{2}$	$\frac{9}{2}$ $4\frac{1}{3}$	$4\frac{3}{16}$ $4\frac{1}{8}$	$\frac{31}{8}$ $4\frac{8}{9}$
$\frac{11}{2}$	$\frac{35}{6}$	$\frac{25}{4}$	$6\frac{2}{7}$	$6\frac{1}{4}$	$\frac{27}{5}$
$\frac{20}{3}$	$5\frac{5}{9}$	$\frac{18}{3}$	$\frac{14}{2}$	$6\frac{1}{2}$	$6\frac{7}{11}$
$\frac{31}{4}$	$\frac{58}{7}$	$7\frac{6}{7}$	$8\frac{5}{8}$	$8\frac{7}{16}$	$\frac{23}{3}$
$8\frac{1}{8}$	$\frac{17}{2}$	$\frac{88}{11}$	$\frac{37}{4}$	$\frac{9}{1}$	$\frac{70}{7}$
$9\frac{1}{16}$	$9\frac{1}{8}$	$9\frac{3}{16}$	$9\frac{2}{5}$	$\frac{59}{6}$	$10\frac{1}{5}$

Receive

APPENDIX D

Science Fair Task

Science Fair

Name ______________________________

Three middle schools are going to hold a science fair in an auditorium. The amount of space given to each school will be based on the number of students in the school. Bret Harte Middle School has about 1000 students, Malcolm X Middle School has about 600 students, and Kennedy Middle School has about 400 students.

1. Suppose that the rectangle below represents the auditorium.
   a. Divide the rectangle to show the amount of space that each school will get. Label each section "BH" (for Bret Harte), "MX" (for Malcolm X), or "K" (for Kennedy).

   b. Explain your mathematical reasons for dividing the rectangle as you did.

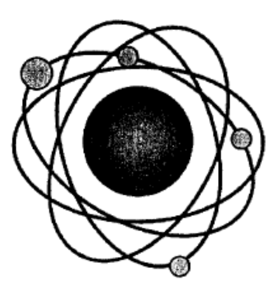
2. What fraction of the space should each school get on the basis of the number of students? Show your mathematical reasoning.

   Bret Harte Middle School ___________

   Malcolm X Middle School ___________

   Kennedy Middle School ___________

SCIENCE FAIR



Science Fair (continued)

Name ______________________________

3. If the schools divide the cost of the science fair according to the number of students at each school, what percentage of the cost will each school pay? Justify your answers.

Bret Harte Middle School __________%

Malcolm X Middle School __________%

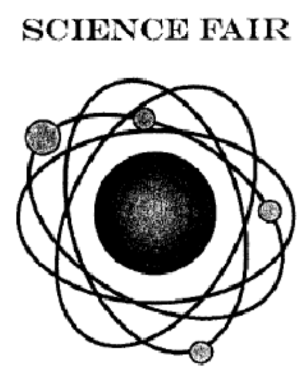
Kennedy Middle School __________%

4. If the cost of the science fair is $300, how much will each school pay? Justify your answers.

Bret Harte Middle School __________

Malcolm X Middle School __________

Kennedy Middle School __________



APPENDIX E

Sample Interview Questions

Interview 1
INT: Okay, so tell me about anything that comes to mind about your experience of participating in the group a few minutes ago.

INT: What do you remember doing to help the group solve the task?

INT: What do you remember each of the other students doing to help the group solve the task?

INT: Was there ever a point where you felt like you could do more to help the group solve the task? Explain.

INT: In some cases, you got behind the group. What happened?

INT: There was one part at the very beginning when you had a disagreement with Patty about a mixed number. What do you think happened there?

INT: Do you remember evaluating or challenging any of the claims that someone else in the group made?

INT: Would you say someone led the group? Explain.

INT: How did you know when to disagree with certain students in the group who made claims. How did you know when to disagree?

INT: Anything else about the group work today that you'd like to share?

Interview 2
INT: What did you see yourself doing to help the group solve the task?

INT: Was there anything else you could have done in addition to help the group solve the task?

INT: Tell me about your interaction with each of the other students.

INT: But if one or two people know all the right answers and can get the group through the task quickly, is it important for everyone else to be equally involved in the discussion? Why?

INT: Do you feel like somebody in the group wasn't as involved as they could be? How do you think you could have helped or someone else could have helped that student get more involved and understand?

INT: When others made contributions to the group, or offered answers, did you evaluate or check the claims that they were making?

INT: It seemed to me like when you were offering your ideas on which fraction you should go to next, you tended to ask the group rather than tell the group what to do. Like, instead of saying "Let's go to 1 and ¾," you'd almost ask, "Should we go to 1 and ¾?" Why ask instead of tell?

INT: Okay, how often do you think the group moved on without you understanding one of the moves that were made?

INT: Okay, what do you like better of these two situations: A) Taking an answer given by another student and just moving on to the next problem OR B) stopping to have a detailed discussion or debate with everybody before moving on to the next problem? Why?

INT: How does the discussion or debate help when you're talking about something that's hard or confusing?

Interview 3

INT: Okay, so what did you think about the task?

INT: How about your discussion of the, of the task – how did that go?

INT: Do you remember a specific case of how you built off each other's thoughts?

INT: How did "testing it out" help you?

INT: What do you remember doing specifically to help the group solve the task aside from that?

INT: Was there anything else that you could have done to help the group the discuss or solve the task?

INT: Would you say that somebody took charge or lead of this discussion? Explain.

INT: Did you evaluate the claims or the observations that the other students made today? Give me an example.

INT: How did you know when to agree or disagree with another student and then what makes you decide whether to speak up and say something about it?

INT: Anything else you want to add?

Interview 4

INT: Tell me about your interaction with each of the other students.

INT: Did you feel like she was listening to the discussion when you were talking?

INT: Who interacted the most with one another?

INT: Okay, did you feel like you were being listened to when you spoke in that clip?
INT: By whom?

INT: I don't see as much interaction between Patty and you, or Marie and you. Why do think that is?

INT: When you look at the tape, what kind of physical things can you see that makes you think that someone's listening to you?

INT: She is so fast at computation. How does that affect your participation in the group when something like that happens?

INT: Does it bother you when she blurts out answers before you've had the chance to solve the problem or because of how much faster she is?

INT: Why do you prefer to ask the teacher for help instead of someone in the group?

INT: There was one part where she says, "Malcolm X has 600 and Kennedy has 400 and so you can divide like that into these, like you can, and you can know that that's half almost, so it's going to be more than half for that one." Did you understand her explanation? Why or why not?

INT: Would you like to say something else about the group discussion today?

APPENDIX F

Whole-class Writing Tasks

Task 1: October 5, 2009
Fraction comparisons and equivalence

Students were instructed to write a letter to a fictitious student who drew a pictorial representation to claim 6/8 is greater than 3/4.

Task 2: October 13, 2009
Fractions that sum to 1 whole

Students wrote what they noticed about the relationship between the numerator and denominator in two different types of fraction addition expressions that sum to 1 whole. Students created the expressions in pairs using a limited set of numbers.

Task 3: October 21, 2009
Subtracting mixed numbers by regrouping

Students wrote a letter to a fourth-grader explaining the similarities and differences between subtracting whole numbers with regrouping and subtracting mixed numbers with regrouping.

Task 4: November 4, 2009
Multiplying mixed numbers with area models

Students explained how an area model represents multiplication.

Task 5: November 10, 2009
Dividing fractions

Students explained why division with whole numbers generally results in a quotient that is less than the dividend but division with fractions typically results in a quotient that is greater than the dividend.

Task 6: December 2, 2009
Finding percent of a number with multiplication algorithm

Students explained a fictitious student's error: 48% of 300 is 6.25. Why is this solution unreasonable?

Task 7: December 3, 2009
Renaming fractions as percents

"Jared incorrectly assumes that 7/16 is equal to 0.716." Students explained why his assumption is unreasonable.