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A COMPREHENSIVE GUIDE TO CGE: THEORY AND PRACTICE

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A COMPREHENSIVE GUIDE TO CGE THEORY AND PRACTICE

1. INTRODUCTION

Computable General Equilibrium (CGE) models use neo-classical behavioural concepts of economic theory such as utility maximisation and cost minimisation, to characterise the workings of the economy. A CGE model which is carefully designed will have a transparent and theoretically consistent structure and will offer a tool for policy appraisal. Indeed, with the availability of appropriate software CGE has become a productive research area over two decades.

A simple model identifies a single 'representative' consumer, who is assumed to own an initial endowment of a number of commodities and factors, and a set of preferences. By maximising utility subject to a budget constraint market demand functions for each commodity can be derived. Market demands must satisfy Walras's Law; that at any set of prices, the total expenditure of consumers equals consumer income. On the production side, technology is typically described by constant returns to scale production functions and producers maximise profits (or minimise costs). A general equilibrium can therefore be characterised by a set of prices and levels of production in each industry which equals market demand across all commodities. And this simple model can be extended to include many other elements like a government sector or an external sector. However, regardless of the level of additional complexity which the market clearing condition is what characterises a general equilibrium. (Greenaway et al., 1993)

Once the model structure is formalised by specifying dimensions of the model and choosing functional forms for production and utility functions the next step is to specify a benchmark data set and calibrate functional forms to this initial equilibrium benchmark data set (see Figure 1). A benchmark data set can be constructed from input-output tables (I-O tables), national income statistics and other major data sources for both a single or representative year and, where necessary, average data over a time period. Calibration can be defined as fitting the model to the benchmark data set and

typically involves choosing and adjusting the parameters of the model. After the calibration stage the model can be used to see the results of specific macroeconomic and trade policy scenarios.

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┌─────────────────────────────────────────┐
│   Basic data for economy for single year │
│   or average of years (national accounts,│
│   household income and expenditure,      │
│   input-output tables, tax data, trade   │
│   and balance of payments)               │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│   Data adjustment: Benchmark equilibrium │
│                data set                  │
└─────────────────────────────────────────┘
                    │
                    ▼                              ┌────────────────────┐
┌─────────────────────────────────────────┐       │ Specification of   │
│   Calibration to benchmark equilibrium   │◄──────│ exogenous elasticity│
└─────────────────────────────────────────┘       │ values             │
                    │                              └────────────────────┘
                    ▼
┌─────────────────────────────────────────┐
│              Policy change               │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│          Counterfactual equilibrium      │
└─────────────────────────────────────────┘
                    │
                    ▼
┌─────────────────────────────────────────┐
│   Policy appraisal based on comparison   │
│   between counterfactual and benchmark   │
└─────────────────────────────────────────┘
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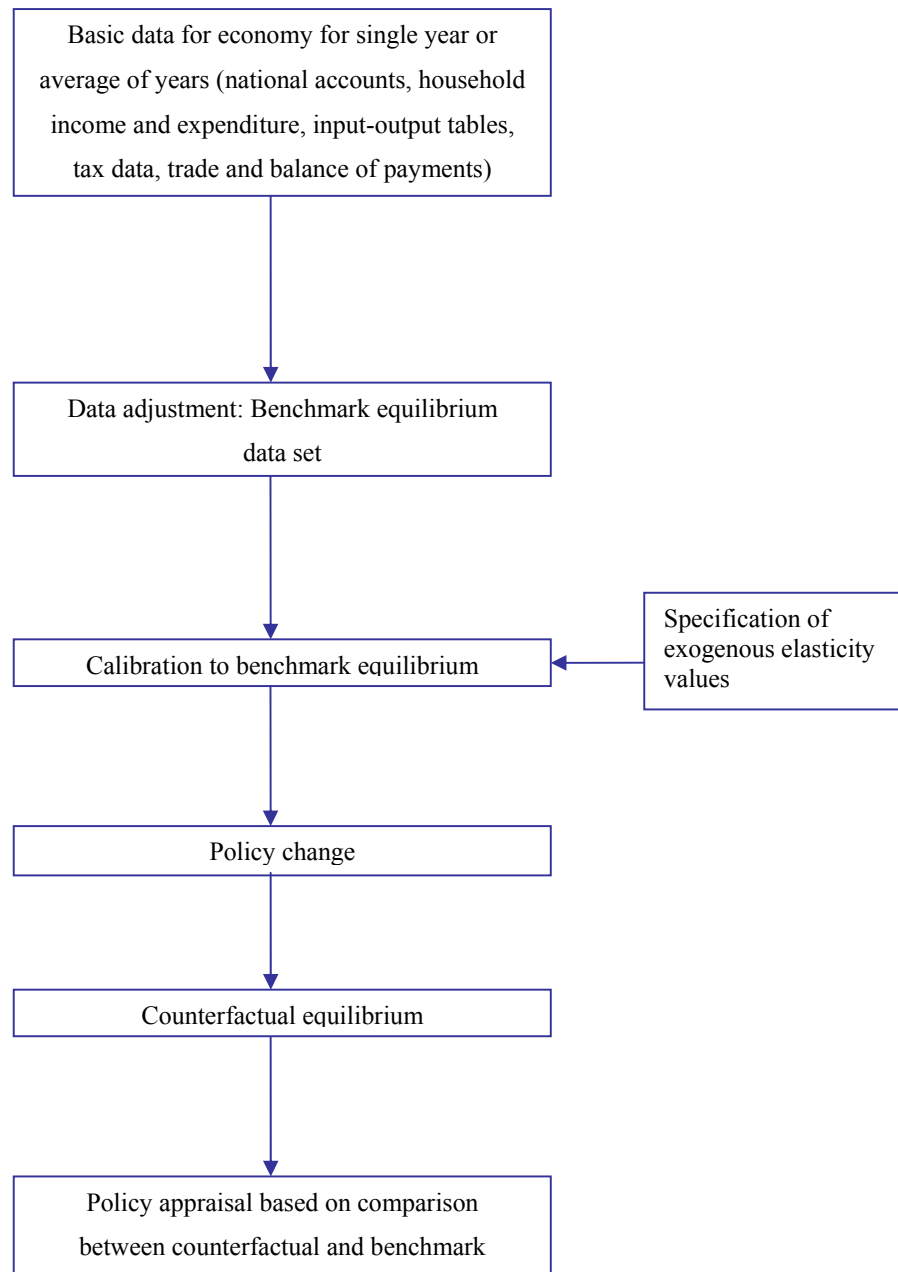
Figure 1 Steps in CGE Modelling (Adapted from Greenaway et al., 1993)

This paper intends to discuss the principles of CGE theory, its practice and also evaluate the methodology. Section 2 gives a detailed discussion of the most popular types of functional forms used in CGE analysis and continues in section 3 with a discussion of the concepts of model representation and solution methodology in CGE modeling including two important issues in CGE modeling; nesting and calibration. Section 4 offers a simple linearised CGE model example to illustrate the basic steps involved in CGE analysis. The paper continues with an explanation of closure (Section 5). The paper continues with section 6, which evaluates the usefulness of CGE modelling for policy appraisal, outlining its strengths and weaknesses. Finally, section 7 concludes the paper. Appendices are provided at the end of the paper.

2. FUNCTIONAL FORMS IN CGE MODELS

There are many factors which affect the choice of functional forms in CGE models. In general, the function chosen should be continuous and homogeneous of degree zero and result in a system of demand in conformity with the Walras's Law (Shoven and Whalley, 1984). These conditions make a function 'convenient' because they ensure equilibrium and ease the analysis of variations in the price resulting from economic policiesIn the following section, the most popular functional forms are reviewed.

2.1. Leontief Function

The Leontief function specifies that there is zero substitution between inputs such that there is only one combination of inputs for any given level of output. For this reason the isoquant is 'L' shaped. The implied L-shaped isoquants of such a production function are shown in Figure 2. In the figure, at any particular output level Q, there is a specified level of X_1 and X_2 at the corner of the isoquants, Q and Q', which shows the best point for the firm to operate at the specified output because it represents the lowest level of input cost. An increase in the usage of one input (e.g., $X_1' \rightarrow X_1''$), holding the other

input fixed ($\overline{X}_2$), will not result in an increase in production, since we remain on the same isoquant Q), although it would increase the cost of production.
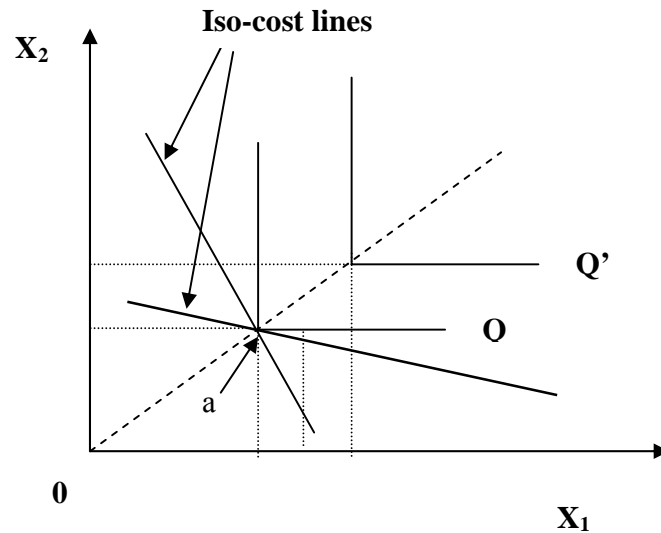
Iso-cost lines

Figure 2 Leontief Isoquant

In the case of 'n' inputs, the Leontief function is algebraically expressed as:

$$Q_j = \min\left[\frac{X_{1,j}}{A_{1,j}}, \frac{X_{2,j}}{A_{2,j}}, \ldots, \frac{X_{n,j}}{A_{n,j}}\right]$$

(LF.1)

where it is supposed that the *minimum* number of units of *all* (intermediate) inputs $X_{i,j}$ required to produce an extra unit of output (Q_j), is given by the parameter $A_{i,j}$. This fixed relationship between output and each input implies constant returns to scale.

The nature of the function implies that to increase output, rational cost minimising producers will only employ the minimum number of input units, giving demand functions:

$$X_{i,j} = A_{i,j}Q_j$$

(LF.2)

where demand for each input 'i' is a function of the fixed input-output parameter $A_{i,j}$. Observe that, Leontief demands remain unaffected by changes in relative prices. This can be illustrated in Figure 2 above, where changes in the slope of the iso-cost line running through optimal production point 'a' (along the ray), does not affect input intensity.

The composite output price over all 'i' inputs (i=1…n), P_j, can be derived by assuming zero profits in industry 'j':

$$P_j Q_j = \sum_{i=1}^{n} R_{i,j} X_{i,j}$$

(LF.3)

where

Q_j - Output in industry 'j'.

P_j - Output price in industry 'j'.

$X_{i,j}$ - Demand for input 'i' in industry 'j'.

$R_{i,j}$ - Price of input 'i' in industry 'j'.

Substituting expression (LF.2) and dividing by Q_j gives:

$$P_j = \sum_{i=1}^{n} A_{i,j} R_{i,j}$$

(LF.4)

The Leontief function is a common specification in many CGE models. In the GTAP (Global Trade Analysis Project) model which is used later in this study, Leontief functions are chosen to characterise (zero) substitution possibilities between composite value added and composite intermediate inputs. In an agricultural context, it may be argued that such a treatment of producer

behaviour is not realistic, where for example a farmer may use a different fertiliser application in response to a relative price change with respect to land. However, with a general lack of data on substitution possibilities between composites of this nature, most CGE applications like GTAP model utilise the Leontief functions.

2.2. More advanced functions

The Cobb-Douglas (CD - Cobb and Douglas, 1928), Constant Elasticity of Substitution (CES - Arrow et al., 1961) and Constant Elasticity of Transformation (CET) are more advanced functions which explain producer/consumer behaviour as they allow for substitution between inputs although they still have restrictions which will be explained in following parts. The nature of these functions implies the shape of the isoquant to be smooth and convex with respect to the origin as shown in Figure 3. Thus, changes in relative input prices, imply substitution between factors (inputs).

In the next two sub-sections, CD and CES production functions are assessed on three criteria:

1. The response of short run output to variation in a single output, all inputs held constant (marginal product and average product)
2. The substitution possibilities of one input for another (applies equivalently to consumer theory)
3. The response of long run output to an equiproportional change in all inputs (returns to scale)

Iso-cost lines

X_2

X_2''

X_2'

Q

X_1'' X_1' X_1
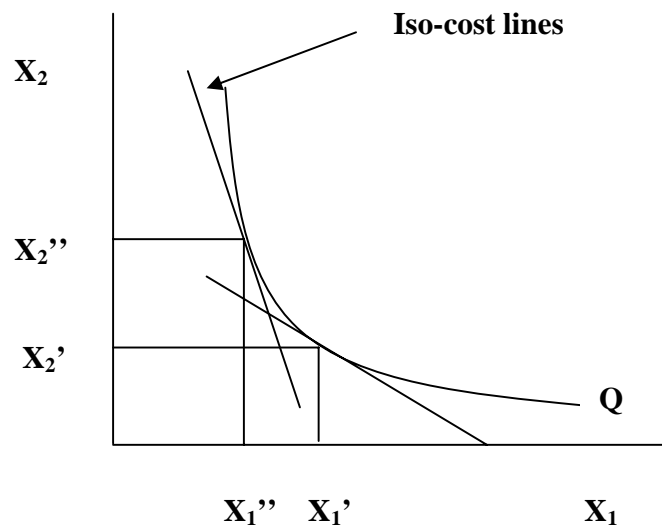
Figure 3 A smooth Convex Isoquant

2.2.1. Cobb-Douglas Function

The standard Cobb Douglas (CD) production function may be expressed as follows:

$$Q = AX_1^{\alpha} X_2^{\beta}$$
(CD.1)

where demands for input 1 and 2 are X_1 and X_2 respectively, Q is output, A is an efficiency parameter and α and β are elasticities. First order *partial* derivatives give short run marginal products:

$$\frac{\partial Q}{\partial X_1} \Rightarrow MP_1 = \alpha AX_1^{\alpha-1} X_2^{\beta}$$
(CD.2)

$$\frac{\partial Q}{\partial X_2} \Rightarrow MP_2 = \beta AX_1^{\alpha} X_2^{\beta-1}$$
(CD.3)

The average product (for input 1) is given as:

$$\frac{Q}{X_1} = AP_1 = AX_1^{\alpha-1}X_2^{\beta}$$
(CD.4)

Substituting (CD.4) into (CD.2) gives the relationship between marginal and average products:

$$MP_1 = \alpha AP_1$$
(CD.5)

In CGE models the production function chosen must obey either concavity or strict concavity (see appendix II) to be consistent with the theory, which limits the range of values that the parameters may assume in the chosen function (see appendix II)[1]. Resulting from these short- and long-run theoretical restrictions, Beattie and Taylor (1985) demonstrate that production functions present three stages of production (see appendix III). Given equation (CD.5) and appendix II, CD functions which are restricted to strict concavity only show stage II of production (i.e., MP<AP) either with respect to each factor (short-run) or with respect to scale (i.e., proportional changes in all inputs, long-run). In a similar way, strict quasi-concavity in CD functions implies stages I or II with respect to either each factor or scale (see Beattie and Taylor, 1985, pp68-69).

The elasticity of substitution measures the curvature of an isoquant. More specifically, the elasticity of substitution measures the percentage change in the factor ratio divided by the percentage change in the marginal rate of substitution (MRS), with output being held fixed (Varian, 1992, pp13). For a two input production function:

$$\sigma = \frac{MRS_{12}}{X_2/X_1} \cdot \frac{d(X_2/X_1)}{d(MRS_{12})}$$
(CD.6)

---

[1] In theoretical terms, 'Short-Run' production functions must show Diminishing Marginal Returns; 'Long-Run' production functions must show some form of returns to scale.

The marginal rate of substitution (MRS) is the ratio of the marginal products, or the slope of the isoquant (Koutsoyiannis, 1979, pp73). Dividing (CD.2) by (CD.3) gives:

$$MRS_{12} = \left(\frac{\alpha}{\beta}\right)\left(\frac{X_2}{X_1}\right)$$

(CD.7)

Substituting into equation (CD.6) it is derived:

$$\sigma = \frac{\left(\frac{\alpha}{\beta}\right)\left(\frac{X_2}{X_1}\right)}{\left(\frac{X_2}{X_1}\right)} \cdot \frac{d\left(\frac{X_2}{X_1}\right)}{\left(\frac{\alpha}{\beta}\right)d\left(\frac{X_2}{X_1}\right)} = 1$$

(CD.8)

Hence, for the Cobb-Douglas function, the elasticity of substitution is always equal to one. Thus, if the MRS$_{12}$ between input 1 and 2 changes by 1%, then the input usage ratio changes by 1% also. In equilibrium, the marginal rate of substitution (slope of the isoquant) is equal to the ratio of input prices (slope of the iso-cost line), so that expression (CD.6) can be rewritten as:

$$\sigma = \frac{(R_1 / R_2)}{(X_2 / X_1)} \times \frac{d(X_2 / X_1)}{d(R_1 / R_2)} = 1$$

(CD.9)

Thus, a 10% increase in the factor (commodity) price ratio (R_1/R_2), leads to a 10% increase in factor (commodity) intensity (X_2/X_1). This implies that in CD functions, the cost (expenditure) shares are fixed.

It is also possible to measure the change in long run output (Q) with changes in scale (i.e. equiproportional change in *all* inputs) by defining the elasticity of scale 'ε' which is the sum of the elasticity of output with respect to proportional changes in all inputs in the function. Then for a two input production function:

$$\varepsilon = \frac{\partial Q}{\partial X_1}\frac{X_1}{Q} + \frac{\partial Q}{\partial X_2}\frac{X_2}{Q} \tag{CD.10}$$

$$\varepsilon = \alpha A X_1^{\alpha-1} X_2^{\beta} X_1 Q^{-1} + \beta A X_1^{\alpha} X_2^{\beta-1} X_2 Q^{-1} \tag{CD.11}$$

$$Q = A X_1^{\alpha} X_2^{\beta}$$
$$\varepsilon = \alpha Q X_1^{-1} X_1 Q^{-1} + \beta Q X_2^{-1} X_2 Q^{-1} = \alpha + \beta \tag{CD.12}$$

According to whether the elasticity of scale for the CD production function is greater than, less than or equal to 1, this implies increasing, decreasing and constant returns to scale as summarised below.

$\alpha + \beta < 1$ − decreasing returns to scale

$\alpha + \beta = 1$ − constant returns to scale (CRS)

$\alpha + \beta > 1$ − increasing returns to scale

As α and β are constant, the elasticity of scale for the CD function is also a constant, so it is invariant to changes in the level of output.

Standard CGE applications employ perfectly competitive structures and also constant returns to scale (CRS) is assumed, implying that long run average cost (TC/Q) is equal to long run marginal cost ($\partial TC/\partial Q$). Given the assumption of long run zero profits, output price equals average unit cost, as well as long run marginal cost (due to CRS), which is a key characteristic of perfectly competitive market structures (Koutsoyiannis, 1979).

When a production function does exhibit constant returns to scale then it is said to be 'linearly homogeneous'. This relationship between homogeneity and returns to scale can be proven mathematically. Considering Q_0 as an initial period, multiplying each of the inputs by a scalar 'c' gives output in period Q_1:

$$Q_1 = A(cX_1)^\alpha (cX_2)^\beta$$
$$Q_1 = c^{\alpha+\beta} Q_0 \qquad (CD.13)$$

where the new output level Q_1 can be expressed as a function of c (to a power $\alpha + \beta$) multiplied by the initial output, Q_0. The power of c is the *degree* of homogeneity of the function where linear homogeneity *in inputs* is established by restricting $\alpha + \beta$ equal to 1.

To derive Hicksian demands, minimise cost subject to the Cobb-Douglas function to give the first order conditions:

$$\frac{\partial L}{\partial X_1} = R_1 - \Lambda \alpha A X_1^{\alpha-1} X_2^\beta = 0 \qquad (CD.14)$$

$$\frac{\partial L}{\partial X_2} = R_2 - \Lambda \beta A X_1^\alpha X_2^{\beta-1} = 0 \qquad (CD.15)$$

$$\frac{\partial L}{\partial \Lambda} = Q - A X_1^\alpha X_2^\beta \qquad (CD.16)$$

where R_i (i=1,2) are input prices, and Λ is the Lagrangian multiplier. Divide (CD.14) by (CD.15), rearrange in terms of X_2 (X_1), and substitute into (CD.16). Rearranging the resulting expression in terms of X_1 (X_2) gives CD Hicksian demands:

$$X_1 = \left(\frac{Q}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{R_2}{R_1}\right)^{\frac{\beta}{\alpha+\beta}} \qquad (CD.17)$$

$$X_2 = \left(\frac{Q}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{R_1}{R_2}\right)^{\frac{\alpha}{\alpha+\beta}} \qquad (CD.18)$$

Note that in consumer theory, there is no income effect in compensated demand functions[2]. Also uniform increases in *all* input prices by x% has no effect on the level of demand (i.e. no money illusion)[3]. This can be shown that the underlying demands of a linearly homogeneous function are zero degree homogeneous in prices. Thus, for (CD.17) increasing the input prices by a scalar 'c' and factorising for 'c' gives the expression:

$$X_1^{t+1} = c^{\left(\frac{\beta}{\alpha+\beta}\right)-\left(\frac{\beta}{\alpha+\beta}\right)} X_1 \Rightarrow c^0 X_1 \qquad (CD.19)$$

Given the assumption of zero profits:

$$PQ = R_1 X_1 + R_2 X_2 \qquad (CD.20)$$

from the equation (CD.20) it is possible to derive the composite output price, P. Substituting Hicksian demands (CD.17) and (CD.18) into (CD.20), simplifying and factorising for prices R_i gives:

$$P = Q^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} A^{-\frac{1}{\alpha+\beta}} \left\{ \left[\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \right] R_1^{\frac{\alpha}{\alpha+\beta}} R_2^{\frac{\beta}{\alpha+\beta}} \right\} \qquad (CD.21)$$

Assuming CRS (i.e. $\alpha + \beta = 1$), the composite output price, P, is linear homogeneous in R_i and zero degree homogeneous in output Q.

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[2] Hicksian final demands are a function of utility and prices only.
[3] Homogeneity proofs can also be shown in the case of other 'convenient' functions (i.e. CES, CET), but this is not done in the text.

The restrictions on the demand elasticities can also be examined: As an example, the input elasticity of demand for input 1 is:

$$\frac{\partial X_1}{\partial R_1} \cdot \frac{R_1}{X_1} = -\left(\frac{\beta}{\alpha + \beta}\right)\left(\frac{Q}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}\left(\frac{R_2}{R_1}\right)^{\left(\frac{\beta}{\alpha+\beta}\right)} X_1^{-1} \qquad \text{(CD.22)}$$

cancelling terms gives:

$$\frac{\partial X_1}{\partial R_1} \cdot \frac{R_1}{X_1} = -\left(\frac{\beta}{\alpha + \beta}\right) = -(1 - \alpha) \qquad \text{(CD.23)}$$

provided $\alpha + \beta = 1$. In a similar manner, the Hicksian compensated own-price elasticity for input 2 is:

$$\frac{\partial X_2}{\partial R_2} \cdot \frac{R_2}{X_2} = -\left(\frac{\alpha}{\alpha + \beta}\right) = -(1 - \beta) \qquad (\alpha + \beta = 1) \qquad \text{(CD.24)}$$

Compensated cross-price elasticities are given as:

$$\frac{\partial X_1}{\partial R_2} \cdot \frac{R_2}{X_1} = \left(\frac{\beta}{\alpha + \beta}\right) = (1 - \alpha) \qquad (\alpha + \beta = 1) \qquad \text{(CD.25)}$$

$$\frac{\partial X_2}{\partial R_1} \cdot \frac{R_1}{X_2} = \left(\frac{\alpha}{\alpha + \beta}\right) = (1 - \beta) \qquad (\alpha + \beta = 1) \qquad \text{(CD.26)}$$

A similar experiment can be conducted for the Marshallian CD demand functions. Thus,

$$\frac{\partial L}{\partial X_1} = \alpha X_1^{\alpha-1} X_2^{\beta} - \Lambda R_1 = 0 \qquad \text{(CD.27)}$$

$$\frac{\partial L}{\partial X_2} = \beta X_1^{\alpha} X_2^{\beta-1} - \Lambda R_2 = 0 \qquad \text{(CD.28)}$$

$$\frac{\partial L}{\partial \Lambda} = Y - R_1 X_1 - R_2 X_2 \qquad \text{(CD.29)}$$

where 'Y' is consumer (household) income. Dividing (CD.27) by (CD.28), rearranging in terms of X_1 and substitute into (CD.29). Rearranging the resulting expression in terms of X_2 gives:

$$X_2 = \frac{P_1}{P_2} \frac{\beta}{\alpha} X_1 \qquad \text{(CD.30)}$$

Substituting (CD.30) into (CD.29) and rearranging in terms of X_1 gives:

$$X_1 = \frac{Y}{P_1} \frac{1}{((\beta/\alpha)+1)} \qquad \text{(CD.31)}$$

Substituting β with $(1-\alpha)$, where $\alpha + \beta = 1$, and simplifying gives the Marshallian Cobb-Douglas household demand function for final commodity 1:

$$X_1 = \frac{Y}{P_1} \alpha \qquad \text{(CD.32)}$$

Using a similar procedure, it is possible to derive the household demand function for commodity 2 as:

$$X_2 = \frac{Y}{P_2} \beta \qquad \text{(CD.33)}$$

With a simple procedure own-price, cross-price and income elasticities can be derived to show that they are -1, 0 and 1 respectively. The income elasticity of demand is restricted to one, which is highly restrictive in light of empirical evidence showing food products to have income elasticities

considerably less than one. Finally, it is obvious from (CD.32) and (CD.33) that the underlying Marshallian demands are zero homogeneous in prices and income.

2.2.2. Constant Elasticity of Substitution (CES) Function

The equation for the CES production function is:

$$Q = A\big[\delta_1 X_1^{-\rho} + (1-\delta_1)X_2^{-\rho}\big]^{-\frac{v}{\rho}} \qquad \text{(CES.1)}$$

where A is an efficiency parameter, δ_1 is a distribution parameter, ρ is an elasticity parameter and v is a scale parameter (discussed further below). First order *partial* derivatives give short run marginal products (assuming v=1 – the significance of v is discussed below):

$$\frac{\partial Q}{\partial X_1} = MP_1 = A\big[\delta_1 X_1^{-\rho} + (1-\delta_1)X_2^{-\rho}\big]^{\left(\frac{v+\rho}{\rho}\right)}.\delta_1 X_1^{-(1+\rho)} \qquad \text{(CES.2)}$$

after simplifying the expression (CES.2):

$$MP_1 = A^{-\rho}Q^{1+\rho}\delta_1 X_1^{-(1+\rho)} \qquad \text{(CES.3)}$$

and similarly for input 2:

$$MP_2 = A^{-\rho}Q^{1+\rho}(1-\delta_1)X_2^{-(1+\rho)} \qquad \text{(CES.4)}$$

Therefore, marginal product is unambiguously positive with positive inputs, outputs, scale and distribution parameters. The average product (Q/X_i) can be related to the marginal product via expressions (CES.3) and (CES.4) as:

$$MP_1 = A^{-\rho}Q^{\rho}\delta_1 X_1^{-\rho}AP_1 \qquad \text{(CES.5)}$$

$$MP_2 = A^{-\rho}Q^{\rho}(1-\delta_1)X_2^{-\rho}AP_2 \tag{CES.6}$$

Like the Cobb-Douglas function, strict concavity in CES functions implies stage II of production either with respect to each factor (short-run) or with respect to scale (long-run). Similarly, strict quasi-concavity in CES functions implies stages I or II only with respect to each factor and scale (Beattie and Taylor, 1985, pp68-69).

The CES marginal rate of substitution between inputs 1 and 2 is given as:

$$MRS_{12} = \left[\frac{\delta_1}{(1-\delta_1)}\right]\left[\frac{X_2}{X_1}\right]^{1+\rho} \tag{CES.7}$$

To derive the elasticity of substitution, it is necessary to differentiate MRS_{12} with respect to input ratio:

$$\frac{dMRS_{12}}{d\left(X_2/X_1\right)} = (1+\rho)\left[\frac{\delta_1}{(1-\delta_1)}\right]\left[\frac{X_2}{X_1}\right]^{\rho} \tag{CES.8}$$

Furthermore, since:

$$\frac{MRS_{12}}{\left(X_2/X_1\right)} = \left[\frac{\delta_1}{(1-\delta_1)}\right]\left[\frac{X_2}{X_1}\right]^{\rho} \tag{CES.9}$$

substitute (CES.9) into (CES.8) to give:

$$\frac{dMRS_{12}}{d\left(X_2/X_1\right)} = (1+\rho)\frac{MRS_{12}}{\left(X_2/X_1\right)} \tag{CES.10}$$

Given the formula for the elasticity of substitution in (CD.6), (CES.10) can be rearranged to give the CES elasticity of substitution as:

$$\sigma = \frac{1}{1+\rho} \qquad \text{(CES.11)}$$

Therefore, the elasticity of substitution of the CES function is constant and depends on the elasticity parameter, ρ, which is constrained to be greater than -1. Thus, with CES production functions the elasticity of substitution can vary, with the consequence that changes in the factor (commodity) price ratio can lead to factor-intensity reversals and which gives more flexibility to CES functions than CD functions (Dinwiddly and Teal, 1988).

As it has been demonstrated with the CD function, it is also possible to measure the change in long run output (Q) with changes in scale. Referring to the two factor CES function (CES.1), assume that $X_2 = \vartheta X_1$ such that the ratio X_2/X_1 is constant with increases in scale. Thus, the CES production function may be rewritten as:

$$Q = X_1^v A \left[\delta_1 + (1-\delta_1)\vartheta^{-\rho} \right]^{-\frac{v}{\rho}} \qquad \text{(CES.12)}$$

where the elasticity of scale with respect to proportional changes in inputs is given as:

$$\frac{dQ}{dX_1} \frac{X_1}{Q} = v \qquad \text{(CES.13)}$$

As it is shown the elasticity of scale is a function of the scale parameter 'v'. Then, according to whether v is greater than, less than or equal to 1, implies increasing, decreasing and constant returns to scale respectively. From the discussion of CD returns to scale above, CES functions are restricted to CRS, so v=1[4]. Given the relationship between homogeneity and returns to scale (see (CD.12)), CES

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[4] In subsequent sections, the value of 'v' in CES (and CET) will be assumed the value of 1.

production functions are homogeneous of degree 'v' in inputs. In addition, it can be proven (this is not done here) that the composite output price function is also homogeneous of degree 'v' in input prices, and compensated demands are homogeneous of degree v-1 in prices. [5]

Minimising cost subject to the CES function gives first order conditions:

$$\frac{\partial Z}{\partial X_1} = R_1 - \left[-\frac{1}{\rho} \Lambda A [\delta_1 X_1^{-\rho} + (1-\delta_1) X_2^{-\rho}]^{-\frac{1}{\rho}-1} - \rho \delta_1 X_1^{-(1+\rho)} \right] = 0 \qquad \text{(CES.14)}$$

$$\frac{\partial Z}{\partial X_2} = R_2 - \left[-\frac{1}{\rho} \Lambda A [\delta_1 X_1^{-\rho} + (1-\delta_1) X_2^{-\rho}]^{-\frac{1}{\rho}-1} - \rho (1-\delta_1) X_2^{-(1+\rho)} \right] = 0 \quad \text{(CES.15)}$$

$$\frac{\partial Z}{\partial \lambda} = Q - A [\delta_1 X_1^{-\rho} + (1-\delta_1) X_2^{-\rho}]^{-\frac{1}{\rho}} = 0 \qquad \text{(CES.16)}$$

Dividing (CES.14) by (CES.15) and rearranging in terms of X_1 gives:

$$X_1 = \left[\frac{R_1 (1-\delta_1)}{R_2 \delta_1} \right]^{-\frac{1}{1+\rho}} X_2 \qquad \text{(CES.17)}$$

Substituting (CES.17) into (CES.16) and simplifying gives:

$$Q = A X_2 \left[\delta_1 \left[\frac{R_1 (1-\delta_1)}{R_2 \delta_2} \right]^{\sigma \rho} + (1-\delta_1) \right]^{-\frac{1}{\rho}} \qquad \text{(CES.18)}$$

where σ is defined in equation (CES.11). Rearranging the equation (CES.18) in terms of X_2 gives the CES Hicksian demand function for input (commodity) 2:

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[5] Marshallian (uncompensated) commodity demands are homogeneous of degree v-1 in prices and income.

$$X_2 = \frac{Q}{A}\left[\delta_1\left[\frac{R_1(1-\delta_1)}{R_2\delta_1}\right]^{\sigma\rho} + (1-\delta_1)\right]^{\frac{1}{\rho}} \tag{CES.19}$$

With a similar procedure, it is possible to derive the CES Hicksian demand function for input (commodity) 1:

$$X_1 = \frac{Q}{A}\left[\delta_1 + (1-\delta_1)\left[\frac{R_2\delta_1}{R_1(1-\delta_1)}\right]^{\sigma\rho}\right]^{\frac{1}{\rho}} \tag{CES.20}$$

Differentiating (CES.19) with respect to R_2:

$$\frac{\partial X_2}{\partial R_2} = \frac{1}{\rho}\frac{Q}{A}\left[\delta_1\left[\frac{R_1(1-\delta_1)}{R_2\delta_1}\right]^{\sigma\rho} + (1-\delta_1)\right]^{\frac{1}{\rho}-1} \cdot -\sigma\rho\delta_1\left[\frac{R_1(1-\delta_1)}{\delta_1}\right]^{\sigma\rho} R_2^{-\sigma\rho-1} \tag{CES.21}$$

Multiplying (CES.21) by R_2/X_2 and substituting X_2, Q and A gives the Hicksian (compensated) own-price elasticity of demand for input 2:

$$\frac{\partial X_2}{\partial R_2}\cdot\frac{R_2}{X_2} = -\sigma\left[\frac{X_2 A}{Q}\right]^{-\rho}\delta_1\left[\frac{R_1(1-\delta_1)}{R_2\delta_1}\right]^{\sigma\rho} \tag{CES.22}$$

Similarly for input 1:

$$\frac{\partial X_1}{\partial R_1}\cdot\frac{R_1}{X_1} = -\sigma\left[\frac{X_1 A}{Q}\right]^{-\rho}(1-\delta_1)\left[\frac{R_2\delta_1}{R_1(1-\delta_1)}\right]^{\sigma\rho} \tag{CES.23}$$

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Compensated cross price elasticities of demand are given as:

$$\frac{\partial X_2}{\partial R_1} \cdot \frac{R_1}{X_2} = \sigma \left[\frac{X_2 A}{Q} \right]^{-\rho} \delta_1 \left[\frac{R_1(1-\delta_1)}{R_2 \delta_1} \right]^{\sigma\rho} \qquad \text{(CES.24)}$$

$$\frac{\partial X_1}{\partial R_2} \cdot \frac{R_2}{X_1} = \sigma \left[\frac{X_1 A}{Q} \right]^{-\rho} (1-\delta_1) \left[\frac{R_2 \delta_1}{R_1(1-\delta_1)} \right]^{\sigma\rho} \qquad \text{(CES.25)}$$

2.2.3. Constant Elasticity of Transformation (CET) Function

The algebraic representation of the CET function is:

$$Z = B \left[\sum_{i=1}^{n} \gamma_i Q_i^{-\rho} \right]^{-\frac{1}{\rho}} \qquad \text{(CET.1)}$$

where B and γ's are positive parameters with $\sum_{i=1}^{n} \gamma_i = 1$ and ρ is a transformation elasticity. Z is a measure of the firm's overall capacity which depends on the quantities of inputs and Q_i is a measure of the output level of each supply activity 'i'. The CET function is identical to the CES function apart from the restriction on ρ. In CES ρ is greater than or equal to -1 where in CET ρ is less than or equal to -1. Therefore, CES is convex with respect to the origin and CET is concave with respect to the origin. As with CES, the CET function is also linearly homogeneous, where a doubling of output from each supply activity (Q) doubles the firms overall capacity (Z).

The derivation of activity supplies is a revenue maximisation process subject to a production possibilities frontier. The mathematical derivations are analogous to the CES function, in which the elasticity of transformation between supply activities is equivalent to the elasticity of substitution in inputs.

3. MODEL REPRESENTATION AND SOLUTION METHODS

In section 2 above, some of the most common functional forms in CGE models are examined. In this section an examination is made of how these functions may be represented within a CGE framework and how solutions may be gained from a large system of equations.

The exposition begins with a discussion of equation 'linearisation' – this format will be the basis of the model application later in this dissertation. Multistage optimisation is then discussed using a technique known as 'nesting'. This process improves the flexibility of CGE models without rendering the model unmanageable. In the latter part of this section various linearised solution algorithms are examined whilst their accuracy vis-á-vis the levels algorithms is briefly commented. The section concludes with a brief discussion of calibration and its reduced role in the case of a linearised model representation.

3.1. Linearisation

This section shows how to derive a linear representation of a levels[6] function. A more complex linearisation example can be found in section 3.4 which provides the derivations of a nested linearised stylised model.

Linearised representation besides being able to get accurate results, has some advantages too. First of all, it offers a more straightforward representation of behavioural relationships and a model which is much simpler to interpret because of being able to obtain results in percentage change forms or in elasticities. (Hertel et al.1992)

---

[6] Reference is made to "levels" forms of equations which were presented in section 2 of the discussion of functional forms

With multivariate functions, the total differential calculates the change in the dependent variable dz at a point brought about by an infinitesimal change in each of the independent variables denoted as dx and dy. Thus, if a multivariate function is given as:

$$z = z(x, y) \qquad\qquad\qquad\qquad \text{(LIN.1)}$$

then the total differential is:

$$dz = \left(\frac{\partial z}{\partial x}\right)dx + \left(\frac{\partial z}{\partial y}\right)dy \qquad\qquad\qquad\qquad \text{(LIN.2)}$$

Equation (LIN.2) measures the change of z with respect to infinitesimal changes in x and y.

More specifically, there are three rules of differentials for obtaining the total differential of each of the equations in levels form. These rules are:

The product rule	$R = PQ \Rightarrow r = p + q$	
The power rule	$R = P^{\alpha} \Rightarrow r = \alpha p$	(LIN.3)
The sum rule	$R = P + Q \Rightarrow r = pS_p + qS_q$	

where r, p and q are percentage changes (or they may be interpreted as changes in logarithms) in R, P and Q, α and β are parameters and Sp and Sq are the shares of P and Q in P+Q.

Using the product rule above, the linearisation error which occurs when Johansen single-step solution (explained in subsection 3.3) method is used can be shown. For example, if the levels variables P and Q are originally valued at 10 and 5, their product is 50. Changing both variables by +10%, gives P and Q values of 11 and 5.5 respectively which is a product increase of 21%, compared to the rule result of 20%. This result occurs because the total differential only looks at infinitesimal changes along

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the curve. But as explained in section 3 below, this kind of linearisation error can be reduced by using different solution methods like Euler's and Gragg's method.

Finally, to illustrate how the total differential can be done, as an example the total differential of the Marshallian Cobb-Douglas demand (CD.32) in the stylised model is given as:

$$dX_i = dY(P_i^{-1}\alpha) - dP(YP_i^{-2}\alpha) \qquad \text{(LIN.4)}$$

To convert from differential changes to linearised percentage changes it is multiplied and divided by respective variables and simplified, which gives:

$$\frac{dX_1}{X_1}X_1 = \frac{dY}{Y}YP_1^{-1}\alpha - \frac{dP_1}{P_1}YP_1^{-1}\alpha \qquad \text{(LIN.5)}$$

Multiplying both sides by 100, dividing by X_i and simplifying gives:

$$x_1 = y - p_1 \qquad \text{(LIN.6)}$$

$$x_1 = \frac{dX_1}{X_1}100 \quad y = \frac{dY}{Y}100 \quad p_1 = \frac{dP_1}{P_1}100 \qquad \text{(LIN.7)}$$

Equations (LIN.6) and (LIN.7) are in percentage form, where the lower case letters are the percentage changes in their respective upper case variables. Because of using the Cobb-Douglas functional form, the linearised Marshallian CD function has an income elasticity of one, and own- and cross-price elasticities of minus one and zero respectively.

3.2 Nesting

As mentioned in the subsection 3.4 when flexible functional forms are chosen, it is necessary to calibrate a larger number of parameters. Therefore, these kinds of requirements lead modellers to use more convenient functional forms like Cobb-Douglas (CD) and Constant Elasticity of Substitution (CES). Using convenient functional forms greatly reduces the number of parameters which limits the degree of flexibility when specifying producer or consumer behaviour within the model. Thus, to build more complicated or realistic models using these simple functions a technique which is known as nesting (or hierarchical functions) is used.

The basic idea behind nesting is very simple; instead of letting goods or inputs enter the utility or production function directly, one uses functions of a subset (or different groups, aggregations) of the original goods or inputs (Petersen, 1997). This assumption of separability allows for subsequent optimisation at several stages, which leads a greater number of elasticity parameters in each stage of the production/utility function. The result is that this increases the flexibility of the model, without increasing the burden of calibration.

3.2.1. Separability and Aggregation

The relationship between separability and multi-stage optimisation can be illustrated with the following theoretical example. Assuming a three factor (x_i i = 1,2,3) production function which is of the form:

$$Y = f(X, x_3) \tag{N.1}$$

where input X is represented as an aggregator function consisting of inputs x_1 and x_2:

$$X = g(x_1, x_2) \tag{N.2}$$

A schematic representation of this two-level nested structure is shown in Figure 5.
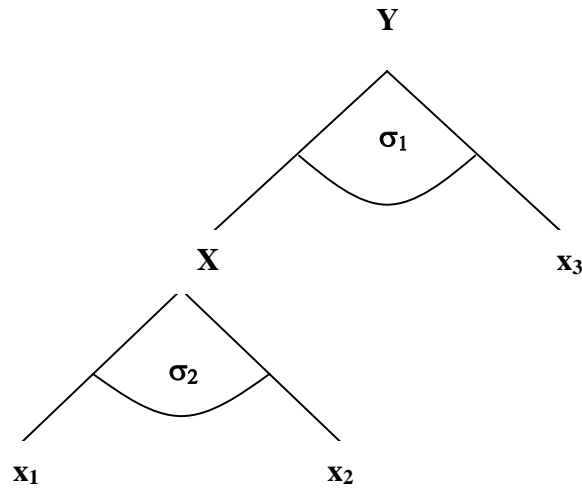
Figure 4 A two level nested production structure

Employing cost minimisation techniques, the producer decides how inputs x_1 and x_2 should be combined into a single 'composite' input X, using the function (N.2). Secondly, the producer decides how this composite input should be combined with input x_3 by using function (N.1). This is called a two stage optimisation procedure.

There are two conditions to meet in order to use a two-stage optimisation procedure (Koschel, 2001). The first condition is that the production function must be weakly separable to permit a partitioning of the inputs. The production function (N.1) is said to be weakly separable if the marginal rate of substitution (MRS) between any two inputs x_i and x_j (in the example x_1 and x_2) from any subset (nest) N_s where s = 1,…,r is independent of the input usage outside of N_s, i.e (Berndt and Christensen, 1973). This can be shown with the following expression:

$$\frac{\partial}{\partial x_3}\left(\frac{\partial X / \partial x_1}{\partial X / \partial x_2}\right) = 0 \tag{N.3}$$

The convenient functions like CD and CES show weak separability. For example, in the case of a two-level nested CD production function:

$$Y = AX_1^\alpha X_2^\beta \qquad and \qquad X_1 = Ax_{11}^\gamma x_{21}^\delta \qquad\qquad \text{(N.4)}$$

The $MRS_{11,21}$ can be shown to be:

$$MRTS_{11,21} = \frac{MP_{11}}{MP_{21}} = \frac{\gamma}{\delta}\frac{x_{21}}{x_{11}} \qquad\qquad \text{(N.5)}$$

Obviously, changes in the level of X_2 in the upper CD nest, has no effect on the MRS between inputs x_{11} and x_{21} in the lower nest. Mathematically:

$$\frac{\partial}{\partial X_2}\left(\frac{\gamma}{\delta}\frac{x_{21}}{x_{11}}\right) = 0 \qquad\qquad \text{(N.6)}$$

The second condition is that aggregator function (N.2) must be linear homogenous with respect to each of inputs. In section 2.2.1, it was demonstrated that the output price composite of a linearly homogeneous function is linearly homogeneous in input prices. Hence, the sum of the prices and quantities of the inputs derived in each nest will be equal to the aggregate quantity and price indices:

$$RX = \sum_{i=1}^{n} r_i x_i \qquad\qquad \text{(N.7)}$$

As shown in the subsection 2.2.1 a basic property of linear homogenous functions is that first order derivatives (i.e. marginal products/utilities) are homogeneous of degree zero. Taking the case of a linearly homogenous Cobb-Douglas production function this property can be shown. Since MP_1 for a two input production function is given as:

$$MP_1 = \frac{\partial Y}{\partial x_1} = \alpha A x_1^{\alpha-1} x_2^\beta \qquad\qquad \text{(N.8)}$$

Multiplying each of the inputs by a scalar, λ, gives:

$$MP_1 = \frac{\partial Y}{\partial x_1} = \alpha A (\lambda x_1)^{\alpha-1} (\lambda x_2)^{\beta}$$

$$MP_1 = \frac{\partial Y}{\partial x_1} = \lambda^{\alpha-1+\beta} \alpha A x_1^{\alpha-1} x_2^{\beta} \qquad\qquad\qquad (N.9)$$

$$MP_1 = \frac{\partial Y}{\partial x_1} = \lambda^0 [\alpha A x_1^{\alpha-1} x_2^{\beta}]$$

Thus, multiplying both inputs by λ, does not affect the marginal product of x_1. In other words the marginal products are zero degree homogenous in inputs. Since the MRS is the ratio of MPs, then proportional increases in both inputs by the scalar value λ implies higher isoquant levels have no effect on the MRS. Therefore, a ray from the origin must cut all isoquants or indifference curves at points of equal slope. According to Green (1971), the isoquants or indifference curves are therefore 'homothetic with respect to the origin' (pp141). In other words, increases in the level of composite (aggregate) output or utility with relative input price ratios fixed has no effect on factor intensity. Thus, increase in the level of composite output means movements onto higher isoquants or indifference curves.

Thus, if the function (N.1) satisfies both weak separability and linear homogeneity, then this function is said to be *weakly homothetically separable* and ensures consistent aggregation. Since the assumption of weak separability assure that the increase in other inputs outside the aggregator nest has no effect on factor usage ratios, and of linear homogeneity assures that increases in the level of composite (aggregate) output or utility with relative input price ratios fixed has no effect on factor intensity, then a weakly homothetically separable function implies that changes in input intensities can only be a function of the relative prices of various types of inputs in the nest. Weak homothetic separability gives the possibility of a two-stage optimisation procedure which implies that the mix of inputs within each aggregate is optimised in the first step, and then the level of each aggregate in the second step (Koschel, 2001).

Finally, Allanson (1989) also notes that relative price changes in one nest can have *indirect* effects on input (commodity) allocations elsewhere in the nest. Using the nested structure in Figure 4, if the price of input x_2 increases, this will affect the optimal combination of x_1 and x_2 in the aggregate nest, but due to the separability restriction, it will not directly affect the optimal use of x_3. But there will be an *indirect* effect on the use of x_3 due to a rise in the composite price of *aggregate* input X. This implies that the firm will substitute x_3 for aggregate X, in the top nest. Moreover, if x_3 was an aggregate input, then as a consequence of linear homogeneity, its increased use would be translated proportionally to all inputs in that nest. An example of the usage of nesting in CGE models can be found in section 4.

3.3. Solution Methods For Linearised Representations

For many years modellers preferred using a levels non-linear algorithm to the linearised algorithm method first employed by Johansen (1960), due to the greater accuracy of the former. However, as Hertel et al. (1992) note and demonstrate, there are ways to improve the accuracy of the results obtained by the early Johansen approach such that the choice between levels and linear solution algorithms is no longer an issue.

To explain the basis of a linear algorithm, consider a simple function, $g(X,Y)$, where X is exogenous and Y is endogenous (see Figure 5). Supposing that the initial (or benchmark) solution of the model is point (X,Y), an exogenous shock from X to X_1 gives a true solution of Y to Y_1 (or A to B). Johansen's method involves calculating the derivative (dY/dX) at A, and then in a "single step" passing from X to X_1, by moving along the tangent to the function at A. This brings us to the point B_1 in Figure 5 which gives the estimate Y_2. This represents the linearised approximation to the non-linear solution at B. Therefore with the Johansen method, the bigger the shock on X the poorer is the quality of the estimation as the tangent gets further from the 'true' solution.

To reduce this "single step" linearisation error, or to obtain more accurate solutions different solution methods can be used like Euler's method, Gragg's method or midpoint method. The main idea behind these methods is to follow the function $g(X,Y)$ more closely. To do this, Euler's method divides

the shock (X, X_1) into a number of equal steps. A 2-step Euler solution is shown in Figure 5 where the path is from A to C to B^* which gives the solution Y^*. At point C, an update procedure of the endogenous variable Y occurs to verify the position of this variable. Therefore with this method the solution obtained by using higher step solution procedure can be much closer to the true solution Y_1.
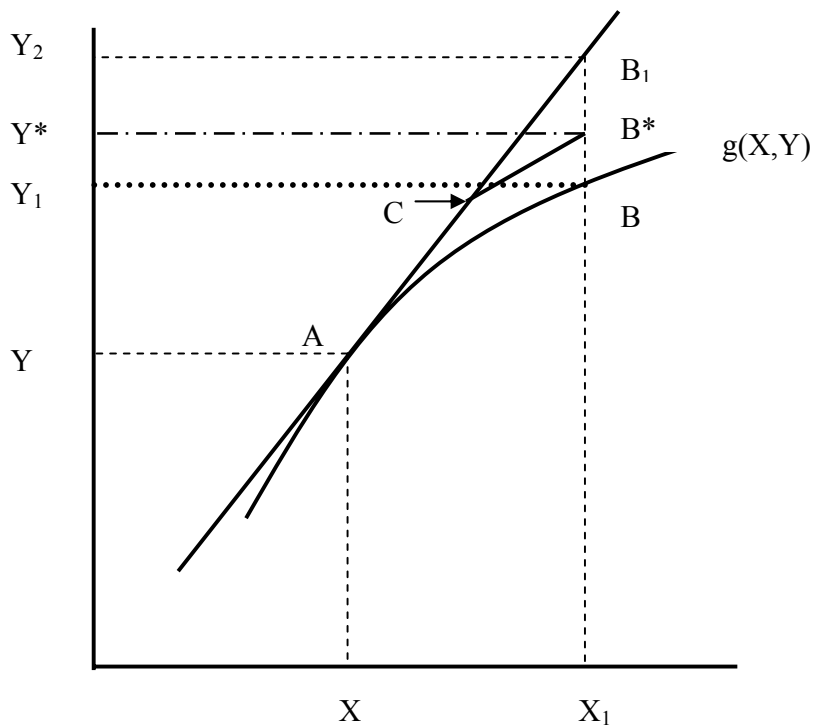
Figure 5 Illustration of Euler's method (Hertel et al., 1992)

Gragg and midpoint methods are identical to Euler's method in step one, following a tangent along the curve from the initial solution. The difference is that Euler's method follows this tangent from the current point (point C in Figure 5) while Gragg's method and the midpoint method follow this direction but from the n-1th point in this simple two step example, point A. This difference can be shown in Figure 6 with a little modification on Figure 5. Each method comes to point C after step-1. At step-2 as shown before Euler's method follows the line CB_1 which gives the solution Y_2, while Gragg and midpoint methods follow line AB_2 which gives the solution Y_3. It can be seen that the solution obtained by using Gragg and midpoint methods is much closer to the true solution Y_1.
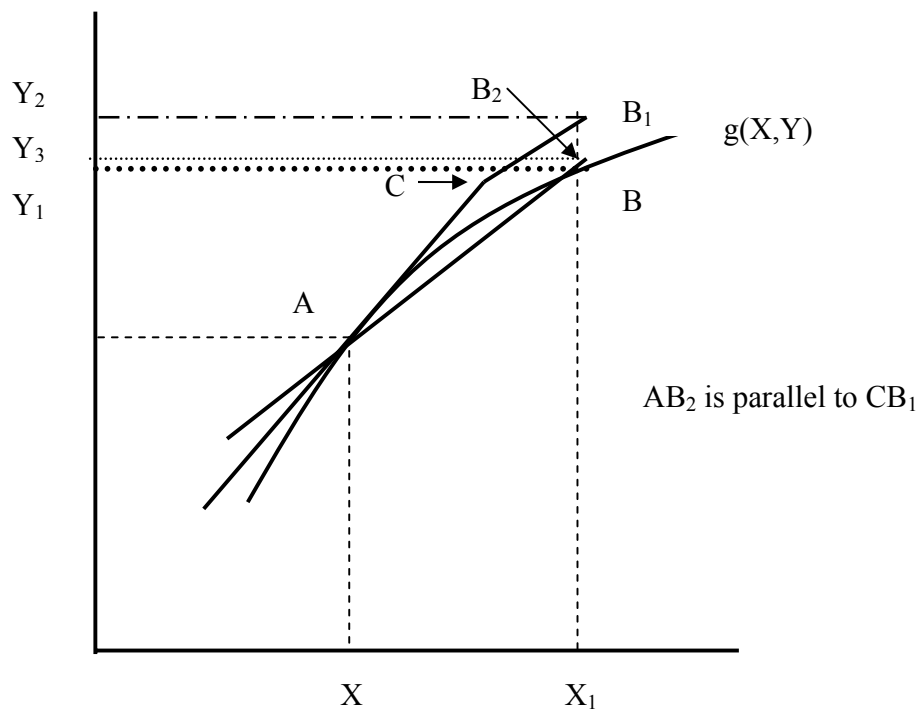
Figure 6 Midpoint and Gragg's method compared to Euler's method

Whilst increasing steps will improve the accuracy, a large number of steps would have a significant computational expense. Another way of increasing accuracy is to make 2 or more different multi-step simulations (e.g. 5-step, 10 step and 20 step) with different numbers of steps and then to calculate the solution as an suitable weighted average of these employing a polynominal function of the necessary degree. This process is known as extrapolation[7].

3.4. Calibration of CGE Models

Greenaway et al. (1993) defines calibration as a procedure which calculates values for unknown parameters of the functional forms used in an applied CGE model from an observed data set. It is assumed that the data set represents an equilibrium for the general equilibrium model under

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[7] More information about the issues discussed in this subsection can be found in appendix I.

consideration (benchmark equilibrium data set). The model is then solved for its unknown parameters as functions of the observed data.

In the linearised model representation, the final model CGE representation used in this study, a large proportion of the benchmark parameters drop out of the equations subordinating the importance of the calibration technique. Indeed, the price and quality variables now represent percentage changes and are therefore not dependent on initial or benchmark values. Notwithstanding, extraneous elasticity of substitution values are still required in the CES functions which has important implications for the responsiveness of the model to a given policy shock[8].

Depending on the choice of functional form, a unique solution of parameters may not be obtained. For example, unless a Cobb-Douglas functional form is chosen, then parameterised values, such as elasticities must be obtained, (usually borrowed from the literature), before the estimation of other parameters. When flexible functional forms are chosen a much greater number of exogenously determined (i.e. income elasticities, elasticities of substitution between factors or between intermediate inputs) parameters are required. Since these values affect the model results, this also implies greater subjectivity of model results.

The process of calibration can be illustrated by a simple numerical example. The derived values of each unknown parameters are based on the hypothetical input-output data which can be found in Table 1. In the table it can be observed that the general equilibrium restrictions apply, where household income and expenditure is equal. More information about the input-output table can be found in section 4, although an underlying principle is that the rows represent the sales of commodities or factors, whilst the columns are intermediate and final demands by firms and households respectively.

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[8] After reading the subsection 3.1 and appendix I it would be easer to understand why there no such calibration is needed.

	Industry		Households	Total Sales
	1	**2**		
Commodity: 1	-	-	2	2
2	-	-	6	6
Primary Factors:				
X_1	1	2	-	3
X_2	1	4	-	5
Production	2	6	8	16

Table 1 Hypothetical Input-Output Data

Using the Cobb-Douglas commodity demand derivations from the subsection 2.2.1 and rearranging these equations in terms of α and β:

$$\alpha = \frac{X_1 P_1}{Y} \tag{CAL.1}$$

$$\beta = \frac{X_2 P_2}{Y} \tag{CAL.2}$$

These parameters, α and β, can be easily calibrated from Table 1 as:

$$\alpha = \frac{2}{8} = 0.25 \qquad \beta = \frac{6}{8} = 0.75 \tag{CAL.3}$$

The calibration of the unknown parameters of the CES function is slightly more complicated. First of all, it is necessary to specify an exogenous value for the elasticity of substitution (σ_1). Assuming that this value is 2 and given that:

$$\sigma_1 = \left(\frac{1}{1+\rho_1} \right) \tag{CAL.4}$$

where ρ_1 is -0.5. The calibrations of the other unknown parameters, the distribution parameter (δ_1) and the scale parameter (A_1), are shown below.

Taking the first order conditions from a two input cost minimisation procedure and dividing, gives:

$$\frac{R_1}{R_2} = \frac{\delta_1}{(1-\delta_1)}\left[\frac{X_1}{X_2}\right]^{-(1+\rho_i)} \tag{CAL.5}$$

Thus, calibration of δ involves substituting in the value flows and the value of ρ into expression (CAL.5). Noting that the price of factors are worth one currency unit, and using the values of X_1 and X_2 for the fist the industry from Table 1 gives:

$$\frac{1}{1} = \frac{\delta_1}{(1-\delta_1)}\left[\frac{1}{1}\right]^{-0.5} \tag{CAL.6}$$

by rearranging (CAL.6) δ_1 can be found 0.5. To find A_1, substitute the value of the parameter values obtained in Table 1 into the CES production function (CES.1). Thus, for the first industry,

$$2 = A_1[0.5\times(1)^{-(-0.5)} + 0.5\times(1)^{-(-0.5)}]^{-\frac{1}{(-0.5)}} \tag{CAL.7}$$

Rearranging the equation (CAL.7) in terms of A_1 gives a value of 2.

4. A STYLISED CGE MODEL WITH LINEARISED REPRESENTATION

This section gives a simple CGE model example which consists of a system of linearised equations characterising the behavioural characteristics of an economy. The aim is to make a summary of what is mentioned in previous sections of this paper. This example offers useful information about general equilibrium theory, major concepts of developing computable general equilibrium models like closure, nesting and finally presents some of possible linearised functional forms typically used in nested CGE models.

In Table 2 the input-output table of the stylised model is given to show the uses of input-output table in CGE modelling. These tables contain detailed data on the flows among different sectors of an economy and this information can be used to build static models or can be used as a benchmark dataset for dynamic models. For the stylised model given in this text, this table serves as a benchmark whilst the software takes initial values from this benchmark to find a solution. These tables represent the value of economic transactions in a given period of time. (Dixon et al., 1992)

		Intermediate Use		Final Use			OUTPUT (TOTAL)
		Manufacturing	Services	Private	Government	Export	
Domestic Production	Manufacturing	10	5	32	15	36	98
	Services	15	9	62	8	0	94
Value added	Man. Tax	0	0	9	0	0	9
	Svces. Tax	0	0	14	0	0	14
	Capital	33	28	0	0	0	61
	Labour	40	16	0	0	0	56
Imports		0	36	0	0	0	36
INPUT (TOTAL)		98	94	117	23	36	368

Table 2 Input-Output Table for the stylised model

To simplify the illustration, the stylised model contains two consumers: private demands by households and public demands by government and two industries, manufacturing and services, each employing two primary production factors (labour and capital). The model also includes inter-industry flows by permitting intermediate good transfer between two industries. The economy is assumed to be 'open' (i.e., external trade) in which the country imports only one of the commodities and exports the other. Government levies commodity taxes to arrange its budget and expenditure. Finally, the model also assumes that labour supply is immobile between the two industries. Behavioural equations are derived from constrained optimisation techniques based on the principles of the neo-classical theory (utility maximisation and cost minimisation).

The structure of the stylised model can also be shown in Table 2 where the rows in the yellow sub-matrix show production sector outputs and columns show sectors which use outputs of production as intermediate inputs. In the orange sub-matrix private consumption, government consumption and exports are shown as final demands for the commodities. The information on total domestic production

is given in the red sub-matrix. Blue sub-matrices give information on imports. Information about payments to labour and capital and indirect taxes are given in the green sub-matrices. For a balanced input-output table the column of the grey sub-matrix should be the same as the rows of the red sub-matrix to reflect the fact that total input usage equals to total output sales for production sectors.

The input-output table which is used for this stylised model are in 'basic' prices where tax rows are (presented explicitly) disaggregated. The table does not include margin costs as is characteristic in standard in input-output (I-O) tables. This is an omission of this simple I-O table. These tables also can be expressed in terms of producer and purchaser prices where taxes and transport costs are also included.

4.1. Notation

L_j	Demand for labour in industry j
K_j	Demand for capital in industry j
XL_j	Labour supply for industry j
W_j	Wage rate in industry j
WCOMP	Composite wage rate
RCOMP	Composite capital rental
X_j	Supply in industry j
$COMPII_j$	Demand for the composite intermediate inputs in industry j
$COMPVA_j$	Demand for the value added composite inputs in industry j
IIM_j	Demand for intermediate manufacturing input in industry j
IIS_j	Demand for intermediate services input in industry j
PII_j	Composite intermediate input price
PVA_j	Composite value added price
C_j	Private demand for industry output in industry j
G_j	Government (public) demand for industry output in industry j
Y	Private consumer income
PEXP	Private consumer expenditure

T	Total government revenue from taxation
GOVT	Total government expenditures
P_j	Basic price in industry j
Q_j	Purchaser's price in industry j
PW_j	World price in industry j (exogenous)
F	Exchange rate (exogenous)
TC_j	Commodity tax on final (private) demands (exogenous)
LAB	Total labour endowment (exogenous)
CAP	Total capital endowment (exogenous)
EXPOR	Exports
IMPOR	Imports
BOT	Balance of trade
GDP	Gross domestic product

Lower case letters which are used in the text are the percentage change equivalent of the upper case 'levels' variable which are shown here.

4.2. Schematic Representation of the Model

This section offers a simple explanation about the model structure by using figures which explain demand (Figure 7) and production (Figure 8) and also immobility of labour (Figure 9) (which is assumed).
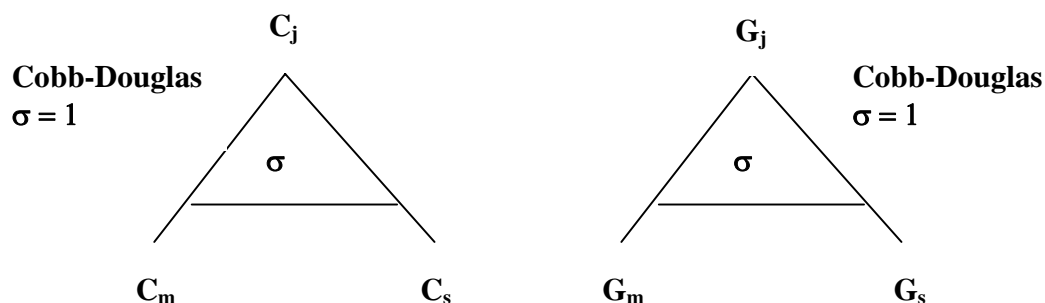
Figure 7 Schematic representation of the demand nest

X$_j$

Leontief ($\sigma=0$)

σ

COMPII$_j$

CES

σ

COMPVA$_j$

CES

σ

IIM$_j$ **IIS$_j$**   **Lj**   **K$_j$**
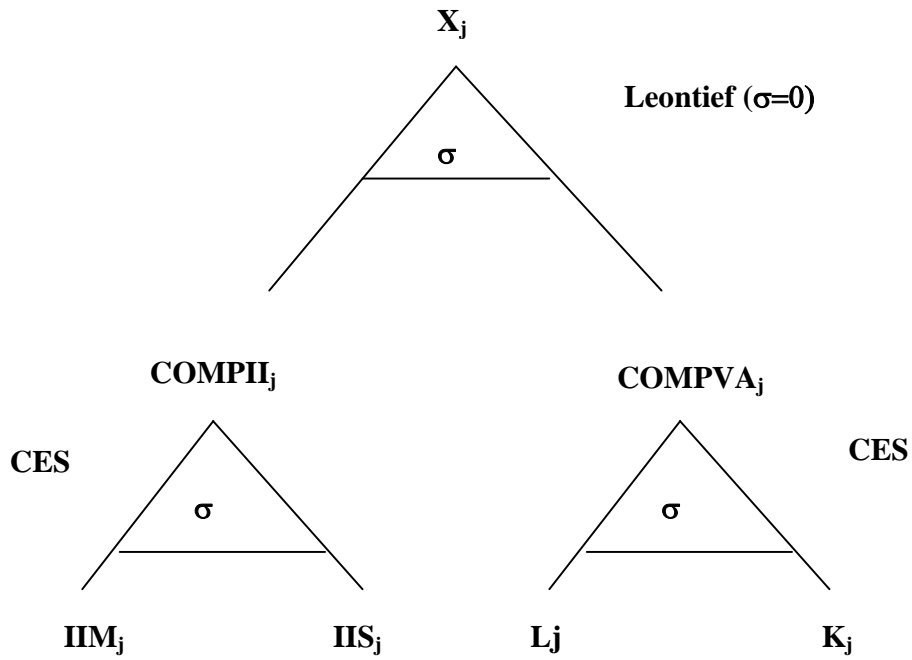
Figure 8 Schematic representation of the production nest

XL$_m$ **XL$_s$**

σ

CET

LAB

Figure 9 Schematic representation of the immobility of labour

4.3. Mathematical derivations of linearised functions

In this subsection a set of general equilibrium equations will be derived in linearised form, with the objective to demonstrate how the equations can be derived in linearised form. The mathematical techniques used in this subsection for linearisation are based on Dixon et al. (1992). In subsections

4.3.1, 4.3.2 and 4.3.4 are shown the derivations of supply, demand functions and the labour supply function respectively.

4.3.1. Production in the model

4.3.1.1. Composite Input Nest

The top nest in the production nest is defined by a single production process Leontief structure. As mentioned before earlier in the text, a Leontief production function implies that the elasticity of substitution (σ) between inputs is zero, hence levels demands for composite inputs are restricted by a fixed share coefficient. Composite intermediate and primary factor demands for industry 'j' are given in equation (CGE.1):

$$COMPII_j = \gamma_j X_j \qquad COMPVA_j = \gamma_j X_j \qquad \text{(CGE.1)}$$

where γ_j are the fixed input-output parameters. Following the same approach which is explained in section 3.1, linearised Leontief demands are:

$$compii_j = x_j \qquad compva_j = x_j \qquad \text{(CGE.2)}$$

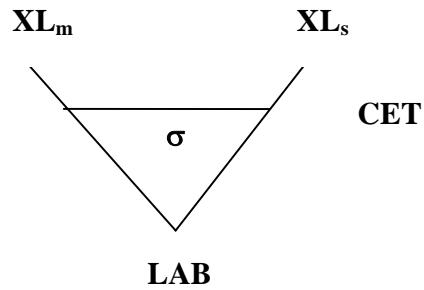There are no price effects in equation (CGE.2), which is a result of the zero value of the elasticity of substitution. Therefore, increases in output are translated as equiproportional changes in demands for each composite input which implies CRS.

4.3.1.2. Primary Factor Set

In the primary factor nest, to specify the production a constant returns to scale (CRS) CES function is used:

$$COMPVA_j = A_j \left[\delta_j K_j^{-\rho} + (1 - \delta_j) L_j^{-\rho} \right]^{-\frac{1}{\rho}} \qquad \text{(CGE.3)}$$

where A_j is an efficiency parameter, δ_j is a distribution parameter with $\sum_i \delta_i = 1$, ρ_j is an elasticity parameter. Minimising cost subject to (CGE.3) gives first order conditions:

$$W_j = \Lambda A_j \left[\delta_j K_j^{-\rho} + (1 - \delta_j) L_j^{-\rho} \right]^{\frac{1+\rho}{\rho}} (1 - \delta_j) L_j^{-(1+\rho)} \qquad \text{(CGE.4)}$$

$$R = \Lambda A_j \left[\delta_j K_j^{-\rho} + (1 - \delta_j) L_j^{-\rho} \right]^{\frac{1+\rho}{\rho}} \delta_j K_j^{-(1+\rho)} \qquad \text{(CGE.5)}$$

$$COMPVA_j = A_j \left[\delta_j K_j^{-\rho} + (1 - \delta_j) L_j^{-\rho} \right]^{-\frac{1}{\rho}} \qquad \text{(CGE.6)}$$

Then, substituting (CGE.6) into (CGE.4) and simplifying gives:

$$W_j = \Lambda A_j^{-\rho} COMPVA_j^{(1+\rho)} (1 - \delta_j) L_j^{-(1+\rho)} \qquad \text{(CGE.7)}$$

where (CGE.4), (CGE.5) and (CGE.6) are in the levels form. Here, the same method in Dixon et al. (1992) (pp124-125) is followed which linearises the first order conditions and solves. Thus linearisation of (CGE.6) gives:

$$compva_j = PVASHRK_j k_j + PVASHRL_j l_j \qquad \text{(CGE.8)}$$

where

$$PVASHRK_j = \frac{\delta_j K_j^{-\rho}}{\delta_j K_j^{-\rho} + (1 - \delta_j) L_j^{-\rho}} \qquad \text{(CGE.9)}$$

$$PVASHRL_j = \frac{(1-\delta_j)L_j^{-\rho}}{\delta_j K_j^{-\rho} + (1-\delta_j)L_j^{-\rho}}$$ (CGE.10)

Rearranging the equation (CGE.7) in terms of δ_j and substituting into (CGE.10) gives:

$$PVASHRL_j = \frac{W_j L_j}{RK_j + W_j L_j}$$ (CGE.11)

With this alternative form of the share $PVASHR_{i,j}$ avoids the process of calibration since it eliminates distribution parameter δ_j.

Furthermore the linearisation of (CGE.7) gives:

$$w_j = \lambda + (1+\rho)compva_j - (1+\rho)l_j$$ (CGE.12)

Thus, equations (CGE.8) and (CGE.12) are linearised first order conditions, where w, l and λ are percentage changes in W, L, and Λ respectively.

Rearranging (CGE.12) in terms of l_j gives:

$$l_j = -\sigma w_j + \sigma\lambda + compva_j$$ (CGE.13)

Repeating this process for capital demands gives:

$$k_j = -\sigma r + \sigma\lambda + compva_j$$ (CGE.14)

where σ is the elasticity of substitution between primary factors (in the example model, labour and capital) in the value added nest:

$$\sigma = \frac{1}{1+\rho} \qquad \text{(CGE.15)}$$

Substituting (CGE.13) and (CGE.14) into (CGE.8) and rearranging in terms of σλ gives:

$$\sigma\lambda = \sigma(PVASHRK_j r + PVASHRL_j w_j) \qquad \text{(CGE.16)}$$

Substituting (CGE.16) into (CGE.14) eliminates the percentage change Langrangian variable λ. Factorising the resulting expression gives linearised CES Hicksian primary factor demands:

$$l_j = compva_j - \sigma\big[w_j - (PVASHRK_j r + PVASHRL_j w_j\big] \qquad \text{(CGE.17)}$$

For consistent aggregation expression (CGE.18) must hold:

$$PVA_j COMPVA_j = RK_j + W_j L_j \qquad \text{(CGE.18)}$$

Linearising (CGE.18), substituting (CGE.8) into linearised (CGE.18) and rearranging gives the derivation of the composite value added price in linearised form:

$$pva_j = PVASHRK_j r + PVASHRL_j w_j \qquad \text{(CGE.19)}$$

where PVASHRLj was defined at the equation (CGE.11). Substituting (CGE.19) into (CGE.17) gives a simplified version of the linearised Hicksian demand function:

$$l_j = compva_j - \sigma\big[w_j - pva_j\big] \qquad \text{(CGE.20)}$$

and using same approach k_j can also be derived as:

$$k_j = compva_j - \sigma\big[r - pva_j\big] \qquad \text{(CGE.21)}$$

Equations (CGE.20) and (CGE.21) implies that in the absence of price changes, all input usage move by the same percentage as output. This reflects the constant returns to scale which is shown by the production function (CGE.3). Moreover, any increase in w_j (or in r) relative to the composite value added price (pva_j), will cause a reduction in the usage of primary factor 'l_j' (or in k_j) relative to other primary factor 'k_j' (or in l_j). The strength of this price substitution effect depends on the size of σ (elasticity of substitution).

4.3.1.3. Intermediate Input Nest

The functional form chosen for the specification of the intermediate input nest is also a CRS CES function as used in primary factor nest. The mathematical derivations of demand and price functions are parallel exactly to the derivations which have done in the primary factor nest. Therefore, in this subsection only derived demand and price functions will be given for not to repeat all steps[9].

Thus, the intermediate input demands for the stylised model are:

$$iim_j = compii_j - \sigma\left[p_m - pii_j\right] \qquad\qquad\text{(CGE.22)}$$

$$iis_j = compii_j - \sigma\left[p_s - pii_j\right] \qquad\qquad\text{(CGE.23)}$$

And the composite price in the nest is:

$$pii_j = PIISHRM_j p_m + PIISHRS_j p_s \qquad\qquad\text{(CGE.24)}$$

where

$$PIISHRM_j = \frac{P_m IIM_j}{P_m IIM_j + P_s IIM_j} \qquad\text{and}\qquad PIISHRS_j = \frac{P_s IIM_j}{P_m IIM_j + P_s IIM_j} \quad\text{(CGE.25)}$$

---

[9] The full list of variables is explained in subsection 4.1.

The interpretation of the equations (CGE.22) and (CGE.23) can be done with a same way as for the primary factor demand functions (CGE.20) and (CGE.21).

4.3.1.4. Immobility of supply of labour

In the stylised model perfect mobility for primary factor capital is assumed and factor immobility is assumed for primary factor labour. As it can be seen in Figure 9 a CET functional form is chosen whilst the mathematical derivations of a CET function are exactly parallel to the CES function.

By repeating the steps from (CGE.4) to (CGE.21) after obtaining the first order conditions for revenue maximisation subject to (CET.1) the functional forms to explain labour supply can be obtained:

$$xl_j = lab + \sigma_j \left[wcomp - w_j \right] \qquad\qquad \text{(CGE.26)}$$

The interpretation of (CGE.26) is similar to that of (CGE.20) and (CGE.21). In the absence of price changes, all output volumes move by same percentage as endowment of the primary factor. The only difference is the positive sign which appears for the elasticity of transformation in (CGE.26). This comes from the restriction on ρ, where in CES ρ is greater than or equal to -1, whilst in CET ρ, is less than or equal to -1. Thus, if the wage rate increases in one industry relative to composite wage rate then the supply of labour in that industry will increase too.

4.3.2. The derivation of demand functions

In the stylised model as it is shown in Figure 7, the Cobb-Douglas function is used to specify the demand functions of private household and government. In this case, the easiest way to derive linearised forms of the demand functions is to derive demand functions in levels form as shown in subsection 2.2.1 and then to linearise the resulting equations by following the same approach (see section 3.1).

Thus, the private household demand can be expressed as:

$$c_j = y - q_j \qquad \text{(CGE.27)}$$

And government or public demand is:

$$g_j = t - p_j \qquad \text{(CGE.28)}$$

4.3.3. The General Equilibrium System of Equations

In addition to the behavioural equations, further market clearing equations are required to satisfy the general equilibrium conditions. These equations appear as equations 5,6,20,21,22,31,32,33,39 in Figure 10. The model assumes perfect competition, perfect capital mobility and immobility across the labour market, whilst commodity and primary factor demands are equal to their supplies.

To turn the model into a closed circular flow economy, accounting equations are introduced so that incomes accrued to the household from ownership from the ownership of the factors of production are equal to total household expenditure (CGE.29) and the total tax revenue is equal to total expenditure of the government (CGE.30). The following equations explain this in level form and in Figure 10 these equations can be found in linearised form (31,32,33,34).

$$Y = R\sum_j K_j + \sum_j W_j L_j \qquad \text{equals to} \qquad PEXP = \sum_j Q_j C_j \qquad \text{(CGE.29)}$$

$$TAXREV = \sum_j (Q_j - P_j) C_j \qquad \text{equals to} \qquad GOVT = \sum_j P_j G_j \qquad \text{(CGE.30)}$$

COMMODITY MARKETS

Demand	*Household (private)*	$c_j = y - q_j$	(1,2)
	Government	$g_j = t - p_j$	(3,4)

Market Clearing

$$VSALE_m(p_m + x_m) = VMANU_m(p_m + iim_m) + VMANU_s(p_m + iim_s)$$
$$+ HHDEMANDB_m(p_m + c_m) + GODEMANDB_m(p_m + g_m) + EXPORTS(\exp or + p_m)$$
(5)

$$VSALE(p_s + x_s) = VSERV_m(p_s + iis_m) + VSERV_s(p_s + iis_s)$$
$$+ HHDEMANDB_s(p_s + c_s) + GODEMANB_s(p_s + g_s)$$
(6)

Price Equations		$q_j = p_j + tc_j$	(7,8)

INPUT MARKETS

Composite Input Demands (Leontief)	$compii_j = x_j$	(9,10)
	$compva_j = x_j$	(11,12)

Primary Factor Market (CES)

Primary Factor Demands	$k_j = compva_j - \sigma[r - pva_j]$	(13,14)
	$l_j = compva_j - \sigma[w_j - pva_j]$	

(15,16)

Composite wage rate	$wcomp = \sum_j REVSHRL_j w_j$	(17)
Labour supply	$xl_j = lab + \sigma_j[w_j - wcomp]$	(18,19)
Labour market clearing	$xl_j = l_j$	(20,21)
Capital market clearing	$cap = REVSHRK_m k_m + REVSHK_s k_s$	

(22)

Figure 10 The Stylised CGE Model

Intermediate Input Market (CES)

Intermediate Input Demands

$$iim_j = compii_j - \sigma\left[p_m - pii_j\right] \qquad (25,26)$$

$$iis_j = compii_j - \sigma\left[p_s - pii_j\right] \qquad (27,28)$$

Composite intermediate input price

$$pii_j = PIISHRM_j\, p_m + PIISHRS_j\, p_s$$

(29,30)

CONSUMER

Income

$$y = \sum_j INCSHRK_j\left(r + k_j\right) + \sum_j INCSHRL_j\left(w_j + l_j\right)$$

(31)

Expenditure

$$p\exp = \sum_j INCSHRE_j\left(q_j + c_j\right) \qquad (32)$$

PUBLIC SECTOR (GOVERNMENT)

Tax Revenue

$$t = \sum_j TAXRSHRP_j\left(q_j + c_j\right) - \sum_j TAXRSHRB_j\left(p_j + c_j\right) \qquad (33)$$

Expenditure

$$govt = \sum_j TAXRSHRG_j\left(p_j + g_j\right) \qquad (34)$$

FOREIGN SECTOR

Importation

$$impor = c_s \qquad (35)$$

Exportation

$$EXPORTS(\exp or + p_m) = IMPORTS(impor + p_s) \qquad (36)$$

Price Equations

$$p_j = f + pw_j \qquad (37,38)$$

Balance of Trade

$$bot = \left(pw_m + \exp or\right) - \left(pw_s + impor\right) \qquad (39)$$

GDP (Gross Domestic Product)

Figure 10 (Cont.) The Stylised CGE Model

where

VSALE$_j$ – The value of domestic supply in industry j

VMANU$_j$ – The value of manufactured intermediate inputs in industry j

VSERV$_j$ – The value of services intermediate inputs in industry j

HHDEMANDB$_j$ – The value of private household demands (basic prices)

GODEMANDB$_j$ – The value of government demands (basic prices)

EXPORTS – The value of (manufacturing) exports

IMPORTS – The value of (services) imports

In Figure 10 the variables which are notated by SHR letters refer to the share coefficients. Some of them are expressed explicitly in the previous parts of this section and below:

$$REVSHRK_j = \frac{RK_j}{\sum_j RK_j} \qquad \text{and} \qquad REVSHRL_j = \frac{W_j L_j}{\sum_j W_j L_j} \qquad \text{(CGE.31)}$$

$$INCSHRK_j = \frac{RK_j}{\sum_j R_j K_j + \sum_j W_j L_j} \qquad \text{and} \qquad INCSHRL_j = \frac{W_j L_j}{\sum_j R_j K_j + \sum_j W_j L_j} \qquad \text{(CGE.32)}$$

$$TAXRSHRP_j = \frac{\sum_j Q_j C_j}{\sum_j Q_j C_j - \sum_j P_j C_j} \qquad \text{and} \qquad TAXRSHRB_j = \frac{\sum_j P_j C_j}{\sum_j Q_j C_j - \sum_j P_j C_j} \qquad \text{(CGE.33)}$$

$$GDSHRB_j = \frac{\sum_j P_j C_j}{\sum_j P_j C_j + \sum_j P_j G_j + EXPORTS - IMPORTS} \qquad \text{(CGE.34)}$$

Also in appendix IV codes for this stylised model are given for the GEMPACK software. GEMPACK (General Equilibrium Modelling PACKage) software is used to solve this stylised model to find the equilibrium solution.

The model consists of 47 variables as given in subsection 4.1 40 of them are endogenous variables and they can be found in Figure 10 and the other 7 variables are used as exogenous variables to ensure correct model closure. More information about closure will be given in the next section.

There are two more important issues in CGE model developing which can be easily shown by using this simple stylised model. The first issue to discuss is Walras' law. This law shows that for a given set of prices, the sum of the excess demands over all markets must be equal to zero. Or in other words if N-1 markets are in equilibrium then the Nth market also will be in equilibrium.

A related issue is the zero homogeneity in prices in the demand functions, where changes in the absolute price level have no effect on the level of demand. Accordingly, CGE models are relative price models, where to establish a relative price base, one price variable known as the "numeraire" must be exogenised and held fixed. Thus, given Walras' law, it is possible to omit the Nth market and exogenise a price variable as a price deflator or numeaire whilst maintaining the closure intact. Therefore, all price movements are interpreted in terms of the numeraire variable (Dinwiddly and Teal, 1988). In the stylised model the variable 'F' (exchange rate) is chosen as the numeraire variable.

5. CLOSURE

Closure is the process of deciding which variables of the model will be treated as endogenous and which variables are to be held exogenous variables of the model[10]. Once exogenous variables are decided, then shocks can be applied to these variables to make a 'counterfactual' database.

Closure also defines the maintained hypothesis about the macro- and microeconomic mechanisms which underlie the workings of the economy. For example in the model employed here 'a small country assumption' is used which implies that the country does not have enough market power

---

[10] Endogenous variables are variables whose values are determined by the solution of the model whilst exogenous variables take values determined by the modeller in order to implement assumed shocks.

to affect world prices. Thus, world commodity prices are typically assumed exogenous, and the transmission mechanism between world and domestic prices is via an exchange rate (F in the stylised model) which adjusts to ensure equilibrium in the balance of payments (Shoven and Whalley, 1992, ch.9). In other words, import and export demands are ascertained by a balance of payments market clearing equation. In fact, this assumption and its implications on the closure decision can be seen in the stylised model (section 4) too. In the stylised model, world prices for both commodities and the exchange rate are selected as exogenous variables to use the small country assumption. On the other hand, for large country single region models, the import and export demand equations may be specified explicitly using a specific functional form, although simultaneous changes in these demands and the exchange rate must still satisfy the balance of payments constraint (Shoven and Whalley, 1992).

Closure can also be used to capture different time horizons. For example, in the stylised model there are no changes in population, productivity or endowments which would be indicative of a short to medium run time period. On the other hand, a closure change, through closure swap could be used to incorporate, for example, capital accumulation (Francois et al., 1996) thereby giving the model a longer run dimension. It is also possible to introduce closure swaps to characterise microeconomic hypothesis. For example, if a tax variable (now endogenous) is swapped with a quantity variable (now exogenous), it could be possible to characterise a production quota.

To summarise, closure choice is a complex issue which defines the key characteristics of the economy. As a result, the choice of endogenous/exogenous variable split has a significant influence on the model result.

6. EVALUATION OF THE METHODOLOGY – CGE

After looking over the principles of CGE modelling, it will be useful to evaluate its utility as a policy appraisal tool. To achieve this objective in this section, the possible strengths and weaknesses of

the CGE methodology are given. This section is based on the study done by Greenaway et al. (1993) and the most important features are emphasised in the section.

6.1. Strengths

As shown in previous sections, the behavioural functions are derived by using microeconomic theory. This becomes one of the key strengths of CGE models allowing the researcher to asses the impacts of policy changes at the micro level. Another important feature of CGE modeling for policy appraisal is that since these models are multisectoral models, they can capture the interaction between different production sectors of the economy. Since all policy changes have distributional consequences, the ability to compute distributional changes at disaggregated sectoral levels is a great strength of this approach. Moreover, these models generally attempt to model welfare changes by using indicators like compensating and equivalent variation. Thus, instead of specifying the price or quantity effects of a given policy shock, net welfare benefits can be identified too. Thus, CGE is a powerful analytical tool and especially useful for assessing the impact of alternative policy interventions.

Although many of the studies that applies CGE models have been concerned with policy issues, they also can be used to address a wide range of subjects such as; macroeconomics, public finance, environmental analysis etc.

In summary, although CGE models are not suitable for the analysis of individual firms and consumers and for macroeconomic forecasting, they are well suited to the economy-wide analysis of the effects of alternative policy scenarios. In general, CGE, for many large-scale studies, is likely to be the most useful tool given its facility for multi-sectoral modeling.

6.2. Weaknesses

In theory these models can adapt any kind of functional forms, but in practice depending on calibration and solution method restrictions, favours the use of convenient functions. Therefore, these models lack flexibility. Moreover, there is no statistical test to evaluate the goodness of fit of the

selected functions and the parameters. As argued before, some of the parameter estimates are based on subjective modeling assumptions like income elasticities and elasticities of substitution, which conceivably compromises the quality of the model's predictions.

CGE models rely too much on secondary data and this reliability can be questionable since the models are calibrated to a benchmark year. Furthermore, these models require a vast amount of data and obtaining a benchmark data set which is consistent with a general equilibrium can be very difficult.

Another contentious issue is the choice of closure. This decision can affect the results and can also constrain the model in important aspects. Finally, in these models monetary sectors are usually quite primitive and monetary flows are ignored.

7. CONCLUSIONS

This paper summarises the principal issues in CGE modeling and implementation. After a brief introduction to the paper, the next section examined the properties of the family of 'convenient' functional forms which are generally used in CGE model structures (section 2).

The paper then proceeded to discuss the concepts of model representation and solution methodology in CGE modeling (section 3). This section then explained the mathematical derivation of linearisation and related issues of nesting and calibration. The section discussed the use of nesting structures (multi-step optimisation technique) as a remedial tool against the lack of functional flexibility in CGE model structures. To help the reader, more information about the Johansen approach, multi-step procedures and extrapolation is presented in appendix I.

The next section offered a stylised CGE open economy model structure to help the reader interpret the mechanisms of linear model representation and nesting, which play an important role in CGE modelling, and to illustrate the usage of functional forms, which are examined in the section 2,

and their linearisation process (section 4). Also the code of this stylised model for GEMPACK software is laid out in appendix IV.

The paper continued with two small sections, while one of them discusses the issue of closure (section 5) and the other one evaluates the CGE modeling approach by giving its strengths, weaknesses (section 6).

APPENDIX I: THE JOHANSEN APPROACH

AI.1. One-Step Johansen Approach

This appendix offers useful information about Johansen approach, multi-step solution and extrapolation (the issues which are discussed in section 3) by giving a simple example which is adapted from Dixon et al. (1992) p73.

In this appendix a class of general equilibrium models is considered in which equilibrium is a vector, V, of length n satisfying a system of equations

$$F(V) = 0 \qquad\qquad (AI.1)$$

where F is a vector function of length m. It is assumed that F is differentiable and the number of variables, n, is more than number of equations m. For simplicity a system will be assumed to explain this approach which consists of two equations and three variables which has the form:

$$V_1^2 V_3 - 1 = 0 \qquad \text{and} \qquad V_1 + V_2 - 2 = 0 \qquad\qquad (AI.2)$$

where V_3 is an exogenous variable and V_1 and V_2 are endogenous variables of the system. The endogenous variables can be expressed as:

$$V_1 = V_3^{-1/2} \qquad \text{and} \qquad V_2 = 2 - V_3^{-1/2} \qquad\qquad (AI.3)$$

With a solution system like (AI.2) which is in levels form, there would be no difficulty to evaluate the effects on the endogenous variables of shifts in the exogenous variable. For example, assume that initially we have

$$V^I = (V_1^I, V_2^I, V_3^I) = (1,1,1) \qquad\qquad (AI.4)$$

a situation which satisfies (AI.1). Assuming an increase in V_3 from 1 to 1.1 in (AI.3) the new values for V_1 and V_2 are 0.9535 and 1.0465. Thus, a 10 percent increase in V_3 induces a 4.65 percent reduction in V_1 and a 4.65 increase in V_2.

After solving the system in levels form, now the same system will be solved in linearised form which is solved by the Johansen approach. To obtain a linearised form, a differential form is derived from (AI.1):

$$A(V)v = 0 \tag{AI.5}$$

where $A(V)$ is an m*n matrix whose components are functions of V. The n * 1 vector is usually interpreted as showing percentage changes or changes in the logarithms of the variables V. (AI.5) can be shown by using the example system (AI.2):

$$\begin{bmatrix} 2V_1V_3 & 0 & V_1^2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \\ dV_3 \end{bmatrix} = 0 \tag{AI.6}$$

In (AI.6) v is interpreted as the vector of changes, and it can be transformed into[11]

$$\begin{bmatrix} 2 & 0 & 1 \\ V_1/2 & V_2/2 & 0 \end{bmatrix} \begin{bmatrix} 100(dV_1)/V_1 \\ 100(dV_2)/V_2 \\ 100(dV_3)/V_3 \end{bmatrix} = 0 \tag{AI.7}$$

---

[11] The equation (AI.7) is derived by dividing the first part of the equation (AI.6) by $V_1^2V_3$ and the second part by 2.

If the (dV_i/V_i)s in (AI.7) are replaced by $(d\ln V_i)$s then, on dividing all equations by 100 gives the system below in which v is the vector of changes in the logarithms of V:

$$
\begin{bmatrix} 2 & 0 & 1 \\ V_1/2 & V_2/2 & 0 \end{bmatrix}
\begin{bmatrix} d\ln V_1 \\ d\ln V_2 \\ d\ln V_3 \end{bmatrix} = 0
\tag{AI.8}
$$

In computations of how endogenous variables will change in response to given changes in the exogenous variables, A(V) is evaluated at $V = V^I$. Then (AI.5) can be rewritten as:

$$
A_\alpha(V^I)v_\alpha + A_\beta(V^I)v_\beta = 0
\tag{AI.9}
$$

where v_α is the m * 1 subvector of endogenous components of v, v_β is the (n − m) * 1 subvector of exogenous components and $A_\alpha(V^I)$ and $A_\beta(V^I)$ are suitable submatrices of $A(V^I)$. By rearranging (AI.9) in terms of v_α gives:

$$
v_\alpha = -A_\alpha^{-1}(V^I)A_\beta(V^I)v_\beta
\tag{AI.10}
$$

or more compactly

$$
v_\alpha = B(V^I)\, v_\beta
\tag{AI.11}
$$

where $B(V^I)$ is defined at the right hand side of the (AI.10). Calculations (AI.9) − (AI.11) can be shown following the same mathematical procedure as in equations (AI.6) − (AI.8). With $V = V^I = (1,1,1)$, (AI.6) becomes

$$
\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} dV_1 \\ dV_2 \\ dV_3 \end{bmatrix} = 0
\tag{AI.12}
$$

By choosing variable 3 as exogenous variable then (AI.12) can be rewritten as:

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} dV_3 = 0 \qquad \text{(AI.13)}$$

That is:

$$\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} = -\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} dV_3 \qquad \text{(AI.14)}$$

Hence:

$$\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} dV_3 \qquad \text{(AI.15)}$$

It is reassuring to note from (AI.3) that $V_3 = 1$

$$\begin{bmatrix} \dfrac{\partial V_1}{\partial V_3} \end{bmatrix} = -\dfrac{1}{2}(V_3)^{-3/2} = -0.5 \qquad \text{and} \qquad \begin{bmatrix} \dfrac{\partial V_2}{\partial V_3} \end{bmatrix} = \dfrac{1}{2}(V_3)^{-3/2} = 0.5$$

According to this, it can be easily seen that the derivates of the endogenous variables with respect to the exogenous variable at V^I is correctly exposed on the right hand side of (AI.15). Applying the same condition ($V = V^I$) for (AI.7) or (AI.8) gives:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -\begin{bmatrix} 2 & 0 \\ 0.5 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_3$$

that is, $\qquad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} v_3 \qquad \text{(AI.16)}$

By using (AI.3) the result can be checked

$$\eta_{1,3} = -\frac{1}{2}(V_3)^{-\frac{1}{2}}/V_1 \qquad \text{and} \qquad \eta_{2,3} = \frac{1}{2}(V_3)^{-\frac{1}{2}}/V_2$$

where $\eta_{1,3}$ and $\eta_{2,3}$ are elasticities of variables 1 and 2 with respect to variable 3. With $V = V^I$, gives $\eta_{1,3} = -0.5$ and $\eta_{2,3} = -0.5$, which confirms the result (AI.16).

To see if the Johansen approach is satisfactory for computing the effects on the endogenous variables of small changes in the exogenous variables a small experiment can be conducted. For example, by using (AI.16), if a 10 percent increase occurs in V_3 it will cause 5 percent reduction in V_1 and a 5 percent increase in V_2. This result is close to the result which was found earlier by substituting into (AI.3).

AI.2. A simple example of a multi-step Johansen approach

In this subsection of the appendix I, a simple example of a multi-step solution of Johansen model is provided. Before the example in Figure AI.1 a diagram for a multi-step solution of a Johansen model is provided to summarise the procedure. To start a multi-step computation for a stylised Johansen model (box 1) the provided input-output data is read (parameter estimates also can be read in case that they are provided). In the initial stage modeller should set the closure, the shocks, the number of steps (denoted by s) to be used and the counter, r, which tracks how many steps have been completed. The mathematics starts in box 2 with evaluating the A matrix using the initial input-output data. In box 3, the shocks to be applied to the exogenous variables are computed. Then, these shocks are used to compute changes in the endogenous variables in box 4. After the completion of the task in box 4, if r + 1 is less than s, then r is increased by 1 (box 5) and with updating the input-output data (box 6) the next step starts. The computations continue as explained above until r +1 is equal to s. Then, the s-step estimates of the values reached by the endogenous variables after the shocks (box 7) are computed.

```
┌──────────────────────────────────────────────────────────────────────┐
│ 1. Set the initial conditions                                          │
│                                                                        │
│        a. Read in the input-output data and parameter estimates        │
│                                                                        │
│        b. Decide the closure (exogenous/endogenous split)              │
│                                                                        │
│        c. Set the exogenous shocks                                     │
│                                                                        │
│        d. Set the number of steps, s.                                  │
│                                                                        │
│        e. Set the counter r at r = 0                                   │
└──────────────────────────────────────────────────────────────────────┘
```

2. Compute $A((V)_{r,s})$ (Evaluate the A matrix by using the initial input-output data

3. Compute $(v_\beta)_{r+1,s}$

4. Compute $(v_\alpha)_{r+1,s} = B((V)_{r,s})(v_\beta)_{r+1,s}$

$r+1 < s$ $r+1 = s$

5. Increase r by 1

6. Update the input-output data according to changes occurring in the previous step

7. Compute the s-step estimates of the values reached by the endogenous variables after the shocks
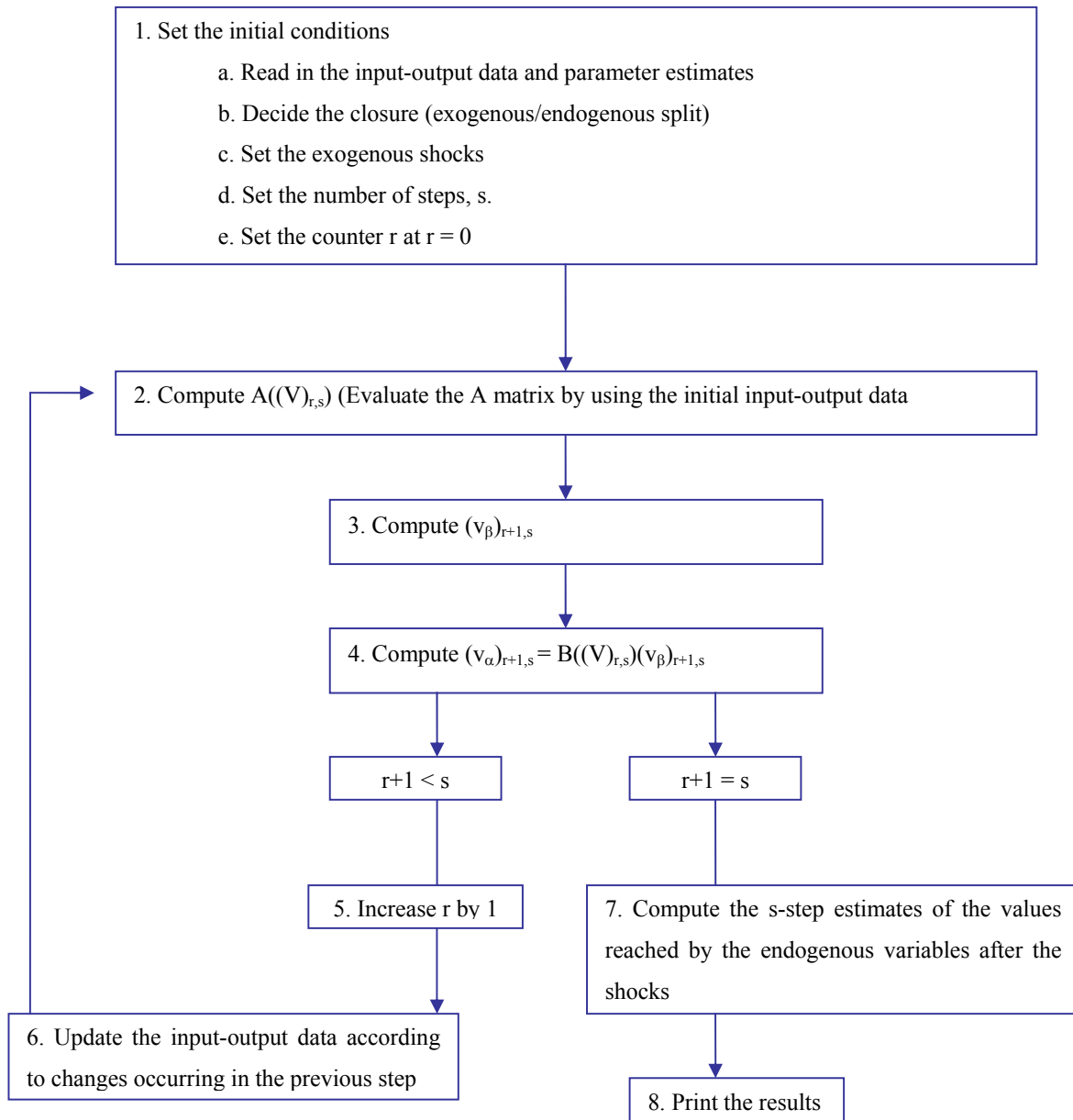
8. Print the results

Figure AI.1 Stages for a multi-step solution of a Johansen model (Dixon et al., 1992)

Using the same system which is used at AI.1 estimations of the values of V_1 and V_2 will be done after a 100 percent increase in V_3 by using one-step, 2-step, 4-step, 8-step and different ways of extrapolation computations. Initial values for the variables will be 0.5 for V_1, 1.5 for V_2 and 4 for V_3.

In this example we have

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = B(V)v_3 \qquad\qquad\qquad\qquad (AI.17)$$

where

$$B(V) = -\begin{bmatrix} 2 & 0 \\ V_1/2 & V_2/2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5V_1/V_2 \end{bmatrix} \qquad\qquad (AI.18)$$

Here the v_is are interpreted as percentage changes. In the first step of the two-step procedure:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{1,2} = B(V^I)50 = \begin{bmatrix} -25 \\ 8.\underline{3} \end{bmatrix}$$

which shows the estimate of the percentage effects on V_1 and V_2 of changing V_3 from 4 to 6. After this step new values are:

$$(V)_{1,2} = (0.375, 1.625, 6)$$

where $(V)_{r,s}$ is the value of V at the end of the rth step of an s-step procedure. In the second step of the two-step procedure:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{2,2} = B((V)_{1,2})33.\underline{3} = \begin{bmatrix} -16.\underline{6} \\ 6.25 \end{bmatrix}$$

which shows the estimate of the percentage effects on V_1 and V_2 of changing V_3 from 6 to 8. Thus, the final estimate of V in the two-step procedure is:

$$(V)_{2,2} = (0.3125, 1.726, 8) \qquad\qquad\qquad (AI.19)$$

Hence, a 100 percent increase in V_3 cause a 37.5 percent reduction in V_1 and nearly 15 percent increase in V_2.

Below the calculations for four-step procedure is given:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{1,4} = \begin{bmatrix} -12.5 \\ 4.1\underline{6} \end{bmatrix} \qquad \text{leading to } (V)_{1,4} = (0.4375, \ 1.5625, \ 5)$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{2,4} = \begin{bmatrix} -10 \\ 2.8 \end{bmatrix} \qquad \text{leading to } (V)_{2,4} = (0.39375, \ 1.60625, \ 6)$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{3,4} = \begin{bmatrix} -8.\underline{3} \\ 2.0428 \end{bmatrix} \qquad \text{leading to } (V)_{3,4} = (0.3609, \ 1.639, \ 7)$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{4,4} = \begin{bmatrix} -7.1429 \\ 1.5728 \end{bmatrix} \quad \text{leading to } (V)_{4,4} = (0.3351, \ 1.6647, \ 8)$$

According to the results of four-step procedure, a 100 percent increase in V_3 cause a 32.98 percent reduction in V_1 and nearly 10.98 percent increase in V_2.

As it was mentioned before another way of increasing accuracy is to use an extrapolation procedure. To explain this the result for variable i from a procedure with a step size h is denoted by $V_i(h)$ (For example, from Table AI.1 $V_1(1/8) = 0.3446$). here two assumptions, the first one is:

$$\lim_{h \to 0} V_i(h) = V_i^T \qquad\qquad\qquad\qquad \text{(AI.20)}$$

where V_i^T, i = 1,2, is the true value for variable i after increasing V_3 to 8. V_1^T and V_2^T can be derived from (AI.1.) and are shown in the last row of Table AI.1 as 0.3535 and 1.6464. This assumption says that the n-procedure converges to the true solution as n becomes large. The second assumption is that $V_i(h)$ can be expressed as:

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$$V_i(h) = \sum_{r=0}^{\infty} a_{ir} h^r \quad , i = 1,2 \tag{AI.21}$$

over the relevant range for h. This assumption relies on the idea that continuous functions can be approximated arbitrarily closely by polynomials of sufficiently high degree. These two (AI.20) and (AI.21) together imply that,

$$V_i^T = a_{i0} \quad , i = 1,2 \tag{AI.22}$$

Now supposing that $V_i(h)$ can be approximated by

$$V_i(h) = a_{i0} + a_{i1} h \quad , i = 1,2 \tag{AI.23}$$

where the higher order terms in (AI.21) can be ignored in the relevant range for h is assumed. The approximations which are done with (AI.23) can permit researcher to achieve appropriate accuracy with multi-step Johansen computations using less number of steps. For example, only using the estimations of one-step computation and two-step computation it can be possible to evaluate $V_i(1)$ and $V_i(1/2)$. Then according to (AI.23), V_i^T can be estimated by solving it for a_{i0} in the equations

$$V_i(1) = a_{i0} + a_{i1}$$
$$V_i(1/2) = a_{i0} + a_{i1}/2$$

Then, V_i^T can be estimated by extrapolation from one using next equation:

$$V_i^T = 2V_i(1/2) - V_i(1) \quad , i = 1,2 \tag{AI.24}$$

The results of applying this equation are given in Table AI.1. To estimate V_i^T by extrapolation from one, two and four step solutions first (AI.23) should be replaced by an improved approximation:

$$V_i(h) = a_{i0} + a_{i1}h + a_{i2}h^2 \qquad\qquad (AI.25)$$

Then, after computing $V_i(h)$, $V_i(h/2)$ and $V_i(h/4)$, the next system of equation should be solved for a_{i0}:

$$V_i(h) = a_{i0} + a_{i1}h + a_{i2}h^2$$
$$V_i(h/2) = a_{i0} + (a_{i1}/2)h + (a_{i2}/4)h^2 \qquad\qquad (AI.26)$$
$$V_i(h/4) = a_{i0} + (a_{i1}/4)h + (a_{i2}/16)h^2$$

To obtain the solution for a_{i0} first equation should be multiplied by -1, the second one by 6 and the last one by -8 and the resulting equations should be added which gives:

$$-V_i(h) + 6V_i(h/2) - 8V_i(h/4) = -3a_{i0}$$

and this will give the extrapolation equation:

$$V_i^T = (8/3)V_i(h/4) - 2V_i(h/2) + (1/3)V_i(h) \qquad\qquad (AI.27)$$

Applying this equation gives the results shown in Table AI.1. And when $V_i(h)$, $V_i(h/2)$, $V_i(h/4)$ and $V_i(h/8)$ are available the approximation can be improved to:

$$V_i(h) = a_{i0} + a_{i1}h + a_{i2}h^2 + a_{i3}h^3 \qquad\qquad (AI.28)$$

And following the same method which has been used to obtain (AI.27) from (AI.24) it can be derived the extrapolation equation for extrapolating from one, two, four and eight step solutions:

$$V_i^T = (64/21)V_i(h/8) - (56/21)V_i(h/4) + (14/21)V_i(h/2) - (1/21)V_i(h) \quad (AI.29)$$

The results of the application of this equation are also given in Table AI.1.

Endogenous Variables	V_1	V_2
Initial Values	0.5	1.5
Estimated values after n increase in V_3 from 4 to 8		
1-step computation	0.25	1.75
2-step computation	0.3125	1.726
4-step computation	0.3351	1.6647
8-step computation	0.3446	1.6560
1,2 step extrapolation [a]	0.375	1.702
1,2,4 step extrapolation [b]	0.3519	1.5705
1,2,4,8 step extrapolation [c]	0.3530	1.6749
Truth [d]	0.3535	1.6464

[a] Computed according to (AI.24)
[b] Computed according to (AI.27)
[c] Computed according to (AI.29)
[d] Computed using (AI.3)

Table AI.1 Solution for V_1 and V_2 when V_3 is moved from 4 to 8

To conclude this appendix, as it can be observed from the results using more step numbers and extrapolation increases the accuracy of the estimations which are done according to the Johansen approach.

APPENDIX II: STRICT AND QUASI CONCAVITY

Following Beattie and Taylor (1985), *strict concavity* can be shown diagrammatically:
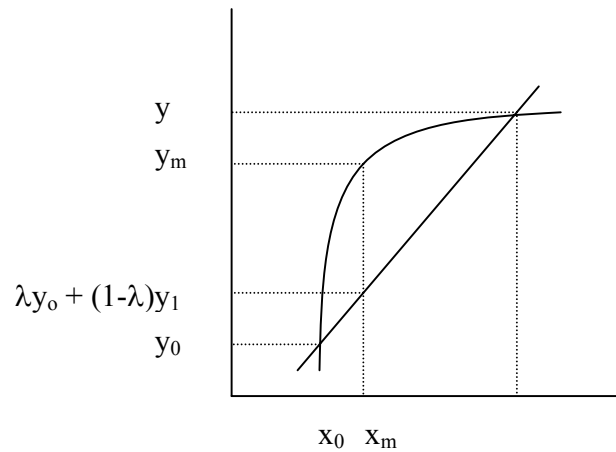

Figure AII.1 Strict concavity

where x_m is a weighted average $(0<\lambda<1)$ of x_0 and x_1:

$$x_m = \lambda x_0 + (1-\lambda)x_1$$

Strict concavity implies that y_m must always be *greater* than a weighted value of y from a linear line connecting two points x_0 and x_1 in the domain. Thus, in the figure, the value of y corresponding to an arbitrary value x_m is:

$$\lambda y_0 + (1-\lambda)y_1$$

where

$$y_m > \lambda y_0 + (1-\lambda)y_1$$

Strict quasi-concavity states that all values of y_m will always be above the minimum value of the function in the domain. Algebraically:

$$y_m > \min(y_0, y_1)$$

Thus, if the minimum value of the function was y_0, then a strictly quasi-concave function would be represented as:
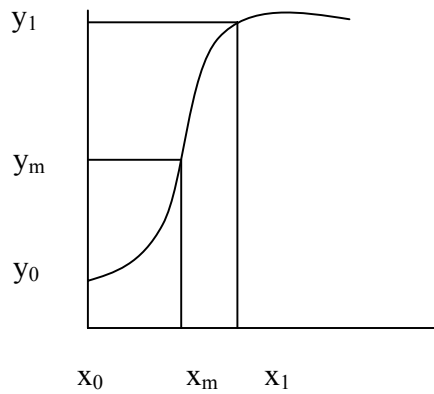

Figure AII.2 Strictly quasi-concave function

where the value of y_m will never fall below the minimum value (in this case y_0), although the shape of the curve does not have to be everywhere concave (for example between x_0 and x_1). Strict quasi-concavity is a more general form of concavity which is inclusive of strict concavity (i.e. all strict quasi-concave functions are strictly concave but not the other way round).

For CD and CES, the restrictions for both forms of concavity are:

	Strict Concavity	Strict quasi-concavity
$Y = AX_1^{\alpha} X_2^{\beta}$	$0 < \alpha < 1$	$\alpha > 0$
	$0 < \beta < 1$	$\beta > 0$
	$0 < (\alpha+\beta) < 1$	$A > 0$
	$A > 0$	
$Y = A[\delta_1 X_1^{-\rho} + (1-\delta_1)X_2^{-\rho}]^{-\frac{v}{\rho}}$	$0 < \delta_1 < 1$	$0 < \delta_1 < 1$
	$A > 0$	$A > 0$
	$\rho > -1$	$\rho > -1$
	$0 < v \leq 1$	$v > 0$

Under constant returns to scale, Cobb-Douglas parameters $\alpha+\beta$ must sum to one, which effectively rules out strict concavity.

APPENDIX III: STAGES OF PRODUCTION

Under neo-classical assumptions of diminishing marginal returns (short run) and returns to scale (long run) a production function (total product (TP) curve) may exhibit an 's' shape which in turn has implications for marginal (MP) and average product (AP) curves. Schematically, the three stages are represented:
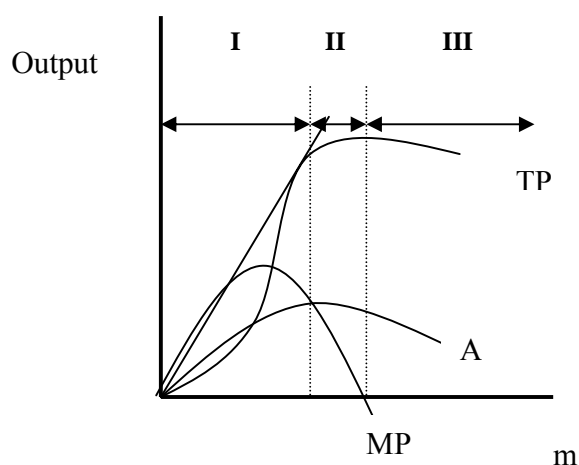

Figure AII.1 Stages of production (Beattie and Taylor, 1985)

where in the short-run 'm' is a single input and in the long-run, 'm' would be a proportional change in all inputs. Thus, stage I is characterised by increases in average productivity up to the point where MP cuts the AP curve at the highest point. Stage II is where MP is positive but everywhere below the AP curve. Stage III characterises negative productivity (i.e., MP is negative). Clearly, it is not sensible for rational producers to be in the third stage of production. Under profit maximising criteria in perfectly competitive input and output markets, equating Marginal Value Product (MVP = MP*P_{output}) with the Marginal Factor Cost (MFC = Average Factor Cost (AFC)) in stage I, will lead to losses where AFC is everywhere above Average Value Product (AVP = AP*P_{output}). Hence, according to the theory, stage II (MP<AP) is the only rational range within which to produce.

APPENDIX IV: GEMPACK CODES FOR THE STYLISED MODEL

```
File        INFILE  # Raw data #;

! Set for comparing COM and IND data !
SET IND # Industries #
(manu, svces);
SET COM # Commodities #
(manu, svces);
Subset IND is subset of COM;
       COM is subset of IND;

!46 variables, 7 exogenous: 38 equations and endogenous variables !
VARIABLE (all,j,IND)
l(j)  # labour demands #;
VARIABLE (all,j,IND)
k(j)  # capital demands #;
VARIABLE (all,j,IND)
xl(j)  # labour supply #;
!VARIABLE (all,j,IND)
xk(j)  # capital supply #;!
!VARIABLE (all,j,IND)
r(j) # capital rental #;!
VARIABLE (orig_level=1.0)
r # capital rental #;
VARIABLE (orig_level=1.0)(all,j,IND)
w(j) # wage rate #;
!VARIABLE
rcomp # composite capital rental #;!
VARIABLE
wcomp # composite wage rate #;
VARIABLE (all,j,IND)
x(j)  # industry supply #;
VARIABLE (all,j,IND)
c(j)  # private demand for industry output #;
VARIABLE (all,j,IND)
g(j)  # public demand for industry output #;
VARIABLE
y  # private consumer income #;
VARIABLE
pexp  # private consumer expenditure #;
VARIABLE (orig_level=1.0)(all,j,IND)
p(j)  # basic price in each industry #;
VARIABLE (orig_level=1.0)(all,j,IND)
q(j)  # purchaser's price in each industry  #;
VARIABLE (orig_level=1.0)(all,j,IND)
tc(j)  # commodity tax on (private) final demands (exogenous)  #;
VARIABLE
lab  # total labour endowment (exogenous) #;
VARIABLE
cap  # total capital endowment (exogenous) #;
VARIABLE
t # total tax revenues to government #;
```

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```
VARIABLE
govt # total government expenditures #;
VARIABLE (orig_level=1.0)(all,j,IND)
pw(j)  # world prices (exogenous)  #;
VARIABLE
f  # exchange rate (exogenous) #;
VARIABLE
expor  # exports #;
VARIABLE
impor  # imports #;
VARIABLE
bot  # balance of trade #;
VARIABLE (all,j,IND)
compii(j)  # composite intermediate inputs #;
VARIABLE (all,j,IND)
compva(j)  # composite value added #;
VARIABLE (all,j,IND)
iim(j)  # intermediate inputs #;
VARIABLE (all,j,IND)
iis(j)  # intermediate inputs #;
VARIABLE (all,j,IND)
pva(j)  # composite value added price #;
VARIABLE (all,j,IND)
pii(j)  # composite intermediate input price #;
VARIABLE
gdp  # produccion bruto interior #;


Coefficient (ge 0)(all,i,IND)
VL(i) # value of labour demands in industry #;
Read (all,i,IND)
VL(i) from file INFILE header "LAB";
Update (all,i,IND)
VL(i) = w(i) * l(i);
Coefficient (ge 0)
VLTOT # total value of labour demands #;
FORMULA
VLTOT = sum(i,IND,VL(i)) ;

Coefficient (ge 0)(all,i,IND)
VK(i) # value of capital demands in industry #;
Read (all,i,IND)
VK(i) from file INFILE header "CAP";
Update (all,i,IND)
VK(i) = r!(i)! * k(i);
Coefficient (ge 0)
VKTOT # total value of capital demands #;
FORMULA
VKTOT = sum(i,IND,VK(i)) ;

! Intermediate demands !
Coefficient (ge 0)(all,j,IND)
VMANU(j) # value of manufactured intermediate inputs by industry #;
Coefficient (ge 0)(all,j,IND)
```

```
VSERV(j) # value of services intermediate inputs by industry #;
Read (all,j,IND)
VMANU(j) from file INFILE header "VMAN";
Read (all,j,IND)
VSERV(j) from file INFILE header "VSER";

Update
VMANU("manu") = p("manu") * iim("manu") ;
Update
VMANU("svces") = p("manu") * iim("svces") ;
Update
VSERV("manu") = p("svces") * iis("manu") ;
Update
VSERV("svces") = p("svces") * iis("svces") ;


Coefficient (ge 0)
IMPORTS # value of (svces) imports #;
Read
IMPORTS from file INFILE header "IMP";
Update
IMPORTS = impor * p("svces") ;
! impor demands (1) !
EQUATION E_imports # import demands #
impor = c("svces")  ;
! assumed that imports are a fixed share of private household expenditure !

! compsite Leontief demands for value added and intermediate inputs !
EQUATION E_compii # composite intermediate inputs # (all,j,IND)
compii(j) = x(j) ;
EQUATION E_compva # composite intermediate inputs # (all,j,IND)
compva(j) = x(j) ;

! Factor demands and supplies (8)!
Coefficient (ge 0)(all,j,IND)
VADDED(j) # total value of factor demands in industry #;
FORMULA (initial) (all,j,IND)
VADDED(j) = VK(j) + VL(j) ;
Update (all,j,IND)
VADDED(j) = pva(j) * compva(j) ;
coefficient (all,j,IND)
    PVASHRK(j);
Formula (all,j,IND)
    PVASHRK(j) = VK(j) / VADDED(j);
coefficient (all,j,IND)
    PVASHRL(j);
Formula (all,j,IND)
    PVASHRL(j) = VL(j) / VADDED(j);
EQUATION E_pva # composite value added price # (all,j,IND)
pva(j) = PVASHRK(j) * r + PVASHRL(j) * w(j);

Coefficient (parameter)(all,j,IND)
SIGVA(j) # elasticity of substitution between factors #;
FORMULA (initial)
```

```
SIGVA("manu") = 2 ;
FORMULA (initial)
SIGVA("svces") = 1.5 ;

! CES demands for the factors of production !
EQUATION E_l1 # labour demands #
l("manu") = compva("manu") - SIGVA("manu") * [w("manu") - pva("manu")] ;

EQUATION E_l2 # labour demands #
l("svces") = compva("svces") - SIGVA("svces") * [w("svces") - pva("svces")] ;

EQUATION E_k1 # capital demands #
k("manu") = compva("manu") - SIGVA("manu") * [r!("manu")! - pva("manu")] ;

EQUATION E_k2 # capital demands #
k("svces") = compva("svces") - SIGVA("svces") * [r!("svces")! - pva("svces")] ;

! CET factor supply function !

Coefficient (parameter)(all,j,IND)
SIGCET(j) # (CET) constant elasticity of transformation between factors #;
FORMULA (initial)
SIGCET("manu") = -120 ;
FORMULA (initial)
SIGCET("svces") = -120 ;

EQUATION E_l11 # labour supply #
xl("manu") = lab + SIGCET("manu") * [wcomp - w("manu")] ;

EQUATION E_l22 # labour supply #
xl("svces") = lab + SIGCET("svces") * [wcomp - w("svces")] ;
!
EQUATION E_k11 # capital supply #
xk("manu") = cap + 0.11 * (rcomp - r("manu")) ;

EQUATION E_k22 # capital supply #
xk("svces") = cap + 0.05 * (rcomp - r("svces")) ;!

! factor market clearing equations (4) !
!
EQUATION E_kkk # capital market clearance # (all,i,IND)
xk(i) = k(i) ;!

EQUATION E_w # labour market clearance # (all,i,IND)
xl(i) = l(i) ;

coefficient (all,j,IND)
    REVSHRK(j);
Formula (all,j,IND)
    REVSHRK(j) = VK(j) / VKTOT;
coefficient (all,j,IND)
    REVSHRL(j);
Formula (all,j,IND)
    REVSHRL(j) = VL(j) / VLTOT;
```

```
! Composite factor price equations (2) !
!EQUATION E_rcomp # labour market clearance #
rcomp = sum(j,IND,REVSHRK(j) * r(j)) ;!

EQUATION E_wcomp # labour market clearance #
wcomp = sum(j,IND,REVSHRL(j) * w(j)) ;

EQUATION E_COMPR # capital factor market #
cap = REVSHRK("manu") * k ("manu") + REVSHRK("svces") * k ("svces");

! intermediate input demands !
Coefficient (ge 0)(all,j,IND)
VINTINP(j) # total value of intermediate demands by industry #;
FORMULA (initial)
VINTINP("manu") = VMANU("manu") + VSERV("manu") ;
FORMULA (initial)
VINTINP("svces") = VMANU("svces") + VSERV("svces") ;
Update
VINTINP("manu") = pii("manu") * compii("manu") ;
Update
VINTINP("svces") = pii("svces") * compii("svces") ;
coefficient (all,j,IND)
    PIISHRM(j);
Formula (all,j,IND)
    PIISHRM(j) = VMANU(j) / VINTINP(j);
coefficient (all,j,IND)
    PIISHRS(j);
Formula (all,j,IND)
    PIISHRS(j) = VSERV(j) / VINTINP(j);

EQUATION E_pii # composite value added price # (all,j,IND)
pii(j) = PIISHRM (j) * p("manu") + PIISHRS (j)* p("svces") ;


! intermediate input demands !
Coefficient (parameter)
SIGII # elasticity of substitution between intermediate inputs #;
FORMULA (initial)
SIGII = 0.87 ;

EQUATION E_ii1 # labour demands #
iim("manu") = compii("manu") - SIGII * [p("manu") - pii("manu")] ;

EQUATION E_ii2 # capital demands #
iis("manu") = compii("manu") - SIGII * [p("svces") - pii("manu")] ;

EQUATION E_ii3 # labour demands #
iim("svces") = compii("svces") - SIGII * [p("manu") - pii("svces")] ;

EQUATION E_ii4 # capital demands #
iis("svces") = compii("svces") - SIGII * [p("svces") - pii("svces")] ;

! final demands !
```

```
Coefficient (ge 0)(all,i,COM)
HHDEMANDB(i) # private household demands (basic prices) #;
Coefficient (ge 0)(all,i,COM)
GODEMANDB(i) # government demands (basic prices)#;
Coefficient (ge 0)(all,i,COM)
HHDEMANDP(i) # private household demands (purchaser's prices) #;

Read (all,j,COM)
HHDEMANDB(j) from file INFILE header "HHDB";
Read (all,j,COM)
GODEMANDB(j) from file INFILE header "GOVB";
Read (all,j,COM)
HHDEMANDP(j) from file INFILE header "HHDP";

UPDATE (all,i,COM)
HHDEMANDB(i) = p(i) * c(i) ;
UPDATE (all,i,COM)
GODEMANDB(i) = p(i) * g(i) ;
UPDATE (all,i,COM)
HHDEMANDP(i) = q(i) * c(i) ;

! Price linkages (1) !
EQUATION E_q # price linkage equation # (all,j,IND)
q(j) = p(j) + tc(j) ;

!Consumer demands (2) !
EQUATION E_c1 # consumer demands #
c("manu") = y - q("manu") ;
EQUATION E_c2 # consumer demands #
c("svces") = y - q("svces") ;

!public demands (2) !
EQUATION E_g1 # consumer demands #
g("manu") = t - p("manu") ;
EQUATION E_g2 # consumer demands #
g("svces") = t - p("svces") ;

Coefficient (ge 0)
TAXREV # commodity tax revenues to government #;
FORMULA
TAXREV = sum(i,COM,HHDEMANDP(i)) - sum(i,COM,HHDEMANDB(i));

! government income from taxes and expenditures on goods (2) !
coefficient (all,i,COM)
    TAXSHRP(i);
Formula (all,i,COM)
    TAXSHRP(i) = HHDEMANDP(i) / TAXREV;
coefficient (all,i,COM)
    TAXSHRB(i);
Formula (all,i,COM)
    TAXSHRB(i) = HHDEMANDB(i) / TAXREV;
coefficient (all,i,COM)
    TAXSHRG(i);
Formula (all,i,COM)
```

```
    TAXSHRG(i) = GODEMANDB(i) / TAXREV;
EQUATION E_t # tax revenue #
t = sum(i,COM,[TAXSHRP(i) * (q(i) + c(i))]) -
sum(i,COM,[TAXSHRB(i) * (p(i) + c(i))]) ;

EQUATION E_govt # government expenditures #
govt = sum(i,COM,[TAXSHRG(i) * (p(i) + g(i))]) ;

!
EQUATION E_COMPW # labour factor market #
VLTOT * lab = sum(i,IND,VL(i) * l(i)) ;!

! keep this out,so that we have a numeraire variable !
!EQUATION E_COMPR # capital factor market #
VKTOT * cap = sum(i,IND,VK(i) * k(i)) ;!

! Household income (1) !
Coefficient (ge 0)
INC # household gross income = household expenditure #;
FORMULA
INC = !sum(i,COM,HHDEMANDP(i))!
VK("manu") + VK("svces") + VL("manu") + VL("svces") ;
coefficient (all,i,COM)
    INCSHRK(i);
Formula (all,i,COM)
    INCSHRK(i) = VK(i) / INC;
coefficient (all,i,COM)
    INCSHRL(i);
Formula (all,i,COM)
    INCSHRL(i) = VL(i) / INC;
coefficient (all,i,COM)
    INCSHRE(i);
Formula (all,i,COM)
    INCSHRE(i) = HHDEMANDP(i) / INC;
EQUATION E_y # private household gross income #
y = sum(i,COM,[INCSHRK(i) * (r + k(i))]) +
sum(i,COM,[INCSHRL(i) * (w(i) + l(i))]) ;

EQUATION E_pexp # private household expenditure #
pexp = sum(i,COM,[INCSHRE(i) * (q(i) + c(i))])
!VK("manu") * [r("manu") + k("manu")] +
VK("svces") * [r("svces") + k("svces")] +
VL("manu") * [w("manu") + l("manu")] +
VL("svces") * [w("svces") + l("svces")] ! ;

! the commodity prices, p(j), are determined by the world price
linkage equations at the bottom of the Tablo !

Coefficient (ge 0)(all,i,IND)
VSALE(i) # value of domestic industry supplies #;
FORMULA
VSALE("manu") = VK("manu") + VL("manu") +
VMANU("manu") + VSERV("manu") ;
FORMULA
```

```
VSALE("svces") = VK("svces") + VL("svces") +
VMANU("svces") + VSERV("svces") + IMPORTS ;

Coefficient (ge 0)
EXPORTS # value of (svces) imports #;
Read
EXPORTS from file INFILE header "EXP";
Update
EXPORTS = expor * p("manu") ;
EQUATION E_exp # export demands #
EXPORTS * [expor + p("manu")] =
IMPORTS * [impor + p("svces")] ;


EQUATION E_x1 # value of commodity demands #
VSALE("manu") * [p("manu") + x("manu")] =
VMANU("manu") * [p("manu") + iim("manu")] +
VMANU("svces") * [p("manu") + iim("svces")] +
HHDEMANDB("manu") * [p("manu") + c("manu")] +
GODEMANDB("manu") * [p("manu") + g("manu")] +
EXPORTS * [p("manu") + expor] ;

EQUATION E_x2 # commodity demands #
VSALE("svces") * [p("svces") + x("svces")] =
VSERV("manu") * [p("svces") + iis("manu")] +
VSERV("svces") * [p("svces") + iis("svces")] +
HHDEMANDB("svces") * [p("svces") + c("svces")] +
GODEMANDB("svces") * [p("svces") + g("svces")] ;
! Here HHDEMANDB("svces") includes imports !


! trade equations (3 equations) !

EQUATION E_p1 # price linkage equation #
p("manu") = f + pw("manu") ;

EQUATION E_p2 # price linkage equation #
p("svces") = f + pw("svces") ;

! balance of payments starts at zero and should stay at zero
to ensure a GE solution !
EQUATION E_BT # balance of payments #
bot = [pw("manu") + expor] - [pw("svces") + impor] ;

! tasa de crecimiento en la economía !
Coefficient (ge 0)
GDPP # PBI #;
FORMULA
GDPP =
sum(i,COM,HHDEMANDB(i)) + sum(i,COM,GODEMANDB(i)) + EXPORTS  - IMPORTS  ;
coefficient (all,i,COM)
    GDPSHRB(i);
Formula (all,i,COM)
    GDPSHRB(i) = HHDEMANDB(i) / GDPP;
```

```
coefficient (all,i,COM)
    GDPSHRG(i);
Formula (all,i,COM)
    GDPSHRG(i) = GODEMANDB(i) / GDPP;
coefficient
    GDPSHRE;
Formula
    GDPSHRE = EXPORTS / GDPP;
coefficient
    GDPSHRI;
Formula
    GDPSHRI = IMPORTS / GDPP;
EQUATION E_gdp # PBI = C + G + X - M #
gdp = sum(i,COM,[GDPSHRB(i) * [p(i) + c(i)]]) +
sum(i,COM,[GDPSHRG(i) * [p(i) + g(i)]]) +
GDPSHRE * [expor + p("manu")] -
GDPSHRI * [impor + p("svces")] ;
```

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