The Roles of Media, Language, and Practice on Solving the Tower of Hanoi Problem

Elizabeth Worth, Angela Mercieri, Alyssa Douglas, Lucas Wong, & Jay Kosegarten
Southern New Hampshire University

Abstract

The current study presents findings of a study conducted on the Tower of Hanoi problem. The Tower of Hanoi problem is a logical puzzle involving recursion in which there are three pegs with discs stacked in ascending order on the left peg. The object is to restack the discs on the right peg in ascending order, moving one disc at a time and never having a larger disc placed on top of a smaller one. The fewer the moves to accomplish this, the more successful the trial is judged to be. Subjects were randomized into one of eight cells and all were measured on their ability to solve the 4-disc version of the Tower of Hanoi in terms of three dependent variables: total number of moves, total moves-to-optimal moves ratio, and completion time. In a 2 X 2 X 2 between subjects factorial design, subjects were assigned to three different conditions with two levels each. Subjects performed either a computerized or physical version of the TOH; verbalized their strategies while doing so or were instructed to remain silent; and were allowed to practice on easier two-disc and three-disc versions or were not given the benefit of doing so. Main effects were found in that subjects completing the physical TOH did so more efficiently than those completing it on the computer. Subjects also benefitted from verbalizing their strategies over remaining silent. Interaction effects were also found for practice and verbalization.

Introduction

The Tower of Hanoi puzzle is a transfer problem in which a set of graduated disks are moved across a group of three pegs to reach a predefined goal state. Progress towards the goal configuration is limited by two rules: only one disk can be moved at a time and larger disks cannot be placed on top of smaller disks. Optimal performance on the task, characterized by using the minimum necessary number of operations, requires adopting a means-ends analysis strategy (Simon & Anzai, 1979).

Experience, metacognitive processes, and the medium used to present the puzzle all significantly impact performance (Ahlum-Heath & Di Vesta, 1986; Noyes & Garland, 2003) and likewise the selection of different problem solving strategies. Examining these factors in terms of the dynamic restructuring and reorganization of representations, based on a framework for distributed cognitive tasks, yields a suitable model for interpreting performance (Zhang & Norman, 1994).

The role of media was studied, comparing performance between subjects using a computer version of the TOH and subjects using a physical version. Performance was moderated by the amount of information that is represented externally and the amount that is represented internally. The computer version contains more information that must be represented internally compared to the physical puzzle.

The role of language was examined by having half of the subjects concurrently verbalize the reasoning behind their moves and the other half asked to remain silent.

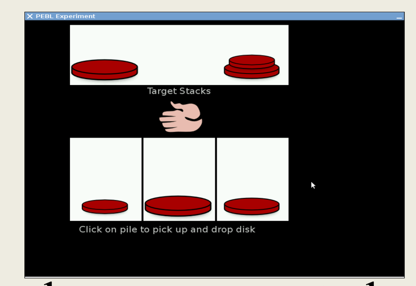
Lastly, half of the subjects completed three-to-five graduated rounds of practice before solving the 4-disc TOH problem. The other half of the subjects had one round of practice before completing the 4-disc TOH.

We hypothesized that the subjects completing the physical Tower of Hanoi would perform better that those completing the computerized version and that practice and verbalization would be advantageous.

Method

Participants. Ninety-five undergraduate students participated in this study.

Materials. The physical model of the Tower of Hanoi puzzle is composed of three vertical pegs that hold a predetermined number of disks of graduated diameters attached to a board.



The computer model of the Tower of Hanoi puzzle was presented on a laptop, using a program called PEBL authored by Shane Mueller (2012) and edited for the purposes of this experiment.

Procedure. A 2 x 2 x 2 between subjects design was used. Participants were randomly assigned to eight conditions: mode of presentation (computer vs. physical), experience (practice vs. no practice), and verbalization (concurrent verbalization vs. no verbalization).

In the training phase participants were given their respective instructions and completed between 1 (no practice) and 5 (practice) multi-disk problems. Then in the test phase the final 4-disk criterion puzzle was completed.

Results

Physical Problem Solving Performance:

Condition	Number of Moves	Proportion of Optimal Moves	Time (s)
Practice	30.26	0.65	111.45
No Practice	29.33	0.71	130.06
Verbalization	27.96	0.71	132.56
No Verbalization	31.54	0.65	109.83

Computer Problem Solving Performance:

Condition	Number of Moves	Proportion of Optimal Moves	Time (s)
Practice	32.42	0.60	102.06
No Practice	42.88	0.57	121.21
Verbalization	31.38	0.61	96.22
No Verbalization	43.92	0.56	127.05

Significant differences for the modalities on the dependent measures, Wilks' Λ = .846, $F(2,85) = 7.764$, $p = .001$, $\eta^2 = .154$

Marginally significant difference for verbalization on the dependent measures, Wilks' Λ = .933, $F(2,85) = 3.071$, $p = .052$, $\eta^2 = .067$

Significant differences for the interaction of experience and verbalization on the dependent measures, Wilks' Λ = .94, $F(2, 85) = 4.017$, $p = .022$, $\eta^2 = .086$

Results Continued

Significant main effect of modality on the number of moves used to solve the problem, $F(1,93) = 10.306$, $p = .002$, $\eta^2 = .107$

Significant main effect of modality on the proportion of optimal moves, $F(1,93) = 10.597$, $p = .002$, $\eta^2 = .110$

Significant main effect of verbalization on the number of moves, $F(1,93) = 5.748$, $p = .019$, $\eta^2 = .063$

Marginally significant interaction between modality and experience for the number of moves, $F(1,93) = 3.513$, $p = .064$, $\eta^2 = .039$

Significant interaction between experience and verbalization for the number of moves, $F(1,93) = 7.503$, $p = .007$, $\eta^2 = .080$

Conclusions

Two main hypotheses were supported by our results: subjects who used the physical Tower of Hanoi outperformed those who were assigned to the computerized version; subjects who verbalized their reasoning performed better than those who were instructed to be silent. Surprisingly, and contrary to our hypothesis, we did not find a main effect for practice. However, there was an interaction effect for practice: participants who practiced the problem showed no difference in performance measures when verbalizing, but participants who did not have that experience completed the task in fewer moves if they made verbalizations. Also with practice, no effects on performance were found for the physical task, but a marginally significant effect was found for the computer task on number of moves. As predicted, participants with no experience exhibited worse performance on the computer puzzle than their counterparts completing the physical model.

References

Ahlum-Heath, M. E., & Di Vesta, F. J. (1986). The effect of conscious controlled verbalization of a cognitive strategy on transfer in problem solving. *Memory & Cognition, 14 (3)*, 281-285.

Anzai, Y., & Simon, H. A. (1979). The Theory of Learning by Doing. *Psychological Review, 86 (2)*, 124-139.

Mueller, S. T. (2012). PEBL: The psychology experiment building language (Version 0.12) [Computer experiment programming language]. Retrieved from http://pebl.sourceforge.net.

Noyes, J. M., & Garland, K. J. (2003). Solving the Tower of Hanoi: does mode of presentation matter? *Computers in Human Behavior, 19*, 579-592.

Zhang, J., & Norman, D. A. (1994). Representations in Distributed Cognitive Tasks. *Cognitive Science, 18*, 87-122.