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The Ultraviolet Behavior of $\mathcal{N}=8$ Supergravity at Four Loops

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We describe the construction of the complete four-loop four-particle amplitude of $\mathcal{N}=8$ supergravity. The amplitude is ultraviolet finite, not only in four dimensions, but in five dimensions as well. The observed extra cancellations provide additional non-trivial evidence that $\mathcal{N}=8$ supergravity in four dimensions may be ultraviolet finite to all orders of perturbation theory.

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An often-expressed sentiment is that point-like quantum field theories based on Einstein's theory of General Relativity, including supersymmetric extensions thereof, are quantum mechanically inconsistent, due to either a proliferation of divergences associated with the dimensionful nature of Newton's constant, or absence of unitarity. A series of recent computations has challenged this widely held belief. In particular, the three-loop four-graviton amplitude [1, 2] in $\mathcal{N}=8$ supergravity [3] exposes cancellations beyond those needed for ultraviolet (UV) finiteness at that order. Novel cancellations occur already in this theory [4, 5] at one loop, related [6, 7] to the remarkably good behavior of gravity tree amplitudes under large complex deformations of external momenta [7, 8], and to the unordered nature of gravity amplitudes [5]. The modern unitarity method [9] implies that extensive UV cancellations occur to *all* loop orders [10], for a class of terms obtained by isolating one-loop sub-amplitudes via generalized unitarity [11], leading to the proposal [6] that the multiloop UV cancellations trace back to the tree-level behavior. These surprising cancellations point to the possible perturbative UV finiteness of the theory.

Interestingly, M theory and string theory have also been used to argue both in favor of the finiteness of $\mathcal{N}=8$ supergravity [12], and that divergences are delayed through nine loops [13, 14]; issues involving the decoupling of certain massive states [15] remain in either case. The non-compact $E_{7(7)}$ duality symmetry of $\mathcal{N}=8$ supergravity [3, 16] may also play a role [7, 17], though this remains to be demonstrated. A mechanism rendering a point-like theory of quantum gravity ultraviolet finite would be novel and should have a profound impact on our understanding of gravity.

Indeed, all studies to date conclude that supersymmetry and gauge invariance alone cannot prevent the onset of UV divergences to all loop orders in four dimensions. In fact, it had been a longstanding expectation that, in generic supergravity theories, four-graviton amplitudes diverge at three loops in four dimensions [18]. Such a divergence would be associated with a counterterm composed of four appropriately contracted Riemann tensors (the square of the Bel-Robinson tensor), denoted by R^4. A recent study [19] explains the known lack of this counterterm [1, 2], both in terms of non-renormalization theorems and an algebraic formalism for constraining counterterms. However, it does predict divergences at $L=5$ loops in dimension $D=4$ and at $L=4$ loops in $D=5$ [31], unless additional cancellation mechanisms beyond supersymmetry and gauge invariance are present.

In contrast, explicit computations of the four-graviton amplitude at successive loop orders have consistently revealed unexpected UV cancellations. Results at two loops strongly suggested [20], and at three loops proved [1, 2] that the R^4 divergence is absent in $\mathcal{N}=8$ supergravity. In addition, UV divergences are absent at three loops in $D<6$. The theory first diverges in $D=6$, and the counterterm has the schematic form $\mathcal{D}^6R^4$, where $\mathcal{D}$ is a space-time derivative acting on the Riemann tensors [2]. The computation described in this letter reveals no UV divergences at four loops in both $D=4$ and $D=5$, specifically ruling out a counterterm of the form $\mathcal{D}^6R^4$ in $D=5$. The origin of the observed UV properties is, however, not yet properly understood.

It is worth noting that more speculative field-theoretic studies have suggested further delays to the onset of divergences. For example, if off-shell superspaces with manifest $\mathcal{N}=6,7$ or 8 supersymmetries were to exist, $D=4$ divergences would be delayed to at least $L=5,6$ or 7 loops, respectively [21, 22]. Locality of counterterms in $\mathcal{N}=8$ light-cone superspace has also been used to argue [17] for an $L=7$ bound. With the additional speculation that all fields respect an eleven-dimensional gauge symmetry, one can even delay the first potential divergence to nine loops [19]. Interestingly, this bound coincides with the one suggested [14] from a string theory non-renormalization theorem [13].

In this letter, we describe the four-loop four-particle amplitude of $\mathcal{N}=8$ supergravity, denoted by $M_4^{\text{4-loop}}$,

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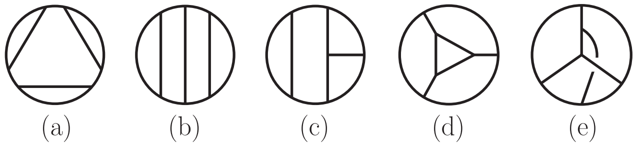

FIG. 1: Vacuum graphs from which one can build the contributing four-point graphs by attaching external legs. They are also useful for classifying the UV divergences.

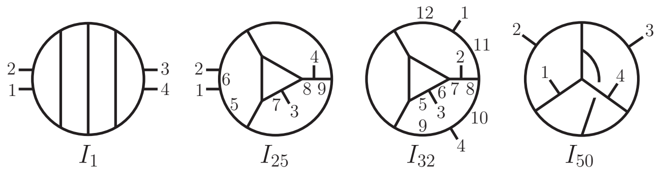

FIG. 2: Four of the 50 distinct graphs corresponding to the integrals composing the result for $M_4^{\text{4-loop}}$.

which we represent as a sum of 50 four-loop integrals I_i,

$$M_4^{\text{4-loop}} = \left(\frac{\kappa}{2}\right)^{10} stu M_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{50} c_i I_i \,. \qquad (1)$$

Here S_4 is the set of 24 permutations of the massless external legs $\{1,2,3,4\}$ with momenta k_i, the c_i are combinatorial factors depending on the symmetries of the integrals, κ is the gravitational coupling, and M_4^{tree} is the corresponding four-point tree amplitude. (All 256^4 four-point amplitudes of $\mathcal{N}=8$ supergravity are related to each other by supersymmetry, which enforces the proportionality of $M_4^{\text{4-loop}}$ to its tree-level counterpart M_4^{tree}.) The Mandelstam invariants are $s=(k_1+k_2)^2$, $t=(k_2+k_3)^2$, $u=(k_1+k_3)^2$. Each integral I_i corresponds to a four-loop graph with 13 propagators and 10 cubic vertices. The 50 graphs may be obtained by attaching four external legs to the edges of the five vacuum graphs in fig. 1. Not all possibilities contribute, however; diagrams containing nontrivial two- or three-point subgraphs, such as all those obtained from fig. 1(a), do not appear in the amplitude. Every integral takes the form

$$I_i = \int \left[\prod_{p=1}^{4} \frac{d^D l_{n_p}}{(2\pi)^D}\right] \frac{N_i(l_j,k_j)}{\prod_{n=1}^{13} l_n^2} \,, \qquad (2)$$

where the propagator momenta l_n are linear combinations of four independent loop momenta l_{n_p} and the external momenta k_j. The numerator polynomial $N_i(l_j,k_j)$ is of degree 12 in the momenta, by dimensional analysis. Generically, we denote loop momenta by l and external momenta by k.

The full amplitude is too lengthy to present in this letter. Rather, we outline its construction and demonstrate some of the relevant UV cancellations. Explicit expressions for the numerators, symmetry factors and propagators may be found online [23]. As examples, the graphs for four integrals, labeled I_1, I_{25}, I_{32} and I_{50} in the online expressions [23], are shown in fig. 2.

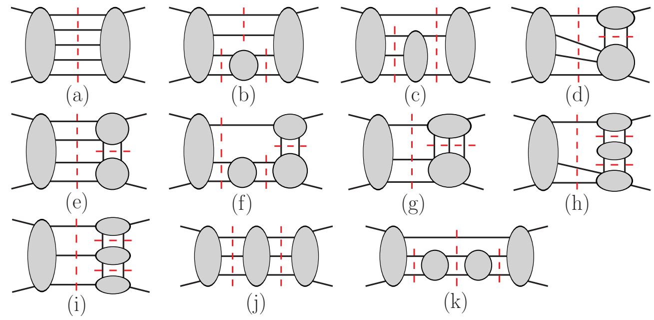

FIG. 3: Evaluating these 11 cuts, along with 15 two-particle reducible cuts, suffices to uniquely determine the four-loop four-point amplitude. Each blob denotes a tree amplitude.

To determine the amplitude, we first construct an ansatz with numerator polynomials $\widetilde{N}_i(l_j,k_j)$ that contain undetermined coefficients. Then we consider generalized unitarity cuts decomposing the four-loop amplitude into a product of tree amplitudes $M_{(i)}^{\text{tree}}$, as shown in fig. 3. Equating the cuts of the ansatz to the corresponding cuts of the amplitude,

$$M_4^{\text{4-loop}}\Big|_{\text{cut}} = \sum_{\text{states}} M_{(1)}^{\text{tree}} M_{(2)}^{\text{tree}} \cdots M_{(n)}^{\text{tree}} \,, \qquad (3)$$

constrains the undetermined coefficients in the ansatz.

As only tree amplitudes enter eq. (3), we follow the strategy [20] of re-expressing the $\mathcal{N}=8$ supergravity cuts in terms of sums of products of related cuts of the four-loop four-gluon amplitude in $\mathcal{N}=4$ super-Yang-Mills (sYM) theory [24, 25]. The strategy relies on the Kawai-Lewellen-Tye (KLT) relations between gravity and gauge theory tree amplitudes [26], facilitated by their recent reorganization in terms of diagrams [27]. While we suspect that a representation of the $\mathcal{N}=8$ amplitude exists in which each N_i is at most of degree four in the loop momenta, it is natural, given the squaring nature of the KLT relations, to first solve the cut constraints with this condition relaxed. We present a solution in which each N_i is at most of degree eight [23]. This representation is sufficient for our purpose of demonstrating UV finiteness in $D=4,5$.

The KLT relations are valid in arbitrary dimensions. Thus, if the $\mathcal{N}=4$ amplitudes are valid in D dimensions, then so are the $\mathcal{N}=8$ amplitudes derived from them. While we do not yet have a complete proof of the $(D>4)$-dimensional validity of the non-planar contributions to the four-loop $\mathcal{N}=4$ amplitudes, we have carried out extensive checks. In particular, ordinary two-particle cuts and cuts isolating four-point subamplitudes extend easily to D dimensions [24, 25, 27]. The full $\mathcal{N}=4$ sYM amplitude, the details of its calculation, and non-trivial consistency checks will be presented elsewhere [25].

Following the method of maximal cuts [2, 28], we first fix those coefficients of the $\widetilde{N}_i(l_j, k_j)$ that contribute when the number of cut propagators is maximal—13 in this case. We then consider cuts with 12 cut lines, fixing the coefficients that appear in terms proportional to single inverse propagators l_n^2 (*i.e.*, contact terms). We continue this procedure down to nine cut lines, considering, in total, 2 906 distinct cuts. At this point, the resulting expression is complete, which we demonstrate using a set of 26 cuts, sufficient to completely determine any four-loop four-point amplitude in any massless theory. The 11 cuts that cannot be straightforwardly verified using lower-loop four-point amplitudes in two-particle cuts are shown in fig. 3.

The UV properties of the amplitude are determined by the numerator polynomials N_i. We decompose them into expressions $N_i^{(m)}$ containing all terms with m powers of loop momenta (and $12 - m$ powers in the external momenta),

$$N_i = N_i^{(8)} + N_i^{(7)} + N_i^{(6)} + \ldots + N_i^{(0)} \,. \qquad (4)$$

There is some freedom in this decomposition, including that induced by the choice of independent l_{n_p} in the loop integral (2). The overall scaling behavior of eq. (2) implies that an integral with $N_i^{(m)}$ in the numerator is finite when $4D-26+m < 0$. For m odd, by Lorentz invariance, the leading divergence trivially vanishes under integration, effectively reducing m by one. Our representation has $m \leq 8$ for all terms; hence the four-loop amplitude is manifestly UV finite in $D = 4$.

Demonstrating UV finiteness in $D = 5$ is more subtle. It requires the cancellation of divergences for $m = 6, 7, 8$. We employ a systematic procedure for extracting divergences from multiloop integrals by expanding in small external momenta [29].

We find that the numerator terms with $m = 8$ can all be expressed solely in terms of inverse propagators l_n^2; those with $m = 7$ have six powers of loop momenta carried by inverse propagators; and those with $m = 6$ have four powers; schematically,

$$\begin{aligned}
&N_i^{(8)} \sim s_a s_b l_j^2 l_n^2 l_p^2 l_q^2 \,, \\
&N_i^{(7)} \sim s_a s_b (k_j \cdot l_n) l_p^2 l_q^2 l_r^2 \,, \\
&N_i^{(6)} \sim s_a s_b (k_j \cdot l_n)(k_p \cdot l_q) l_r^2 l_w^2 + s_a s_b s_c (l_j \cdot l_n) l_p^2 l_q^2 \,,
\end{aligned} \qquad (5)$$

where each s_a denotes s, t or u. After expanding in small external momenta, potential UV divergences enter through vacuum integrals, just as at three loops [1]. Vacuum integrals also exhibit infrared singularities, which we regularize by injecting two fictitious off-shell external momenta at appropriate locations in the graph.

Only 12 of the 50 integrals have a nonvanishing $N_i^{(8)}$; all of them are associated with vacuum diagrams (d) and (e) of fig. 1. For example, the $k^4 l^8$ terms in the numerators of the integrals I_{25} and I_{32} in fig. 2 are

$$\begin{aligned}
N_{25}^{(8)} &= \frac{1}{8} l_5^2 l_6^2 l_7^2 \Big[(30s^2 + 13t^2 + 13u^2) l_9^2 \\
&\qquad - (32s^2 + 19t^2 + 19u^2) l_8^2 \Big] \,, \\
N_{32}^{(8)} &= \frac{1}{8} \Big\{ 2(7s^2 + 7t^2 + 6u^2) l_5^2 l_8^2 l_{10}^2 l_{12}^2 \\
&\qquad + l_9^2 \Big[12(2s^2 - t^2 + 2u^2) l_6^2 l_7^2 l_{12}^2 \\
&\qquad - (24s^2 + 19t^2 + 19u^2) l_5^2 l_8^2 l_{11}^2 \Big] \Big\} \,.
\end{aligned} \qquad (6)$$

All of the l_n^2 factors in eq. (6) cancel propagators in the integrals. Thus, to leading order in the expansion in small external momenta, the $k^4 l^8$ terms in I_{25} and I_{32} reduce to the vacuum diagram $V^{(\mathrm{d})}$ of fig. 1(d),

$$\begin{aligned}
I_{25} &\to -14(s^2 + t^2 + u^2) V^{(\mathrm{d})} + O(k^5) \,, \\
I_{32} &\to +14(s^2 + t^2 + u^2) V^{(\mathrm{d})} + O(k^5) \,.
\end{aligned} \qquad (7)$$

Here we have summed over the S_4 permutations of external legs in eq. (1). Because their combinatorial factors c_{25} and c_{32} are equal [23], the I_{25} and I_{32} contributions cancel at leading order. Similarly, all $k^4 l^8$ contributions in the remaining diagrams cancel, independent of D.

As the $k^5 l^7$ terms cannot generate a leading divergence, we need only inspect the $k^6 l^6$ term to determine the UV properties of the amplitude in $D = 5$. It is necessary to expand all integrands down to $k^6 l^6$. For the 12 the integrals starting at $O(k^4 l^8)$, two derivatives are required with respect to the external momenta k_i, acting on propagators of the form $1/(l_j + K_n)^2$ (where K_n denotes a sum of external momenta). The numerators obtained by expanding the integrals to this order have the schematic form,

$$N_i^{(6)} + N_i^{(7)} \frac{K_n \cdot l_j}{l_j^2} + N_i^{(8)} \left(\frac{K_n^2}{l_j^2} + \frac{K_n \cdot l_j \; K_q \cdot l_p}{l_j^2 l_p^2} \right) . \qquad (8)$$

The additional denominators can lead to doubled or even tripled propagators for the graphs in fig. 1. Vacuum integrals with $l_i^\mu l_j^\nu$ in the numerator can be reduced using Lorentz invariance, $l_i^\mu l_j^\nu \to \eta^{\mu\nu} l_i \cdot l_j / D$, with $D = 5$. After this reduction, the potential UV divergence is described by 30 vacuum integrals. Of these, 23 possess no loop momenta in the numerator, while seven have an $(l_i + l_j)^2$ numerator factor that cannot be reduced to inverse propagators using momentum conservation. There are many ways to expand the original 50 integrals I_i. Shifting the loop momenta in eq. (2) by $d^D l_{n_p} \to d^D (l_{n_p} + k_j)$ leads to different representations of the terms proportional to $N_i^{(7)}$ and $N_i^{(8)}$ in eq. (8), and hence to different forms of the UV divergences in terms of the 30 vacuum integrals. Requiring that the different forms are equal generates identities between vacuum integral divergences. These

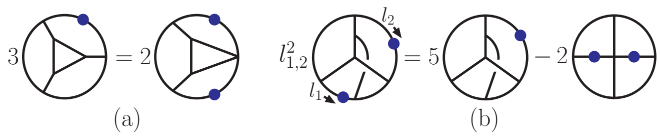

FIG. 4: Two of the vacuum relations used to analyze the $D = 5$ divergence. They are valid in $D = 5 - 2\epsilon$ to order $1/\epsilon$. Dots denote doubled propagators and $l_{1,2}^2 = (l_1 + l_2)^2$ represents a numerator factor inside the integral.

identities suffice to demonstrate cancellation of the $k^6 l^6$ divergence in $M_4^{4\text{-loop}}$.

Independently, we verified the identities by evaluating all 30 vacuum integrals analytically in $D = 5 - 2\epsilon$. To do this we injected external off-shell momenta and factorized the resulting four-loop propagator integrals into the product of one-loop and three-loop propagator integrals, much as we did at three loops [2]. Integration by parts [30] was used to reduce the three-loop propagator integrals to master integrals.

Both the vacuum integral identities and the direct integral evaluation lead to the exact cancellation of the potential $D = 5$ UV divergence. It is striking that this cancellation can be demonstrated using only the consistency of the small momentum expansion. Fig. 4 displays two of the 16 vacuum integral identities needed to demonstrate finiteness in $D = 5$.

The $k^6 l^6$ cancellation rules out a $\mathcal{D}^6 R^4$ counterterm in $D = 5$. It implies that the first potential divergence is proportional to k^8 (since a divergence must have an even power of k), corresponding to $D = 11/2$. As the four-loop four-point $\mathcal{N} = 4$ sYM amplitude diverges in $D = 11/2$ [24, 25], the corresponding $\mathcal{N} = 8$ supergravity amplitude behaves no worse.

In summary, the results presented here demonstrate that the four-loop four-particle amplitude of $\mathcal{N} = 8$ supergravity is UV finite in $D < 11/2$. Finiteness in $5 \leq D < 11/2$ is a consequence of nontrivial cancellations, beyond those already found at three loops [1, 2]. From a traditional vantage point of supersymmetry [18, 19, 22], our results are surprising and lend additional support to the possibility that $\mathcal{N} = 8$ supergravity is a perturbatively consistent quantum theory of gravity.

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