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Dark Energy in the Dark Ages

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Non-negligible dark energy density at high redshifts would indicate dark energy physics distinct from a cosmological constant or "reasonable" canonical scalar fields. Such dark energy can be constrained tightly through investigation of the growth of structure, with limits of $\lesssim 2\%$ of total energy density at $z \gg 1$ for many models. Intermediate dark energy can have effects distinct from its energy density; the dark ages acceleration can be constrained to last less than 5% of a Hubble e-fold time, exacerbating the coincidence problem. Both the total linear growth, or equivalently σ_8, and the shape and evolution of the nonlinear mass power spectrum for $z < 2$ (using the Linder-White nonlinear mapping prescription) provide important windows. Probes of growth, such as weak gravitational lensing, can interact with supernovae and CMB distance measurements to scan dark energy behavior over the entire range $z = 0 - 1100$.

I. INTRODUCTION

At recent times the expansion of the universe has accelerated and dark energy has dominated the total energy density. Earlier, the universe was in a matter dominated, decelerating expansion epoch. This can be seen directly through precision distance-redshift measurements of Type Ia supernovae (SN Ia) at redshifts $z \sim 1$ and precision cosmic microwave background (CMB) measurements at $z \sim 1000$, and indirectly through the presence of large scale structure such as galaxies and clusters of galaxies that require a matter dominated epoch in order to form.

In this article we consider the extent of our knowledge about the behavior of dark energy in the dark ages at $z \approx 2 - 1000$. This is sometimes phrased in terms of "early dark energy" − the fraction of the total energy density at the CMB last scattering surface due to dark energy, though we will present a more general treatment.

While canonical dark energy models with near cosmological constant behavior do not predict any substantial dark energy effect at $z > 2$, limits on early dark energy are already important for cosmological parameter estimation. For the parameter constraints by [1] using weak gravitational lensing data, [2] pointed out that a prior on the early dark energy fraction is needed to remove a second likelihood peak far from the concordance cosmology.

We look to the growth of structure to provide a third window on the nature of dark energy, especially in the intermediate epoch. We suppose that SN Ia will provide accurate characterization of dark energy at $z < 2$, so our focus is to probe for unexpected behavior at higher redshifts where canonical dark energy models have negligible influence. High redshift distance measures are not the appropriate probe for early dark energy, however, as even a drastic jump from cosmological constant equation of state behavior to matter behavior at $z = 1.7$ imparts less than 1% effect to the distance measured to $z = 3$. Similarly, the CMB temperature power spectrum does not care about the dark energy density as such, but rather the expansion history, so an early smooth dark component with an equation of state acting like matter does not disturb the CMB. Therefore we concentrate on the growth of structure for probing dark energy in the dark ages, since growth measurements at $z < 2$ are sensitive to changes at $z > 2$, and to an early smooth dark component regardless of equation of state.

In §II we consider early dark energy where the contribution to the total energy density at CMB last scattering is much larger than the canonical 10^{-9} of the cosmological constant case (ΛCDM). Intermediate redshift effects of "dark age" dark energy are treated in §III. We discuss general limits on the dark energy contribution in §IV, along with assessment of the measurements needed for such constraints.

II. EARLY DARK ENERGY

Constraints on the amount of dark energy at early times necessarily depend on the dark energy model and its evolution over a wide range of redshifts. This can be approached through phenomenological parameterizations, e.g. along the lines of [3, 4, 5]. For their specific models they find limits of a few percent on the contribution of dark energy to the total energy density. We follow this phemenological approach with some variations, focusing on the physics where possible.

The major influence of early dark energy is on the growth of matter density perturbations $\delta \equiv \delta\rho/\rho$. (Note we consider modes on scales below the horizon where dark energy inhomogeneity is negligible due to the sound speed for quintessence being the speed of light.) We solve the linear growth equation for general dark energy evolution [6] with scale factor $a = 1/(1+z)$ by a 4th order Runge-Kutta scheme to find the growth history $g(a) = \delta/a$. A purely matter dominated universe has $g = 1$. Of particular interest are the quantities $g_0 = g(a=1)$, the total growth by the present, and the ratio $R \equiv g(a=0.35)/g_0$. The first measures the linear growth amplitude, proportional to the mass fluctuation amplitude σ_8, and the second provides an excellent indicator of the nonlinear power spectrum [7], at least for canonical dark energy.

One approach to investigating variations in growth behavior from early dark energy is to take the $z \lesssim 2$ cosmology to be essentially fixed by accurate observations such as SN Ia distance measurements; we denote this as "with respect to Λ" and keep the matter density Ω_m (and the present equation of state ratio w_0 where appropriate) the same for the early dark energy and the fiducial (flat, $\Omega_m = 0.28$) ΛCDM case. Another is to compare models where the distance to CMB last scattering, $d_{\rm lss}$, is held fixed, denoted "with respect to CMB" (so Ω_m is changed, and we can also alter the Hubble constant h so as to preserve the quantity $\Omega_m h^2$ and hence essentially the CMB temperature power spectrum). We comment on the differences between the two for each model class.

Note that the amplitude of growth today g_0 is degenerate with the present mass fluctuation amplitude σ_8, or the primordial density perturbation amplitude δ_H. These in turn are correlated with other CMB parameters such as the scalar tilt and reionization optical depth, so this points up the importance of CMB polarization measurements for fuller understanding of dark energy.

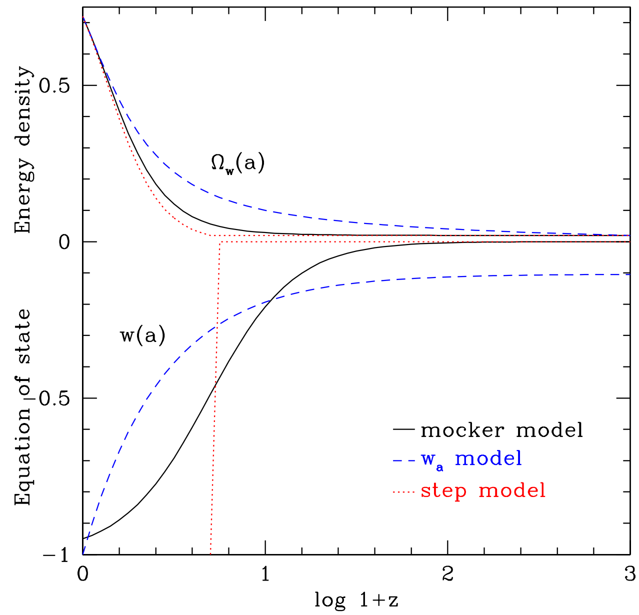

FIG. 1: The dark energy density, and equation of state, shown as a function of redshift for the three classes of models considered here. The fraction of dark energy density today and at the CMB last scattering surface ($z = 1089$) are held fixed.

A. Mocker models

The dark energy density evolution and equation of state (EOS), or pressure to density ratio $w(z)$ are tied together. To attain a substantial level of early dark energy one requires the early equation of state not to be appreciably negative, for a monotonic evolution. Rather than parameterize directly in terms of the dark energy density, we can adopt the physical characteristics of certain dark energy equation of state models. Mocker models [8] of dark energy have the requisite behavior, acting similar to matter ($w \approx 0$) at early times before evolving to an accelerating component and eventually cosmological constant ($w = -1$) behavior. In the equation of state phase plane of w and $w' = dw/d\ln a$ these have the dynamics $w' = Cw(1+w)$ with solutions

$$w(a) = -1 + \left[1 - \frac{w_0}{1+w_0} a^C\right]^{-1}, \qquad (1)$$

$$\rho_{\rm de}(a) = \rho_{\rm de}(1)\, \left[(1+w_0)\, a^{-C} - w_0\right]^{3/C}. \qquad (2)$$

At early times in the matter dominated era, the mocker models have an energy density that scales as matter and so contribute a fixed fraction Ω_e of the total energy density (the dark energy fraction as a function of redshift is shown in Fig. 1 for the three classes of models considered in this section). Rather than using $\{\Omega_{\rm de}, w_0, C\}$ as the parameters we can use $\{\Omega_m, w_0, \Omega_e\}$ and investigate the effects of Ω_e, or the nearly equivalent $\Omega_{\rm de}(z_{\rm lss})$, on observational quantities. We find that for $10^{-6} \lesssim \Omega_e \lesssim 0.1$, the value of $C \approx 0.5 - 2$. Interestingly, this puts the mocker model squarely within the freezing region of the phase space [9].

Figure 2 shows the impact on the growth measures as a function of the early dark energy density. We compare the growth to the cosmological constant case, for the fixed low redshift cosmology and fixed CMB approaches. The greatest deviation is in g_0, but this total growth can be renormalized by a shift in the mass fluctuation amplitude σ_8 (see [10] for a detailed discussion of σ_8 in early dark energy models). Still, values of g_0 or σ_8 differing from the ΛCDM value by more than 10% are already disfavored [11, 12, 13], and future observations should limit the uncertainty to less than 3%. The dark energy density for a mocker model is thus currently limited to less than 1.5% of the total for $w_0 > -0.95$ (as w_0 approaches −1, the mocker model looks more and more like a cosmological constant for the entire matter domination era and constraints vanish). If $w_0 = -0.85$, then the density would be bounded below 0.4% to give less than 10% growth deviation.

Note that using the CMB matching rather than the low redshift matching does not significantly affect the amount of early dark energy density tolerated for the growth. The low z matching in turn still gives $d_{\rm lss}$ within 1.7% for $\Omega_e = 0.02$ (current constraints are 1.8%, derived here from [14]). The mocker models can also be distinguished relative to the constant EOS models with the same present value w_0. (Even for $\Omega_e = 10^{-20}$, $C = 0.15$, not 0, so the mocker model approaches the constant EOS limit very slowly.)

In the absence of bounds on g_0 or σ_8, the constraints from R and the nonlinear power spectrum have a shallower dependence on Ω_e. The deviation in R is at the

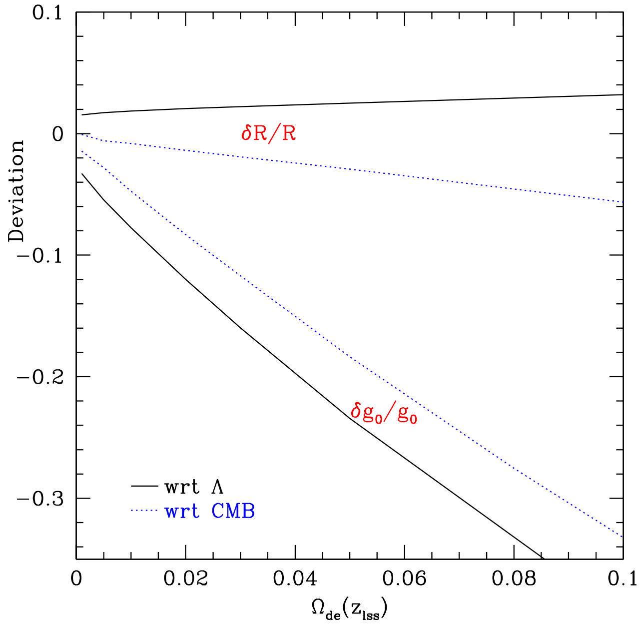

FIG. 2: Increasing the dark energy density at high redshifts causes strong deviations in the total linear growth achieved by today, g_0, as well as changes in the evolution parameter R, for mocker dark energy (here with $\Omega_m = 0.28$, $w_0 = -0.95$). Solid curves show deviations with respect to the ΛCDM case with the same low redshift expansion (i.e. Ω_m), while dotted curves have the same distance to last scattering as the ΛCDM case.

2-6% level (for $w_0 = -0.95$), or roughly a 4-12% effect in the nonlinear power spectrum. From the Linder-White [7] nonlinear mapping prescription, a variation $\Delta R/R$ around ΛCDM translates as

$$\Delta R/R \approx -0.75\,\Delta\Omega_m + 0.21\,\Delta w_0 + 0.052\,\Delta w_a, \qquad (3)$$

where $w(a) = w_0 + w_a(1-a)$ is the standard EOS parametrization. So a measured deviation of 2% in R has the equivalent effect as a change in the cosmological model by 0.027 in Ω_m, or 0.095 in w_0, or 0.38 in w_a. Such a deviation again gives limits around the 2% level in early dark energy density.

While we have assumed that $z < 2$ cosmology will be accurately mapped by SN Ia distance measurements, one should check that uncertainties in Ω_m and w_0 do not significantly blur the bounds on Ω_e. In fact, allowing for covariance between the parameters degrades the estimation of Ω_e by less than 50%: Ω_e is very weakly correlated with the other parameters (for example the correlation coefficient with Ω_m is 0.12). As discussed in §IV, we make no claims for high precision in constraining Ω_e, our aim is factor of two, or even order of magnitude constraints, on non-canonical dark energy.

B. Standard models

One can attain early dark energy within the standard parametrization $w(a) = w_0 + w_a(1-a)$, which has been shown to fit wide varieties of dark energy models, including early dark energy [15]. When the early EOS $w(a \ll 1) = w_0 + w_a$ approaches zero, the early dark energy density can contribute a non-negligible fraction. Note that this is not a constant fraction at high redshift, unlike the mocker case, so we quote $\Omega_{\rm de}(z_{\rm lss})$. Here, a 10% deviation in g_0 is attained for a much lower Ω_e than in the mocker case (due to the evolution of the density fraction, or conversely longer persistence of acceleration effects), except in the CMB-matched approach where Ω_e is again near 1%. (With the low z cosmology matching instead, $d_{\rm lss}$ deviates by 2.3% from the Λ case, for $w_a = 0.6$.) Because of the increased dynamic range of Ω_e, we illustrate the growth deviations in g_0 and R as a function of w_a in Figure 3. It is the presence of early dark energy density that causes the deviation, not any "breakdown" in the w_a parametrization.

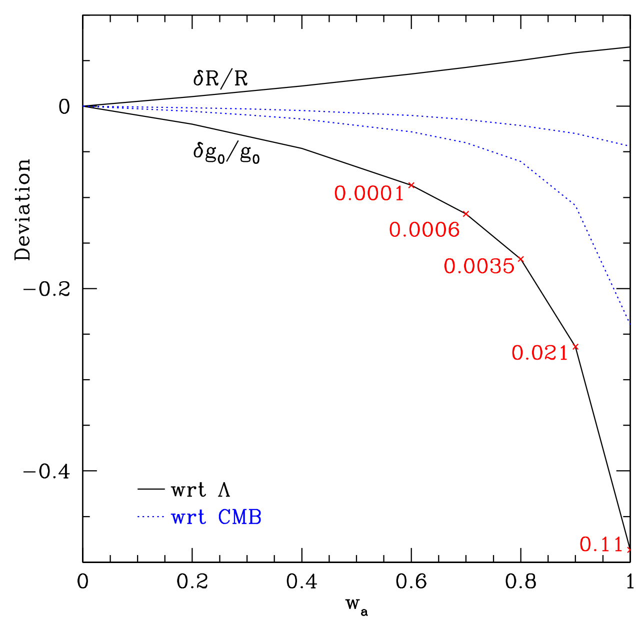

FIG. 3: Dark energy with EOS of the standard form $w(a) = w_0 + w_a(1-a)$ can have a significant effect on growth even with small energy density at high redshift. The dark energy not only does not contribute to clumping, but opposes it through a negative EOS (here $w_0 = -1$). The influence on growth comes from both effects. Values of $\Omega_{\rm de}(z_{\rm lss})$ are indicated at the red x's.

Recall that in Eq. (3) we discussed the change in the key growth ratio R arising from a change in the cosmological parameters; here we see that the change $\Delta R/R$ is linear for a wide range of Δw_a, not just a small perturbation around $w_a = 0$. We also very clearly see one of the advantages of the Linder-White prescription [7], that matching the growth matches the CMB temperature power spectrum. They arranged the canonical w_a model such that the growth ratio R stayed fixed (and hence obtained a highly accurate nonlinear mass power spectrum) and found that the distance to CMB last scattering automatically was nearly perfectly preserved. Here we show the converse, that matching $d_{\rm lss}$ almost perfectly matches the growth ratio R, as illustrated by the upper dotted line in Fig. 3.

C. Step models

The tight bounds on early dark energy density in the previous two models could be a product of the slow evolution of the dark energy properties from matter-like to cosmological constant-like. Here we consider a rapid transition, in the form of a step function where $w(z \geq z_c) = 0$ and $w(z < z_c) = -1$. This will preserve the property that Ω_e is a constant at high redshift in the matter dominated era. (Note that if one makes a step in $\Omega_{\rm de}(z)$ instead, one will have a singularity in $w(z)$.)

Restricting the allowed deviation in g_0 to below 10% implies that $\Omega_e < 2.5\%$, similar to the mocker models. This also corresponds to requiring $z_c > 3.65$. Figure 4 plots the deviations in g_0 and R with respect to ΛCDM, and with respect to the ΛCDM model when $d_{\rm lss}$ is held fixed. Note that in the former case (matching low redshift cosmology) the deviations in R are negligible for a transition $z_c > 2$ since R involves the ratio between growth at $z = 1.86$ and 0. Aside from this, the constraints on Ω_e from the two approaches are very similar. The red x's are labeled with z_c (purely a function of Ω_e), for the low redshift cosmology matching model.

Note that even when the dark energy has $w = 0$, it is not equivalent to matter because it is not allowed to clump. This brings up an interesting point about the impact of dark energy on reducing the source term in the matter growth equation vs. its impact through accelerating the expansion. We address this distinction in the next section.

III. INTERMEDIATE DARK ENERGY

Dark energy has two distinct effects on growth – it changes the Hubble friction term by accelerating the expansion, and it changes the source term by reducing the fraction of the total energy density subject to gravitational clumping and structure formation (recall we assume it itself does not clump). A more subtle consequence of these effects involves the indirect influence of dark energy on the growth "velocity"

$$f = \frac{d\ln\delta}{d\ln a} = 1 + \frac{d\ln g}{d\ln a}. \qquad (4)$$

This last effect effectively resets the boundary conditions, meaning an intermediate, even transient, period of acceleration can strongly influence later growth.

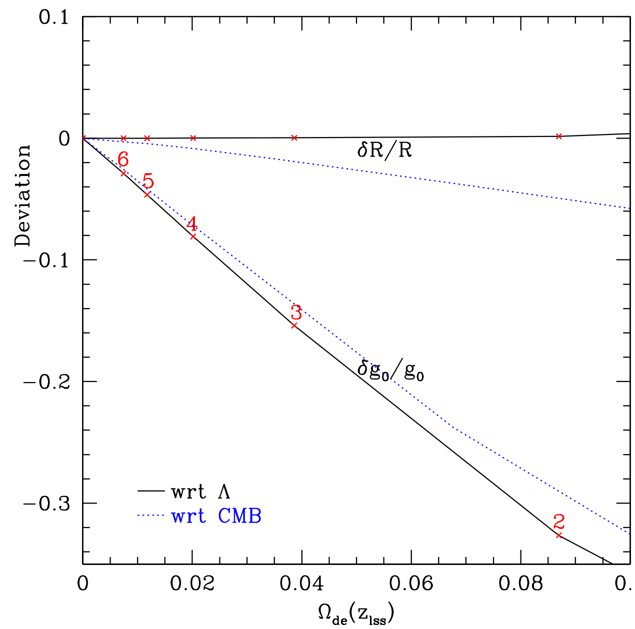

FIG. 4: As Fig. 2 but for stepped dark energy. The EOS jumps from -1 at $z < z_c$ to 0 at $z \geq z_c$; this gives a conservative limit relative to a smoother transition or more negative early EOS. The values of z_c corresponding to $\Omega_{\rm de}(z_{\rm lss})$ are indicated at the red x's.

The models of the previous section affected growth by possessing a substantial dark energy density at early times – where substantial is relative to the $\Omega_{\rm de}(z_{\rm lss}) \sim 10^{-9}$ level of ΛCDM. Without early dark energy it is difficult to significantly affect growth through the first two effects. For example, if we adapt the step model to form a box model, where we take ΛCDM but have a jump to $w(z) = 0$ (the matter value) between z_u and z_d, then we have a period where we diminish the source term of energy density that can clump. This will still have an early dark energy density that is much smaller than one ($10^{-6} - 10^{-9}$), and the growth is negligibly affected[1]. For example, with $w(z) = 0$ between $z = 2-20$, the deviation of g_0 is less than 10% (recall that for the step model with $z_c = 2$ the deviation was 33% and $\Omega_e = 0.087$). This sort of intermediate epoch transition has little observational influence.

We *can* attain strong deviation in the growth without early dark energy through the velocity effect. To investigate this we take a box model of dark energy, but in terms of the total equation of state. This acts like

[1] An early, general analysis of negative equation of state and smooth components appears in [16], extending [17]. The analytic formula for an additional smooth matter component is $g \sim a^{(-5+\sqrt{25-24\Omega_e})/4} \approx a^{-3\Omega_e/5}$.

ΛCDM everywhere except that between z_u and z_d the total equation of state of the universe is set to -1. This gives a transient epoch of strong acceleration that will shut down matter perturbation growth. Once the period is over, the perturbations are free to grow again, but from a state where the growth velocity f had decayed from the matter dominated value of unity. For one indication of how important the growth velocity is, note that cold dark matter perturbations can grow in the radiation dominated epoch if given initial velocity: $\delta \sim \ln a$ (and indeed $\delta \sim a$ if $w_{\rm tot} = +1$).

Figure 5 shows that the length of this transient acceleration epoch can be quite short, a fraction of an e-fold $\Delta \ln a$ ($\equiv \ln[(1+z_d)/(1+z_u)]$), and still cause appreciable deviation in growth behavior. For a transition at $z_u > 4$, a duration of $\Delta \ln a > 0.05$, i.e. 5% of that epoch's Hubble time, changes the present growth by more than 10%. This tightly constrains models of oscillating or stochastic dark energy [18, 19], which have intermediate epochs of dark age dark energy domination[2]. Even recent transitions, $z_u > 0.5$, would still show 10% deviations for what might be considered short durations $\Delta \ln a = 0.3 \ll 1$. Constraints from R can be even tighter and give direct limits on the length of dark energy domination $\Delta \ln a$.

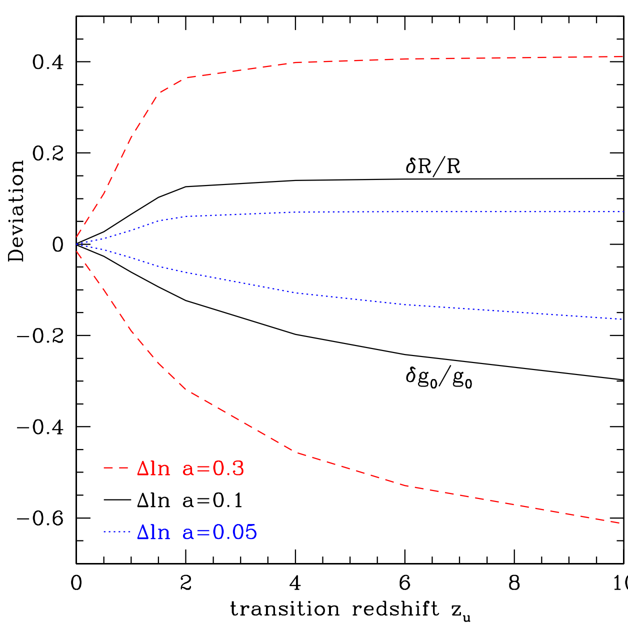

FIG. 5: An intermediate period of domination by dark energy in the dark ages can have a significant effect on growth. This model takes a LCDM universe and steps the total equation of state down to -1 for a period $\Delta \ln a$ ending at a_u. This halts the growth of structure, and even when the universe returns to matter domination the growth is slower than $\delta\rho/\rho \sim a$. This shows the influence of the "velocity" of growth rather than of dark energy density.

[2] Such models attempt to ameliorate the coincidence problem (that dark energy is dominating during our one e-fold epoch of observations out of the perhaps 10^{54} times expansion since the beginning of inflation) by saying that acceleration happens periodically.

IV. CONCLUSION

The nature of dark energy is so little known that we should test its behavior however possible, even in the "dark ages" $z \approx 2 - 1000$ where standard models predict no effect. The growth of large scale structure provides key windows on this epoch and has the potential to see early dark energy density or transitions in its equation of state. Because of the uncertainty in the mass fluctuation amplitude, i.e. σ_8, we should employ not only the total growth factor but the growth rate, through the evolution of the mass power spectrum.

For measurements of 10% precision in total growth or, say, 5% in the power spectrum evolution – the latter related to the growth ratio $R = g(a = 0.35)/g(1)$ through the Linder-White nonlinear mapping prescription – the analysis for a variety of physical behaviors indicates that early dark energy density of less than $\sim 2\%$ of the total energy density does not affect observations. This is a fairly conservative limit, as we have considered the extreme cases of dark energy behavior, e.g. both slow and instantaneous transitions, and parameter degeneracies will also degrade limits (though we find this is to be a surprisingly weak effect).

Early dark energy can also affect the details of nonlinear structure, such as halo formation, concentration, cluster abundances, and lensing statistics – see [20] and references therein. The cosmic microwave background can be sensitive to early dark energy (if it does not act like matter), but not necessarily dark age dark energy (at $z < 1000$), since the CMB relies on the integrated dark energy density, which is generally dominated by low redshifts (an exception is the early time Sachs-Wolfe effect). The CMB plays an important complementary role through constraining σ_8 for use of the total growth factor. Apart from this, we find that fixing the distance to CMB last scattering does not greatly affect growth deviations (except for standard dark energy where this matching was pointed out by [7]).

Intermediate dark energy, where there is insignificant early dark energy density but a dark ages period of acceleration, can be tightly constrained. This impacts models that try to solve the coincidence problem through periodic episodes of dark energy domination. Such periods must last a seemingly unnatural 5% or less of a Hubble (e-fold) time.

These analyses of physical behaviors of dark energy in the dark ages indicate that one can quantitatively use the observations of the growth of structure, in conjunction with accurate supernovae distance and CMB measurements, to have confidence that the usual scenario of dark energy as a late time phenomenon is valid.

Acknowledgments

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