Covariance Spectroscopy for Fissile Material Detection

DOE/NV/25946--722

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Abstract

Nuclear fission produces multiple prompt neutrons and gammas at each fission event. The resulting daughter nuclei continue to emit delayed radiation as neutrons boil off, beta decay occurs, etc. All of the radiations are causally connected, and therefore correlated. The correlations are generally positive, but when different decay channels compete, so that some radiations tend to exclude others, negative correlations could also be observed. A similar problem of reduced complexity is that of cascades radiation, whereby a simple radioactive decay produces two or more correlated gamma rays at each decay. Covariance is the usual means for measuring correlation, and techniques of covariance mapping may be useful to produce distinct signatures of special nuclear materials (SNM). A covariance measurement can also be used to filter data streams because uncorrelated signals are largely rejected. The technique is generally more effective than a coincidence measurement. In this poster, we concentrate on cascades and the covariance filtering problem.

Goals and Relevance to NA-22

- Develop and implement covariance techniques to enhance signal-to-noise ratios of nuclear radiation data
- Model a covariance measurement of gamma cascades as a first step towards fission modeling
- Model covariance measurements of correlated radiations emanating from nuclear fission (spontaneous and stimulated)
- Attempt to implement covariance signatures as a means of identifying SNM

These goals are pertinent to the NA-22 portfolios "Detecting SNM Movement / Radiation Sensing" (NN2001-03) and "Signatures and Observables" (NN2001-09).

Deliverables

FY 2009
- Annotated bibliography
- Covariance model of nuclear cascades and validation
- Preliminary covariance model of spontaneous fission

FY 2010
- Refined model of spontaneous fission and validation
- Preliminary covariance model of stimulated fission

FY 2011
- Validation of covariance model for active neutron interrogation
- Covariance signatures of SNM via active neutron interrogation

Some Background

A covariance technique was used in 1956 [1], and again in 1980 [2,3], to measure lifetimes of intermediate states of nuclear cascades. For some reason, the technique did not catch on, presumably because of computational limitations of contemporarily existing hardware.

In the early 1990s, covariance mapping [4] gained popularity in the atomic physics community. These applications illustrated that "hidden" information can be extracted from spectra.

In the late 1990s, a similar technique of higher order statistical signatures for fissile materials began at Oak Ridge National Laboratory [5]. The high multiplicity of particles emitted from the fission process readily suggests that covariance techniques might be invaluable.

Present-day computational capabilities now make real-time software implementations of covariance measurements viable, and the techniques can be used for filtering and "fingerprinting." Used in conjunction with other techniques, such as Kalman filtering and Wald's sequential testing, significant enhancements of existing radiation detectors become possible without additional hardware.

Covariance is a Measure of Correlation

1. $Cov(X,Y) = r\,\sigma_X\,\sigma_Y$
   from the probability distribution: r is the correlation coefficient, the σ's are variances
2. $Cov(X,Y) = \langle (X-\langle X\rangle)(Y-\langle Y\rangle)\rangle = \langle XY\rangle - \langle X\rangle\langle Y\rangle$
   estimate from the sample: brackets mean expectation value, $\langle XY\rangle$ is the coincidence term

The term $\langle XY\rangle$ is the statistic measured in coincidence experiments.

Combining 1 and 2 and solving for the coincidence yields:

$$\langle X,Y\rangle = r\,\sigma_X\,\sigma_Y + \langle X\rangle\langle Y\rangle$$

The coincidence term contains contributions from noise and accidentals.
Therefore, coincidence is not a direct measure of correlation.

Covariance is a direct measure of correlation!

Covariance for a Poisson Probability Distribution

Let $X = A + B$, and $Y = A + C$, where A, B, and C are random variables governed by Poisson distributions. One way of writing the joint probability distribution function for X and Y is:

$$P(X_k, Y_m) = e^{-(\lambda_A+\lambda_B+\lambda_C)}\frac{\lambda_B^k}{k!}\frac{\lambda_C^m}{m!}\sum_{i=0}^{\min(k,m)}\binom{k}{i}\binom{m}{i}i!\left(\frac{\lambda_A}{\lambda_B\lambda_C}\right)^i$$

where it is understood that $X_k = k$, etc. The expectation values are:

$$\langle X\rangle = \lambda_A + \lambda_B \qquad \langle Y\rangle = \lambda_A + \lambda_C \qquad \langle XY\rangle = \lambda_A(1+\lambda_A) + \lambda_A\lambda_B + \lambda_A\lambda_C + \lambda_B\lambda_C$$

And the covariance is $Cov(X,Y) = \lambda_A$

This result is true even if B and C are not Poissonian. In general, the covariance is the variance of A (i.e., $Cov(X,Y) = \sigma_A$).

Combined Probability Distributions

- The initial radioactive decay is Poissonian, *f(n)*, but the gamma releases and their detections involve other distributions.
- Both gammas have energy distributions: *g(E)*
- The gammas have exponential time distributions: *h(t)*
- The gammas are usually geometrically correlated: *w(θ)*
- The detectors have solid angle and energy efficiencies: *e(E,Ω)*

The Poisson distribution for the decay is most important for considerations of the covariance measurement, and the other distributions are either integrated, or are sampled, over a small region. Those other distributions can, however, complicate the problem.

Probabilities for Data Acquired in Time Slices

(such as data acquired from oscilloscope traces)

γ_1 and γ_2 in the same time slice:

γ_1 is uniformly distributed $\rightarrow p_1 = dt/\delta t$

γ_2 is exponentially distributed $\rightarrow p_2 = \lambda e^{-\lambda(t'-t)}\,dt'$

$$P(\lambda\delta t) = \int_0^{\delta t}\frac{dt}{\delta t}\int_t^{\delta t}\lambda e^{-\lambda(t'-t)}dt'$$

$$P(\lambda\delta t) = \frac{\lambda\delta t - 1 + e^{-\lambda\delta t}}{\lambda\delta t}$$

γ_1 and γ_2 in different time slices:

γ_1 is uniformly distributed $\rightarrow p_1 = dt/\delta t$

γ_2 is exponentially distributed $\rightarrow p_2 = \lambda e^{-\lambda(t'-t)}\,dt'$

The second time slice begins at t_2.

$$P(\lambda\delta t) = \int_0^{\delta t}\frac{dt}{\delta t}\int_{t_2}^{t_2+\delta t}\lambda e^{-\lambda(t'-t)}dt'$$

$$P(\lambda\delta t) = 4\frac{e^{-\lambda t_2}}{\lambda\delta t}\sinh^2\left(\frac{\lambda\delta t}{2}\right)$$

This work was done by National Security Technologies, LLC, under Contract No. DE-AC52-06NA25946 with the U.S. Department of Energy.

Precision and Scaling Behavior of Covariance

The covariance of the compound Poisson process is the variance of the underlying common variable. We can estimate its precision.

A maximum likelihood argument for its estimation: $P(\lambda_A \mid \{N\}) = \prod_i e^{-\lambda_A}\frac{\lambda_A^{n_i}}{n_i!}$

$$\frac{d}{d\lambda_A}\log(P(\lambda_A \mid \{N\})) = \frac{d}{d\lambda_A}\left(\sum_i -\lambda_A + n_i\log(\lambda_A) - \log(n_i!)\right) = 0$$

$$\overline{\lambda_A} = \frac{\sum_i n_i}{N} \qquad \frac{d^2}{d\lambda_A^2}\left(\log(P(\lambda_A \mid \{N\}))\right) = \frac{-N}{\overline{\lambda_A}}$$

And adding in the detection efficiencies: $S/N \sim \sqrt{\varepsilon_X\varepsilon_Y\overline{\lambda_A}N}$

Obviously, high detection efficiencies and large samples are desirable.

Monte Carlo Simulations

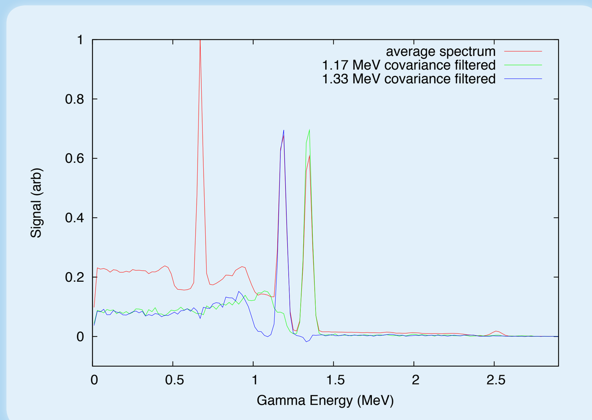
To start, we have written a simple Monte Carlo program to model the cascades problem. It makes use of F8 tallies from MCNP5 as discrete probability distributions for detector response functions.

PoliMi [6] (an MCNP4c derivative) will be used for some of the later work.

Ultimately, we intend to use GEANT4 [7] to do the fission simulations.

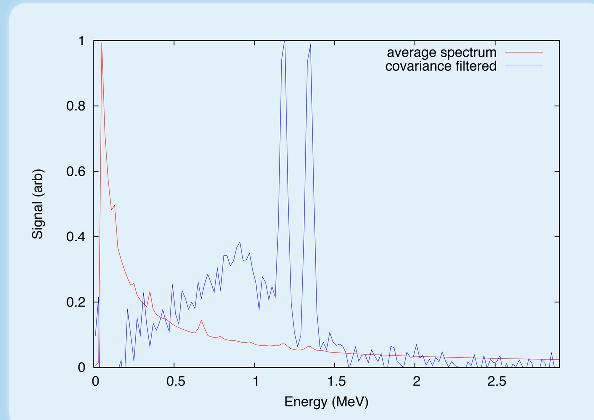
The following results were obtained with our Monte Carlo program for cascades.

Monte Carlo Simulations of Data Filtering



^{60}Co – 1.17 and 1.33 MeV
^{137}Cs – 0.66 MeV

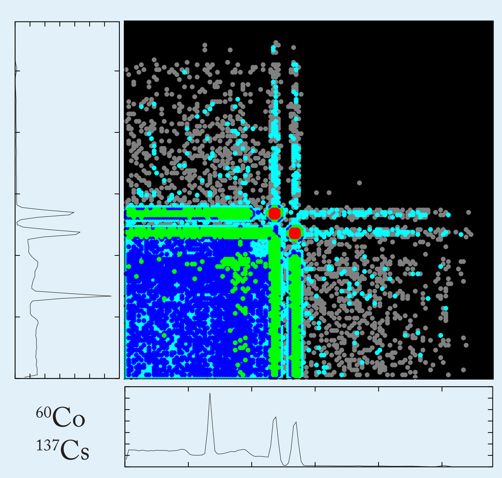
The ^{60}Co gammas are correlated, thus they can be separated from the rest of the spectra. Note that the sum peak at 2.5 MeV is suppressed.



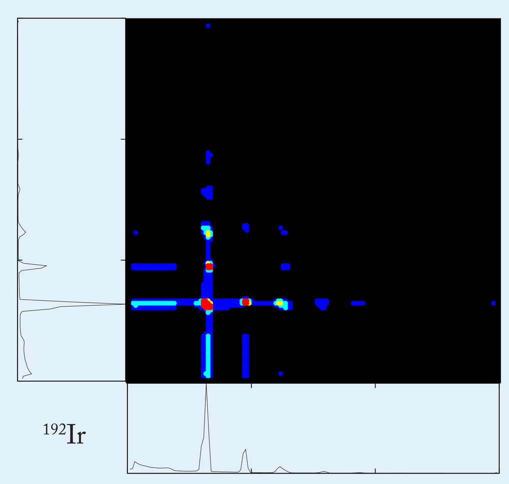
^{60}Co – 1.17 and 1.33 MeV
^{152}Eu – 10 gammas included
^{137}Cs – 0.66 MeV
Noise – 1/E

The spectrum has been covariance-filtered to extract the two lines from ^{60}Co.

Covariance Maps as Fingerprints?



^{60}Co
^{137}Cs



^{192}Ir

Multidimensional covariance maps should allow such techniques as pattern matching, principle component analysis, etc., to exploit data to identify sources. False color-coded covariance maps offer an intuitive presentation for 2-dimensional tables, but the inclusion of extra dimensions (e.g., energy-energy-temporal) presents a real challenge for data interpretation. Dimensional reduction, or projection, will probably be required for decision making to become reliable and efficient. Bayesian prediction methods should play a key role.

Data for Validation

Data with preserved correlations are somewhat rare, but we have found a couple candidates. The utilization of other people's data, however, is challenging. The data sets are archived in non-standard binary formats, so special software needs to be written to extract the data. Documentation of the measurements is often lacking.

An interesting data set from the early 1990s called BIGOH has calibration spectra with correlated gammas, and the data set contains spectra from significant quantities of SNM in real-world scenarios.

ORNL has a significant quantity of potentially useful data, but a certain effort will be required to locate the data and its accompanying documentation for the experimental conditions. These data, of course, are in non-standard binary formats.

Conclusions

Monte Carlo simulations suggest that covariance techniques can be used to filter nuclear data streams to extract correlated gamma peaks. The enhancement of signal-to-noise can be a few orders of magnitude, depending upon other characteristics in the spectra. The precision of a covariance measurement scales in the manner of Poisson counting statistics. Certain systematics often found in average spectra, such as gamma sum peaks, are largely suppressed.

Another covariance application under investigation is the use of generalized covariance maps as signatures of correlated features of fission emissions. The energy, temporal, and spatial correlations should provide distinct multi-dimensional "fingerprints" that can be used for positive identification (maybe even quantification) of SNM. Multidimensional fingerprints will require some sort of semantic reduction to be useful.

References

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