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A Comparison of Approximate Reasoning Results Using Information Uncertainty

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Abstract. Approximate Reasoning (AR) is a useful alternative for modeling linguistic values provided by subject matter experts; however, AR models can produce many competing results. Associated with each competing AR result is a vector of linguistic values and a respective degree of membership in each value. A suitable means to compare and segregate AR results would be an invaluable tool to analysts and decisions makers. A viable method would be to quantify the information uncertainty present in each AR result then use the measured quantity comparatively. One issue of concern for measuring the information uncertainty involved with fuzzy uncertainty is that previously proposed approaches focus on the information uncertainty involved within the entire fuzzy set. This paper proposes extending measures of information uncertainty to AR results, which involve only one degree of membership for each fuzzy set included in the AR result. An approach to quantify the information uncertainty in the AR result is presented.

Key words: Information measure, uncertainty measure, information entropy, linguistic values, inference model, fuzzy logic

1 Introduction

An Approximate Reasoning (AR) model is a useful alternative to a probabilistic model when there is a need to draw conclusions from information that is qualitative. For certain systems, much of the information available is elicited from subject matter experts (SME). One such example is the risk of attack on a particular facility by a pernicious adversary. In this example there are several avenues of attack, i.e. scenarios, and AR can be used to model the risk of attack associated with each scenario. The qualitative information available and provided by the SME is comprised of linguistic values which are well suited for an AR model but meager for other modeling approaches. Natural language tends to be interpreted slightly differently by various individuals [13, 8]. The linguistic values used by SME's are no different and have a tendency to be vague and imprecise. There is an uncertainty associated with natural language which is commonly called *fuzzy uncertainty* [12, 13, 8]. For example, an SME may indicate that the likelihood the adversary smuggles a device through a border crossing is "high" or that it is "somewhat likely". The exact meaning of "high" or "somewhat likely" may be interpreted slightly differently by different individuals; however, these linguistic values are often the values the SME is most confident in and comfortable providing. Fuzzy uncertainty is different from *random uncertainty*, where random uncertainty arises due to chance and deals with specific and well defined values such as the number on the top face of a die that is thrown. Random uncertainty is referred to as an aleatoric uncertainty and fuzzy uncertainty is referred to as an epistemic uncertainty. Epistemic uncertainty may be reduced to aleatoric uncertainty where as, aleatoric uncertainty is non reducible uncertainty [10, 16].

Linguistic values such as "high", "medium", and "low" describe several states or conditions and are considered sets. The boundary that defines any one of these sets is unclear or fuzzy and thus these sets are called *fuzzy sets*. Vague and imprecise uncertainty is represented marvelously using fuzzy sets. The degree of membership of a particular state, i.e. element, provides an indication of the fuzzy set's ability to describe the element. The degree of membership for all the elements in a fuzzy set are defined by a membership function. A general overview of fuzzy sets and AR is provided in Section 2, an in depth discussion of fuzzy set theory and AR can be found in: [9, 12, 13, 17–19]. An AR model uses the degree of membership of a elements in fuzzy sets to draw conclusions about a system such as risk of attack on a facility. The AR result is comprised of a vector of various fuzzy sets used to describe risk and a respective degree of membership in each fuzzy set. Each attack scenario thus has an associated vector of risk values. Decision makers are interested in the confidence associated with each of the competing alternatives. The quantity of uncertainty present in the result is related to the confidence [3]. That is, the less uncertainty present in the resulting alternative the more confidence one can have in the result. By measuring the information uncertainty present in each resulting alternative, the possible alternatives can be ranked ordered and the most credible alternatives can be determined.

The quantification of information uncertainty for random uncertainty was addressed by Shannon [14]. Klir [6] elaborates on Shannon's measure of information uncertainty and identifies *conflict* as the basis for the information uncertainty measured by Shannon. The measure of information uncertainty proposed by Shannon works as follows: there exists a regular die with six faces all of which are equally to be thrown and there exists a six sided trick die with one side being twice as likely to be thrown than the rest. The regular die has more information uncertainty than the trick die because all sides are equally likely to occur in the regular die. The trick die is less uncertain because one side is twice as likely to be thrown than each of the remaining five; thus, one can have more confidence in the result. De Luca and Termini [2] extended Shannon's measure of information uncertainty to fuzzy uncertainty in a fuzzy set while others also presented alternative measures, see Yager [15], and Higashi and Klir [5]. Pal and Bezdek [11] provide a good summary of many of the approaches used to measure fuzzy uncertainty. These approaches are intended to quantify the information uncertainty contained in a fuzzy set using the membership function for all the elements in the set; however, an AR result only involves one degree of membership in each fuzzy set. Section 3 discusses the quantification of information uncertainty in a fuzzy set and introduces a modified approach to quantify information uncertainty in an AR result. This paper then concludes with a simple application of the proposed method in Section 4.

2 Approximate Reasoning

Often the only information available for modeling some systems is qualitative. For example, an SME may indicate that the occurrence of a particular result is "highly likely", "somewhat likely", or "negligible" and the resulting consequences are "extremely costly", "moderately costly", or "insignificant". These expressions are called propositions and the kind of uncertainty associated with these propositions is from vagueness, imprecision, and/or a lack of information regarding a particular element of the system. This type of uncertainty is collectively called fuzzy uncertainty. Fuzzy set theory provides a means for representing the type of uncertainty contained in these propositions. Propositions of this type are commonly referred to as *fuzzy propositions* and express subjective ideas that can be interpreted slightly differently by various individuals. Reasoning using fuzzy propositions is referred to as approximate reasoning [13, 8]. This section briefly describes fuzzy set theory and the logical expressions or propositions known as fuzzy logic that form the basis for AR, the reader is referred to [12] for an in depth description of each.

2.1 Fuzzy Set Theory

A collection of objects having similar characteristics defines a universe of discourse, X. The individual elements in X are denoted as x, with the same notations used for Y and y, and Z and z. The elements can be grouped into various sets, such as $\tilde{\mathrm{A}}$, $\tilde{\mathrm{B}}$, or $\tilde{\mathrm{C}}$. The value of $\tilde{\mathrm{A}}$, $\tilde{\mathrm{B}}$, and $\tilde{\mathrm{C}}$ may represent something like "high" which are approximate values, that is, it is not precise or well defined. The elements of a fuzzy set can be mapped to a universe of membership values using a function theoretic form. If an element x is a member of the set $\tilde{\mathrm{A}}$, then this mapping is given by Eq. (1). A typical mapping of $\tilde{\mathrm{A}}$ is shown in Fig. 1.

$$\mu_{\tilde{\mathrm{A}}}(x) \in [0,1] \,. \tag{1}$$

The complement of $\tilde{\mathrm{A}}$ is defined as:

$$\mu_{\bar{\tilde{\mathrm{A}}}}(x_i) = 1 - \mu_{\tilde{\mathrm{A}}}(x_i). \tag{2}$$

The mapping for the complement is also shown in Figure 1.

2.2 Fuzzy Set Theory and AR

Now suppose that an SME indicates that values $\tilde{\mathrm{A}}$ and $\tilde{\mathrm{B}}$ for elements x_i and y_j, respectively, infers a particular value $\tilde{\mathrm{F}}$ for z_k. The information provided is

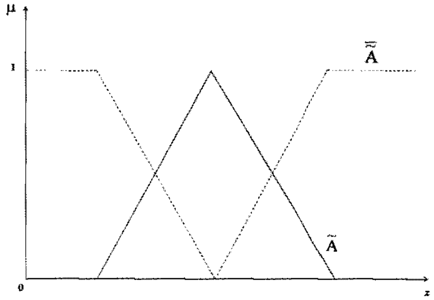

Fig. 1. Membership function for Ã and complement of Ã (Courtesy of Ross 1995)

considered a rule governing the outcome z_k and can be presented as follows:

Rule 1 IF x_i is Ã and y_j is B̃ THEN z_k is Ẽ

All the rules governing the particular outcome z_k involving values for x_i and y_j can be grouped together into a rule base, see Table 1. Now consider the situation when both x_i and y_j can be described by more than on value. In such a situation x_i and y_j have membership in each value that describes them and the extent of this membership is represented using the degree of membership. The values of x_i and y_j are used to identify the governing rule and infer the value of z_k. The inferred value of z_k will have an associated degree of membership which results from the conjunction ($\wedge$), i.e. taking the minimum value, of the degree of membership for x_i and y_j in the values included in the governing rule. Take for example the rule specified above with $\mu_{\tilde{A}}(x_i) = 0.3$ and $\mu_{\tilde{B}}(y_j) = 0.6$, results in a $mu_{\tilde{E}}(z_k) = 0.3$. Another applicable governing rule may be:

Rule 2 IF x_i is B̃ and y_j is B̃ THEN z_k is Ẽ

with $\mu_{\tilde{B}}(x_i) = 0.7$ and $\mu_{\tilde{B}}(y_j) = 0.6$, which results in $\mu_{\tilde{E}}(z_k) = 0.7$. Both Rule 1 and Rule 2 result in the value Ẽ but there are now two different values for the degree of membership. That is, either Rule 1 or Rule 2 is applicable and the disjunction ($\vee$), i.e. taking the maximum value, of $\mu_{\tilde{E}}(z_k) = 0.3$ and $\mu_{\tilde{E}}(z_k) = 0.7$ results in $\mu_{\tilde{E}}(z_k) = 0.7$. The conjunction and disjunction operations are used when the logical *and* and *or* are encountered. In each of the rules the logical *and* is encountered and the conjunction operation is used to determine the resulting degree of membership. The logical *or* is encountered in the example because either Rule 1 *or* Rule 2 in result Ẽ. Additional logical operations can be found in [13].

Table 1. Rule Base

Rule Base		Universe of Discourse X		
		Ã	B̃	C̃
Universe of Discourse Y	Ã	F̃	Ẽ	G̃
	B̃	F̃	Ẽ	Ẽ
	C̃	Ẽ	G̃	G̃

3 Quantification of Information Uncertainty in Approximate Reasoning

The quantification of information uncertainty is often referred to as a measure of *entropy* [7]. Claud E. Shannon introduced the concept of entropy as a measure of the average information content missing when the value of a random variable is unknown [14]. Klir proves that Shannon entropy measures conflict in a probability distribution [6]. Shannon's measure of conflict has the form

$$S(p) = -\sum_{x \in X} p(x) \log_2 p(x), \tag{3}$$

Others have extended Shannon's entropy measure from random uncertainty to fuzzy uncertainty [2, 11]. De Luca and Termini's [2] measure for the entropy of a fuzzy set is similar to Shannon's but conceptually different. Shannon measures the conflict due to random uncertainty while De Luca measures the conflict due to the fuzzy uncertainty. The measure of conflict due to fuzzy uncertainty in a fuzzy set can be determined from the membership functions for the fuzzy set and its complement; as De Luca and Termini's proposed in the following equation [2]:

$$D(\tilde{\mathrm{A}}) = -\sum_{i=1}^{n} \mu_{\tilde{\mathrm{A}}}(x_i) \log_2 \mu_{\tilde{\mathrm{A}}}(x_i) + \mu_{\bar{\tilde{\mathrm{A}}}}(x_i) \log_2 \mu_{\bar{\tilde{\mathrm{A}}}}(x_i), \tag{4}$$

Pal and Bezdek [11] present several previously proposed alternative approaches to measure fuzzy uncertainty in a fuzzy set.

Another type of entropy, known as nonspecificity, reflects the ambiguity in specifying the exact solution [11]. Hartley [4] first proposed measuring the lack of specificity which is simply related to the number of alternatives present. Klir [6] simply defines the Hartley measure of uncertainty as

$$H(f_E) = \log 2|E|, \tag{5}$$

where f_E is any function of the subset E. The nonspecificity of an AR result can be determined using Equation 5 and considering that f_E instead represents vector consisting of values in the AR result . The Hartley measure has been extended to probability distribution functions and membership function which are not discussed here and the reader is referred to [6, 7] for an in depth discussion.

As discussed in the previous section, AR uses the degree of membership for a linguistic values to predict the outcome of a system. The outcome resulting from the AR is expressed as a vector of linguistic values and a respective degree of membership. That is, only one membership value results for each linguistic value. The conflict due to fuzzy uncertainty as quantified from methods such as De Luca and Termini [2] and those summarized by Pal and Bezdek [11] rely on the degree of membership for all the elements in the fuzzy set. In an AR model the conflict is not among one fuzzy set but several, that is, there is conflict among all the fuzzy sets that have a degree of membership greater than 0. We propose to quantify the conflict present in the AR results rather than the conflict in anyone particular fuzzy set. The approach presented is similar to De Luca's but conceptually different in that both approaches are concerned with fuzzy uncertainty although the conflict is among competing fuzzy sets included in the AR result. A familiar similar to that used in Equation 4 is used to quantify the conflict in the AR result

$$C(\boldsymbol{R}) = -\sum_{i=1}^{n} \mu_{\tilde{\mathrm{R}}_i}(x) \log_2 \mu_{\tilde{\mathrm{R}}_i}(x) + \mu_{\bar{\tilde{\mathrm{R}}}_i}(x) \log_2 \mu_{\bar{\tilde{\mathrm{R}}}_i}(x), \tag{6}$$

where $\boldsymbol{R}$ is the vector consisting of the degree of membership for each fuzzy set in the AR result, and C is the conflict, $\tilde{\mathrm{R}}_i(x)$ are the degree of membership in the fuzzy set $\tilde{\mathrm{R}}_i$.

The nonspecificity in an AR result can also be measured using Equation 5 and the number of fuzzy sets with a non-zero degree of membership. Random uncertainty may be present in available information elicited from an SME but it is at an epistemic level and captured in the linguistic values provided by the SME. As a result the conflict due to random uncertainty is captured by 6. Both 5 and 6 have units of bits of information from the use of the logarithm base 2; therefore, the values derived from Equations 5 and 6 are summed to obtain the bits of information uncertainty for each AR result.

4 Application of the Proposed Approach

In each scenario, an adversary attempts to attack the United States (US) with a highly enriched uranium (HEU) weapon. The weapon originates from outside of the US and the adversary chooses to enter the US through a cargo vessel water venue. The water venue does not have detection devices, therefore the likelihood of detecting and interdicting the device is assumed to be extremely unlikely. The adversary is limited to either having no prior information regarding the defensive architecture or having limited information on the defensive architecture. The defensive architecture has an intelligence warning level that is either weak or not available and a characteristic threat response level that is either green or yellow.

In Scenario 1, the adversary has no prior information regarding the defensive architecture; therefore his choice of cargo vessel and the location of the water venue are less clear. Additionally, the economic consequences of a successful attack are assumed to be high. In Scenario 2, the adversary has retained some information regarding the defensive architecture and his choices are more clear. Since the adversary has more information it is assumed that the adversary would maximize his likelihood of success by choosing a venue that is assumed to be monitored less frequently, but have lower consequences than Scenario 1. Scenario 3 is similar to Scenario 2, except the defensive architecture has no prior intelligence regarding the adversarys choices, therefore it is assumed the adversary could choose and attack any type of venue ranging from low to high consequences.

The rule base of Table 2 is used to infer likelihoods using the information as displayed in Figure 2, for scenario 1. The risk rule base of Table 3 is then used to infer risk from the final resulting likelihood and the consequence of the current scenario.

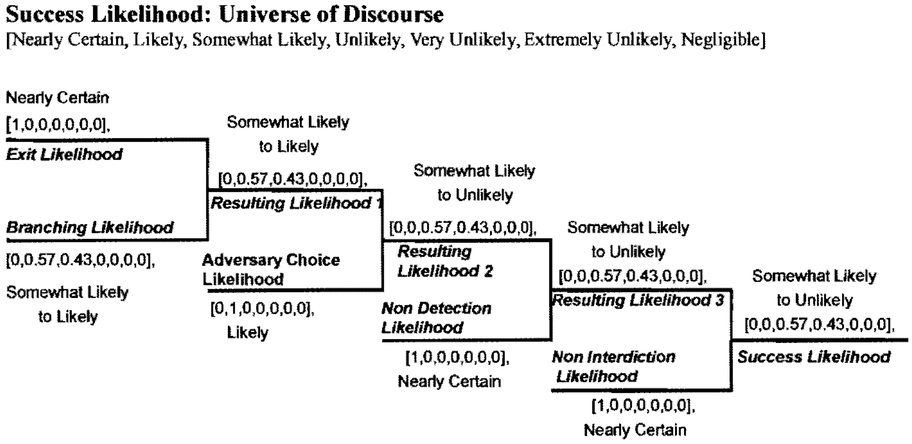

Fig. 2. Likelihood Reasoning for Scenario 1

4.1 Scenario 1, S1

Radiological/nuclear attack on a target in the contiguous United States. Threat device characteristics: Device Type - *HEU*, Shielding is - *minimal*, Adversary Characteristics: Adversary capability is - *nominal*, Adversary Decision Process: *no prior data*, Defensive State: Intelligence Warning Level is: *weak*, Existing threat response level is: *yellow*, Device enters: As Planned Architecture. Architecture Layer 1: *Through a Border*, specifically a *Water Border*: [The Exit Likelihood is: "Nearly Certain"], Using a Cargo Vessel with Detection and Interdiction Nodes: [The Branching Likelihood is: "Somewhat Likely" to "Likely"], Crossing at W1: *Radiological and Nuclear Instrumentated Port*, Port area has a *high* population [The Adversary Choice Likelihood is: "Likely"], Device is not

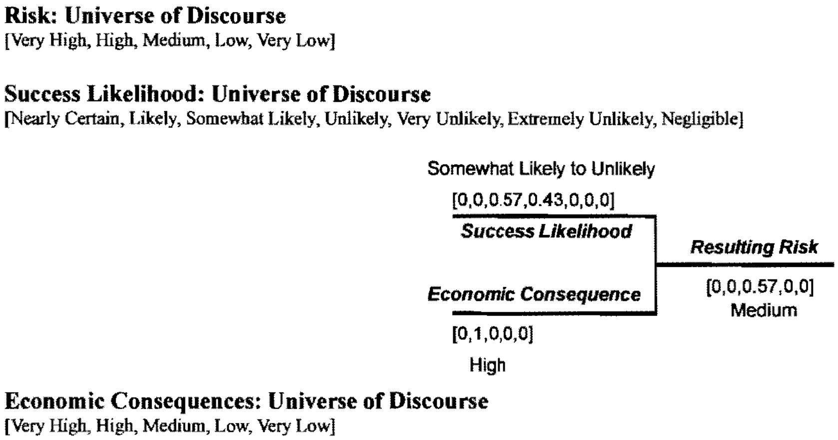

Fig. 3. Risk Reasoning with Likelihoods and Consequences for Scenario 1

Table 2. Likelihood Rule Base

		Initial Likelihood						
		Negligible	Extremely Unlikely	Very Unlikely	Unlikely	Somewhat Likely	Likely	Nearly Certain
Subsequent Likelihood	Negligible	Negligible	Negligible	Negligible	Negligible	Negligible	Negligible	Negligible
	Extremely Unlikely	Negligible	Negligible	Negligible	Negligible	Negligible	Negligible	Extremely Unlikely
	Very Unlikely	Negligible	Negligible	Negligible	Negligible	Negligible	Extremely Unlikely	Very Unlikely
	Unlikely	Negligible	Negligible	Negligible	Negligible	Extremely Unlikely	Very Unlikely	Unlikely
	Somewhat Unlikely	Negligible	Negligible	Negligible	Extremely Unlikely	Very Unlikely	Unlikely	Somewhat Likely
	Likely	Negligible	Negligible	Extremely Unlikely	Very Unlikely	Unlikely	Somewhat Likely	Likely
	Nearly Certain	Negligible	Extremely Unlikely	Very Unlikely	Unlikely	Somewhat Likely	Likely	Nearly Certain

Table 3. Risk Rule Base

		Economic Consequences				
		Very Low	Low	Medium	High	Very High
Success Likelihood	Negligible	Very Low	Very Low	Very Low	Very Low	Very Low
	Extremely Unlikely	Very Low	Very Low	Very Low	Very Low	Low
	Very Unlikely	Very Low	Very Low	Very Low	Low	Medium
	Unlikely	Very Low	Low	Low	Medium	Medium
	Somewhat Likely	Very Low	Low	Low	Medium	Medium
	Likely	Low	Low	Medium	High	Very High
	Nearly Certain	Low	Low	Medium	High	Very High

detected at W1. [The Non Detection Likelihood is: "Nearly Certain"], Device not interdicted at W1. [The Non Interdiction Likelihood is: "Nearly Certain"], Device detonates at W1. [The Economic Consequences are: "High"]

4.2 Scenario 2, S2:

Radiological/nuclear attack on a target in the contiguous United States. Threat device characteristics: Device Type - *HEU*, Shielding is - *minimal*, Adversary Characteristics: Adversary capability - *nominal*, Adversary Decision Process: *prior data*, Defensive State: Intelligence Warning Level is: *weak*, Existing threat response level is: *yellow*, Device enters: As Planned Architecture. Architecture Layer 1: *Through a Border*, specifically a *Water Border*: [The Exit Likelihood is: "Nearly Certain"], Using a Cargo Vessel with Detection and Interdiction Nodes: [The Branching Likelihood is: "Nearly Certain" to "Likely"], Crossing at W1: *Radiological and Nuclear Non Instrumented Port*, Port area has a *medium to high* population [The Adversary Choice Likelihood is: "Nearly Certain" to "Likely"], Device is not detected at W1. [The Non Detection Likelihood is: "Nearly Certain"], Device not interdicted at W1. [The Non Interdiction Likelihood is: "Nearly Certain"], Device detonates at W1. [The Economic Consequences are: "Medium" to "High"]

4.3 Scenario 3, S3:

Radiological/Nuclear attack on a target in the contiguous United States. Threat device characteristics: Device Type - *HEU*, Shielding is - *minimal*, Adversary Characteristics: Adversary capability -*nominal*, Adversary Decision Process: *prior data*, Defensive State: Intelligence Warning Level is: *none*, Existing threat response level is: *green*, Device enters: As Planned Architecture. Architecture Layer 1: *Through a Border*, specifically a *Water Border*: [The Exit Likelihood is: "Nearly Certain"], Using a Cargo Vessel with Detection and Interdiction Nodes: [The Branching Likelihood is: "Nearly Certain" to "Somewhat Likely"], Crossing at W1: *A Radiological and Nuclear Non Instrumented Port*, Port are has a *low to high* population [The Adversary Choice Likelihood is: "Nearly Certain" to"Somewhat Likely"], Device is not detected at W1. [The Detection Likelihood is: "Nearly Certain"], Device not interdicted at W1. [The Non Interdiction Likelihood is: "Nearly Certain"], Device detonates at W1. [The Economic Consequences are: "Low" to "High"]

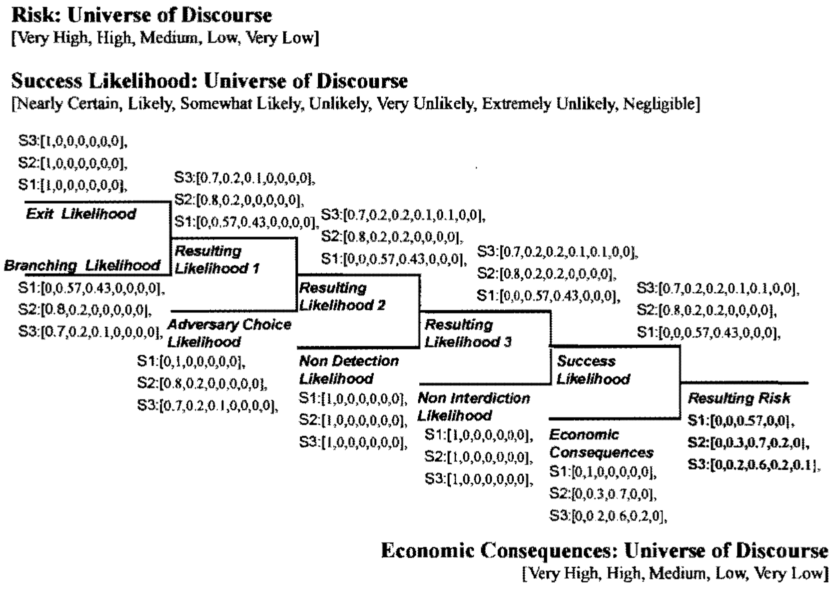

Fig. 4. Risk Reasoning with Likelihoods and Consequences for Scenarios 1, 2, 3

The quantification of information uncertainty for each scenario is determined by applying Equations 5 and 6 to the Risk Results. Equation 6 is used to quantify information uncertainty in the AR results related to conflict and Equation 5 is used to quantify the information uncertainty associated with nonspecificity. The calculated values are provided in Table 4. There is only one fuzzy set in the resulting vector with a degree of membership greater than zero; therefore, the resulting value of risk is not in conflict with any other values for risk in the vector. The value for nonspecificity is 0 in scenario 1 and the conflict is lower than that of scenario 2 and 3. Scenario 2 has degree of membership in "high", "medium", and "low" which results in conflict among these three resulting fuzzy sets. There is less conflict in scenario 3 than 2 because the degree of membership in "medium" risk is substantially higher than the remaining three fuzzy sets for scenario 3. Scenario 2 is more specific than scenario 3 and thus has a lower value for the nonspecificity.

Table 4. Information Uncertainty

Scenario Risk	Conflict (C)	Nonspecificity (H)	$\sum C + H$
S1:[0, 0, 0.57, 0, 0]	0.985815	0.0	0.985815
S2:[0, 0.3, 0.7, 0.2, 0]	2.883802	1.0	3.883802
S3:[0, 0.2, 0.6, 0.2, 0.1]	2.484510	1.5849625	4.069472

5 Conclusion

A greater amount of confidence can be placed in the results with a lower value of uncertainty. Each result of an AR model contains a quantifiable measure of information uncertainty associated with the conflict and nonspecificity due to the fuzzy uncertainty. This paper proposes measuring the conflict and nonspecificity in each AR result and using these quantities to compare the AR results. For situations involving fuzzy uncertainty, earlier research in the area of conflict and nonspecificity have focused on measuring these quantities using the degree of membership for all the elements within the fuzzy set. The results of AR models involve only one degree of membership for each linguistic value and not the degree of membership for all the elements within the fuzzy set. The current research extends the quantification of information uncertainty to fuzzy uncertainty in AR. The quantification of information uncertainty in AR is a valuable contribution and the importance is evident in the comparison of the information uncertainty in competing AR results. The results of this study can be extend by relating a measure of confidence to the measure of information uncertainty.

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