LA-UR- 02-5313

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Title: Evaluation of a Semi-Implicit Numerical Algorithm for a Rate-Dependent Ductile Failure Model

Author(s): Marvin A. Zocher, X-7
Quihai K. Zuo, T-3
Thomas A. Mason, MST-8

Submitted to: Shock Waves in Condensed Matter
St. Petersburg, Russia
September 1-6, 2002

Evaluation of a Semi-Implicit Numerical Algorithm for a Rate-Dependent Ductile Failure Model

Marvin A. Zocher, Quihai K. Zuo, Thomas A. Mason
Los Alamos National Laboratory

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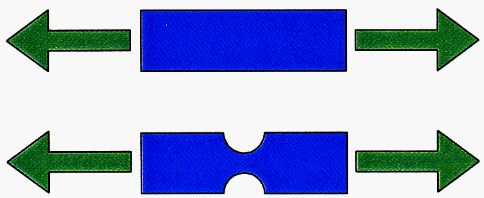
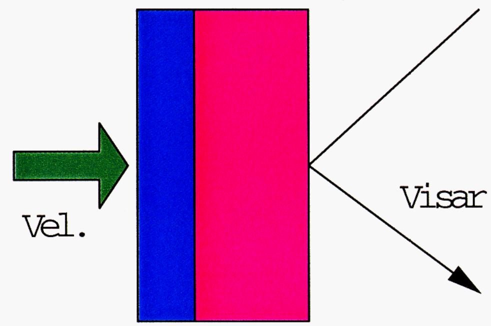
Outline

- History of the TEPLA model
- Current Version of the model
- Results

TEPLA History - 1
Motivation

- A survey conducted in the mid-80's revealed that the mathematical descriptions of ductile fracture tended to apply to either tensile tests or spall tests.





- plane stress
- little void growth
- large shear strain

- plane strain
- significant void growth
- little shear strain

- The objective behind the development of the TEPLA was then a unification of these disparate phenomena into a single model.

TEPLA History - 2

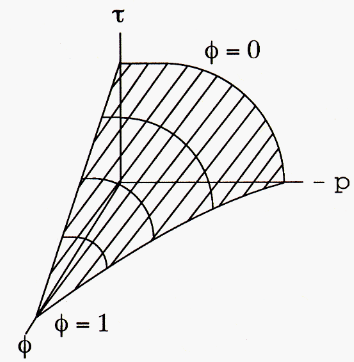
Johnson, J.N., and Addessio, F.L., "Tensile Plasticity and Ductile Fracture",
J. of Appl. Phys., Vol. 64, No. 12, 1988, pp. 6699-6712

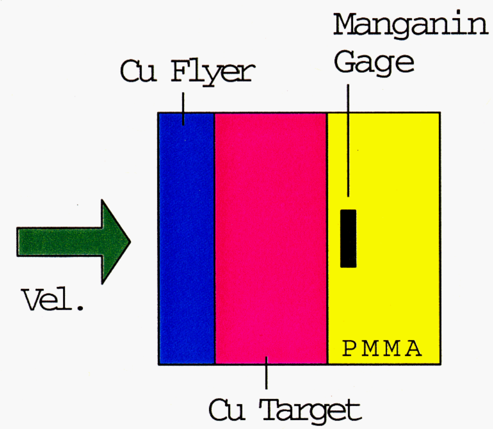
Gurson Flow Surface

$$\tau = \sqrt{\tfrac{3}{8} s_{ij} s_{ij}}$$

$$\delta = -\tfrac{3}{2}\frac{p}{Y}$$

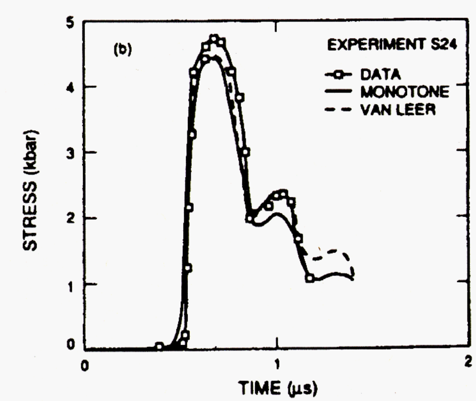
$$F(s_{ij}, p, Y, \phi) = 4\tau^2 - Y^2\left[1 + \phi^2 - 2\phi\cosh\delta\right]$$





Coupled Shear/Porosity Failure Criterion

$$\gamma_f = \gamma_0 + \Delta\gamma \exp\left[a\left(\frac{p}{2\tau}\right)\right] \qquad \left(\frac{\phi}{\phi_f}\right)^2 + \left(\frac{\gamma}{\gamma_f}\right)^2 = 1$$



Coupled Evolution Rules for:

Deviator $\quad d\tau = \frac{3\mu}{4\tau}\left(s_{ij}de_{ij} - s_{ij}de^{p}_{ij}\right)$

Porosity $\quad d\phi = (1-\phi)d\epsilon^{p}_{kk}$

Pressure $\quad dp = -(1-\phi)B_s d\ln v + \rho\Gamma_s T ds + \left[(1+\Gamma_s)p - (1-\phi)B_s\right] d\ln(1-\phi)$

TEPLA History - 3

Addessio, F.L., Johnson, J.N., and Maudlin, P.J., "The Effect of Void Growth on Taylor Cylinder Impact Experiments," *J. of Appl. Phys.*, Vol. 73, No. 11, 1993, pp. 7288-7297

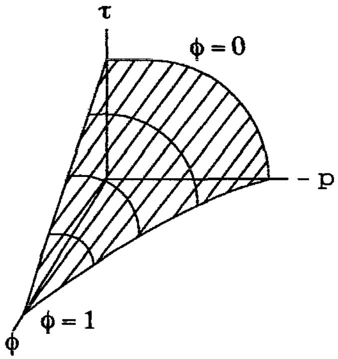
Gurson Flow Surface

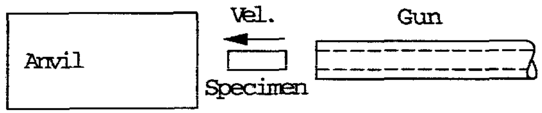
$$\tau = \frac{1}{2}\sqrt{\frac{3}{2}s_{ij}s_{ij}} \qquad \delta = -\frac{3}{2}\frac{p}{Y}$$

$$\dot{Y} = H\sqrt{\frac{2}{3}\dot{e}^{p}_{ij}\dot{e}^{p}_{ij}}$$

$$Y = \left[Y_0 + H\left(e^p\right)^{a_n}\right]\left[1 + b_n \ln\left(\dot{e}^p\right)\right]$$

$$F(s_{ij}, p, Y, \phi) = 4\tau^2 - Y^2\left[1 + (q\phi)^2 - 2q\phi\cosh\delta\right]$$



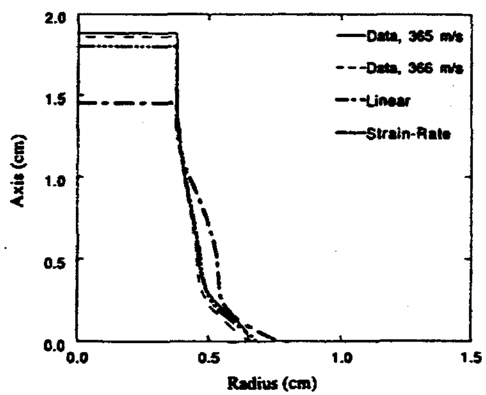


Coupled Evolution Rules for:

Deviator $\quad \check{s}_{ij} = 2G\left(\dot{e}_{ij} - \dot{e}^{p}_{ij}\right)$

Porosity $\quad \dot{\phi} = (1-\phi)\dot{\epsilon}^{p}_{kk}$

Pressure $\quad \dot{P} = \left[B - (1+\Gamma_s)P\right]\dot{\epsilon}^{p}_{kk} - B\dot{\epsilon}^{p}_{kk} + \Gamma_s s.$



Updating the plastic strain rate Three methods investigated (associative flow, return, hybrid)

Strain Softening

Problem: Softening leads to a change in the set of governing equations for the dynamic IBVP from hyperbolic to elliptic and the problem becomes ill-posed.

Manifestation (Simo 1989)

- The strains localize to a narrow band (set of measure zero)
- Classical local dissipation becomes meaningless since no dissipation can take place in a localized set of zero Borel measure
- Numerical simulation of softening materials exhibit a totally spurious mesh dependency
- For elastic and rate independent materials, the governing equations exhibit a local loss of ellipticity which precludes wave propagation

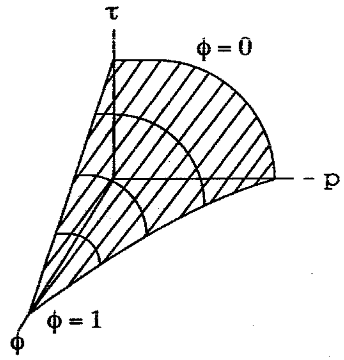
Possible Fixes (Simo 1989)

- Mesh dependent modulus H^h
- Nonlocal methods (higher-order spatial derivatives)
- Viscoplasticity (Higher order temporal derivatives)

TEPLA History - 4

Addessio, F.L., and Johnson, J.N., "Rate-Dependent Ductile Failure Model,",
J. of Appl. Phys., Vol. 74, No. 3, 1993, pp. 1640-1648

Gurson Flow Surface



$$\tau = \sqrt{\tfrac{3}{2} s_{ij} s_{ij}}$$

$$\delta = -\frac{3}{2}\frac{p}{Y}$$

$$F(s_{ij}, p, Y, \phi) = \tau^2 - Y^2 \left[1 + (q\phi)^2 - 2q\phi \cosh \delta\right]$$

Coupled Shear/Porosity Failure Criterion

$$\gamma_f = \gamma_0 + \Delta\gamma \exp\left[a\left(\frac{p}{2\tau}\right)_f\right] \qquad \left(\frac{\phi}{\phi_f}\right)^2 + \left(\frac{\gamma}{\gamma_f}\right)^2 = 1$$

Coupled Evolution Rules for:

Deviator $\check{s}_{ij} = 2G\left(\dot{e}_{ij} - \dot{e}^p_{ij}\right)$

Porosity $\dot{\phi} = (1 - \phi)\dot{\epsilon}^p_{kk}$

Pressure $\dot{P} = \Gamma_s s_{ij} \dot{e}_{ij} - B\dot{\epsilon}^p_{kk} + \alpha\dot{\epsilon}^p_{kk}$

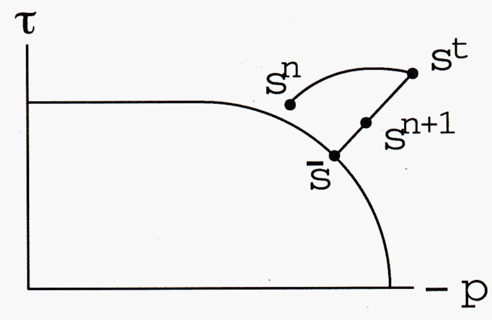
TEPLA History - 5

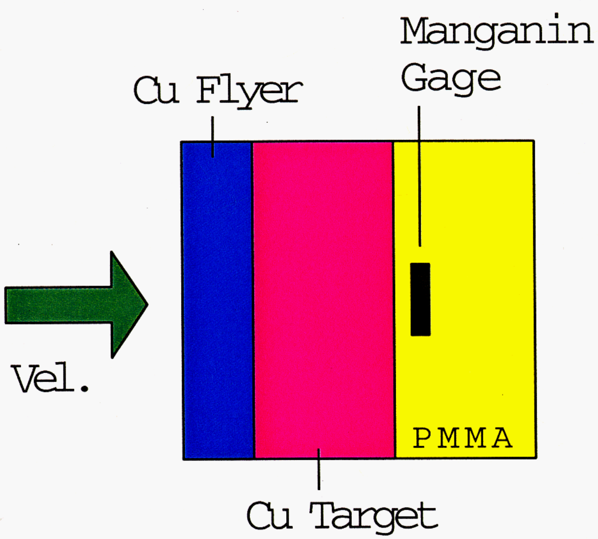
Viscoplasticity

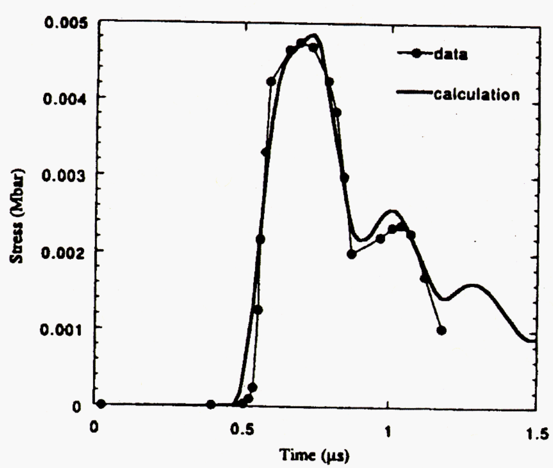
$$\dot{\sigma}_{ij} = f'\dot{\epsilon} + \tau_r \ddot{\epsilon}$$

Length scale: $l = k\frac{\tau_r c}{E}$

Rate parameter: $\tau_r = \eta \left(\frac{1-\phi_0}{\phi_0}\right)^{2/3} + \left(\frac{1-\phi}{\phi}\right)^{1/3}$

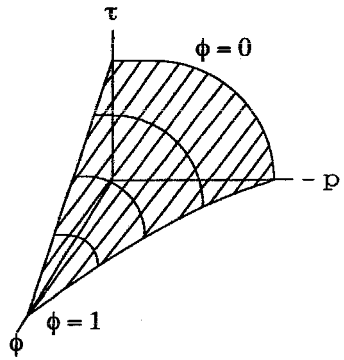






Current TEPLA Model

Gurson Flow Surface



$$\tau = \sqrt{\tfrac{3}{2} s_{ij} s_{ij}}$$

$$\delta = -q_2 \tfrac{3}{2} \tfrac{p}{Y}$$

$$F(s_{ij}, p, Y, \phi) = \left(\tfrac{\tau}{Y}\right)^2 - \left[1 + q_3 \phi^2 - 2 q_1 \phi \cosh \delta\right]$$

Coupled Shear/Porosity Failure Criterion

$$\epsilon_f = \sqrt{1 - \left(\frac{\phi}{\phi_f}\right)^2} \left[D_1 + D_2 \exp\left(D_3 \tfrac{P}{Y}\right)\right]$$

Coupled Evolution Rules for:

Deviator $\dot{s}_{ij} = 2G\left(\dot{e}_{ij} - \dot{e}^p_{ij}\right) - s_{ik} W_{kj} + W_{ik} s_{kj}$

Porosity $\dot{\phi} = (1 - \phi)\dot{\epsilon}^p_{kk}$

Pressure $\dot{P} = \Gamma_s s_{ij} \dot{e}_{ij} - B \dot{\epsilon}^p_{kk} + \alpha \dot{\epsilon}^p_{kk}$

1. Solve for the trial state

$$s_{ij}^{t} = s_{ij}^{n} + 2G\dot{e}_{ij}\Delta t + \dot{r}_{ij}\Delta t \qquad P^{t} = P^{n} - B\dot{\epsilon}_{kk}\Delta t \qquad \phi^{t} = \phi^{n}$$

2. Solve for the equilibrium state

Implicit time integration leads to four coupled nonlinear equations which must be solved simultaneously:

$$C = \left(1 + \frac{6G}{Y}\frac{\lambda}{Y}\right)\frac{\overline{\tau}}{Y} - \frac{\tau^{t}}{Y} = 0$$

$$D = \overline{\delta} + \frac{3}{2}q_2\frac{\lambda}{Y_f}\left[3\frac{q_1 q_2}{Y_f}\alpha\overline{\phi}\sinh\overline{\delta} + 2\Gamma_s\left(\frac{\overline{\tau}}{Y}\right)^2\right] - \delta^{t} = 0$$

$$E = \overline{\phi} - \phi^{t} - 3(1-\overline{\phi})q_1 q_2\frac{\lambda}{Y_f}\overline{\phi}\sinh\overline{\delta} = 0$$

$$F = \overline{F} = \left(\frac{\overline{\tau}}{Y}\right)^2 - \left(1 + q_3\overline{\phi}^2 - 2q_1\overline{\phi}\cosh\overline{\delta}\right) = 0$$

Which must be solved simultaneously for independent variables: $\frac{\overline{\tau}}{Y}$, $\overline{\delta}$, $\overline{\phi}$, $\frac{\lambda}{Y}$.

Implicit Algorithm - 2

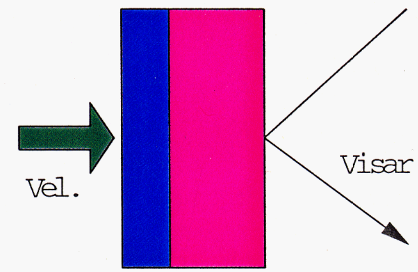
3. Solve for the final state

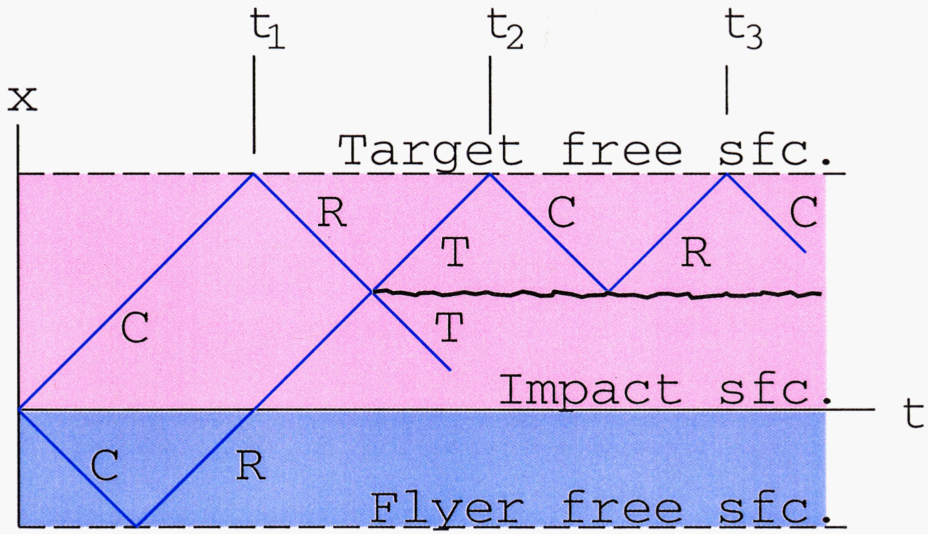
$$s_{ij}^{n+1} = \frac{\frac{\xi}{\Delta t} + \frac{\overline{\tau}}{\tau^t}}{\frac{\xi}{\Delta t} + 1} \; s_{ij}^{t}$$

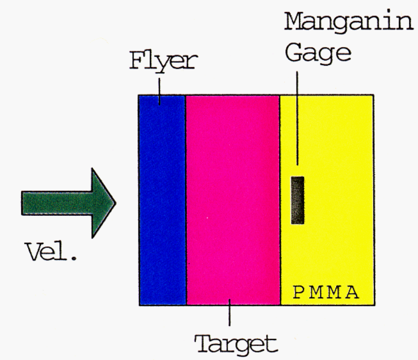
$$P^{n+1} = \frac{P^t + \frac{\alpha \Delta t}{\tau_r}\overline{P} + \frac{\Gamma_s}{3G}\left(\tau^t - \tau^{n+1}\right)\tau^{n+1}}{1 + \frac{\alpha \Delta t}{\tau_r}}$$

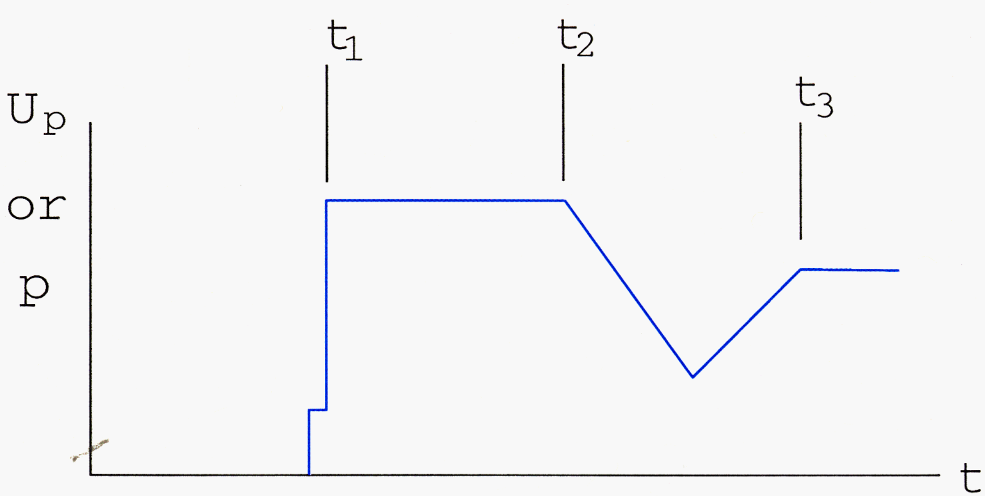
$$\phi^{n+1} = \frac{\phi^n + \dot{\epsilon}_{kk}^{p}\Delta t}{1 + \dot{\epsilon}_{kk}^{p}\Delta t}$$

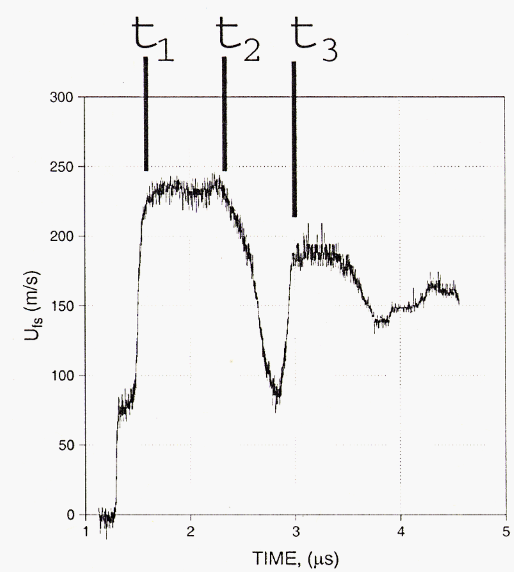
Flyer Plate Experiment



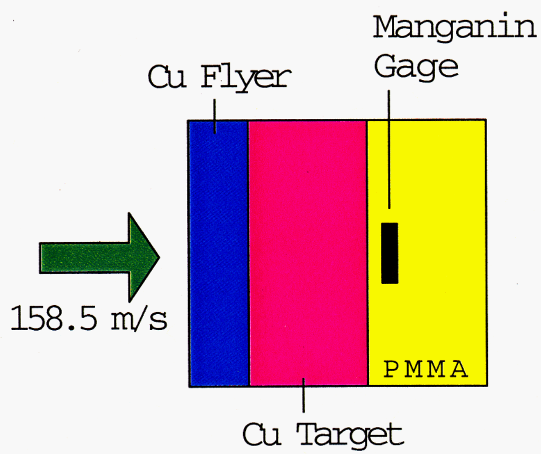


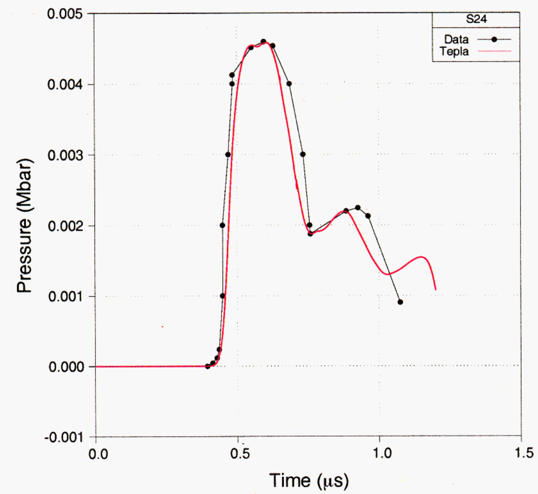


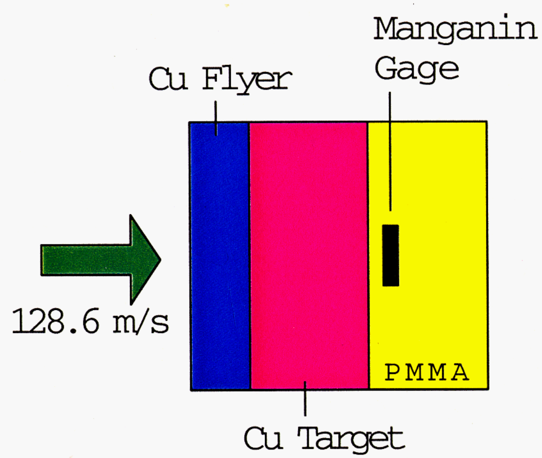


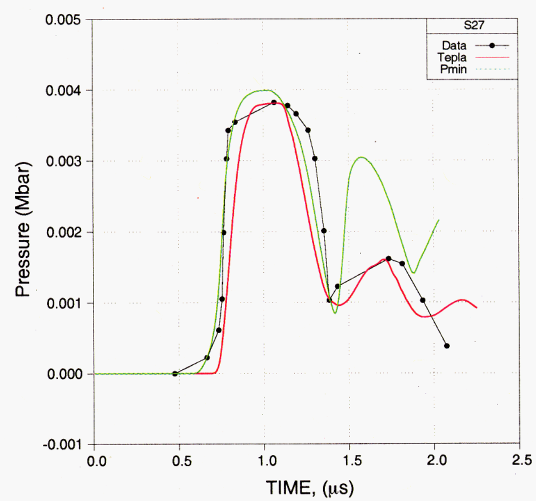


Copper Results

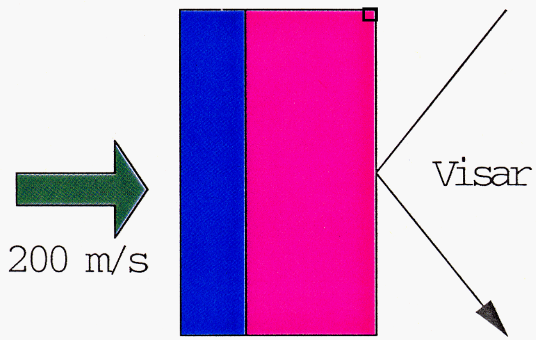


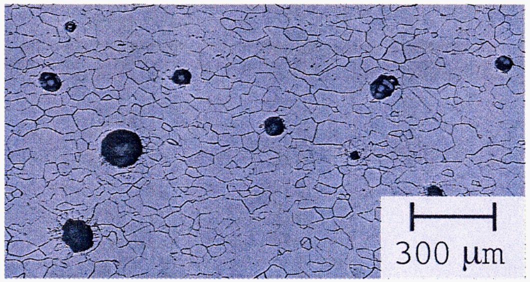


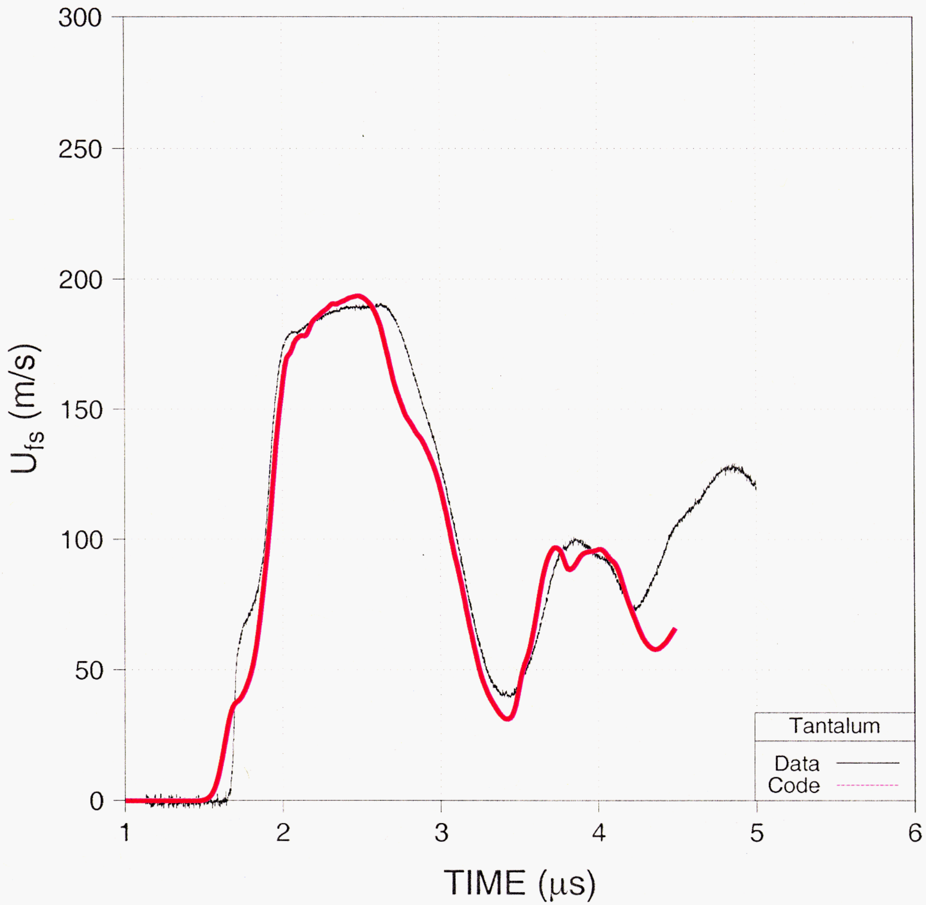




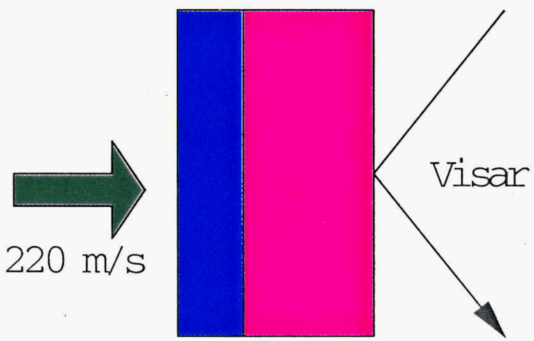
Tantalum Results – 1

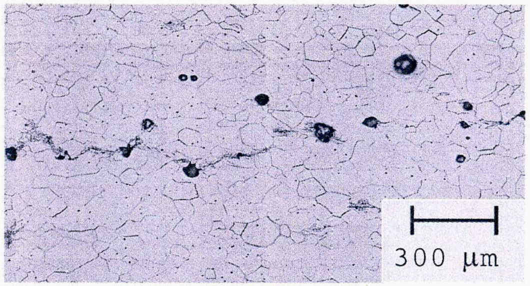


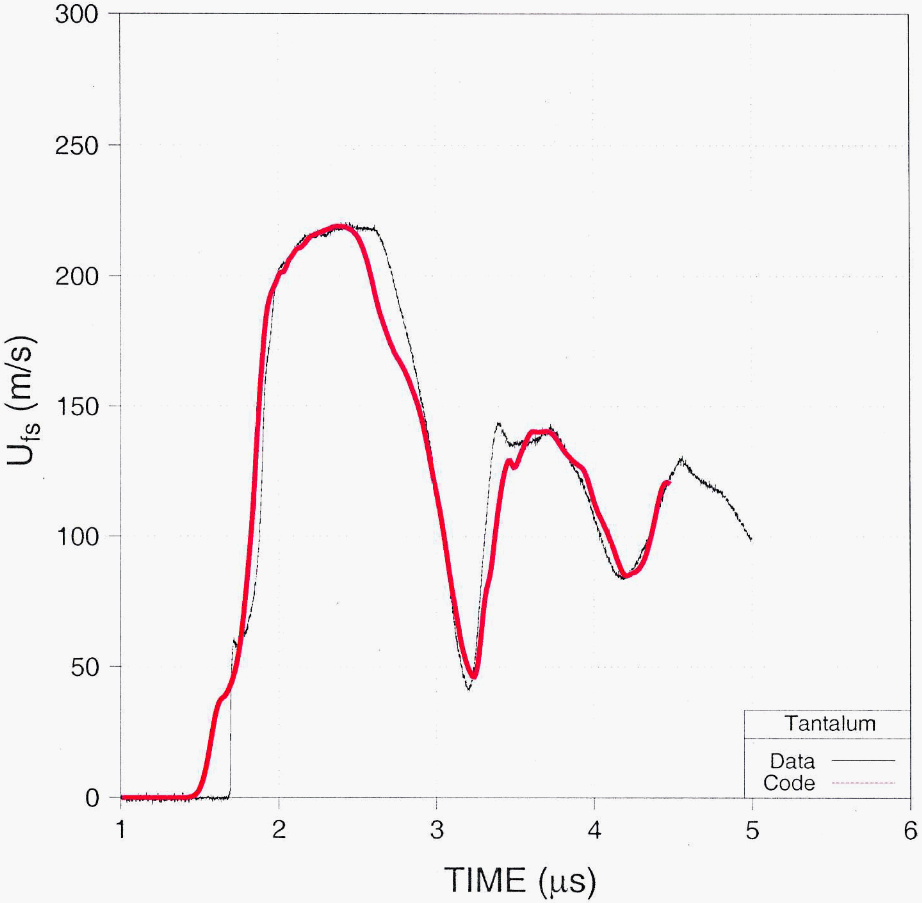




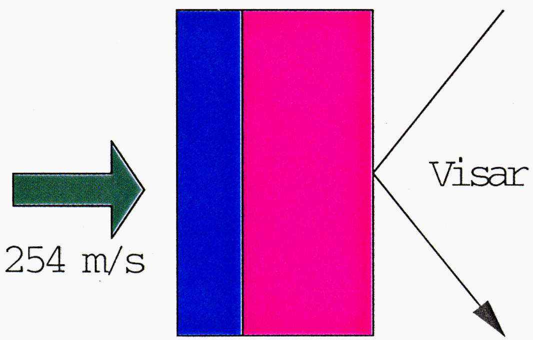
Tantalum Results – 2

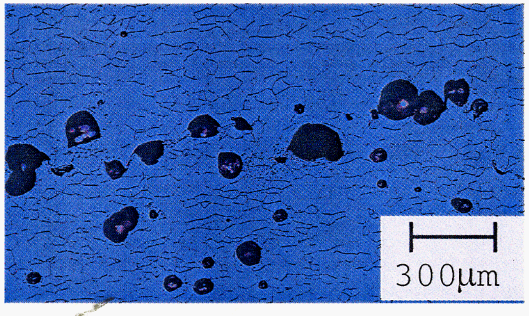


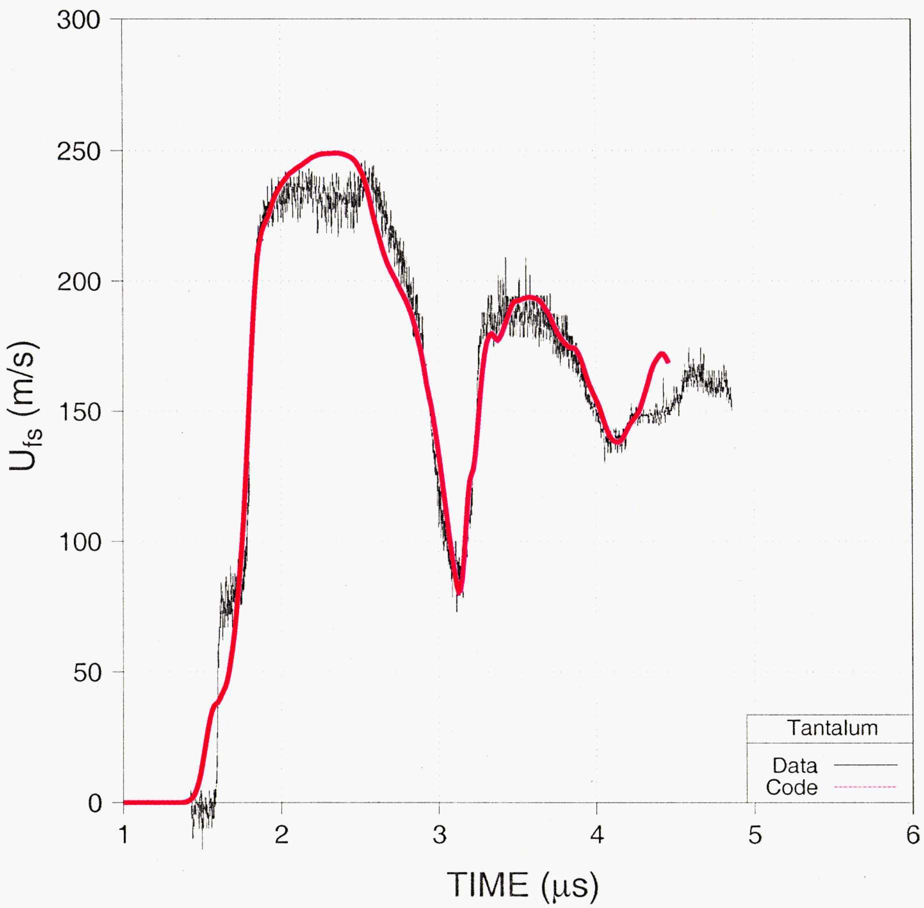




Tantalum Results – 3







Conclusions

- Few results shown
  Much more needed for validation
- Time step problem overcome – yes
  Quantification needed
- 1d, 2d, 3d nuances
- Parameter set for variety of materials