Low-frequency dilatational wave propagation through unsaturated porous media containing two immiscible fluids

Wei-Cheng Lo[1,2], Garrison Sposito[2,3], Ernest Majer[2]

[1]Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Tainan 701, Taiwan
[2]Department of Geophysics, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
[3]Department of Civil and Environmental Engineering, University of California, Berkeley, California 94720-1710, USA

Abstract

An analytical theory is presented for the low-frequency behavior of dilatational waves propagating through a homogeneous elastic porous medium containing two immiscible fluids. The theory is based on the Berryman-Thigpen-Chin (BTC) model, in which capillary pressure effects are neglected. We show that the BTC model equations in the frequency domain can be transformed, at sufficiently low frequencies, into a dissipative wave equation (telegraph equation) and a propagating wave equation in the time domain. These partial differential equations describe two independent modes of dilatational wave motion that are analogous to the Biot fast and slow compressional waves in a single-fluid system. The equations can be solved analytically under a variety of initial and boundary conditions.

The stipulation of "low frequency" underlying the derivation of our equations in the time domain is shown to require that the excitation frequency of wave motions be much smaller than a critical frequency. This frequency is shown to be the inverse of an intrinsic time scale that depends on an effective kinematic shear viscosity of the interstitial fluids and the intrinsic permeability of the porous medium. Numerical calculations indicate that the critical frequency in both unconsolidated and consolidated materials containing water and a nonaqueous phase liquid ranges typically from kHz to MHz. Thus engineering problems involving the dynamic response of an unsaturated porous medium to low excitation frequencies (e.g. seismic wave stimulation) should be accurately modeled by our equations after suitable initial and boundary conditions are imposed.

Key words: dilatational waves, immiscible fluid flow, poroelastic behavior

1. Introduction

The quantitative description of elastic wave propagation in a porous medium containing a single fluid is one of the classic problems in the physics of flow through porous materials [Biot, 1956, 1962; Biot and Willis, 1957; Rice and Cleary, 1976; Chandler and Johnson, 1981; Dvorkin and Nur, 1993; Donskoy *et al.* 1997]. An extension of the theory to include the effects of two immiscible pore fluids on the behavior of elastic waves was proposed long ago by Brutsaert [Brutsaert, 1964; Brutsaert and Luthin, 1964] as a generalization of the seminal Biot [1962] poroelasticity model for a single-fluid system. Notable among many others following this approach, Berryman *et al.* [1988] developed a general Lagrangian formulation of elastic wave behavior in unsaturated porous media under the assumption that the wave excitation is of long enough wavelength that the effects of capillary pressure changes are negligible. This assumption allowed the two fluids in a porous medium to be represented by a single multiphase fluid whose material properties are volume-weighted averages of those of its components.

All poroelasticity models feature partial differential equations that are coupled in the terms describing inertia, viscous damping, and applied stresses. Decoupling of these equations into normal coordinates representing independent modes of wave motion is highly desirable for determining analytical solutions of boundary value problems. An exact decoupling of the Biot [1962] model equations for a single-fluid system has been achieved in the frequency domain [Dutta and Ode, 1979; Berryman, 1983], with the two resulting complex-valued normal coordinates each found to satisfy a Helmholtz equation featuring complex frequency-dependent eigenvalues. Chandler and Johnson [1981] demonstrated that, when inertial terms are neglected in the Biot model, decoupling also can be achieved in the time domain with two real-valued normal coordinates that satisfy a diffusion equation and a Laplace equation, respectively. If inertial terms are included, decoupling with the Chandler-Johnson normal coordinates is still possible, but requires imposing a constraint relation between elasticity coefficients and inertial parameters in the Biot model [Lo *et al.*, 2002].

Berryman *et al.* [1988] decoupled their model equations for unsaturated porous media following the frequency-domain method used by Berryman [1983] for the single-fluid case. However, the resulting pair of Helmholtz equations cannot be converted directly to partial differential equations in the time domain. Thus, closed-form analytical solutions of these equations in space and time variables cannot be obtained. The aim of the present paper is to show that the decoupled model equations of Berryman *et al.* [1988] in the frequency domain can in fact be converted directly to the time domain in a low-frequency limit that is consistent with neglect of capillary pressure changes. Consistency is assured because the wavelength in this limit is large enough for the two fluids to experience the same pressure change under excitation of the porous medium [Berryman *et al.*, 1988].

The low-frequency decoupled equations, which apply to dilatational wave motions in an elastic porous medium containing two immiscible fluids, have the mathematical form of a dissipative wave equation (telegraph equation) and a propagating wave equation. The precise definition of "low frequency" is established and shown to be equivalent to requiring the angular frequency of wave motions to be much smaller than a critical frequency ω_c equal to the inverse of an intrinsic time scale in the two-fluid system. Illustrative numerical calculations performed for unsaturated porous media containing water and a nonaqueous phase liquid (NAPL) indicate that ω_c will lie typically in the kHz to MHz range. Thus our decoupled equations should be applicable to acoustic wave phenomena at seismic frequencies in partially-saturated porous media [Li *et al.*, 2001]. Interest in these phenomena is being stimulated as a consequence of their frequent appearance in problems of biomechanics [Cowin, 1999], geophysical exploration [Kearey *et al.*, 2002], hydrocarbon extraction [Beresnev and Johnson, 1994; Kouznetsov *et al.,* 1998], groundwater remediation [Roberts *et al.,* 2001], seabed stability [Mei and Foda, 1981], and soil consolidation [Lewis and Schrefler, 1998]. The key practical question to be addressed in these applications is how to describe acoustic wave motions as the relative content of each fluid changes.

2. Model equations

Using a Lagrangian variational approach, Berryman *et al.* [1988] derived a set of coupled partial differential equations that describe elastic wave propagation and attenuation through unsaturated porous media under the assumption that *changes* in capillary pressure have negligible effect on wave motions. Lo *et al.* [2002] developed more general model equations based on the continuum theory of mixtures [Truesdell, 1984] that, under the same physical assumptions as made by Berryman *et al.* [1988], reproduced their model equations in the form:

$$\rho_1 \frac{\partial^2 \vec{u}_s}{\partial t^2} + [\frac{\rho_1}{\theta_1} - \frac{A_{11}}{\theta_1^2}] \frac{\partial^2 \vec{w}_1}{\partial t^2} - \frac{R_{11}}{\theta_1^2} \frac{\partial \vec{w}_1}{\partial t} - \frac{A_{12}}{\theta_1 \theta_2} \frac{\partial^2 \vec{w}_2}{\partial t^2} = -\vec{\nabla} p_f \,, \tag{1.1}$$

$$\rho_2 \frac{\partial^2 \vec{u}_s}{\partial t^2} + [\frac{\rho_2}{\theta_2} - \frac{A_{22}}{\theta_2^2}] \frac{\partial^2 \vec{w}_2}{\partial t^2} - \frac{R_{22}}{\theta_2^2} \frac{\partial \vec{w}_2}{\partial t} - \frac{A_{21}}{\theta_1 \theta_2} \frac{\partial^2 \vec{w}_1}{\partial t^2} = -\vec{\nabla} p_f\,, \tag{1.2}$$

$$\rho \frac{\partial^2 \vec{u}_s}{\partial t^2} + \rho_1 \frac{\partial^2 \vec{w}_1}{\partial t^2} + \rho_2 \frac{\partial^2 \vec{w}_2}{\partial t^2} = \vec{\nabla} \cdot \overset{=}{\sigma}\,, \tag{1.3}$$

where ρ_α is the mass density of phase α; θ_α is its volume fraction, the subscript α designating the three immiscible phases: wetting fluid ($\alpha = 1$; fluid 1), non-wetting fluid ($\alpha = 2$; fluid 2), and solid phase ($\alpha = s$); $\rho = \sum \rho_\alpha \theta_\alpha$; $\vec{u}_s$ and $\vec{u}_\xi$ ($\xi = 1, 2$) are displacement vectors for the solid and fluid phases, respectively; $\vec{w}_\xi = \theta_\xi (\vec{u}_\xi - \vec{u}_s)$ refers to the displacement vector of fluid ξ relative to the solid phase; $\overset{=}{\sigma}$ is the total stress, i.e. the stress applied to the solid phase in the porous medium plus the stress acting on the fluids [Biot, 1962; Lo *et al.*, 2002]; p_f is fluid pressure (the same for both fluids because of the assumption about capillary pressure changes); A_{11}, A_{12}, A_{21}, and A_{22} are constitutive coefficients related to inertial drag [Lo *et al.*, 2002]; R_{11} and R_{22} are constitutive coefficients pertinent to viscous drag [Gray, 1983; Lo *et al.*, 2002]. The viscous drag tensor element $R_{\xi\xi} = -\frac{\theta_\xi^2 \eta_\xi}{k_s k_{r\xi}}$, i.e. each fluid flow obeys Darcy's law [Garg and Nayfeh, 1986; Berryman *et al.*, 1988; Santos *et al.*, 1990], where η_ξ is the dynamic viscosity of the fluid phase ξ, k_s is the intrinsic permeability of the porous medium, and $k_{r\xi}$ is the relative permeability of the medium to fluid phase ξ. Relative permeability, the ratio of the effective permeability to a fluid at a given value of saturation to the effective permeability to that fluid at full saturation, has values ranging between zero and one [Bear, 1988].

The assumption $R_{12} = R_{21} = 0$ has been imposed implicitly in (1) to reflect the common observation that cross-coupling caused by viscous drag can be ignored in two-phase flow through unsaturated porous media [Dullien, 1992]. This hydrodynamic phenomenon, known as the Yuster [1951] effect, has been debated extensively in the literature [Ehrlich, 1993]. Jerauld and Salter [1990] claimed that, despite the possibility that one fluid flowing in response to a pressure gradient could drag the other fluid into motion, little empirical evidence points to the importance of this hydrodynamic interaction. It was concluded by Avraam and Payatakes [1995] that the interaction is very difficult to isolate through macroscopic measurements and that it can be incorporated into the relative permeability, $k_{r\xi}$.

Following Biot [1962], we can express the stress-strain relations in an isotropic porous medium as:

$$\overline{\overline{\sigma}} = 2G\overline{\overline{e}} + (\lambda_c e - C\zeta)\overline{\overline{\delta}}, \tag{2.1}$$

$$p_f \overline{\overline{\delta}} = (-Ce + M\zeta)\overline{\overline{\delta}}, \tag{2.2}$$

where the solid-phase strain tensor $\overline{\overline{e}} = \frac{1}{2}(\vec{\nabla}\vec{u}_s + \vec{\nabla}\vec{u}_s^{\,T})$, the superscript *T* denoting the transpose; $\overline{\overline{\delta}}$ is a unit tensor; *G* is the shear modulus of the porous medium frame; *C*, *M*, and λ_c are elastic coefficients that can be expressed in terms of *G*, the porosity ϕ, and the bulk moduli of the porous medium frame K_b, the multiphase interstitial fluid K_f, and the solid grains K_s [Biot and Willis, 1957; Biot, 1962; Stoll, 1974; Johnson, 1986]. The variable ζ is the linearized increment of fluid content as defined for a two-fluid system by Berryman *et al.* [1988]:

$$\zeta = -\vec{\nabla} \cdot \sum_{\xi} [\theta_\xi (\vec{u}_\xi - \vec{u}_s)], \tag{3}$$

and the average fluid bulk modulus K_f is defined by the harmonic mean [Berryman *et al.*, 1988]:

$$\frac{\phi}{K_f} = \frac{\theta_1}{K_1} + \frac{\theta_2}{K_2}, \tag{4}$$

where K_1 and K_2 are the bulk moduli of fluid phases 1 and 2, respectively.

Next we define the induced mass densities ρ_{1s} and ρ_{2s}:

$$\rho_{1s} = \frac{\rho_1}{\theta_1} - \frac{A_{11}}{\theta_1^2} = \frac{\alpha_1 \rho_1}{\theta_1}, \tag{5.1}$$

$$\rho_{2s} = \frac{\rho_2}{\theta_2} - \frac{A_{22}}{\theta_2^2} = \frac{\alpha_2 \rho_2}{\theta_2}. \tag{5.2}$$

The quantities ρ_{1s} and ρ_{2s} reflect inertial drag interactions between the solid and fluid phases [Biot, 1956, 1962; Johnson, 1986; Berryman *et al.*, 1988; Santos *et al.*, 1990]; the parameters α_1 and α_2 (which are always greater than 1) are tortuosity factors used to quantify the impedance to fluid flow that occurs when the fluids are accelerated [Biot, 1962; Stoll, 1974; Johnson, 1986].

Similarly, interactions of one pore fluid with another pore fluid lead to an inertial effect of importance in an unsaturated porous medium [Berryman *et al.*, 1988; Santos *et al.*, 1990], this coupling effect being represented by the induced mass densities:

$$\rho_{12} = \frac{A_{12}}{\theta_1 \theta_2}, \tag{6.1}$$

$$\rho_{21} = \frac{A_{21}}{\theta_1 \theta_2}, \tag{6.2}$$

where $\rho_{12} = \rho_{21}$ [Berryman *et al.*,1988; Santos *et al.*, 1990]. Finally, substituting (2) into (1), taking account of (5) and (6), then applying the divergence operation to both sides of the result, we obtain the set of coupled linear partial differential equations for a two-fluid system in the absence of changing capillary pressure:

$$C\nabla^2 e - M\nabla^2 \zeta = \rho_1 \frac{\partial^2 e}{\partial t^2} + \rho_{1s} \frac{\partial^2 \zeta_1}{\partial t^2} - \rho_{12} \frac{\partial^2 \zeta_2}{\partial t^2} + \frac{1}{k_s b_1} \frac{\partial \zeta_1}{\partial t}, \tag{7.1}$$

$$C\nabla^2 e - M\nabla^2 \zeta = \rho_2 \frac{\partial^2 e}{\partial t^2} - \rho_{21} \frac{\partial^2 \zeta_1}{\partial t^2} + \rho_{2s} \frac{\partial^2 \zeta_2}{\partial t^2} + \frac{1}{k_s b_2} \frac{\partial \zeta_2}{\partial t}, \tag{7.2}$$

$$H\nabla^2 e - C\nabla^2 \zeta = \rho \frac{\partial^2 e}{\partial t^2} + \rho_1 \frac{\partial^2 \zeta_1}{\partial t^2} + \rho_2 \frac{\partial^2 \zeta_2}{\partial t^2}, \tag{7.3}$$

where $H = \lambda_c + 2G$, $\zeta_1 = \vec{\nabla} \cdot \vec{w}_1$, $\zeta_2 = \vec{\nabla} \cdot \vec{w}_2$. The parameters $b_1 = \frac{k_{r1}}{\eta_1}$, and $b_2 = \frac{k_{r2}}{\eta_2}$ are termed the relative mobilities of the fluid phases 1 and 2, respectively [Lake, 1989; Lo *et al.*, 2005].

3. Decoupling the model equations

After postulating that the strain parameters *e*, ζ_1, and ζ_2 have a time dependence of the form of $\exp(-i\omega t)$, Berryman *et al.* [1988] collapsed (7) into two coupled linear partial differential equations whose dependent variables are *e* and ζ, with decoupling accomplished subsequently. Application of Fourier transformation [$f(\vec{x},t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(\vec{x},\omega) \exp(-i\omega t) d\omega$] to (7) leads to:

$$C\nabla^2 \tilde{e} - M\nabla^2 \tilde{\zeta} + \rho_1 \omega^2 \tilde{e} = -q_1 \omega^2 \tilde{\zeta}_1 + \rho_{12} \omega^2 \tilde{\zeta}_2, \tag{8.1}$$

$$C\nabla^2 \tilde{e} - M\nabla^2 \tilde{\zeta} + \rho_2 \omega^2 \tilde{e} = \rho_{21} \omega^2 \tilde{\zeta}_1 - q_2 \omega^2 \tilde{\zeta}_2, \tag{8.2}$$

$$H\nabla^2\tilde{e} - C\nabla^2\tilde{\zeta} + \rho\omega^2\tilde{e} = -\rho_1\omega^2\tilde{\zeta}_1 - \rho_2\omega^2\tilde{\zeta}_2\,, \tag{8.3}$$

where $q_1 = \rho_{1s} + \frac{i}{\omega}\frac{1}{k_s b_1}$, $q_2 = \rho_{2s} + \frac{i}{\omega}\frac{1}{k_s b_2}$, and ω is angular frequency. Next let us express (8.1) and (8.2) in matrix form:

$$\begin{bmatrix} C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \rho_1\omega^2\tilde{e} \\ C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \rho_2\omega^2\tilde{e} \end{bmatrix} = -\omega^2 \begin{bmatrix} q_1 & -\rho_{12} \\ -\rho_{21} & q_2 \end{bmatrix} \begin{bmatrix} \tilde{\zeta}_1 \\ \tilde{\zeta}_2 \end{bmatrix}. \tag{9}$$

Inverting the coefficient matrix on the right side of (9), we have

$$\begin{bmatrix} q_2 & r_1 \\ r_2 & q_1 \end{bmatrix} \begin{bmatrix} C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \rho_1\omega^2\tilde{e} \\ C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \rho_2\omega^2\tilde{e} \end{bmatrix} = -\omega^2(q_1 q_2 - r_1 r_2) \begin{bmatrix} \tilde{\zeta}_1 \\ \tilde{\zeta}_2 \end{bmatrix}, \tag{10}$$

where $r_1 = \rho_{12}$ and $r_2 = \rho_{21}$. Combination of the two equations in (10) gives

$$(s_1 + s_2)(C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta}) + (s_1\rho_2 + s_2\rho_1)\omega^2\tilde{e} = (q_1 q_2 - r_1 r_2)\omega^2\tilde{\zeta}\,, \tag{11}$$

with $s_1 = q_1 + r_1$, $s_2 = q_2 + r_2$, and $\tilde{\zeta} = -(\tilde{\zeta}_1 + \tilde{\zeta}_2)$. The definitions,

$$\rho_{wu} = \frac{(s_1\rho_2 + s_2\rho_1)}{(s_1 + s_2)} = \frac{(\rho_2\rho_{1s} + \rho_2\frac{1}{k_s b_1}\frac{i}{\omega} + \rho_2\rho_{12}) + (\rho_1\rho_{2s} + \rho_1\frac{1}{k_s b_2}\frac{i}{\omega} + \rho_1\rho_{21})}{[\rho_x + (\frac{b_1 + b_2}{k_s b_1 b_2})\frac{i}{\omega}]}\,, \tag{12.1}$$

$$\rho_{ww} = \frac{(q_1 q_2 - r_1 r_2)}{(s_1 + s_2)} = \frac{\rho_{1s}\rho_{2s} + \frac{1}{k_s b_1}\rho_{2s}\frac{i}{\omega} + \frac{1}{k_s b_2}\rho_{1s}\frac{i}{\omega} - \frac{1}{k_s^2 b_1 b_2}\frac{1}{\omega^2} - \rho_{12}\rho_{21}}{[\rho_x + (\frac{b_1 + b_2}{k_s b_1 b_2})\frac{i}{\omega}]}\,, \tag{12.2}$$

$$\rho_x = \rho_{1s} + \rho_{12} + \rho_{21} + \rho_{2s}\,, \tag{12.3}$$

enable (11) to be reorganized compactly:

$$C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \omega^2(\rho_{wu}\tilde{e} - \rho_{ww}\tilde{\zeta}) = 0\,. \tag{13}$$

Now we multiply the first row in (10) by ρ_1 and the second row by ρ_2. The result is:

$$\begin{bmatrix} \rho_1 q_2 & \rho_1 r_1 \\ \rho_2 r_2 & \rho_2 q_1 \end{bmatrix} \begin{bmatrix} C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \rho_1\omega^2\tilde{e} \\ C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta} + \rho_2\omega^2\tilde{e} \end{bmatrix} = -\omega^2(q_1 q_2 - r_1 r_2) \begin{bmatrix} \rho_1\tilde{\zeta}_1 \\ \rho_2\tilde{\zeta}_2 \end{bmatrix}. \tag{14}$$

The two equations in (14) are then summed to yield

$$\begin{aligned} &[(q_2 + r_1)\rho_1 + (q_1 + r_2)\rho_2](C\nabla^2\tilde{e} - M\nabla^2\tilde{\zeta}) \\ &\qquad + (q_2\rho_1^2 + r_1\rho_1\rho_2 + r_2\rho_1\rho_2 + q_1\rho_2^2)\omega^2\tilde{e} = -\omega^2(q_1 q_2 - r_1 r_2)(\rho_1\tilde{\zeta}_1 + \rho_2\tilde{\zeta}_2)\,. \end{aligned} \tag{15}$$

Replacing the term $(C\nabla^2\tilde{e}-M\nabla^2\tilde{\zeta})$ in (15) from (11), we get

$$[(q_2+r_1)\rho_1+(q_1+r_2)\rho_2][\frac{(q_1q_2-r_1r_2)}{(s_1+s_2)}\omega^2\tilde{\zeta}-\frac{(s_1\rho_2+s_2\rho_1)}{(s_1+s_2)}\omega^2\tilde{e}]$$

$$+(q_2\rho_1^2+r_1\rho_1\rho_2+r_2\rho_1\rho_2+q_1\rho_2^2)\omega^2\tilde{e}=-\omega^2(q_1q_2-r_1r_2)(\rho_1\tilde{\zeta}_1+\rho_2\tilde{\zeta}_2)\,. \tag{16}$$

Or, after some algebra,

$$(\rho_1\tilde{\zeta}_1+\rho_2\tilde{\zeta}_2)=-\frac{(q_2+r_1)\rho_1+(q_1+r_2)\rho_2}{(s_1+s_2)}\tilde{\zeta}-\frac{(\rho_1-\rho_2)^2}{(s_1+s_2)}\tilde{e}\,. \tag{17}$$

Finally, substitution of (17) into (8.3) yields the expression:

$$H\nabla^2\tilde{e}-C\nabla^2\tilde{\zeta}+\omega^2(\rho_{uu}\tilde{e}-\rho_{uw}\tilde{\zeta})=0\,, \tag{18}$$

where ρ_{uu} and ρ_{uw} are given by

$$\rho_{uu}=\rho-\frac{(\rho_1-\rho_2)^2}{(s_1+s_2)}=\frac{\rho[\rho_x+(\frac{b_1+b_2}{k_sb_1b_2})\frac{i}{\omega}]-(\rho_1-\rho_2)^2}{[\rho_x+(\frac{b_1+b_2}{k_sb_1b_2})\frac{i}{\omega}]}\,, \tag{19.1}$$

$$\rho_{uw}=\frac{(q_2+r_1)\rho_1+(q_1+r_2)\rho_2}{(s_1+s_2)}$$

$$=\frac{(\rho_1\rho_{2s}+\rho_1\frac{1}{k_sb_2}\frac{i}{\omega}+\rho_1\rho_{12})+(\rho_2\rho_{1s}+\rho_2\frac{1}{k_sb_1}\frac{i}{\omega}+\rho_2\rho_{21})}{[\rho_x+(\frac{b_1+b_2}{k_sb_1b_2})\frac{i}{\omega}]}\,. \tag{19.2}$$

The transformed coupled equations, (13) and (18), can be written even more compactly as:

$$(\overline{\overline{\delta}}\nabla^2+\overline{\overline{B}})\begin{bmatrix}\tilde{e}\\ \tilde{\zeta}\end{bmatrix}=0\,, \tag{20}$$

where $\overline{\overline{B}}=\begin{bmatrix}b & c\\ d & f\end{bmatrix}$ with the matrix elements [Berryman *et al.*, 1988]:

$$b=\omega^2(M\rho_{uu}-C\rho_{wu})\Delta^{-1}, \tag{21.1}$$

$$c=\omega^2(C\rho_{ww}-M\rho_{uw})\Delta^{-1}, \tag{21.2}$$

$$d=\omega^2(C\rho_{uu}-H\rho_{wu})\Delta^{-1}, \tag{21.3}$$

$$f=\omega^2(H\rho_{ww}-C\rho_{uw})\Delta^{-1}, \tag{21.4}$$

$$\Delta=MH-C^2\,. \tag{21.5}$$

Decoupling of (20) is achieved by considering it as an eigenvalue problem for the matrix $\overline{\overline{B}}$. The resulting complex-valued, frequency-dependent eigenvalues $\lambda_{\pm}$ and eigenvectors $\tilde{\Phi}_{\pm}$ are [Berryman *et al.*, 1988]:

$$\lambda_{\pm} = \frac{1}{2}\{(b+f) \pm [(b-f)^2 + 4cd]^{\frac{1}{2}}\}, \tag{22.1}$$

$$\Phi_{\pm} = \Gamma_{\pm}\tilde{e} + \tilde{\zeta}, \tag{22.2}$$

where

$$\Gamma_{\pm} = d(\lambda_{\pm} - b)^{-1} = (\lambda_{\pm} - f)c^{-1} = \frac{1}{2c}\{(b-f) \pm [(b-f)^2 + 4cd]^{\frac{1}{2}}\}. \tag{22.3}$$

The decoupled partial differential equations in the frequency domain are then the Helmholtz equations:

$$(\nabla^2 + \lambda_{\pm})(\Gamma_{\pm}\tilde{e} + \tilde{\zeta}) = 0. \tag{23}$$

4. Low-frequency limit

Although Berryman *et al*. [1988] formally discussed the low-frequency behavior of the eigenvalues and eigenvectors in (23), the corresponding decoupled partial differential equations were neither presented explicitly nor considered in the time domain. To examine (23) at low values of ω, we employ MacLaurin expansions of the coefficients $\Gamma_{\pm}$:

$$\begin{aligned}\Gamma_{+} &\approx \Gamma_{+}\big|_{\omega=0} + \omega\left.\frac{d\Gamma_{+}}{d\omega}\right|_{\omega=0} + \mathrm{O}(\omega^2) \\ &= i[1 - \frac{C}{H}\frac{(b_1+b_2)\rho}{(\rho_1 b_1 + \rho_2 b_2)}]k_s(\rho_1 b_1 + \rho_2 b_2)\omega + \mathrm{O}(\omega^2),\end{aligned} \tag{24.1}$$

$$\begin{aligned}\Gamma_{-} &\approx \Gamma_{-}\big|_{\omega=0} + \omega\left.\frac{d\Gamma_{-}}{d\omega}\right|_{\omega=0} + \mathrm{O}(\omega^2) \\ &= -\frac{H}{C} + i\frac{\Delta}{C^2}[1 - \frac{C}{H}\frac{(b_1+b_2)\rho}{(\rho_1 b_1 + \rho_2 b_2)}]k_s(\rho_1 b_1 + \rho_2 b_2)\omega + \mathrm{O}(\omega^2).\end{aligned} \tag{24.2}$$

To zero-order in ω, $\Gamma_+ \approx 0$ and $\Gamma_- \approx -\frac{H}{C}$. It follows that, to the same order in ω, we have $\rho_{uu} \approx \rho$, $\rho_{uw} \approx \frac{(\rho_1 b_1 + \rho_2 b_2)}{(b_1 + b_2)}$, $\rho_{wu} \approx \frac{(\rho_1 b_1 + \rho_2 b_2)}{(b_1 + b_2)}$, and $\rho_{ww} \approx \frac{\rho_{1s} b_1 + \rho_{2s} b_2}{(b_1 + b_2)} + \frac{1}{k_s (b_1 + b_2)} \frac{i}{\omega}$ after expanding (12) and (19), with the consequence that the eigenvalues of the matrix $\overline{\overline{B}}$ become:

$$\lambda_+ \approx f = \frac{\omega^2}{\Delta} [\frac{(\rho_{1s} b_1 + \rho_{2s} b_2)}{(b_1 + b_2)} H - \frac{(\rho_1 b_1 + \rho_2 b_2)}{(b_1 + b_2)} C + \frac{1}{k_s (b_1 + b_2)} H \frac{i}{\omega}], \tag{25.1}$$

$$\lambda_- \approx \omega^2 \frac{\rho}{H}. \tag{25.2}$$

Thus the two Helmholtz equations in (23) can be rewritten as:

$$\{\nabla^2 + \frac{\omega^2}{\Delta} [\frac{(\rho_{1s} b_1 + \rho_{2s} b_2)}{(b_1 + b_2)} H - \frac{(\rho_1 b_1 + \rho_2 b_2)}{(b_1 + b_2)} C + \frac{1}{k_s (b_1 + b_2)} H \frac{i}{\omega}]\} \tilde{\zeta} = 0, \tag{26.1}$$

$$(\nabla^2 + \omega^2 \frac{\rho}{H})(\frac{H}{C} \tilde{e} - \tilde{\zeta}) = 0. \tag{26.2}$$

The inverse relative mobility term $\frac{1}{b_1 + b_2}$ inside the square brackets in (26.1) can be interpreted physically as an effective dynamic shear viscosity $\eta_{eff} = \frac{1}{b_1 + b_2} = \frac{\eta_1 \eta_2}{k_{r1} \eta_2 + k_{r2} \eta_1}$ for a two-fluid system [Berryman *et al.*, 1988; Lo *et al.*, 2005].

The validity of (26) requires that the angular frequency ω be very small. A more precise statement of this requirement is possible after inspecting (24) and noting that the condition of low frequency is satisfied generally if the dimensionless product of ω and $k_s (\rho_1 b_1 + \rho_2 b_2)$ is much smaller than unity, i.e. $k_s (\rho_1 b_1 + \rho_2 b_2)\, \omega << 1$. [Typically the dimensionless term in square brackets in (24) is *O*(1) for both consolidated and unconsolidated porous media, and the dimensionless ratio $\frac{\Delta}{C^2} < 1$.] The small dimensionless parameter in (24) that has been identified reveals an intrinsic time scale in our model equations (7): $\tau = k_s (\rho_1 b_1 + \rho_2 b_2) = k_s \rho_1 b_1 + k_s \rho_2 b_2 = \tau_1 + \tau_2$, which is the sum of "damping time scales" for the two pore fluids. The condition of very low frequency that underlies the derivation of (26) thus requires the wave excitation frequency ω to be much smaller than the inverse of the intrinsic time scale τ ($\omega \ll \frac{1}{\tau}$).

The time-domain representation of (26) can be deduced immediately after inverse Fourier transformation:

$$\nabla^2 \zeta = \frac{1}{(MH - C^2)}\left[\frac{(\alpha_1 \rho_1 / \theta_1) b_1 + (\alpha_2 \rho_2 / \theta_2) b_2}{(b_1 + b_2)} H - \frac{(\rho_1 b_1 + \rho_2 b_2)}{(b_1 + b_2)} C\right] \frac{\partial^2 \zeta}{\partial t^2}$$

$$+ \frac{H}{(MH - C^2)} \frac{\eta_{eff}}{k_s} \frac{\partial \zeta}{\partial t}, \tag{27.1}$$

$$\nabla^2 \left(\frac{H}{C} e - \zeta\right) = \frac{\rho}{H} \frac{\partial^2}{\partial t^2} \left(\frac{H}{C} e - \zeta\right). \tag{27.2}$$

These two partial differential equations are recognized as a dissipative wave equation (27.1) for the linearized increment of fluid content and a propagating wave equation (27.2) for the normal coordinate $(\frac{H}{C} e - \zeta)$. They describe independent modes of wave motion (“slow” and “fast” compressional waves, respectively) that exist on time scales which are long when compared to the intrinsic time scale τ. Equation (27.2) is consistent with (125) in Berryman *et al.* [1988], which indicates that, in the low-frequency limit, the fast compressional wave in a two-fluid system propagates with speed $(\frac{H}{\rho})^{\frac{1}{2}}$, thus generalizing the expression derived by Biot [1962] for the fast compressional wave in a porous medium containing a single fluid. Comparing our results in (27.1) with those in Berryman *et al.* [1988], we find that, on expanding their function R in a MacLaurin series, the factor R_1 defined in their (119) actually equals zero because $f_1 = \rho_{ww}^{(1)} = 0$ in their notation. This means that the eigenvalue k_-^2 in their (124) contains only ω and ω^2 to lowest order. Therefore, the corresponding decoupled partial differential equation in the time domain must be a dissipative wave equation. We note in passing that (27.1) is symmetric in the labeling of the fluids, a result of the neglect of capillary pressure changes which otherwise would serve to distinguish the fluids physically.

The governing equations for compressional wave propagation and attenuation in single-fluid and two-fluid systems have similar mathematical forms in the low-frequency limit. The decoupled Biot model equations for a fluid-saturated porous medium in this limit also take the form of a dissipative wave equation and a propagating wave equation [Lo *et al.*, 2006]:

$$\nabla^2 \zeta = \frac{1}{(MH - C^2)} \left(\frac{\alpha_s \rho_f}{\phi} H - \rho_f C \right) \frac{\partial^2 \zeta}{\partial t^2} + \frac{H}{(MH - C^2)} \frac{\eta_f}{k_s} \frac{\partial \zeta}{\partial t} , \tag{28.1}$$

$$\nabla^2 \left(\frac{H}{C} e - \zeta \right) = \frac{\rho}{H} \frac{\partial^2}{\partial t^2} \left(\frac{H}{C} e - \zeta \right) . \tag{28.2}$$

Comparison of (27) with (28) provides insight as to the physical behavior of dilatational wave motions in either saturated or unsaturated porous media. One can see that, when a second fluid is present, the governing equations are formulated in terms of a single interstitial fluid having average properties of the two fluids [e.g. K_f in (4)]. For example, the coefficient of the first time-derivative in (27.1) contains the effective dynamic shear viscosity η_{eff}. Similarly, in the coefficient of the second time-derivative in (27.1), the first term contains a relative mobility-weighted average inertial coupling parameter, while the second term contains a relative mobility-weighted average fluid density.

5. Conditions for "low frequency"

In order to illustrate the range of validity of (27), $\tau^{-1} = \omega_c$ was studied numerically as a function of water saturation S_1 $\left(= \frac{\theta_1}{\phi} \right)$ for two different unconsolidated porous media, one containing a LNAPL (light nonaqueous phase liquid), the other a DNAPL (dense nonaqueous phase liquid).

5.1. Water-oil mixture in Columbia fine sandy loam

Material properties necessary for determination of the value of ω_c are listed in Table 1 [Chen *et al.*, 1999]. The van Genuchten [1980]-Mualem [1976] model was applied to evaluate the relative permeability of water and oil at different water volume fractions θ_1:

$$k_{r1}(S_1^e) = (S_1^e)^{\varepsilon} \{1 - [1 - (S_1^e)^{\frac{1}{m}}]^m\}^2 , \tag{29.1}$$

$$k_{r2}(S_1^e) = (1 - S_1^e)^{\varepsilon} [1 - (S_1^e)^{\frac{1}{m}}]^{2m} , \tag{29.2}$$

$$S_1^e = \frac{\theta_1 - \theta_1^r}{\theta_1^s - \theta_1^r}, \tag{29.3}$$

where the quantities ε, m, θ_1^r, and θ_1^s are model parameters obtained by fitting experimental data; θ_1^r and θ_1^s are the residual and saturated volume fractions of water; S_1^e is the effective water saturation [van Genuchten, 1980]. The calculation of ω_c begins by calculating the relative permeabilities k_{r1} and k_{r2} at a given S_1 using (29). Next the "damping coefficients" b_1 and b_2 are computed and, once they are known, the value of ω_c is simply equal to $\frac{1}{k_s(\rho_1 b_1 + \rho_2 b_2)} = \frac{\eta_1 \eta_2}{k_s(\rho_1 k_{r1} \eta_2 + \rho_2 k_{r2} \eta_1)} = \frac{\nu_{eff}}{k_s}$, where ν_{eff} is an effective kinematic shear viscosity defined analogously to η_{eff} [Berryman *et al.*, 1988]. Numerical results are given in Figure 1, which shows that $\omega_c \geq 1254$ kHz, the value found at full water saturation. A wave excitation frequency ω then must be much smaller than 1254 kHz for (27) to be applicable to this system.

5.2. Water-hydrofluoroether mixture in Toyoura sand

One of the most common halogenated DNAPL contaminants in groundwater aquifers is trichloroethylene (TCE), which is used primarily as a solvent, both in metal-cleaning and in dry-cleaning operations [Wartenberg *et al.*, 2000]. In the absence of published data describing the dependence of relative permeability on water saturation for a porous medium containing TCE and water, we used experimental data published for water and hydrofluoroether (HFE-7100) in Toyoura sand [Kamon *et al.*, 2003]. This DNAPL fluid has a dynamic viscosity (0.00058 Ns/m^2) and mass density (1520 kg/m^3) very similar to those of TCE (viscosity = 0.00057 Ns/m^2, mass density = 1480 kg/m^3). The fitting coefficient ε in (29) was reported as $1/2$ for (29.1) and $1/3$ for (29.2) by Kamon *et al.* [2003]. Table 2 lists material parameters for Toyoura sand and HFE-7100. A graph of ω_c as a function of water saturation is presented in Figure 2. It can be seen that $\omega_c \geq 26$ kHz, the value at the residual volume fraction of water. This value is two orders of magnitude lower than that obtained for Columbia fine sandy loam containing oil and water.

5.3. Discussion

The illustrative examples suggest that the minimum value of ω_c is always the smaller of the values that occur at either 0 or 1 water saturation, i.e. the "end-member" values of the critical frequency are sufficient to determine the minimum critical frequency. This trend can be demonstrated rigorously by expressing ω_c in terms of the effective kinematic viscosity ν_{eff} :

$$\omega_c = \frac{1}{\tau} = \frac{\nu_{eff}}{k_s}. \tag{30}$$

Equation (30) tells us that ω_c has a minimum value when ν_{eff} is minimal. Let us consider the two values of ν_{eff} corresponding respectively to full saturation for each fluid [although strictly, residual water still exists at full NAPL saturation, according to (29.3)]. When $k_{r1} = 1$ and $k_{r2} = 0$, ν_{eff} is equal to ν_1. By contrast, when $k_{r1} = 0$ and $k_{r2} = 1$, ν_{eff} is equal to ν_2. At an arbitrary water saturation, if $\nu_1 > \nu_2$, then $\frac{1}{\nu_{eff}} = \frac{k_{r1}}{\nu_1} + \frac{k_{r2}}{\nu_2} < \frac{k_{r1}}{\nu_2} + \frac{k_{r2}}{\nu_2} = \frac{k_{r1} + k_{r2}}{\nu_2} \le \frac{1}{\nu_2}$, since $k_{r1} + k_{r2} \le 1$. Thus, under the condition $\nu_1 > \nu_2$, ν_2 is the minimum value of ν_{eff} . In the opposite case, $\nu_2 > \nu_1$, we can verify that ν_1 is the minimum value of ν_{eff} in a similar way.

This line of reasoning leads to the general conclusion that the fluid with the smaller kinematic viscosity will yield the minimum ν_{eff}, leading to the minimum ω_c for a given porous medium. Hence, the minimum critical frequency is simply:

$$\omega_c(\min) = \frac{\nu_i}{k_s}, \tag{31}$$

where the subscript *i* denotes the fluid which has the smaller kinematic shear viscosity. Now we can find $\omega_c(\min)$ for an unsaturated porous medium without having to calculate values of ω_c over the whole range of water saturation. For example, a water-oil mixture in Massilon sandstone ($k_s = 9 \times 10^{-13} \mathrm{m}^2$) or Lost Hills diatomite ($k_s = 3 \times 10^{-13} \mathrm{m}^2$), two consolidated porous media of relevance to enhanced oil recovery, yields $\omega_c \ge$ 1114 kHz and $\omega_c \ge$ 3343 kHz, respectively, each value corresponding to full water saturation because the kinematic shear viscosity of water is smaller than that of oil.

6. Conclusions

We have found the exact low-frequency limit in the time domain for the decoupled partial differential equations derived by Berryman *et al.* [1988] in the frequency domain to describe dilatational wave propagation with negligible capillary pressure change in an elastic porous medium containing two immiscible fluids. In this limit, the decoupled equations can be reduced to a propagating wave equation and a dissipative wave equation (telegraph equation) representing two independent modes of dilatational wave motion. These modes for a two-fluid system are analogous respectively to the well known "fast" and "slow" compressional waves that occur in an elastic porous medium containing a single fluid [Biot, 1962; Stoll, 1974; Johnson, 1986]. Their existence is assured for all dilatational waves whose excitation frequency is very small when compared to the inverse of the sum of "damping time scales" for the two fluids: $\tau = \tau_1 + \tau_2 = \frac{k_s}{\nu_{eff}}$. The parameter τ is an intrinsic time scale for the two-fluid system.

Numerical calculations were performed to determine the magnitude of $\tau^{-1} \equiv \omega_c$ for representative LNAPL-water and DNAPL-water mixtures in unconsolidated sediments, the main geological materials in shallow aquifers. It was found that ω_c was typically in the kHz-MHz range, depending on the material properties of the two-fluid system. Generalizing these results, we showed that the minimum value of ω_c always occurs at zero (or residual) water saturation for NAPLs whose kinematic viscosity is smaller than that of water (e.g. TCE) and at full water saturation for NAPLs whose kinematic viscosity is larger than that of water (e.g. oil). For example, the minimum ω_c value is in the MHz range for water-oil mixtures in representative consolidated porous media and is determined solely by the kinematic viscosity of water and the permeability of the material. These values are well above the frequency range of interest in a number of engineering applications, e.g. seismic wave stimulation technology (1-250 Hz) used for the remediation of contaminated groundwater [Roberts *et al.*, 2002] and enhanced oil recovery [Kouznetsov *et al.*, 1998]. Therefore, the decoupled partial differential equations in (27) should be accurate for modeling elastic wave motions in a porous medium under low-frequency seismic stimulation, after appropriate initial and boundary conditions are imposed.

Acknowledgments

Gratitude is expressed for financial support to the Assistant Secretary for Fossil Energy, Office of Natural Gas and Petroleum Technology, through the National Petroleum Technology Office, Natural Gas and Oil Technology Partnership under U.S. Department of Energy Contract No. DE-AC02-05CH11231 to Lawrence Berkeley National Laboratory. The first author also acknowledges financial support for this work from the Department of Civil and Environmental Engineering, University of California at Berkeley and from the Department of Hydraulic and Ocean Engineering, National Cheng Kung University.

References

Avraam, D. G. and Payatakes, A. C.: 1995, Generalized relative permeability coefficients during steady-state two-phase flow in porous media, and correlation with the flow mechanisms, *Transp. Porous Media* **20**, 135-168.

Bear, J.: 1988, *Dynamics of fluids in porous media*, Dover: New York.

Beresnev, I. A. and Johnson, P. A.: 1994, Elastic-wave stimulation of oil production: a review of methods and results, *Geophysics* **59**(6), 1000-1017.

Berryman, J. G.: 1983, Dispersion of extensional waves in fluid-saturated porous cylinders at ultrasonic frequencies, *J. Acoust. Soc. Am.* **74**(6), 1805-1812.

Berryman, J. G., Thigpen, L. and Chin, R. C. Y.: 1988, Bulk elastic wave propagation in partially saturated porous solids, *J. Acoust. Soc. Am.* **84**(1), 360-373.

Biot, M. A.: 1956, Theory of propagation of elastic waves in a fluid-saturated porous solid, I. Low-frequency range, II. Higher frequency range, *J. Acoust. Soc. Am.* **28**(2), 168-191.

Biot, M. A. and Willis, D. G.: 1957, The elastic coefficients of the theory of consolidation, *J. Appl. Mech.* **24**, 594-601.

Biot, M. A.: 1962, Mechanics of deformation and acoustic propagation in porous media, *J. Appl. Phys.* **33**(4), 1482-1498.

Brutsaert, W.: 1964, The propagation of elastic waves in unconsolidated unsaturated granular mediums, *J. Geophys. Res.* **69**(2), 243-257.

Brutsaert, W. and Luthin, J. M.: 1964, The velocity of sound in soils near the surface as a function of the moisture content, *J. Geophys. Res.* **69**(4), 643-252.

Chandler, R. N. and Johnson, D. L.: 1981, The equivalence of quasistatic flow in fluid-saturated porous media and Biot's slow wave in the limit of zero frequency, *J. Appl. Phys.* **52**(5), 3391-3395.

Chen, J., Hopmans, J. W. and Grismer, M. E.: 1999, Parameter estimation of two-fluid capillary pressure-saturation and permeability functions, *Adv. Water Resour.* **22**(5), 479-493.

Cowin, S. C.: 1999, Bone poroelasticity, *J. Biomechanics* **32**, 217-238.

Donskoy, D. M., Khashanah, K. and McKee, T. G.: 1997, Nonlinear acoustic waves in porous media in the context of Biot's theory, *J. Acoust. Soc. Am.* **102**(5), 2521-2528.

Dullien, F. L. A.: 1992, *Porous media: Fluid Transport and Pore Structure*, Academic Press: San Diego.

Dutta, N. C. and Ode, H.: 1979, Attenuation and dispersion of compressional waves in fluid-filled porous rocks with partial gas saturation (White model) – Part I: Biot theory, *Geophysics* **11**, 1777-1788.

Dvorkin, J. and Nur, A.: 1993, Dynamic poroelasticity - a unified model with the squirt and the Biot mechanisms**,** *Geophysics* **58**(4), 524-533.

Ehrlich, R.: 1993, Viscous coupling in two-phase flow in porous media and its effect on relative permeabilities, *Transp. Porous Media* **11**, 201-218.

Garg, S. K. and Nayfeh, A. H.: 1986, Compressional wave propagation in liquid and/or gas saturated elastic porous media, *J. Appl. Phys.* **60**(9), 3045-3055.

Gray, W. G.: 1983, General conservation equations for multi-phase systems: 4 Constitutive theory including phase change, *Adv. Water Res.* **6**,130-140.

Jerauld, G.R. and Salter, S. J.: 1990, The effect of pore-structure on hysteresis in relative permeability and capillary pressure: pore level modeling, *Transp. Porous Media* **5**, 103-151.

Johnson, D. L.: 1986, Recent developments in the acoustic properties of porous media, *Proceedings of the International School of Physics "Enrico Fermi" Course XCIII, Frontiers in physical acoustics*, edited by D. Sette, pp. 255-290, North Holland: Amsterdam.

Kamon, M., Endo, K. and Katsumi, T.: 2003, Measuring the *k-S-p* relations on DNAPLs Migration, *Eng. Geol.* **70**, 351-363.

Kearey, P., Brooks, M. and Hill, I.: 2002, *An introduction to geophysical exploration*, Oxford: Blackwell Science.

Kouznetsov, O. L., Simkin, E. M., Chilingar, G. V. and Katz, S. A.: 1998, Improved oil recovery by application of vibro-energy to waterflooded sandstones, *J. Pet. Sci. Eng.* **19**(3-4), 191-200.

Lake, L. W.: 1989, *Enhanced oil recovery*, Prentice-Hall: Englewood Cliffs.

Lewis, R. W. and Schrefler, B. A.:1998, *The finite element method in the static and dynamic deformation and consolidation of porous media*, John Wiley & Sons: Chichester.

Li, X., Zhong, L. and Pyrak-Nolte, L. J.: 2001, Physics of partially saturated porous media: Residual saturation and seismic-wave propagation, *Annu. Rev. Earth Planet. Sci*. **29**, 419-460.

Lo, W.-C., Sposito, G. and Majer, E.: 2002, Immiscible two-phase fluid flows in deformable porous media, *Adv. Water Resour.* **25**(8-12), 1105-1117.

Lo, W.-C., Sposito, G. and Majer, E.: 2005, Wave propagation through elastic porous media containing two immiscible fluids, *Water Resour. Res.* **41**, W02025.

Lo, W.-C., Sposito, G. and Majer, E.: 2006, Low-frequency dilatational wave propagation through fully-saturated poroelastic media, *Adv. Water Resour.* in press.

Mei, C. C. and Foda, M. A.:1981, Wave-induced responses in a fluid-filled poro-elastic solid with a free-surface - a boundary-layer theory, *Geophysical Journal of the Royal Astronomical Society*, **66**(3), 597-631.

Mualem, Y.: 1976, A new model for predicting the hydraulic conductivity of unsaturated porous media, *Water Resour. Res.* **12**(3), 513-522.

Rice, J. R. and Cleary, M. P.: 1976, Some basic stress diffusion solutions for fluid-saturated elastic porous-media with compressible constituents, *Rev. Geophys.* **14**(2), 227-241.

Roberts, P. M., Sharma, A., Uddameri, V., Monagle, M., Dale, D. E. and Steck, L. K.: 2001, Enhanced DNAPL transport in a sand core during dynamic stress stimulation, *Environ. Eng. Sci.* **18**(2), 67-79.

Santos, J. E., Corbero, J. M. and Douglas, J.: 1990, Static and dynamic behavior of a porous solid saturated by a two-phase fluid, A model for wave propagation in a porous medium saturated by a two-phase fluid, *J. Acoust. Soc. Am.* **87**(4), 1428-1448.

Stoll, R. D.: 1974, Acoustic waves in saturated sediments, *Physics of Sound in Marine Sediments*, edited by L. Hampton, pp. 19-39, Plenum Press: New York.

Truesdell, C.: 1984, Rational thermodynamics, New York: Springer-Verlag.

van Genuchten, M. T.: 1980, A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am. J.* **44**(5), 892-898.

Wartenberg, D., Reyner, D. and Scott, C. S.: 2000, Trichloroethylene and cancer: Epidemiologic evidence, *Environ. Health Perspect.* **108**(Suppl. 2), 161-176.

Yuster, S. T.:1951, Theoretical considerations of multiphase flow in idealized capillary systems, *World Petroleum Congress Proceeding*, Section II, 436-445, The Hague.

Table 1 Material parameters for Columbia fine sandy loam saturated by an oil – water mixture [Chen *et al.*, 1999]

Input parameter	Notation	Value Selected
Fitting parameter	m	0.5090
Fitting parameter	ε	0.5
Mass density of oil	ρ_2	762 kg/m^3
Mass density of water	ρ_1	997 kg/m^3
Permeability	k_s	8×10^{-13} m^2
Porosity	ϕ	0.45
Residual water volume fraction	θ_1^r	0.0723
Saturated water volume fraction	θ_1^s	0.45
Viscosity of oil	η_2	0.00144 Ns/m^2
Viscosity of water	η_1	0.001 Ns/m^2

Table 2 Material parameters for Toyoura sand saturated by a HFE-7100 – water mixture [Kamon *et al.*, 2003]

Input parameter	Notation	Value Selected
Fitting parameter	m	0.726
Fitting parameter	ε	0.5 or 1/3
Mass density of HFE-7100	ρ_2	1520 kg/m^3
Mass density of water	ρ_1	997 kg/m^3
Permeability	k_s	1.4825×10^{-11} m^2
Porosity	ϕ	0.38
Residual water volume fraction	θ_1^r	0.0532
Saturated water volume fraction	θ_1^s	0.38
Viscosity of HFE-7100	η_2	0.00058 Ns/m^2
Viscosity of water	η_1	0.001 Ns/m^2

Figure 1 Graph of ω_c versus water saturation for Columbia fine sandy loam containing an oil - water mixture

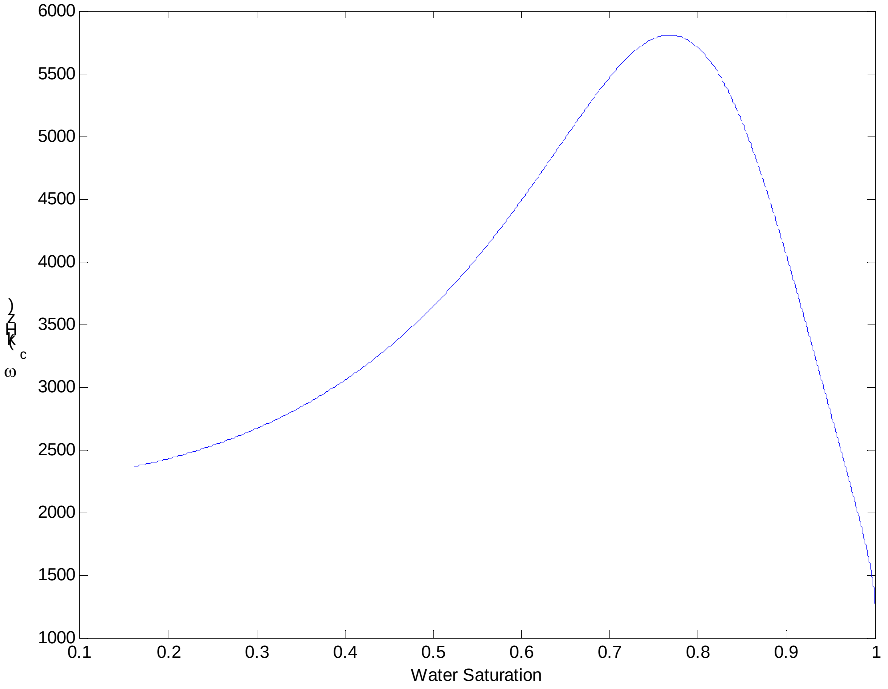

Figure 2 Graph of ω_c versus water saturation for Toyoura sand containing a HFE-7100 – water mixture