Princeton Plasma Physics Laboratory

PPPL- 4369

PPPL- 4369

Use of Helical Fields to Allow a Long Pulse Reversed Field Pinch

A.H. Boozer and N. Pomphrey

November, 2008

Prepared for the U.S. Department of Energy under Contract DE-AC02-76CH03073.

Use of helical fields to allow a long pulse Reversed Field Pinch

Allen H. Boozer

Columbia University, New York, NY 10027, ahb17@columbia.edu

and

Neil Pomphrey

Princeton Plasma Physics Laboratory, Princeton, N.J. 08540, pomphrey@pppl.gov

(Dated: November 7, 2008)

The maintenance of the magnetic configuration of a Reversed Field Pinch (RFP) is an unsolved problem. Even a toroidal loop voltage does not suffice to maintain the magnetic configuration in axisymmetry but could if the plasma had helical shaping. The theoretical tools for plasma optimization using helical shaping have advanced, so an RFP could be relatively easily designed for optimal performance with a spatially constant toroidal loop voltage. A demonstration that interesting solutions exist is given.

I. INTRODUCTION

Experiments on the Madison Symmetric Torus (MST) experiment have shown [1], [2], [3] that if the magnetic configuration of a Reversed Field Pinch (RFP) is maintained by loop voltages the confinement is similar to that of a tokamak of comparable size and current–a very optimistic result. Unfortunately, both a poloidal V_p and a toroidal V_t loop voltage are required, Appendix. The poloidal loop voltage drains toroidal magnetic flux from the plasma and can, therefore, be maintained for only a short period of time.

By helically shaping the RFP [4], the reversal of the toroidal field can be maintained with a spatially constant toroidal loop voltage, which means an electric field of the form $\vec{E} = \vec{\nabla}(V_t\varphi) - \vec{\nabla}\Phi$. With this electric field, the magnetic field in the plasma region is independent of time, although the magnetic flux through a transformer solenoid must change to produce the toroidal loop voltage V_t. In a large aspect ratio RFP, the flux in the transformer can be very large compared to the toroidal magnetic flux in the plasma. Consequently, the RFP is consistent with a long pulse if the magnetic configuration is maintained by a spatially constant toroidal loop voltage.

Since the early work on helical shaping of the RFP [4], which led to the Ohmically Heated Toroidal Experiment (OHTE) [5], little research has been done on controlled helical shaping. However, RFP plasmas in the axisymmetric Reversed Field eXperiment (RFX) [6] are observed [7], [8] to self-organize into a long-lived helical state with greatly improved transport coefficients. The ability to design helically-shaped plasmas to achieve given physics goals has greatly improved over the last quarter century [9], [10], [11]. The improved theoretical tools, and the experiments on MST and RFX imply that another study is timely.

Here a preliminary study is reported that shows that relatively weak helical fields, which are within the technical capabilities of the existing RFX device [12], are consistent with the net parallel current profile,

$$k = \mu_0 \frac{\langle \vec{j}\cdot\vec{B}\rangle}{\langle B^2\rangle}, \tag{1}$$

vanishing at the plasma edge while maintaining an RFP-like safety factor profile, Section (II). The safety factor q is the number of toroidal circuits a magnetic field line makes during one poloidal circuit. In an RFP, the safety factor passes through zero near the plasma edge. For the case that was studied a helical field with poloidal mode number $m = 1$ and toroidal mode number $n = 8$ of approximately 8% of the average toroidal field was shown to be consistent with an RFP-like q profile with $k = 0$ at the edge.

An important result is that the helical field that reduces the required k at the edge has the opposite helicity to that of the magnetic field lines, so magnetic islands should not be an issue.

The Variational Moments Equilibrium Code (VMEC) [13] was used in the study. The radial coordinate of the VMEC three-dimensional equilibrium code is the toroidal magnetic flux, so a full reversal in the sign of q could not be enforced. Nonetheless using VMEC, the safety factor at the plasma edge could be made very small compared to the central value.

A more complete study would involve a VMEC optimization [14] of the shape of the outer boundary of an RFP with a fixed safety factor profile to obtain: (1) consistency of the current profile with a spatially constant toroidal loop voltage, (2) stability within ideal magnetohydrodynamics (MHD), and (3) sufficiently small neoclassical transport. In addition, only a small external field should be required since a major attraction of the RFP is that the magnetic field field that must be produced by coils is small.

A large helical wobble of the plasma can be produced by adding a weak external magnetic field that has the approximate form

$$\vec{B}_{ext} = (\hat{R}\cos n\varphi + \hat{Z}\sin n\varphi)B_h, \tag{2}$$

where (R, φ, Z) are cylindrical coordinates. This is intuitively obvious. When the toroidal field is zero and the aspect ratio is large, even an infinitesimal field B_h will shift the magnetic axis of the poloidal magnetic field. That is, the field of Equation (2) should produce a strong helical wobble of the plasma with little distortion of the plasma cross section in a constant-φ plane. It should be noted that the magnetic field of Equation (2) is neither curl nor divergence free, but it approximates such a field over the minor radius a of the plasma if $na/R << 1$, where R is the major radius of the torus. A simple analytic calculation gives the helical wobble and the rotational transform produced by the external field of Equation (2) when acting on a toroidal field B_φ. The answers are $n\delta/R_{00} \approx B_h/B_\varphi$ and $\iota_{ext} \approx n^3\delta^2/2R_{00}^2$. Numerically, the reduction in the edge value of k scales somewhat more like $n^4\delta^2$ than $n^3\delta^2$. If the analytic calculation is used to estimate the required field B_h for the case considered, then $B_h/\langle B_\varphi\rangle \approx (na/R_{00})\delta$, which is about twice the numerical results.

II. DEMONSTRATION OF EXISTENCE

The effect of a helical wobble of the plasma has been studied using fixed boundary VMEC [13] to demonstrate that the parallel plasma current can be made zero at the plasma edge for an RFP-like profile of the safety factor q. The radial coordinate in VMEC is s, the fraction of the toroidal magnetic flux each magnetic surface encloses; $s = 1$ is the plasma boundary. A VMEC equilibrium is specified by giving the plasma boundary $\vec{x}_b(\theta, \varphi)$, the pressure profile $p(s)$, and the rotational transform profile $\iota(s) \equiv 1/q(s)$. For maximal simplicity of the demonstration, the plasma pressure was made zero.

A helical wobble of the plasma can be produced by a weak external magnetic field, so the plasma boundary

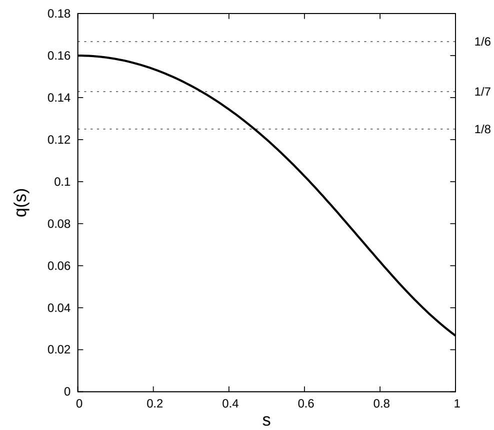

FIG. 1: The safety factor profile used in the VMEC calculations, where s is labels the magnetic surfaces by the fraction of enclosed toroidal magnetic flux.

$\vec{x}_b(\theta,\varphi)$ was chosen to be circular in cross section with a minor radius a, an average major radius R_{00}, and a helical wobble δ. In (R,φ,Z) cylindrical coordinates this boundary is $\vec{x}_b(\theta,\varphi) = R(\theta,\varphi)\hat{R}(\varphi) + Z(\theta,\varphi)\hat{Z}$, where $R(\theta,\varphi) = R_{00} + \delta\cos n\varphi + a\cos\theta$ and $Z(\theta,\varphi) = \delta\sin n\varphi + a\sin\theta$. We choose $R_{00} = 2.0$m and $a = 0.46$m (aspect ratio 4.35), which is consistent with RFX. The sign conventions imply the helicity of the wobble is in the opposite sense to that of the magnetic field lines. This sense is found to reduce the edge current for a fixed q profile while the opposite sense increases the edge current.

In the standard version of VMEC, the rotational transform profile, $\iota(s) \equiv 1/q(s)$ is input as a polynomial. The profile used was $\iota(s) = 6.25(1 + s^2 + s^4 + s^6 + s^8 + s^{10})$, which is the first six terms in a Taylor expansion of an ι profile with $q(s) = q_0(1 - s^2)$. The q profile that was used is illustrated in Figure (1). A plasma current of 1.0MA is assumed. With the given q profile and plasma shape, this corresponds to an enclosed toroidal flux $\langle B_\varphi\rangle\pi a^2 = 0.214$ Webers.

The calculation consists of varying δ for various toroidal mode numbers n and calculating the parallel current, or more precisely k, at the plasma edge. The results are given in Figs (2) and (3). The required wobble decreases with increasing n. The parallel current distribution in the axisymmetric equilibrium, $\delta = 0$, and the $n = 8$ equilibrium for which k vanishes at the edge, $\delta \approx 0.08$m, are shown in Figure (2).

The external magnetic field required to produce the helical wobble of the plasma is calculated using the Neumann Solver for fields produced by external Coils (NESCOIL) code [15, 16]. A toroidal surface (TS) is chosen that encloses the plasma. Any current distribution on that surface can be represented by a single scalar current potential $\kappa(\theta,\varphi)$ in the form $\vec{j} = \delta(r - r_c)\vec{\nabla}r \times \vec{\nabla}\kappa(\theta,\varphi)$ where r is any well-behaved radial coordinate and $r = r_c$ gives the TS. The current flows along constant-κ contours because $\vec{j}\cdot\vec{\nabla}\kappa = 0$. The current potential is found by minimizing the mean square of the normal field on the desired plasma boundary.

RFX has an array of 192 saddle coils mounted in grooves on the outer surface of the axisymmetric Toroidal Support Structure (48 along the toroidal direction and 4 along the poloidal direction)[12]. For consistency with RFX capabilities the current potential was calculated for $n = 6$ and $n = 8$ plasma wobbles on a TS with R=2.00m, a=0.65m. The

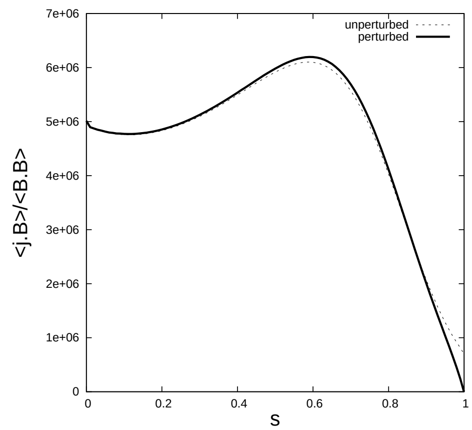

FIG. 2: The parallel current distribution is given for both the unperturbed axisymmetric case $\delta = 0$ and for the $n = 8$ perturbed case with $\delta \approx 0.08$m.

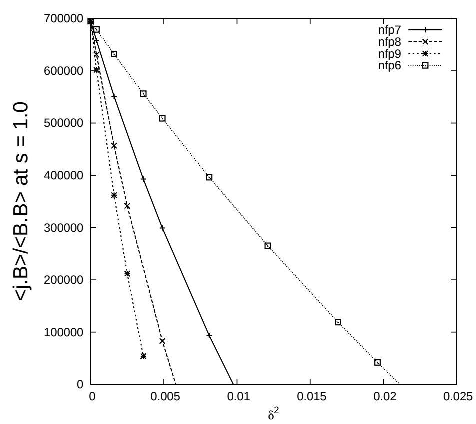

FIG. 3: The parallel current at the plasma edge is reduced as the helcial wobble δ is increased.

vertical magnetic field B_z^n due to the current potential κ for mode n was evaluated on an axisymmetric toroidal loop at the center of the unperturbed plasma (R=2.00m, Z = 0.00m).

Figure (4) shows the vertical field B_z^n required to zero the edge parallel current. The vertical field, $B_z^n = B_v + B_h \cos(n\varphi)$, is the sum of an axisymmetric field B_v and an oscillating helical field B_h. For an axisymmetric equilibrium, $B_v = 0.138T$ but is increased slightly to $0.14T$ for $n = 8$ and to $0.147T$ for $n = 6$. The required helical field B_h is $0.034T$ for $n = 6$, but only 0.025 for $n = 8$. The RFX saddle coils [12] are capable of producing $0.04T$ in $n = 8, m = 1$. The average toroidal field in the axisymmetric equilibrium is the $\langle B_\varphi \rangle = 0.322T$, while the toroidal field coils of RFX can produce $0.7T$.

Figure (5) shows the current potential. The $n = 0$ terms have been removed from the plot since these contributions are small ($\sim 0.002T$) and can be provided by adjusting the currents in the axisymmetric poloidal field coils. Contours

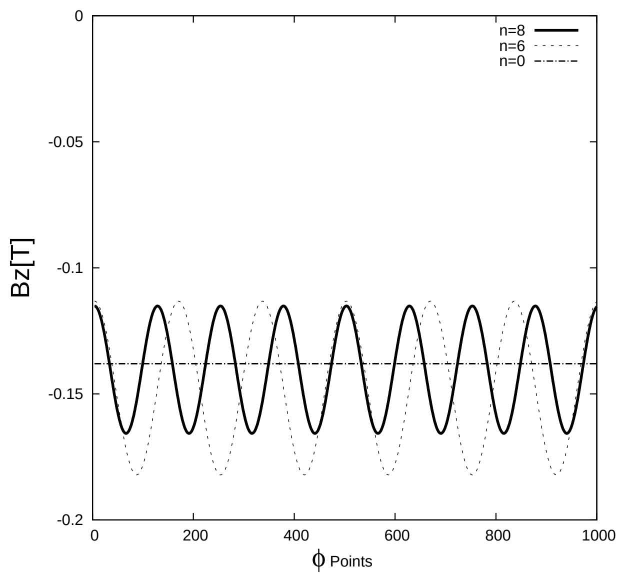

FIG. 4: Vertical field around midplane loop $R = R_{00} = 2.0$m, $Z = 0.0$m required to zero the edge parallel current by inducing $n = 6$ (dash) and $n = 8$ (solid) helical wobbles. The horizontal (dash-dot) line corresponds to the calculated vertical field for the unperturbed axisymmetric equilibrium.

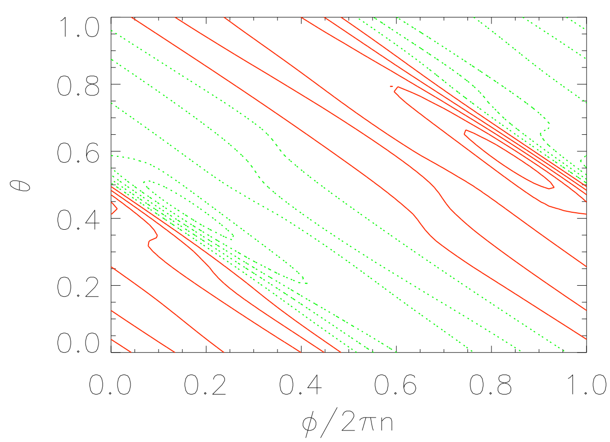

FIG. 5: Contours of current potential over 1/8 of a torus calculated by NESCOIL for providing $n = 8$ wobble to zero the edge parallel current. Positive and negative contour values are differentiated by line type.

of κ can be interpreted as a discrete coil representation of the current distribution required to produce the plasma wobble, each contour representing a one-turn coil. The change in κ between contours gives the required current in each coil turn. The current potential contours approximate the straight lines that would be expected from an $n = 8, m = 1$ potential. The departure from straight lines comes from the assumption of a simple plasma shape. Little difference in the plasma properties would be expected if the current potential were replaced by a pure $n = 8, m = 1$ potential–just a small deformation of the plasma cross section from a wobbling circle.

Although the calculations indicate that reversal may be achieved for an $n = 8, m = 1$ helical field of only 8% of the average toroidal field, this number is dependent upon the assumed q profile, Figure (1), which required a relatively low value for the current distribution, k, in axisymmetry, Figure (2). The reduction of the required edge current density k for reversal was found to scale approximately as the helical field squared. Even if the q profile required a current profile, k, that was approximately constant in axisymmetry, a helical field of about 20% should be consistent with reversal. This study did not consider the issue of start-up. The two limiting cases for start-up are turning on the helical field along with the toroidal field before the plasma is initiated and turning on the helical field after a standard RFP is formed.

The experimental issue is whether a quiescent plasma can be maintained for as long as a loop voltage is supplied. For this demonstration, a high plasma current is not required since the more plasma skin times quiescence can be maintained the more convincing the result. For a plasma pulse longer than the skin time of the wall, the current in the toroidal field coils must be reversed from the initial direction. The current in the toroidal field coils at the time of plasma initiation determines the magnitude and sign of the toroidal flux in the plasma.

Acknowledgments

AHB would like to acknowledge support by the U.S. Department of Energy grant ER54333.

Appendix: Condition for a long pulse RFP

The difficulty of maintaining the magnetic configuration of an RFP can be understood by considering a model, a cylindrical, force-free $\mu_0\vec{j} = k(r)\vec{B}$, RFP of length $2\pi R$. The analogue of the toroidal angle is $\varphi \equiv z/R$. Ampere's law implies $dB_z/dr = -kB_\theta$ and $d(rB_\theta)dr = rk(r)B_z$. The best known RFP equilibrium [17] has $k = k_0$, a constant. For this equilibrium $B_z = B_0J_0(k_0r)$, and $B_\theta = B_0J_1(k_0r)$, where B_0 is a constant and J_0 and J_1 are ordinary cylindrical Bessel functions. The condition for reversal of B_z is $k_0a > 2.404$. In any case, the equation $dB_z/dr = -k(r)B_\theta$ implies $k(r)$ must be non-zero at the radial location at which B_z passes through zero.

The evolution of the current profile $k(r)$ can be obtained from the component of Ohm's law along the magnetic field, $\vec{B}\cdot\vec{E} = \eta(r)\vec{B}\cdot\vec{j}$ and the expression for the electric field $\vec{E} = V_p(r,t)\vec{\nabla}\theta/2\pi + V_t(r,t)\vec{\nabla}z/2\pi R - \vec{\nabla}\Phi(r,t)$. The current profile in the cylindrical model is

$$k(r,t) = \frac{\mu_0}{\eta B^2}\left(\frac{V_p}{2\pi r}B_\theta + \frac{V_t}{2\pi R}B_z\right). \tag{3}$$

Faraday's law gives the flux evolution equations $\partial\psi_t^{(in)}(r,t)/\partial t = -V_p$ for the toriodal magnetic flux inside the radius r and $\partial\psi_p^{(out)}(r,t)/\partial t = V_t$ for the poloidal magnetic flux outside the radius r. The current profile vanishes at the radial location at which $B_z = 0$ unless the poloidal loop voltage V_p is non-zero. Consequently, a poloidal loop voltage is required in the cylindrical model to maintain reversal; the required poloidal loop voltage V_p drains toroidal flux from the plasma.

The magnetic field in a toroidal plasma is independent of time if the electric field has the form $\vec{E} = \vec{\nabla}(V_t\varphi/2\pi) - \vec{\nabla}\Phi$ with the toroidal loop voltage independent of position. Since the magnetic flux in the transformer solenoid Ψ_s can be very large in a large aspect ratio RFP, the magnetic configuration could be maintained for a very long period if the magnetic configuration could be maintained by a toroidal loop voltage alone.

The general constraint that must be obeyed to maintain the magnetic configuration with a toroidal loop voltage alone can be examined by considering a force-free RFP, $\mu_0\vec{j} = k(\vec{x})\vec{B}$. The profile of the parallel current, $k(\vec{x})$, must be constant within the magnetic surfaces in order for $\vec{\nabla}\cdot\vec{j}$ to be zero. The Ohm's law, $\vec{E}+\vec{v}\times\vec{B} = \eta\vec{j}$ then implies

$$k = \frac{\mu_0 V_t}{\eta}\left\langle \frac{B_\varphi}{2\pi R B^2}\right\rangle, \tag{4}$$

where the average over a magnetic surface is defined by

$$\left\langle \frac{B_\varphi}{2\pi R B^2}\right\rangle \equiv \frac{\oint \frac{\vec{B}\cdot\vec{\nabla}\varphi}{2\pi B^2}\frac{da}{|\vec{\nabla}\psi_p|}}{\oint \frac{da}{|\vec{\nabla}\psi_p|}},$$

and ψ_p is the poloidal flux outside of a magnetic surface. Since the toroidal field B_φ vanishes at the plasma edge, so must k. A helical wobble of the magnetic surfaces allows B_φ to pass through zero even on a surface on which $k = 0$.

---

[1] B. E. Chapman, J. K. Anderson, T. M. Biewer, D. L. Brower, S. Castillo, P. K. Chattopadhyay, C.-S. Chiang, D. Craig, D. J. Den Hartog, G. Fiksel, P. W. Fontana, C. B. Forest, S. Gerhardt, A. K. Hansen, D. Holly, Y. Jiang, N. E. Lanier, S. C. Prager, J. C. Reardon, and J. S. Sarff, Phys. Rev. Lett. **87**, 205001 (2001).

[2] J.S. Sarff, A.F. Almagri, J.K. Anderson, T.M. Biewer, A.P. Blair, M. Cengher, B.E. Chapman, P.K. Chattopadhyay, D. Craig, D.J. Den Hartog, F. Ebrahimi, G. Fiksel, C.B. Forest, J.A. Goetz, D. Holly, B. Hudson, T.W. Lovell, K.J. McCollam, P.D. Nonn, R. O'Connell, S.P. Oliva, S.C. Prager, J.C. Reardon, M.A. Thomas, M.D. Wyman, D.L. Brower, W.X. Ding, S.D. Terry, M.D. Carter, V.I. Davydenko, A.A. Ivanov, R.W. Harvey, R.I. Pinsker, and C. Xiao, Nucl. Fusion **43**, 1684 (2003).

[3] J. K. Anderson, J. Adney, A. Almagri, A. Blair, D. L. Brower, M. Cengher, B. E. Chapman, S. Choi, D. Craig, D. R. Demers, D. J. Den Hartog, B. Deng, W. X. Ding, F. Ebrahimi, D. Ennis, G. Fiksel, C. B. Forest, P. Franz, J. Goetz, and R. W. Harvey, Phys. Plasmas **12**, 056118 (2005).

[4] T. Ohkawa, M. Chu, C. Chu, M. Schaffer, Nuclear Fusion **20**, 1464 (1980).

[5] R. Goforth, T. Carlstrom, C. Chu, G. Jackson, R. La Haye, T. Ohkawa, M. Shaffer, T. Tamano, P. Taylor, N. H. Brooks, and R. Chase, Nucl. Instruments and Methods in Phys. Research **207**, 87 (1983).

[6] P. Sonato, G. Chitarin, P. Zaccaria, F. Gnesotto, S. Ortolani, A. Buffa, M. Bagatin, W. R. Baker, S. Dal Bello, P. Fiorentin, L. Grando, G. Marchiori, D. Marcuzzi, A. Masiello, S. Peruzzo, N. Pomaro, G. Serianni, Fusion Eng. Des., **66**, 161 (2003).

[7] P. Martin, L. Marrelli, A. Alfier, F. Bonomo, D. F. Escande, P. Franz, L. Frassinetti, M. Gobbin, R. Pasqualotto, P. Piovesan, D. Terranova, and the RFX-mod Team, Plasma Phys. and Controlled Fusion **49**, A177, (2007).

[8] R. Lorenzini, D. Terranova, A. Alfier, P. Innocente, E. Martines, R. Pasqualotto, and P. Zanca P, Phys. Rev. Lett. **101**, 025005 (2008).

[9] C. G. Beidler, G. Grieger, F. Herrnegger, E. Harmeyer, J. Kisslinger, W. Lotz, H. Maassberg, P. Merkel, J. Nuhrenberg, F. Rau, J. Sapper, F. Sardei, R. Scardovelli, A. Schluter, and H. Wobig, Fusion Technol. **19**, 148 (1990).

[10] M. C. Zarnstorff, L. A. Berry, A. Brooks, E. Fredrickson, G-Y. Fu, S. Hirshman, S. Hudson, L-P. Ku, E. Lazarus, D. Mikkelsen, D. Monticello, G. H. Neilson, N. Pomphrey, A. Reiman, D. Spong, D. Strickler, A. Boozer, W. A. Cooper, R. Goldston, R. Hatcher, M. Isaev, C. Kessel, J. Lewandowski, J. F. Lyon, P. Merkel, H. Mynick, B. E.Nelson, C. Nuehrenberg, M. Redi, W. Reiersen, P. Rutherford, R. Sanchez, J. Schmidt, and R. B. White, Plasma Phys. Controlled Fusion **43**, A237 (2001).

[11] D. J. Strickler, S. P. Hirshman, D. A. Spong, M. J. Cole, J. F. Lyon, B. E. Nelson, D. E. Williamson, and A. S. Ware, Fusion Science and Technology **45**, 15 (2004).

[12] G. Chitarin, S. Dal Bello, L. Grando, and S. Peruzzo, Fusion Eng. Des. **66**, 1055 (2003).

[13] S. P. Hirshman, W. I. van Rij, and P. Merkel, Comput. Phys. Commun. **43**, 143 (1986).

[14] D. A. Spong, S. P. Hirshman, J. C. Whitson, D. B. Batchelor, B. A. Carreras, V. E. Lynch, and J. A. Rome, Phys. Plasmas **5**, 1752 (1998).

[15] P. Merkel, Nucl. Fusion **27**, 867 (1987).

[16] M. Drevlak, Fusion Technology **33**, 106 (1998).

[17] J. B. Taylor, Phys. Rev. Lett. **33**, 1139 (1974).

The Princeton Plasma Physics Laboratory is operated
by Princeton University under contract
with the U.S. Department of Energy.

Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2750
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: http://www.pppl.gov