
Princeton Plasma Physics Laboratory

PPPL-4139

PPPL-4139

Interchange and Infernal Fishbone Modes in Plasmas with Tangentially Injected Beams

Ya.I. Kolesnichenko, V.S. Marchenko,
and R.B. White

January 2006



Prepared for the U.S. Department of Energy under Contract DE-AC02-76CH03073.

Princeton Plasma Physics Laboratory

Report Disclaimers

Full Legal Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party's use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Trademark Disclaimer

Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors.

PPPL Report Availability

Princeton Plasma Physics Laboratory

This report is posted on the U.S. Department of Energy's Princeton Plasma Physics Laboratory Publications and Reports web site in Fiscal Year 2006.

The home page for PPPL Reports and Publications is:

http://www.pppl.gov/pub_report/

Office of Scientific and Technical Information (OSTI):

Available electronically at: <http://www.osti.gov/bridge>.

Available for a processing fee to U.S. Department of Energy and its contractors, in paper from:

U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831-0062

Telephone: (865) 576-8401

Fax: (865) 576-5728

E-mail: reports@adonis.osti.gov

Interchange and infernal fishbone modes in plasmas with tangentially injected beams

Ya.I. Kolesnichenko¹, V.S. Marchenko¹, R.B. White²

¹*Institute for Nuclear Research, Prospect Nauky 47, Kyiv, 03680, Ukraine*

²*Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton,*

New Jersey, 08543, USA

(January 3, 2006)

Abstract

New energetic particle mode instabilities of fishbone type are predicted. The considered instabilities are driven by the circulating energetic ions. They can arise in plasmas of tokamaks and spherical tori with weak magnetic shear in the wide core region and strong shear at the periphery, provided that the central safety factor is close to the ratio m/n , where m and n are the poloidal mode number and toroidal mode number, respectively. The instability with $m = n = 1$ has interchange-like spatial structure, whereas the structure of instabilities with $m/n > 1$ is similar to that of the infernal MHD mode (except for the region in vicinity of the local Alfvén resonance).

I. INTRODUCTION

Fishbone oscillation is an important collective phenomenon caused by the energetic ions in tokamak plasmas.¹ This phenomenon is associated with perturbations of the Alfvén type. The perturbations have the frequency, ω , equal to either the bulk ion diamagnetic frequency, ω_{*i} , or a characteristic frequency determined by the energetic ions. In the latter case, the instability is called the Energetic Particle Mode (EPM). Instabilities of the EPM type are presumably responsible for fishbones with bursting character and strong frequency chirping. The first theoretical explanation of experimentally observed fishbone oscillations was based on a prediction of an EPM instability.² This instability was caused by the trapped energetic ions; it was actually a rigid $m = n = 1$ kink displacement (m and n are the poloidal mode number and toroidal mode number, respectively) and had the frequency of approximately the precession frequency of the energetic ions, ω_D^t . A little later an instability with the frequency $\omega \approx \omega_{*i}$ and a similar spatial structure was discovered.³ Fishbone instabilities with the same structure were studied also in subsequent works. However, trapped-particle-induced EPM fishbones can have another, interchange-like, structure. This was shown for the case when magnetic shear, \hat{s} , is small in the plasma core and the magnetic field strength, B , is low, which is typical for spherical tori.⁴ On the other hand, circulating-particle-induced EPM fishbone instability in systems with small shear and low magnetic field has not been considered yet. Moreover, a conventional circulating-particle-induced EPM fishbone instability was considered only recently.^{4,5}

Fishbone instability of the EPM type is actually an Alfvén instability with the frequency lying in the Alfvén continuum region [in contrast to “Alfvén gap modes” such as Toroidicity-induced Alfvén Eigenmodes (TAE)]. It differs from another mode in the Alfvén continuum region, Global Alfvén Eigenmode (GAE), whose frequency is determined by the bulk plasma and lies below the Alfvén continuum branch corresponding to a dominant Fourier harmonic of the mode (the

GAE frequency intersects the Alfvén continuum of satellite harmonics only). Being a continuum mode, EPM fishbones suffer from continuum damping; therefore, this instability arises when the pressure of the energetic ions exceeds a threshold value associated with this damping.

In the case of the conventional $m = n = 1$ fishbone instability associated with trapped particles, there are two points of the local Alfvén resonance, $r_{res}^{(1)}$ and $r_{res}^{(2)}$. Both these points are located close to the $q = 1$ radius [$q(r)$ is the safety factor], $r_{res}^{(1)} = r_s - \Delta$ and $r_{res}^{(2)} = r_s + \Delta$, with $\Delta \ll r_s$ and r_s is defined by $q(r_s) = m/n$. The small magnitude of Δ is explained by the fact that $\omega/\omega_{AC}(r = 0) \ll 1$, where $\omega_{AC} \approx |\iota(r) - 1|v_A(r)/R$ is the Alfvén continuum frequency, v_A is the Alfvén velocity, R is the large radius of the torus, and $\iota = q^{-1}$ is the rotational transform. This implies that finite frequency of the mode, $\omega \sim \omega_D^t$, affects the structure of MHD perturbations with $\omega = 0$, i.e., internal kink mode, only in the region where the mode amplitude is quickly decreasing. When B is low, and/or q is close to unity inside the $q = 1$ radius, the ratio $\omega/\omega_{AC}(0)$ is not small because $\omega/\omega_{AC}(0) \propto \omega_D^t/\omega_{AC}(0) \propto (\iota_0 - 1)^{-1}B^{-2}$, with $\iota_0 = \iota(r = 0)$. Because of this, $r_{res}^{(2)}$ is shifted to the right, where the mode amplitude is much less than that at $r = 0$, whereas the resonance point $r_{res}^{(1)}$ may disappear.⁴ In this case, the structure of the ideal MHD perturbations with $\omega = 0$ is again not essentially affected by the energetic ions, but this structure has nothing to do with the rigid kink displacement.⁴

Circulating-ion-induced EPM fishbone instability in large-shear systems has the frequency $\omega \sim (\hat{s}v_\alpha^2)/(r_s R \omega_{c\alpha}) = 2\hat{s}\omega_D^t$, where v_α is the birth velocity of the energetic ions, $\omega_{c\alpha}$ is the energetic ion gyrofrequency.^{4,5} We conclude from this that $\omega^c \sim \omega^t$ for $\hat{s} \sim 1$, where ω^t and ω^c are frequencies of the modes associated with trapped ions and circulating ions, respectively. Therefore, finite mode frequency weakly affects the rigid kink structure, which justifies the approach used in Refs. 4, 5. The mentioned instability is caused by only those particles which intersect the $q = 1$ surface, as in the case of the fishbone instability with $\omega \approx \omega_{*i}$ considered in Ref. 6.

The situation changes when the magnetic shear at $r < r_s$ is small and $q_0 \approx m/n$.

Then the MHD stability numerical calculations predict the existence of pressure-driven "infernal" modes in plasmas with a shearless core.^{7–10} A particular case of these instabilities is a quasi-interchange mode with $m = n = 1$, which was studied analytically in Ref. 11–13. The eigenfunction of this instability has convective, "cellular" character, in contrast to the rigid kink displacement in the finite shear case.¹⁴ This mode structure will not change essentially in the presence of the circulating energetic ions when the points of the local Alfvén resonance are located in the periphery region, where the shear is not small.

The non-rigid character of perturbations in low-shear systems has an important consequence: It provides a strong energy exchange between the energetic ions and perturbations through the resonance $\omega = k_{\parallel} v_{\parallel}$ [$k_{\parallel} = (m\ell - n)/R$ is the longitudinal wavenumber, v_{\parallel} is the particle velocity along the magnetic field], which is a particular case of the resonance $\omega = [k_{\parallel} + s/(qR)]v_{\parallel}$ (s is an integer). The latter resonance immediately follows from the equation $\overline{d\varepsilon/dt} \propto \overline{\delta\vec{E} \cdot \vec{v}_D} \neq 0$, where ε is the energy of well circulating particles, $\delta\vec{E} = \delta\vec{E}[r(\theta), \theta, \phi]$ is the perturbed electric field, θ and ϕ are the poloidal and toroidal angles, respectively, $r(\theta)$ describes the particle orbit, \vec{v}_D is the velocity of the toroidal drift, the line over the magnitudes means time averaging. Due to small shear, the resonance provides wave-particle interaction in a wide plasma region (rather than in the region $r_s - \Delta_b < r < r_s$, where Δ_b is the particle orbit width, which is the case when the mode represents rigid kink displacement⁶). Moreover, due to this resonance a possible mode frequency is $\omega \sim k_{\parallel}(0)v_{\alpha}$ and, thus, $\omega > \omega_A(0)$ when $v_{\alpha} > v_A$, which implies that the Alfvén resonance points are located at the periphery. Note that the above mentioned general resonance condition provides also interaction of the circulating energetic ions with perturbations having higher frequency, $\omega \gg k_{\parallel}(0)v_{\alpha}$, which is the case when $s \neq 0$.

The purpose of this work is to consider possible EPM fishbone instabilities driven by circulating energetic ions in plasmas with the safety factor close to unity

or other low-order rationals in a core region surrounded by a region with large magnetic shear. Such behavior of $q(r)$ is typical for spherical tori.¹⁵ In addition, $q(r)$ is close to unity in the plasma core in many tokamak experiments; this also will be the case in the ITER third operational scenario, the so called "hybrid" regime.^{16,17} Note that fishbone oscillations with $m \neq n \neq 1$ were observed in plasmas with $q(0) \sim 2$ of the National Spherical Torus Experiment (NSTX).¹⁸ We consider both the modes with $m = n = 1$ and $m \neq n$. Our analysis will be based on the approach of Refs. 12, 13 extended to the case when the plasma contains a small number of the energetic ions. This will be done with the assumption that only kinetic (non-adiabatic) response of the energetic ions is important.

The structure of the paper is as follows. In Sec. II a dispersion relation describing an EPM fishbone mode with $m = n = 1$ in low-shear systems is derived and analyzed. In Sec. III the modes with $m \neq n \neq 1$ are considered. In Sec. IV the obtained results are summarized.

II. INTERCHANGE FISHBONE MODE

We consider a plasma with a strong-shear periphery and shearless core that contains a small number of well circulating energetic ions with the distribution function, F_α , given by

$$F_\alpha(\bar{r}, \varepsilon, \Lambda) = \frac{m_\alpha^{3/2}}{2\sqrt{2}\pi\varepsilon_\alpha} p_\alpha(\bar{r}) H(r_0 - r) H(\varepsilon_\alpha - \varepsilon) \varepsilon^{-3/2} \delta(\Lambda), \quad (1)$$

where \bar{r} is the average radius of a particle during its orbital motion, $\Lambda = \mu B_0/\varepsilon$, μ is the particle magnetic moment, B_0 is the magnetic field at the magnetic axis, ε_α is the birth energy, $p_\alpha(\bar{r}) = \int d^3v m_\alpha v_\parallel^2 F_\alpha$ is the beam particle pressure, $H(x)$ is the unit step function, $\delta(x)$ is the Heaviside δ -function, r_0 is the radius restricting the shearless core.

Below we derive an equation describing an $m = n = 1$ EPM fishbone mode. We proceed from the energy functional, $\delta\mathcal{E}$, written as

$$\delta\mathcal{E} = \frac{R}{\pi^2 B_0^2} (\delta W_{MHD} + \delta W_k) - \frac{\omega^2}{\omega_A^2} N, \quad (2)$$

where δW_{MHD} is the ideal MHD potential energy^{12–14,19,20}, δW_k is the kinetic part of the potential energy, which describes the resonant energy exchange between the energetic ions and fishbone mode, $\omega_A = v_A/R$, v_A is the Alfvén velocity. The last term in Eq. (2) represents the kinetic energy. In this term

$$N = \frac{1}{2\pi^2 R} \int d^3r |\vec{\xi}_\perp|^2, \quad (3)$$

with $\vec{\xi}_\perp$ the transverse plasma displacement. The magnitude δW_k is given by (we used Refs. 21, 22)

$$\delta W_k \equiv \frac{1}{2} \int \vec{\xi}_\perp^* \cdot \nabla \delta \Pi_\alpha^k d^3r = -\frac{\pi^2 m_\alpha}{\omega_{c\alpha}} \times \sum_{\sigma_v} \int v^3 dv \int d\bar{r} \int d\Lambda \tau_b \frac{\partial F_\alpha}{\partial \varepsilon} \frac{\omega - \omega_{*\alpha}}{\omega - k_\parallel v_\parallel} \left| \left\langle \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right) \vec{\xi}_\perp \cdot \vec{\kappa} \exp[i(\omega - k_\parallel v_\parallel)t] \right\rangle \right|^2, \quad (4)$$

where $\delta \Pi_\alpha^k = \delta p_{\perp\alpha}^k \hat{I} + (\delta p_{\parallel\alpha}^k - \delta p_{\perp\alpha}^k) \vec{b}\vec{b}$ is the pressure tensor, \hat{I} is the identity tensor, $\delta p_{\parallel/\perp\alpha}^k$ is the parallel/perpendicular pressure perturbation associated with the non-adiabatic response of energetic ions, $\sigma_v = v_\parallel/|v_\parallel|$, $\vec{\kappa}$ is the field line curvature, τ_b is the particle transit time, $\omega_{*\alpha}$ is the diamagnetic drift frequency of the energetic ions, and $\langle \dots \rangle$ denotes the orbit averaging.

First of all, we perform orbit averaging and calculate the velocity integral in Eq. (4). Omitting the term odd in θ in $\vec{\xi}_\perp \cdot \vec{\kappa}$ in the integrand of Eq. (4) (this term does not contribute to δW_k) we obtain

$$\vec{\xi}_\perp \cdot \vec{\kappa} = -\frac{1}{R} \xi_1 \{r[\theta(t)]\} \cos[\theta(t)] \exp\{i[\theta(t) - \phi(t) - \omega t]\}, \quad (5)$$

where ξ_1 is the amplitude of the $m = 1$ radial displacement,

$$r[\theta(t)] = \bar{r} + \Delta_\alpha \cos[\theta(t)], \quad \theta(t) = \frac{v_\parallel}{q(\bar{r})R} t, \quad \phi(t) = \frac{v_\parallel}{R} t, \quad (6)$$

$\Delta_\alpha = (q(\bar{r})/v_\parallel \omega_{c\alpha}) (0.5v_\perp^2 + v_\parallel^2)$, $\Delta_\alpha \ll \bar{r}$. Due to Eq. (6) we can expand $\xi_1[r(\theta)]$ in Eq. (5) in a Taylor series at the point \bar{r} . Substituting the result into Eq. (4) and using Eq. (1) we obtain after the calculation of the integrals:

$$\delta W_k = -\pi B_0^2 \rho_\alpha^3 F \left(\frac{\omega}{k_{\parallel 0} v_\alpha} \right) \int_0^a d\bar{r} \left| \frac{d\xi_1}{d\bar{r}} \right|^2 \frac{d\beta_\alpha}{d\bar{r}}, \quad (7)$$

where $\rho_\alpha = v_\alpha / \omega_{c\alpha}$, $k_{\parallel 0} = k_{\parallel}(0)$, $\beta_\alpha = 8\pi p_\alpha / B_0^2$ ($\beta_\alpha = 0$ for $r > r_0$),

$$F(\Omega) \equiv \frac{1}{5} + \frac{\Omega}{4} + \frac{\Omega^2}{3} + \frac{\Omega^3}{2} + \Omega^4 + \Omega^5 \ln \left(1 - \frac{1}{\Omega} \right), \quad (8)$$

and $\omega \ll \omega_{*\alpha}$ has been assumed. It follows from Eq. (7) that when the mode is characterized by $d\xi_1/dr \neq 0$ in a wide region, the energy exchange between the mode and energetic particles is most effective. Below we will show that such a mode really exists. Note that, in contrast to this, $d\xi_1/dr = 0$ inside the $q = 1$ radius for the rigid kink displacement during the conventional fishbone instability.

Using Eq. (7) and taking for simplicity β_α in the form $\beta_\alpha = \beta_{\alpha 0} [1 - (\bar{r}/r_0)^4] H(r_0 - \bar{r})$ we obtain the following Euler equations from Eq. (2) (\bar{r} is replaced with r):

$$\frac{d}{dr} \left\{ \left[(\iota - 1)^2 + l_\alpha(\omega, r) - \frac{\omega^2}{\omega_A^2} \right] r^3 \frac{d\xi_1}{dr} \right\} - G\{\xi_1\} = \hat{C}\{\xi_2\}, \quad (9)$$

$$\frac{d}{dr} \left[\left(\iota - \frac{1}{2} \right)^2 r^3 \frac{d\xi_2}{dr} \right] - 3 \left(\iota - \frac{1}{2} \right)^2 r \xi_2 = \hat{C}^+\{\xi_1\}, \quad (10)$$

where

$$l_\alpha(\omega, r) \equiv \frac{4\rho_\alpha^3 R}{\pi r_0^4} \beta_{\alpha 0} F \left(\frac{\omega}{k_{\parallel 0} v_\alpha} \right) H(r_0 - r), \quad (11)$$

ξ_2 is the amplitude of the $m = 2$ radial displacement (the $m = 2$ harmonic is coupled with the $m = 1$ harmonic due to toroidicity), G is the toroidal driving term, \hat{C} and \hat{C}^+ are the toroidal coupling operators. The explicit forms of G and \hat{C} are given in Ref. 12. The operator \hat{C}^+ is adjoint to \hat{C} :

$$\int_0^a dr f(r) \hat{C}\{g(r)\} = \int_0^a dr g(r) \hat{C}^+\{f(r)\}. \quad (12)$$

We assume that $\omega/\omega_A = O(\epsilon)$, $|\iota - 1| = O(\epsilon)$, $\beta = O(\epsilon^2)$, $G(\xi_1) = O(\epsilon^2 \xi_1)$, $\hat{C}(\xi) = O(\epsilon \xi)$, $\xi_2 = O(\epsilon) \xi_1$ ($\epsilon = a/R$, a the plasma radius) in the plasma core.

Due to the mentioned ordering, we can take $\iota = 1$ at $r \leq r_0$ in the terms G , \hat{C} and \hat{C}^+ . Then Eqs. (9), (10) can be written as [cf. Eqs. (44a,b) of Ref.¹²]:

$$\begin{aligned} \frac{d}{d\tilde{r}} \left\{ \epsilon^{-2} \left[(\iota_0 - 1)^2 + l_\alpha(\omega, \tilde{r}) - \frac{\omega^2}{\omega_A^2} \right] \tilde{r}^3 \frac{d\xi_1}{d\tilde{r}} \right\} - 4 \left(\frac{\tilde{r}}{4} \beta_p' + \beta_p \right)^2 \tilde{r}^3 \xi_1 \\ = \left(\frac{\tilde{r}}{4} \beta_p' + \beta_p \right) \frac{d}{d\tilde{r}} (\tilde{r}^3 \hat{\xi}_2), \end{aligned} \quad (13)$$

$$\frac{d}{d\tilde{r}} \left(\tilde{r}^3 \frac{d\hat{\xi}_2}{d\tilde{r}} \right) - 3\tilde{r} \hat{\xi}_2 = -4\tilde{r}^3 \frac{d}{d\tilde{r}} \left[\left(\frac{\tilde{r}}{4} \beta_p' + \beta_p \right) \xi_1 \right], \quad (14)$$

where $\tilde{r} = r/a$, $\hat{\xi}_2$ is defined by $\xi_2 \equiv \epsilon \hat{\xi}_2$, prime denotes the radial derivative,

$$\beta_p(\tilde{r}) = -\frac{8\pi R^2}{a^2 \tilde{r}^4 B_0^2} \int_0^{\tilde{r}} \hat{r}^2 \frac{dp_c}{d\hat{r}} d\hat{r}. \quad (15)$$

The general solution of Eq. (14), which is regular on the axis, is given by¹²

$$\hat{\xi}_2 = \tilde{r}^{-3} \int_0^{\tilde{r}} \hat{r}^4 \beta_p(\hat{r}) \frac{d\xi_1}{d\hat{r}} d\hat{r} + [C_1 - \beta_p(\tilde{r}) \xi_1(\tilde{r})] \tilde{r}, \quad (16)$$

where C_1 is an integration constant. Putting Eq. (16) into Eq. (13) and integrating, we find:

$$\frac{d\xi_1}{d\tilde{r}} = \frac{\epsilon^2 C_1 \tilde{r} \beta_p}{(\iota - 1)^2 + l_\alpha(\omega, \tilde{r}) - \omega^2/\omega_A^2}. \quad (17)$$

The dispersion relation can be obtained by matching the solution in the inner (shear-free) region to the solution in the outer (sheared) region. In the latter region $|\iota - 1| \sim 1$, therefore, $\xi_1 \sim \epsilon^2$, as follows from Eq. (17). Due to this, we can neglect the toroidal coupling in Eq. (10) and write the mentioned equation as follows:

$$\frac{d}{dr} \left[\left(\iota - \frac{1}{2} \right)^2 r^3 \frac{d\hat{\xi}_2}{dr} \right] - 3 \left(\iota - \frac{1}{2} \right)^2 r \hat{\xi}_2 = 0. \quad (18)$$

Equation (18) has the following asymptotic solution in the shear-free region:

$$\hat{\xi}_2 \propto \frac{r}{r_2} + \sigma \left(\frac{r}{r_2} \right)^{-3}, \quad (19)$$

where r_2 is defined by $\iota(r_2) = 1/2$ and the constant σ can be determined by integrating Eq. (18) over the outer region. To calculate σ , $\iota(r)$ is to be specified. In particular, when $\iota(r)$ is given by¹²

$$\iota = \frac{1}{2} + \left(\iota_0 - \frac{1}{2}\right) \left[1 - \left(\frac{r}{r_2}\right)^{2\lambda}\right], \quad (20)$$

with $\lambda \geq 3$, then

$$\sigma \approx \frac{1}{3} \left(1 - \frac{2}{\lambda}\right). \quad (21)$$

Matching Eq. (19) with the asymptotic form of Eq. (16) in the outer region, we obtain the dispersion relation [cf. Eq.(50) of Ref.¹²] as follows:

$$\sigma = \left(\frac{r_2}{a}\right)^2 \int_0^a \frac{[\epsilon\beta_p(r)]^2}{(\iota - 1)^2 + l_\alpha(\omega, r) - \omega^2/\omega_A^2} \left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2}. \quad (22)$$

The integrand in Eq. (22) has the pole at the Alfvén resonance $1 - \iota = \omega/\omega_A$ in the outer region. The residue at this pole gives the continuum damping of the fishbone mode. Away from the resonance the integrand is negligible in the sheared region. Taking this into account and assuming $v_A(r) = \text{const}$, we can write Eq. (22) in the form:

$$\sigma = \left(\frac{r_2}{a}\right)^2 \frac{\epsilon^2}{(\iota_0 - 1)^2 + l_\alpha(\omega) - \omega^2/\omega_A^2} \int_0^{r_0} \beta_p^2(r) \left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2} + i\sigma_{res}(\omega), \quad (23)$$

or

$$\left[(\iota_0 - 1)^2 \left(1 - \frac{v_\alpha^2 \Omega^2}{v_A^2}\right) + \hat{\beta}_\alpha F(\Omega)\right] (\sigma - i\sigma_{res}) = \left(\frac{r_2}{a}\right)^2 \int_0^{r_0} [\epsilon\beta_p(r)]^2 \left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2}, \quad (24)$$

where $F(\Omega)$ is given by Eq. (8),

$$\hat{\beta}_\alpha \equiv \frac{4\rho_\alpha^3 R}{\pi r_0^4} \beta_{\alpha 0}, \quad (25)$$

$$\begin{aligned} \sigma_{res}(\omega) &= \left(\frac{r_2}{a}\right)^2 \epsilon^2 \beta_p^2(r_A) \left(\frac{r_A}{r_2}\right)^5 \lim_{\eta \rightarrow 0} \int_{r_A-0}^{r_A+0} \frac{1}{(\iota - 1)^2 - (\omega + i\eta)^2/\omega_A^2} \frac{dr}{r_2} \\ &= \pi \left(\frac{r_2}{a}\right)^2 [\epsilon\beta_p(r_A)]^2 \frac{(r_A/r_2)^5}{r_2 |\partial/\partial r(\iota - 1)^2|_{r=r_A}}, \end{aligned} \quad (26)$$

and r_A is defined by

$$[\iota(r_A) - 1]^2 = \frac{\omega^2}{\omega_A^2}. \quad (27)$$

We assume that the bulk plasma is marginally stable, i.e., β_p is close to a certain value, β_p^{marg} , determined by

$$(\iota_0 - 1)^2 \sigma = \left(\frac{r_2}{a}\right)^2 \int_0^{r_0} [\epsilon \beta_p^{marg}(r)]^2 \left(\frac{r}{r_2}\right)^5 \frac{dr}{r_2}. \quad (28)$$

Then, using Eq. (24), we obtain the following equations that determine threshold beta of the energetic ions (for which $\text{Im } \Omega = 0$) and the mode frequency:

$$\hat{\beta}_\alpha^{crit} = \frac{\sigma(\iota_0 - 1)^2 \Omega^2}{\sigma \text{Re}F(\Omega) + \sigma_{res} \text{Im}F(\Omega)} \cdot \left(\frac{v_\alpha}{v_A}\right)^2, \quad (29)$$

$$\sigma \Omega^2 v_\alpha^2 [\sigma \text{Im}F(\Omega) - \sigma_{res} \text{Re}F(\Omega)] = \sigma_{res} v_A^2 \left[1 - \Omega^2 \left(\frac{v_\alpha}{v_A}\right)^2\right] [\sigma \text{Re}F(\Omega) + \sigma_{res} \text{Im}F(\Omega)]. \quad (30)$$

Let us consider a specific example. We assume that

$$p_c = p_0 \left[1 - \left(\frac{r}{a}\right)^{2\nu}\right] \quad (31)$$

and $\iota(r)$ is given by Eq. (20). Then it follows from Eq. (27) that $r_A \simeq r_2(2\omega/\omega_A)^{\frac{1}{2\lambda}}$, and Eq. (26) is reduced to

$$\sigma_{res}(\omega) = \frac{\pi}{\lambda} \left(\frac{4\nu}{\nu+1}\right)^2 \beta(0)^2 \left(\frac{R}{a}\right)^2 \left(\frac{r_2}{a}\right)^{4\nu-2} \left(\frac{2\omega}{\omega_A}\right)^{\frac{2\nu+1}{\lambda}-2}. \quad (32)$$

It follows from Eq. (32) that σ_{res} does not depend on ω when $2\nu + 1 - 2\lambda = 0$. To satisfy this condition, we take $\nu = 5/2, \lambda = 3$. In addition, we take $v_\alpha = v_A$, which leads to $\omega/\omega_A = (\iota_0 - 1)\Omega$. Using Eq. (20), $\lambda = 3$ and assuming $r_0 \simeq r_1$ with $\iota(r_1) = 1, \iota(a) = 0$, we obtain $r_2/a = 0.9$ and $r_0/r_2 \simeq 0.7$. Using Eq. (21), we have $\sigma = 1/9$. We specify the aspect ratio of the torus and central rotational transform: $R/a = 3$ and $\iota_0 - 1 \simeq 0.05$. Now we can find β_0^{marg} from Eqs. (28), (31): $\beta_0^{marg} \simeq 8.7 \times 10^{-2}$. Substituting this value to Eq. (32) we obtain $\sigma_{res} \simeq 0.25$. Finally, we have from Eqs. (29), (30):

$$\Omega \simeq 0.8, \quad \hat{\beta}_\alpha^{crit} \simeq 5.5 \times 10^{-4}. \quad (33)$$

We conclude from here that the mode frequency is $\omega = 0.04\omega_A$. However, in the considered example it was assumed that $v_\alpha = v_A$, whereas in many cases of practical

importance v_α exceeds v_A by a factor of two or more. Therefore, in those cases the mode frequency will be higher, although it will be less than the frequency of Alfvén eigenmodes, such as TAE ($\omega_{TAE} = 0.5\omega_A$). To evaluate the instability threshold, we take into account that the factor $\rho_\alpha^3 R/r_0^4$ in Eq. (25) varies from $\sim 10^{-2}$ in conventional tokamaks to $\sim 10^{-1}$ in spherical tori. We conclude from this and Eq. (33) that $\beta_{\alpha 0}^{crit}$ is of the order of several per cent in tokamaks and about 10^{-3} in spherical tori. This difference is explained by the fact that relative orbit width, Δ/a , is larger in spherical tori.

III. INFERNAL FISHBONE MODE (ARBITRARY MODE NUMBERS)

In this section we eliminate the assumption $m = n = 1$, i.e., we consider a plasma with $q_0 \simeq m/n$, where the mode numbers are arbitrary. First of all, we write the following equations in the shear-free core [cf. Eqs. (13), (14) and Eqs. (56a,b) of Ref.¹²]

$$\begin{aligned} & \frac{d}{d\tilde{r}} \left\{ \left[\left(\frac{\iota}{n} - \frac{1}{m} \right)^2 + \frac{l_\alpha}{(mn)^2} - \left(\frac{\omega}{\omega_{Amn}} \right)^2 \right] \tilde{r}^3 \frac{d\xi_m}{d\tilde{r}} \right\} \\ & - (m^2 - 1) \left[\left(\frac{\iota}{n} - \frac{1}{m} \right)^2 - \left(\frac{\omega}{\omega_{Amn}} \right)^2 \right] \tilde{r} \xi_m - \frac{\epsilon^2}{m^2} \left[\frac{1}{2} (\tilde{r} \beta'_p + 4\beta_p)^2 \right. \\ & \left. + \left(1 - \frac{n^2}{m^2} \right) (\tilde{r} \beta'_p + 4\beta_p) \right] \tilde{r}^3 \xi_m = \frac{\epsilon^2 n}{2m^2(m+1)} \tilde{r}^{1+m} (\tilde{r} \beta'_p + 4\beta_p) \frac{d}{d\tilde{r}} (\tilde{r}^{2+m} \hat{\xi}_{m+1}), \quad (34) \end{aligned}$$

$$\begin{aligned} & \frac{1}{m^2(1+m)^2} \left[\frac{d}{d\tilde{r}} \left(\tilde{r}^3 \frac{d\hat{\xi}_{m+1}}{d\tilde{r}} \right) - [(m+1)^2 - 1] \tilde{r} \hat{\xi}_{m+1} \right] = \\ & - \frac{1}{2nm^2(1+m)} \tilde{r}^{2+m} \frac{d}{d\tilde{r}} [(\tilde{r} \beta'_p + 4\beta_p) \tilde{r}^{1+m} \xi_m], \quad (35) \end{aligned}$$

where $\xi_{m+1} \equiv \epsilon \hat{\xi}_{m+1}$, $\beta'_p = d\beta_p/d\tilde{r}$. The general solution of Eq. (35), which is regular on the magnetic axis, can be written as

$$n \hat{\xi}_{m+1} = - \frac{(1+m)}{2\tilde{r}^{2+m}} \int_0^{\tilde{r}} d\hat{r} (\hat{r} \beta'_p + 4\beta_p) \hat{r}^{2+m} \xi_m + C_m \tilde{r}^m. \quad (36)$$

where $C_m(r) = const.$ Putting Eq. (36) into Eq. (34) we obtain:

$$\begin{aligned}
& \frac{d}{dr} \left\{ \left[\left(\frac{\iota}{n} - \frac{1}{m} \right)^2 + \frac{l_\alpha}{(mn)^2} - \left(\frac{\omega}{\omega_A mn} \right)^2 \right] r^3 \frac{d\xi_m}{dr} \right\} \\
& - (m^2 - 1) \left[\left(\frac{\iota}{n} - \frac{1}{m} \right)^2 - \left(\frac{\omega}{\omega_A mn} \right)^2 \right] r \xi_m - \frac{\epsilon^2}{m^2} \left(1 - \frac{n^2}{m^2} \right) \frac{d}{dr} (r^4 \beta_p) \xi_m \\
& = \frac{C_m \epsilon^2}{m^2} \frac{d}{dr} (r^4 \beta_p) r^{m-1}. \tag{37}
\end{aligned}$$

One can see that $\beta_p(r) \propto r^{2\nu-2}$ when the pressure profile is described by Eq. (31). Using this fact and assuming $|m - nq_0| \sim \epsilon$, we can conclude that the ratio of the last term in the left-hand side (LHS) of Eq. (37) (this term represents the stabilizing effect of the average magnetic well) to the second term is of the order of $(r_0/a)^{2\nu} \ll 1$. When the last term on the LHS is neglected, Eq. (37) can be easily integrated. Imposing the boundary condition $\xi_m(a) = 0$, we find:

$$\xi_m = \frac{2\epsilon^2 e \hat{\beta}_p (\nu + 1) (r^{2\nu} - 1) r^{m-1}}{[(2\nu + m)^2 - 1] l_\alpha(\omega, r) / n^2 + 4\nu(\nu + m) [(m/nq - 1)^2 - (\omega/\omega_A n)^2]}. \tag{38}$$

The magnitude ξ_m is negligible in the sheared region, except for the region in the vicinity of the local Alfvén resonance. The dispersion relation can be obtained by matching the asymptotic form of a solution of Eq. (36) with ξ_m given by Eq. (38) in the sheared region, to the shear-free limit of Eq. (35) (with the right-hand side neglected):

$$\hat{\xi}_{m+1} \propto \left(\frac{r}{r_{m+1}} \right)^m + \sigma_m \left(\frac{r}{r_{m+1}} \right)^{-(2+m)}, \tag{39}$$

where $\iota(r_{m+1}) = n/(m+1)$. We find

$$\begin{aligned}
\sigma_m &= \frac{1+m}{n(\nu+m)} \left(\frac{r_{m+1}}{a} \right)^{-2(m+1)} \left(\frac{r_0}{a} \right)^{2(\nu+m)} \\
&\times \frac{\epsilon^2 \hat{\beta}_p^2 (\nu+1)^2}{[(2\nu+m)^2 - 1] l_\alpha(\omega) / n^2 + 4\nu(\nu+m) [(m/nq_0 - 1)^2 - (\omega/\omega_A n)^2]} + i\sigma_{res,m}. \tag{40}
\end{aligned}$$

For the ι -profile given by

$$\iota = \frac{n}{m+1} + \left(\iota_0 - \frac{n}{m+1} \right) \left[1 - \left(\frac{r}{r_{m+1}} \right)^{2\lambda} \right]$$

with $\lambda \geq m + 2$, we have¹²

$$\sigma_m \simeq \frac{m}{m+2} \left(1 - \frac{m+1}{\lambda}\right), \quad (41)$$

and the expression for $\sigma_{res,m}$ takes the form

$$\sigma_{res,m} = \pi \frac{(m+1)^3 \epsilon^2 \hat{\beta}_p^2 (\nu+1)^2}{8\lambda n \nu(\nu+m)} \left(\frac{r_{m+1}}{a}\right)^{2(\nu-1)} \left[\frac{(m+1)\omega}{n\omega_A}\right]^{\frac{\nu+m}{\lambda}-2}. \quad (42)$$

where $\hat{\beta}_p = \beta_p r^{2-2\nu}$.

Let us consider a possibility of the destabilization of infernal fishbones in NSTX plasmas with $q_0 \lesssim 2$. This is of interest because bursting fishbone instabilities with $m/n > 1$ were observed in NSTX.¹⁸ We use the following parameters: $R \simeq 100$ cm, $a = 65$ cm, $B = 0.3$ T, $\varepsilon_\alpha = 90$ keV, $v_\alpha/v_A = 3$, $m = 2$, $n = 1$, $q_0 = 1.7$, $\nu = 6$, $\lambda = 4$. Then $\sigma_2 = 1/8$, $\sigma_{res,2}(\omega) = const$, $r_3/a \simeq 0.8$, $r_0/a \simeq r_2/a \simeq 0.6$. We assume that the plasma is at the margin of the MHD stability in the absence of fast ions [$l_\alpha = 0$, $\omega = 0$ in Eq. (40)], and take into account that $\hat{\beta}_p = (\beta_0/\epsilon^2)(m/n)^2[\nu/(\nu+1)]$. This leads to $\beta_0^{margin} \simeq 0.35$. Equation (40) then yields at the margin of the fishbone stability ($Im\Omega = 0$):

$$\hat{\beta}_\alpha^{crit} \approx \frac{9\Omega^2(2\iota_0 - 1)^2}{ReF(\Omega) + (\sigma_{res,2}/\sigma_2)ImF(\Omega)}, \quad (43)$$

$$ImF(\Omega) \left[9\Omega^2 \left(\frac{\sigma_{res,2}}{\sigma_2} + \frac{\sigma_2}{\sigma_{res,2}} \right) - \frac{\sigma_{res,2}}{\sigma_2} \right] = ReF(\Omega), \quad (44)$$

where $\sigma_{res,2} \approx 0.55$. We find from Eqs. (43), (44) that $\Omega \simeq 0.5$, $\hat{\beta}_\alpha^{crit} \simeq 6 \times 10^{-2}$. Taking into account that $r_0 \simeq 0.6a \simeq 40$ cm, $\rho_\alpha \simeq 20$ cm, we obtain $R\rho_\alpha^3/r_0^4 \simeq 0.31$. Using the obtained magnitude of $\hat{\beta}_\alpha^{crit}$ and Eq. (25), we obtain that the threshold β of the energetic ions is $\beta_{\alpha 0}^{crit} \simeq 15\%$. To calculate the mode frequency we take into account that $\varepsilon_\alpha = 90$ keV, $q_0 = 1.7$, and $R = 100$ cm. This leads to $f \equiv \omega/(2\pi) = 0.5(2\iota_0 - 1)v_\alpha/(2\pi R) \sim 40kHz$. The obtained threshold magnitude of β_α and the mode frequency are quite reasonable. In particular, a bursting fishbone instability with $m = 2$ and the initial frequency in the plasma frame $f \simeq 45kHz$ was observed when the central safety factor was $q_0 \lesssim 2$ in the NSTX shot #106218.¹⁸

IV. SUMMARY AND CONCLUSIONS

We have predicted new circulating-ion-driven fishbone instabilities in toroidal plasmas with $q_0 \sim m/n \geq 1$ and low magnetic shear in the core region. The instabilities have the frequency determined by the energetic particles, $\omega \lesssim k_{\parallel 0} v_\alpha$, i.e., they are of the EPM type. This implies that the considered instabilities are potentially dangerous, being able to expel energetic ions from the core region to the wall or plasma periphery. The mode numbers, m and n , are not necessarily equal to unity, and determine the spatial structure of the modes. When $m = n = 1$, the structure of the considered mode strongly differs from the rigid kink displacement taking place during conventional fishbone oscillations. Although our instability is caused by the same resonance ($\omega = k_{\parallel} v_{\parallel}$) as the conventional one, its physics differs from that of the conventional instability. The reason is that all the energetic ions in the core region (rather than only particles crossing the $q = 1$ surface) contribute to the destabilization of the mode.

Because the considered instabilities involve most energetic ions, $\varepsilon \lesssim \varepsilon_\alpha$, they can lead to a strong change of fusion reactivity and neutron emission from the plasma. Therefore, it is of interest to apply the developed theory to experiments where oscillations of neutron emission during fishbone activity took place. In particular, such oscillations were observed in NSTX.¹⁸ An example relevant to NSTX was considered in the paper. It was found that there is agreement between the experimentally observed frequency of the $m/n = 2$ fishbones and the calculated frequency. However, we have to note that our analysis was made on a qualitative level. A more detailed consideration is required to be able to identify the observed instability, especially because the trapped-ion population arising mainly due to Coulomb pitch angle scattering (partly slowed down particles) can also lead to fishbone instability in the considered shot.^{18,23}

ACKNOWLEDGEMENTS

This work was supported in part by the CRDF Project UKP2-2643-KV-05, and by the U.S. Department of Energy Grant DE-FG03-94ER54271.

REFERENCES

- ¹ K. McGuire, R. Goldston, M. Bell, *et al.*, Phys. Rev. Lett. **50**, 891 (1983).
- ² L. Chen, R.B. White, M.N. Rosenbluth, Phys. Rev. Lett. **52**, 1122 (1984).
- ³ B. Coppi and F. Porcelli, Phys. Rev. Lett. **57**, 2272 (1986).
- ⁴ Ya.I. Kolesnichenko, V.V. Lutsenko, V. S. Marchenko, R.B. White, 20th IAEA Fusion Energy Conference 2004, IAEA-CSP-25/CD, Vilamoura, Portugal, 1-6 November 2004, Report TH/P4-42 (on CD-ROM).
- ⁵ Ya.I. Kolesnichenko, V.V. Lutsenko, V. S. Marchenko, R.B. White, Phys. Plasmas **11**, 1803 (2004).
- ⁶ R. Betti and J.P. Freidberg, Phys. Rev. Lett. **70**, 3428 (1993).
- ⁷ L.E. Zakharov, Nucl. Fusion **18**, 335 (1978).
- ⁸ J. Manickam, N. Pomphrey, and A.M.M. Todd, Nucl. Fusion **27**, 1461 (1987).
- ⁹ L.A. Charlton, R.J. Hastie, and T.C. Hender, Phys. Fluids **B1**, 798 (1989).
- ¹⁰ L.A. Charlton, B.A. Carreras, and V.E. Lynch, Phys. Fluids **B2**, 1574 (1990).
- ¹¹ J.A. Wesson, Plasma Phys. Control. Fusion **28**, 243 (1986).
- ¹² F.L. Waelbroeck and R.D. Hazeltine, Phys. Fluids **31**, 1217 (1988).
- ¹³ R.J. Hastie and T.C. Hender, Nucl. Fusion **28**, 585 (1988).
- ¹⁴ M.N. Bussac *et al.*, Phys. Rev. Lett. **35**, 1638 (1975).
- ¹⁵ J.E. Menard *et al.*, Nucl. Fusion **43**, 330 (2003).
- ¹⁶ X. Litaudon *et al.*, Plasma Phys. Control. Fusion **46**, A19 (2004).
- ¹⁷ M.R. Wade *et al.*, Nucl. Fusion **45**, 407 (2005).
- ¹⁸ E.D. Fredrickson, C.Z. Cheng, L. Chen, *et al.*, 29th EPS Conference on Plasma Phys. and Contr. Fusion, Montreux (2002).

- ¹⁹ G.B. Crew and J.J. Ramos, Phys. Fluids **26**, 2621 (1983).
- ²⁰ L.E. Zakharov, Sov. J. Plasma Phys. **4**, 503 (1979).
- ²¹ T.M. Antonsen, Jr. and Y.C. Lee, Phys. Fluids **25**, 132 (1982).
- ²² F. Porcelli, R. Stankevich, W. Kerner, and H.L. Berk, Phys. Plasmas **1**, 470 (1994).
- ²³ E. Fredrickson, L. Chen, R. White, Nucl. Fusion, **43** 1258 (2003).

External Distribution

Plasma Research Laboratory, Australian National University, Australia
Professor I.R. Jones, Flinders University, Australia
Professor João Canalle, Instituto de Fisica DEQ/IF - UERJ, Brazil
Mr. Gerson O. Ludwig, Instituto Nacional de Pesquisas, Brazil
Dr. P.H. Sakanaka, Instituto Fisica, Brazil
The Librarian, Culham Science Center, England
Mrs. S.A. Hutchinson, JET Library, England
Professor M.N. Bussac, Ecole Polytechnique, France
Librarian, Max-Planck-Institut für Plasmaphysik, Germany
Jolan Moldvai, Reports Library, Hungarian Academy of Sciences, Central Research
Institute for Physics, Hungary
Dr. P. Kaw, Institute for Plasma Research, India
Ms. P.J. Pathak, Librarian, Institute for Plasma Research, India
Dr. Pandji Triadyaksa, Fakultas MIPA Universitas Diponegoro, Indonesia
Professor Sami Cuperman, Plasma Physics Group, Tel Aviv University, Israel
Ms. Clelia De Palo, Associazione EURATOM-ENEA, Italy
Dr. G. Grosso, Istituto di Fisica del Plasma, Italy
Librarian, Naka Fusion Research Establishment, JAERI, Japan
Library, Laboratory for Complex Energy Processes, Institute for Advanced Study,
Kyoto University, Japan
Research Information Center, National Institute for Fusion Science, Japan
Professor Toshitaka Idehara, Director, Research Center for Development of Far-Infrared Region,
Fukui University, Japan
Dr. O. Mitarai, Kyushu Tokai University, Japan
Mr. Adefila Olumide, Ilorin, Kwara State, Nigeria
Dr. Jiangang Li, Institute of Plasma Physics, Chinese Academy of Sciences, People's Republic of China
Professor Yuping Huo, School of Physical Science and Technology, People's Republic of China
Library, Academia Sinica, Institute of Plasma Physics, People's Republic of China
Librarian, Institute of Physics, Chinese Academy of Sciences, People's Republic of China
Dr. S. Mirnov, TRINITI, Troitsk, Russian Federation, Russia
Dr. V.S. Strelkov, Kurchatov Institute, Russian Federation, Russia
Kazi Firoz, UPJS, Kosice, Slovakia
Professor Peter Lukac, Katedra Fyziky Plazmy MFF UK, Mlynska dolina F-2, Komenskeho Univerzita,
SK-842 15 Bratislava, Slovakia
Dr. G.S. Lee, Korea Basic Science Institute, South Korea
Dr. Rasulkhozha S. Sharafiddinov, Theoretical Physics Division, Institute of Nuclear Physics, Uzbekistan
Institute for Plasma Research, University of Maryland, USA
Librarian, Fusion Energy Division, Oak Ridge National Laboratory, USA
Librarian, Institute of Fusion Studies, University of Texas, USA
Librarian, Magnetic Fusion Program, Lawrence Livermore National Laboratory, USA
Library, General Atomics, USA
Plasma Physics Group, Fusion Energy Research Program, University of California at San Diego, USA
Plasma Physics Library, Columbia University, USA
Alkesh Punjabi, Center for Fusion Research and Training, Hampton University, USA
Dr. W.M. Stacey, Fusion Research Center, Georgia Institute of Technology, USA
Director, Research Division, OFES, Washington, D.C. 20585-1290

The Princeton Plasma Physics Laboratory is operated
by Princeton University under contract
with the U.S. Department of Energy.

Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2750
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: <http://www.pppl.gov>