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Practical Issues in Component Aging Analysis

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PRACTICAL ISSUES IN COMPONENT AGING ANALYSIS

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ABSTRACT

This paper examines practical issues in the statistical analysis of component aging data. These issues center on the stochastic process chosen to model component failures. The two stochastic processes examined are repair same as new, leading to a renewal process, and repair same as old, leading to a nonhomogeneous Poisson process. Under the first assumption, times between failures can treated as statistically independent observations from a stationary process. The common distribution of the times between failures is called the renewal distribution. Under the second process, the times between failures will not be independently and identically distributed, and one cannot simply fit a renewal distribution to the cumulative failure times or the times between failures. The paper illustrates how the assumption made regarding the repair process is crucial to the analysis. Besides the choice of stochastic process, other issues that are discussed include qualitative graphical analysis and simple nonparametric hypothesis tests to help judge which process appears more appropriate. Numerical examples are presented to illustrate the issues discussed in the paper.

Key Words: Aging, failure with repair

1 INTRODUCTION

Analysis of component data with respect to aging involves more complex stochastic models than the typical binomial and Poisson distributions used to model component failures in the majority of PSA applications. When components fail, they may be replaced with new components or repaired. If a component is replaced, then (neglecting the time required for replacement), we have a stochastic point process in which the component is restored to new after each failure. This is called a *renewal process*. To adopt a metaphysical metaphor, the component is reincarnated in a new state after each failure. The failure rate between failures may be constant, decreasing (reliability growth), or increasing (aging).

Times between failures under this assumption are treated as statistically independent observations from a stationary process. The distribution of the times between failures is called the *renewal distribution*. If the failure rate is constant over either operating time or time in standby (not calendar time), depending on which is being modeled, then the renewal distribution is an exponential distribution. A Weibull or gamma distribution allows for monotonically increasing or decreasing failure rates, depending on whether the shape parameter is > 1 or < 1, respectively (when the shape parameter equals 1, both distributions reduce to the exponential). Another popular renewal distribution is the lognormal distribution. The lognormal distribution does not have a shape parameter, and its failure rate increases quickly, and then decreases monotonically with operating or standby time. It can be useful when early failures dominate, causing an initially increasing failure rate.

The other extreme considered is when repair returns the component to the same state it was in just prior to the failure. This is referred to as repair *same-as-old*. Continuing our metaphysical metaphor from above, this type of process is equivalent to resuscitating the component after failure. The stochastic point process in this case is referred to as a *nonhomogeneous Poisson process* or NHPP, because the Poisson intensity changes with time. With repair same-as-old, we adopt the terminology of Ascher and Feingold [1] and refer to the intensity of the process as the *rate of occurrence of failure* or ROCOF. This helps to distinguish a seemingly similar parameter from the failure rate of a renewal process.

Just as there are a number of possible renewal distributions, there are many different functional forms for the intensity of an NHPP. Common functional forms are

Linear: $$\lambda = a + bt \tag{1}$$

Loglinear: $$\log(\lambda) = a + bt \tag{2}$$

Power-law: $$\lambda = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \tag{3}$$

The third of these is also sometimes referred to as a Weibull process, because the time to first failure has a Weibull distribution with shape parameter β and scale parameter α.

Note that in the case of repair same-as-old, the times between failures will not be a random sample from a single renewal distribution, as they were in the case of a renewal process. Instead, if the ROCOF is changing with time, say increasing if aging is taking place, then later times between failures will tend to be shorter than earlier times. In this case, one cannot simply fit a single renewal distribution to the cumulative failure times or the times between failures.

2 RENEWAL PROCESS

With a renewal process, the failure rate does not change with *calendar* time, as pointed out above, only with operating time or time in standby, according to the situation being considered. Likewise, *cumulative* times to failure are not the inputs to a statistical analysis; it is the times *between* failures that are treated as a random sample from a renewal distribution. Furthermore, because a renewal process is stationary, a plot of the cumulative number of failures versus cumulative failure time will be approximately a straight line, so a cumulative failure plot is not useful in deciding how the failure rate is changing with time. The figures below illustrate this plot for three cases of simulated failure times: constant failure rate, increasing failure rate, and decreasing failure rate. The times in the constant case are simulated from an exponential distribution with mean time to failure of 350 arbitrary time units. The increasing and decreasing cases were simulated from Weibull distributions with shape parameters of 2 and 0.5, respectively, and a scale parameter of 350 arbitrary time units, corresponding to a mean time between failures of 310 and 700 time units, respectively. Note the linearity displayed in all three cases, illustrating the behavior typical of the cumulative failure plot when the times between failures are generated by a renewal (i.e., repair same as new) process.

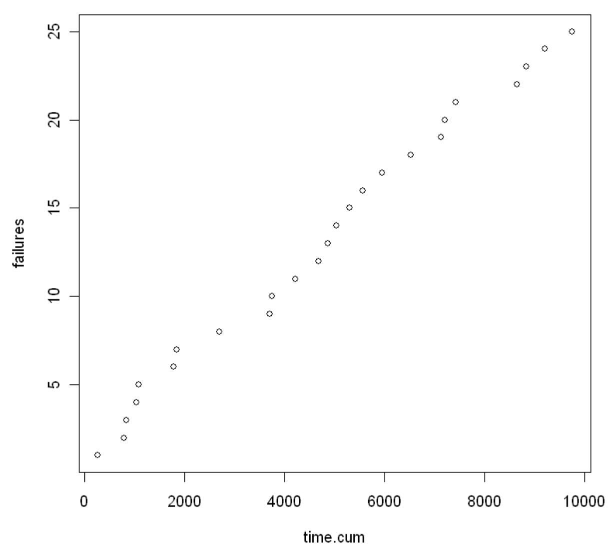

Figure 1 Cumulative failure plot for 25 times between failures from exponential distribution (constant failure rate)

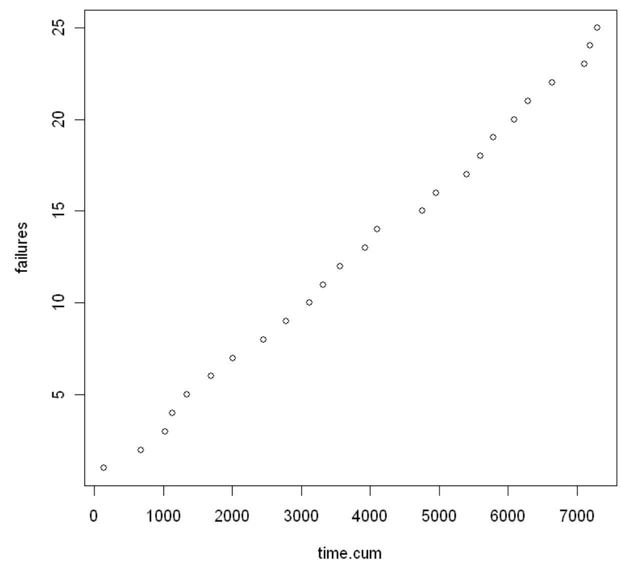

Figure 2 Cumulative failure plot for 25 times between failures for renewal distribution with increasing failure rate

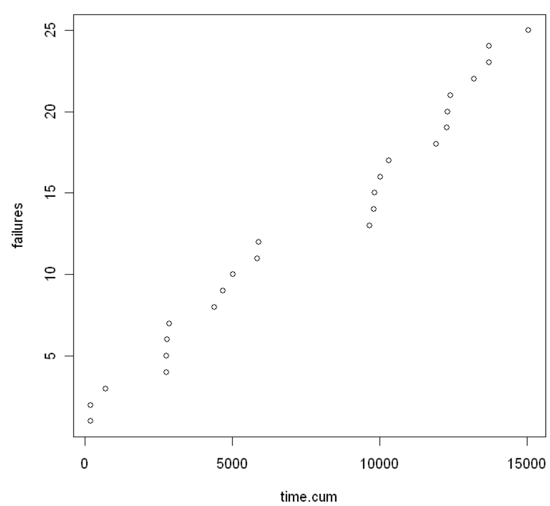

Figure 3 Cumulative failure plot for 25 times between failures for renewal distribution with decreasing failure rate

The plots below show cumulative failures versus cumulative time for 1,000 simulated failure times from two different renewal processes, one in which failure rate is decreasing with increasing operating time, the other where failure rate is increasing with operating time. Note in both cases that the cumulative failure plot produces a straight line, reinforcing the conclusion that this plot cannot detect a time-dependent failure rate under the same-as-new repair assumption.

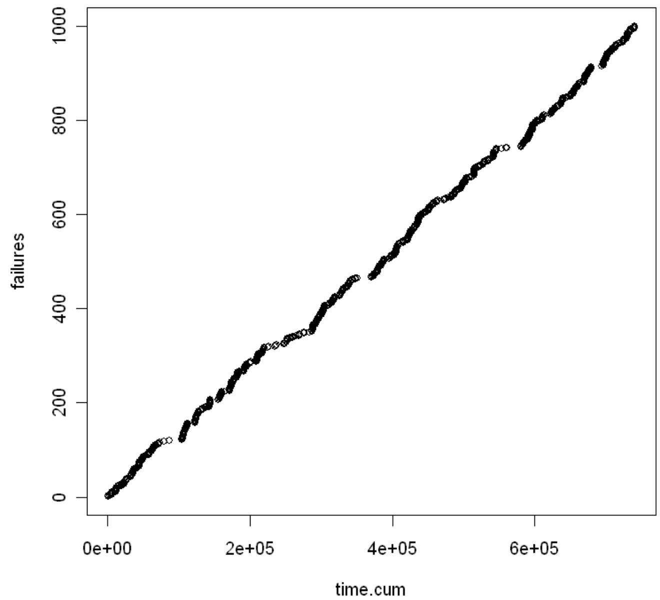

Figure 4 Cumulative failure plot for 1,000 simulated failure times from renewal process with decreasing failure rate

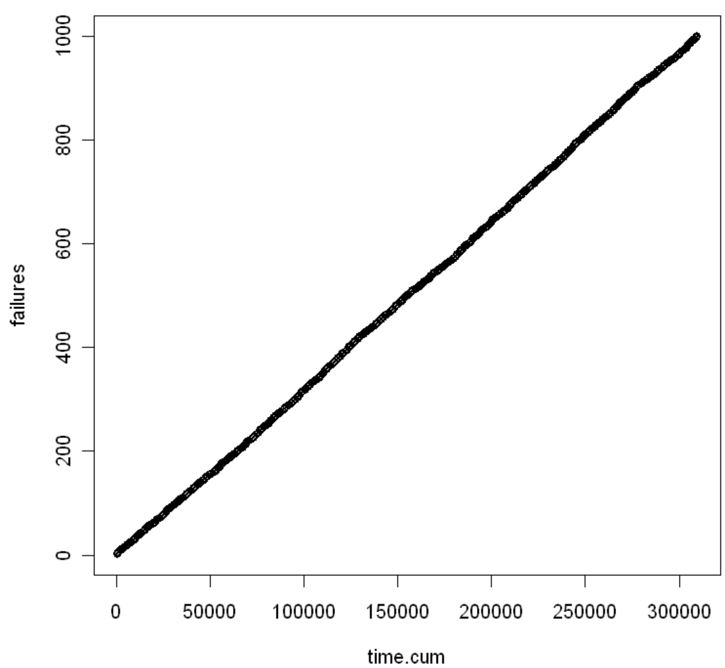

Figure 5 Cumulative failure plot for 1,000 simulated failure times from renewal process with increasing failure rate

2.1 Graphical Check for Aging in a Renewal Process

A plot that is useful for the renewal process is a *cumulative hazard plot*. If the failure rate is constant in a renewal process, then the times between failures are exponentially distributed. If one plots the *ranked* times between failures on the x-axis, and $1/n_t$ on the y-axis, where n_t is the number of components still operating at time t, the result should be approximately a straight line if the failure rate is constant (i.e., the renewal distribution is exponential). If the slope is increasing (decreasing) with time, this suggests a renewal process whose failure rate is likewise increasing (decreasing) with time. The figures below illustrate this plot for the three cases of simulated failure times described above.

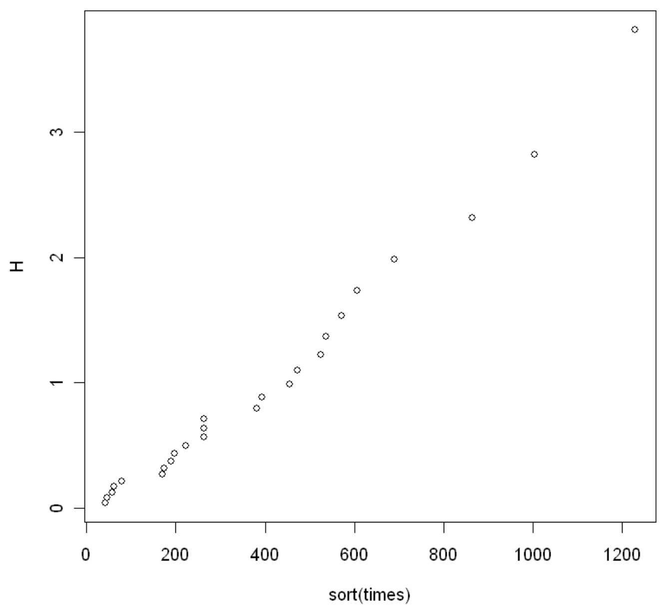

Figure 6 Cumulative hazard plot for 25 times between failures from exponential distribution (constant failure rate)

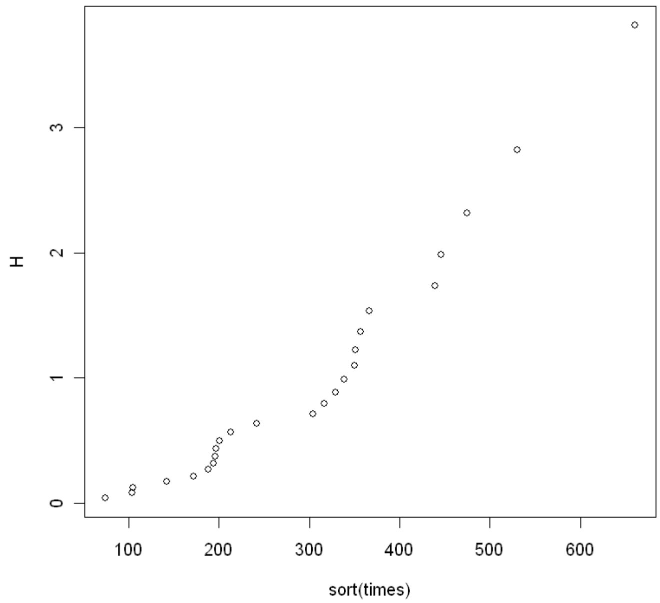

Figure 7 Cumulative hazard plot for 25 times between failures for renewal distribution with increasing failure rate

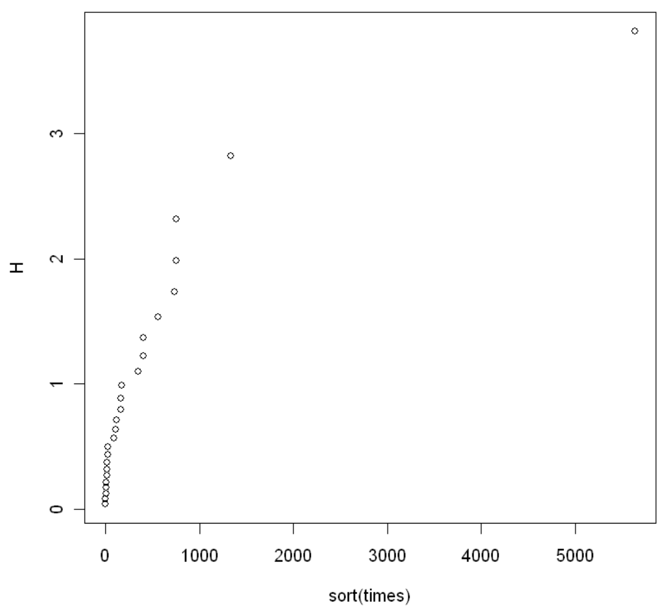

Figure 8 Cumulative hazard plot for 25 times between failures for renewal distribution with decreasing failure rate

3 NONHOMOGENEOUS POISSON PROCESS

There is also a qualitative check for an increasing or decreasing trend in the ROCOF under the same-as-old repair assumption, meaning that our stochastic process for failure is an NHPP. It is simply the cumulative failure plot, which, as shown above, is not an appropriate check for a renewal process. However, the NHPP is not stationary, so this check can be helpful for identifying an increasing or decreasing ROCOF. We plot the cumulative number of failures versus the cumulative time to failure. The slope of this plot is an estimate of the ROCOF, so an increasing slope corresponds to aging and vice versa. The figures below show these plots for simulated data from a power-law NHPP with increasing and decreasing ROCOF.

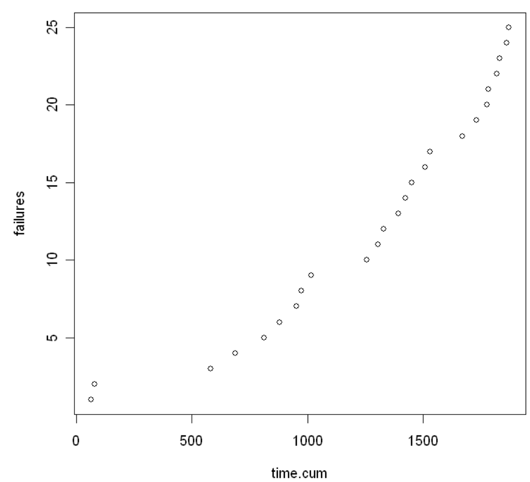

Figure 9 Cumulative failure plot for simulated data from power-law NHPP with increasing ROCOF

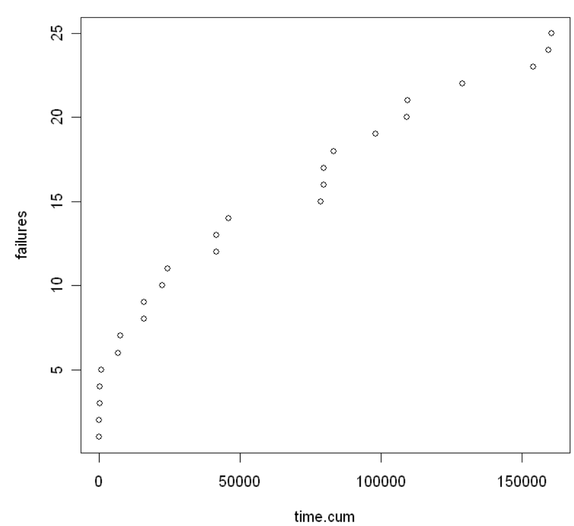

Figure 10 Cumulative failure plot for simulated data from power-law NHPP with decreasing ROCOF

3.1 Nonparametric Test to Distinguish NHPP from Renewal Process

There is a non-parametric statistical test that can be helpful in distinguishing between a renewal process (same-as-new repair) and a nonhomogeneous Poisson process (NHPP) representing same-as-old repair. It is very similar to the so-called Laplace test, and it is simple to implement in a spreadsheet or other software. It uses the cumulative times to failure. The formula depends on whether the observation period is for a fixed period of time, or terminates at the time of the last failure. If it is for a fixed period of time, τ, and there are n observed failure times, one first computes the Laplace statistic, U, according to the following formula:

$$U = \frac{\sum_{i=1}^{n} t_i / \tau - n/2}{\sqrt{n/12}} \tag{4}$$

If the observation period is only up until the last observed cumulative failure time, t_n, U is computed by

$$U = \frac{\sum_{i=1}^{n} t_i / t_n - \frac{n-1}{2}}{\sqrt{\frac{n-1}{12}}} \tag{5}$$

This statistic quickly approaches a standard normal distribution under the null hypothesis of a homogeneous Poisson process. If the process is actually NHPP with increasing ROCOF, too many of the failures will occur after the midpoint of the observation period, and U will be too large. Conversely, if the process is NHPP with decreasing ROCOF, too many failures will occur before the midpoint of the observation period, and U will be too small. At a 5% significance level, we reject the null hypothesis if U is larger than 2 or smaller than -2.

As pointed out above, the null hypothesis for the Laplace test is a homogeneous Poisson process. A slight modification to this test allows for a null hypothesis of a renewal process with any renewal distribution, not just the exponential distribution that is the null hypothesis for the Laplace test. We divide U by the estimated coefficient of variation of the *times between failures* (ratio of sample standard deviation to sample mean). Asymptotically, if the renewal distribution is exponential, this new statistic will equal U. Again, this statistic quickly approaches a standard normal distribution under the null hypothesis of a renewal process.

Applying this test to the simulated data shown in Figures 1-3, we obtain the following values of the modified Laplace statistic. The two-sided p-values are shown in parentheses. As expected, since these times were simulated from a renewal process, we cannot reject the null hypothesis of a renewal process at any reasonable significance level.

Constant failure rate: -0.78 (0.43)

Increasing failure rate: 0.71 (0.48)

Decreasing failure rate: -0.10 (0.92)

Now let us examine this test for the data shown in Figures 9 and 10, which were cumulative failure times simulated from a power-law NHPP with increasing and decreasing ROCOF, respectively. The values of the statistic from our nonparametric test (Laplace U divided by estimated coefficient of variation) are shown below, with the two-sided p-values in parentheses. The null hypothesis of a renewal process would just be rejected at a 0.05 significance level in both of these cases.

Increasing ROCOF: 1.99 (0.046)

Decreasing ROCOF: -1.99 (0.046)

4 IMPACT OF ASSUMPTION REGARDING STOCHASTIC PROCESS

The assumption made regarding repair (same as old versus same as new) is crucial to the analysis. Consider times to failure being produced by a process with increasing ROCOF, corresponding to aging with repair same as old. As time progresses, times between failure will tend to decrease, and there will be a preponderance of short times between failures in a sample. If the process is assumed (erroneously) to be a renewal process, with times between failures described by, for example, a Weibull distribution, the preponderance of short times between failures will cause the estimate of the Weibull shape parameter to be less than one, corresponding to an apparent *decreasing* failure rate, the opposite of what is actually happening. This can be illustrated by simulation. We generated 1,000 cumulative failure times for a system whose repair is same-as-old, described by a power-law process with shape parameter of 2 and scale parameter of 350. The cumulative failure plot below shows the increasing trend in ROCOF with time.

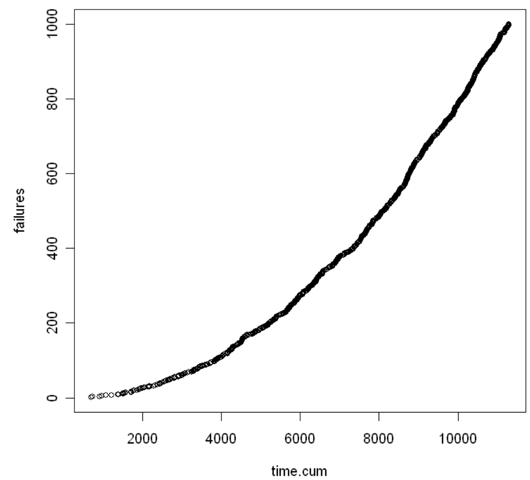

Figure 11 Cumulative failure plot for 1,000 times simulated from power-law process with shape parameter of 2, illustrating increasing ROCOF

The histogram below of the times between failures shows the preponderance of short times between failures caused by the increasing ROCOF.

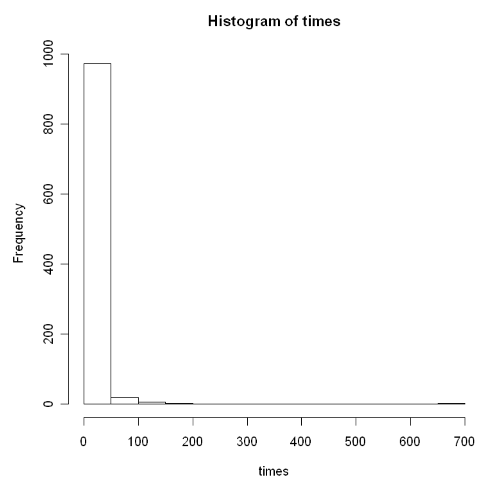

Figure 12 Histogram of times between failures for simulated failure times from power-law process with increasing ROCOF

Assuming the repair is same-as-new instead of same-as-old and fitting a Weibull distribution to these times between failures using either the method of maximum likelihood or a Bayesian update of diffuse prior distributions, one estimates a Weibull shape parameter of about 0.8, which would incorrectly suggest a failure rate that is decreasing with time.

Conversely, if the repair is same-as-new, with the failure rate increasing with operating time or time in standby (whichever is being modeled), an erroneous assumption of same-as-old repair will, as suggested by Figure 5, lead to an estimate near one for the shape parameter of the power-law process. This again will tend to mask the aging that is taking place.

5 CONCLUSIONS

The analysis of component aging data depends crucially on assumptions regarding repair of failed components. If failed components are replaced with new ones, then a renewal process is likely an appropriate model. In this case, it is the times between failures that are of interest. The analysis proceeds on the assumption that these times constitute a random sample from a common renewal distribution. The cumulative hazard plot is a useful graphical indicator of how the failure rate behaves as a function of operating time or time in standby. We presented a modification of the Laplace test, described in more detail in [1], that can be used to test the null hypothesis of a renewal process.

For the case for complex components, where failure involves repair or replacement of a small subset of the piece parts, leaving the component in nearly the state it was in just before failure occurred, a renewal process may not be a good stochastic model. In this case, it is the cumulative failure times that are of interest, and these times no longer can be assumed to be a random sample from a common renewal distribution. A useful check on whether aging is taking place is the cumulative failure plot. The nonparametric test above can be used as a quantitative test.

Making the wrong assumption about the underlying type of repair (same-as-new vs. same-as-old) can have a drastic impact on the conclusions. If repair is same-as-old, and aging is occurring, analyzing the data as if repair were same-as-new can lead to the opposite conclusion that reliability growth is occurring. Similarly, assuming that a renewal process is repair same-as-old can lead to the erroneous conclusion of constant failure rate.

6 ACKNOWLEDGMENTS

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7 REFERENCES

1. Harold Ascher and Harry Feingold, *Repairable Systems Reliability*, Marcel-Dekker, New York and Basel (1984).