INL/CON-08-14181 PREPRINT

Improving Attack Graph Visualization Through Data Reduction and Attack Grouping

VizSEC 2008 Workshop on Visualization for Computer Security

John Homer Ashok Varikuti Xinming Ou Miles A. McQueen

September 2008

This is a preprint of a paper intended for publication in a journal or proceedings. Since changes may be made before publication, this preprint should not be cited or reproduced without permission of the author. This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, or any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for any third party's use, or the results of such use, of any information, apparatus, product or process disclosed in this report, or represents that its use by such third party would not infringe privately owned rights. The views expressed in this paper are not necessarily those of the United States Government or the sponsoring agency.

The INL is a U.S. Department of Energy National Laboratory operated by Battelle Energy Alliance





Improving Attack Graph Visualization through Data Reduction and Attack Grouping

John Homer¹, Ashok Varikuti¹, Xinming Ou¹, and Miles A. McQueen²

Kansas State University, USA
Idaho National Laboratory, USA

Abstract. Various tools exist to analyze enterprise network systems and to produce attack graphs detailing how attackers might penetrate into the system. These attack graphs, however, are often complex and difficult to comprehend fully, and a human user may find it problematic to reach appropriate configuration decisions. This paper presents methodologies that can 1) automatically identify portions of an attack graph that do not help a user to understand the core security problems and so can be trimmed, and 2) automatically group similar attack steps as virtual nodes in a model of the network topology, to immediately increase the understandability of the data. We believe both methods are important steps toward improving visualization of attack graphs to make them more useful in configuration management for large enterprise networks. We implemented our methods using one of the existing attack-graph toolkits. Initial experimentation shows that the proposed approaches can 1) significantly reduce the complexity of attack graphs by trimming a large portion of the graph that is not needed for a user to understand the security problem, and 2) significantly increase the accessibility and understandability of the data presented in the attack graph by clearly showing, within a generated visualization of the network topology, the number and type of potential attacks to which each host is exposed.

Key words: attack graph, attack graph visualization, dominator, graph clustering, network security analysis

1 Introduction

Attack graphs have been developed to aid in identification and correction of misconfigurations in enterprise network systems, by providing a visual representation of potential attack paths [1–8]. Much work has already been done in the generation of attack graphs, producing more efficient techniques for building them [6, 7]. Attack graphs, however, are difficult for a human to utilize effectively because of their complexity [9–11]. Even a network of moderate size can have dozens of possible attack paths, overwhelming a human user with the amount of information presented. It is not easy for a human to determine from the information in the attack graph which configuration settings should be changed to best address the identified security problems. Without a clear understanding of the existing security problems, it is difficult for a human user to evaluate possible configuration changes and to verify that optimal changes are made.

Previous works have introduced improvements in the visualization of attack paths and the overall presentation of attack graph data. Noel, *et al.* suggested that complexity can be reduced through the use of protection domains to represent groups of machines with unrestricted interconnectivity [9, 10]. Lippmann, *et al.* introduced visualization approaches to emphasize critical attack steps while clearly showing host-to-host reachability [11].

In this paper, we show that by utilizing the logical semantics of an attack graph, one can 1) distinguish attack steps based on their usefulness for a human to quickly understand the core security problems in an enterprise network, and trim those that do not contribute much for this purpose; 2) identify attack steps that share similar semantics, and thus can be grouped and presented as a single virtual node. These techniques can further improve the visualization of attack graphs to make them more useful in practice.

For our implementation, we use the MulVAL attack graph tool suite [7, 12], which provides reasonable performance and scalability for enterprise networks of a realistic size. MulVAL produces complete logical

attack graphs, which are easily mapped back to a visualization of the network topology, based on the input data.



Fig. 1. An example enterprise network

Figure 1 depicts an example enterprise network that is based on a real (and much bigger) system; we will return to this example throughout the paper. The network includes three subnets: a DMZ (Demilitarized Zone), an internal subnet, and an EMS (Energy Management System) subnet, which is a control-system network for power grids. In this example, we will assume that host-grouping has already been applied, based on similar configurations; the workStation node, for example, might be an abstracted grouping of one hundred workstation machines with comparable setups. Both the web server and the VPN server are directly accessible from the Internet. The web server can access the file server through the NFS file-sharing protocol; the VPN server is allowed access to all hosts in the internal subnet. Outside access to the EMS subnet is only allowed from the Citrix server in the internal subnet, and even then only to the data historian. In this example, we assume that the attacker's goal is to gain privileges to execute code on the commServer. From the commServer, an attacker could send commands to physical facilities such as power-generating turbines, which can cause grave damage to critical infrastructures.

A visualization of the MulVAL attack graph is shown in Figure 2, identifying a large number of potential attack paths in this network. This visualization is produced by mapping the full MulVAL logical attack graph (shown in Appendix A) to the network subnet topology, using clustering techniques similar to Noel *et al.*'s approach [10]. The black solid lines represent connectivity between subnets and gateways (router, firewall, *et al.*). Machines in the same subnet (represented as a rectangular cluster) have unrestricted access to each other. The red dotted lines indicate attack propagation paths as mapped from the full logical attack graph. This visualization omits a large amount of information from the original full attack graph, such as pre- and postconditions for each attack step. We believe a simple visualization of attack paths directly on the network



Fig. 2. Attack graph visualization

topology will be useful in practice, since it relates the information conveyed by the attack graph to the concrete entities in an enterprise network, and a system administrator will likely find it easier to understand than the full attack graph (see Appendix A). Further information about the attacks can be displayed upon request through user interaction.

Even after this simplification, the attack paths in the visualization may still overwhelm a user. In this paper we will focus on how to further reduce the complexity through two techniques. First, we observe that there are a number of logically valid attack steps that are not "useful" to an attacker. For example, an attacker who has gained privileges on the workStation machine in the internal subnet has opportunity to exploit the webServer in the DMZ. Though possible, this attack step is intuitively unhelpful to an attacker attempting to reach "deeper" into the network, since the attacker would have to compromise DMZ to gain privilege on workStation in the first place. Attacking "back" to DMZ does not add any additional privileges useful for the attacker. Another example of an intuitively useless attack step would be attacking the fileServer from the citrixServer; an attacker with privileges on the citrixServer already has all of the necessary privileges to attack the EMS subnet through the dataHistorian and gains nothing useful by further attacking the file-Server. Second, we observe that the complexity of the attack graph does not necessarily reflect complexity in security vulnerabilities. Employing a compromised user account, an attacker can access the citrixServer from the vpnServer, workStation, and fileServer; using a Trojan horse attack, an attacker can gain access to the citrixServer from the fileServer. (The edge from fileServer to citrixServer represents both potential attack steps). Although there are four distinct attack steps that can enable an attacker to compromise the citrixServer, these attack steps utilize only two distinct exploitations. This fact is obscured in the attack graph by the separate attack steps leading to citrixServer from different host machines.

We believe that the attack graph complexity can be further reduced. In order to make an attack graph a useful tool for configuration management, we identify as a research challenge the need for presenting the security problems expressed by an attack graph in a manner that enables a human user to more quickly grasp the core of the security problem. Our contributions are:

- 1. We developed an algorithm to identify portions of an attack graph that are not helpful for a user to understand the core security problems, and reduce the amount of data presented to the user by trimming those portions. When the amount of information presented in the attack graph is reduced, we believe that core security problems will be more quickly identifiable from the attack graph.
- 2. We developed a method to create virtual nodes to represent groupings of similar exploitations. In this approach, each attack step edge leading into a host represents a unique attack on that machine. We believe that this approach will increase the understandability of the attack graph data by more clearly displaying the exploitability of each host.

Our approach to trimming attack steps ensures that all distinct attack paths will be retained in the trimmed attack graph, while removing data not beneficial to the understanding of core security problems. Host-grouping techniques have already been shown to be effective for reducing complexity [6, 9, 10]. We show that further gains can be made in grouping similar exploits from multiple sources, which makes clearly visible the number of exploits available on a given machine and all of the possible sources for each potential exploit. It is easy then to see all attack steps that can be eliminated by resolving the vulnerability enabling a specific exploit. When both approaches are applied, the attack graph from Figure 2 will be converted to Figure 3, which we believe to be more easily understandable to a human user.

In Figure 3, attack steps that are not helpful in gaining a better understanding of network vulnerabilities have been removed from the graph, reducing the total number of edges and thus reducing graph complexity. The trimming algorithm for this approach is presented in Section 2. Our exploit grouping approach has also

been applied in this figure, so the "principal compromised" attack that can provide an attacker access to citrixServer is shown only once, with edges leading from all possible source nodes to the abstract representation of the exploit and only one edge representing this attack leading to citrixServer. The other attack edge leading to citrixServer from the fileServer represents the Trojan horse attack that can only source from the fileServer. Shown this way, we believe that it is easier to see and understand the actual vulnerabilities in the graph. The exploit grouping approach is presented in Section 3. We will discuss related work in Section 4 and conclude with a discussion of future work on these approaches in Section 5.



Fig. 3. Attack graph with abstracted exploitation nodes and trimming applied

2 Identifying and Removing "Useless" Attack Steps

In examining the attack graph, we found that many of the attack steps, while valid from a logical point of view, are not helpful for a human user to comprehend the core security problems in the network configurations. They share a common characteristic which is they do not reveal the most important vulnerability in the system since the attacker does not penetrate "deeper" into the enterprise network along those steps. While these steps contain important information that would be useful if one wished to block every possible attack path, they can also be distracting to a human reader and often hides the root causes of the security problems. It is thus beneficial to remove these less useful attack steps from the attack graph so that the security problems become easier to grasp for a human reader.

We refer to attack steps that are not useful for a human reader to understand the underlying security problems as "useless" attack steps. Intuitively, "useless" attack steps are those that would not gain useful, new privileges for an attacker. Generally, these "useless" attack steps involve an attack on a machine further from the goal machine than the machine from which the attack is made. In the example described earlier, one valid attack step enables an attacker with privileges on the workStation to gain privileges on the webServer, but this attack would not bring the attacker any closer to the presumed goal of accessing the commServer.

It is important to emphasize that those attack steps are valid and important to consider when determining upon appropriate countermeasures. However, when the user is first presented with the attack graph, understanding these paths is not crucial for understanding overall security threats. It would be more beneficial if the user can quickly understand the core security problems from a simplified attack graph. We will refer to attack steps that contribute to this understanding as "useful" and to all other attack steps as "useless." For example, in Figure 4 the attacker's starting machine is host 1, and his goal is to compromise host 4. There are two paths: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ and $1 \rightarrow 3 \rightarrow 4$. Intuitively the attack step from machine 3 to machine 2 is not useful, since it does not help the attacker to reach his goal. We would like to trim those steps.



Fig. 4. Example useless attack steps

The immediately obvious solution for identifying these "useless" attack steps is to implement a breadthfirst search algorithm to compute the distance of machine from the goal machine, as measured by the minimum number of inter-host attack steps necessary to reach the goal from that machine. Machine 2 in the above example would have a distance of 2 and machine 3 would have a distance of 1. Attack steps that move from a machine to another machine with the same or greater distance (*e.g.* $3 \rightarrow 2$) could then be labelled "useless" and trimmed. While this approach would work in some cases, in many cases useful attack paths would be trimmed. For example, this algorithm would also trim the step $1 \rightarrow 2$, and thus lose the complete attack path $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$. Actually only attack paths with the shortest length will be preserved and all other paths will be trimmed. This will lose too much information, especially in cases when an attacker actually will prefer a longer path due to ease of exploit or better stealthiness along the longer path. Another seemingly correct solution is to perform a simple depth-first search and trim the back edges in the DFS tree. However, depending on the order of traversing a node's multiple children, both edge $2 \rightarrow 3$ and $3 \rightarrow 2$ could be back edges in the DFS tree. So this method does not work either.

We have developed a two-level approach to identifying and removing "useless" attack steps³. First, we create a directed graph with subnets as nodes and possible inter-subnet attack steps as edges. From this directed graph, we then construct a dominator tree to recognize dominance and post-dominance relationships between subnets with respect to presumed attacker location and his goal. The subnet where the attacker is located will be the source node and the subnet where the goal machine is located will be the sink node. Let d, n be vertices in a directed graph. d dominates n if every path from the source node to n must go through d. We write d dom n for this fact. p post-dominates n if every path from n to the sink node must go through p. We write p postdom n for this fact. In the subnet graph of Figure 5, assuming that INTERNET is the source and EMS is the goal, some examples of dominance relationships are DMZ dom CORP and CORP dom EMS. We also have EMS postdom CORP and CORP postdom DMZ.



Fig. 5. Subnet graph, built from example in Section 1

For any two subnets X and Y, we then identify and trim all inter-subnet attack steps $X \to Y$ where $Y \operatorname{dom} X$ or $X \operatorname{postdom} Y$. If $Y \operatorname{dom} X$, then an attacker who has gained privileges in subnet X must already have privileges in subnet Y (or the attacker would not have been able to transition to X). If X postdom Y, then moving from X to Y will not help the attacker either since he would have to return to X in order to reach his goal.

We will eliminate these attack steps because they are distracting for a human reader trying to comprehend other, more enlightening attack paths. In the attack graph shown in Figure 2, every transition from the CORP subnet to the DMZ is useless since DMZ dominates CORP. On the other hand, the transition from CORP to the control network subnet EMS is useful. Figure 6 shows the application of this first-level trimming approach to the sample network.

After applying the inter-subnet transition trimming, we then address intra-subnet transitions. An attack step between two machines $A \rightarrow B$ in the same subnet is useful in only two cases. First, if the subnet contains the goal machine, the transition is useful only if B is the goal. Any other transition within this subnet is intuitively useless, since an attacker already has opportunity to attack the goal machine. Second, if the subnet does not contain the goal machine, the transition is useful only if B would provide an attacker with access to another subnet that would be deemed useful according to the subnet dominator tree, and even then only if that same access is not available from A. In the attack graph shown in Figure 2, the transition from

 $^{^{3}}$ Formal definitions for this approach are presented in Appendix B.



Fig. 6. Attack graph with inter-subnet trimming



Fig. 7. Attack graph with both inter- and intra-subnet trimming applied

fileServer to workStation is useless, since the workStation would not provide an attacker with new, useful access; however, the transition from fileServer to citrixServer is useful, since, from citrixServer, an attacker could access the EMS subnet. Figure 7 shows the application of this second-level trimming approach to the sample network, after the inter-subnet trimming has been applied. The attack graph now shows three key attack paths to reach the Citrix server, from which the control network subnet EMS is accessible:

 $\begin{array}{l} Internet \rightarrow web \; server \rightarrow file \; server \rightarrow Citrix \; server \\ Internet \rightarrow VPN \; server \rightarrow workstation \rightarrow Citrix \; server \\ Internet \rightarrow VPN \; server \rightarrow Citrix \; server \end{array}$

A careful reader might ask why the second attack attack path is retained, given the existence of a shorter path (3). As mentioned above, shorter attack paths cannot always subsume longer ones, since the exploits along the path may be different. The exploit-grouping technique discussed in the next section can help further reduce redundancies arising from similar exploits.

3 Abstraction of Attack Traces

Even after identifying and removing "useless" attack steps, many edges will likely remain in the visualization. Humans assessing the data presented in the attack graph will benefit from the reduced amount of data, but still face other obstacles to clear and straightforward understanding of the underlying security issues in the current network configuration. One hindrance to easy understanding of attack graph data can be the number of edges directed into a single host machine in the attack graph. In Figure 7, for example, the citrixServer node in the Corp subnet is the destination point of four attack steps (shown in three edges), even after "useless" attack steps have been trimmed from the graph. It is not immediately clear to the user how many different possible exploitations are being represented, and how many sources for exploitations of the citrixServer are repetitions of a single attack type.

Our solution to this difficulty is to create an abstraction of each exploitation with multiple sources, from which only one edge will lead into the exploited node. In this way, it is much easier for a human user to see how many exploitations are possible on a given host and what potential attack steps could be eliminated by resolving the conditions enabling a specific exploitation. Potential exploits with only one source, on the other hand, will be represented by a direct attack step edge between the two machines, to maintain as much simplicity in the graph as possible.

In the attack graph shown in Figure 7, three of the edges that lead to the citrixServer represent different source points but only a single security issue in the network, namely the uncertain reliability of the user with account "ordinaryUser." If this user account is compromised, an attacker could gain access to the citrixServer from any of the three host machines with edges leading to the abstracted exploitation node. By creating a virtual node to represent the existence of this security concern, it is much easier to see now that if the reliability of this user account can be verified, most of the possible attacks leading to citrixServer will be eliminated.

The full attack graph, shown in Figure 2, included a number of "useless" attack steps that are removed by the trimming algorithm. The trimming also reduces the number of multiple-source attacks and thus the number of abstract exploit nodes that can be created. For optimal effectiveness, this exploit abstraction technique is applied only to the trimmed attack graph. The final attack graph is shown again in Figure 8.



Fig. 8. Attack graph, trimmed, with virtual exploitation nodes

4 Related Work

A number of other previous works addressed the problem of how to use attack graphs to better manage the security of enterprise networks [13–17]. The observations and insights from these previous works helped us develop the approach in this paper, and our work either complements or improves upon them. Our contribution is the development of formal, logic-based approaches to simplifying an attack graph for a human to better understand.

Noel, *et al.* proposed a number of techniques for reducing complexity in attack graphs [10]. We adopted some of the approaches in our work, such as using clustering techniques to show the subnet topology of the network, and enabling user interaction to get more details of the attack graphs. Our approaches address complementary problems in visualization, namely identification and removal of attack paths that are not useful for a human to better understand the core security problems, and better represent attacks by grouping similar exploits targeted at a single host. Noel, *et al.* also presents a notion of graph trimming, by removing redundant exploits and allowing them to be implicitly conveyed in the graph. However, they do not systematically address how to identify and trim the "useless" attack steps described in this paper.

Lippmann, *et al.* have built on the multiple-prerequisite graphs produced by the NetSPA system with a goal of reducing attack graph complexity [11]. Their visualization employs spatial grouping and color-coding to represent levels of potential compromise. Groups of machines with similar levels of exploitability can then be collapsed, reducing the overall complexity of the graph. Our approach differs in that we do not group machines with similar vulnerabilities, but rather create abstract representations of attacks, with edges leading to the potentially affected machines.

5 Conclusion

We have proposed two techniques for improving visualization of attack graphs — reducing the amount of data by identifying attack steps that are not crucial for a human to quickly understand the core security problems, and grouping similar attacks targeted at a single host to better represent the number and type of security problems. These techniques, in combination with visualization techniques developed by previous researchers, will display attack graphs to a more human-readable manner. This is crucial for using attack graphs to further automate enterprise network security management. A human can only trust a tool if she/he understands its output. In order to further use attack graphs to automatically find the best way to fix the security problems, one must let the human user first understand the security problems. Our techniques will help a human user quickly identify the core security problems in an enterprise network without being overwhelmed by the amount of information contained in the full attack graphs.

References

- 1. Laura P. Swiler, Cynthia Phillips, David Ellis, and Stefan Chakerian. Computer-attack graph generation tool. In DARPA Information Survivability Conference and Exposition (DISCEX II'01), volume 2, June 2001.
- 2. Oleg Sheyner, Joshua Haines, Somesh Jha, Richard Lippmann, and Jeannette M. Wing. Automated generation and analysis of attack graphs. In *Proceedings of the 2002 IEEE Symposium on Security and Privacy*, pages 254–265, 2002.
- 3. Paul Ammann, Duminda Wijesekera, and Saket Kaushik. Scalable, graph-based network vulnerability analysis. In *Proceedings* of 9th ACM Conference on Computer and Communications Security, Washington, DC, November 2002.
- 4. Sushil Jajodia, Steven Noel, and Brian O'Berry. Topological analysis of network attack vulnerability. In V. Kumar, J. Srivastava, and A. Lazarevic, editors, *Managing Cyber Threats: Issues, Approaches and Challanges*, chapter 5. Kluwer Academic Publisher, 2003.
- 5. Richard Lippmann and Kyle W. Ingols. An annotated review of past papers on attack graphs. Technical report, MIT Lincoln Laboratory, March 2005.
- 6. Kyle Ingols, Richard Lippmann, and Keith Piwowarski. Practical attack graph generation for network defense. In 22nd Annual Computer Security Applications Conference (ACSAC), Miami Beach, Florida, December 2006.
- 7. Xinming Ou, Wayne F. Boyer, and Miles A. McQueen. A scalable approach to attack graph generation. In *13th ACM Conference on Computer and Communications Security (CCS)*, pages 336–345, 2006.
- 8. Wei Li, Rayford B. Vaughn, and Yoginder S. Dandass. An approach to model network exploitations using exploitation graphs. *SIMULATION*, 82(8):523–541, 2006.
- Steven Noel and Sushil Jajodia. Managing attack graph complexity through visual hierarchical aggregation. In *VizSEC/DMSEC* '04: Proceedings of the 2004 ACM workshop on Visualization and data mining for computer security, pages 109–118, New York, NY, USA, 2004. ACM Press.
- 10. Steven Noel, Michael Jacobs, Pramod Kalapa, and Sushil Jajodia. Multiple coordinated views for network attack graphs. In *IEEE Workshop on Visualization for Computer Security (VizSEC 2005)*, 2005.
- 11. Richard Lippmann, Leevar Williams, and Kyle Ingols. An interactive attack graph cascade and reachability display. In *IEEE Workshop on Visualization for Computer Security (VizSEC 2007)*, 2007.
- 12. Xinming Ou, Sudhakar Govindavajhala, and Andrew W. Appel. MulVAL: A logic-based network security analyzer. In *14th* USENIX Security Symposium, 2005.
- 13. Somesh Jha, Oleg Sheyner, and Jeannette M. Wing. Two formal analyses of attack graphs. In *Proceedings of the 15th IEEE Computer Security Foundations Workshop*, pages 49–63, Nova Scotia, Canada, June 2002.
- Richard P. Lippmann, Kyle W. Ingols, Chris Scott, Keith Piwowarski, Kendra Kratkiewicz, Michael Artz, and Robert Cunningham. Evaluating and strengthening enterprise network security using attack graphs. Technical Report ESC-TR-2005-064, MIT Lincoln Laboratory, October 2005.
- 15. Richard Lippmann, Kyle Ingols, Chris Scott, Keith Piwowarski, Kendra Kratkiewicz, Mike Artz, and Robert Cunningham. Validating and restoring defense in depth using attack graphs. In *Military Communications Conference (MILCOM)*, Washington, DC, U.S.A., October 2006.
- 16. Vaibhav Mehta, Constantinos Bartzis, Haifeng Zhu, Edmund Clarke, and Jeannette Wing. Ranking attack graphs. In *Proceedings of Recent Advances in Intrusion Detection (RAID)*, September 2006.
- 17. Lingyu Wang, Anoop Singhal, and Sushil Jajodia. Measuring network security using attack graphs. In *Third Workshop on Quality of Protection (QoP)*, 2007.
- 18. Thomas Lengauer and Robert Endre Tarjan. A fast algorithm for finding dominators in a flowgraph. *ACM Trans. Program. Lang. Syst.*, 1(1):121–141, 1979.

A MulVAL Logical Attack Graph



Fig. 9. Full logical attack graph, as generated by MulVAL

B Formal Definitions

In considering the attack paths displayed in Figure 2, we found that many of the attack steps, while valid from a logical point of view, are not helpful for a human user to comprehend the core security problems in the configurations. Consider the following examples:

- 1. The attacker, after compromising the VPN server, can further compromise the web server.
- 2. The attacker, after compromising the workstation, can further compromise the web server.
- 3. The attacker, after compromising the Citrix server, can further compromise the file server.

Useless intra-subnet transitions In one type of situations, the attack steps indicate an attacker's privilege on machine A can enable him to further compromise machine B in the same subnet, but privileges on machine B cannot help the attacker penetrate deeper into the enterprise system. Case (1) and (3) in the example are in this category. Since the VPN server can already access all the machines in the internal subnet, moving from the VPN server to the web server does not increase the attacker's ability to attack machines in the internal subnet. Similarly, since the file server cannot directly access the control network, attacking it from the Citrix server, which can access the control network, does not increase the attacker's ability to further compromise the control network.

Useless inter-subnet transitions In the second type of situations, the attacker's privileges on machine A in subnet S_1 enables him to attack a machine B in another subnet S_2 , but S_1 is "deeper" into the enterprise network than S_2 . Case (2) in the example is in this category. Intuitively, the internal subnet is "deeper" than the DMZ subnet in the enterprise system. Thus, attacking from the workstation in the internal subnet to the web server in the DMZ does not increase an attacker's ability to penetrate deeper into the system.

We need a few more definitions to formally characterize what transitions are useless.

Definition 1 A subnet graph of an enterprise network is (V, E), where V is the set of subnets⁴ and E is the set of edges between subnets. $(S_1, S_2) \in E$ if there is a machine H_1 in S_1 such that H_1 can reach H_2 in S_2 using some protocol and port.



Fig. 10. A subnet graph

Figure 10 shows an example subnet graph. We assume that the attacker is initially located in subnet Internet and he would like to gain some privileges in a "goal" subnet (e.g., Goal). This can be generalized

⁴ We adopt the normal meaning of a subnet as a group of machines that are physically connected behind a switch with unrestricted access to each other (except for simple filtering to prevent source-IP spoofing or ARP spoofing).

to include cases where an attacker has multiple start locations and multiple goals in mind. In such cases we only need to introduce a dummy source node in the graph that has an edge to all the initial-location subnets, and a sink node that has an edge from each goal subnet. The information needed to build the subnet graph is most likely already available from an attack-graph toolkit, since it needs to analyze the topology of the enterprise network along with the firewalls, router configurations, etc., to compute reachability relations between hosts. We actually wrote very simple code to derive the subnet graph structure from the input tuples to the MulVAL reasoning engine.

From the subnet graph, it can be seen that there is some ordering on the subnets. For example, S1 and S2 are deeper than DMZ, since machines from the Internet can only access the DMZ, and machines in S1 and S2 can only be directly accessed from the DMZ. S1 is also deeper than S2 since S1 can directly access the Goal subnet. When producing attack graphs, such topological order is not taken into account, which is the reason why many useless and distracting attack steps are included.

We compute the dominator and post-dominator trees on the subnet graph to identify such topological order. Let d, n be vertices in a directed graph. d dominates n if every path from the source node to n must go through d. We write d dom n for this fact. p post-dominates n if every path from n to the sink node must go through p. We write p postdom n for this fact. In the subnet graph of Figure 5, assuming that Internet is the source and Goal is the sink, we have DMZ dom S1 and DMZ dom S2. We also have S1 postdom S2 and S1 postdom DMZ. Computing the dominators and post-dominators in a graph can be done in almost linear time [18].

Definition 2 A transition between subnets $S_1 \rightarrow S_2$ is useless if either S_2 dom S_1 or S_1 postdom S_2 .

Intuitively, if an attacker moves from subnet S_1 to one of its dominators S_2 , this is not a useful step since he already must have reached S_2 in order to be in S_1 . Likewise, if S_1 post-dominates S_2 , the attacker will necessarily return to S_1 at some later point.

Definition 3 An outgoing set of a host H is the set of tuples (host, protocol, port) such that host is not in the same subnet as H and can be reached directly from H through protocol and port. We use outgoing(H) to denote this set.

Definition 4 A transition between two hosts $H_1 \rightarrow H_2$ in the same subnet S is useful if either H_2 is one of the target machines, or there exists $s = (host, protocol, port), s \in outgoing(H_2), s \notin outgoing(H_1)$, host is in subnet S' and $S \rightarrow S'$ is useful.

Definition 5 A transition between two hosts $H_1 \rightarrow H_2$ in two different subnets S_1 and S_2 is useful if the transition $S_1 \rightarrow S_2$ is useful.

The transitions between hosts that are not useful, based on the above definitions, are considered useless and will be trimmed from the attack graph. In Section 1's example, the transition from the VPN server to the web server is useless since the two machines are in the same subnet (DMZ), and no machine outside the DMZ can be reached by the web server and not the VPN server. The transition from the workstation to the web server is also useless since the transition from the internal subnet to the DMZ subnet is useless. But the transition from the web server to the file server is useful since the transition from the DMZ to the internal subnet is useful. And the transition from the file server to the Citrix server is useful since the Citrix server can reach the data historian in the control network subnet and the file server cannot, and the transition from the internal subnet to control network subnet is useful.