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Prelude

In accord with recent custom, the organizing committee for this conference has scheduled this review paper on beam instabilities. In view of the various review papers which already exist[1-4] and the fact that the fundamentals of the subject have even been treated in a textbook,[5] I thought this paper might best be devoted to a limited part of the rather large field of beam instabilities. Thus, I have selected only an aspect of the general subject, but an aspect which has during the last years been very much at the center of activity, and will--if my judgment is correct--be even more so in the years to come. I wish to concentrate, here, on the interaction of a relativistic particle beam with itself which is a result of the coupling of the beam with its surroundings.

Before approaching this topic, a few remarks on the existing review papers are in order. A comprehensive treatment of beam instabilities may be found in Ref. 1, where, also, the reader will find some 48 references to the original literature. In Refs. 2 and 3, the general subject is approached from other points of view. Reference 4 is concerned with some special topics, but treats them in depth; and the text of Ref. 5 closely follows the original papers.

I. Static Self-Field Phenomena: Three Examples

The interaction of a beam with its surroundings will influence the static self-field of the beam and consequently the incoherent behavior of individual particles, whose motion is determined by the combination of externally applied fields and self-consistent beam fields. Thus the equilibrium properties of beams --including the possibility of the absence of an equilibrium configuration--is determined, in part, by beam-surrounding interactions.

1.1 Transverse Oscillations

One example of this phenomenon is supplied by the transverse (betatron) oscillations of particles in a circular accelerator. It was first observed by Kerst[6] that the beam self-fields reduce the betatron oscillation frequency ν by an amount $\Delta\nu$, from the zero-beam value.

If the beam self-fields are computed for a uniform (straight) beam in free space then[6]

$$\Delta\nu = \frac{N R r_0 F}{\nu \beta^2 \gamma^3 2\pi a^2} , \qquad (1)$$

where N is the number of particles in a torus of major radius R and minor radius a, βc is the particle velocity, the associated relativistic factor is γ, r_0 is the classical radius of the particles, and the factor F equals unity.

The important influence of beam surroundings was pointed out by Laslett[7] who showed--for example--that for a uniform beam between conducting walls (vacuum tank) with separation $2h$, and iron (magnet) surfaces of separation $2g$ the factor F in (1) becomes

$$F = 1 + \frac{2a^2}{h^2} \left(\frac{\pi^2}{48} \right) \left[(1 + \beta^2\gamma^2) + \beta^2\gamma^2 \frac{h^2}{g^2} \right] . \qquad (2)$$

In a strong focusing machine the beam will be unstable if ν is shifted to the nearest resonance and thus $\Delta\nu$ is typically limited to a value $\Delta\nu \approx 1/4$. Thus (1) may be employed to estimate a limiting value of N. The influence of surroundings may, from (2), be seen to be dominant at high energies where the limit on N varies only as γ, in contrast with the γ^3 dependence given by (1) when F is set equal to unity. We shall not further pursue, here, the literature in which this subject is more fully developed.

1.2 Longitudinal Oscillations

Another example, of the influence of surrounding media upon the self-field of a relativistic beam, is supplied by the diffraction radiation reaction force on an electron ring moving in an acceleration column.

It is well known that the diffraction radiation by an electron ring in the acceleration column of an electron ring accelerator (ERA) is a strong function of the geometry of the acceleration column and, furthermore, is an important effect insofar as it can cause significant loss of energy of the ring.[8] The diffraction radiation reaction effect upon the axial stability of rings has been evaluated in a recent paper,[9] where it has been shown to be only a small defocusing effect and unimportant in comparison even with the rather weak axial focusing supplied by ions and images.

Although the above effect is unimportant, it is a clear--yet complicated--example of the self-interaction of a beam via interaction with surrounding media; and actually in this problem the small answer was by no means obvious prior to an extensive calculation.

1.3 Bunch Length in Electron Storage Rings

As a final example of a static self-field phenomenon ---or at least a phenomenon which could have this as its root cause--consider the azimuthal length of bunches in electron-positron storage rings. At Orsay and Frascati the bunch length is observed to be a function of stored current, although no such effect has been observed at Stanford or Novosibirsk.[10]

These observations have stimulated considerable theoretical effort.[11-15] Theories based upon coherent synchrotron radiation[11,12] predicted a shortening of bunches with increasing current, in contradiction with the observations. Resonances associated with clearing-field electrodes[13] and resonances associated with radio-frequency cavities[14] have been suggested as the source of the phenomenon.

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All the above theories are equilibrium calculations; that is, they are theories in which the effective azimuthal potential well is modified in strength as a result of the high beam current. One analysis[15] suggests that the bunch lengthening is due to an instability of the internal coherent synchrotron oscillations, but the parametric dependence of the bunch lengthening in this theory is not in good agreement with the observations. The equilibrium theories have, recently, all been incorporated into a general formulation of the problem in which the beam-surrounding interaction is explicitly exhibited.[16]

Following Ref. 16, the beam self-interaction may be described by a Green's function $\mathcal{G}(\sigma)$ such that if $\lambda(\sigma)$ is the linear charge density of the bunch as a function of the distance σ from the synchronous particle, then the energy change of a particle due to self-forces per revolution, eU, is given by

$$eU(\sigma) = -e\int_{-\infty}^{\infty} d\sigma' \, \lambda(\sigma')\, \mathcal{G}(\sigma' - \sigma). \tag{3}$$

The gradient of $U(\sigma)$, evaluated at the synchronous particle $\sigma = 0$, produces a change in the synchrotron oscillation frequency from Ω_s^2 to Ω^2, where

$$\Omega^2 - \Omega_s^2 = -\frac{R\,\eta\,\omega_s^2\, e\,\frac{dU(\sigma)}{d\sigma}}{2\pi\,\beta^2\,E_s}, \tag{4}$$

E_s is the total energy, ω_s is the revolution frequency, and R is the orbit radius of the synchronous particle. The dispersion coefficient η is given by

$$\eta = \beta^2 \frac{E}{\omega}\frac{d\omega}{dE}\bigg|_{E=E_s}. \tag{5}$$

The mean square bunch length Δ^2 is related to the mean square bunch length in the limit of zero bunch current Δ_0^2 by

$$\Delta^2 = \Delta_0^2\left[1 + \frac{\Omega^2 - \Omega_s^2}{\Omega_s^2}\right]^{-1}. \tag{6}$$

The combining of (6), (5), (4), (3), and the formula

$$\lambda(\sigma) = \frac{Ne}{(2\pi)^{1/2}\Delta}\exp(-\sigma^2/2\Delta^2) \tag{7}$$

yields an explicitly formula for bunch length in terms of $\mathcal{G}(\sigma)$ or--often more conveniently--in terms of its Fourier transform, the impedance $Z(\omega)$ defined by

$$Z(\omega) = \int_{\infty}^{\infty} \mathcal{G}(\sigma)\, e^{i(\omega\sigma/\beta c)}\,\frac{d\sigma}{\beta c}. \tag{8}$$

The reader interested in the subject of bunch length in storage rings should consult Ref. 16, where extensive discussion may be found of the theory and the observations. For our purposes, here, it suffices to merely discuss the functions $\mathcal{G}(\sigma)$ and $Z(\omega)$ for some typical structures which are always present in storage rings--such as rf cavities, pickup electrodes, and clearing electrodes.

For a smooth chamber approximated by two parallel perfectly conducting infinite planes separated by distance H,

$$\mathcal{G}(\sigma) = -\frac{2\pi\beta c}{\omega_s}\left[\frac{1}{\gamma^2}\left(1 + 2\ln\frac{2H}{\pi a}\right) + \left(\frac{\beta H}{\pi R}\right)^2\right]\delta'(\sigma), \tag{9}$$

where a is the beam minor radius, R is the beam major radius, and $\delta'(\sigma)$ is the first derivative of the Dirac delta function.[12] Thus, for this case

$$Z(\omega) = \frac{2\pi i}{\beta c\omega_s}\left[\frac{1}{\gamma^2}\left(1 + 2\ln\frac{2H}{\pi a}\right) + \left(\frac{\beta H}{\pi R}\right)^2\right]\omega, \tag{10}$$

which is valid only for frequencies $\omega < \pi\beta c/H$. This capacitive impedance leads, above transition, to bunch shortening.

For an rf cavity[17,18] of radius b and length L,

$$Z(\omega) = \frac{-8i}{b^2Lv^2}\sum_{s=1}^{\infty}\sum_{p=0}^{\infty}$$

$$\times\frac{(1+\delta_{p0})\omega\left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi p}{L}\right)^2\right]\left[1 - (-1)^p\cos\frac{\omega L}{v}\right]}{J_1^{\,2}(\mu_s b)\left[\left(\frac{p\pi}{L}\right)^2 - \left(\frac{\omega}{v}\right)^2\right]^2\left[\left(\frac{\omega}{c}\right)^2 - \omega_{sp}^{\,2}\right]}, \tag{11}$$

where $v = \beta c$,

$$\omega_{ps}^{\,2} = c^2\left[\mu_s^{\,2} + \left(\frac{p\pi}{L}\right)^2\right], \tag{12}$$

and μ_s is determined by

$$J_0(\mu_s b) = 0, \qquad \text{for } s = 1, 2, \cdots. \tag{13}$$

When L and b are much smaller than the length of a bunch, then the term $s = 1$, $p = 0$ dominates in the sum and

$$Z(\omega) \approx \frac{-8iv^2}{b^2L}\,\frac{\left[1 - \cos\frac{\omega L}{v}\right]}{J_1^{\,2}(\mu_1 b)\omega^2}\left[\frac{1}{\omega + c\mu_1} + \frac{1}{\omega - c\mu_1}\right] \tag{14}$$

where $\mu_1 b = 2.41$ and $J_1(\mu_1 b) = 0.52$. In the case in which the range of interesting frequency is such that $\omega \ll c\mu_1$, $Z(\omega)$ becomes

$$Z(\omega) \approx \frac{-8iL\omega}{c^2\left[\mu_1 b J_1(\mu_1 b)\right]^2}, \tag{15}$$

which is of the same functional form obtained by Robinson.[14] Note that the rf cavity at frequencies ω, less than its resonant frequency, is inductive which--above transition--leads to bunch lengthening.

The first study of the interaction of a beam with clearing electrodes was by Laslett.[19] It has been followed by many papers, including computations for general electrodes by Ruggiero, Strolin, and Vaccaro.[20] For the special case of a pickup electrode which extends around the full chamber, is of length ℓ, and is terminated in its characteristic impedance Z_0, the Green's function for a relativistic beam is

$$\mathcal{G}(\sigma) = \frac{Z_0}{2}\left[\delta\left(\frac{\sigma}{\beta c}\right) - \delta\left(\frac{\sigma}{\beta c} - \frac{2\ell}{c}\right)\right]. \qquad (16)$$

For electrodes short compared to bunch length, the interaction produces bunch lengthening.

II. Dynamic Self-Field Phenomena: The Coherent Longitudinal Instability of a Uniform Beam

It is in connection with dynamic self-field effects, which subject includes beam instabilities, that beam-surrounding interactions have been most closely studied and most clearly of great importance. I think you will agree that I am not exaggerating about the importance, when you recall Amman's report,[10] at the last meeting of this conference, that upon removing the clearing electrodes from the ADONE storage ring the coherent transverse instability threshold increased by a factor of 15!

I can not do justice, in the present review, to the extensive literature, but shall attempt to indicate the range and nature of the activity by considering one topic in detail; namely the longitudinal instability of azimuthally uniform beams.

Of course there has been significant work done on transverse coherent instabilities--much of it very beautiful work--but I must forego reviewing that material here, as I believe my goals are best accomplished by going--in depth--into the one subject.

In both proton storage rings and electron ring accelerators the maintenance of stability of the stored beam against longitudinal bunching is of great importance and--probably--the single most difficult feat to be accomplished. Since the last few years has seen considerable work devoted to high-current effects in the ISR at CERN and the ERA's in Dubna and Berkeley, there now exists a relatively highly developed theory on the subject under discussion.

2.1 Classical Period

In the very first papers on the negative mass instability[21,22] the threshold and growth rate were expressed in terms of a geometrical factor called g. Taking a "rectangular distribution" for the beam energy, the threshold is given by

$$N = \frac{\pi}{2}\left(\frac{R}{r_0}\right)\frac{\gamma}{g}\left(\frac{E}{\omega}\frac{d\omega}{dE}\right)_s\left(\frac{\Delta E}{E}\right)^2, \qquad (17)$$

where ΔE is the full energy spread in a beam of N particles circulating at (angular) frequency ω_s on an orbit of radius R with total energy E_s. The geometrical factor--which, in fact, describes the beam-surrounding coupling--was originally evaluated for a beam of minor radius a, inside a perfectly conducting tube of radius b as

$$g = \frac{1}{\gamma^2}\left[1 + 2\ \ell n(b/a)\right]. \qquad (18)$$

As long as ten years ago, a study was made of the azimuthal stability of a uniform beam passing through an externally unexcited rf cavity.[23] It was shown that the beam would be stable; i.e., not spontaneously bunch on the nth azimuthal mode, provided the shunt impedance Z_n of the cavity (at frequency $n\omega_s$) satisfied

$$\left|\frac{Z_n}{n}\right| < Z_0\, 2\pi\gamma\left(\frac{R}{r_0}\right)\frac{1}{N}\left(\frac{E}{\omega}\frac{d\omega}{dE}\right)_s\left(\frac{\Delta E}{E}\right)^2, \qquad (19)$$

where Z_0 is the impedance of free space (equal to 377 ohms in mks units, and $4\pi/c$ in cgs units).

2.2 Medieval Period

It has been seen that even in the earliest papers, beam-surrounding factors were isolated. However, in those days it was not appreciated how sensitively these factors depended upon the nature of the surroundings. It was the work of Briggs and Neil[24] which first emphasized the importance of wall materials, and even suggested walls which would dramatically increase the threshold of both transverse and longitudinal coherent instabilities. For example, they showed that a layer of thickness τ and dielectric constant ϵ inside a tube of radius b would have a geometrical factor

$$g = \frac{1}{\gamma^2}\left[1 + 2\ \ell n(b/a)\right] - \frac{2\kappa\tan\kappa\tau}{\epsilon\,\kappa^2\, b}, \qquad (20)$$

with

$$\kappa^2 = \frac{n^2}{R^2}\left(\beta^2\epsilon - 1\right). \qquad (21)$$

Since for a loss-less dielectric the negative mass instability is only present for positive g, one can see, from (20), that in this case--and especially for ultra-relativistic beams-- it is possible to eliminate the instability by means of a thin dielectric layer.

The first attempt to isolate--in a general way--the beam-surrounding interaction factor appeared in 1967, having been stimulated by the work of Ref. 24.[25] In this paper the beam dynamics (dispersion analysis) was done in general, and the stability limit expressed in terms of a beam coupling impedance Z_n. The impedance Z_n is defined by

$$2\pi R\, E_{Zn} = -Z_n I_n, \qquad (22)$$

where I_n is the nth Fourier component of beam current and E_{Zn} is the nth component of the azimuthal electric field at the beam. The result of careful beam dynamics yielded (19), but with a factor of (0.7/8) inserted on the right.

In Ref. 25, the electrodynamics was also done in general by splitting Z_n into two terms. One term describes the impedance of a beam inside a perfectly conducting pipe of radius b; namely, as in (18)--or (10)--

$$Z_{n1} = \frac{2\pi i n}{\beta c}\left(1/\gamma^2\right)\left[1 + 2\ \ell n\ b/a\right]. \qquad (23)$$

The other term Z_{n2} was shown to be related to the wall impedance per unit length $\tilde{Z}_n$ by

$$Z_{n2} = 2\pi R \tilde{Z}_n \left[\frac{1}{qb} \frac{J_1(qb)}{J_0(qb)}\right] , \qquad (24)$$

where $q = in/\gamma R$. In the limit of long wave-length perturbations it can be seen, from (24), that $Z_{n2} = 2\pi R\tilde{Z}_n$.

It was emphasized--and, in fact, it was the main point of Ref. 25--that $\tilde{Z}_n$ described the impedance of the wall elements and was, thus, amenable to computation--or measurement--by means of all the standard techniques employed in electrical engineering. For example, if the outer wall were resistive with conductivity σ, then clearly

$$\tilde{Z}_n = \frac{1 - i}{2\pi b \ \delta \ \sigma} , \qquad (25)$$

with the skin depth δ given by

$$\delta = c(2/4\pi\sigma \omega)^{1/2} , \qquad (26)$$

and $\omega = n\beta c/R$. Combining (23), (24), (25), and (26) yields--in the long wave-length limit--the well-known result[26]

$$Z_n = Z_{n1} + Z_{n2} = \frac{2\pi \ in}{\beta c \ \gamma^2} \ [1 + 2 \ \ell n \ b/a] + \frac{4\pi \ R}{cb} \mathcal{R}(1 - i), \qquad (27)$$

where

$$\mathcal{R} = (\omega/8\pi\sigma)^{1/2} .$$

This "engineering technique" was applied to a number of problems--such as helical conducting walls[23] --and allowed complicated structures to be readily analyzed. For example, the impedances presented in Section 1.3 may be employed to study the azimuthal stability of beams interacting with various elements such as pickup electrodes.

2.3 Modern Period

In 1969, Keil and Schnell[28] observed that "By combining formulae in various papers it is, however, possible to arrive at an expression which is very easily understood.", and proceeded to derive a formula which was identical--except for the numerical factor of (0.7/8)--with the long-since-forgotten formula (19). Their work was, however, important for it stimulated a large program at CERN of computation and measurement of the elements which were to go into the new proton storage ring (ISR).

As part of the ISR program, Ruggiero, Strolin, and Vaccaro presented a comprehensive theoretical investigation of clearing electrodes and pickup electrodes.[20] Their results are, in general, very complicated; one limiting case of their work has already been presented in (16).

The effect of laminated vacuum chamber walls (for example, layers of metalized ceramic inside an outer conductor) was comprehensively studed by Zotter,[29] whose work, subsequently, proved of great value to the ERA Group in Berkeley. Shortly thereafter, Keil and Zotter, in an impressive set of papers, evaluated the coupling impedance of corrugated vacuum chambers.[30]

On the experimental side, Schnell and Thorndahl, using an analogue model, measured the coupling impedance of the ISR clearing electrodes.[31]

The coupling impedance for the beam of an electron ring accelerator has been the subject of very extensive study, especially by the Dubna Group. Since a contribution on this very topic is to be presented in this session (Contribution M-18), no further remarks will be made.

Recently, Ruggiero has--evidently independently--rediscovered the approach of Ref. 25,[32] and then applied it to a number of problems. In particular, he has studied the effect of the NAL-Booster magnetic laminations by means of a radial transmission line model.[33] The same problem has also been studied by Snowdon[34] (and I think it was analyzed by Chirikov, in 1964, in connection with a plasma betatron, but I can not locate the reference). The work of Ruggiero[33] can be recommended as an excellent example of the application of the "engineering technique" to the solution of beam instability problems.

III. The Main Theme--Succinctly Stated

Emphasis has been directed in this review to the interaction of a beam with its surroundings. Concentration upon this particular subject has not meant to imply that deep understanding and insight is not also required into the other aspects of beam dynamics; in fact, one needs only think of the very beautiful work required for the development of the theory of the head-tail effect[35] to see how wrong such a view would be.

Rather, the point has been that for many phenomena the beam dynamics part can readily be done --and, in fact, has been done--and the resulting formulas involve factors which alone describe the beam-surrounding coupling. (See, for example, the factor F in (1) and (2); or the factor $Z(\omega)$ which determines bunch length via (3) through (8); or the coupling impedance Z_n of (19) and (27).) For such phenomena the "weakest link" in the theory is the beam-surrounding coupling factor, which factor it is worth isolating.

One reason for the importance of isolating the beam-surrounding coupling factors is that they are often extremely difficult to calculate in that they depend sensitively on the beam surroundings. Once isolated, however, they may be measured or intuitively estimated. I especially look forward to a much greater application to this subject--in the future--of an engineering approach, which should result in the development of a suitable highly-convenient phenomenology.

More importantly, these factors can influence beam behavior--for example, instability thresholds--in a very major way; in fact, usually more so than any other factor under the machine designer's control. Once isolated, however, the beam-surrounding coupling factors can be included in the over-all optimization of the design.

Finally, it is necessary to remark that in recent years much attention has been devoted--on the theoretical side--to beam-surrounding interactions: good and bad geometries have been identified, and devices have been designed on the basis of these theories. There is, however, a great paucity of experimental confirmation of these calculations. One hopes that the next few years will see a change in this situation, and consequently the development of a substantial base upon which particle handling devices of ever greater performance capabilities may be confidently designed, successfully constructed, and effectively employed.

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