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Statistical Sampling Plans for Prior Measurement Verification and Determination of the SNM Content of Inventories

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Pacific Northwest Laboratory
Operated by
Battelle Memorial Institute

Prepared for
U.S. Nuclear Regulatory Commission

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ABSTRACT

Current regulations require that prior information on the Special Nuclear Material (SNM) content of a population of containers be verified and that periodic measurements of the SNM inventory of a facility be performed. This report develops and describes statistical sampling plans for accomplishing these tasks and compares results obtained by sampling to those obtained by the current practice of performing a census (100% sampling).

EXECUTIVE SUMMARY

This report develops and describes statistical sampling plans for verifying prior information on the Special Nuclear Material (SNM) content of a facility and for estimating the SNM inventory of a facility. All SNM is assumed to be held in discrete, accessible containers. The current rule and practice is to verify or estimate the SNM content of an inventory using all containers in a facility.

Section 1 includes a general discussion of the steps in developing a statistical sampling plan and introduces several basic sampling techniques. Sections 2 and 3 consider inventory verification sampling plans, while Sections 4 and 5 consider inventory determination sampling plans. Section 6 compares inventory determination and verification results for sampling versus those from a complete census (100% sampling).

The major contributions of this report are: 1) the presentation of several sampling plans (and associated formulas) for the verification and determination of the SNM inventory of a facility, and 2) the illustration of the use and relative usefulness of these sampling plans through examples. Examining these contributions, the following general conclusions are reached:

- An inventory determination or verification is subject to error (measurement, transcription, other human error) even when a complete census is performed. Sampling may reduce the impact of these errors since fewer containers are involved; however unless these errors are many or are relatively large, this reduction due to sampling will probably be small relative to the additional error caused by basing decisions or estimates on only a portion of the containers in the population.
- Although a complete census of a population of containers will provide the most accurate results, statistical sampling can reduce the verification or determination effort considerably with only a minor reduction in accuracy. For inventory determination, "accuracy" refers to the closeness of the inventory estimate to the unknown true value. The investigator can control accuracy (through sample size) by specifying the maximum difference

between the estimate and the true value, and the probability that the maximum difference condition will be met. For inventory verification, "accuracy" refers to correctly detecting or not detecting a specified loss of SNM. The investigator can control accuracy by specifying the loss detection goal and the probabilities of incorrectly detecting or not detecting the specified loss of SNM.

- As expected, sampling compares more favorably (increased accuracy of inventory determination and verification results) with a complete census as the sampling fraction increases. If the frequency and magnitude of measurement and other errors are small, sampling may compare quite favorably to a complete census even for sampling fractions considerably below 100%. In these situations, diminishing returns in accuracy (for inventory determination) and probability of detection (for inventory verification) are gained by increasing the sampling fraction beyond a certain level.
- When inefficient measurement and detection techniques are used, more inaccuracy (for inventory determination) and lower probabilities of detection (for inventory verification) result for both sampling and census methods. In these situations sampling compares poorly with census results unless the sampling fraction is near 100%.
- For inventory verification, sampling compares quite well with a complete census when the number of defective containers in the population is large (assuming efficient detection techniques are used). Probabilities of not detecting a specified loss can be quite small even with sampling fractions well below 100%.
- For inventory determination, sampling can significantly reduce the effort required from census determination when accuracy requirements on the inventory estimate are not too restrictive. How restrictive this is will depend upon the particular situation. If the accuracy requirement is too restrictive, the sampling fraction will approach 100%.

Several comments are made in the report concerning the relative merits of specific sampling plans. Most of these hold true, whether the sampling plan is designed for inventory verification or determination. Some of the more important are:

- If a facility contains several types of SNM or if for any one type, containers can be split into relatively homogeneous groups, then stratification should be incorporated into the sampling plan.
- Simple random sampling is easy to perform, but can involve excessive container location times. Cluster sampling techniques can reduce the time and cost of locating individual containers, but these techniques often require more containers to be sampled to retain the same level of accuracy or detection capability.
- Simple random sampling assumes each container or cluster of containers in a facility is equally important. If this is not the case, probability proportional to size (PPS) sampling techniques should be incorporated into the sampling plan, where size might be measured by the quantity of SNM in a container.
- For inventory verification applications, sequential sampling techniques can be incorporated into sampling plans based on any of the other sampling techniques (simple random, cluster, stratified, PPS). Curtailed sequential sampling techniques assume a decision (on whether a loss has occurred) must be made upon reaching a preselected maximum number of sampled containers, if not sooner.
- Allocation by one technique given a sample size developed using another technique is not generally recommended, since the accuracy or probability of detection obtained may differ drastically from the values specified in developing the sample size.

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STATISTICAL SAMPLING PLANS FOR PRIOR MEASUREMENT VERIFICATION AND DETERMINATION OF THE SNM CONTENT OF INVENTORIES

1.0 INTRODUCTION

1.1 OBJECTIVE OF THE STUDY

The purpose of this study is to develop and describe statistical sampling plans for verifying prior information on the Special Nuclear Material (SNM) content of containers and for estimating the SNM inventory of a facility. The current rule and practice is to verify prior measurement data for all unsealed items and check the seal integrity of all tamper-safed items at the time of a facility-wide physical inventory. In current practice the physical inventory of a facility consists of a total census, **i.e.,** a 100% inventory.

1.2 REGULATORY BACKGROUND

The determination of the SNM content of an inventory and the verification of prior location and content information for SNM inventories are requirements of the Code of Federal Regulations, Title 10, Part 70, Section 70.51 (CFR, 1981). Particular requirements include:

70.51 (f)(2)

"Establish inventory procedures for sealed sources and containers or vaults containing special nuclear material that provide for:

(i) Identification and location of all such items;

(ii) Verification of the integrity of the tamper-safing devices for such items;

(iii) Reverification of identity and quantity of contained special nuclear material for each item not tamper-safed, or whose tamper-safing is found to have been compromised;

(iv) Verification of the correctness of the inventory records of identity and location for all such items."

70.51 (f)(3)

"Establish inventory procedures for special nuclear material in process that provide for:

(i) Measurement of all quantities not previously measured by the licensee for element and fissile isotope; and

(ii) For all material whose content of element and fissile isotope has been previously measured by the licensee but for which the validity of such previously made measurements has not been assured by tamper-safing, verification of the quantity of contained element and fissile isotope by remeasurement."

70.51 (f)(4)

"Conduct physical inventories according to written inventory instructions..."

Future amendments to 10 CFR 70 can be expected to revise the requirements for verification and determination of inventories of SNM. The Material Control and Accounting (MC&A) Task Force (NRC, 1978) recommended SSNM[(a)] control and accounting goals pertaining to inventory verification and determination. Amendments (Federal Register, September 1981) based on some of these goals are being considered by the NRC. Some of the MC&A Task Force goals are:

MA 6

"Detect, with high assurance, based upon a periodic measured physical inventory, a loss of five formula kilograms of SSNM from a facility, or, if not achievable for an entire facility, from smaller accounting units comprising the entire facility. Establish, for the latter case, controls to preclude theft by the same adversary from two or more accounting units and to preclude falsification of records of more than one accounting unit by any individual having access to material."

(a) SSNM refers to the isotope uranium-235 (contained in uranium enriched to 20% or more in the uranium-235 isotope), the isotope uranium-233, or plutonium.

MA 14

"Provide for bimonthly physical inventories, based on measurements, to provide a highly reliable record of quantities and locations of all SSNM at a facility. Reconcile and adjust book inventories to the results of the physical inventory within 30 days from its beginning."

MC 8

"Detect within one shift, with high assurance, a loss of five formula kilograms of SSNM in the form of items or sealed containers accessible to theft. Detect within 24 hours, with high assurance, a loss of five formula kilograms of SSNM in bulk form accessible to theft from any controllable unit of a facility."

MC 10

"Detect, with high assurance, the cumulative loss of five formula kilograms of SSNM from any controllable unit of a facility within the interval between physical inventories."

MC 13

"Based upon a statistical sampling plan designed for the detection with high assurance of a composite loss of five formula kilograms of SSNM, confirm during each shift the specific location and the integrity of items or sealed containers of SSNM accessible to theft."

MC 14

"Based upon procedures and analyses designed for detection with high assurance of a composite loss of five formula kilograms of SSNM in bulk form from any controllable unit of a facility, confirm each day the presence of SSNM accessible to theft."

Rules aimed at achieving these goals would require much more frequent inventory estimates and tests of material status than is now required. They would also require specific detection probability and detection quantity goals in any tests of material status by statistical sampling methods.

1.3 BASIC SAMPLING TECHNIQUES

In the following sections of this report various sampling plans will be discussed. It will be helpful to briefly define and explain several basic sampling techniques upon which later discussions are based.

1. Simple Random Sampling - A procedure in which each element of the population has an equal chance of being selected on any one selection. Elements may be chosen from the population with or without replacement.

2. Systematic Sampling - A procedure in which the first element Is randomly selected from among the first k elements of the population with every k^{th} element following the first selected subsequently.

3. Stratified Sampling - A procedure in which a population is divided into mutually exclusive and exhaustive strata and a sample is selected from each strata.

4. Cluster Sampling - A procedure in which groups (clusters) of population elements are selected instead of individual elements. Multistage cluster sampling involves the sampling of subclusters or individual elements from within clusters or other subclusters.

5. Probability Proportional to Size (PPS) Sampling - A procedure in which population elements or clusters of elements are chosen with probabilities of selection proportional to size. "Size" might be a measurable characteristic of a population element or a count of the number of elements within a cluster.

6. Sequential Sampling - An iterative procedure applied after the selection of a sample point which decides to either stop sampling or to choose another sample point based on a predetermined decision rule. Sequential sampling may include or be composed of any of the other sampling techniques discussed above.

7. Curtailed Sequential Sampling - A special case of sequential sampling where a maximum sample size is specified. Sampling cannot proceed indefinitely as in unconstrained sequential sampling.

Each sampling technique discussed above should be considered as a basic or "building block" technique. Developing a sampling plan involves considering the strengths and weaknesses of basic sampling techniques and then combining the techniques into a plan best suited to the problem and its underlying constraints. The strengths and weaknesses of the basic sampling techniques are given in Table 1.1.

Several of the comments in Table 1.1 concern the estimation of parameters based on data collected by the given sampling technique. These comments refer to situations where estimates of parameters, e.g., quantity and variance of SNM content, are required, such as in determining a total inventory or in verification by variables testing. Comments concerning the feasibility of and the effort required for a specific sampling technique apply to samples required for both inventory determination and attribute or variables verification testing.

1.4 DEVELOPING A SAMPLING PLAN

Developing a sampling plan involves several related steps. The major steps are presented below with explanations of each, how they relate to each other, and how they combine to form a sampling plan.

1. Specify Population. The population of interest must be defined. The characteristics of elements in the population must be known. A list and the number of population elements should be available.

2. State Reason for Data Collection. The reason for collecting data is an important factor in developing a sampling plan. This study considers two such reasons: 1) prior measurement verification and 2) determination of the SNM content of an inventory. The latter reason will require that quantitative estimates of SNM and variances of these estimates be developed While this will also be true for some of the verification work, other parts will only require a qualitative estimate or decision.

3. State Objectives. In verification work, the objective is to detect a loss of SNM. If different groups of SNM exist in a facility, is the objective

TABLE 1.1. Strengths and Weaknesses of Basic Sampling Techniques

Sampling Technique	Strengths	Weaknesses
1. Simple Random	• Easy to choose samples. • Unbiased sample statistics,	• Requires all population elements be identified and labeled prior to sampling. • Can be time consuming/expensive. • May not adequately represent important subgroups that comprise a small proportion of the population.
2. Systematic	• Easy to choose samples. • Does not require knowledge of total number of population elements.	• Requires ordering and labeling. • Simplifying assumptions required to estimate parameters. • May produce biased parameter estimates.
3. Stratified	• Includes elements from each stratum to insure sample is representative of the population. • Allows separate decisions or parameter estimates for each stratum as well as for total population. • Increased precision (smaller standard errors) under certain conditions. • Stratification can be applied after the sample is collected using another sampling technique.	• Strata should be relatively homogeneous. • Time consuming to identify sample elements. • More sample elements may be required than for other techniques.
4. Cluster	• Does not require a list of all population elements to select sample units. • Saves time/cost. • Multistage clustering for large populations with naturally occurring hierarchical structure.	• May ignore certain groups of elements of interest. • May yield large standard errors for some parameter estimates if the elements in the clusters are homogeneous.
5. Probability Proportional to Size	• Allows sampling according to some measure of importance to insure coverage of the areas of interest in the population.	• Biased parameter estimates are possible. • Highly dependent upon measure of importance used.
6. Sequential Sampling	• Smaller sample sizes (usually).	• Not applicable in all situations. • Requires involved calculations after acquiring each sample point. • Sampling can continue indefinitely. • The average sample size required to reach a decision depends upon the decision rule (statistical test) used.
7. Curtailed Sequential Sampling	• Maximum sample size is specified. Cannot continue indefinitely as can unconstrained sequential sampling. • Smaller sample sizes.	• Not applicable in all situations • Requires involved calculations after acquiring each sample point.

to detect a certain loss in each group or only in the overall inventory? Information relating to the objective such as loss detection goal, probability of detection, and probability of a false alarm (Type I error) should also be specified.

In inventory determination work, the objective is to estimate the quantity of SNM in a facility. If different groups of SNM exist, is the objective to estimate the amount in each group or only in the total facility? Information relating to the objective such as the accuracy required of the estimate and the probability the accuracy is achieved should be specified.

4. Identify Statistical Tools. The statistical tests or estimation procedures required to satisfy the objective should be specified. The determination of sample size is highly dependent on this step.

5. Determine Preliminary Sampling Technique. The objectives and statistical tests or estimation procedures may indicate a preliminary sampling technique should be employed. A preliminary technique may affect sample size determination. Stratification and cluster sampling are examples.

6. Determine Sample Size. This will depend on all of the above steps. For sampling techniques such as sequential or curtailed sampling, the sample size may be variable and dependent upon intermediate results.

7. Allocate Sample Size. Once the sample size (or sizes) is chosen, additional sampling techniques must be applied to allocate the sample size to various portions of the population (e.g., strata and clusters).

8. Select Sample. The specific units (items, containers, or batches) must be selected according to the allocation and additional sampling techniques chosen in step 7. (Following selection, the measurements that are made may involve sampling in a different sense, i.e., bulk material sampling to obtain a representative sample of the contents of an item for a measurement of the composition.)

In the following sections, various sampling plans for verification and inventory determination will be considered. Several of the above steps are similar for all. Individual sampling plans will basically be characterized in terms of steps 4-8, although the other items will be discussed as necessary.

2.0 VERIFICATION OF PRIOR INFORMATION CONCERNING A POPULATION OF CONTAINERS

2.1 AN OUTLINE OF THE VERIFICATION PROCESS

Given that a nuclear materials accounting system is in place at a facility, it is desirable to verify prior information concerning the population of SNM containers both as a periodic test of item controls and as a means to redetermine the inventory of a facility. A verification plan is developed by considering the ways a diverter could take a specified amount of SNM. There are two major diversion possibilities: 1) the diverter attempts to cover up the diversion by falsifying data or by substituting other material for the SNM, or 2) the diverter does not falsify the data or make substitutions but relies on the inherent noise level of the system to obscure the diversion. For both possibilities, diversion can range from partial to complete removal of the contents of a container.

The possible diversion strategies must be taken into account when planning a verification procedure. In this respect, the first consideration is whether a diversion from a container is large or small.

- A large diversion is one that can be detected with certainty by a simple observation or single measurement.
- A small diversion cannot be detected with certainty by a simple observation or a single measurement.

Large diversions include removals of total items, the total contents of containers and part of the contents of containers. In the case of partial removals, the quantity removed from each container is sufficient to be detected with certainty with the measurement device chosen for verification. (The measurement can be referred to as an attribute-type measurement.) Large diversions include removal of tamper-safed items. However, if the container has been unsealed and part of the contents removed, the quantity diverted may be either large or small.

Small diversions are partial removals from containers, where detection by the measurement device chosen for verification is not certain in a single item measurement. The diverter's strategy is to remove small quantities from many containers. The detection strategy is to 1) measure the contents of many containers and detect the cumulative deficiency of SNM, and 2) use a measurement method that is sufficiently precise to detect with certainty any significant total diversion. A summary of the diversion possibilities and some verification strategies is given in Table 2.1.[a] The diversion strategy of falsifying data is not included because inventory verification techniaues by thernselves do not detect diversion when the data, i.e., the inventory records or accounts, have been falsified.[b]

2.2 CHARACTERISTICS OF A POPULATION OF CONTAINERS

The number and variety of items or containers of SNM in a nuclear processing facility vary considerably between facilities of any one type and size. The range in inventory size of all licensed production facilities may be an order of magnitude. Containers of fuel rods, pins or elements constitute the largest number of items in fuel fabrication plants. Other materials are feed and intermediate product material, scrap, and waste in containers of various sizes. Table 2.2 shows some characteristics of plant inventories. These inventories do not represent any particular facility but are reasonable for operating plants of the type and size commercially feasible at this time.

The composition of the day-to-day inventory of items in a facility differs from that at the time of a scheduled plant-wide physical inventory. The clean-out activities that precede a scheduled inventory produce many containers of waste, scrap and intermediate products that would normally be in bulk form in

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(a) Not all of the verification strategies need to be used because the more complete and sensitive measurements detect diversions of lesser sophistication as well as those for which the strategy was designed. For example, if material substitution is feasible, the no-substitution strategy need not be considered because tests for removals with substitution cover both strategies.

(b) Records and accounting internal controls and records security, records auditing and frequent or routine cross checking and independent verifications of source data are means of preventing and/or detecting data falsification. These methods are also useful for minimizing innocently caused errors in the accounting records.

TABLE 2.1. Diversion and Verification Strategies

Diversion Strategy[a]			Verification Strategy
Removal		Cover-up Method	Test and Measurement Methods
1.	Entire item (or items) or entire contents	None	Item location and identity check
		Replace with empty container	Weight check or SNM test, e.g., NDA
		Replace with inert material	Test for SNM, e.g., NDA
2.	Partial contents of one or more items	None	Weight check or SNM assay[b]
3.	Partial contents of one or more items (large quantity)[c]	Replace with inert material	Measurement for SNM defect[b]
4.	Partial contents of many items (small quantities)	None	Weight check or precise assay sensititve to small defects[d]
		Replace with inert material	Precision SNM determination (sensitive NDA or chemical analysis)[e]

(a) Listed in the order of increasing sophistication of the diversion strategy.
(b) The assay test must be able to detect the defect with certainty (e.g., a 100 g removal would be detected with a NDA method having a standard deviation of 25 g or less.
(c) The quantity removed is sufficient to be detected with certainty by the test or measurement used.
(d) A defect in net weight or SNM content is detected by testing the sum or average of the sample set.
(e) The NDA measurement detects the defect in SNM content or the chemical analysis detects either the defect in percent SNM or the presence of the additive. Sums of SNM defects in the inventory are generally required. (A measurement bias may be indistinguishable from a diversion.)

TABLE 2.2. Some Characteristics of Fuel Fabrication Plant Inventories

Materials	Form of Containment LEU[a]	HEU[b]	Typical Quantity of SNM/Container LEU, kg	HEU, kg	Number of Containers
UF_6 Feed	Cylinders	Cylinders	~1400	~10-15	10-50
UF_6 Residues	Cylinders	Cylinders	1-10	a.l	10-50
Feed Oxides	Orums, 5-15 gallons	Cans, 1-39.	15-30	1-3	100-500
Intermediate Product	Cans up to 5 gallons	up to 1 gallon	5-10	1-3	100-500
Pellets	Trays	Trays	5-10	1-3	500-5000
Scrap Powder	Cans up to 5 gallons	1-3R	5-15	1-3	50-200
Scrap Pellets	Cans, ~1 gallon	1-2 gallons	5-15	1-3	50-200
Sludges	Cans, ~1 gallon	1-2 gallons	5-15	1-3	25-100
Cleanup Powders	Cans, ~1 gallon	1-311	1-15	up to 2	10-50
Filter Cake	Cans, ~1 gallon	1-2 gallons	5-15	up to 2	25-100
Air Filters	Cartons, ~1 cubic meter	Cartons and 5 gallon cans	up to 3	up to 0.2	10-50
Filter Media	Cans up to 5 gallons	5 gallon cans	up to 3	up to 0.2	25-100
Samples	Bottles and cans, 1 gallon	1-2 gallons	up to 2	up to 0.5	up to 500[c]
Solid Waste	Drums and cartons, up to 55 gallons	Orums and cartons	up to 5	<0.1	up to 500
Liquid Waste	Tanks	Tanks	up to 2	<0.2	5-10
Fuel Rods or Plates	Zr alloy clad items	Zr alloy clad items	2-4	0.1 to 0.2	2000-5000
Scrap Solutions	Tanks up to 50011	up to 100ℓ	up to 50	up to 10	10-20 (HEU:10-50)

(a) LEU: low enriched uranium.
(b) HEU: high enriched uranium.
(c) Assume retention samples in sealed cans at up to 50 per can.

temporary storage or in processing equipment at other times. However, sampling plans for inventory verification will not be significantly affected by these differences because products and feed stocks, which generally constitute over 80% of the items, are not affected by shutdown and cleanout schedules. However, some verification procedures, especially sampling and assays, are affected by the operating status. If inventory verification is to be made frequently (more than once a month), shutdown and cleanout of the process equipment will probably not be economically feasible. In that case, verification of in-process quantities of SNM will require determination of quantities held in equipment, distributed in processing stations as spillage, and in heterogeneous forms. This will result in much greater variances than would be the case if the material were containerized and directly weighable.

Most of the inventory is located in vaults, vault-type rooms, and secure storerooms. The containers are stored on open shelves and pallets or in cabinets, cubicles and bins. Usually some items are in the processing lines as material awaiting processing or in intermediate products being held in temporary in-line storage. In batch plutonium processing such as conversion and fabrication, considerable buffer storage for intermediates is provided within the process lines to minimize transfers into and out of the process glove boxes and to decrease the risk of contamination by plutonium. The largest category of material in items outside vaults are fuel rods, pins, plates or elements that are in the finishing, testing and inspection stages of fuel fabrication. Items in this category are moved frequently from one machine or test station to another, and their location at any time may be traceable, at best, to a particular table, cart, or movable rack.

Storerooms, vaults and other item control areas (ICAs) generally segregate types of material to some degree. Separate storage is likely to be provided for sealed containers of waste, scrap and finished products. Dry waste is commonly packaged in barrels and stored on pallets in a separate storage building or fenced yard.

Cans, barrels, jars, and bottles of SSNM may have permanent markings identifying the type of material and a container number. Glued-on labels are usually used for the specific data such as the item number (if item numbers are assigned), type and quantity of contents, lot and batch numbers, dates, and other information that the licensee desires. In addition, tamper indicating seals have a unique number, while paper seals sometimes also have space for writing in other information, i.e., seals may also serve as labels. The commonly used seals are self-adhesive paper, wire seals, such as E-seals, and band seals. Two per container are needed on some types of closures. Identification of an item for inventory verification purposes would be by seal number(s) or by item number. Fuel rods, pins, elements and assemblies, and sealed sources of other kinds are fabricated as sealed items. They have unique item numbers stamped or embossed on them so additional seals or labels are usually not applied. Additional information (such as item content, lot number, etc.), which is available in the item records file, is not needed for the verification of such tamper-safed items.

As required by the material control and accounting regulations as well as by production control demands, licensees keep detailed records of each item. This information does not necessarily identify an item's location more specifically than by bin, box, shelf, or pallet location. In the case of items in a processing or inspection area, the records may identify a general processing or inspection area, room, or rack, but the specific item distributions within racks, bins, boxes, and shelves are often random. Fuel rods and elements are usually stored in sets, such as batches or lots, and the number per storage unit may be up to 100. Shelves of cans up to about one gallon usually have less than 10 each. If the storage unit contains more than a few (5 to 10) items, the searching time for a specific item can be a substantial part of the total time required for verification.

The status of items in the inventory of a facility may change in several ways; for example:

- new items are created, such as by receiving feed material, generating waste and scrap, and forming fuel elements
- items are removed from inventory, such as by disposing of waste and shipping fuel elements
- items are moved to and from an ICA, to a process Material Balance Area (MBA), to laboratories and between storage locations
- items such as fuel rods are moved from step to step in an inspection and finishing process.

These activities occur continuously. In large fabrication and recovery processes, hundreds of item location changes can occur each 24 hour period.

2.3 THE VERIFICATION SENSITIVITY GOAL

Here, the goals for verification of prior measurements are a high power of detecting a loss of 5 kg of ^{235}U in high enriched uranium and 75 kg of ^{235}U in low enriched uranium. These detection goals are similar to the goals recommended by the MC&A Task Force (NRC 1978), quoted in Section 1.0. A "high power" of detection will be defined in this study as equal to or greater than 90 percent probability of detection. Values of 90, 95, and 99 percent will be investigated in terms of impact on sample sizes required to meet the detection goal.

3.0 VERIFICATION SAMPLING PLANS

The kind of sampling plan chosen for prior measurement verification will depend upon the type of potential diversion to be protected against. As discussed in Section 2.1:

1. Large Diversions are diversions large enough to be detected by a simple observation or a single item measurement.
2. Small Diversions are diversions too small to be detected by a simple observation or a single item measurement.

A sampling plan developed to detect large diversions will be referred to as an attributes sampling plan. A plan developed to detect small diversions will be referred to as a variables sampling plan. The sample size and allocation of sample items over the population (using sampling techniques discussed in Section 1.3) for each type of sampling plan will depend on the statistical techniques employed to detect the various potential diversions.

3.1 ATTRIBUTES SAMPLING PLANS

The first level of the attributes verification effort investigates whether a goal quantity G of SNM has been removed from a facility by one or more large diversions from individual containers. Since large diversions are detectable by a simple observation or a single measurement, the underlying statistical test is quite simple. A container either has the attribute of being defective or not being defective, where here, defective means involving a large diversion. Sample sizes will be developed for these attributes (yes-no) tests based on the Bernoulli class of probability distributions.

Hough, Schneider, Stewart, et al. (1974) presented a procedure for selecting attribute sample sizes if the population of containers can be classified into strata on the basis of type of SNM and uniform SNM content. Stratification has favorable sampling properties (see Table 1.1); stratified attribute sampling plans will be considered (Sections 3.1.1 and 3.1.2). However, in some situations (see Good, Griffith, and Hamlin 1978 and Hamlin 1981) stratification may not be appropriate. Certainly, if a facility contains different types of SNM it will usually be of interest to stratify and separately verify information on each type. However, once the scope is reduced to a single type of SNM, it may not be possible to efficiently stratify using other variables such as the amount of SNM per container or the number of containers per cluster. These variables may vary so widely that stratification would result in a very large number of strata and essentially 100% sampling of the population. Sampling plans for such situations will be considered in Sections 3.1.3 and 3.1.4.

3.1.1 Stratified Attribute Sampling Plans

It is assumed that the population of containers in a facility is stratified according to type and content of SNM. The containers in each stratum are assumed to have a uniform SNM content within a reasonable limit, such as ±25%. Recall that we want to detect whether a goal quantity G of SNM has been removed from a facility. The sampling plan is developed from the stratum viewpoint on the premise that the entire goal quantity G could be removed from any one stratum.

The attribute sampling plan within each stratum will be designed to have a high probability (greater than or equal to 90%) of including at least one defect when the number of large defects are such that the total amount removed in a stratum is G. An attribute sample size is determined by the detection goal (G), the amount of material per container, and the risk of not including at least one defect in the sample. The following symbols will be utilized:

n = sample size for stratum,

N = total number of containers in stratum,

β = probability of not obtaining at least one defect (out of d) in the sample,[(a)]

d = $G/\gamma A$ = number of containers needed to obtain G units of SNM by removing γA units from each (if $G/\gamma A$ is not an integer, round to the next highest integer),

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(a) β may also be interpreted as the probability of nondetection of at least one defect in the sample. This assumes the probability of identifying a defect, given it is in the sample, is one. See Section 6.1 for more on this subject.

G = goal quantity,

A = amount of SNM in each container in stratum,

γ = fraction of A to be removed from each container.

Sample Size Determination - Zero Defects

In attribute testing, it is usually assumed that a known defect is detected with 100% certainty (a container is determined to be either defective or not defective). A distribution from the Bernoulli class of statistical distributions will provide the basis for determining the sample size. Since sampling will be performed on the containers within a stratum without replacement, the hypergeometric distribution is used to calculate a sample size [Jaech, 1973; Sherr, 1972]:

$$n = \left(N - \frac{d-1}{2}\right)\left(1 - \beta^{\frac{1}{d}}\right). \tag{3.1}$$

Often a conservative approximation

$$n = N\left(1 - \beta^{\frac{1}{d}}\right) \tag{3.2}$$

based on the binomial distribution (sampling with replacement) is substituted [Hough, Schneider, Stewart, et al., 1974]. This sample size is conservative because it is larger than that based on the hypergeometric distribution.

In practice, there is often a need to detect very large defects quickly. Non-destructive instruments are often used to do this. They are not as sensitive as destructive techniques but do provide a timely response. Those large defects big enough to be detected immediately will be referred to as gross defects.

If immediate detection of gross defects is required, a sampling plan and sample size separate from the overall sampling plan to detect large diversions are required. If the timeliness of gross defect detection is not required, only one sampling plan and sample size for detecting large defects is required (because destructive techniques will detect any large defect).

Assume a large defect is detectable by a measurement device which is sensitive to removals from a container larger than $\gamma_0 A$ (where γ_0 is a fraction of the amount A in a container). Assume a gross defect is detectable by a measurement device which is sensitive to removals from a container larger than $\gamma_1 A$ (where $\gamma_1 > \gamma_0$ is a fraction of the amount A in a container).

It is clear that attribute sample sizes to detect large (including gross) defects should be chosen to combat a diverter's optimal strategy. Since by definition a large defect can be detected with certainty by a simple observation or a single measurement, the diverter should minimize the number of removals made within a stratum. Equivalently, the fraction γ removed from each container should be maximized. For gross defects, this implies $\gamma = 1$ and $\gamma = \gamma_1$ for all large defects. The specific value for γ_1 will depend on the sensitivity of the attributes measurements device to be employed. Hough, Schneider, Stewart, et al. (1974) suggest:

$$\gamma_1 = \frac{4\sigma_T}{A}$$

where σ_T is the combined systematic and random error standard deviation of the attributes measurement instrument and A is the amount of SNM in a container.

The amount A of SNM in a container plays an important role in sample size calculation. Hough, Schneider, Stewart, et al. (1974) suggest choosing A to be the average amount of SNM for all containers in a stratum, where it is assumed all containers are within ±25% of this average. More conservative (larger) sample sizes are obtained from formulas (3.1) and (3.2) by choosing A to be the largest amount in any container in the stratum.

The sample size formulas (3.1) and (3.2) are based on an acceptance number of zero,[a] which means that one defect in the sample is sufficient to indicate

(a) This basis will also be referred to as "zero defect level".

a diversion of the goal quantity G. Such a sampling scheme controls the risk of making one of the two types of statistical errors--the error of concluding everything is in order when in fact something is wrong. This is the β in formulas (3.1) and (3.2) and will be equal to or less than 10% for this report. In statistical quality control, this risk has many names; some of the more common are beta error, type II error and acceptance quality level (AQL). The second type of statistical error is that of concluding something is wrong when in fact everything is in order. Names used for this type of error are alpha error, type I error, and rejection quality level (RQL). In choosing a zero defect level, we ignore the effects of this type of error. Emphasis is placed only on the power of the test $(1-\beta)$ and sample sizes are minimized as a result. In most SNM accounting systems, even a single gross or partial defect is not acceptable. If an error is made by identifying a nonexistent defect, it can be reconciled and sampling may continue. However, if the time or expense of reconciliation is excessive, a plan which allows sampling to immediately continue may be of interest.

Sample Size Determination - One or More Defects

Sampling schemes that allow one to control the alpha error as well as the beta error are widely used in industrial quality control. These are also based on hypergeometric or binomial distributions. The sample size calculations are quite complex, and hence published tables are available (Sherr, 1972; Mil Std-105D, 1963). If the tables do not cover a particular case of interest, approximation formulas are available (Jaech 1973, pp. 320-321).

Table 3.1 illustrates the effect on sample size of controlling the type I (alpha) error. Sample sizes for zero defect (acceptance number = 0) were calculated using formula (3.1)

$$n = \left(N - \frac{d-1}{2}\right)\left(1 - \beta^{\frac{1}{d}}\right)$$

with $\beta = 0.10$ and $d = 5$. Non-integer results are rounded up to the next integer. The table values taken from Sherr (1972) are based on the hypergeometric distribution with $a \leq 0.05$, $\beta = 0.10$ and the detection goal of five defectives in each

TABLE 3.1. Sample Size Required for Detection of Five Defects at 90% Probability

Population Size, N	Sample Size Calculated for Acceptance Number = 0 (a not specifiable)	Sample Size Table Values[a] for Acceptance Number = 1 ($\alpha \leq 5\%$)
50	18	28
100	37	58

(a) Values for N = 50, 100 from Sherr (1972), Tables VI.A and XIV.B respectively.

of the populations. At these population sizes, one defect in the sample is acceptable but two defects are not. Note that the sample sizes required to achieve a limitation of $a \leq 5\%$ are considerably larger than those from the zero defect formula.

The probabilities of a type I error (a) for sample sizes based on a zero acceptance number can be calculated using the formula[a] given by Jaech (1973, p. 321):

$$\alpha = 1 - \left(1 - \frac{2n}{2N - D + 1}\right)^D$$

where D is the number of defects in the population. An illustration for several values of D (using the example of Table 3.1) is given in Table 3.2. The a values are

$$a = P[\text{rejecting } H_0 \text{ in favor of } H_A \text{ given } H_0 \text{ is true}]$$

where H_0: Number defective = D
H_A: Number defective = 5

(a) Based on the hypergeometric distribution.

TABLE 3.2. Probabilities of Type I Error (a) for Acceptance Numbers of 0 and 1

Acceptance Number = 0

N	n	a for D = 1	2	3	4
50	18	.36	.60	.75	.84
100	37	.37	.61	.75	.85

Acceptance Number = 1

N	n	a for D = 1	2	3	4
50	28	0	.31	.59	.78
100	58	0	.33	.62	.80

Although the results in Table 3.2 are only an example, increasing the acceptance number of the sampling plan will generally decrease a through increased sample sizes. The increase in sample size can be substantial and hence the greater sampling costs must be compared to the resultant costs associated with investigating a type I error. In SNM accounting, any confirmed defects, even if only a fraction of the detection goal, are usually not acceptable. Since we assume large defects within the sample are detected with 100% certainty, the zero defect (acceptance number = 0) sampling plan should usually be chosen.

When comparing zero acceptance sampling to sampling with higher acceptance numbers, one additional point should be considered. Appendix A.l shows, that for zero acceptance sampling, the probability of detecting diversion by the strategy of removing part of the goal quantity from each of several strata is at least as high as detecting the diversion of the entire goal quantity from a single stratum. This is not necessarily true for sampling schemes with acceptance numbers of one or higher. Thus, zero acceptance sampling better combats the diversion strategy of taking a portion of the goal quantity G from each of several strata.

Allocation of Sample

Referring to Section 1.4, steps 3-6 of developing a sampling plan have been discussed for stratified attribute sampling. Options on the choice of distribution (hypergeometric or binomial), acceptance number (0, 1, or more), and container content (average container content or largest content) and effects on sample size were discussed. The next step in developing the sampling plan is to allocate the sample size n within a stratum.

The sample size formulas previously discussed are based on the binomial or hypergeometric distribution, and thus are inherently based on simple random sampling. Simple random sampling may be quite time consuming and expensive as discussed below.

Most of a facility's inventory may be located in storerooms, where containers are stored on shelves or pallets, or in cabinets, cubicles or bins. An important characteristic of SNM storage systems is that the inventory records show which cabinet, bin, or shelving unit contains any specific container or item. Since individual container location is identified only by storage unit in the inventory records, the entire storage unit must be inventoried to confirm that an item is missing. Also, to locate one specific item that is not missing, half the storage unit, on the average, will be inventoried before the item is found. Thus, significant effort may be required for simple random sampling when items are stored in groups (clusters). The storage mode and characteristics of containers in the stratum may suggest other sampling techniques (such as cluster sampling in this type of situation).

Before considering methods other than simple random sampling for allocating a sample size, note that they may not yield the same probability of detection specified in the sample size calculation. This is because the previously discussed sample size calculations implicitly assume simple random sampling.

Let us consider one cluster sampling allocation plan in which the sample size and the number of items per storage unit determine the number of units (clusters) to sample, i.e., the number that contain at least the required sample size. The specific storage units are chosen by a randomization procedure. The inventory files are then examined to obtain a complete list of items in each of the chosen units, a trivial task for a computerized system. The inventory teams then perform the actual verifications for every container in each cluster chosen.

While a certain efficiency is achieved by sampling all containers in a storage unit, a loss of detection capability may result for some diversion strategies. For example, it is clear that a diverter's optimal strategy against the above cluster allocation plan is to take all of the containers desired from as few storage units as possible. The following simple numerical example shows that the power of detection specified in the sample size calculation is not retained using the above cluster sample size allocation.

Let a stratum contain N = 100 items stored in k storage units (clusters) with $\frac{100}{k}$ items in each cluster. It is desired to detect at least one of 10 defective items with 90% probability. Using the cluster allocation plan described above, the probability of detection will be investigated for several values of k (note that k = 100 is equivalent to simple random sampling):

$$n = \left(N - \frac{d-1}{2}\right)\left(1 - \beta^{\frac{1}{d}}\right)$$

where $d = 10$

$\beta = 0.1$

$n = 19.64 \simeq 20$

The probabilities of detecting at least one defect are summarized in Table 3.3. Note that the probability of detection is much lower than 90% when the number of clusters in a stratum is small. The probability of detection increases as the numbers of clusters increases. The case when each container is a cluster is equivalent to simple random sampling and yields 90% probability of detection.

The above example has pointed out a diversion strategy where the probability of detection may be much less than specified if the previously calculated sample size is distributed using cluster sampling allocation. The diversion strategy assumed the diverter would take all of the SNM from every container in a storage unit (cluster) and would do so for as many clusters as necessary to obtain the goal quantity G. While this is an optimal strategy against cluster sampling allocation, it may not be preferred if other methods are employed which easily

TABLE 3.3. Probabilities of Detecting at Least One Defect for Cluster Sample Allocation Example

Number of Clusters (kg)	Number of Items/ Cluster	Number of Clusters in Sample	Probability of Detection
10	10	2	.20
20	5	4	.37
50	2	10	.69
100[a]	1	20	.90

(a) Equivalent to simple random sampling.

detect concentrated large losses of this type. Other reasonable diversion strategies exist for which the cluster sampling allocation would provide probabilities of detection higher than the prespecified 90%.

To summarize, allocation of a predetermined stratum sample size by cluster sampling can significantly reduce sample location and collection times. However, since the original sample size was calculated based on simple random sampling, the probability of detection may suffer using a cluster sampling allocation scheme. The choice of whether to use this scheme requires consideration of the benefits and risks mentioned above. In addition, the structure of other portions of the verification effort will impact this decision.

Other allocation schemes can be considered (see Section 1.3). Systematic sampling offers no advantages over simple random sampling in this application and is hence rejected. Given the assumption of homogeneous strata with respect to SNM content, allocation with PPS is basically equivalent to simple random sampling.

While not true allocation schemes, sequential sampling techniques are applicable. Specifically, curtailed sequential sampling appears most appropriate. A maximum sample size is calculated and an allocation scheme specified. One at a time, containers are sampled according to the allocation scheme and verification testing (either for gross or all large defects) is performed. If a defect is found, sampling stops at that point for corrective action. If not, sampling proceeds. If no defects[a] are found by the time the maximum sample size is reached, the risk level for the specified goal quantity G is satisfied. Since verification stops whenever one defect[a] is found or when a total number of containers is verified without defects, this scheme is statistically a type of curtailed sequential sampling (Cohen, 1970; Guenther, 1969). More information on sequential sampling can be found in Wald (1947).

3.1.2 Stratified-Cluster Attribute Sampling Plans

This type of sampling plan assumes the population is composed of clusters of elements and stratified based on type and content of SNM (as spelled out in Section 3.1.1). Further, it is assumed that all clusters will contain the same (or approximately the same) number of containers. A sample size for each stratum will be calculated based on a simple one-stage cluster sampling allocation scheme. This is in contrast to the sampling plan discussed in Section 3.1.1, where a simple one-stage cluster sampling allocation was used but the sample size n was calculated based on a simple random sample. Some additional notation beyond that utilized in Section 3.1.1 is required:

M = number of clusters in the stratum,
m = number of clusters sampled from the stratum,
N_c = number of containers per cluster.

Each cluster is assumed to have approximately the same number of containers, thus the problem of calculating n reduces to that of determining m, because

$$n = m N_c .$$

Since a single-stage cluster sample involves verifying every container in a cluster, the existence of at least one defective container in a cluster implies

(a) Assuming a zero acceptance number sampling plan. The acceptance and rejection numbers will be higher for other plans.

that the cluster is defective. Thus, the probability of not obtaining at least one defective container (out of d in the population of N containers) in a sample of n containers is equal to the probability of not obtaining at least one defective cluster (out of c in the population of M clusters) in a sample of m clusters. Hence, based on formula (3.1) and the hypergeometric distribution, the number of clusters m required for the sample is

$$m = \left(M - \frac{c-1}{2}\right)\left(1 - \beta^{\frac{1}{c}}\right). \tag{3.3}$$

A conservative approximation based on formula (3.2) and the binominal distribution is

$$m = M\left(1 - \beta^{\frac{1}{c}}\right) \tag{3.4}$$

where β = probability of not obtaining at least one defective cluster (out of c) in the sample,

$c = d/N_c = G/\gamma AN_c$ = number of clusters needed to obtain G units of SNM by removing γAN_c units from each cluster. If $G/\gamma AN_c$ is not an integer, round to the next highest integer.

If m includes a fractional amount, the next higher integer is used for m. Note that the form for c,

$$c = \frac{G}{\gamma AN_c}$$

assumes that if a cluster is defective, every container in the cluster is defective. Clearly this is the optimal strategy for a diverter given the one-stage cluster sampling plan.

At this point, the situation is analogous to that for stratified attribute sampling (discussed in Section 3.1.1). The major points are reiterated here with additional comments based on the cluster structure.

- If required, a separate sample size for the timely detection of gross defects is computed by choosing $\gamma = 1$ in formula (3.3) or (3.4). This provides a sample size large enough to combat the optimal diversion strategy for gross defects which involves diverting the total contents of each container in a cluster for enough clusters to attain the desired goal quantity G.
- If the timely detection of gross defects is not required, a single sample size is calculated by choosing γ (the fraction of each container to be diverted) according to the sensitivity of the attributes measurement device (see Section 3.1.1).
- The amount of SNM in a container (A) can be chosen to be the average amount in all containers in a cluster, or the amount in the largest container (which yields a more conservative sample size). Likewise, if the number of containers in each cluster is not the same, N_c may be chosen to be either the average or largest number. Choosing the largest number yields a more conservative (larger) sample size than does the average.
- The cluster sample size formulas (3.3) and (3.4) are based on an acceptance number of zero. Sample sizes for larger acceptance numbers are available by modifying the techniques referenced in Section 3.1.1.

Allocation of Cluster Sample Size

Once the cluster sample size m is computed, the m clusters to be sampled must be chosen. One possibility is to choose the clusters by simple random sampling. This is the most reasonable choice when the number of containers per cluster is approximately the same.

If the number of containers per cluster varies widely, the preceding techniques may be applied, taking care that a sufficent number of clusters are chosen to preserve the chosen value of β. This may require a modification of the value of m obtained from formula (3.3) or (3.4). A PPS allocation scheme is suggested in this situation (see Section 3.1.3 or refer to Levy and Lemeshow 1980, p. 247).

Whichever situation or allocation scheme, a curtailed sequential sampling technique is applicable. Sampling of clusters proceeds until too many defects are observed or until the maximum sample size is reached.

3.1.3 A Probability Proportional to Size Sampling Plan - Verification by Population Total

The attribute sampling plans considered in Sections 3.1.1 and 3.1.2 relied on having a stratified population of containers. However, stratification may not always be possible or desirable. Good, Griffith and Hamlin (1978) and Hamlin (1981) present examples of such situations and suggest sampling plans which involve sampling with probability proportional to size (PPS). In other situations, stratification may be part of the sampling plan, but allocation schemes which assume equal weighting for containers or clusters of containers may not be appropriate. Such an example was noted in Section 3.1.2. Natural clusters of containers may exist in a facility, where the number of containers per cluster varies widely. Sampling clusters with probability proportional to size (number of containers in the cluster) may be applicable.

The development of a sampling plan with probability proportional to size proceeds as follows. The first requirement is a listing of all containers in the population (or stratum) showing the identity and location of all containers, the amounts of SNM in each container, and the cumulative sum for the total amount (T) present.

The next step in developing the sampling plan is to select the variable which will measure "size." Variables such as the quantity or strategic value of SNM might be used to measure the size of a container. For a cluster of containers, the same variables or the number of containers in the cluster could be used to measure size. While these variables are often measured in common units, Good, Griffith and Hamlin (1978) suggest it can be meaningful to use the smallest defect size detectable with certainty as the measurement unit. Both the size variable and its measurement unit significantly impact the sample size calculation and selection of the sample.

Once the size variable measurement unit (U) is chosen, the individual and cumulative container contents are expressed in terms of measurement units. The population size in measurement units is given by $N^* = T/U$.

Sample Size Determination

The PPS attribute sample size should be large enough to provide a high probability of including at least one defect when the number of large defects is such that the total amount removed is G. Formulas (3.1) and (3.2) are still applicable, given that the calculations must be performed in the new units of measurement. Specifically, we have sample size formulas analogous to (3.1) and the conservative approximation (3.2):

$$n^* = \left(N^* - \frac{d^*-1}{2}\right)\left(1 - \beta^{*\frac{1}{d^*}}\right) \qquad (3.5)$$

and

$$n^* = N^* \left(1 - \beta^{*\frac{1}{d^*}}\right) \qquad (3.6)$$

where n^* = sample size of new population units,

N^* = T/U = population size in new units of measurement,

T = total amount SNM present in population (or stratum) in orginal units of measurement,

U = new unit of measurement,

β^* = probability of not obtaining at least one defect (out of d^* new unit defects) in the sample,

d^* = G/U = number of new units of SNM needed to obtain G original units,

G = goal quantity in original units.

If timely detection of gross defects is required, a separate sample size (and sampling plan) is developed by choosing a new unit of measurement to be the defect size detectable with certainty by the instrument used for detection of gross defects.

Allocation of Sample Size

The first step in the allocation of the sample size is to randomly select n* different numbers between 1 and N* and arrange them in ascending order. Using the cumulative list of SNM units, the containers associated with the n* numbers are determined. There is no need to select an alternative container if more than one of the n* numbers is associated with any container.

The number of containers (n) sampled with this sampling plan and allocation scheme is random, depending upon the size of each container and the measurement unit chosen. Sequential and curtailed sequential sampling techniques are applicable as the verification inspection effort progresses.

3.1.4 A Probability Proportional to Size Sampling Plan - Verification by Containers

The PPS sampling approach considered in Section 3.1.3 differs significantly from the previous sampling plans we have considered. By not assuming stratification, it was natural to verify the total amount of SNM in the population. In doing so, little attention was given to individual containers. This is in contrast to the approaches of sampling plans in Sections 3.1.1 and 3.1.2 which seek to verify SNM by container. There is a subtle difference between the two approaches, which can affect sample size. A PPS sampling plan based on container verification is presented in this section.

Sample Size Determination

Formulas (3.1) and (3.2) have been the basis of all the attribute sampling plan sample sizes discussed thus far. In their development (Section 3.1.1), it was assumed that the importance of each container in the population or stratum was approximately the same. A PPS sampling plan, however, implicitly assumes that some containers are more "important" than others. While there is clearly a conflict,[a] let us consider the implications of ignoring this conflict and utilizing formula (3.1) or (3.2) to determine a PPS sample size.

(a) Note that there is no conflict in Section 3.1.3 since there the new measurement units are the items to be sampled, and they are equally important due to their common size.

In utilizing formula (3.1) or (3.2), a value must be chosen for A, the amount of SNM in each container.[a] Here the amount is not uniform, but varies widely across the population (or stratum). Choosing A as the amount in the largest container yields a conservative sample size (i.e., $\beta \leq$ chosen value). However, if the difference between the largest and smallest containers is great, then the required sample size may be quite a bit larger than is needed for the smaller containers. On the other hand, if A is chosen to be the average amount of material in a container, the sample size may be too small to provide the desired probability of detection. The probability of detection depends upon the goal quantity G, and the diversion strategy. In this situation the optimal diversion strategy is to divert from the less important containers, since they are not as likely to be included in the sample.

Formulas (3.1) and (3.2) may be used to calculate sample sizes when it is desired to verify container information by PPS sampling. However, there are some subjective decisions required, such as choosing a value for A. Choosing A to be the amount in the largest container will yield a conservative sample size, i.e. one that provides at least the desired probability of detection. Depending upon the specific population and the variation in containers, the above choice for A may yield a very conservative sample size (too large). In such a case, the inspector could choose a smaller sample size (by choosing A to be the average container amount or some value between the average and the maximum amount). In making this choice, the nominal probability of detection will be less since fewer containers will be verified. However, if there is little chance or concern that a diversion took place in less important containers, then this loss in probability of detection may not be of concern. If the sample size is reduced, an estimate of the new probability of detection is given by

$$1 - \beta \simeq 1 - \left(1 - \frac{2n}{2N - d + 1}\right)^{d} \tag{3.7}$$

where n is now the reduced sample size (Jaech 1973, pg. 327).

(a) Recall that $d = G/\gamma A$ in formulas (3.1) and (3.2).

Allocation of Sample Size

Once the number of containers to sample is determined, the PPS allocation scheme must be outlined. The preliminary requirements are identical to those in Section 3.1.3: 1) a measure of size or importance for each container, 2) a listing of all containers with their associated size and cumulative size, and 3) the total combined size of all containers.

To select the sample, begin by letting N* represent the number of measurement units in the total combined size of all containers. Then a number between 1 and N* is randomly selected and its associated container is found using the cumulative size column in the listing. This container is then in the sample. The process continues as numbers between 1 and N* are randomly selected. If a selected number is associated with a container already in the sample, another number is selected. This procedure[a] continues until the sample size n is reached, where n different containers are in the sample.

Again, sequential and curtailed sequential techniques are applicable. In most situations the sample size and sample containers will be determined prior to beginning the verification effort. It is the actual verification effort that is sequential. There are three choices on how to verify the containers in the sample: 1) in random order, 2) in the order selected, or 3) in the order determined by the size or importance of each. Options 2 and 3 will be similar and can even be identical. With any choice, sampling stops when a defect is found or when the maximum sample size is reached. The benefit of choosing 2 or 3 is the potential for more timely detection of a defect if a diverter is indeed interested in the more important containers.

There is one final point which should be made. Any sampling plan involving sampling with PPS includes the implicit assumption that some containers are more "important" than others in verifying the total population. The "importance" may be measured by physical or strategic size. If all containers of a population (or stratum) are equally important, sampling with PPS is equivalent to simple random sampling.

(a) If a computer is available, a more efficient algorithm can be devised.

3.1.5 Other Attribute Sampling Plans

The attribute sampling plans discussed thus far are by no means exhaustive of all possible attribute sampling plans. Many other variations could be developed with slight modifications in existing plans. The plans we have considered involve the basic sampling techniques most applicable to SNM verification work: stratification, clustering, PPS, simple random, and sequential techniques. Through the illustration of these techniques in the sampling plans discussed, the development of other plans should be clear.

3.1.6 Comparison of Stratified Attribute Sampling Plans

The sampling plans discussed in Sections 3.1.1 and 3.1.2 will be compared here in terms of required sample size. Specifically, a zero defect, simple random sampling plan (Section 3.1.1) will be compared to a one-stage cluster sampling plan (Section 3.1.2), where clusters are chosen by simple random sampling. To simplify the comparisons, the conservative approximation sample size formulas (3.2) and (3.4) will be used. With these formulas, it is easy to compute the sampling fraction and thus avoid dependence on N. The sampling fraction (of N) for formula (3.2) is given by

$$1 - \beta^{\frac{1}{d}} \tag{3.8}$$

and

$$1 - \beta^{\frac{1}{c}} \tag{3.9}$$

for formula (3.4). The symbol definitions are as given in Sections 3.1.1 and 3.1.2 respectively. Recall that d and c are integers and may be approximated by $G/\gamma A$ and $G/\gamma AN_c$ respectively. If these quantities are not integers, they are rounded to the next highest integer. The following example will be used to provide a basis for the comparisons.

An Example

A population of containers is stratified according to the content of ^{235}U in high enriched uranium (HEU) in each container. Consider one stratum in which there are N_i containers, each containing approximately the same amount (A) of HEU. The containers are stored in storage cabinets, each of which holds N_c containers. It is desired to verify this stratum so that a diversion of quantity G would be detected with high probability ($1-\beta \geq 0.90$).

Sampling fractions (of N_i) for detecting removal of G = 5 kg of ^{235}U in HEU for various combinations of β, γ, and A are given in Table 3.4 (based on

TABLE 3.4. Sampling Fractions for a Simple Random Sampling Attribute Verification Example

A (kg)[a]	β	γ 1.0	γ 0.50	γ 0.25
0.5	.10	.21	.11	.06
	.05	.26	.14	.07
	.01	.37	.21	.11
1.0	.10	.37	.21	.11
	.05	.45	.26	.14
	.01	.60	.37	.21
3.0	.10	.68	.44	.28
	.05	.78	.53	.35
	.01	.90	.68	.48
5.0	.10	.90	.68	.44
	.05	.95	.78	.53
	.01	.99	.90	.68

(a) Note that the sampling fractions in this table are the same for any problem where the A/G [see formula (3.8)] ratios are the same as those utilized here.

Section 3.1.1, simple random sampling) and Table 3.5 (based on Section 3.1.2, simple random cluster sampling). It is clear when comparing the tables that many more containers must be sampled using simple random cluster sampling than with simple random sampling to maintain the desired value of β.

TABLE 3.5. Sampling Fractions for a Simple Random Cluster Sampling Attribute Verification Example

		N_c = 2			N_c = 5			N_c = 10		
		γ			γ			γ		
A (kg)[a]	β	1.0	0.50	0.25	1.0	0.50	0.25	1.0	0.50	0.25
0.5	.10	.37	.21	.11	.68	.44	.25	.90	.68	.44
	.05	.45	.26	.14	.78	.53	.31	.95	.78	.53
	.01	.60	.37	.21	.90	.68	.44	.99	.90	.68
1.0	.10	.54	.37	.21	.90	.68	.44	.90	.90	.68
	.05	.63	.45	.26	.95	.78	.53	.95	.95	.78
	.01	.78	.60	.37	.99	.90	.68	.99	.99	.90
3.0	.10	.90	.68	.44	.90	.90	.68	.90	.90	.90
	.05	.95	.78	.53	.95	.95	.78	.95	.95	.95
	.01	.99	.90	.68	.99	.99	.90	.99	.99	.99
5.0	.10	.90	.90	.68	.90	.90	.90	.90	.90	.90
	.05	.95	.95	.78	.95	.95	.95	.95	.95	.95
	.01	.99	.99	.90	.99	.99	.99	.99	.99	.99

(a) Note that the sampling fractions in this table are the same for any problem where the A/G ratios are the same as those utilized here.

3.1.7 Comparison of Nonstratified Attribute Sampling Plans

The two PPS sampling plans discussed in Sections 3.1.3 and 3.1.4 will be compared now in terms of sample sizes required for verification applications. Recall that the verification by population total plan in Section 3.1.3 utilizes new measurement units as the basic sampling unit and verifies the population as a whole. The number of containers sampled is random. The plan in Section 3.1.4 utilizes a container as the basic sampling unit and verifies the population by individual container. Formulas (3.5) and (3.1) [or (3.6) and (3.2)] provide the sample sizes. The following example will be used to provide a basis for the comparisons and will also serve to illustrate the application of the sampling techniques.

An Example

A population consists of 200 containers of various scrap, waste and cleanup materials. Each container contains up to 14 kg of ^{235}U in low enriched uranium (LEU), with the following summary of all containers.

Range (kg)	Number of Containers
0 - 2	10
2 - 4	15
4 - 6	20
6 - 8	25
8 - 10	30
10 - 12	50
12 - 14	50

A sampling plan with a high probability of detecting 75 kg of ^{235}U in LEU is desired.

To simplify the problem, assume prior records show that the containers in each class above contain the midpoint amount of ^{235}U. That is, assume the prior records show the 10 containers in the 0-2 kg class contain 1 kg, the 15 containers in the 2-4 kg class contain 3 kg, etc. Due to the measurement device used, a measurement unit of 0.1 kg will be used to indicate the importance or "size" of each container.

First the calculation of the PPS sample size for the sampling plan of Section 3.1.3 is presented. The following table summarizes some of the information necessary to illustrate the technique.

Range (kg)	Number of Containers	SNM Content (kg of ^{235}U)	Size (0.1 kg)	Cumulative Size (0.1 kg)
0 - 2	10	10	100	100
2 - 4	15	45	450	550
4 - 6	20	100	1,000	1,550
6 - 8	25	175	1,750	3,300
8 - 10	30	270	2,700	6,000
10 - 12	50	550	5,500	11,500
12 - 14	50	650	6,500	18,000

Table 3.6 gives the cumulative container size used to determine which containers are sampled.

TABLE 3.6. Listing of Containers with Cumulative Sizes

#	Cum. Size	#	Cum. Size	#	Cum. Size	#	Cum. Size	#	Cum. Size	#	Cum. Size	#	Cum. Size	#	Cum. Size
1	10	26	600	51	1970	76	3840	101	6110	126	8860	151	11630	176	14880
2	20	27	650	52	2040	77	3930	102	6220	127	8970	152	11760	177	15010
3	30	28	700	53	2110	78	4020	103	6330	128	9080	153	11890	178	15140
4	40	29	750	54	2180	79	4110	104	6440	129	9190	154	12020	179	15270
5	50	30	800	55	2250	80	4200	105	6550	130	9300	155	12150	180	15400
6	60	31	850	56	2320	81	4290	106	6660	131	9410	156	12280	181	15530
7	70	32	900	57	2390	82	4380	107	6770	132	9520	157	12410	182	15660
8	80	33	950	58	2460	83	4470	108	6880	133	9630	158	12540	183	15790
9	90	34	1000	59	2530	84	4560	109	6990	134	9740	159	12670	184	15920
10	100	35	1050	60	2600	85	4650	110	7100	135	9850	160	12800	185	16050
11	130	36	1100	61	2670	86	4740	111	7210	136	9960	161	12930	186	16180
12	160	37	1150	62	2740	87	4830	112	7320	137	10070	162	13060	187	16310
13	190	38	1200	63	2810	88	4920	113	7430	138	10180	163	13190	188	16440
14	220	39	1250	64	2880	89	5010	114	7540	139	10290	164	13320	189	16570
15	250	40	1300	65	2950	90	5100	115	7650	140	10400	165	13450	190	16700
16	280	41	1350	66	3020	91	5190	116	7760	141	10510	166	13580	191	16830
17	310	42	1400	67	3090	92	5280	117	7870	142	10620	167	13710	192	16960
18	340	43	1450	68	3160	93	5370	118	7980	143	10730	168	13840	193	17090
19	370	44	1500	69	3230	94	5460	119	8090	144	10840	169	13970	194	17220
20	4.00	45	1550	70	3300	95	5550	120	8200	145	10950	170	14100	195	17350
21	430	46	1620	71	3390	96	5640	121	8310	146	11060	171	14230	196	17480
22	460	47	1690	72	3480	97	5730	122	8420	147	11170	172	14360	197	17610
23	490	48	1760	73	3570	98	5820	123	8530	148	11280	173	14490	198	17740
24	520	49	1830	74	3660	99	5910	124	8640	149	11390	174	14620	199	17870
25	550	50	1900	75	3750	100	6000	125	8750	150	11500	175	14750	200	18000

Formula (3.5) provides the sample size of measurement units; where $d^* = G/U = (75\ kg/0.1\ kg) = 750$ units:

$$n^* = \left(N^* - \frac{d^*-1}{2}\right)\left(1 - \beta^{*\frac{1}{d^*}}\right)$$

and

$$n^* = \left(18000 - \frac{750-1}{2}\right)\left(1 - \beta^{*\frac{1}{750}}\right) .$$

For each of three values of β^*, a sample size n^* was generated, and the containers to be sampled were determined using the listing in Table 3.6. The values of n^* and n, for each of three values of B^*, are given in Table 3.7. Note that n is random, due to repeated sampling of some containers. The values in Table 3.7 are the result of one simulation.

TABLE 3.7. Sample Sizes for PPS Sampling Plan--Verification by Population Total

β^*	n^*	n
.10	55	44
.05	71	56
.01	108	78

Now the calculation of sample size for the sampling plan of Section 3.1.4 is presented. Formula (3.1) may be used to calculate sample size, where some care must be taken in the choice of **A**, the amount of SNM in a container. Choosing A to be the largest amount in any container (13 kg) yields a conservative sample size for the β chosen. The choice of A as the average amount **(9** kg) will also be considered. Using formula (3.1) with various values of γ and the two values of A noted above, the sample sizes found in Table 3.8 are obtained.

TABLE 3.8. Sample Sizes for PPS Sampling Plan--Verification by Containers

A (kg)	β	γ = 1.0	0.50	0.25
9	.10	45	25	13
	.05	56	32	16
	.01	79	46	24
13	.10	63	34	18
	.05	78	43	23
	.01	106	62	33

In comparing the sample sizes of the two methods, the results in Table 3.8 for A = 9 kg and $\gamma = 1$ are almost the same as the results in Table 3.7. This observation is somewhat expected, since the method used for Table 3.7 (as developed in Section 3.1.3) assumes a diverter will not make removals smaller than the unit of measurement chosen to measure size, an equivalent assumption to choosing $\gamma = 1$ in Table 3.8. Table 3.8 illustrates that smaller sample sizes are required when $\gamma < 1$. The method of Section 3.1.3 could be modified to incorporate values of γ less than one and the results could be expected to be similar to those in Table 3.8 when A = 9 kg. The conservative sample sizes in Table 3.8 for A = 13 kg are larger as expected, but still considerably less than 100% sampling.

In summary, given equivalent assumptions, the two PPS sampling plans of Sections 3.1.3 and 3.1.4 perform similarly. The sampling plan of Section 3.1.4 is presented in a more general manner. While this generality provides more flexibility, the flexibility requires the investigator to make additional decisions.

3.2 VARIABLES SAMPLING PLANS

Two distinct types of variables sampling plans will be considered in this section. One type investigates whether a goal quantity G of SNM has been removed from a facility (or portion thereof) by small diversions from several containers. Recall that a small diversion is defined as a diversion too small to be detected by a simple observation or a single item measurement. The second type of variable sampling plan investigates whether a diverter has inflated the random error variance (of an individual container difference) in order to reduce the probability of detecting a loss of the goal quantity through small diversions.

The development of any sampling plan, especially sample size determination, depends strongly on the underlying statistical test. The first type of variables sampling plan considers removal of G by small diversions from several containers. Jaech (1973, pp. 331-350) and Hough, Schneider, Stewart, et al. (1974, pp. 51-55) develop and discuss the applicable statistical test. The test can be considered as a hypothesis test

$$H_0: \mu_D = 0$$
$$H_A: \mu_D \neq 0$$

with test statistic

$$D^* = \frac{\hat{D}}{\hat{\sigma}_{\hat{D}}} \quad (3.10)$$

where μ_D = true unknown population mean of the difference between the prior and current SNM content of a container,

$\hat{D}$ = an estimate of μ_D based on a sample,

$\hat{\sigma}_{\hat{D}}$ = sample estimate of the standard deviation of $\hat{D}$.

The test statistic D^* is assumed to be normally distributed with zero mean and unit standard deviation under the null hypothesis.[a] The test consists of calculating the test statistic from the sample collected and comparing the

(a) Note that the null hypothesis is equivalent to stating that the goal quantity G has not been diverted.

result to a tabulated value ($z_{1-\alpha/2}$) of the standard normal distribution. If the absolute value of D* exceeds $z_{1-\alpha/2}$, the null hypothesis H_0 is rejected in favor of the alternative H_A, with probability a of a type I error. If the absolute value of D* does not exceed the table value, fail to reject H_0 with probability β of a type II error. A sample size formula will be developed based on the test statistic D* and choices for a, β and G (the detection goal).

The second type of variables sampling plan investigates whether a diverter has inflated the random error variance (of an individual container difference) in order to reduce the probability of detection of G through small diversions. Hough, Schneider, Stewart, et al. (1974, pp. 48-50) develop and discuss the applicable statistical test. The hypotheses

$$H_0: \sigma^2_{dr} = s^2_{dr}$$
$$H_A: \sigma^2_{dr} < s^2_{dr}$$

are tested with test statistic

$$\chi^* = \frac{(n-1)\, s^2_{dr}}{\sigma^2_{dr}} = \frac{\sum_{j=1}^{n} (d_j-\bar{d})^2}{\sigma^2_{dr}} \qquad (3.11)$$

where d_j = difference between prior and current SNM content value for the j^{th} container in the sample,

$\bar{d}$ = sample mean of the d_j, i=1, 2, ..., n,

n = sample size,

s^2_{dr} = sample variance of the d_j values,

σ^2_{dr} = expected variance of the d_j values from prior information.

Under the null hypothesis χ^* is distributed as a chi-square distribution with (n-1) degrees of freedom. The calculated value of χ^* is compared with the critical value $\chi^2_{1-\alpha}$ in a chi-square table to see if $s^2_{dr} = \sigma^2_{dr}$. If this is rejected, there is probability α of a type I error, while there is probability β of a type II error if it is not rejected. A sample size formula will be developed based on α, β, and the test statistic.

The two situations above have a comnon element: the underlying statistical tests require data and measurements over several containers. In this sense containers cannot be individually verified. The verification effort is directed at a group of containers, such as a stratum or the entire population.

As with the attributes sampling plans, it may or may not be possible to stratify a population of containers on the basis of type and amount of SNM. Stratification of this type is frequently appropriate and carries favorable sampling properties (see Table 1.1); stratified variables sampling plans will be considered in Sections 3.2.1-3.2.2. Sampling plans when stratification is not appropriate are discussed in Section 3.2.3.

3.2.1 A Stratified Variables Sampling Plan for Small Diversions - Verification by Population Total

It is assumed that the population of containers in a facility is stratified according to type and content of SNM. The containers in each stratum are assumed to have a uniform SNM content within a reasonable limit, such as ±25%. This assumption is required to assure the homogeneity of measurement variances of containers within a stratum, so that estimates of diversion and variances of these estimates can be obtained for each stratum.

Sample Size Determination

The sample size for detecting diversion of a goal quantity G of SNM through small diversions is based on the use of the test statistic

$$D^* = \frac{D}{\hat{\sigma}_{\hat{D}}}$$

For a stratified population, the difference statistic $\hat{D}$ over the whole population is

$$\hat{D} = \sum_{i=1} N_i \bar{d}_i$$

where N_i = number of containers in stratum i (i = 1, 2, ..., L),

$\bar{d}_i = \sum_{j=1}^{n_i} d_{ij}/n_i$ = the mean contribution to $\hat{D}$ from stratum i,

d_{ij} = difference between current and prior content values for the j^{th} container in stratum i,

n_i = sample size for stratum i,

$n = \Sigma n_i$ = sample size for population.

The sample size n is chosen to detect with high probability $(1-\beta)$ a total difference G over all strata.

An equation for calculating the total sample size over all strata is

$$z_{1-\alpha/2}\sqrt{\hat{\sigma}^2_{\hat{D}s} + \hat{\sigma}^2_{\hat{D}r|H_0}} = G - z_{1-\beta}\sqrt{\hat{\sigma}^2_{\hat{D}s} + \hat{\sigma}^2_{\hat{D}r|H_A}} \qquad (3.12)$$

where z_v = the v^{th} percentile of the standard normal distribution,

$\hat{\sigma}^2_{\hat{D}s}$ = systematic component of $\hat{D}$ variance,

$\hat{\sigma}^2_{\hat{D}r|H_0}$, $\hat{\sigma}^2_{\hat{D}r|H_A}$ = random component of $\hat{D}$ variance which may change from H_0 to H_A,

G = goal quantity,

α = probability of rejecting H_0 when H_0 is true,

β = probability of accepting H_0 when H_A is true.

An equivalent form of equation (3.12) is

$$z_{1-\alpha/2}\sqrt{\hat{\sigma}^2_{\hat{D}s} + \sum_{i=1}^{L} \frac{N_i^2}{n_i}\hat{\sigma}^2_{dr_i}} = G - z_{1-\beta}\sqrt{\hat{\sigma}^2_{\hat{D}s} + (f)\sum_{i=1}^{L} \frac{N_i^2}{n_i}\hat{\sigma}^2_{dr_i}} \qquad (3.13)$$

where N_i = number of containers in stratum i,

n_i = sample size in stratum i,

$\hat{\sigma}^2_{dr_i}$ = estimate of random error variance for individual container differences in stratum i,

L = number of strata in the population,

f = random error variance inflation factor if perpetrated by diverter.

The details of the development of equations (3.12) and (3.13) are given in Appendix A.2. Equation (3.13) can be solved given α, β, and G only if

$$\frac{G}{\hat{\sigma}_{\hat{D}s}} \geq z_{1-\alpha/2} + z_{1-\beta} \qquad (3.14)$$

This restriction is developed in Appendix A.2. If it is satisfied, equation (3.13) can be solved for $\hat{\sigma}^2_{\hat{D}r}$, a function of the n_i. The solution is

$$\hat{\sigma}^2_{\hat{D}r} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad (3.15)$$

where

$$A = 2(z_{1-\alpha/2})^2(z_{1-\beta})^2 f - \qquad)^4 - (z_{1-\beta})^4 f^2$$

$$B = 2(z_{1-\alpha/2})^2(z_{1-\beta})^2\hat{\sigma}^2_{\hat{D}s}(f+1) + 2(z_{1-\alpha/2})^2 G^2 + 2(z_{1-\beta})^2 G^2 f - 2(z_{1-\alpha/2})^4(\hat{\sigma}^2_{\hat{D}s}) - 2(z_{1-\beta})^4\hat{\sigma}^2_{\hat{D}s} f$$

$$C = 2(z_{1-\alpha/2})^2(z_{1-\beta})^2(\hat{\sigma}^2_{\hat{D}s})^2 + 2(z_{1-\alpha/2})^2 G^2\hat{\sigma}^2_{\hat{D}s} + 2(z_{1-\beta})^2 G^2\hat{\sigma}^2_{\hat{D}s} - (z_{1-\alpha/2})^4(\hat{\sigma}^2_{\hat{D}s})^2 - (z_{1-\beta})^4(\hat{\sigma}^2_{\hat{D}s})^2 - G^4 .$$

Note that the desired n_i (and $n = \Sigma n_i$) are implicit in

$$\hat{\sigma}^2_{\hat{D}r} = \sum_{i=1}^{L} \frac{N_i^2}{n_i} \hat{\sigma}^2_{dr_i} \tag{3.16}$$

and cannot be explicitly and uniquely solved for. Formula (3.16) can be solved by trial and error for the sample size n_i required in each stratum. Any of a number of nonunique sets of n_i may be obtained. The question of how to allocate the total sample size n [implicit in (3.16)] is thus raised; are any of the non-unique sets "better" than the others? The answer is certainly yes, but depends upon the criterion chosen. One criterion is to allocate n among the strata to minimize the denominator of the test statistic

$$D^* = \frac{D}{\hat{\sigma}_{\hat{D}}} .$$

This is equivalent to minimizing $\hat{\sigma}^2_{\hat{D}r}$ over the strata subject to the constraint $n = \Sigma n_i$. This yields

$$n = \frac{\left(f \sum_{i=1}^{L} N_i \hat{\sigma}_{dr_i}\right)^2}{\hat{\sigma}^2_{\hat{D}r|H_A}} = \frac{\left(\sum_{i=1}^{L} N_i \hat{\sigma}_{dr_i}\right)^2}{\hat{\sigma}^2_{\hat{D}r}} \tag{3.17}$$

where $\hat{\sigma}^2_{\hat{D}r}$ is given by equation (3.15).

The allocation of n into strata is given by

$$n_i = n\left(\frac{N_i \hat{\sigma}_{dr_i}}{\sum_{i=1}^{L} N_i \hat{\sigma}_{dr_i}}\right). \tag{3.18}$$

The formulas for n and allocation of n among strata specified in equations (3.17) and (3.18) do not consider the cost of sampling a container from a stratum. "Cost," as used here, includes both the cost of locating the container and collecting the sample as well as the cost of subsequent analytical work required to verify its contents. Often this cost may vary widely from stratum to stratum, so it may be reasonable to choose an allocation scheme which considers the cost of sampling within a stratum. Cochran (1977) suggests the following cost equation for many applications:

$$C = c_o + \sum_{i=1}^{L} c_i n_i \tag{3.19}$$

where C = total cost of sample of size n,
c_o = overhead cost,
c_i = cost per sample unit in stratum i.

With this cost function, two allocation criteria are possible: 1) choosing the n_i to minimize the variance $\hat{\sigma}^2_{\hat{D}r}$ (equation 3.15) for a fixed cost C or 2) choosing the n_i to minimize the cost C for a fixed variance $\hat{\sigma}^2_{\hat{D}r}$. For both criteria, the allocation[(a)] of n_i is given by

$$n_i = n \left(\frac{N_i \hat{\sigma}_{dr_i} / \sqrt{c_i}}{\sum_{i=1}^{L} (N_i \hat{\sigma}_{dr_i} / \sqrt{c_i})} \right), \tag{3.20}$$

but n is computed differently. If cost is fixed,

(a) It is noted that formulas (3.18) and (3.20) may yield $n_i > N_i$. In this case n_i is set equal to N_i and the remaining $(n-n_i)$ items are optimally allocated among the other strata as discussed by Cochran (1977, p. 104).

$$n = \frac{(C-c_o) \sum_{i=1}^{L} (N_i\hat{\sigma}_{dr_i}/\sqrt{c_i})}{\sum_{i=1}^{L} (N_i\hat{\sigma}_{dr_i}\sqrt{c_i})} , \tag{3.21}$$

while

$$n = \frac{1}{\hat{\sigma}^2_{\hat{D}r}} \left[\sum_{i=1}^{L} (N_i\hat{\sigma}_{dr_i}\sqrt{c_i})\right] \left[\sum_{i=1}^{L} (N_i\hat{\sigma}_{dr_i}/\sqrt{c_i})\right] \tag{3.22}$$

if $\hat{\sigma}^2_{\hat{D}r}$ is fixed. See Cochran (1977, pp. 96-98) for additional details.

The previous formulas for determining n and allocating it into n_i are based on satisfying equation (3.14). When G is not much larger than the systematic error variance $\hat{\sigma}^2_{\hat{D}s}$, close to 100% sampling may be required. Appendix A.2 considers this case where the sample sizes required may be reduced by accepting larger values of a and β. Imposing the constraint

$$\hat{\sigma}_{\hat{D}r|H_A} \geq \left(\frac{1}{2}\right) \hat{\sigma}_{\hat{D}s} \tag{3.23}$$

limits n because $\hat{\sigma}_{\hat{D}r}$ decreases with increased sample size. This point is chosen as one of diminishing returns. This is explained in more detail in Appendix A.2. Restriction (3.23) implies that

$$\frac{G}{\hat{\sigma}_{\hat{D}s}} \geq 1.03z_{1-\alpha/2} + 1.12z_{1-\beta} \tag{3.24}$$

must be satisfied for equation (3.13) to have a solution. If it is satisfied, formulas (3.17) through (3.22) are applicable. If not, then $\hat{\sigma}_{\hat{D}r|H_A}$ is set equal to $\frac{1}{2}\hat{\sigma}_{\hat{D}s}$, resulting in

$$\hat{\sigma}^2_{\hat{D}r} = \frac{1}{4f}\hat{\sigma}^2_{\hat{D}s} \qquad (3.25)$$

as the solution for $\hat{\sigma}^2_{\hat{D}r}$. Then n and its allocation into strata are determined as directed by equations (3.17) through (3.22).

As noted earlier, when it is necessary to set $\hat{\sigma}_{\hat{D}r|H_A} = \frac{1}{2}\hat{\sigma}_{\hat{D}s}$, the prespecified values for a and β are not retained. This restriction lowers the sample size, causing either one or both of *a* and β to increase. Some flexibility exists since the person performing the test can choose *a* and β within the bounds allowed by restriction (3.23):

$$\left(\frac{G}{\hat{\sigma}_{\hat{D}s}} - 1.12\, z_{1-\beta}\right)^2 = \left(z_{1-\alpha/2}\right)^2 \left(1 + \frac{1}{4f}\right) \qquad (3.26)$$

Before leaving the topic of allocation of n among the strata, allocation with probability proportional to size (PPS) of the strata is considered. Note that the allocation schemes based on the cost function (3.19) are PPS schemes. Strata containing more containers are sampled more, while strata where it is expensive to sample are sampled less. Both cost per container and the number of containers per strata are used as measures of "size." It is possible to base other PPS allocation schemes on the strategic value or other measures of size.

An Example

Consider the following stratified population of containers containing ^{235}U in high enriched uranium (HEU).

	Stratum	N_i	$\hat{\sigma}_{dr_i}$ (kg)	$N_i\hat{\sigma}_{dr_i}$
1.	Feed and Intermediate Oxides	900	0.004	3.60
2.	Product	4000	0.002	8.00
3.	Scrap	450	0.04	18.00
4.	Fuel Rods	3000	0.0015	4.50
	Totals	8350		34.10

Assuming that the systematic component of the variance of $\hat{D}$ is

$$\hat{\sigma}^2_{\hat{D}s} = 1.32\ (\text{kg}\ ^{235}\text{U})^2,$$

we want to verify that an amount G = 5 kg ^{235}U has not been diverted from the population through small defects, with a = 0.05 and β = 0.05. Assume that if a diverter has inflated the random error variance component $\hat{\sigma}^2_{\hat{D}r}$, the inflation factor is two or less (f = 2).

The required sample size is given by formula (3.17), given restriction (3.24) is satisfied. Since

$$\frac{G}{\hat{\sigma}_{\hat{D}s}} = \frac{5}{\sqrt{1.32}} = 4.35$$

is greater than or equal to

$$1.03\ z_{.975} + 1.12\ z_{.95} = 1.03(1.96) + 1.12(1.645) = 3.86,$$

restriction (3.24) is satisfied. Note also that restriction (3.14) is satisfied, and equation (3.15) provides a solution for $\hat{\sigma}^2_{\hat{D}r}$ [required as input for formula (3.17)] as follows:

$$\hat{\sigma}^2_{\hat{D}r} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where $A = 2(1.96)^2(1.645)^2(2) - (1.96)^4 - (1.645)^4(2)^2 = -2.47$

$$B = 2(1.96)^2(1.645)^2(1.32)(2+1) + 2(1.96)^2(5)^2 + 2(1.645)^2(5)^2(2) - 2(1.96)^4(1.32) - 2(1.645)^4(1.32)(2) = 467.39$$

$$C = 2(1.96)^2(1.645)^2(1.32)^2 + 2(1.96)^2(5)^2(1.32) + 2(1.645)^2(5)^2(1.32) - (1.96)^4(1.32)^2 - (1.645)^4(1.32)^2 - (5)^4 = -195.10.$$

Then

$$\hat{\sigma}^2_{\hat{D}r} = \frac{-467.39 \pm \sqrt{(467.39)^2 - 4(-2.47)(-195.10)}}{2(-2.47)}$$

$$= \frac{467.39 \pm 465.32}{4.94}$$

$$= 0.42 \text{ or } 188.81$$

The desired answer is $\hat{\sigma}^2_{\hat{D}r} = 0.42$, the other value being an extraneous root of the algebraic process used to solve equation (3.13).

Thus, using formulas (3.17) and (3.18), the sample size n and its allocation into strata are

Stratum	N_i	n_i
1	900	292
2	4000	650
3	450	1462
4	3000	365
	N=8350	n=2769

Note that $n_3 > N_3$ (see the footnote on p. 3.32). To resolve this, n_3 is set equal to N_3 and the remaining $(n-n_3)$ containers are optimally allocated as in Cochran (1977, p. 104). This produces the following revised table.

Stratum	N_i	n_i
1	900	519
2	4000	1152
3	450	450
4	3000	648
	N=8350	n=2769

Allocation of Stratum Sample Sizes

Once the total population sample size n and its allocation of n_i to each stratum is determined, the question arises of how to allocate (or select) the n_i containers within each stratum. The desired probabilities of type I and II errors are known to hold for simple random sampling within each stratum. However, as noted in Section 3.1.1, cluster sampling allocation may be less costly and time consuming. The effects of cluster sampling allocation on the power of the test[a] under optimal small diversion strategies are unknown. While these effects are of interest, their investigation is beyond the scope of this study. The effects would be expected to be smallest when the goal quantity G is removed through small diversions spread over many strata.

Sequential Sampling Techniques

Sequential sampling techniques can be very useful in variables sampling plans for small diversions. If a diverter has taken more than G, the goal quantity to be detected, it is highly desirable to know that a total diversion of at least G has occurred without collecting the total sample. A curtailed sequential sampling superstructure can be imposed on any of the previously discussed techniques for determining n and the n_i. One drawback to this is the time required to obtain the difference estimate d_{ij} (j^{th} container in i^{th} stratum) for each container sampled. Weighing and chemical analysis times can be significant for nontamper-safed containers. Further, the calculations to perform the statistical test require a computer with terminals readily available to verification personnel after each container is sampled. However, if these drawbacks are not serious for a given facility, a curtailed sequential sampling plan framework is certainly recommended.

3.2.2 A Stratified Variables Sampling Plan for Small Diversions - Verification by Stratum

The sampling plan in the previous section approached the verification problem from a population viewpoint. That is, the plan was developed to detect a

(a) The test defined by the test statistic given in equation (3.10).

removal of G units taken by small diversions from the population of containers. However, if it is possible for a diverter to acquire the goal quantity G from a single stratum or a small number of strata, then the sampling plan presented in Section 3.2.1 may be more conservative than specified by the desired α and β. Thus, it is necessary to develop a sampling plan from the stratum viewpoint, so that each stratum will be verified.

Sample Size Determination

The sample size for detecting diversion of a goal quantity G of SNM through small diversions in a single stratum is based on the use of the test statistic

$$D_i^* = \frac{\hat{D}_i}{\hat{\sigma}_{\hat{D}_i}}$$

where $\hat{D}_i = N_i \bar{d}_i$ = difference statistic for stratum i,

N_i = number of containers in stratum i,

$\bar{d}_i = \sum_{j=1}^{n_i} d_{ij}/n_i$ = average difference for containers in the stratum,

d_{ij} = difference between prior and current content values for the j^{th} container in stratum i,

n_i = sample size for stratum i.

Analogous to equations (3.12) and (3.13) in Section 3.2.1, a formula for calculating the sample size n_i is

$$z_{1-\alpha/2}\sqrt{\hat{\sigma}^2_{ds_i} + \hat{\sigma}^2_{dr_i|H_0}} = G - z_{1-\beta}\sqrt{\hat{\sigma}^2_{ds_i} + \hat{\sigma}^2_{dr_i|H_A}} \qquad (3.27)$$

which is equivalent to

$$z_{1-\alpha/2}\sqrt{\hat{\sigma}^2_{ds_i} + \frac{N_i^2}{n_i}\hat{\sigma}^2_{dr_i}} = G - z_{1-\beta}\sqrt{\hat{\sigma}^2_{ds_i} + f\left(\frac{N_i^2}{n_i}\right)\hat{\sigma}^2_{dr_i}} \qquad (3.28)$$

where N_i = number of containers in stratum i,

n_i = sample size in stratum i,

$\hat{\sigma}^2_{dr_i}$ = estimate of random error variance for individual container differences in stratum i,

$\hat{\sigma}^2_{ds_i}$ = systematic component of variance of $\hat{D}_i$,

L = number of strata in the population,

f = random error variance inflation factor if perpetrated by diverter.

Note that equation (3.28) is identical to equation (3.13) except for the missing summation sign (Σ). Thus, equation (3.15) can be used to solve (3.28) for $\hat{\sigma}^2_{\hat{D}r_i}$, where

$$\hat{\sigma}^2_{\hat{D}r_i} = \left(\frac{N_i^2}{n_i} \right) \hat{\sigma}^2_{dr_i} \qquad (3.29)$$

in equation (3.28). A formula for n_i is obtained by solving (3.29),

$$n_i = \frac{N_i^2 \hat{\sigma}^2_{dr_i}}{\hat{\sigma}^2_{\hat{D}r_i}} \qquad (3.30)$$

where $\hat{\sigma}^2_{\hat{D}r_i}$ is the solution obtained from (3.15).

An Example

Consider the example in Section 3.2.1, where we wish to verify that an amount G = 5 kg of ^{235}U in HEU has not been diverted from each stratum. For illustration we will choose stratum 3 (scrap). Again, we choose a = 0.05, β = 0.05, f = 2 and assume $\hat{\sigma}^2_{ds_3}$ = 0.70 (kg $^{235}U)^2$

Utilizing a modified formula (3.24), note that

$$\frac{G}{\hat{\sigma}_{ds_3}} = \frac{5}{0.70} = 7.14$$

is greater than

$$1.03\, z_{.975} + 1.12\, z_{.95} = 3.86.$$

Hence, formula (3.28) can be solved for $\hat{\sigma}^2_{\hat{D}r_3}$ using formula (3.15), where the $\hat{\sigma}^2_{\hat{D}s}$ in the formulas for B and C are replaced by $\hat{\sigma}^2_{ds_3}$. Then

$$\hat{\sigma}^2_{\hat{D}r_3} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-465.18 \pm \sqrt{(465.18)^2 - 4(-2.47)(-396.46)}}{2(-2.47)}$$

$$= \frac{465.18 \pm 460.95}{4.94}$$

$$= 0.86 \text{ or } 187.48$$

The desired value is $\hat{\sigma}^2_{\hat{D}r_3} = 0.86$. Using formula (3.30), n_3 is obtained:

$$n_3 = \frac{N_3^2\, \hat{\sigma}^2_{dr_3}}{\hat{\sigma}^2_{\hat{D}r_3}} = \frac{(450)^2(0.04)^2}{0.86} \simeq 377.$$

Since we are still looking for a loss of G but restricting our search to stratum 3, we should expect to see a lower sample size than that obtained in the example from Section 3.2.1.

Allocation of Stratum Sample Size

We now consider the allocation of the stratum sample size within the stratum. The desired values of α and β are known to be supported by simple random sampling within a stratum. It has been previously noted that cluster sampling can often be less costly and time consuming, but for the framework under which the sample size was developed, its effects on α and β have not been studied. If a stratum contains a large number of clusters and the goal quantity G could be removed through small diversions in one cluster, cluster sampling could substantially affect the power of the test as noted in Table 3.3. On the other hand, if a diverter is forced to make removals from many clusters in a stratum to accumulate the goal quantity G, then cluster sampling could be expected to not seriously affect the specified α and β.

Sequential Sampling Techniques

The comments made under this heading in Section 3.2.1 are valid here. The change from verifying the population as a whole to verifying it by strata does not alter the effectiveness of the curtailed sequential sampling framework suggested there.

3.2.3 A Probability Proportional to Size Variables Sampling Plan

The variables sampling plans considered in Sections 3.2.1 and 3.2.2 relied on having a stratified population of containers. Stratification, however, may not always be possible or desirable. A sampling plan which selects containers to be sampled with PPS will be considered. This plan could also be applied to a population of containers grouped in clusters, where the clusters are sampled with PPS.

The development of a variables PPS sampling plan for detecting removal of G through small diversions in a population proceeds as follows. The first requirement is a listing of all containers in the population. In addition to the identity and location of all containers, the listing should include the amounts of SNM in each container and the cumulative sum for the total amount (T) present. The next step in developing the sampling plan is to select the variable which will measure "size." Variables such as the quantity or strategic value of SNM could be used to measure the size of a container. For a cluster of containers, the same variables or the number of containers in the cluster could be used to measure size. While these variables are often measured in common units, it can be meaningful to use the smallest defect size detectable with certainty as the measurement unit. This defect detectable with certainty is by definition a large defect. Although we are currently interested in removals due to small defects, this measurement unit still has some appealing features. (See Hough, Schneider, Stewart, et al. 1973, p. 51.) Both the size variable and its measurement unit significantly impact the sample size calculation and selection of the sample.

Once the size variable measurement unit (U) is chosen, express the container contents, individually and cumulatively, in terms of measurement units. The population size in measurement units is given by $N^* = T/U$.

Sample Size Determination

All of the formulas in Section 3.2.2 are applicable here, given that the calculations are performed in the new units of measurement. Hence

$$n^* = \frac{(N^*)^2 \hat{\sigma}^{2*}_{dr}}{\hat{\sigma}^{2*}_{\hat{D}r}} \tag{3.31}$$

where n^* = sample size in new population units,

N^* = population size in new population units,

$\hat{\sigma}^{2*}_{dr}$ = sample estimate of variability of container differences from prior content information,

$\hat{\sigma}^{2*}_{\hat{D}r}$ = solution of equation (3.28) using equation (3.15), where new measurement units are used.

Allocation of Sample Size

The first step in the allocation of the sample size is to randomly select n^* different numbers between 1 and N^* and arrange them in ascending order.

Then, using the cumulative list of SNM units, the containers associated with the n^* numbers are determined. There is no need to select an alternative container if more than one of the n^* numbers is associated with any container.

The number of containers (n) sampled with this sampling plan and allocation scheme is random, depending upon the number of containers selected more than once. This duplication depends on the measurement unit chosen and the size of each container. Sequential and curtailed sequential sampling techniques are applicable as the verification inspection effort progresses.

3.2.4 A Variables Sampling Plan for the Detection of Inflated Random Error Variance

It was seen in the previous three sections that the ability to detect small diversions lies in minimizing the random error variance of $\hat{D}$ through increased sample size. As noted in the introductory comments of Section 3.2, there is a need to protect against the possibility that a diverter may inflate the random error variance of $\hat{D}$, thus lowering the probability of detecting a removal through small diversions. A sampling plan for this purpose is developed here.

A sample size n is desired which provides high probability of detecting a ratio

$$\frac{\hat{\sigma}^2_{dr}}{\sigma^2_{dr}}$$

as large as 4.0, for specified a and β. The sample size n is that which satisfies

$$\chi^2_{1-\alpha}(n-1) = 4\chi^2_{\beta}(n-1), \qquad (3.32)$$

where $\chi^2_{1-\alpha}(n-1)$ and $\chi^2_{\beta}(n-1)$ are $1-\alpha$ and β percentiles respectively from a chi-square distribution with (n-1) degrees of freedom. This is easily solved with a table of percentile values for the chi-square distribution such as the one in Appendix C. Values of n [obtained by solving (3.32)] for several combinations of a and β are given in Table 3.9.

TABLE 3.9. Sample Sizes Required to Detect an Inflated Random Error Variance

α	β .10	β .05	β .01
.10	9	11	17
.05	10	13	20
.01	14	17	25

If the population is stratified, then the n obtained above is the sample size to be collected in each stratum to verify that the random error variance component has not been inflated. If the population is not stratified, then only n containers need be selected from the entire population.

3.3 OVERALL VERIFICATION SAMPLING PLAN

Individual sampling plans for the following verification activities have been discussed:

1. detection of gross defects with an attributes sampling plan
2. detection of large defects with an attributes sampling plan
3. detection of a removal by small defects with a variables sampling plan
4. detection of an inflated random error variance (of D) with a variables sampling plan.

In the formulation of an overall verification sampling plan it is not necessary to collect separate samples to perform each of the above verification activities. The first activity involves the timely detection of very large defects. If this is desired, a separate sample size is usually chosen. Since the last three activities all rely on the same type of measurements, the maximum of the sample sizes from individual sampling plans for activities 2-4 is chosen. An overall allocation plan can be worked out by referring to the relevant discussions and examples in the preceding sections of this chapter.

As an example of overall sampling size choice, consider the problem in Hough, Schneider, Stewart, et al. (1974, Section D), which has the following sample sizes for one stratum:

Purpose	Sample Size
Gross Defects	26
All Large Defects	8
Small Defects	37
Inflated Random Error	13

From these values, the inventory sample sizes for that stratum would be 26 attribute samples and 37 variables samples.

4.0 DETERMINING THE SNM CONTENT OF A POPULATION OF CONTAINERS

4.1 AN OUTLINE OF THE INVENTORY ESTIMATION PROCESS

Given that a nuclear materials accounting system is in place at a facility, an estimate of the amount of SNM in a population of containers is desired. The current practice is to conduct an inventory by census, **i.e.,** measure every container. This portion of the study will investigate sampling approaches to inventory determination; specifically it will show how to

- select a sample size
- obtain an estimator for the amount of SNM in the population of containers
- determine the variance of the estimator
- develop confidence intervals based on the estimator.

The physical collection of data will depend upon the nature of each container. If a container is sealed and tamper-safed, the amount of SNM for that container is the recorded amount (assuming it could not be falsified). If a container is not sealed or tamper-safed, the data collection may involve weighing and assay measurements to determine the amount of SNM.

4.2 CHARACTERISTICS OF A POPULATION OF CONTAINERS

The reader may want to refer back to Section 2.2; the characteristics of a population of containers given there are still applicable. In general, containers may come in various shapes and sizes and contain varying amounts of SNM. Containers can often be stratified into groups by similar types and amount of SNM. Other groupings (clusters) of containers may occur naturally due to physical proximity. Most of a typical inventory is located in vaults or storerooms. Individual containers are stored on shelves, racks, and pallets or in cabinets, cubicles and bins. As noted earlier, individual containers or groups of containers may be secured and tamper-safed while others may not be.

4.3 INVENTORY ACCURACY REQUIREMENTS

The accuracy required of an inventory estimate may depend upon the size and characteristics of the specific population of containers. A general framework will be adopted, allowing the user to choose the accuracy required for the specific application. We choose the general requirement to be a limit ε on the maximum relative difference allowed between the estimate and the true unknown population value. That is, if X is the true population value and x our estimate, we require

$$\left| \frac{x-X}{X} \right| \leq \varepsilon. \tag{4.1}$$

5.0 INVENTORY DETERMINATION SAMPLING PLANS

All of the inventory sampling plans discussed in this section are derived from a single probabilistic statement:

$$Pr\left[\left|\frac{x-X}{X}\right| \geq \varepsilon\right] = \alpha \qquad (5.1)$$

where x = sample estimate of total SNM content of the population,

X = true unknown SNM content of the population,

ε = maximum relative difference limit,

α = probability that the absolute relative difference between the sample estimate and population value is greater than ε.

Equation (5.1) may also be written as

$$Pr\left[|x-X| \geq \varepsilon X\right] = \alpha \ . \qquad (5.2)$$

Formulas for sample size will be developed from equation (5.2), in which n is implicit in the sample estimate x. The sample estimate x will be a function of the sum of the estimates for each sampled container; hence the implicit existence of n in equation (5.2). Since x will be obtained by summation of component estimates, it will be assumed to be approximately normally distributed. This assumption allows calculation of the probability α in equation (5.2). The specific sample size formulas will depend upon the type of basic sampling techniques (see Section 1.3) utilized.

The inventory sampling plans will be classified into two major classes determined by whether it is possible or appropriate to stratify the population of containers.

5.1 NONSTRATIFIED INVENTORY SAMPLING PLANS

It may not always be possible to efficiently stratify a population of containers according to amount of SNM. Amounts in containers may vary widely, in which case the variance of an inventory estimator based on nonhomogeneous strata could be quite large; in fact, larger than the variance of estimators based on other sampling techniques. Nonstratified inventory sampling plans are considered below.

5.1.1 A Simple Random Sampling Plan for Estimating a Population Inventory

Simple random sampling has the advantages of being very easy to perform and yielding easy formulas for the inventory estimator, its variance, and sample size. When the population is not amenable to other sampling techniques, simple random sampling is the preferred technique.

The estimator for total population inventory and its estimated standard error ($\hat{SE}$)[a] under simple random sampling are given by Levy and Lemeshow (1980, Chapter 3):

$$x = \frac{N \sum_{i=1}^{n} x_i}{n} \tag{5.3}$$

$$\hat{SE}(x) = N\left(\frac{N-n}{N}\right)^{1/2} \frac{s_x}{\sqrt{n}} \tag{5.4}$$

where n = number of containers in sample,

N = number of containers in population,

x = estimator of total population inventory,

x_i = estimated inventory of the i^{th} sample container,

$s_x = \left(\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)\right)^{1/2}$ = standard deviation of the sample x_i, $i=1, 2, \ldots n$.

(a) The term standard error refers to the variability of an estimator formed by summing or averaging random variables.

A (1-a)% confidence interval on the population inventory X is given by

$$x \pm z_{1-\alpha/2}\ N\left(\frac{N-n}{N}\right)^{\frac{1}{2}} \frac{\hat{}x}{\sqrt{n}} \qquad (5.5)$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)^{th}$ percentile of the standard normal distribution.

A sample size formula to achieve prespecified a and ε is

$$n = \frac{(z_{1-\alpha/2})^2 N \hat{V}_X^2}{(z_{1-\alpha/2})^2 \hat{V}_X^2 + (N-1)\varepsilon^2} \qquad (5.6)$$

where $\hat{V}_X^2$ = an estimate of $\frac{\sigma^2_X}{\overline{X}^2}$, the population coefficient of variation squared (from previous knowledge or the most recent sample estimate),

ε = maximum relative difference limit on $\left|\frac{x-X}{X}\right|$,

a = probability that the absolute relative difference between the sample estimate and population value is greater than ε.

The details of the development of equation (5.6) are found in Appendix B.1. An example illustrating the use of formulas 5.3, 5.4 and 5.6 is found in Section 6.2. Extending this example to investigate the effects of a and ε on sample size yields the following sample sizes (out of the population of 1000 containers).

ε(a)	α .10	.05	.01
.0133	22	31	52
.0067	81	111	177
.0033	265	338	469
.0007	889	919	952

(a) These values of ε correspond to being within 20, 10, 5, and 1 kg ^{235}U of the population total of 1500 kg.

Assuming a computer is available, the simple random sample of size n given in equation (5.6) is easily chosen. If not, the sample can be chosen using random number tables or some other random device to select containers from a listing of all containers in the population.

5.1.2 Cluster Sampling Plans for Estimating a Population Inventory

It was noted in Section 4.2 that clusters of containers naturally occur in facilities due to physical storage methods. In addition, as was pointed out in Section 3.1.1, it can be very time consuming to locate individual containers in a population. Records for each container usually only identify the cluster (room, shelf, etc.) it is located in and not the precise position of the container within the cluster. Hence, using clusters as the sampling units and performing an inventory of all containers in each cluster sampled can provide large savings in the time and cost required to perform an inventory of a population.

Many types of cluster sampling are possible. The technique outlined above where all containers in a selected cluster are sampled is called a one-stage cluster sampling technique. Two-stage cluster sampling involves sampling only a portion of the containers in each cluster selected in the first stage. For both one and two-stage cluster sampling, any of the other sampling techniques may be used to select the subclusters or containers within clusters to be sampled. For example, simple random or PPS sampling is often used to select clusters in one-stage cluster sampling (or the first stage of two-stage cluster sampling). These same sampling techniques are common for selecting containers in the second stage of two-stage cluster sampling. One- and two-stage cluster sampling plans based on simple random sampling are discussed and variations of these are considered.

Simple One-Stage Cluster Sampling

Simple one-stage cluster sampling selects the clusters from the population of clusters by simple random sampling. The estimators for total population inventory and its standard error are given by Levy and Lemeshow (1980, pg. 178):

$$x = \frac{M}{m} \sum_{i=1}^{m} x_i \tag{5.7}$$

and

a

$$\hat{SE}(x) = \frac{M}{\sqrt{m}} \hat{\sigma}_{1X} \left(\frac{M-m}{M-1}\right)^{\frac{1}{2}} \tag{5.8}$$

where m = number of clusters in sample,

M = number of clusters in the population,

X = estimator of total population inventory,

x_i = inventory of cluster **i,**

$$\hat{\sigma}_{1X} = \left[\frac{\sum_{i=1}^{m} (x_i - \bar{x}_{clu})^2}{m-1}\right]^{\frac{1}{2}} \left[\frac{M-1}{M}\right]^{\frac{1}{2}} .$$

Note that formulas (5.7) and (5.8) do not require the number of containers in each cluster be the same. In fact, the number of containers sampled

$$n = \sum_{i=1}^{m} N_i$$

is random and depends upon the m clusters chosen. This scheme gives equal weight to each cluster, regardless of the number N_i of containers in each cluster. Clusters can be chosen with unequal weights; often they are chosen with probability proportional to some measure of size (such as the number of containers per cluster). Probability proportional to size (PPS) sampling plans for containers or clusters are discussed in Section 5.1.3.

A (1-a)% confidence interval on the population inventory X is given by

$$x \pm z_{1-\alpha/2} \frac{M}{\sqrt{m}} \hat{\sigma}_{1X} \left(\frac{M-m}{M-1}\right)^{\frac{1}{2}} \tag{5.9}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)^{th}$ percentile of the standard normal distribution.

A sample size formula to achieve prespecified a and ε with simple single stage cluster sampling is

$$m = \frac{(z_{1-\alpha/2})^2 M V_{1X}^2}{(z_{1-\alpha/2})^2 V_{1X}^2 + (M-1)\varepsilon^2} \tag{5.10}$$

where $V_{1X}^2 = \frac{\sigma^2_{1X}}{\overline{X}^2} = \frac{\sigma^2_{1X}}{\left(\sum_{i=1}^{M} X_i/M\right)^2}$ = population coefficient of variation squared,

$\sigma^2_{1X} = \frac{\sum_{i=1}^{M} (X_i-\overline{X})^2}{M}$ = population variance of inventory over all clusters,

X_i = population inventory of cluster i,

ε = maximum relative difference limit on $\left|\frac{x-X}{X}\right|$,

a = probability that the absolute relative difference between the sample estimate and population value is greater than ε,

and m, M, $z_{1-\alpha/2}$, and X are as previously defined. Formula (5.10) is not useful practically since it contains the population coefficient of variation V_{1X}^2. In practice it should be replaced with a good estimate $\hat{V}_{1X}^2$ from past knowledge or a sample (or census) estimate from a recent inventory. The application of equation (5.10) parallels that of equation (5.6), since the equations are the same except for the cluster basis of (5.10).

Simple Two-Stage Cluster Sampling

Simple two-stage cluster sampling selects clusters from the population by simple random sampling, and then selects part of the containers within each cluster to sample by simple random sampling. This type of plan is not as economical in saving time required to locate specific containers as is a simple one-stage cluster sampling plan, but it does provide the flexibility to inventory, at least partially, more clusters for the same fixed number of containers to be sampled.

The estimators for total population inventory and its standard error are given by Levy and Lemeshow (1980, p. 204):

$$x = \frac{M}{m}\frac{\overline{N}}{\overline{n}} \sum_{i=1}^{m} x_i \tag{5.11}$$

$$\hat{SE}(x) = \left(\frac{M}{\sqrt{m}}\frac{\overline{N}}{\overline{n}}\right) \left[\frac{\sum_{i=1}^{m} (x_i - \overline{x})^2}{m-1}\right]^{\frac{1}{2}} \left(\frac{N-n}{N}\right)^{\frac{1}{2}} \tag{5.12}$$

where m = number of clusters in the sample,
M = number of clusters in the population,
$N = \overline{N}M$ = number of containers in the population,
$\overline{N} = \frac{N}{M}$ = average number of containers per cluster in the population,
$n = \overline{n}m$ = total number of containers in the sample,
$\overline{n}$ = number of containers sampled from each cluster,
x_i = sample total inventory for cluster i,
$\overline{x} = \sum_{i=1}^{m} x_i/m$ = sample average inventory over clusters in the sample.

A $(1-\alpha)$% confidence interval on the population inventory X is given by

$$x \pm z_{1-\alpha/2}\hat{SE}(x) \tag{5.13}$$

where $z_{1-\alpha/2}$ is as previously defined and $\hat{SE}(x)$ is given in equation (5.12).

A sample size formula to achieve prespecified α and ε with simple two-stage cluster sampling requires choosing the number m of clusters to sample in the first stage and $\bar{n}$, the number of containers to sample (in the second stage) from each cluster chosen in the first stage. The development of these formulas and the formulas themselves are quite complicated, and thus are not given here. If interested refer to the work of Levy and Lemeshow (1980, pp. 212-221), Cochran (1977, pp. 280-285), and Beetle (1978).

In general, one should exercise care in selecting the correct sampling techniques for the problem at hand. For example, the simple two-stage cluster sampling plan discussed above is designed for populations which have clusters containing approximately the same number of containers.[a] In addition, the technique assumes each container within a cluster is equally important. If this is not the case, some sort of two-stage technique utilizing PPS sampling should be employed.

5.1.3 Probability Proportional to Size Sampling Plans for Estimating a Population Inventory

Probability proportional to size (PPS) sampling plans choose containers or clusters of containers for the sample based on some measure of size. The "size" of a container might be its SNM content or strategic value. Either of these variables could be used to measure the size of a cluster; also the number of containers in the cluster is sometimes used.

The development of an inventory sampling plan with PPS follows. The first requirement is a listing of all containers (or clusters) in the population. In addition to information concerning the identity and location of all containers, the listing should include the amounts of SNM in each container and the cumulative sum for the total amount (T) present. Then using the size variable measurement unit (U), the individual and cumulative container contents are expressed in terms of measurement units. The population size in measurement units is given by $N^* = T/U$.

(a) Two-stage sampling plans that are valid when there are unequal numbers of items in clusters are discussed by Levy and Lemeshow (1980, Chapter 12) and by Cochran (1977, Chapters 9A, 10, 11).

Sampling begins by randomly selecting a number between 1 and N*. Then, using the cumulative list of SNM units, the container associated with the chosen random number is found. This container is in the sample. At this point, the question of how to proceed arises. The same procedure used to select the first container could be used, with the possibility that the same container might be chosen again. This is commonly referred to as sampling with replacement. Sampling without replacement does not allow the same container to be chosen again, and requires generating a new cumulative list with the selected container deleted.

Sampling with replacement is easy to perform and provides a relatively simple theoretical background for the development of the desired formulas. Allowing a container to be sampled more than once[a] reduces the number of containers in the sample. This is desirable because sampling costs are reduced, but undesirable because the variance of the population inventory estimator is increased[b] over the case when sampling is without replacement. Formulas for inventory estimators, estimated standard errors, and confidence intervals are presented for each type of sampling method.

Sampling With Replacement

The estimator for total population inventory and its estimated standard error under PPS sampling with replacement are[c]

$$x = \sum_{i=1}^{n} \frac{x_i}{np_i} \qquad (5.14)$$

and

(a) When a specific container is selected more than once, it is not necessary to reinventory it. Its one inventory result is given increased influence in the final total population estimate due to its multiple selection.
(b) For a fixed sample size n.
(c) These formulas can be developed using results in Cochran (1977, Chapter 9A).

$$\hat{SE}(x) = \left[\frac{1}{n(n-1)} \sum_{i=1}^{n} \left(\frac{x_i}{p_i} - x \right)^2 \right]^{1/2} \tag{5.15}$$

where x = estimator of total population inventory,

x_i = estimated inventory of the i^{th} sample container,

n = number of containers in the sample,

$p_i = \frac{y_i}{Y}$ = probability of selecting the i^{th} container,

y_i = "size" of the i^{th} container (in new measurement units)

$Y = \sum_{i=1}^{n} y_i$ = total size of the population,

N = number of containers in the population.

A $(1-\alpha)\%$ confidence interval on the true population inventory X is given by

$$x \pm z_{1-\alpha/2} \hat{SE}(x) \tag{5.16}$$

where x and $\hat{SE}(x)$ are given in formulas (5.14) and (5.15).

A sample size formula to achieve prespecified α and ε for PPS sampling with replacement is

$$n = \frac{(z_{1-\alpha/2})^2 \sum_{i=1}^{N} \frac{1}{p_i} \left(\frac{x_i}{X} - p_i \right)^2}{\varepsilon^2} \tag{5.17}$$

where ε = maximum relative difference limit on $\left| \frac{x-X}{X} \right|$,

α = probability that the absolute relative difference between the sample estimate and population value is greater than ε.

The details of the development are given in Appendix B.2.

Formula (5.17) is not practical because it contains X and x_i, $i=1, 2, \ldots, N$, the unknown population inventory and its components. However, reasonable estimates from prior knowledge (such as from a recent census) may be substituted to make formula (5.17) useful. The use of formula (5.17) is illustrated in the following example.

An Example

A population consists of 500 containers of various scrap, waste and cleanup materials. Each container contains up to 10 kg of ^{235}U in low enriched uranium (LEU), with the following summary of container contents.

Range (kg)	Number of Containers
0-2	25
2-4	25
4-6	100
6-8	150
8-10	200

Assume that the population actually contains 3444 kg of ^{235}U, and that the size of each container will be measured in units of 1 kg.

Formula (5.17) provides the sample size required to satisfy the prespecified accuracy and confidence values ε and a. Using a simulated population, and a census estimate of the inventory, the following quantity was obtained

$$\sum_{i=1}^{N} \frac{1}{p_i} \left(\frac{x_i}{X} - p_i \right)^2 = 0.00032.$$

Then formula (5.17) yields the following values of n for various combinations of a and ε.

ε[(a)]	α .10	.05	.01
.0058	26	37	64
.0029	103	147	253
.0015	385	500	500

(a) These values of ε correspond to being within 20, 10, and 5 kg of the population total of 3444 kg.

Sampling Without Replacement

The calculation of the desired formulas for PPS sampling without replacement is not straight forward, since the probability of selecting each container changes each time some other container is chosen to be in the sample. Several methods to simplify the development have been proposed (Cochran 1977, pp. 258-270). The Rao, Hartley, Cochran (RHC) method has several favorable features and is presented here.

The RHC method consists of two steps:

(i) Split the population at random into n groups, each of size N_i, where $\sum_{i=1}^{n} N_i = N$.

(ii) Draw one container from each group with probability proportional to size (within the group). The selection of a container within each group is performed independently.

These two steps provide a simple calculational foundation since the probability of selecting any container remains fixed throughout the sample selection. Other methods which skip step (i) are simple to carry out, but provide a very difficult calculational foundation.

The estimator and its estimated standard error for the total population inventory using the RHC method are (a)

$$x = \sum_{i=1}^{n} \frac{x_i P_i}{p_i} \tag{5.18}$$

$$\hat{SE}(x) = \left\{ \frac{\sum_{i=1}^{n} N_i^2 - N}{N^2 - \sum_{i=1}^{n} N_i^2} \left[\sum_{i=1}^{n} P_i \left(\frac{x_i}{p_i} - x \right)^2 \right] \right\}^{1/2} \tag{5.19}$$

where x = estimator of total population inventory,

= estimated inventory of the container sampled from the i^{th} group,

n = number of containers in the sample,

N = number of containers in the population,

N_i = number of containers in the group from which the i^{th} container was selected,

p_i = probability of selection (over the population) of the container selected from the i^{th} group,

$P_i = \sum_{j=1}^{N_i} p_j$ = the sum of the probabilities of selection for all containers in group i.

It can be shown that the standard error given in equation (5.19) is minimized by choosing the N_i such that

a) $N_1 = \ldots = N_n = \frac{N}{n} = R$ if R is an integer

(a) These formulas can be developed using results in Cochran (1977, pg. 266-267).

b) $N_1 = \ldots = N_k = [R] + 1$ if R is not an integer, where $N = nR+k$ and $0 < k < n$, and [R] is the greatest integer in R. (5.20)

$N_{k+1} = \ldots = N_n = [R]$

Note the RHC method does not choose the sample with PPS over the whole population directly, but within the randomly formed groups. The method is simple and always provides an estimator with smaller standard error than obtained by sampling with replacement. In addition, the standard error of the estimator becomes small when the measure of size is proportional to the true inventory for a given container. This is very helpful since the previous recorded inventories of containers are often used to measure "size;" hence the measure of size and inventory are approximately equal when no diversion has occurred.

A (1-a)% confidence interval on the total population inventory X using the RHC method is given by

$$x \pm z_{1-\alpha/2} \hat{SE}(x) \tag{5.21}$$

where x and $\hat{SE}(x)$ are given in formulas (5.18) and (5.19).

A sample size formula to achieve prespecified a and ε for PPS sampling without replacement using the RHC method is given by

$$n = \frac{(z_{1-\alpha/2})^2 \frac{(N^2-k^2)}{N(N-1)} \sum_{i=1}^{N} \frac{1}{p_i}\left(\frac{x_i}{X} - p_i\right)^2}{\varepsilon^2 + (z_{1-\alpha/2})^2 \left(\frac{N-k}{N(N-1)}\right) \sum_{i=1}^{N} \frac{1}{p_i}\left(\frac{x_i}{X} - p_i\right)^2} \tag{5.22}$$

with all notation as previously defined. This formula is based on the optimal choice of the N_i [as presented in equation (5.20)] to minimize equation (5.19). The details of its development are given in Appendix B.3. Again, this formula is not practical; since k and the quantity

$$\sum_{i=1}^{N} \frac{1}{p_i}\left(\frac{x_i}{X} - p_i\right)^2 \tag{5.23}$$

are unknown. The quantity in formula (5.23) must be replaced by a reasonable estimate. In practice, k can be set equal to zero to provide an approximate sample size. The use of formula (5.22) is illustrated in the following example.

An Example

The example outlined earlier in this subsection will be utilized here to illustrate the use of formula (5.22). An estimate of 0.00032 for formula (5.23) was obtained in that example. The effect of assorted choices of k on the sample size can be considered and a conservative sample size chosen. Consider the values $\alpha = 0.05$ and $\varepsilon = 0.0029$; the following values of n are obtained from formula (5.22).

k	n
0	114
25	115
50	115
75	115
100	114

The range of sample sizes obtained is not large. A choice of n = 115 is reasonable and would be conservative for some specific situations. In general, k can be chosen as zero in formula (5.22) to provide an approximate sample size. For $k = 0$, the effects of α and ε on sample size are displayed in the following table.

	α		
ε	.10	.05	.01
.0058	25	35	57
.0029	86	114	168
.0015	218	262	328

5.2 STRATIFIED INVENTORY SAMPLING PLANS

Often it is possible to stratify a population of containers on the basis of type and amount of SNM. Stratification has favorable sampling properties when strata are homogeneous (see Table 1.1), hence stratified inventory Sampling plans are considered in this section.

5.2.1 Simple Stratified Random Sampling

Simple stratified random sampling is merely simple random sampling within each stratum of the population. The estimator for total population inventory and its estimated standard error are given by Levy and Lemeshow (1980, pp. 104, 110):

$$x = \sum_{i=1}^{L} \left(\frac{N_i}{n_i} \sum_{j=1}^{n_i} x_{ij} \right) \quad (5.24)$$

$$\hat{SE}(x) = \left[\sum_{i=1}^{L} (N_i^2) \left(\frac{s_{ix}^2}{n_i} \right) \left(\frac{N_i - n_i}{N_i} \right) \right]^{1/2} \quad (5.25)$$

where x = estimator of total population inventory,

x_{ij} = estimated inventory of the j^{th} container in the i^{th} stratum,

L = number of strata,

N_i = number of containers in i^{th} stratum,

n_i = number of containers sampled from the i^{th} stratum,

$s_{ix}^2 = \frac{\sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2}{n_i - 1}$ = sampled container inventory variance for i^{th} stratum.

A $(1-a)\%$ confidence interval on the total population inventory X using simple stratified random sampling is given by

$$x \pm z_{1-\alpha/2}\, \hat{SE}(x) \tag{5.26}$$

where x and $\hat{SE}(x)$ are given in formulas (5.24) and (5.25).

A sample size formula to achieve prespecified a and ε for simple stratified random sampling is given by

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \sum_{i=1}^{L} N_i^2 \left(\frac{\sigma_{iX}^2}{n_i}\right)\left(\frac{N_i-n_i}{N_i-1}\right) \tag{5.27}$$

where ε = maximum relative difference limit on $\left|\frac{x-X}{X}\right|$,

a = probability that the absolute relative difference between the sample estimate and population value is greater than ε.

This formula is not practically useful for two reasons. The first is that X (the population inventory) and σ_{iX}^2 (the variance for each stratum) are unknown. The second reason is that n (where $n=\Sigma n_i$) cannot be solved for explicitly. Further, there are many sets of n_i which solve (5.27). Thus the question arises whether any one of the nonunique sets of n_i is "better" than the others. The answer is certainly yes, but depends upon the criterion chosen to make the decision. One criterion is to allocate n among the strata to minimize the population variance of the estimator x. The allocation of n to the n_i is then given by

$$n_i = n \left[\frac{N_i\, \sigma_{iX}}{\sum_{i=1}^{L} N_i \sigma_{iX}}\right] \tag{5.28}$$

where

$$n = \frac{(z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^2}{N_i - 1}\right) (\sigma_{iX}) \left(\sum_{i=1}^{L} N_i \sigma_{iX}\right)}{\varepsilon^2 X^2 + (z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^2}{N_i - 1}\right) \sigma_{iX}^2} \qquad (5.29)$$

The details of the development of formulas (5.27) through (5.29) are given in Appendix B.4. To use formulas (5.28) and (5.29), the population parameters X and σ_{iX} must be replaced with reasonable values.

Formulas (5.28) and (5.29) do not take into account the cost of sampling and measuring a container in each stratum. If desired, a cost function and related formulas can be developed [see Section 3.2.1 or Cochran (1977, p. 98.)]

5.2.2 Other Stratified Sampling Plans

In Section 5.2.1, a stratified sampling plan based on simple random sampling within the strata was considered. The following question arises: Can other sampling techniques be used to allocate n into n_i [where n and the n_i are given by formulas (5.28) and (5.29)]? The answer to this question is generally "no". Although estimators, confidence intervals, and sample size formulas can be developed for any sampling plan imaginable, these equations can be quite difficult and time consuming to derive. Thus, the practice of using the formulas at hand while modifying the sample gathering techniques looks inviting. However, this practice has not been studied in depth and in general should be avoided.

Stratified sampling plans using single or multi-stage cluster techniques or using PPS techniques are certainly to be considered and not dismissed due to the effort required to derive the formulas for the population inventory estimate, its estimated standard error, inventory confidence interval, and sample size. Familiarity with the techniques is sufficient for a user to formulate a sampling plan tailored to a specific facility; a statistician can then derive the formulas for such a sampling plan. The following stratified sampling plans and extensions thereof appear suited to inventory determination problems:

1) probability proportional to size (PPS) sampling of containers within strata

2) single-stage cluster sampling within strata and: a) clusters chosen by simple random sampling, or b) clusters chosen with PPS

3) two-stage cluster sampling within strata/first stage cluster sampled as in (2a) or (2b) and: a) second stage sample of containers chosen by simple random sampling, or b) second stage sample of containers chosen with PPS

Many other sampling plans may also be suitable.

5.3 CHOOSING AN INVENTORY SAMPLING PLAN

Several sampling plans for estimating the SNM inventory of a population of containers have been presented and discussed in Sections 5.1 and 5.2. This section will discuss how to choose a sampling plan to fit a specific situation.

Several factors impact the choosing of a sampling plan. These factors can be grouped into four major areas, which are described below.

- Characteristics of the population of containers - Are there several types of SNM in the population? For each type of SNM, are there many containers with approximately the same amount of SNM, or do the amounts vary widely? How are the containers physically stored? Are there natural groupings of containers such as rooms or storage units?
- Sampling Cost - How expensive is it to locate and inventory a specific container? Does this cost vary by container or groups of containers? If so, how much variability is there?
- Accuracy Requirement - How accurate an inventory estimate is required?
- Confidence Requirement - How much confidence is required that the accuracy requirement is met?

The first two areas are concerned with which sampling techniques are appropriate or preferable, while the last two incorporate confidence and accuracy requirements for the sample inventory estimate. Note that sample size comes to play in the last two areas; more containers must be sampled to achieve higher confidence and/or accuracy. Thus sample size is often one of the factors deciding which of several sampling plans is chosen. In practice, all of the major factors impacting the choice of a sampling plan must be weighed, and often compromises must be made in some areas to meet needs in the others.

An outline of the steps in choosing a sampling plan are given below.

1. State the population of interest.

   If a facility contains more than one type of SNM, each type may be specified as a population to be inventoried separately.

2. Decide if a stratified sampling plan is indicated.

   If the population contains groups of many containers where each container in a group has approximately the same amount of SNM, then a stratified sampling plan should be chosen. Benefits (in reduced standard errors) are greatest with a small number of homogeneous strata.

3. Consider sampling and inventory costs.

   If the cost of locating individual containers is high, sampling plans involving cluster sampling may be indicated. This must be weighed against the disadvantage of cluster sampling, which is increased standard errors for a fixed sample size. This disadvantage is the greatest when the containers within the clusters are homogeneous.

   If the cost of locating and inventorying containers changes by container or groups of containers, then a sampling plan involving the PPS technique is indicated.

4. Decide if all containers or groups of containers are equally important.

   If some containers or groups of containers are more important (maybe much larger) than others, then a sampling plan involving PPS sampling is indicated.

5. State accuracy and confidence requirements.

Steps 1-4 will basically indicate the form of the sampling plan required. Accuracy and confidence requirements will define the sample size required. Often this is an iterative process; if the required sample size is too large it may be reduced by modifying the accuracy and or confidence requirements.

6.0 SAMPLING VERSUS CENSUS

Sampling has some favorable aspects for the tasks of estimating the SNM inventory and verifying prior content information for a population of containers. For both, sampling involves a savings in time and money since fewer containers are handled and measured. In addition, sampling may reduce the impact of certain errors, since no errors are made for containers not sampled. However, each of these positive aspects has a companion negative aspect. Compared to a complete census, additional time and cost may be incurred to locate and handle only part of the containers instead of all of them. Further, although no measurement or transcription errors are made on containers not sampled, errors due to inferences about the whole population based on a sample occur.

A general point made above should be kept in mind; even though performing an inventory or verification by census involves all containers, errors can still occur. These may be measurement errors, transcription errors, or many others. These errors and the fact that sampling may reduce some of these errors (and not affect others) should be kept in mind in the following discussions.

Sampling and census results will be compared separately for inventory determination and verification. Since precise comparisons based on general situations are hard to make, examples will be utilized to provide numerical comparisons.

6.1 COMPARISON FOR PRIOR MEASUREMENT VERIFICATION

The purpose of prior measurement verification is to detect if a diversion has taken place. For the verification effort, diversions are classified as stemming from large or small defects. A large defect is a removal detectable by a simple observation or a single measurement, while a small defect can not be detected by investigating a single container. Let us consider large defects first.

For both census and sampling methods, it is assumed by the definition of a large defect that a defective container will be detected with certainty. If it is not, because of instrument errors for example, then the failure to detect a defect is as likely for a container chosen by sampling as by complete census. That is, sampling does not increase the chance of detection errors for containers under investigation. For containers not chosen in a sampling plan, there is no chance of detection errors; on the other hand there is no chance of detecting a defect either. The increase in the probability of nondetection due to not including a defective container in the sample usually far outweighs the small decrease in the probability of detection errors when sampling.

An analytic comparison of probability of detection for sampling versus census methods is now considered. The following notation will be useful:

N = number of containers in the population,

n = number of containers in the sample,

$f = \frac{n}{N}$ = the fraction of containers sampled,

d = the number of large defects in the population,

p = probability that a defective container in the sample will be detected,

β = probability that the sample does not contain a defective container,

β' = probability of not detecting at least one defective container in the population.

For illustration, assume a simple random sampling plan with replacement. Then

$$\beta' = (1 - pf)^d \tag{6.1}$$

and

$$\beta = (1 - f)^d \tag{6.2}$$

Note that $f = 1$ represents a complete census, and that $\beta' = \beta$ when $p = 1$. The probabilities of nondetection for census and sampling methods are

$$\beta'_{census} = (1 - p)^d \tag{6.3}$$

$$\beta'_{sampling} = (1 - pf)^d \qquad (6.4)$$

Often β is mistakenly considered as the probability of nondetection (β'). It is of interest to see how far apart β and β' can be, and at the same time investigate the difference in probability of detection for sampling and for complete census methods. An example for a population of 100 and varying values of p, f, and d is presented in Table 6.1. As expected, the lower the value of p, the further $\beta'_{sampling}$ is from β. The differences between β'_{census} and $\beta'_{sampling}$ are relatively small for larger values of p, f, and d. For example, referring to Table 6.1 (c) when d = 10 and p = 0.95, a sampling fraction of 0.37 only causes a 0.01 increase in the probability of nondetection over the complete census situation.

The above example illustrates several general conclusions.

- Higher probabilities of nondetection for both sampling and census methods result from inefficient detection techniques (techniques with low p values). Sampling compares poorly with a complete census for small sampling fractions when detection techniques are inefficient.
- As expected, sampling compares more favorably with a complete census as the sampling fraction is increased. The sampling fraction required for a high probability of detection will depend upon factors such as the detection efficiency and number of defectives in the population.
- Assuming efficient detection techniques are used, sampling compares quite well with a complete census when the number of defects in the population is large. It can also compare quite well when the number of defects in the population is small, if larger sampling fractions are used.

6.2 COMPARISON FOR INVENTORY DETERMINATION

Inventory determination involves the estimation of the SNM content of a population. For a population of containers, estimates of the SNM in each container investigated must be made. This involves performing weight and assay

TABLE 6.1. A Comparison of Probabilities of Non-Detection for Sampling Versus Census Verification

(a) d = 2

			P			
			.50	.75	.90	.95
•• census			.25	.06	.01	.003
	f	β				
β′ sampling	.55	.20	.53	.35	.26	.23
	.68	.10	.44	.24	.15	.13
	.78	.05	.37	.17	.09	.07
	.90	.01	.30	.11	.04	.02

(b) d = 5

			p			
			.50	.75	.90	.95
•• census			.03	.001	-0	-0
	f	β				
β′ sampling	.28	.20	.47	.31	.23	.21
	.37	.10	.36	.20	.13	.11
	.45	.05	.28	.13	.07	.06
	.60	.01	.17	.05	.02	.01

(c) d = 10

			P			
			.50	.75	.90	.95
•• census			.001	≈0	≈0	≈0
	f	β				
β′ sampling	.15	.20	.46	.30	.23	.21
	.21	.10	.33	.18	.12	.11
	.26	.05	.25	.11	.07	.06
	.37	.01	.13	.04	.02	.01

Note: Calculations assume simple random sampling and are based on formulas (6.3) and (6.4).

measurements, which are subject to error. Hence, even inventory estimates based on a complete census will be in error. We are interested in how much additional error is involved when choosing to sample instead of performing a complete census. This will depend upon the sampling plan and sample size.

Recall that for any particular sampling technique, the sample size is calculated based on a requirement that the relative difference between the sample estimate (x) and the true population value (X) of the inventory is less than some specified value ε with probability $1 - \alpha$, i.e.,

$$P\left[\left|\frac{x - X}{X}\right| \leq \varepsilon\right] = 1 - \alpha. \tag{6.5}$$

Assuming measurement variability is small compared to sampling variability, the additional uncertainty in the estimate (over measurement uncertainty) due to sampling is obtained indirectly from formula (6.5). It is obtained directly from the standard error formulas for a specific sampling plan found in Chapter 5. Recall that the standard error formulas are dependent upon the choice of α and ε in formula (6.5).

An Example

As a simple example, consider a population of 1000 trays of high enriched uranium (HEU) fuel pellets. Assume the trays contain between 1.4 and 1.6 kg of ^{235}U in HEU with a mean content of 1.5 kg and a standard deviation of 0.057 kg.[a] For a complete census of the population assume the inventory estimate is 1500 kg with standard error 0.1 kg. We are interested in how much additional variability results from taking a sample. Again, keeping the example simple, consider a simple random sampling plan where we want an inventory estimate within 10 kg of the true value with probability 0.95. Then,

$$\varepsilon = \frac{10}{1500} = 0.0067,$$

(a) In general, the population mean and standard deviation are not known. In practice reasonable estimates are used, often being based on past information.

$$\alpha = 0.05,$$

and from formula (5.6)

$$n = \frac{(z_{1-\alpha/2})^2 \, N \, \hat{V}_X^2}{(z_{1-\alpha/2})^2 \, \hat{V}_X^2 + (N-1)\,\varepsilon^2}$$

$$= \frac{(1.96)^2 \, (1000) \left(\frac{0.057}{1.5}\right)^2}{(1.96)^2 \left(\frac{0.057}{1.5}\right)^2 + 999(0.0067)^2}$$

$$= 110.08 \simeq 111.$$

A population meeting the above assumptions was simulated and a sample of size 111 chosen without replacement. The following was produced using formulas (5.3-5.4) :

$$x = \frac{1000(166.07)}{111} = 1496.13$$

$$\hat{SE}(x) = (1000)\sqrt{\frac{1000-111}{1000}} \left(\frac{0.059}{\sqrt{111}}\right) = 5.28$$

Thus, for the sample chosen, an additional standard error of 5.28 kg is estimated due to sampling and measurement and analysis error (which was included in the simulation). The latter errors amount to 0.03 kg, indicating sampling reduced the census standard error by 0.07 kg (due to fewer measurements and analyses). Sampling increased the census standard error by 5.25 kg, yielding the net effect standard error of 5.28 kg. Before leaving the example, note that $x = 1496.13$ kg is within 10 kg of $X = 1500$ kg as specified (with a = 0.05).

In considering the above example, the 5.25 kg increase in the standard error (from the initial 0.1 kg) seems large. Yet, only slightly more than one-tenth of the population was inventoried. The advantages and disadvantages of sampling must be weighed in deciding what is needed. Note that the above is only one example and even it would have looked different for different values of ε, α and the population standard error (0.1 kg used above). The methodology of comparing sampling and census results illustrated above is important however, and can be used as a guide for other situations and the investigation of other sampling plans.

6.3 THE EFFECT OF SAMPLING ON INVENTORY DIFFERENCE

One of the uses of inventory estimates is in the control of SNM, where Inventory Difference (ID) is a basic quantity of interest. For any one material balance period, ID is defined as

$$ID = B + A - R - E$$

where B = beginning inventory,
A = additions,
R = removals,
E = ending inventory.

It is the beginning and ending inventories which would be affected by sampling instead of a complete census; additions and removals must be checked by a complete census.

Of interest is the quantitative impact of sampling on the variance of ID. The impact will come from using a sampling plan to estimate the beginning and ending inventories, i.e.

$$\mathrm{Var}(ID)_{\text{sampling}} - \mathrm{Var}(ID)_{\text{census}} = \mathrm{Var}(B-E)_{\text{sampling}} - \mathrm{Var}(B-E)_{\text{census}} . \qquad (6.6)$$

Formula (6.6) assumes that arrivals and removals are independent of the beginning and ending inventories; this is often a reasonable assumption. If B and E are independent,[a] formula (6.6) becomes

$$\mathrm{Var(ID)}_{\text{sampling}} - \mathrm{Var(ID)}_{\text{census}} = \left(\mathrm{Var(B)}_{\text{sampling}} - \mathrm{Var(B)}_{\text{census}}\right) + \left(\mathrm{Var(E)}_{\text{sampling}} - \mathrm{Var(E)}_{\text{census}}\right) \quad (6.7)$$

Formula (6.7) shows that the effect of sampling on the variance of ID is felt twice, once for the beginning inventory sampling and once for the ending inventory sampling. If the variances of B and E are equal for sampling and for a census, formula (6.7) becomes

$$\mathrm{Var(ID)}_{\text{sampling}} - \mathrm{Var(ID)}_{\text{census}} = 2\left(\mathrm{Var(B)}_{\text{sampling}} - \mathrm{Var(B)}_{\text{census}}\right) \quad (6.8)$$

Literally then, the sampling impact on the variance of ID is twice the impact on the variance of an individual inventory estimate. The standard error factor is then $\sqrt{2} \simeq 1.4$.

The actual magnitude of the impacts in formulas (6.7) or (6.8) will depend upon the sampling plan and the values of n, a and ε selected for estimating a given inventory.

(a) While this is not as reasonable an assumption as the previous one, in practice ID is often calculated with values of B, A, R and E modified to reduce or eliminate sources of covariance.

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APPENDIX A

DEVELOPMENT OF VERIFICATION SAMPLING RESULTS

APPENDIX A

DEVELOPMENT OF VERIFICATION SAMPLING RESULTS

A.1 THE PROBABILITY OF DETECTING DIVERSION FROM SEVERAL STRATA WHEN INSPECTING INVENTORIES BY ATTRIBUTE SAMPLING[a]

When zero defect attribute sampling is used to select sample sizes for verification of inventories to detect losses equal to or exceeding a specified goal quantity G, each stratum or class of material is sampled on the premise that the entire goal quantity is taken from that class. It can be shown that the probability of detecting diversion by the strategy of removing part of G from each of several strata is at least as high as detecting the diversion of the entire quantity G from a single stratum.

Let the inspection sample size for each of i inventory strata be chosen with the binomial sample size equation as follows:

$$n_i = N_i \left(1 - \beta^{1/d_i}\right)$$

where n_i = sample size from stratum i,

N_i = total number of containers in i^{th} stratum,

β = the probability of not obtaining at least one defect (out of d_i) in the sample,

d_i = $G/\gamma A$ = number of containers needed to obtain G units from the i^{th} stratum (by removing γA units from each container),

G = the goal quantity,

A = the amount of SNM per container in the i^{th} stratum,

γ = fraction of A removed from each container.

Assume there are L strata and G is obtained by diversion of v_i items from each, where the v_i are not necessarily equal. Then:

(a) Based on work by K. B. Stewart of Pacific Northwest Laboratory (see Stewart and Jung 1980).

$$G = v_1\gamma_1 A_1 + v_2\gamma_2 A_2 + \ldots + v_L\gamma_L A_L$$

If n_i items from each stratum are inspected, as specified by the binomial equation, the probability of not detecting the loss of G is equal to or less than β since:

$$\left.\begin{array}{c}\text{Probability}\\ \text{of Nondetection}\\ \text{in Stratum } i\end{array}\right\} = Q_i = \prod_{j=1}^{v_i}\left(\frac{N_i - n_i - j + 1}{N_i - j + 1}\right) = \prod_{j=1}^{v_i}\left(1 - \frac{n_i}{N_i - j + 1}\right)$$

$$\left.\begin{array}{c}\text{Probability}\\ \text{of Nondetection}\\ \text{in all L Strata}\end{array}\right\} = Q = \prod_{i=1}^{L} Q_i$$

now

$$Q_i = \prod_{j=1}^{v_i}\left(1 - \frac{n_i}{N_i - j + 1}\right) \leq \left(1 - \frac{n_i}{N_i}\right)^{v_i}$$

and

$$\left(1 - \frac{n_i}{N_i}\right)^{v_i} = \left(1 - \frac{N_i(1-\beta^{1/d_i})}{N_i}\right)^{v_i} = \beta^{v_i/d_i}$$

then

$$Q = \prod_{i=1}^{L} Q_i \leq \prod_{i=1}^{L} \beta^{v_i/d_i} = \prod_{i=1}^{L} \beta^{v_i/(G/\gamma_i A_i)}$$

$$= \prod_{i=1}^{L} \beta^{v_i\gamma_i A_i/G}$$

$$= \beta^{(v_1\gamma_1 A_1 + v_2\gamma_2 A_2 + \ldots + v_L\gamma_L A_L)/G}$$

Thus,

$$Q \leq \beta$$

A2 DEVELOPMENT OF A SAMPLE SIZE EQUATION FOR VERIFICATION WITH A VARIABLES SAMPLING PLAN

The sample size equation for the verification variables sampling plan given in Section 3.2.1 is developed here. The sample size equation is based on the hypothesis to be tested:

$$H_0: \mu_D = 0$$
$$H_A: \mu_D \neq 0$$

with test statistic

$$D^* = \frac{\hat{D}}{\hat{\sigma}_{\hat{D}}} \qquad (A.1)$$

where μ_D = true unknown population mean of the difference between the prior and current SNM content of a container,

$\hat{D}$ = an estimate of μ_D based on a sample,

$\hat{\sigma}_{\hat{D}}$ = sample estimate of the standard deviation of $\hat{D}$.

Under the null hypothesis, D^* is assumed to be distributed approximately standard normal (mean = 0, variance = 1). We are interested in a two-tailed test with the probability of a type I error equal to a and with probability 1 - β of detecting a difference of G. For a positive G (removal of SNM) the test is represented graphically in Figure A.1.

An equation which involves n, a, and β is easily obtained by noting

$$z_{1-\alpha/2}\hat{\sigma}_{\hat{D}|H_0} = G + z_\beta \hat{\sigma}_{\hat{D}|H_A}$$

$$z_{1-\alpha/2}\sqrt{\hat{\sigma}^2_{\hat{D}s} + \hat{\sigma}^2_{\hat{D}r|H_0}} = G - z_{1-\beta}\sqrt{\hat{\sigma}^2_{\hat{D}s} + \hat{\sigma}^2_{\hat{D}r|H_A}} \qquad (A.2)$$

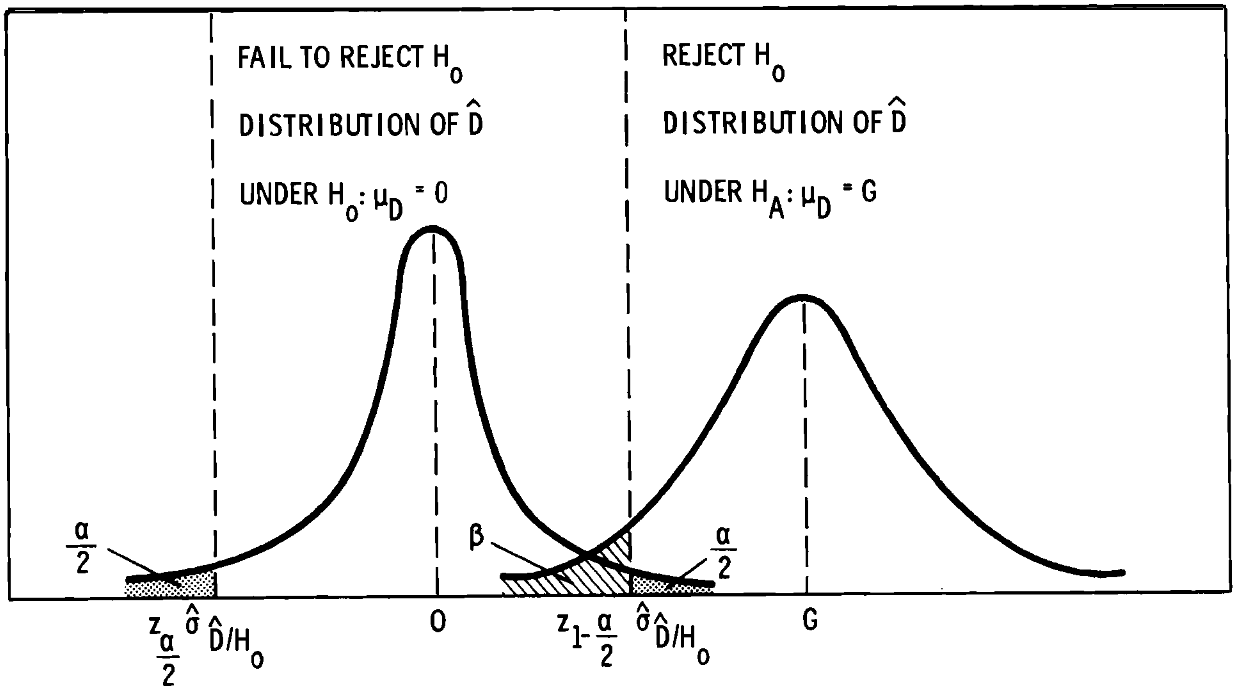


FIGURE A.1. Graphical Development of Variables Verification Sample Size Equation

where z_v = the v^{th} percentile of the standard normal distribution,

$\hat{\sigma}^2_{\hat{D}s}$ = systematic component of $\hat{D}$ variance,

$\hat{\sigma}^2_{\hat{D}r|H_0}$, $\hat{\sigma}^2_{\hat{D}r|H_A}$ = random component of $\hat{D}$ variance which may change from H_0 to H_A,

G = goal quantity to be detected,

α = probability of a type I error,

β = probability of a type II error.

Because of symmetry of the normal distribution, equation (A.2) is also appropriate for the other tail test (when the true difference G is negative).

Now, expressing $\hat{\sigma}^2_{\hat{D}r|H_A}$ as some multiple f of $\hat{\sigma}^2_{\hat{D}r|H_0}$ in equation (A.2) and letting $\hat{\sigma}^2_{\hat{D}r} = \hat{\sigma}^2_{\hat{D}r|H_0}$, we have

$$z_{1-\alpha/2}\sqrt{\hat{\sigma}^2_{\hat{D}s} + \hat{\sigma}^2_{\hat{D}r}} = G - z_{1-\beta}\sqrt{\hat{\sigma}^2_{\hat{D}s} + f\hat{\sigma}^2_{\hat{D}r}}$$

and

$$z_{1-\alpha/2}\sqrt{1+R} = \frac{G}{\hat{\sigma}_{\hat{D}s}} - z_{1-\beta}\sqrt{1+fR} \qquad \text{(A.3)}$$

where

$$R = \frac{\hat{\sigma}^2_{\hat{D}r}}{\hat{\sigma}^2_{\hat{D}s}} .$$

It is assumed that $1 \leq f \leq 4$, i.e., that a diverter will not inflate the random error variance by more than a factor of four, since beyond a certain point a small diversion would become a large diversion.

Note that equation (A.3) contains the desired sample size n implictly through the random error variance component. Since $\hat{D} = \Sigma N_i\bar{d}_i$, the random error variance of $\hat{D}$ is

$$\hat{\sigma}^2_{\hat{D}r} = \text{Var}\left(\sum_{i=1}^{L} N_i\bar{d}_i\right)$$

$$= \sum_{i=1}^{L} N_i^2\text{Var}(\bar{d}_i)$$

$$= \sum_{i=1}^{L} N_i^2\left(\frac{\text{Var}(d_i)}{n_i}\right)$$

$$= \sum_{i=1}^{L} N_i^2 \left(\frac{\hat{\sigma}^2_{dr_i}}{n_i} \right) \quad (A.4)$$

where N_i = number of containers in stratum i,

n_i = sample size in stratum i,

L = number of strata in the population,

$\hat{\sigma}^2_{dr_i}$ = estimate of random error variance for individual container differences in stratum i.

The problem of solving equation (A.3) for n and the n_i yet remains. In the ideal case where $R = 0$, equation (A.3) can only be solved (given α, β, and G) if

$$\frac{G}{\hat{\sigma}_{\hat{D}s}} \geq z_{1-\alpha/2} + z_{1-\beta} \quad (A.5)$$

Otherwise the goal G is too small relative to systematic error for even 100% sampling to provide a test with power $1-\beta$ for the chosen value of α.

The power to detect small diversions is directly related to the variance of the difference statistic ($\hat{D}$). Since the variance is made up of both random and systematic error components, both components influence the probability of detection. However, only the random component reduces with increasing sample size. As a result, there is a point where a further reduction in the size of the random error variance component has little effect on the overall measurement uncertainty of the difference statistic. The relationship

$$\sigma_{total} = \sqrt{\sigma^2_{systematic} + \frac{\sigma^2_{random}}{n}}$$

where n is the verification sample size, shows how the random error component decreases with increasing n while the systematic error component remains constant. Hough, Schneider, Stewart, et al. (1974) suggest the point of diminishing returns is when the random error component standard deviation is one-half the systematic error standard deviation, since this point is within 12 percent of the minimum, e.g.

$$\sigma_{total} = \sqrt{[(1)^2 + (\tfrac{1}{2})^2]\ \sigma^2_{systematic}} \tag{A.6}$$

$$= 1.12\ \sigma_{systematic}$$

If the lower limit for total error given in equation (A.6) is adopted and it is assumed that $1 \leq f \leq 4$, then equation (A.3) can only be solved (given a, β and G) if

$$\frac{G}{\hat{\sigma}_{\hat{D}s}} \geq z_{1-\alpha/2}\sqrt{1+(\tfrac{1}{2})^2(\tfrac{1}{4})} + z_{1-\beta}\sqrt{1+(\tfrac{1}{2})^2}$$

$$\frac{G}{\hat{\sigma}_{\hat{D}s}} \geq 1.03z_{1-\alpha/2} + 1.12z_{1-\beta} \tag{A.7}$$

If equation (A.3) can not eventually be solved for n because of restriction (A.7), then a and/or β must be reduced or G increased. This assumes $\hat{\sigma}_{\hat{D}s}$ is fixed. A specific example of this general solution was presented by Hough, Schneider, Stewart, et al. (1974), who suggested solving the equation

$$fR = \frac{1}{4} \tag{A.8}$$

in place of equation (A.3). This equation is obtained by choosing

$$\hat{\sigma}_{\hat{D}r|H_A} = \frac{1}{2}\hat{\sigma}_{\hat{D}s} ,$$

as discussed earlier in this section.

APPENDIX B

DEVELOPMENT OF INVENTORY SAMPLING PLAN SAMPLE SIZES

APPENDIX B

DEVELOPMENT OF INVENTORY SAMPLING PLAN SAMPLE SIZES

B.l SAMPLE SIZE FOR SIMPLE RANDOM SAMPLING

In general, we desire a sample size n which satisfies

$$Pr\,[|x-X| \geq \varepsilon X] = \alpha \qquad (B.1)$$

where X = total population inventory,
x = estimator of X based on a sample,
ε = prespecified accuracy requirement on x,
α = prespecified confidence level on accuracy requirement.

Given a population and values of a and ε, n is obtained by solving

$$\varepsilon X = (z_{1-\alpha/2})\ SE(x) \qquad (B.2)$$

where $SE(x)$ is the population standard error for the total inventory estimator.

For simple random sampling, we have from Levy and Lemeshow (1980, p. 43).

$$SE(x) = N\left(\frac{N-n}{N-1}\right)^{\frac{1}{2}} \frac{\sigma_X}{\sqrt{n}} \qquad (B.3)$$

where σ_X is the population standard deviation. Then formula (8.2) may be used for calculating the sample size n given a, ε and σ_X:

$$\varepsilon X = z_{1-\alpha/2} N\left(\frac{N-n}{N-1}\right)^{\frac{1}{2}} \frac{\sigma_X}{\sqrt{n}}\ . \qquad (B.4)$$

Solving this for n gives

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \; N^2 \left(\frac{N-n}{N-1}\right) \frac{\sigma_X^2}{n}$$

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \; N^2 \left(\frac{N}{N-1} - \frac{n}{N-1}\right) \frac{\sigma_X^2}{n}$$

$$\frac{1}{n} (z_{1-\alpha/2})^2 \left(\frac{N^3}{N-1}\right) \sigma_X^2 = \varepsilon^2 X^2 + (z_{1-\alpha/2})^2 \left(\frac{N^2}{N-1}\right) \sigma_X^2$$

$$n = \frac{(z_{1-\alpha/2})^2 \left(\frac{N^3}{N-1}\right) \sigma_X^2}{\varepsilon^2 X^2 + (z_{1-\alpha/2})^2 \left(\frac{N^2}{N-1}\right) \sigma_X^2}$$

$$n = \frac{(z_{1-\alpha/2})^2 \; N \; \sigma_X^2}{\varepsilon^2 \overline{X}^2 \; (N-1) + (z_{1-\alpha/2})^2 \; \sigma_X^2}$$

$$n = \frac{(z_{1-\alpha/2})^2 N V_X^2}{(z_{1-\alpha/2})^2 V_X^2 + (N-1)\varepsilon^2} \qquad \text{(B.4)}$$

where $V_X^2 = \frac{\sigma_X^2}{\overline{X}^2}$ = the population coefficient of variation squared.

This equation is not useful due to the presence of V_X^2; **if** X and σ_X^2 were known, we would not need a sample to estimate them. However, from previous inventories, a reasonable estimate of V_X^2 should be available. Letting $\ddot{V}_X^2$ represent this estimate, equation (B.4) becomes

$$n = \frac{(z_{1-\alpha/2})^2 N \hat{V}_X^2}{(z_{1-\alpha/2})^2 \hat{V}_X^2 + (N-1)\varepsilon^2} \; . \qquad \text{(B.6)}$$

This is formula (5.6) in Section 5.1.1.

B.2 SAMPLE SIZE FOR PPS SAMPLING WITH REPLACEMENT

The desired sample size is given by formula (B.2) or, equivalently,

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \, \text{Var}(x) \tag{B.7}$$

where, for PPS with replacement,[a]

$$\text{Var}(x) = \frac{1}{n} \sum_{i=1}^{N} p_i \left(\frac{x_i}{p_i} - X \right)^2 . \tag{B.8}$$

Then, substituting equation (B.8) into (B.7) and solving we have

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \left[\frac{1}{n} \sum_{i=1}^{N} p_i \left(\frac{x_i}{p_i} - X \right)^2 \right]$$

$$n = \frac{(z_{1-\alpha/2})^2 \sum_{i=1}^{N} p_i \left(\frac{x_i}{p_i} - X \right)^2}{\varepsilon^2 X^2}$$

$$n = \frac{(z_{1-\alpha/2})^2 \sum_{i=1}^{N} \frac{1}{p_i} \left(\frac{x_i}{X} - p_i \right)^2}{\varepsilon^2} \tag{B.9}$$

This is formula (5.17) in Section 5.1.3.

(a) This formula can be developed using results in Cochran (1977, Chapter 9A).

B.3 SAMPLE SIZE FOR PPS SAMPLING WITHOUT REPLACEMENT USING THE RHC METHOD

The desired sample size formula for PPS sampling without replacement using the Rao, Hartley, Cochran (RHC) method is given by formula (B.7) where[(a)]

$$\mathrm{Var}(x) = \left[1 - \frac{n-1}{N-1} + \frac{k(n-k)}{N(N-1)}\right]\left[\frac{1}{n}\sum_{i=1}^{N} p_i\left(\frac{x_i}{p_i} - X\right)^2\right]. \qquad (B.10)$$

Then, substituting formula (B.IO) into (B.7) and solving, we have

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2\left[1- \frac{n-1}{N-1} + \frac{k(n-k)}{N(N-1)}\right]\left[\frac{1}{n}\sum_{i=1}^{N} p_i\left(\frac{x_i}{p_i} - X\right)^2\right]$$

$$\varepsilon^2 = (z_{1-\alpha/2})^2\left[\frac{1}{n}\left(\frac{N^2-k^2}{N(N-1)}\right) - \frac{N-k}{N(N-1)}\right]\left[\sum_{i=1}^{N}\frac{1}{p_i}\left(\frac{x_i}{X} - p_i\right)^2\right]$$

$$n = \frac{(z_{1-\alpha/2})^2\left(\frac{N^2-k^2}{N(N-1)}\right)\sum_{i=1}^{N}\frac{1}{p_i}\left(\frac{x_i}{X} - p_i\right)^2}{\varepsilon^2 + (z_{1-\alpha/2})^2\left(\frac{N-k}{N(N-1)}\right)\sum_{i=1}^{N}\frac{1}{p_i}\left(\frac{x_i}{X} - p_i\right)^2} \qquad (B.11)$$

This is formula (5.22) in Section 5.1.3.

B.4 SAMPLE SIZE FOR SIMPLE STRATIFIED RANDOM SAMPLING

The desired sample size formula for simple stratifed random sampling is given by formula (B.7) where, from Levy and Lemeshow (1980, p. 108)

$$\mathrm{Var}(x) = \sum_{i=1}^{L} N_i^2\left(\frac{\sigma_{iX}^2}{n_i}\right)\left(\frac{N_i-n_i}{N_i-1}\right). \qquad (B.12)$$

(a) This formula can be developed using results in Cochran (1977, Chapter 9A).

Substituting (8.12) into (8.7) yields

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \left[\sum_{i=1}^{L} N_i^2 \left(\frac{\sigma_{iX}^2}{n_i}\right)\left(\frac{N_i - n_i}{N_i - 1}\right)\right]. \tag{B.13}$$

Formula (B.13) does not have a unique solution. Choosing the criterion that a sample size n is desired which minimizes the population variance of the estimator x yields (see Cochran 1977, pp. 96-98) the allocation scheme

$$n_i = n \left[\frac{N_i \sigma_{iX}}{\sum_{i=1}^{L} N_i \sigma_{iX}} \right]. \tag{B.14}$$

This is formula (5.28) in Section 5.2.1. Substituting (B.14) into (B.12), the minimum variance becomes

$$\mathrm{Var}(x) = \sum_{i=1}^{L} \left(\frac{N_i^2}{N_i - 1}\right)\left(\sigma_{iX}^2\right)\left[\left(\frac{N_i}{n}\right)\left(\frac{\sum_{i=1}^{L} N_i \sigma_{iX}}{N_i \sigma_{iX}}\right) - 1\right]$$

$$= \sum_{i=1}^{L} \left(\frac{N_i^2}{N_i - 1}\right)\left(\sigma_{iX}^2\right)\left(\frac{\sum_{i=1}^{L} N_i \sigma_{iX}}{n \sigma_{iX}} - 1\right). \tag{B.15}$$

Substituting **(0.15)** in formula (B.7) and solving for n yields

$$\varepsilon^2 X^2 = (z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^2}{N_i - 1}\right)\left(\sigma_{iX}^2\right)\left(\frac{\sum_{i=1}^{L} N_i \sigma_{iX}}{n \sigma_{iX}} - 1\right)$$

$$(z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^{\,2}}{N_i-1}\right)\left(\sigma_{iX}^2\right)\left(\frac{\sum_{i=1}^{L} N_i\sigma_{iX}}{n\sigma_{iX}}\right) = \varepsilon^2 X^2 + (z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^{\,2}}{N_i-1}\right)\left(\sigma_{iX}^2\right)$$

$$n = \frac{(z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^{\,2}}{N_i-1}\right)\left(\sigma_{iX}\right)\left(\sum_{i=1}^{L} N_i\sigma_{iX}\right)}{\varepsilon^2 X^2 + (z_{1-\alpha/2})^2 \sum_{i=1}^{L} \left(\frac{N_i^{\,2}}{N_i-1}\right)\sigma_{iX}^2} . \qquad \text{(B.16)}$$

This is formula (5.29) in Section 5.2.1.

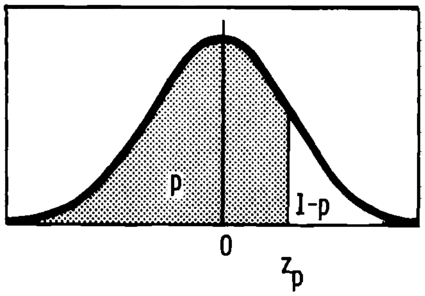
APPENDIX C

SELECTED STATISTICAL DISTRIBUTION TABLES

APPENDIX C

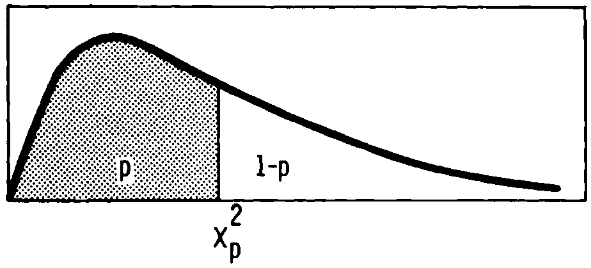
SELECTED STATISTICAL DISTRIBUTION TABLES

TABLE C.1. Percentile Values (z_p) for the Standard Normal Distribution



z_p	p	z_p	p	z_p	p
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.988
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.960	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999

TABLE C.2. **Percentile Values (χ^2) for the Chi-Square Distribution with ν Degrees of Freedom P**



ν	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.25}$	$\chi^2_{.50}$	$\chi^2_{.75}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$	$\chi^2_{.999}$
1	.0000	.0002	.0010	.0039	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88	10.8
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6	13.8
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8	16.3
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9	18.5
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7	20.5
6	.676	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5	22.5
7	.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3	24.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0	26.1
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6	27.9
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2	29.6
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8	31.3
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3	32.9
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8	34.5
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3	36.1
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8	37.7
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3	39.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7	40.8
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2	42.3
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6	43.8
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0	45.3
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4	46.8
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8	48.3
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2	49.7
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6	51.2
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9	52.6
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	54.1
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	55.5
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	56.9
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	58.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	59.7
40	20.7	22.2	24.4	26.5	29.1	33.7	39.3	45.6	51.8	55.8	59.3	63.7	66.8	73.4
50	28.0	29.7	32.4	34.8	37.7	42.9	49.3	56.3	63.2	67.5	71.4	76.2	79.5	86.7
60	35.5	37.5	40.5	43.2	46.5	52.3	59.3	67.0	74.4	79.1	83.3	88.4	92.0	99.6
70	43.3	45.4	48.8	51.7	55.3	61.7	69.3	77.6	85.5	90.5	95.0	100	104	112
80	51.2	53.5	57.2	60.4	64.3	71.1	79.3	88.1	96.6	102	107	112	116	125
90	59.2	61.8	65.6	69.1	73.3	80.6	89.3	98.6	108	113	118	124	128	137
100	67.3	70.1	74.2	77.9	82.4	90.1	99.3	109	118	124	130	136	140	149

APPENDIX D

GLOSSARY

APPENDIX D

GLOSSARY

D.1 SAMPLING TERMINOLOGY AND NOTATION

Attributes Sampling Plan:	A verification sampling plan designed to detect containers with large defects.
Census:	A complete accounting of a population, i.e. 100% sampling.
Cluster:	A group of elements from a population related by physical proximity. The elements may or may not have similar characteristics.
Curtailed Sequential Sampling:	A sequential sampling plan with a predetermined maximum sample size. A decision for the related test is made after collecting the last sample unit if not done so previously.
Inventory Sampling Plan:	A sampling plan designed to produce a quantitative estimate of the SNM content (inventory) of a population of containers.
One-stage Cluster Sample:	A sample composed of groups of population elements. Every element in each group (cluster) is contained in the sample.
Population:	The original set of elements of interest.
Probability Proportional to Size (PPS) Sampling:	A procedure in which population elements or clusters of elements are chosen with probability proportional to their size. Size may be in terms of a measurable or countable variable.
Random Sample:	A sample in which each element is selected from the population by chance.
Sample:	A subset of elements from the population.
Sampling With Replacement:	A sample selection technique which permits a population element to be included in the sample more than once.
Sampling Without Replacement:	A sample selection technique which does not permit a population element to be included in the sample more than once.

Sequential Sampling: An iterative procedure applied after the selection of a sample point which decides to either stop sampling or to choose an additional sample point based on a predetermined decision rule.

Simple Random Sample: A random sample in which each element has an equal chance of being selected.

Stratified Random Sample: A stratified sample in which a simple random sample is selected within each stratum.

Stratum: A group of elements from a population with similar characterisitics.

Stratified Sample: A sample composed of samples from each of several mutually exclusive and exhaustive strata into which a population has been divided.

Systematic Sample: A sample in which the first element is randomly selected from the first k elements of the population, and every k^{th} element of the population following the first element is subsequently selected.

Two-stage Cluster Sample: A sample selected in two stages. In the first stage, a sample of clusters of elements is chosen from a population of clusters. In the second stage, elements are sampled from each of the clusters chosen in the first stage.

Variables Sampling Plan: A verification sampling plan designed to detect a removal (obtained through small defects in several containers) of SNM in a population of containers.

α: The probability of a type I error. For inventory determination, it is also interpreted as the probability that the sample estimate (x) is further from the population value (X) than εX.

β: The probability of not obtaining a detectable removal in a sample. For verification attribute sampling it is the probability of not obtaining at least one large defect in a sample.

β': The probability of a type II error. This is equal to β assuming the probability of identifying a defect, given it is in the sample, is one.

n, n_i: The number of elements to be sampled from a population or from the i^{th} group (stratum or cluster) within the population.

N, N_i: The number of elements in the population or i^{th} group (stratum or cluster) within the population.

n^*, N^*, β^*: These symbols have the same meanings as n, N, and β where the asterisk indicates values for a PPS sampling plan where a measure of size different than the original units is utilized.

L: Number of strata in a stratified population.

The number of clusters to be sampled for a cluster sampling plan.

The number of clusters in a population (or stratum, for a stratified population).

N_c: The number of containers per cluster (when this number is the same for each cluster).

D.2 MISCELLANEOUS TERMINOLOGY AND NOTATION

Gross Defect: Larger "large defects" which are detectable with nondestructive techniques.

Large Defect: Any removal of SNM from a container detectable by a simple observation or single measurement.

Small Defect: Any removal of SNM from a container which cannot be detected by a simple observation or single measurement.

SNM: Special nuclear material, which is any enriched fissile isotope.

SSNM: Strategic special nuclear material, which is the isotope uranium-235 (contained in uranium enriched to 20% or more in the uranium-235 isotope), the isotope uranium-233, or plutonium. SSNM is a subset of SNM.

Tamper-safed: The property of a container sealed so that the seal cannot be tampered with without detection.

H_0: Null hypothesis in a statistical hypothesis test.

H_A:	Alternative hypothesis in a statistical hypothesis test.
G:	The goal quantity under the alternative hypothesis that a diversion has occurred. The quantity is used in designing attribute and variables sample sizes so that a diversion of amount G will be detected at a preassigned probability level.
A:	Amount of SNM in each container in a population, stratum, or cluster.
γ, γ_0, γ_1:	γ (gamma) is a fraction of SNM in a container hypothesized to be removed in a diversion. γ_0 is the cutoff point for detecting gross defects with certainty, while γ_1 is the cutoff point for detecting any large defect with certainty.
d_{ij}:	The difference between current and prior content values for the j^{th} container in stratum i.
$\bar{d}_i$:	$\sum_{j=1}^{n_i} d_{ij}/n_i$ = the mean contribution to $\hat{D}$ from stratum i.
d_j, $\bar{d}$:	Corresponding values of the above two items if the population is not stratified.
s^2_{dr}:	Sample estimate of the random error variance of the d_j values.
$\hat{D}$:	An estimate of the population mean difference between prior and current SNM content based on a sample.
D*:	The test statistic $\hat{D}/\hat{\sigma}_{\hat{D}}$ used to test for a diversion in the population through small defects.
$\hat{\sigma}^2_{\hat{D}}$:	A sample based estimate of the variance of $\hat{D}$ where $\hat{\sigma}^2_{\hat{D}} = \hat{\sigma}^2_{\hat{D}s} + \hat{\sigma}^2_{\hat{D}r}$.
$\hat{\sigma}^2_{\hat{D}s}$, $\hat{\sigma}^2_{\hat{D}r}$:	The systematic and random error variance components respectively, of the estimated variance of $\hat{D}$.

$\hat{\sigma}^2_{\hat{D}r|H_0}$: An estimate of the random error variance component of the statistic $\hat{D}$ under the null hypothesis that the material is in control.

$\hat{\sigma}^2_{\hat{D}r|H_A}$: An estimate of the random error variance component of the statistic $\hat{D}$ under the alternative hypothesis that the material is not in contol.

D^*, $\hat{\sigma}^2_{\hat{D}_i}$, $\hat{\sigma}^2_{ds_i}$, $\hat{\sigma}^2_{dr_i}$, $\hat{\sigma}^2_{dr_i|H_0}$, $\hat{\sigma}^2_{dr_i|H_A}$: The respective notations for the previous items when the population is stratified. The subscript indicates the values for the i^{th} stratum.

X: True population inventory.

x: Estimator of total population inventory based on sample information.

$\hat{SE}(x)$: Estimated standard error of x.

ε: Prespecified accuracy limit on the relative difference of x and X, i.e. $\left|\frac{\hat{X} - X}{X}\right| \leq \varepsilon$.

$z_{1-\alpha/2}$: The $(1-\alpha/2)^{th}$ percentile of the standard normal distribution (the normal distribution with mean zero and variance one).

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16. ABSTRACT (200 words or less)

Current regulations require that prior information on the Special Nuclear Material (SNM) content of a population of containers be verified and that periodic measurement of the SNM inventory of a facility be performed. This report develops and describes statistical sampling plans for accomplishing these tasks and compares results obtained by sampling to those obtained by the current practice of performing a census (100 percent sampling).

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