Vector Unparticle Contributions to Lepton g-2

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Abstract

The generic unparticle propagator may be modified in two ways. Breaking the conformal symmetry effectively adds a mass term to the propagator, while considering vacuum polarization corrections adds a width-like term. Both of these modifications result naturally from the coupling of the unparticle to standard model (SM) fields. We explore how these modifications to the propagator affect the calculation of the lepton anomalous magnetic moment using an integral approximation of the propagator that is accurate for $d \lesssim 1.5$, where d is the unparticle dimension. We find that for this range of d and various values of the conformal breaking scale μ, the value of $g - 2$ calculated when allowing various SM fermions to run in the unparticle self-energy loops does not significantly deviate from the value of $g-2$ when the width term is ignored. We also investigate the limits on a characteristic mass scale for the unparticle sector as a function of μ and d.

1. Introduction

Unparticle physics [1, 2] represents an intriguing possibility for physics beyond the standard model. In this scenario, there is a new sector which is conformal below a mass scale $\Lambda_{\mathcal{U}}$. It can be described by an effective field theory of so-called "unparticle" operators, which in some respects imitate a non-integer number of massless particles. The phenomenology of unparticles has been studied in great detail. In particular, the anomalous magnetic moments of the electron and the muon have been calculated in various unparticle scenarios [3, 4, 5, 6, 7]. Here we explore the effect of adding a width-like term to the denominator of the propagator on the calculation of electron and muon $g - 2$. We also study in detail the relationship between the conformal breaking scale μ and these quantities.

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The propagator for unparticles can be obtained from the spectral representation

$$P_{\mathcal{U}} = \int d^4x \; e^{ipx} \langle \mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}^\dagger(0)\rangle = \int_0^\infty \frac{dM^2}{2\pi}\rho_{\mathcal{U}}(M^2)\frac{i}{p^2 - M^2 + i\epsilon}, \quad (1)$$

where $\mathcal{O}_{\mathcal{U}}$ is an unparticle operator and $\rho_{\mathcal{U}}(M^2)$ is the unparticle spectral function. If $O_{\mathcal{U}}$ has scaling dimension d, conformal invariance determines

$$\rho_{\mathcal{U}}(M^2) = A_d(M^2)^{d-2} , \quad (2)$$

where

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}}\frac{\Gamma(d+1/2)}{\Gamma(d-1)\Gamma(2d)} \quad (3)$$

is chosen so that the $d \to 1$ limit gives the propagator for a normal particle. Following [2] we will take $d \in (1,2)$. Inserting this spectral function into Eq. 1 and evaluating the integral yields the propagator

$$P_{\mathcal{U}}(p^2) = \frac{iA_d}{2\sin(\pi d)}(-p^2 - i\epsilon)^{d-2} . \quad (4)$$

The above is valid for a scalar unparticle. For vector unparticles, which we consider in this paper, this propagator must be multiplied by the projector $-(g_{\mu\nu} + bp^\mu p^\nu/p^2)$. The constant b is in principle undetermined because the unparticle theory need not have a gauge symmetry, but it will not affect the results of any of our calculations so we set it to zero from now on. The propagator is then $P_{\mathcal{U}}^{\mu\nu}(p^2) = -g^{\mu\nu}P_{\mathcal{U}}(p^2)$.

To get a feel for the behavior of this unusual propagator, we show the imaginary and real parts of $\Delta_{\mathcal{U}}(p^2) = -iP_{\mathcal{U}}(p^2)$ for positive p^2 in Figs. 1a and 1b, respectively. It will be convenient for us to use $\Delta_{\mathcal{U}}$ rather than $P_{\mathcal{U}}$ in our analysis and we will often refer to $\Delta_{\mathcal{U}}$ as "the propagator." It should be noted that the imaginary part of $\Delta_{\mathcal{U}}(p^2)$ is zero for negative p^2, while the real part is not.

Following [8] we shall write the coupling of vector unparticles to Standard Model (SM) fermions in the form

$$\Delta\mathcal{L} = i\frac{c}{\Lambda_3^{d-1}}\bar{\psi}\gamma_\mu\psi\mathcal{O}_{\mathcal{U}}^\mu . \quad (5)$$

The parameter Λ_3 is not fundamental; it can be expressed as the combination

$$\Lambda_3 = \left(\frac{\Lambda_{\mathcal{U}}}{M}\right)^{\frac{d_{UV}-1}{d-1}}\Lambda_{\mathcal{U}} , \quad (6)$$

where M and d_{UV} are parameters of the underlying UV theory as described in [8]. Since the coupling c and the scale Λ_3 only appear in the combination given in Eq. 5, and we have no theoretical guidance as to what c may be (other than naturalness, which might suggest that c is $\mathcal{O}(1)$), in our analysis we will take $c = 1$ and let the coupling strength vary by varying Λ_3.
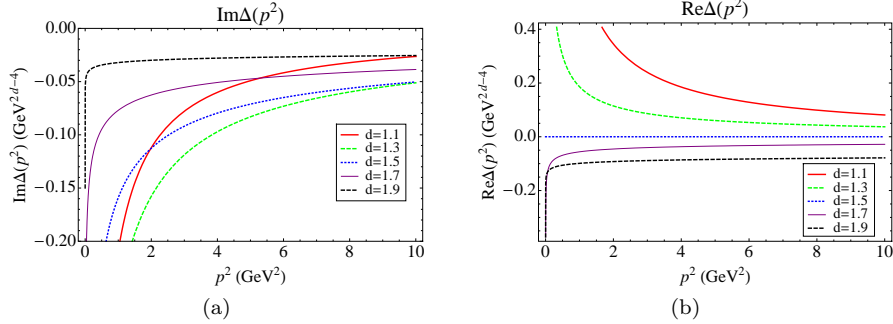
Figure 1: The imaginary part (left panel) and real part (right panel) of the unparticle propagator $\Delta_\mathcal{U}(p^2)$ for various values of unparticle dimension d, in the absence of unparticle sector conformal symmetry breaking.

An interesting modification of the unparticle scenario involves breaking the conformal symmetry of the unparticle sector at a scale μ. This can result from interactions between the unparticle and the Higgs, for example. An ansatz for the spectral function in this case essentially consists in removing modes below μ^2 from the spectrum, yielding [9]

$$\rho_\mathcal{U}(M^2) = A_d \theta(M^2 - \mu^2)(M^2 - \mu^2)^{d-2} . \tag{7}$$

With this spectral function, the propagator becomes

$$P_\mathcal{U}(p^2) = \frac{iA_d}{2\sin(\pi d)}(-(p^2 - \mu^2) - i\epsilon)^{d-2} . \tag{8}$$

For the fractional power, we take the standard branch cut such that

$$(\mu^2 - p^2 - i\epsilon)^{2-d} = \begin{cases} (\mu^2 - p^2)^{2-d} & \text{if } \mu^2 \geq p^2, \\ (p^2 - \mu^2)^{2-d}e^{i\pi d} & \text{if } p^2 > \mu^2 . \end{cases} \tag{9}$$

In Fig. 2a, we show the imaginary part of $\Delta_\mathcal{U}(p^2) = -iP_\mathcal{U}(p^2)$ for $d = 1.3$ and different choices of μ. It rises steeply near $p^2 = \mu^2$ and vanishes for $p^2 < \mu^2$. In Fig. 2b we display $\mathrm{Re}\Delta_\mathcal{U}(p^2)$ for $\mu = 4$ GeV for different values of d. Unlike the imaginary part, the real part is nonzero for $p^2 < \mu^2$, though for $d = 1.5$ it is zero for $p^2 > \mu^2$.

It has been pointed out that some of the strongest experimental constraints on unparticle physics can be evaded with a sufficiently large value of μ.

It has also been suggested [9, 10] that the propagator given in Eq. 8 should be further modified by summing the contributions to the propagator from diagrams containing an arbitrary number of vacuum polarization bubbles, as is done when finding the renormalized mass and width of a "normal" particle.

In this paper we calculate the contribution to muon $g-2$ from a vector unparticle both with and without vacuum polarization corrections to the propagator.
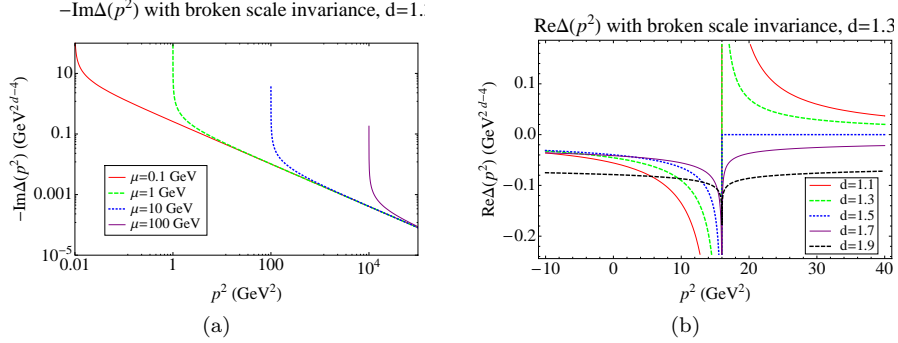
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Figure 2: The negative imaginary part (left panel) and real part (right panel) of the unparticle propagator with broken conformal invariance $\Delta_{\mathcal{U}}(p^2)$ for $d = 1.3$. The negative imaginary part is shown for four different values of the conformal breaking scale μ, while the real part is shown for five different values of the unparticle dimension d.

We obtain bounds on d, μ, and Λ_3 from the experimental limits on muon and electron $g-2$, and investigate how these limits are affected by these corrections. Specifically we will consider the modification to the unparticle propagator from vacuum polarization graphs with fermions running in the loop. In particular, we will consider the following 3 cases:

1. The unparticle couples only to the lepton for which we are calculating $g-2$.
2. The unparticle couples to all charged leptons with equal strength, but to no other particles.
3. The unparticle couples to all charged standard model fermions with equal strength, but to no other particles.

The calculation of the unparticle contribution to $g - 2$ in these cases will require obtaining an approximate dispersion integral representation of the modified propagator, whose validity and accuracy we will investigate.

2. Vacuum polarization corrections to the propagator

We will first consider the modification of the propagator from vacuum polarization diagrams. While in principle we could consider decays to scalar or vector particles, and possibly even other species of unparticle, for simplicity we confine our attention to loops involving standard model fermions. For a given fermion in the loop, the amplitude for the vacuum polarization graph is

$$q \;\;\text{(loop diagram)}\;\; = \; i(g^{\mu\nu} - q^\mu q^\nu/q^2)\Pi(q^2) \;, \tag{10}$$

where

$$\Pi(q^2) = -\frac{1}{2\pi^2} \left(\frac{c}{\Lambda_3^{d-1}}\right)^2 q^2 B\left(\frac{q^2}{4m_f^2}\right) , \tag{11}$$

where $B(\beta)$ is the Feynman parameter integral

$$B(\beta) = \int_0^1 dx\, x(1-x) \log(1/(1-4\beta x(1-x))) . \tag{12}$$

Our unparticle theory is not a gauge theory. Since the rest of the g-2 calculation involves only QED, however, the contribution proportional to $q^\mu q^\nu$ will vanish by the Ward identity.

Summing over diagrams with an arbitrary number of vacuum polarization loop insertions yields the propagator $\tilde{P}_\mathcal{U}^{\mu\nu}(p^2) = -g^{\mu\nu}\tilde{P}_\mathcal{U}(p^2)$, with

$$\tilde{P}_\mathcal{U}(p^2) = \frac{iA_d}{2\sin(\pi d)(\mu^2 - p^2 - i\epsilon)^{2-d} - A_d\Pi(p^2)} . \tag{13}$$

To see the effect of this modification to the propagator, in Fig. 3a we show $-\mathrm{Im}\tilde{\Delta}_\mathcal{U}(p^2) = -\mathrm{Im}(-i\tilde{P}_\mathcal{U}(p^2))$ for $d = 1.1$, $\mu = 10$ GeV, and $\Lambda_3 = 1$ TeV with four different choices of vacuum polarization corrections. The peak near $p^2 = \mu^2$ is broadened considerably by the corrections, showing that the $A_d\Pi(p^2)$ term in the denominator of the propagator has an effect similar to that of a decay width for a normal particle. In Fig. 3b, on the other hand, we show $-\mathrm{Im}\tilde{\Delta}_\mathcal{U}(p^2)$ again for $d = 1.1$ and $\mu = 10$ GeV, but now we only consider vacuum polarization corrections from all charged SM fermions and let Λ_3 take on four different values. Because decreasing Λ_3 effectively increases the unparticle-SM coupling, the propagator peak is broadened more for lower Λ_3.

3. Calculating $g - 2$ using a dispersion integral

Now that we have the form for the unparticle propagator with vacuum polarization corrections, we turn our attention to the question of how to calculate the one-loop contribution to $g-2$ from unparticles both with and without these corrections to the propagator. This contribution comes from the diagram in Fig. 4. The amplitude $i\mathcal{M}^\mu$ corresponding to this diagram satisfies

$$i\mathcal{M}^\mu \propto \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')(\slashed{k}\gamma^\mu\slashed{k}' + m^2\gamma^\mu - 2m(k+k')^\mu)u(p)}{(k^2 - m^2 + i\epsilon)(k'^2 - m^2 + i\epsilon)D_\mathcal{U}} , \tag{14}$$

where $D_\mathcal{U}$ is given by

$$D_\mathcal{U} = (\mu^2 - (p-k)^2 - i\epsilon)^{2-d} \tag{15}$$

for an unparticle propagator without vacuum polarization corrections, and

$$D_\mathcal{U} = (\mu^2 - (p-k)^2 - i\epsilon)^{2-d} - \frac{A_d}{2\sin(\pi d)}\Pi((p-k)^2) \tag{16}$$
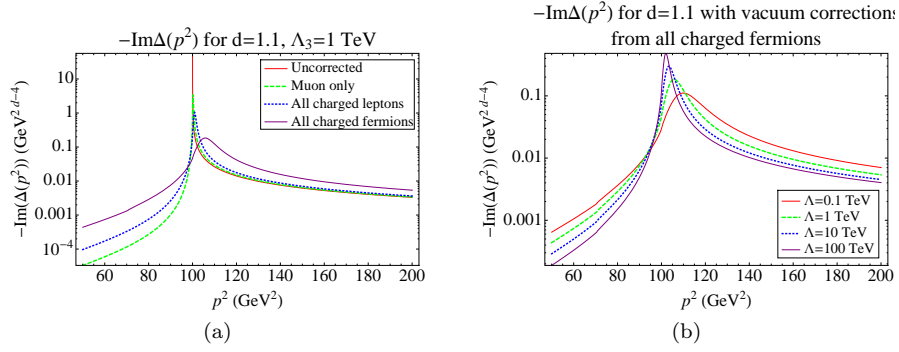
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Figure 3: The negative imaginary part of the propagator, for $d = 1.1$ and $\mu = 10$ GeV, for $\Lambda_3 = 1$ TeV and different choices for loop corrections (left panel) and for various values of Λ_3 (right panel). In the left panel, the "uncorrected" curve shows the propagator in the absence of vacuum polarization corrections, while the other three curves show the value of the propagator if only muons, all charged leptons, or all charged SM fermions are allowed to run in the loops of the self-energy terms which lead to a width-like term in the unparticle propagator. In the right panel, different curves represent different values of Λ_3; since this parametrizes the fermion-unparticle coupling, it affects the shape as well as the normalization of the propagator if the propagator includes the width-like factor from vacuum polarization corrections.
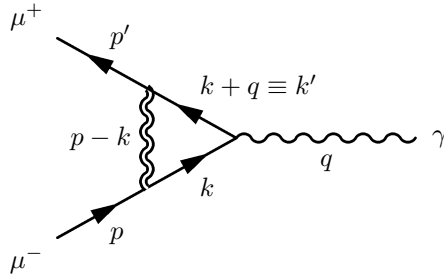

Figure 4: The unparticle correction to the muon $g - 2$. The photon momentum q is incoming and the muon momenta follow the fermion lines.

for an unparticle propagator with vacuum polarization corrections.

Note that the numerator is identical to that which one obtains in the calculation of the QED one-loop contribution to $g-2$. Since we are only considering vector (not axial vector) unparticles, we can write the amplitude as

$$i\mathcal{M}^\mu = ie\bar{u}(p') \left(\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}}{2m}F_2(q^2) \right) u(p) \,. \tag{17}$$

The unparticle contribution to the quantity $a = g/2 - 1$ is then given by $a^\mathcal{U} = F_2(0)$.

When calculating the QED one-loop contribution to $g-2$ (where the particle in the loop is a photon rather than an unparticle, and hence $D_\mathcal{U} \to (p-k)^2 + i\epsilon$), we can use the method of Feynman parameters to combine the three factors in D and shift the integration variable to $l = k + yq - zp$. The result is

$$F_2(0) = 8ie^2m^2 \int \frac{d^4l}{(2\pi)^4} \int_0^1 dx\,dy\,dz \frac{z(1-z)}{(l^2 - \Delta + i\epsilon)^3} \,. \tag{18}$$

Note that there are no powers of l in the numerator of the integrand. As a result, the integral is convergent. The simplifications (such as determining that terms proportional to l^2 contribute to F_1 rather than F_2) that make this explicit require the use of Feynman parameters.

In the case of unparticles without vacuum polarization corrections to the propagator, the Feynman parameter method can still be used, though in this case the denominator is raised to the power $4 - d$. With the corrected propagator, however, it is impossible to combine the terms in the denominator into an expression quadratic in momentum variables. Hence, we cannot use Feynman parameters in this case to obtain a relatively simple, manifestly convergent integral expression for $F_2(0)$ analogous to Eq. 18. It is difficult to imagine that one can calculate the integral in Eq. 14 analytically in this case. One could hope to perform the integral numerically, but without the simplifications obtained using Feynman parameters, the integral has divergent contributions whose cancellation would be difficult to guarantee in a numerical calculation. We therefore seek to recast the integral in a form amenable to the use of Feynman parameters.

To do this, we try writing the unparticle propagator as a dispersion integral (cf. Eq. 1) which contains the standard vector boson propagator. Because $\rho(M^2)$ is independent of the loop momentum, it can be pulled out of the loop momentum integral. The vector boson propagator, however, remains under the loop integral and we end up with

$$a^\mathcal{U} = \int_0^\infty \frac{dM^2}{2\pi} \rho(M^2) I(M^2), \tag{19}$$

where $\rho(M^2)$ is the spectral function and $I(M^2)$ is the contribution to $g-2$ at one loop from a vector boson with mass M and coupling to fermions given by Eq. 5. Specifically we have that $I(M^2)$ is given by the Feynman parameter integral

$$I(M^2) = \frac{c^2}{4\pi^2\Lambda_3^{2-2d}} \int_0^1 dx \frac{x^2(1-x)^2m^2}{x^2m^2 + (1-x)M^2} \,, \tag{20}$$

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where m is the mass of the fermion for which we are calculating $g-2$ (generally the muon), and as noted above we will set c=1 in what follows. The Feynman parameter integral in Eq. 20 can be performed to obtain an analytical expression for $I(M^2)$, leaving the M^2 integral in Eq. 19 to be done numerically.

This method of expressing the propagator as a dispersion integral clearly works in the case without vacuum polarization corrections, since we already have expressions given above for $\rho(M^2)$ (Eqs. 2 and 7). Of course in this case, as mentioned above, one could do the calculation using Feynman parameters [3, 6] without recourse to the dispersion integral. This provides a useful check on our results.

To use this method when we include vacuum polarization corrections to the unparticle propagator, however, we still need to express that propagator as a dispersion integral. In the next section, we will study whether this can be done.

4. Expressing the propagator as a dispersion integral

We need to find an expression for the spectral function of the vacuum polarization-corrected unparticle propagator. To this end, we first note that the spectral function for the "tree-level" propagator in both the unbroken (Eq. 2) and broken (Eq. 7) conformal symmetry cases satisfies the relation $\rho(M^2) = 2\mathrm{Im}\Delta(M^2)$. In other words, the spectral function is (twice) the imaginary part of the propagator. This is actually due to a general result in complex analysis, which states that for any function of a complex variable $f(z)$,

$$ f(z) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, f(z)}{z - z' + i\epsilon} dz' \, , \tag{21} $$

if f is analytic in the upper half-plane. (The difference between the $\frac{1}{\pi}$ factor in this equation and the $\frac{1}{2\pi}$ in Eq. 1 accounts for the factor of 2 in the relation $\rho(M^2) = 2\mathrm{Im}\Delta(M^2)$, while the i in the numerator of Eq. 1 combined with the i in $\Delta = \mathrm{Im}\, P$ lead to the lack of a relative sign.) The uncorrected unparticle propagator is indeed analytic in the upper half-plane in both the broken and unbroken conformal invariance cases.

It therefore seems like a simple matter to obtain the spectral function for our vacuum polarization corrected unparticle propagator, Eq. 13. It should be $\tilde{\rho}_{\mathcal{U}}(M^2) = 2\mathrm{Im}\tilde{\Delta}_{\mathcal{U}}(M^2)$. The problem is that it is not clear if the corrected propagator $\tilde{\Delta}_{\mathcal{U}}(M^2)$ is analytic in the upper half-plane. If it is not, this relation may not hold.

We would like to test if this condition holds exactly, indicating that the propagator is analytic in the upper half-plane. It may also be possible that the propagator is not analytic in the upper half-plane, but is close enough to an analytic function that the dispersion integral with $\tilde{\rho}_{\mathcal{U}}(M^2) = 2\mathrm{Im}\tilde{\Delta}_{\mathcal{U}}(M^2)$ serves as a good approximation to the propagator. To investigate these possibilities, we adopt a numerical strategy. We will *assume* $\tilde{\rho}_{\mathcal{U}}(M^2) = 2\mathrm{Im}\tilde{\Delta}_{\mathcal{U}}(M^2)$ as a putative definition for the spectral function associated with the vacuum

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polarization-corrected unparticle propagator. We then want to know if the dispersion relation

$$\tilde{\Delta}_{\mathcal{U}}(p^2) \stackrel{?}{=} -\int_0^\infty \frac{dM^2}{\pi} \text{Im}\tilde{\Delta}_{\mathcal{U}}(M^2) \frac{1}{p^2 - M^2 + i\epsilon} \qquad (22)$$

holds. Note that the lower range of integration may be taken to be 0 because $\text{Im}\tilde{\Delta}_{\mathcal{U}}(p^2)$ is zero for $p^2 < 0$. The left-hand side of this equation is the true value of the propagator, which we can simply evaluate using Eq. 13. The right hand side is our putative spectral representation for the propagator, and we can compute its value by carrying out the integral numerically. We can then compare the two sides. If they do not agree, we learn that, for the loop-corrected propagator, the relation $\tilde{\rho}_{\mathcal{U}}(p^2) = 2\text{Im}\tilde{\Delta}_{\mathcal{U}}(p^2)$ does not in fact hold (or, equivalently, that Eq. 21 does not hold), because the propagator is not analytic in the upper-half plane. If the two sides of Eq. 22 do agree to a high precision, we can conclude that the propagator is analytic (or at least well-enough approximated by an analytic function) in the upper half-plane, so that the RHS provides the correct spectral representation (or at least good approximation) of the true propagator. If this is the case, we can use $\tilde{\rho}_{\mathcal{U}}(p^2)$ as a good approximation to the spectral function in the $g - 2$ calculation.

It should be noted that, following [11], when the mass m of the fermion in the loop satisfies $2m > \mu$, there will be a value of M^2 for which the denominator of $\tilde{\Delta}_{\mathcal{U}}(M^2)$ is zero. This leads to an additional delta function term in $\tilde{\rho}_{\mathcal{U}}(M^2)$. Such a value can only occur when the loop contribution $\Pi(M^2)$ and the first term in the denominator of the propagator, $(2\sin(\pi d)/A_d)(\mu^2 - p^2 - i\epsilon)^{2-d}$, are both real. The loop contribution from a given fermion with mass m is real for $M^2 < 4m^2$ while the first term in the denominator of the propagator is real for $M^2 < \mu^2$. Hence, the denominator cannot vanish for $\mu^2 > M^2 > 4m^2$, where m is the mass of the lightest fermion running in the loop, unless there were some fortuitous cancellation of the imaginary parts of the loop contributions from the various fermion species. It can be easily verified that there is no such cancellation. It may be further shown that, given the signs of the imaginary parts of the various terms in the denominator of the propagator, it also cannot have a zero for $M^2 > \mu^2$. For $M^2 < 4m^2$, where m is the mass of the lightest fermion running in the loop, both terms in the denominator are strictly real. In the situations we consider, however, where the lightest fermion in the loop is either an electron or a muon, and μ takes values of 10 or 100 GeV, the absolute value of the first term in the propagator, in this region, is always much larger than the absolute value of the loop correction term, so no cancellation can occur. Thus, in the situations we consider, there are no delta function contributions to the unparticle spectral function.

In Table 1, we compare the actual loop-corrected propagator (Eq. 13) to its putative spectral representation (RHS of Eq. 22) for a large number of choices for d, μ, and p^2. For each choice of these parameters, we show the fractional error between the spectral integral approximation for the propagator and its true value. That error is obtained by first, using *Mathematica* [12], performing the numerical integration to evaluate the RHS of Eq. 22 (which we'll denote

$d,\ \mu,\ p^2$	None	Muon	Leptons	All
$1.1, 10, 5$	2.5×10^{-7}	7.2×10^{-10}	1.9×10^{-7}	1.6×10^{-5}
$1.1, 10, 500$	2.2×10^{-7}	1.2×10^{-12}	3.2×10^{-7}	5.3×10^{-5}
$1.1, 10, 50000$	1.3×10^{-7}	9.5×10^{-12}	1.8×10^{-7}	3.6×10^{-3}
$1.1, 100, 5$	2.5×10^{-7}	5.3×10^{-11}	1.8×10^{-8}	1.1×10^{-3}
$1.1, 100, 500$	2.5×10^{-7}	5.6×10^{-13}	7.9×10^{-9}	$1. \times 10^{-3}$
$1.1, 100, 50000$	2.2×10^{-7}	7.8×10^{-12}	3.6×10^{-8}	2.8×10^{-3}
$1.3, 10, 5$	$7. \times 10^{-15}$	4.9×10^{-7}	5.4×10^{-6}	2.1×10^{-4}
$1.3, 10, 500$	1.1×10^{-13}	1.3×10^{-6}	1.5×10^{-5}	5.6×10^{-4}
$1.3, 10, 50000$	$9. \times 10^{-14}$	3.9×10^{-5}	4.4×10^{-4}	1.6×10^{-2}
$1.3, 100, 5$	1.8×10^{-12}	1.3×10^{-5}	1.4×10^{-4}	5.4×10^{-3}
$1.3, 100, 500$	1.8×10^{-12}	1.2×10^{-5}	1.4×10^{-4}	5.2×10^{-3}
$1.3, 100, 50000$	5.4×10^{-14}	3.4×10^{-5}	3.7×10^{-4}	1.4×10^{-2}
$1.5, 10, 5$	2.1×10^{-16}	1.6×10^{-4}	4.7×10^{-4}	2.4×10^{-3}
$1.5, 10, 500$	1.5×10^{-13}	3.3×10^{-4}	9.6×10^{-4}	4.8×10^{-3}
$1.5, 10, 50000$	1.1×10^{-13}	3.6×10^{-3}	1.1×10^{-2}	5.4×10^{-2}
$1.5, 100, 5$	$9. \times 10^{-14}$	1.6×10^{-3}	4.8×10^{-3}	2.4×10^{-2}
$1.5, 100, 500$	9.3×10^{-14}	1.6×10^{-3}	4.7×10^{-3}	2.4×10^{-2}
$1.5, 100, 50000$	6.8×10^{-14}	3.3×10^{-3}	9.6×10^{-3}	4.9×10^{-2}
$1.7, 10, 5$	1.7×10^{-15}	8.6×10^{-3}	1.4×10^{-2}	2.8×10^{-2}
$1.7, 10, 500$	1.2×10^{-13}	1.3×10^{-2}	2.1×10^{-2}	4.3×10^{-2}
$1.7, 10, 50000$	8.9×10^{-14}	5.6×10^{-2}	$9. \times 10^{-2}$	1.8×10^{-1}
$1.7, 100, 5$	1.9×10^{-7}	3.5×10^{-2}	5.6×10^{-2}	1.1×10^{-1}
$1.7, 100, 500$	1.9×10^{-7}	3.4×10^{-2}	5.5×10^{-2}	1.1×10^{-1}
$1.7, 100, 50000$	5.2×10^{-14}	5.2×10^{-2}	8.4×10^{-2}	1.7×10^{-1}
$1.9, 10, 5$	2.5×10^{-8}	2.3×10^{-1}	2.6×10^{-1}	3.1×10^{-1}
$1.9, 10, 500$	4.6×10^{-14}	2.7×10^{-1}	$3. \times 10^{-1}$	3.6×10^{-1}
$1.9, 10, 50000$	2.4×10^{-7}	4.4×10^{-1}	4.9×10^{-1}	5.9×10^{-1}
$1.9, 100, 5$	3.8×10^{-10}	3.7×10^{-1}	4.2×10^{-1}	$5. \times 10^{-1}$
$1.9, 100, 500$	$4. \times 10^{-10}$	3.7×10^{-1}	4.2×10^{-1}	$5. \times 10^{-1}$
$1.9, 100, 50000$	2.4×10^{-7}	4.3×10^{-1}	4.8×10^{-1}	5.7×10^{-1}

Table 1: The fractional error δ^{disp} (see text) in the dispersion integral approximation for the unparticle propagator. An entry in the first column gives the choice of parameters used for that row, with μ given in GeV and p^2 in GeV^2. The remaining columns give the fractional error for the dispersion integral approximation for the propagator $\tilde{\Delta}_{\mathcal{U}}^{\mathrm{disp}}(p^2)$ using different choices for vacuum polarization corrections. "None" means no vacuum polarization corrections, "Muon" means only the muon loop is included, "Leptons" means the vacuum polarization corrections from all charged SM leptons are included, and "All" means the corrections from all charged SM fermions are included.

by $\tilde{\Delta}_{\mathcal{U}}^{\text{disp}}(p^2)$). We then compare this to the actual value of the propagator, obtaining the fractional error

$$\delta^{\text{disp}} \equiv \left| \frac{\tilde{\Delta}_{\mathcal{U}}^{\text{disp}}(p^2) - \tilde{\Delta}_{\mathcal{U}}(p^2)}{\tilde{\Delta}_{\mathcal{U}}(p^2)} \right| . \tag{23}$$

This fractional error is obtained for four cases. First, for the case of no loop corrections to the propagator. In this case, we know the spectral representation with $\tilde{\rho}_{\mathcal{U}}(p^2) = 2\text{Im}\tilde{\Delta}_{\mathcal{U}}(p^2)$ to be exact, so we expect zero error, and indeed find errors small enough to be attributed to inaccuracy in the numerical integration. In the last three columns, the fractional error is shown with three different sets of fermions allowed to run in the loop in the unparticle propagator—muon only, charged leptons only, and all charged SM fermions. We notice that for all these cases and for all values of $d \lesssim 1.5$, the fractional error is at most a few percent. Thus for these values of d, we can use $\tilde{\rho}_{\mathcal{U}}(p^2) = 2\text{Im}\tilde{\Delta}_{\mathcal{U}}(p^2)$ to carry out the calculation of the unparticle contribution to $g-2$ with confidence that the result will be accurate.

5. Results

We now have a dispersion representation for the unparticle propagator. It is exact in the cases with unbroken and broken conformal symmetry but without vacuum polarization corrections. In the case with vacuum polarization corrections, it is a good approximation for $d \lesssim 1.5$. We can then numerically integrate Eq. 19 to obtain the unparticle contribution to $g-2$.

Doing this, we find, in fact, that the effect of the propagator corrections on the $g-2$ contribution is small. Table 2 shows the relative difference between the unparticle contribution to $g-2$ with and without the inclusion of vacuum polarization corrections to the unparticle propagator. The difference is negligible for all parameter choices for which the dispersion representation of the propagator is accurate. Therefore, in the rest of this section, we will discuss results using the uncorrected unparticle propagator, knowing they will still hold when the vacuum polarization corrections to the propagator are included.[2] It is possible, of course, that $\Pi(p^2)$ is significantly modified at high p^2 by heavy particles whose presence we have not taken into account, for example from other hidden sectors to which the unparticle is coupled. We note that this possibility could cause a larger effect on $g-2$ and other observables from vacuum polarization corrections.

Strong experimental constraints exist on $g-2$ for both muons and electrons. In the case of muons, the measured value of a is three standard deviations larger

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[2]In fact we are only certain this is true for $d \lesssim 1.5$. Since for $d \gtrsim 1.5$, we do not have an accurate approximation for the spectral function for the unparticle propagator, we technically have not ruled out the possibility that the effects of loop corrections to the propagator on $g-2$ are large in this case.

$d,\ \mu,\ \Lambda_3$	$a_\mu^{\mathcal{U}}$	$\delta a_\mu^{\mathcal{U}}$
$1.1, 0, 500$	1.3×10^{-3}	7.8×10^{-3}
$1.1, 0, 1000$	1.1×10^{-3}	6.8×10^{-3}
$1.1, 0, 2000$	9.6×10^{-4}	$6. \times 10^{-3}$
$1.1, 1, 500$	1.7×10^{-5}	1.9×10^{-3}
$1.1, 1, 1000$	1.5×10^{-5}	1.6×10^{-3}
$1.1, 1, 2000$	1.3×10^{-5}	1.4×10^{-3}
$1.1, 10, 500$	2.9×10^{-7}	$9. \times 10^{-5}$
$1.1, 10, 1000$	2.5×10^{-7}	9.4×10^{-5}
$1.1, 10, 2000$	2.2×10^{-7}	$9. \times 10^{-5}$
$1.3, 0, 500$	1.5×10^{-5}	1.5×10^{-4}
$1.3, 0, 1000$	$1. \times 10^{-5}$	9.9×10^{-5}
$1.3, 0, 2000$	6.6×10^{-6}	6.5×10^{-5}
$1.3, 1, 500$	6.7×10^{-7}	6.6×10^{-5}
$1.3, 1, 1000$	4.4×10^{-7}	4.8×10^{-5}
$1.3, 1, 2000$	2.9×10^{-7}	3.4×10^{-5}
$1.3, 10, 500$	2.9×10^{-8}	-4.7×10^{-4}
$1.3, 10, 1000$	1.9×10^{-8}	-2.1×10^{-4}
$1.3, 10, 2000$	1.2×10^{-8}	-8.8×10^{-5}
$1.5, 0, 500$	2.3×10^{-7}	-5.5×10^{-5}
$1.5, 0, 1000$	1.1×10^{-7}	$-3. \times 10^{-5}$
$1.5, 0, 2000$	5.7×10^{-8}	-1.6×10^{-5}
$1.5, 1, 500$	2.9×10^{-8}	-4.7×10^{-4}
$1.5, 1, 1000$	1.4×10^{-8}	-2.5×10^{-4}
$1.5, 1, 2000$	7.1×10^{-9}	-1.4×10^{-4}
$1.5, 10, 500$	$3. \times 10^{-9}$	-4.5×10^{-3}
$1.5, 10, 1000$	1.5×10^{-9}	-2.4×10^{-3}
$1.5, 10, 2000$	7.5×10^{-10}	-1.3×10^{-3}

Table 2: The relative difference $\delta(a_\mu^{\mathcal{U}})$ between the unparticle contribution to muon $g-2$ with and without vacuum polarization corrections to the unparticle propagator. An entry in the first column gives the choice of parameters used for that row, with μ and Λ_3 given in GeV. The second column gives $a_\mu^{\mathcal{U}}$ from unparticles with no vacuum polarization corrections. The remaining column gives the fractional difference $\delta(a_\mu^{\mathcal{U}}) = (\tilde{a}_\mu^{\mathcal{U}} - a_\mu^{\mathcal{U}})/(a_\mu^{\mathcal{U}})$, where $\tilde{a}_\mu^{\mathcal{U}}$ takes into account vacuum polarization correction to the unparticle propagator from all charged SM fermions.
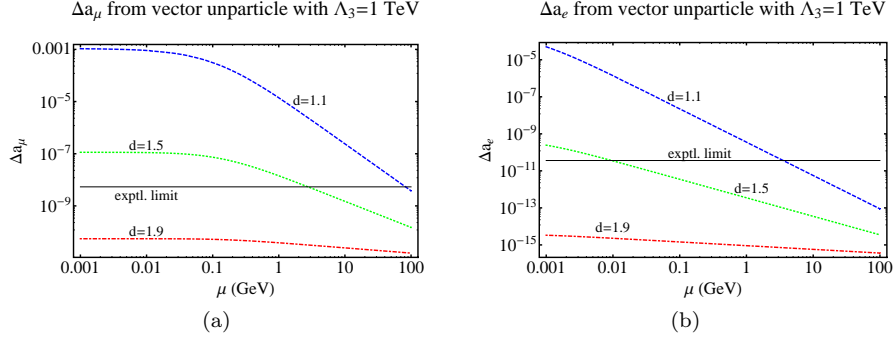
Figure 5: The unparticle contribution to the muon anomalous magnetic moment $(a_\mu^\mathcal{U})$ (left panel), and to the electron anomalous magnetic moment $(a_e^\mathcal{U})$ (right panel), as a function of the conformal breaking scale μ. Here, $\Lambda_3 = 1$ TeV and we show curves for $d = 1.1$, $d = 1.5$, and $d = 1.9$. Shown for comparison in each case is the experimental limit (see text).

than the calculated SM value. Specifically,

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 2.95 \pm 0.81 \times 10^{-9} \ . \tag{24}$$

As a very conservative upper bound, we require that the unparticle contribution, which is always positive in the case of vector unparticles, be no more than three standard deviations above the central value, i.e.

$$a_\mu^\mathcal{U} < 5.38 \times 10^{-9} \ . \tag{25}$$

In the case of electrons, the measured and calculated values are in good agreement. We will take the conservative bound [6] $a_e^\mathcal{U} < 1.5 \times 10^{-11}$, which, we note, is over five times the error on the most recent experimental measurement [13].

In Fig. 5a, we display the unparticle contribution to muon $g - 2$ in the case of broken conformal symmetry as a function of μ and compare it to the experimental bounds. As described above, this contribution is obtained by numerically integrating Eq. 19, again using *Mathematica*. The contribution begins to drop quickly as μ is increased above m_μ, and the statement that for $\mu \gtrsim 1$ GeV, the experimental bound is evaded, is shown to be true for, e.g., $\Lambda_3 = 1$ TeV and $d = 1.5$. It is interesting to note the large hierarchy between the $d = 1.1$, 1.5, and 1.9 contributions.

Finally, in Fig. 6a, we show the lower bounds on d and μ from muon $g - 2$ for various choices of Λ_3. We note the very strong dependence on d and μ. In Fig. 6b, we display the same bounds but from electron $g - 2$. It is qualitatively similar though the bounds are much weaker.
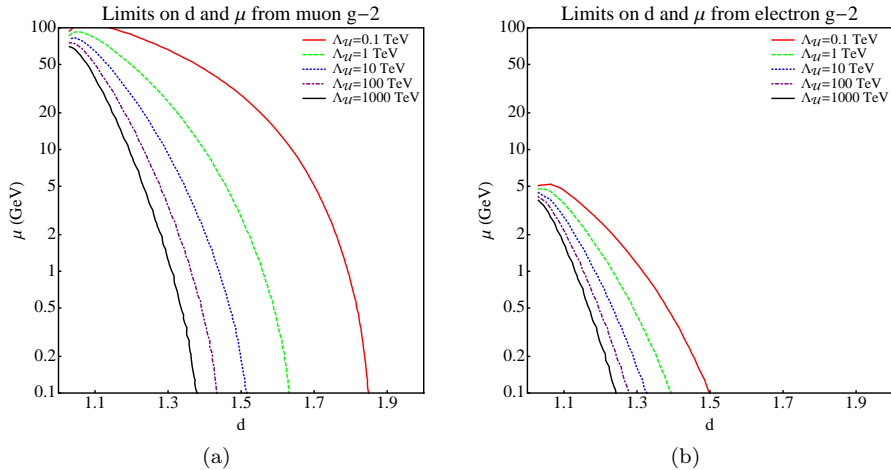
Figure 6: Lower bounds on d and μ from muon (left panel) or electron (right panel) $g - 2$ for various values of Λ_3. The region above the curves is allowed.

6. Conclusions

In this paper we have examined the correction to lepton $g - 2$ in several unparticle scenarios. In particular, we studied the model in which in addition to breaking the conformal symmetry of the unparticle sector at a scale μ, we also added a width-like term from vacuum polarization to the propagator. To calculate the effects of this term we utilized an approximation based on dispersion integral techniques, which is exact when there are no vacuum polarization corrections to the propagator. This approximation is accurate for $d \leq 1.5$; it breaks down for greater values of d.

We found that adding the width-like term to the propagator significantly changed the the value of the propagator, but made little difference in the calculation of $g - 2$. Specifically, for the points we analyzed numerically using our dispersion integral approximation, we found that adding this term never changed the value of $g - 2$ by more than 1%. It would be interesting to learn whether this is the case for other observables or whether $g - 2$ is exceptionally insensitive to this term in the propagator.

We also obtained limits on the unparticle dimension d and the conformal scale μ for various values of the unparticle scale Λ_3 by assuming that the dimensionless coupling c is equal to unity. In the muon case, we find that even for Λ_3 as high as 1000 TeV, there are strong bounds on μ for $d \lesssim 1.3$. In the electron case, the bounds are much weaker, though we still find that for $\Lambda_3 = 1000$ TeV, $\mu \gtrsim 1$ GeV for $d \lesssim 1.2$.

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