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Kahler Independence of the G_2-MSSM

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The G_2-MSSM is a model of particle physics coupled to moduli fields with interesting phenomenology both for colliders and astrophysical experiments. In this paper we consider a more general model - whose moduli Kahler potential is a *completely arbitrary* G_2-holonomy Kahler potential and whose matter Kahler potential is also more general. We prove that the vacuum structure and spectrum of BSM particles is largely unchanged in this much more general class of theories. In particular, gaugino masses are still supressed relative to the gravitino mass and moduli masses. We also consider the effects of higher order corrections to the matter Kahler potential and find a connection between the nature of the LSP and flavor effects.

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I. INTRODUCTION

From several theoretical points of view, the existence of moduli fields seems inevitable. For instance, supersymmetry may be the mechanism resonsible for stabilizing the scale of the Standard Model. Supersymmetry requires supergravity, whose only (known) reasonable UV completion seems to be String theory; and along with string theory come extra dimensions and their moduli. In fact, since string/M theory contains no dimensionless parameters, moduli appear necessary to explain the observed values of various couplings in nature. From the bottom up, moduli appear in various theories with "dynamical couplings" as well as in Inflation – the inflaton field is usually a neutral scalar field aka a modulus. For all of these reasons and more, moduli physics and phenomena must be considered seriously.

In a series of papers, [1], [2], [3], a very detailed model of moduli physics coupled to matter has been described. The G_2-MSSM model, largely inspired by M theory compactifications on manifolds of G_2-holonomy, is a model in which strong gauge dynamics in the hidden sector generates a potential which both stabilizes all the moduli fields and simultaneously generates a hierarchically small scale – thus solving (most of) the hierarchy problem. The model has an interesting spectrum: moduli have masses in the 50-100 TeV region, scalar superpartners and higgsinos have masses in the 10's of TeV region, whilst gauginos, which are the lightest BSM particles have masses of order 100's of GeV. Direct production of gluinos and electroweak gauginos are the dominant new physics channels at the LHC. The nature of the LSP is also very interesting as it is a neutral Wino. Moreover, its production in the early universe is dominated by the decays of the moduli fields (ie non-thermal production) and can naturally account for the observed fraction of dark matter today. The moduli and gravitino problems are avoided in an interesting way due to hierarchical structure in the moduli spectrum. One drawback of the model is the fine tuning between the 10's of TeV scale and M_Z and is the reason the model solves most of the hierarchy problem and not all of it.

However, the G_2-MSSM model, as defined in [2], is based on some specific assumptions about the moduli and matter Kahler potentials, albeit with the claim that these are general enough to incorporate all of the essential ingredients of more general Kahler potentials (and hence G_2-manifolds). Thus far, there has been no serious study of these assumptions and it is the main aim of this paper to undertake this. The main result that we prove here is that the mass spectrum of the theory depends very weakly on the specific form of the *moduli* Kahler potential; in fact the spectrum depends on the Kahler potential for moduli *only* through the fact that it is the Log of a homogeneous function (the volume of the extra dimensions); the precise nature of this homogeneous function is fairly irrelevant as we will see. Regarding the Kahler potential for *matter* – we provide two consistent arguments for calculating the moduli dependence of the matter kinetic terms in 4d Einstein frame. Whilst non-trivial, these modifications do not change the results of [1], [2] much. More importantly, we consider higher order terms in the matter Kahler potential, in particular the terms which are usually considered troublesome for flavour physics in theories of gravity mediated susy breaking. Whilst we expect that such operators will be suppressed, if they enter with large coefficients they can affect the mass spectrum: whilst they do not affect the scalar and higssino masses, which are large, they can alter the nature of the LSP. In particular, we find that the LSP can be a Bino. This provides a connection between flavor physics and the nature of the LSP in models of this sort.

The paper is organized as follows. The next section describes some simple properties of the moduli space metric for general G_2-manifolds which will be important for our later considerations. Section III is devoted to the Kahler potential for charged matter fields. Following this, we re-do the analysis of Moduli stabilization from [1] in this much more general context. In section V we compute the mass spectrum and susy breaking couplings in the minimum of the potential and demonstrate that it is almost identical to that of the original G_2-MSSM. In section VI we renormalise this Lagrangian down to the Electroweak scale.

II. GENERAL PROPERTIES OF MODULI SPACE METRICS ON G_2 HOLONOMY MANIFOLDS

In this section we describe some very general and simple properties of the moduli space metric of G_2-manifolds. It is these simple properties which will allow us to draw very general conclusions.

The complexified moduli space $\mathcal{M}$ of a G_2 holonomy compactification manifold X has holomorphic coordinates z_i given by

$$z_i = t_i + i s_i \,, \tag{1}$$

where s_i are geometric moduli corresponding to the perturbations of the internal metric and represent volumes of three cycles inside manifold X and t_i are the axions corresponding to the fluctuations of the three form. The moduli space $\mathcal{M}$ has dimension $N = b^3(X)$ and the classical moduli space metric (not including possible quantum corrections) can be derived from the following Kahler potential [4]

$$\hat{K} = -3 \ln 4\pi^{1/3} V_X \,, \tag{2}$$

where the dimensionless volume $V_X \equiv Vol(X)/l_M^7$ is a homogeneous function of s_i of degree 7/3 and l_M is the 11d Planck length. The homogeneity of V_X is the key property that we will utilise in what follows.

Define the following derivatives wrt moduli

$$\hat{K}_i \equiv \frac{\partial \hat{K}}{\partial s_i} \quad \text{and} \quad \hat{K}_{ij} \equiv \frac{\partial^2 \hat{K}}{\partial s_i \partial s_j} \,. \tag{3}$$

The matrix $\hat{K}_{ij}$, the Hessian of $\hat{K}$ is related to the actual Kahler metric $\hat{G}_{i\bar{j}}$ which controls the kinetic terms as $4\hat{G}_{i\bar{j}} = \hat{K}_{i\bar{j}}$, where in the Hessian we simply replace index j with $\bar{j}$. Since V_X is a homogeneous function of degree 7/3, the first derivative of $\hat{K}$ defined above has the following property

$$\sum_{i=1}^{N} s_i \hat{K}_i = -7 \,. \tag{4}$$

Differentiating (4) with respect to s_j we obtain an important property of the metric $\hat{K}_{ij}$

$$\sum_{i=1}^{N} s_i \hat{K}_{ij} = -\hat{K}_j \,, \text{ and since } \hat{K}_{ij} \text{ is symmetric } \sum_{j=1}^{N} s_j \hat{K}_{ij} = -\hat{K}_i \,. \tag{5}$$

Introducing new variables $\tilde{a}_i$ defined by

$$\tilde{a}_i \equiv -\frac{1}{3} s_i \hat{K}_i \,, \text{ no sum over i} \,. \tag{6}$$

We see that $\tilde{a}_i$

$$\sum_{i=1}^{N} \tilde{a}_i = \frac{7}{3} \,. \tag{7}$$

Differentiating the $\tilde{a}_i$ allows us to introduce the matrix

$$P_{ij} \equiv -s_j \frac{\partial \tilde{a}_i}{\partial s_j} \,, \text{ no sum over j} \,. \tag{8}$$

which has components

$$P_{ij} = \frac{1}{3} \delta_{ij} s_j \hat{K}_i + s_i s_j \frac{1}{3} \hat{K}_{ij} \,, \text{ no sum over i, j} \,. \tag{9}$$

P_{ij} has the following contraction properties, which follow from (9) and (5)

$$\sum_{i=1}^{N} P_{ij} = 0 \,, \text{ and } \sum_{j=1}^{N} P_{ij} = 0 \,. \tag{10}$$

We can then write

$$\hat{K}_{ij} = \frac{3\tilde{a}_j}{s_i s_j} \Delta_{ij} \,, \tag{11}$$

where the matrix Δ_{ij} is defined as

$$\Delta_{ij} \equiv \delta_{ij} + \frac{P_{ij}}{\tilde{a}_j} \,, \tag{12}$$

and satisfies the following contraction properties

$$\sum_{i=1}^{N} \Delta_{ij} = 1\,, \text{ and } \sum_{j=1}^{N} \Delta_{ij}\tilde{a}_j = \tilde{a}_i\,, \tag{13}$$

where we used (10) to derive (13). Note that parameters $\tilde{a}_i$ defined in (6) are the components of an eigenvector $\tilde{a}$ of the non-Hermitian matrix Δ with unit eigenvalue.

We can compute the formal inverse of the Hessian metric, $\hat{K}^{ij}$. By definition of the inverse it must satisfy

$$\sum_{j=1}^{N} \hat{K}^{ij}\hat{K}_{jk} = \delta^i_k\,, \tag{14}$$

and using (11) it can be expressed as

$$\hat{K}^{ij} = \frac{s_i s_j}{3\tilde{a}_i}(\Delta^{-1})^{ij}\,, \tag{15}$$

where the inverse matrix $(\Delta^{-1})^{ij}$ satisfies

$$\sum_{j=1}^{N} (\Delta^{-1})^{ij}\Delta_{jk} = \delta^i_k\,. \tag{16}$$

Symbolically we can express Δ^{-1} as

$$\Delta^{-1} = \frac{1}{1+\frac{P}{\tilde{a}}}\,, \tag{17}$$

which in terms of components translates into

$$(\Delta^{-1})^{ij} = \delta^{ij} - P_{ij}\frac{1}{\tilde{a}_j} + P_{il}\frac{1}{\tilde{a}_l}P_{lj}\frac{1}{\tilde{a}_j} - P_{il}\frac{1}{\tilde{a}_l}P_{lm}\frac{1}{\tilde{a}_m}P_{mj}\frac{1}{\tilde{a}_j} + \ldots\,. \tag{18}$$

Using (10) and (18) we derive the following properties of the inverse matrix Δ^{-1}

$$\sum_{i=1}^{N} (\Delta^{-1})^{ij} = 1\,, \text{ and } \sum_{j=1}^{N} (\Delta^{-1})^{ij}\tilde{a}_j = \tilde{a}_i\,, \tag{19}$$

which could have also been obtained directly from (13). Note that although we do not have a closed form expression for the components $(\Delta^{-1})^{ij}$, the contraction properties in (19) are what will ultimately allow us to derive explicit expressions for the terms in the soft breaking lagrangian – since such couplings depend only on the contractions and not the precise details of the functional form of V_X. Before going on to the details of these calculations, we first must consider the Kahler potential for matter fields in M theory.

III. KAHLER POTENTIAL FOR CHIRAL MATTER

In this section we re-visit the Kahler potential for charged matter fields in M theory. In practice, the absence of a useful microscopic formulation makes it difficult to compute the moduli dependence of the Kahler potential for these fields in general. Below we outline two arguments for the structure of the Kahler potential - first from dimensional reduction and the second based on the holomorphy of the superpotential. Happily, the two methods agree.

A. Kahler potential from dimensional reduction

In M theory, charged, chiral matter is localized near conical singularities [5], [6], [7], [8]. These are literally points in the seven extra dimensions. Because of this, we expect that the kinetic terms for the chiral matter fields should be "largely independent of bulk moduli fields" that the G_2 manifold X has. They could, of course depend on local moduli inherent to the conical singularity, but, since, in a supersymmetric theory, a single chiral multiplet in a complex representation of the gauge group usually has no D or F-flat directions [9], there are typically no such local moduli.

There is a subtely in the above general arguments. Since, in four dimensions, a scalar field kinetic term is not invariant under Weyl rescalings of the metric, one has to pick a Weyl gauge. We will argue that the correct Weyl gauge for the statement is NOT the 4d Einstein frame. Therefore, the kinetic term for chiral matter will be non-trivial in the 4d Einstein frame, which is the standard one in which to define the Kahler potential.

Since the physics of a conical singularity in M theory does not introduce any new scale, asides from the 11d Planck scale, the only reasonable Weyl frame is the 11d Einstein frame. In fact, the action for a codimension four conical (orbifold) singularity in M theory – where non-Abelian gauge fields are supported – is canonical in 11d Einstein frame [10]. Thus, we will follow our noses and also assume that the codimension seven, localized matter has a canonical kinetic term in 11d Einstein frame as well. As we will see, the results are consistent with other considerations.

Therefore the lagrangian density in 11d frame is

$$L \sim {M_{11}}^9 \sqrt{g_{11}} R + \delta_7 \wedge \partial_M \phi \partial_N \phi g^{MN} + \ldots , \tag{20}$$

where δ_7 is a delta function peaked at the position of the matter multiplet containing the scalar field ϕ and has mass dimension 7. Integrating this over X leads to a 4d density

$$L_4 \sim V_X M_{11}^2 \sqrt{g_4} R_4 + \partial_\mu \phi \partial_\nu \phi g_4^{\mu\nu} , \tag{21}$$

where V_X is the volume of the extra dimensions in 11d units. This is the Lagrangian in 11d Einstein frame. If we now Weyl rescale into the 4d Einstein frame we find

$$L_4 \sim \sqrt{g_E} R_E + \frac{1}{V_X} \partial_\mu \phi \partial_\nu \phi g_E^{\mu\nu} , \tag{22}$$

where the subscript E indicates we are using the 4d Einstein frame metric.

We have only considered the Einstein-Hilbert and kinetic terms of the matter fields. Including all the other terms would give the 4d supergravity Lagrangian in Einstein frame. In particular from this we would read off that the Kahler metric for the multiplet containing ϕ is

$$K_{\phi\bar{\phi}} = \frac{1}{V_X} . \tag{23}$$

As we will see, this is consistent with the arguments given in the next subsection.

B. Kahler potential from holomorphy of the superpotential

The second argument for the stucture of the Kahler potential for chiral matter is based on the requirement of holomorphy of the superpotential. Since the calculation is rather tedious, uninterested readers may prefer to move on to the next subsection.

Let us consider a hidden sector gauge theory which is $\mathcal{N}=1$ super QCD with $SU(N)\times U(1)$ gauge group and N_f flavors of chiral matter fields. Following the notation of Friedmann and Witten [10], the RGE for the $SU(N)$ gauge coupling including the threshold corrections is

$$\frac{16\pi^2}{g_a^2(\Lambda)} = \frac{16\pi^2}{g_a^2(\mu)} - (3N - N_f)\ln\left(\frac{\mu^2}{\Lambda^2}\right) + \mathcal{S}_a \,. \tag{24}$$

The threshold corrections from the Kaluza-Klein modes were computed in [10] and are given by

$$\mathcal{S}_a = -3N\ln\left(\frac{1}{V_{\mathcal{Q}}^{2/3}\mu^2}\right) + \mathcal{S}'_a \,, \tag{25}$$

where $\mathcal{S}'_a$ is given in terms of Ray-Singer torsion classes $\mathcal{T}_i$ corresponding to different $U(1)$ representations

$$\mathcal{S}'_a = 2\sum_i \mathcal{T}_i \mathrm{Tr}_{\mathcal{R}_i} Q_a^2 \,, \tag{26}$$

where Q_a is a generator of the $SU(N)$ gauge group of the hidden sector. From (24), the scale at which the theory becomes strongly coupled is

$$\Lambda^{3N-N_f} = \mu^{3N-N_f} e^{-\frac{8\pi^2}{g_a^2(\mu)}} e^{-\frac{S_a}{2}} \,. \tag{27}$$

Following the recipe outlined in [11], we need to insert an additional factor of $(g_a(\mu))^{-2N}$ to make the combination

$$\tilde{\Lambda}^{3N-N_f} = \mu^{3N-N_f} (g_a(\mu))^{-2N} e^{-\frac{8\pi^2}{g_a^2(\mu)}} e^{-\frac{S_a}{2}} \,, \tag{28}$$

renormalization group invariant. The authors of [11] point out that the origin of this extra loop factor probably arises as a left-over of the cancellation between bosonic and fermionic determinants when the gauge bosons and gauginos are not canonically normalized. Next, using the relation $l_m = 2\pi/M_{11}$, we define the dimensionless volume $\tilde{V}_{\mathcal{Q}}$ of the associative cycle supporting the gauge fields as

$$V_{\mathcal{Q}} \equiv \tilde{V}_{\mathcal{Q}} l_m^3 = \frac{(2\pi)^3}{M_{11}^3}\tilde{V}_{\mathcal{Q}} \,, \tag{29}$$

so that the $SU(N)$ gauge coupling $g_M \equiv g_a(M_{11})$ at the scale M_{11} can be identified as

$$\frac{g_M^2}{4\pi} = \frac{1}{\tilde{V}_{\mathcal{Q}}} \,. \tag{30}$$

According to [10] the cutoff scale μ is of order M_{11}. Hence, using (25) and setting $\mu = M_{11}$ we obtain

$$e^{-\frac{S_a}{2}} = \frac{1}{V_{\mathcal{Q}}^N \mu^{3N}} e^{-\frac{S'_a}{2}} = \frac{(g_M)^{2N}}{2^N (2\pi)^{4N}} e^{-\frac{S'_a}{2}} \,. \tag{31}$$

Substituting (31) into (28) we then obtain

$$\tilde{\Lambda}^{3N-N_f} = \frac{M_{11}^{3N-N_f}}{2^N (2\pi)^{4N}} e^{-\frac{8\pi^2}{g_M^2}} e^{-\frac{S'_a}{2}} \,. \tag{32}$$

Of course, one should not assume that the overall numerical constant in the above expression gives a correct normalization. In what follows, we will simply parametrize the unknown overall normalization by a single parameter. In supergravity, the Affleck-Dine-Seiberg effective superpotential [12] W should be identified with

$$e^{\hat{K}/2}W = \frac{(N-N_f)\tilde{\Lambda}^{\frac{3N-N_f}{N-N_f}}}{\det(Q\tilde{Q})^{\frac{1}{N-N_f}}}\,, \tag{33}$$

and using (32), up to an overall numerical constant we obtain

$$e^{\hat{K}/2}W \sim (N-N_f)M_{11}^{\frac{3N-N_f}{N-N_f}}\det(Q\tilde{Q})^{-\frac{1}{N-N_f}}\,e^{-\frac{8\pi^2}{(N-N_f)g_M^2}}\,e^{-\frac{S_a'}{2(N-N_f)}}\,. \tag{34}$$

In the above expression, the Kahler potential is given by

$$\hat{K} = -3\ln(4\pi^{1/3}V_X)\,, \tag{35}$$

where V_X is the volume of the internal manifold X and g_M^2 is the holomorphic coupling. From dimensional reduction, we have the following relation between the four dimensional reduced Planck scale m_{pl} and M_{11}

$$M_{11}^2 = \frac{\pi m_{pl}^2}{V_X}\,. \tag{36}$$

Therefore, the contribution of the bulk Kahler potential boils down to

$$e^{\hat{K}/2} = \frac{1}{8\pi^{1/2}V_X^{3/2}} = \frac{M_{11}^3}{8\pi^2 m_{pl}^3}\,. \tag{37}$$

Combining (34) with (37), we obtain, up to an overall constant

$$W \sim \frac{m_{pl}^3}{M_{11}^3}(N-N_f)M_{11}^{\frac{3N-N_f}{N-N_f}}\det(Q\tilde{Q})^{-\frac{1}{N-N_f}}\,e^{-\frac{8\pi^2}{(N-N_f)g_M^2}}\,e^{-\frac{S_a'}{2(N-N_f)}}\,. \tag{38}$$

Because the above superpotential has to be a holomorphic function of the fields, including the moduli, it should not depend on the volume V_X which enters through M_{11} in (36). In the above expression, the chiral matter fields Q and $\tilde{Q}$ have mass dimension one and if we define dimensionless fields $\hat{Q}$ and $\hat{\tilde{Q}}$ as

$$Q = \hat{Q}M_{11}\,, \quad \tilde{Q} = \hat{\tilde{Q}}M_{11}\,, \tag{39}$$

the superpotential in (38) indeed becomes holomorphic since the dependence on M_{11} completely drops out

$$W = \tilde{C}\,m_{pl}^3(N-N_f)\det(\hat{Q}\hat{\tilde{Q}})^{-\frac{1}{N-N_f}}\,e^{-\frac{8\pi^2}{(N-N_f)g_M^2}}\,e^{-\frac{S_a'}{2(N-N_f)}}\,, \tag{40}$$

where $\tilde{C}$ is an overall numerical constant. In our further notation, we also define the following constants

$$C \equiv \tilde{C}\,e^{-\frac{S_a'}{2(N-N_f)}}\,, \quad \text{and} \quad A \equiv (N-N_f)\,C\,. \tag{41}$$

So that the superpotential can be written as

$$W = A\,m_{pl}^3\det(\hat{Q}\hat{\tilde{Q}})^{-\frac{1}{N-N_f}}\,e^{-\frac{8\pi^2}{(N-N_f)g_M^2}}\,. \tag{42}$$

In $\mathcal{N}=1$ $D=4$ supergravity, canonically normalized Kahler potential for one flavor $N_f=1$ of chiral matter fields in N and $\bar{N}$ of $SU(N)$ is given by

$$\tilde{K}=Q^\dagger Q+\tilde{Q}^\dagger\tilde{Q}=m_{pl}^2 Q_c^\dagger Q_c+m_{pl}^2\tilde{Q}_c^\dagger\tilde{Q}_c\,. \tag{43}$$

where matter fields Q_c and $\tilde{Q}_c$ are dimensionless and measured in units of m_{pl}. Clearly, the eleven-dimensional frame fields $\hat{Q}$ and $\hat{\tilde{Q}}$ appearing inside the superpotential cannot be directly identified as Q_c and $\tilde{Q}_c$. However, we can rewrite the above Kahler potential as

$$\tilde{K}=Q^\dagger Q+\tilde{Q}^\dagger\tilde{Q}=M_{11}^2\hat{Q}^\dagger\hat{Q}+M_{11}^2\hat{\tilde{Q}}^\dagger\hat{\tilde{Q}}=m_{pl}^2\left(\frac{M_{11}^2}{m_{pl}^2}\right)\hat{Q}^\dagger\hat{Q}+m_{pl}^2\left(\frac{M_{11}^2}{m_{pl}^2}\right)\hat{\tilde{Q}}^\dagger\hat{\tilde{Q}}. \tag{44}$$

Using (36) we can then recast the Kahler potential as

$$\tilde{K}=m_{pl}^2\frac{\hat{Q}^\dagger\hat{Q}}{V_X}+m_{pl}^2\frac{\hat{\tilde{Q}}^\dagger\hat{\tilde{Q}}}{V_X}\,, \tag{45}$$

where we have also rescaled the fields by a factor of $\sqrt{\pi}$ which can be further absorbed into the overall normalization constant C in (41). Hence, the canonically normalized dimensionless fields Q_c and $\tilde{Q}_c$ of $\mathcal{N}=1$ $D=4$ supergravity are expressed in terms of the eleven-dimensional frame fields $\hat{Q}$ and $\hat{\tilde{Q}}$ appearing inside the superpotential as

$$Q_c=\frac{\hat{Q}}{\sqrt{V_X}}\,,\ \text{and}\quad \tilde{Q}_c=\frac{\hat{\tilde{Q}}}{\sqrt{V_X}}\,. \tag{46}$$

Let us now consider the case of $N_f=1$ flavors. Introducing an effective meson degree of freedom

$$\phi\equiv\sqrt{2\hat{Q}\hat{\tilde{Q}}}\,, \tag{47}$$

we can rewrite the superpotential in terms of ϕ as

$$W=A\,m_{pl}^3\phi^{-\frac{2}{N-1}}e^{-\frac{8\pi^2}{(N-1)g_M^2}}\,, \tag{48}$$

where we have absorbed the factor of $2^{1/(N-1)}$ into the normalization constant C. Along the D-flat direction we have $\hat{Q}=\hat{\tilde{Q}}$ and the Kahler potential can be rewritten as

$$\tilde{K}=m_{pl}^2\frac{\bar{\phi}\phi}{V_X}\,. \tag{49}$$

C. Higher order terms

In our derivation of the Kahler potential we so far neglected possible higher order contributions to the visible sector matter Kahler potential of the form

$$\delta\tilde{K}=c_{\alpha\bar{\beta}}Q_c^\alpha\bar{Q}_c^{\bar{\beta}}\phi_c\bar{\phi}_c+\ldots=c_{\alpha\bar{\beta}}\frac{Q^\alpha\bar{Q}^{\bar{\beta}}}{V_X}\frac{\phi\bar{\phi}}{V_X}+\ldots, \tag{50}$$

which gravitationally couple the hidden sector meson to the visible sector fields Q_α. In the above expression, the subscript c denotes canonically normalized matter fields in the 4-dimensional Einstein frame. Such couplings can create problems if the meson F-term is quite large (which is true in the G_2-MSSM) because they can induce flavor changing neutral currents. This is the flavor problem of gravity mediated susy breaking models. These terms were neglected in our previous work [2].

Technically, computing the unknown coefficients $c_{\alpha\bar{\beta}}$ from the underlying theory is difficult, goes well beyond the scope of this work and our aim here is *not* to explain the flavor structure of the supersymmetry breaking Lagrangian. Rather, we would like to understand the effect that the presence of such terms might have on other sectors of the theory, eg their effect on superpartner masses and couplings. For these purposes it is sufficient to ignore flavor and consider the simpler

$$c_{\alpha\bar{\beta}} = \frac{c}{3}\,\delta_{\alpha\bar{\beta}}\,. \tag{51}$$

Whilst this does not introduce any flavor violation, the point will be that the effect of such terms on the mass spectrum will be similar even if we introduced flavor violating terms, as should become clear eventually.[1]

Actually, such a form might arise from an expansion of the Kahler potential if the visible and hidden sectors were completely sequestered. Though we do not expect M theory to be sequestered, it can be useful to think of the sequestering as an extreme limit in a more general model.

A sequestered Kahler potential has the form

$$K^{seq} = -3\ln\left(4\pi^{1/3}V_X - \frac{1}{3}\phi^2 - \frac{1}{3}\delta_{\alpha\bar{\beta}}Q^{\alpha}\bar{Q}^{\bar{\beta}}\right)\,, \tag{52}$$

and the Kahler metric for the visible sector is given by

$$K^{seq}_{\alpha\bar{\beta}} = \frac{\delta_{\alpha\bar{\beta}}}{4\pi^{1/3}V_X - \frac{1}{3}\phi^2}\,. \tag{53}$$

Absorbing the factor of $4\pi^{1/3}$ into the definition of the fields and expanding the above expression in powers of ϕ^2/V_X we obtain

$$K^{seq}_{\alpha\bar{\beta}} = \frac{\delta_{\alpha\bar{\beta}}}{V_X}\left(1 + \frac{\phi^2}{3V_X}\right) + \dots\,. \tag{54}$$

Comparing the above expression with (50) we can read off the coefficients

$$c^{seq}_{\alpha\bar{\beta}} = \frac{1}{3}\delta_{\alpha\bar{\beta}}\,, \tag{55}$$

which corresponds to (51) when $c = 1$. Hence, parameter c in (51) is the measure of deviation of the matter Kahler potential from the exactly sequestered form. As was pointed out in [14], sequestering is not at all generic in string/M theory and presumably G_2 compactifications of M theory are no exception. We thus will regard c as a parameter and consider the theory for various values of $0 \leq c \leq 1$.

Combining all of the previous considerations, our visible sector matter Kahler potential takes the form

$$\tilde{K} = \tilde{K}_{\alpha}\delta_{\alpha\bar{\beta}}Q^{\alpha}\bar{Q}^{\bar{\beta}} = \frac{\delta_{\alpha\bar{\beta}}Q^{\alpha}\bar{Q}^{\bar{\beta}}}{V_X}\left(1 + c\frac{\phi^2}{3V_X}\right)\,, \tag{56}$$

[1] Generically, the absence of flavor changing neutral currents implies that the off-diagonal entries in $c_{\alpha\beta}$ are suppressed eg less than 1/10, though in particular models even stronger constraints are possible depending on the spectrum and A-terms [13].

where parameter c takes values in the range

$$0 \le c \le 1\,. \tag{57}$$

The Kahler metric for the visible sector matter is flavor-universal and is given by

$$\tilde{K}_\alpha = \frac{1}{V_X}\left(1 + c\frac{\phi^2}{3V_X}\right)\,. \tag{58}$$

In computing the scalar masses, the trilinears and the anomaly mediated contribution to the gaugino masses, it will be necessary to compute various derivatives of $\ln \tilde{K}_\alpha$. For this purpose, it turns out that it is very convenient to express $\ln \tilde{K}_\alpha$ as

$$\ln \tilde{K}_\alpha = -\ln V_X + \ln\left(1 + c\frac{\phi^2}{3V_X}\right) \approx \frac{1}{3}K + (c-1)\frac{\phi^2}{3V_X} + \mathrm{const}\,, \tag{59}$$

where K is the Kahler potential in (63).

IV. MODULI STABILIZATION

In this section we reconsider the problem of moduli stabilization with the much more general moduli and matter Kahler potentials introduced in the previous section. We will be working in the framework of $\mathcal{N} = 1$ $D = 4$ effective supergravity and will demonstrate that all the moduli can be stabilized self-consistently in the regime where the supergravity approximation is valid. Recall that in the compactifications we study here, non-Abelian gauge fields arise from co-dimension four singularities [15], [16], [17], [18], [5]. In other words, there exist three-dimensional submanifolds $\mathcal{Q}$ inside the G_2-manifold X, along which there is an orbifold singularity of A-D-E type. Here we assume there exist two hidden sectors with gauge groups $SU(P+N_f)$ and $SU(Q)$ where the first hidden sector gauge theory is a super QCD with $N_f = 1$ flavor of quarks Q and $\tilde{Q}$ transforming in a complex (conjugate) representation of $SU(P+1)$ (the corresponding associative cycle $\mathcal{Q}$ contains two isolated singularities of co-dimension seven) and the second hidden sector with the gauge group $SU(Q)$ is a "pure glue" super Yang-Mills theory. One can easily consider more general gauge groups without much qualitative difference. One can also consider a setup with charged matter in both hidden sectors. However, as was demonstrated in [1], in such cases, one of the two F-terms coming from the matter fields in the hidden sectors is always zero and thus does not contribute to the quantities relevant for phenomenology. A single hidden sector gauge theory is also enough to stabilise the moduli, though the vacuum is not in a place where supergravity is trustable.

Therefore, the nonperturbative effective superpotential generated by the strong gauge dynamics in the hidden sectors is given by

$$W = A_1 \phi^a e^{ib_1 f} + A_2 e^{ib_2 f}\,. \tag{60}$$

The matter field ϕ represents an effective meson degree of freedom defined in (47) in terms of the chiral matter fields $\hat{Q}$ and $\hat{\tilde{Q}}$. The coefficients b_1, b_2 and a are

$$b_1 = \frac{2\pi}{P}\,, \quad b_1 = \frac{2\pi}{Q}\,, \quad a = -\frac{2}{P}\,. \tag{61}$$

In [1] it was explained that if one uses a superpotential of the form (60), de Sitter vacua arise only when $Q > P$ (if we include matter in both hidden sectors dS vacua exist without such condition). Hence, we will keep this in mind from now on.

In (60) we explicitly assumed that the associative cycles supporting both hidden sectors are in proportional homology classes which results in the gauge kinetic function being given by essentially the same integer combination of the moduli z_i for both hidden sectors

$$f = \sum_{i=1}^{N} N_i z_i \,. \tag{62}$$

Here $\mathrm{Im}(f) = V_{\mathcal{Q}}$ is the volume of the corresponding associative cycles. While one can certainly consider possibilities where the gauge kinetic functions f_1 and f_2 are not proportional, the results in [1] taught us that unless $f_1 \propto f_2$ it is very difficult to stabilize all the moduli in the regime where the supergravity approximation is valid. Thus, obtaining solutions which we can trust is the main reason for choosing to consider the case where $f_1 = f_2 = f$. Obviously, progress in the more general cases would be welcome.

Typical examples for 3-cycles supporting non-Abelian gauge fields in G_2-manifolds are spheres and their quotients such as Lens spaces $S^3/\mathbf{Z_q}$ considered in [10]. The expression in (60) can in principle contain many additional non-perturbative contributions if X contains other associative cycles. In that respect, the two terms included in (60) should be regarded as the leading order exponentials. As long as Q and P are large enough compared to the casimirs from the other gauge groups, the remaining terms will be exponentially suppressed in general. This is particularly true for the membrane instanton corrections to (60) which come with exponentials containing $b_i = 2\pi$. On the other hand, some such instantons induce Yukawa interactions among the visible sector matter fields and are therefore implicitly assumed to be part of the full superpotential.

The total Kahler potential - moduli plus hidden sector matter, is given by

$$K = -3\ln 4\pi^{1/3} V_X + \frac{\bar{\phi}\phi}{V_X} \,. \tag{63}$$

In general, (63) must include the contributions to the Kahler potential from all matter sectors including the visible sector as described in the previous section. However, since the visible sector fields will obtain zero vev's, they can be dropped for the purposes of stabilizing moduli.

The standard $\mathcal{N} = 1$ $D = 4$ supergravity scalar potential is given by

$$V = e^K \left(K^{i\bar{j}} \, F_i F_{\bar{j}} \, - 3|W|^2 \right) , \tag{64}$$

where the F-terms are

$$\begin{aligned} F_i &= \partial_i W + W \partial_i K = i N_i e^{i b_2 \vec{N}\cdot\vec{t}} \left(-b_1 A_1 \phi_0^a e^{-b_1 \vec{N}\cdot\vec{s}} + b_2 A_2 e^{-b_2 \vec{N}\cdot\vec{s}} \right) \\ &+ i \frac{3\tilde{a}_i}{2 s_i} \left(1 + \frac{\phi^2}{3 V_X} \right) e^{i b_2 \vec{N}\cdot\vec{s}} \left(-A_1 \phi^a e^{-b_1 \vec{N}\cdot\vec{s}} + A_2 e^{-b_2 \vec{N}\cdot\vec{s}} \right) \end{aligned} \tag{65}$$

$$\begin{aligned} F_\phi &= \partial_\phi W + W \partial_\phi K = -e^{i b_2 \vec{N}\cdot\vec{t} - i\theta} a A_1 \phi_0^{a-1} e^{-b_1 \vec{N}\cdot\vec{s}} \\ &+ \frac{\phi_0}{V_X} e^{i b_2 \vec{N}\cdot\vec{t} - i\theta} \left(-A_1 \phi^a e^{-b_1 \vec{N}\cdot\vec{s}} + A_2 e^{-b_2 \vec{N}\cdot\vec{s}} \right) \,. \end{aligned} \tag{66}$$

In the above we used

$$\frac{\partial K}{\partial z_i} = \frac{1}{2i} \frac{\partial K}{\partial s_i} \,, \tag{67}$$

together with the definition of $\tilde{a}_i$ in (6) in combination with

$$\frac{\partial}{\partial s_i} \frac{1}{V_X} = \frac{\hat{K}_i}{3 V_X} \,. \tag{68}$$

We also parametrized the meson field ϕ as

$$\phi = \phi_0 e^{i\theta}\,, \tag{69}$$

and fixed one combination of the axions and the meson phase θ

$$\cos((b_1 - b_2)\vec{N}\cdot\vec{t} + a\theta) = -1\,. \tag{70}$$

In order to compute the scalar potential we need to compute the inverse Kahler metric. Using the Kahler potential (63) together with (6), (11), (67) and (68) we first obtain the following components for the Kahler metric

$$\begin{aligned} K_{i\bar{j}} &= \frac{3\tilde{a}_{\bar{j}}}{4s_i s_{\bar{j}}}\left(1+\frac{\phi_0^2}{3V_X}\right)\Delta_{i\bar{j}} + \frac{\tilde{a}_i\tilde{a}_{\bar{j}}}{4s_i s_{\bar{j}}}\frac{\phi_0^2}{V_X} \\ K_{i\bar{\phi}} &= i\frac{\tilde{a}_i}{2s_i}\frac{\phi}{V_X} \\ K_{\phi\bar{j}} &= -i\frac{\tilde{a}_{\bar{j}}}{2s_{\bar{j}}}\frac{\bar{\phi}}{V_X} \\ K_{\phi\bar{\phi}} &= \frac{1}{V_X}\,. \end{aligned} \tag{71}$$

Note that on the right hand side of the above expressions $\tilde{a}_{\bar{j}}$ and $\Delta_{i\bar{j}}$ are the same real quantities defined previously with index j replaced by $\bar{j}$.

The inverse Kahler metric must satisfy the following set of equations

$$\begin{aligned} K^{i\bar{j}}\,K_{\bar{j}\,k} + K^{i\bar{\phi}}K_{\bar{\phi}k} &= \delta^i_k \\ K^{i\bar{j}}\,K_{\bar{j}\,\phi} + K^{i\bar{\phi}}K_{\bar{\phi}\phi} &= 0 \\ K^{\phi\bar{j}}\,K_{\bar{j}\,\phi} + K^{\phi\bar{\phi}}K_{\bar{\phi}\phi} &= 1\,. \end{aligned} \tag{72}$$

After a little bit of work we obtain the following components for the inverse Kahler metric

$$\begin{aligned} K^{i\bar{j}} &= \frac{4s_i s_{\bar{j}}}{3\tilde{a}_i}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}} \\ K^{i\bar{\phi}} &= i\frac{2}{3}\frac{s_i\bar{\phi}}{1+\frac{\phi_0^2}{3V_X}} \\ K^{\phi\bar{j}} &= -i\frac{2}{3}\frac{s_{\bar{j}}\phi}{1+\frac{\phi_0^2}{3V_X}} \\ K^{\phi\bar{\phi}} &= V_X\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi_0^2}{3V_X}}\frac{\phi_0^2}{3V_X}\right)\,. \end{aligned} \tag{73}$$

Note that despite the fact that the matter part of the Kahler potential in (63) is only given up to the quadratic order in $\frac{\phi_0^2}{V_X}$, we decided to keep all the higher order terms inside the inverse Kahler metric. This is self-consistent as long as the combination $\frac{\phi_0^2}{3V_X}$ appearing in the inverse Kahler metric is stabilized at a value sufficiently smaller than one such that the quartic and higher order terms are suppressed.

Now, putting all the pieces together we obtain the scalar potential

$$
\begin{aligned}
V \;=\; & \frac{e^{\phi_0^2/V_X}}{64\pi V_X^3}\Big(\frac{4}{3}\sum_{i=1}^{N}\sum_{\bar{j}=1}^{N}\frac{s_i s_{\bar{j}}\,N_i N_{\bar{j}}}{\tilde{a}_i}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}}\left(b_1A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-b_2A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)^2 \\
&+\;4\vec{N}\cdot\vec{s}\left(b_1A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-b_2A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-A_2e^{-b_2\vec{N}\cdot\vec{s}}\right) \\
&+\;7\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)^2\left(1+\frac{\phi_0^2}{3V_X}\right) \\
&-\;\frac{4}{3}\left(\frac{b_1A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-b_2A_2e^{-b_2\vec{N}\cdot\vec{s}}}{1+\frac{\phi_0^2}{3V_X}}\vec{N}\cdot\vec{s}+\frac{7}{2}\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)\right) \\
&\times\;\left(aA_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}+\frac{\phi_0^2}{V_X}\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)\right)+\frac{V_X}{\phi_0^2}\left(1+\frac{7}{3}\frac{1}{1+\frac{\phi_0^2}{3V_X}}\frac{\phi_0^2}{3V_X}\right) \\
&\times\;\left(aA_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}+\frac{\phi_0^2}{V_X}\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)\right)^2 \\
&-\;3\left(A_1\phi_0^a e^{-b_1\vec{N}\cdot\vec{s}}-A_2e^{-b_2\vec{N}\cdot\vec{s}}\right)^2\Big)\,.
\end{aligned}
\tag{74}
$$

To understand the minima of the potential we will use the techniques developed earlier in [1]. Namely, we will work in the regime when the volume of the hidden sector associative cycle $V_{\mathcal{Q}} = \vec{N}\cdot\vec{s}$ is large and expand our solutions in the inverse powers of this volume. This is equivalent to an expansion in the UV weak hidden sector gauge coupling. Since much of the details follow closely [1] in this long and tedious procedure we only need list the key steps and results. We reintroduce the notation of [1]

$$\alpha \equiv \frac{A_1\phi_0^a}{A_2}e^{-(b1-b2)\vec{N}\cdot\vec{s}}\,,\quad x\equiv\alpha-1\,,\quad y\equiv b_1\alpha-b_2\,,\quad z\equiv b_1^2\alpha-b_2^2\,, \tag{75}$$

and make the following ansatz for the moduli vevs at the minimum

$$s_i = \frac{\tilde{a}_i}{N_i}\frac{x}{y}L\,. \tag{76}$$

In this notation, the volume of the associative cycles supporting the hidden sector gauge groups is given by

$$V_{\mathcal{Q}} = \vec{N}\cdot\vec{s} = \frac{x}{y}L\sum_{i=1}^{N}\tilde{a}_i = \frac{7}{3}\frac{x}{y}L\,. \tag{77}$$

which can be rewritten as

$$s_i = \frac{\tilde{a}_i}{N_i}\frac{3}{7}V_{\mathcal{Q}}\,. \tag{78}$$

Let us first assume that L is non-zero and finite when $y\to 0$. Then, we get from (77) and the definitions above

$$
\begin{aligned}
&V_{\mathcal{Q}}\to\infty \quad\Rightarrow\quad y\to 0 \quad\Rightarrow\quad \alpha=\frac{b_1}{b_2}+\mathcal{O}(\frac{1}{V_{\mathcal{Q}}}) \\
&\Rightarrow V_{\mathcal{Q}}=\vec{N}\cdot\vec{s}=\frac{1}{b_1-b_2}\ln\left(\frac{b_1A_1\phi_0^a}{b_2A_2}\right)=\frac{1}{2\pi}\frac{PQ}{Q-P}\ln\left(\frac{QA_1\phi_0^a}{PA_2}\right)\,.
\end{aligned}
\tag{79}
$$

This fixes the allowed value of the volume of the hidden sector cycle $V_{\mathcal{Q}}$. Now the moduli vevs must be determined. For this, we put our expressions for s_i (78) into the definition of $\tilde{a}_i$ in (6) to get a system of N transcendental equations which then completely determine $\tilde{a}_i$ in principle

$$\frac{\tilde{a}_i}{N_i}\frac{V_{\mathcal{Q}}}{7}\hat{K}_i|_{s_i=\frac{\tilde{a}_i}{N_i}\frac{3}{7}V_{\mathcal{Q}}} + \tilde{a}_i = 0\,, \text{ no sum over i}\,. \tag{80}$$

Since $N_i > 0$ and $V_{\mathcal{Q}} > 0$, for the moduli s_i in (78) to be positive the solutions for $\tilde{a}_i$ from (80) must all be positive

$$\tilde{a}_i > 0\,,\ \forall i\,,\ i = \overline{1,N}\,. \tag{81}$$

In addition, one also needs to make sure that the Kahler metric is positive definite so that the corresponding kinetic terms are positive.

Obviously, obtaining general analytic solutions for $\tilde{a}_i$ from (80) is impossible in practice, since V_X has not been specified. However, precisely because the moduli vevs at the minumum are given by (76), it turns out that in order to compute the quantities relevant for particle physics, one does not need to know the values of $\tilde{a}_i$ explicitly. All one actually needs to know are the contraction properties (7) and (19).

Therefore, the results we derive will be valid for any singular manifold of G_2 holonomy for which solutions of the system (80) exist such that $\tilde{a}_i > 0$ and conditions on the positive definiteness of the Kahler metric are met. By explicitly checking in explicit examples, both numerically and analytically it seems that, for a given form of V_X, an isolated solution indeed exists.

In order to illustrate how the system (80) is realized in practice we give a couple of explicit examples, though we stress that we have checked many more general examples than just those given here. Let us first consider a particulatly simple N-parameter family of Kahler potentials consistent with G_2 holonomy where the volume V_X is given by

$$V_X = \prod_{i=1}^{N} s_i^{a_i}\,, \text{ where } \sum_{i=1}^{N} a_i = \frac{7}{3}\,. \tag{82}$$

In this case the solutions to (80) are simply constants given by

$$\tilde{a}_i = a_i\,. \tag{83}$$

In fact, this example represents the class of Kahler potentials considered in the previous work [1] and the solutions are discussed in detail there.

One can easily construct more complicated examples such as

$$V_X = \sum_k V_k\,, \text{ where } V_k \equiv c_k \prod_{i=1}^{N} s_i^{a_i^k}\,, \text{ such that} \,\forall k\ \sum_{i=1}^{N} a_i^k = \frac{7}{3}\,. \tag{84}$$

In this case system (80) translates into

$$\sum_k \left(a_i^k - \tilde{a}_i\right) c_k \prod_{m=1}^{N}\left(\frac{\tilde{a}_m}{N_m}\right)^{a_m^k} = 0\,. \tag{85}$$

In these cases one can check numerically that, for very generic sets of parameters $\{a_i^k,\, c_k,\, N_i\}$, the system of equations (85) yields positive solutions for $\tilde{a}_i$ where the Kahler metric is indeed positive definite.

For example, choosing $N_1 = 1,\ N_2 = 1,\ N_3 = 1,\ N_4 = 1$ we numerically compute $\tilde{a}_i$ for the following two cases with four moduli

$$V_X = s_1^{\frac{7}{9}} s_2^{\frac{7}{9}} s_3^{\frac{7}{18}} s_4^{\frac{7}{18}} - \frac{1}{3} s_1^1 s_2^{\frac{2}{3}} s_3^{\frac{1}{3}} s_4^{\frac{1}{3}} - \frac{1}{2} s_1^{\frac{1}{3}} s_2^1 s_3^{\frac{1}{2}} s_4^{\frac{1}{2}} \tag{86}$$

$$\Rightarrow\ \tilde{a}_1 \approx 1.038\,,\ \tilde{a}_2 \approx 0.648\,,\ \tilde{a}_3 \approx 0.324\,,\ \tilde{a}_4 \approx 0.324$$

$$V_X = s_2^{\frac{14}{9}} s_3^{\frac{7}{18}} s_4^{\frac{7}{18}} + \frac{1}{3} s_1^1 s_2^{\frac{2}{3}} s_4^{\frac{2}{3}} + \frac{1}{2} s_1^{\frac{1}{3}} s_2^1 s_3^1$$

$$\Rightarrow\ \tilde{a}_1 \approx 0.051\,,\ \tilde{a}_2 \approx 1.478\,,\ \tilde{a}_3 \approx 0.459\,,\ \tilde{a}_3 \approx 0.344\,.$$

In addition we present two randomly picked but fairly complicated examples of homogeneous functions of degree 7/3 for three moduli where $\tilde{a}_i$ were found numerically. For both examples we chose $N_1 = 1,\ N_2 = 2,\ N_3 = 1$

$$V_X = \left(2 s_1^{10} s_2^{\frac{40}{3}} + 3 s_1^{\frac{25}{3}} s_2^{15} + s_2^{\frac{35}{3}} s_3^{\frac{35}{3}} + s_3^{\frac{70}{3}}\right)^{\frac{1}{10}} - s_1^{\frac{25}{12}} s_2^{\frac{1}{8}} s_3^{\frac{1}{8}} \tag{87}$$

$$\Rightarrow\ \tilde{a}_1 \approx 2.102\,,\ \tilde{a}_2 \approx 0.104\,,\ \tilde{a}_3 \approx 0.127$$

$$V_X = \left(2 s_1^{10} s_2^{\frac{40}{3}} \exp\left[\left(\frac{s_1}{s_2}\right)^{3/7} + \sin\left(\frac{s_2}{s_1}\right)\right] + 3 s_1^{\frac{25}{3}} s_2^{15} + s_2^{\frac{35}{3}} s_3^{\frac{35}{3}} + s_3^{\frac{70}{3}}\right)^{\frac{1}{10}} + s_3^{\frac{7}{3}} \ln\left(1 + \frac{s_1}{s_3}\right)$$

$$\Rightarrow\ \tilde{a}_1 \approx 0.896\,,\ \tilde{a}_2 \approx 1.019\,,\ \tilde{a}_3 \approx 0.418\,,$$

which explicitly demonstrates that having positive solutions for $\tilde{a}_i$ is fairly generic. We now go on to determining the moduli vevs.

To verify our above assumption about L in the limit $y \to 0$ we need to find L self-consistently in this limit and demonstrate explicitly that one of the possible solutions is indeed non-zero and finite in this limit. After minimizing the potential with respect to the moduli s_i we essentially follow the steps outlined in section VI(C) of [1] and obtain the following equation

$$\begin{aligned} &2\frac{y^2}{x^2} \sum_{i=1}^{N} \sum_{\bar{j}=1}^{N} \frac{s_i s_{\bar{j}}\, N_i N_{\bar{j}}}{\tilde{a}_i} (\Delta^{-1})^{i\bar{j}} + \frac{3y}{x} (\vec{N} \cdot \vec{s}) \left(1 - \frac{a\alpha}{3x}\right) - (\vec{N} \cdot \vec{s}) \frac{b_1 a\alpha y^2}{x^2 z} \\ &- \frac{7 b_1 a\alpha y}{2xz} \left(1 + \frac{\phi_0^2}{3V_X}\right) + \frac{3 b_1 a\alpha y}{2xz} \left(1 + \frac{10}{9} \frac{\phi_0^2}{V_X}\right) \left(\frac{a\alpha V_X}{\phi_0^2 x} + 1\right) = 0\,, \end{aligned} \tag{88}$$

which is analogous to the equation in the second line in (126) of [1] but before substituting the ansatz (76). At first sight it appears that finding an analytic expression for L from (88) is hopeless since a closed form for $(\Delta^{-1})^{i\bar{j}}$ is unknown and $\tilde{a}_i$ have not been determined explicitly. However, upon further examination we notice that in order to find L from (88) we only need to know the contraction rules (7) and (19). Indeed, using the the ansatz (76) and applying (7) and (19) we obtain the following equation for L from (88)

$$\begin{aligned} &\frac{2}{3} L^2 + L \left(1 - \frac{a\alpha}{3x}\right) - L \frac{b_1 a\alpha y}{3xz} - \frac{b_1 a\alpha y}{2xz} \left(1 + \frac{\phi_0^2}{3V_X}\right) \\ &+ \frac{3 b_1 a\alpha y}{14xz} \left(1 + \frac{10}{9} \frac{\phi_0^2}{V_X}\right) \left(\frac{a\alpha V_X}{\phi_0^2 x} + 1\right) = 0\,. \end{aligned} \tag{89}$$

Solving (89) to the first subleading order in y results in

$$L = -\frac{3}{2}\left(1 - \frac{a\alpha}{3x}\right) + y\frac{3b_1 a\alpha}{14xz}\frac{1 + \frac{a\alpha V_X}{\phi_0^2 x}}{1 - \frac{a\alpha}{3x}}\left(1 + \frac{\phi_0^2}{3V_X}\right). \tag{90}$$

Hence, we see that this solution is non-zero and finite when $y \to 0$ and therefore is self-consistent. This is the solution describing the minimum of the potential. We must note that there is another possible solution of (89) for which $L \sim y \to 0$. In fact this other solution corresponds to the extremum at the top of the potential barrier and we will not discuss it further. Using (90) we can now compute the first subleading order correction to α to obtain

$$\begin{aligned} \alpha &= \frac{P}{Q} + \frac{7P\,(3(Q-P)-2)}{12\pi Q}\frac{1}{V_Q} \\ &= \frac{P}{Q} + \frac{7(Q-P)^2}{2Q^2}\left(1 - \frac{2}{3(Q-P)}\right)\frac{P}{P_{eff}}, \end{aligned} \tag{91}$$

where we have introduced

$$P_{eff} \equiv P\ln\left(\frac{QA_1\phi_0^a}{PA_2}\right). \tag{92}$$

Using (91) we can express the solution for L from (90) as

$$\begin{aligned} L &= -\frac{3}{2}\left(1 - \frac{2}{3(Q-P)}\right) + \frac{7}{2P_{eff}}\left(1 - \frac{2}{3(Q-P)}\right) \\ &+ \frac{3}{2P_{eff}}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(1 + \frac{\phi_0^2}{3V_X}\right). \end{aligned} \tag{93}$$

In the leading order, the moduli vevs are given by

$$s_i = \frac{\tilde{a}_i}{N_i}\frac{3QP_{eff}}{14\pi(Q-P)}. \tag{94}$$

We note that since $\tilde{a}_i$, N_i are positive, we need $P_{eff} > 0$ if $Q > P$, so that there exists a local minimum with $s_i > 0$.

The next step is to determine the vev of the effective meson field by minimizing the potential with respect to ϕ_0. Let us first compute the potential at the minimum as a function of the meson. The result is given in equation (107) and the reader not interested in its derivation may proceed directly there. It turns out that since the moduli vevs at the minumum are proportional to $\tilde{a}_i/N_i$ as in (78), explicit computation of the F-terms at the minumum and various contractions thereof while using the rules (7) and (19) becomes possible. Let us demonstrate some of these computations in detail. First we need to identify the gravitino mass in terms of our notation in (75). Using the usual definition in combination with (75) we have

$$m_{3/2} = e^{K/2}|W| = e^{K/2}|x|A_2 e^{-b_2\vec{N}\cdot\vec{s}}. \tag{95}$$

Because the existence of de Sitter vacua requires $Q - P > 0$ (see [1] for details) we obtain using (91) that

$$x \approx \frac{P}{Q} - 1 < 0. \tag{96}$$

On the other hand, since $m_{3/2} > 0$ we can express the following combination in terms of the gravitino mass

$$e^{K/2} x A_2 e^{-b_2 \vec{N}\cdot\vec{s}} = -m_{3/2}\,. \tag{97}$$

We now multiply F_i in (65) by $e^{K/2}$ and using (75) and (97) express

$$\begin{aligned} e^{K/2} F_i &= i N_i e^{i\gamma_W}\left(-y - x\frac{3\tilde{a}_i}{2s_i N_i}\left(1+\frac{\phi_0^2}{3V_X}\right)\right) e^{K/2} A_2 e^{-b_2\vec{N}\cdot\vec{s}} \\ &= i N_i e^{i\gamma_W}\left(\frac{y}{x} + \frac{3\tilde{a}_i}{2s_i N_i}\left(1+\frac{\phi_0^2}{3V_X}\right)\right) m_{3/2}\,, \end{aligned} \tag{98}$$

where γ_W denotes the overall phase of the superpotential. Using the ansatz (76) for s_i we obtain from (98)

$$\begin{aligned} e^{K/2} F_i &= i N_i e^{i\gamma_W}\left(\frac{y}{x} + \frac{3y}{2xL}\left(1+\frac{\phi_0^2}{3V_X}\right)\right) m_{3/2} \\ &= i N_i e^{i\gamma_W}\frac{7}{3V_{\mathcal{Q}}}\left(L + \frac{3}{2}\left(1+\frac{\phi_0^2}{3V_X}\right)\right) m_{3/2}\,, \end{aligned} \tag{99}$$

where in the second line we used

$$\frac{x}{y} L = \frac{3}{7} V_{\mathcal{Q}}\,, \tag{100}$$

obtained from (77). Similarly, we find from (65) using (75) together with (97)

$$e^{K/2} F_\phi = e^{i(\gamma_W - \theta)}\left(\frac{a\alpha}{\phi_0 x} + \frac{\phi_0}{V_X}\right) m_{3/2}\,. \tag{101}$$

Before computing $e^{K/2} F^i$ we would like to express the $K^{i\bar{j}}$ components of the inverse Kahler metric at the minimum using the ansatz (76) for $s_{\bar{j}}$ as follows

$$K^{i\bar{j}} = \frac{4 s_i s_{\bar{j}}}{3\tilde{a}_i}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}} = \frac{xL}{y}\frac{4 s_i \tilde{a}_{\bar{j}}}{3\tilde{a}_i N_{\bar{j}}}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}} = V_{\mathcal{Q}}\frac{4 s_i \tilde{a}_{\bar{j}}}{7\tilde{a}_i N_{\bar{j}}}\frac{(\Delta^{-1})^{i\bar{j}}}{1+\frac{\phi_0^2}{3V_X}}\,. \tag{102}$$

Contracting (99) and (101) with the inverse Kahler metric and using the solution for L from (90) we then obtain

$$\begin{aligned} e^{K/2} F^i &= e^{K/2} K^{i\bar{j}}\,\bar{F}_{\bar{j}} + e^{K/2} K^{i\bar{\phi}} \bar{F}_{\bar{\phi}} \\ &= -i e^{-i\gamma_W}\frac{4 s_i}{3\tilde{a}_i}\sum_{\bar{j}=1}^{N}\tilde{a}_{\bar{j}}\,(\Delta^{-1})^{i\bar{j}}\left(\frac{L}{1+\frac{\phi_0^2}{3V_X}} + \frac{3}{2}\right) m_{3/2} + i e^{-i\gamma_W}\frac{2}{3}\frac{s_i}{1+\frac{\phi_0^2}{3V_X}}\left(\frac{a\alpha}{x} + \frac{\phi_0^2}{V_X}\right) m_{3/2} \\ &= -i s_i e^{-i\gamma_W}\frac{2 y b_1 a\alpha}{7xz}\frac{1+\frac{a\alpha V_X}{\phi_0^2 x}}{1-\frac{a\alpha}{3x}} m_{3/2} \approx -i e^{-i\gamma_W}\frac{2 s_i}{P_{eff}}\left(1+\frac{a\alpha V_X}{\phi_0^2 x}\right) m_{3/2}\,, \end{aligned} \tag{103}$$

where in the last line we used (91) to plug into x, y, and z defined by (75) except for the combination $\left(1+\frac{a\alpha V_X}{\phi_0^2 x}\right)$ and kept the leading term in $1/P_{eff}$. Note that in order to get from the second to third line in (133) we used the second contraction property in (19). Similarly, contracting (99) and (101) with the corresponding components of the inverse Kahler metric (73) we obtain

$$e^{K/2}F^{\phi} = e^{-i\gamma_W}\,\phi\left(1-\frac{7}{3P_{eff}}\right)\left(1+\frac{a\alpha V_X}{\phi_0^2 x}\right)m_{3/2}\,. \tag{104}$$

Using the results (99), (101), (103) and (104) together with (90) and (91) we can compute the following contributions

$$\begin{aligned} e^K F^i F_i &= \frac{7}{P_{eff}}\left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\left(\frac{V_X}{\phi_0^2}\right)^2\left[\frac{\phi_0^2}{3V_X}+\frac{1}{P_{eff}}\left(1+\frac{\phi_0^2}{3V_X}\right)\right]m_{3/2}^2 \\ e^K F^{\phi}F_{\phi} &= \left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\frac{V_X}{\phi_0^2}\left(1-\frac{7}{3P_{eff}}\right)m_{3/2}^2\,, \end{aligned} \tag{105}$$

where we also used $\vec{N}\cdot\vec{s}=V_Q$ while performing the computations in the first line of (105).

Then, the potential at the minimum is given by

$$\begin{aligned} V_0 &= e^K(F^iF_i+F^{\phi}F_{\phi}-3|W|^2) \\ &= \left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\frac{V_X}{\phi_0^2}m_{3/2}^2+\frac{7}{P_{eff}^2}\left(\frac{a\alpha}{x}+\frac{\phi_0^2}{V_X}\right)^2\left(1+\frac{\phi_0^2}{3V_X}\right)\left(\frac{V_X}{\phi_0^2}\right)^2 m_{3/2}^2-3m_{3/2}^2\,. \end{aligned} \tag{106}$$

Using (91) and dropping the terms of order $\mathcal{O}(1/P_{eff}^2)$ we obtain the following expression for the tree-level vacuum energy as a function of the meson field

$$V_0 = \left[\left(\frac{2}{Q-P}+\frac{\phi_0^2}{V_X}\right)^2+\frac{14}{P_{eff}}\left(1-\frac{2}{3(Q-P)}\right)\left(\frac{2}{Q-P}+\frac{\phi_0^2}{V_X}\right)-3\frac{\phi_0^2}{V_X}\right]\frac{V_X}{\phi_0^2}m_{3/2}^2\,. \tag{107}$$

The polynomial in the square brackets in (107) is quadratic with respect to the canonically normalized meson vev squared $\phi_c^2\equiv\phi_0^2/V_X$ with the coefficient of the $(\phi_0^2/V_X)^2$ monomial being positive (+1) and therefore, the minimum V_0 is positive when the corresponding discriminant is negative. Tuning the cosmological constant to zero is then equivalent to setting the discriminant of the above polynomial to zero, which boils down to a simple condition

$$P_{eff} = \frac{14(3(Q-P)-2)}{3(3(Q-P)-2\sqrt{6(Q-P)})}\,. \tag{108}$$

Note that P_{eff} defined in (92) is actually dependent on ϕ but because of the smallness of a and the Log dependence, it was safe to use the approximation $P_{eff}\approx const.$ This approximation turned out to be self-consistent since P_{eff} is fairly large. From (108) we see immediately that

$$P_{eff}>0\;\Rightarrow\;Q-P\geq 3\,. \tag{109}$$

Minimizing (107) with respect to ϕ_c^2 we obtain the meson vev at the minimum in the leading order

$$\phi_c^2=\frac{\phi_0^2}{V_X}\approx\frac{2}{Q-P}+\frac{7}{P_{eff}}\left(1-\frac{2}{3(Q-P)}\right)\,. \tag{110}$$

If we tune the tree-level vacuum energy and set $Q-P=3$ we obtain

$$P_{eff}\approx 63.5\,,\quad\frac{\phi_0^2}{V_X}\approx 0.75\,. \tag{111}$$

We find numerically that for the minimum value $Q-P=3$, the tuning of the cosmological constant by varying the constants A_1 and A_2 inside the superpotential results in fixing the value of P_{eff} at

$$P_{eff} \approx 61.648\,, \tag{112}$$

while the canonically normalized meson vev squared is stabilized at

$$\phi_c^2 = \frac{\phi_0^2}{V_X} \approx 0.746\,, \tag{113}$$

thus confirming the analytical results above. For example, we obtain numerically (112) and (113) for the following examples with two moduli

$$P=27\,,\ Q=30\,,\ A_1=27\,,\ A_2=2.1544\,,\ N_1=N_2=1\,, V_X = s_1^{\frac{7}{6}} s_2^{\frac{7}{6}} + \frac{1}{3} s_1^1 s_2^{\frac{4}{3}} + \frac{1}{2} s_1^{\frac{1}{3}} s_2^2 \tag{114}$$

$$\Rightarrow s_1 \approx 34.52\,,\quad s_2 \approx 63.13$$

$$P=27\,,\ Q=30\,,\ A_1=27\,,\ A_2=2.14126\,,\ N_1=N_2=1\,, V_X = s_1^{\frac{7}{3}} + \frac{1}{3} s_2^{\frac{7}{3}} + s_1^{\frac{1}{3}} s_2^2$$

$$\Rightarrow s_1 \approx 49.99\,,\quad s_2 \approx 47.66\,.$$

As we will see in the computations that follow, the value of P_{eff} will enter into many quantities relevant for particle physics, such as tree-level gaugino masses, etc. Here we would like to point out a very crucial fact. Namely, while changing the value of P_{eff} in the range $61 \le P_{eff} \le 62$ hardly affects the values of the soft breaking terms, as will be evident from the corresponding explicit formulas, such small changes in P_{eff} result in vastly different values of the vacuum energy: $-\mathcal{O}(m_{3/2}m_p)^2 \lesssim V_0 \lesssim +\mathcal{O}(m_{3/2}m_p)^2$. Therefore, once we coarsely tune P_{eff} to its approximate value, the cosmological constant problem becomes completely decoupled from the rest of particle physics. Even though this should be the case, it is satisfying to see it explicitly in a complete example of moduli coupled to matter.

Although the value in (113) is not much smaller than one, the combination $\frac{\phi_0^2}{3V_X}$ inside the inverse Kahler metric (73) has a value

$$\frac{\phi_0^2}{3V_X} \approx 0.25\,, \tag{115}$$

which is small enough to make the quartic and higher order terms which we kept inside the inverse Kahler metric much smaller.

Note also that in the original paper [1] we obtained $P_{eff} \approx 83$. This is due to the different matter Kahler potential considered there. As we will see, this numerical difference will result in slightly different values for the soft breaking terms if compared to those obtained in [1],[2]. Using (112) the volume of the hidden sector associative cycle for $Q-P=3$ is given by

$$V_Q = \frac{QP_{eff}}{2\pi(Q-P)} \approx \frac{10Q}{\pi}\,, \tag{116}$$

which completely determines the scale of gaugino condensation

$$\Lambda \sim m_{pl} e^{-\frac{2\pi}{3Q}V_Q} \approx m_{pl} e^{-20/3} \approx 3\times 10^{15}\text{GeV}\,. \tag{117}$$

Recall that for a stable minimum to exist it is necessary that $Q-P \ge 3$. We have seen that when $Q-P=3$ and the minimum of the potential is tuned to zero, the value of $P_{eff} \approx 60$ which ensures that the moduli (94) can be reliably fixed at values large enough to satisfy the supergravity approximation. On the other hand, when $Q-P=4$, from (108) we get $P_{eff}\approx 20$, in which case if Q is fixed, the moduli vevs become smaller by about a factor of four. Thus, unless the ranks of hidden sector gauge groups are incredibly large, situations when $Q-P>3$ may put our solutions well outside of the supergravity approximation. Therefore, from now on we will only consider the case when $Q-P=3$ to ensure the validity of the regime where our construction is reliable.

V. MASSES AND SOFT SUPERSYMMETRY BREAKING TERMS

A. Gravitino mass

In supergravity the bare gravitino mass is defined as

$$m_{3/2}=m_{pl}e^{K/2}|W|=e^{K/2}|x|A_2e^{-b_2V_Q}\,, \tag{118}$$

and can now be computed since we stabilized V_Q explicitly. It is given by

$$m_{3/2}=m_{pl}\frac{e^{\frac{\phi_0^2}{2V_X}}}{8\sqrt{\pi}V_X^{3/2}}|P-Q|\frac{A_2}{Q}e^{-\frac{P_{eff}}{Q-P}}\,. \tag{119}$$

When the cosmological constant is tuned such that (112) is satisfied, for $Q-P=3$ we obtain

$$m_{3/2}\approx 9\times 10^5(\text{TeV})\frac{C_2}{V_X^{3/2}}\,, \tag{120}$$

where $C_2\equiv A_2/Q$ was defined in (41). Calculating C_2 goes beyond the scope of this paper. Here we will treat C_2 as a phenomenological parameter with values $C_2\sim\mathcal{O}(0.1-1)$ since it may experience a mild exponential suppression as in (41).

On the other hand, the actual value at which the volume V_X must be stabilized can be almost uniquely determined from the scale of Grand Unification. In particular, we can use equation (4.12) in [10] to express

$$G_N=\frac{1}{8\pi m_p^2}=\frac{\alpha_{GUT}^3V_Q^{7/3}L(Q)^{2/3}}{32\pi^2M_{GUT}^2V_X}\,, \tag{121}$$

where the factor $L(Q)$ is due to the threshold corrections from the Kaluza Klein modes and is given by

$$L(Q)=4q\sin^2(5\pi w/q)\,, \tag{122}$$

such that $5w$ is not divisible by q. For typical values

$$\alpha_{GUT}=\frac{1}{V_Q}=\frac{1}{25}\,,\quad M_{GUT}=2\times 10^{16}\,\text{GeV}\,, \tag{123}$$

we obtain

$$V_X=137.4\times L(Q)^{2/3}\,. \tag{124}$$

In Table I we list a few typical benchmark values for the volume and the resulting gravitino mass up to the overall factor C_2.

Quite amazingly, the gravitino mass scale naturally turns out to be constrained to $m_{3/2}\sim\mathcal{O}(10)$TeV. While this is presumably large enough to alleviate the gravitino problem, it is also small enough to give the superpartners masses which can be easily accessible at the LHC energies. As we will see below, this is possible because of the significant suppression of the tree-level gaugino masses relative to $m_{3/2}$.

	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
q	2	3	4	4	6	6	6
w	1	1, 2	1, 3	2	1, 5	2, 4	3
V_X	549.6	594.5	549.6	872.4	453.7	943.7	1143.2
$m_{3/2}/C_2$	70 TeV	62 TeV	70 TeV	35 TeV	93.0 TeV	31 TeV	23 TeV

TABLE I: Typical values of V_X and $m_{3/2}$ divided by C_2 for different values of q and w.

B. Moduli masses

In order to compute the masses of the moduli we first need to evaluate the matrix V_{mn} with $m, n = \overline{1, N+1}$, with components given by

$$V_{ij} = \frac{\partial^2 V}{\partial s_i \partial s_j}\,, \quad V_{i\,N+1} = \frac{\partial^2 V}{\partial s_i \partial \phi_0}\,, \quad V_{N+1\,N+1} = \frac{\partial^2 V}{\partial \phi_0 \partial \phi_0}\,, \tag{125}$$

at the minimum of the potential. However, because the Kahler metric in (71) is not diagonal, we also need to find a unitary transformation U which diagonalizes the Kahler metric. We denote all the components of the Kahler metric as $K_{m\bar{n}}$. Then, by diagonalizing $K_{m\bar{n}}$ we obtain

$$K_k \delta_{k\bar{l}} = U^\dagger_{km} K_{m\bar{n}} U_{\bar{n}\bar{l}}\,. \tag{126}$$

After that, we need to rescale the fluctuations of the moduli around the minimum by the corresponding $1/\sqrt{2K_k}$ factors so that the new *real* scalar fields have canonical kinetic terms. At the end, finding the moduli mass squared eigenvalues boils down to diagonalizing the fillowing matrix

$$M^2_{kl} = \frac{1}{2}\frac{1}{\sqrt{K_k K_l}} U^\dagger_{km} V_{mn} U_{nl}\,. \tag{127}$$

Unlike most of the other masses, the detailed form of the moduli mass matrix does depend upon the detailed form of V_X. Therefore we have resorted to numerical analyses in this case and found that there is one heavy modulus whose mass mainly depends on Q and for $Q = 30$

$$M \sim O(200 - 300) \times m_{3/2}\,, \tag{128}$$

and N lighter moduli with masses

$$m_i \sim O(1) \times m_{3/2}\,, \quad i = \overline{1, N}\,. \tag{129}$$

The heavy modulus arises from the fluctuation which deforms the volume of the three-cycle V_Q, while N-1 light moduli originate from the fluctuations approximately preserving the volume and tangential to the hyperplane defined by

$$\vec{N} \cdot \vec{s} - V_Q = 0\,. \tag{130}$$

The remaining light modulus represents the fluctuations of the hidden sector meson ϕ mixed with the geometric moduli.

The important point is that there is indeed a hierarchy of moduli masses and that there can be phase space supression for moduli to decay into gravitinos. This alleviates the moduli and gravitino problems, as discussed in great detail in [3]. In particular, it was demonstrated that non-thermal production of dark matter from moduli decays in G_2-MSSM can successfully account for the observed dark matter abundance, explicitly implementing ideas of Moroi-Randall [19].

C. Gaugino masses

The universal tree-level contribution to the gaugino masses can be computed from the standard supergravity formula [20]

$$m_{1/2}^{tree} = \frac{e^{K/2} F^i \partial_i f_{vis}}{2i \mathrm{Im} f_{vis}} , \tag{131}$$

where the visible sector gauge kinetic function is another integer combination of the moduli

$$f_{vis} = \sum_{i=1}^{N} N_i^{vis} z_i . \tag{132}$$

Plugging the solution for α (91) into (103) while using the definitions (75) we obtain

$$e^{K/2} F^i \approx -i \frac{2 s_i}{P_{eff}} \left(1 + \frac{2 V_X}{(Q-P) \phi_0^2} \right) m_{3/2} , \tag{133}$$

where we dropped the overall phase factor $e^{-i\gamma_W}$. It is now straightforward to compute the tree-level gaugino mass

$$m_{1/2}^{tree} \approx -\frac{1}{P_{eff}} \left(1 + \frac{2 V_X}{(Q-P) \phi_0^2} + \mathcal{O}\left(\frac{1}{P_{eff}} \right) \right) m_{3/2} . \tag{134}$$

It is interesting to note that this formula is identical to the leading order expression previously obtained in [1] when one replaces the combination ϕ_0^2 / V_X by the canonically normalized meson field. Here, again the suppression coefficient is completely independent of the number of moduli N as well as the integers N_i (N_i^{vis}) appearing inside either the hidden sector (62) or the visible sector (132) gauge kinetic functions. Moreover, all the detailed dependence on the individual moduli is completely buried inside the volume V_X and the gravitino mass $m_{3/2}$ (which also depends on V_X) and therefore expression (134) is universally valid for any G_2 manifold that yields positive solutions of the system of equations in (80). Hence, despite the presence of a huge number of unknown microscopic parameters, the tree-level gaugino masses in (134) depend on very few of them. Moreover, when the cosmological constant is tuned to a small value and $Q - P = 3$, the gaugino mass suppression coefficient becomes completely fixed! Indeed, using (112) and (113) for $Q - P = 3$ we obtain

$$m_{1/2}^{tree} \approx -0.0307 \times m_{3/2} . \tag{135}$$

This result gets slightly corrected by the threshold corrections to the gauge kinetic function from the Kaluza-Klein modes computed in [10]

$$\alpha_{GUT}^{-1} = f_{vis} + \frac{5}{2\pi} \mathcal{T}_\omega . \tag{136}$$

In the above formula, $\mathcal{T}_\omega$ is a topological invariant (Ray-Singer torsion)

$$\mathcal{T}_\omega = \ln \left(4 \sin^2 (5\pi w / q) \right) , \tag{137}$$

where w and q are integers such that $5w$ is not divisible by q. In this case, the tree-level gaugino mass is given by

$$m_{1/2}^{tree} \approx -0.0307\, \eta \times m_{3/2} , \tag{138}$$

where

$$\eta = 1 - \frac{5\, g_{GUT}^2}{8\pi^2} \mathcal{T}_\omega . \tag{139}$$

D. Anomaly mediated contribution to the gaugino masses

Because of the substantial suppression of the universal tree-level gaugino mass, it makes sense to take into account the anomaly mediated contributions which appear at one-loop. The anomaly mediated contributions are given by the following general expression [21]

$$m_a^{AM} = -\frac{g_a^2}{16\pi^2}\left[-\left(3C_a - \sum_\alpha C_a^\alpha\right)e^{K/2}W^* + \left(C_a - \sum_\alpha C_a^\alpha\right)e^{K/2}F^n K_n + 2\sum_\alpha\left(C_a^\alpha e^{K/2}F^n\partial_n \ln\tilde{K}_\alpha\right)\right], \tag{140}$$

where C_a and C_a^α are the quadratic Casimirs of the a-th gauge group and $\tilde{K}_\alpha$ is the Kahler metric for the visible sector fields (58). Assuming the MSSM particle content, we have the following values for the Casimirs

$$\begin{aligned} U(1):&\quad C_a = 0 &\quad \sum_\alpha C_a^\alpha = \frac{33}{5} \\ SU(2):&\quad C_a = 2 &\quad \sum_\alpha C_a^\alpha = 7 \\ SU(3):&\quad C_a = 3 &\quad \sum_\alpha C_a^\alpha = 6\,. \end{aligned} \tag{141}$$

Plugging the solution for α (91) into (104) while using the definitions (75) we obtain

$$e^{K/2}F^\phi \approx \phi\left(1-\frac{7}{3P_{eff}}\right)\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)m_{3/2}\,, \tag{142}$$

where we dropped the overall phase factor $e^{-i\gamma_W}$. Combining (133), (142) and using (63) and (58) we now compute the contributions

$$\begin{aligned} e^{K/2}F^n K_n &= e^{K/2}F^i K_i + e^{K/2}F^\phi K_\phi = \left(\frac{\phi_0^2}{V_X}+\frac{7}{P_{eff}}\right)\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)m_{3/2} \\ e^{K/2}F^n\partial_n\ln\tilde{K}_\alpha &= \frac{1}{3}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(c\frac{\phi_0^2}{V_X}+\frac{7}{P_{eff}}\right)m_{3/2}\,. \end{aligned} \tag{143}$$

In the above we also used (67) and (68) together with the definition of $\tilde{a}_i$ in (6) as well as its contraction property (7).

Using the definition (140) we then obtain the following expression for the anomaly mediated contributions to the gaugino masses

$$\begin{aligned} m_a^{AM} \approx\ & -\frac{\alpha_{GUT}}{4\pi}[-\left(3C_a-\sum_\alpha C_a^\alpha\right)\left(1-\frac{1}{3}\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X}+\frac{7}{P_{eff}}\right)\right) \\ & + (c-1)\left(1+\frac{2V_X}{(Q-P)\phi_0^2}\right)\frac{2\phi_0^2}{3V_X}\sum_\alpha C_a^\alpha]\times m_{3/2}\,, \end{aligned} \tag{144}$$

where we have explicitly separated the conformal anomaly contribution from the Konishi anomaly term using (59).

Notice the appearence of the coefficient c which controls the size of the higher order corrections to the matter Kahler potential. As expected, the Konishi anomaly vanishes in the exactly sequestered case [22], i.e. when $c = 1$. Again, in the leading order in $1/P_{eff}$, when $c = 0$ the result obtained above is almost the same as the one in [1]. Just like in the case with tree-level gaugino masses, the above result is completely independent of the detailed moduli dependence of the volume V_X and therefore is completely general.

When we set $Q-P=3$, tune the tree-level vacuum energy by imposing the constraint (112), use (113) and combine the above formula with the tree-level contribution (138), we obtain the following expression for the total gaugino masses

$$M_a \approx \left[-0.0307\,\eta + \alpha_{GUT}\left(0.0364\left(3C_a - \sum_\alpha C_a^\alpha\right) + 0.0749\,(1-c)\sum_\alpha C_a^\alpha\right)\right] \times m_{3/2}\,. \tag{145}$$

Note that as was previously pointed out in [22], in the limit when $c \to 1$ we obtain a particular type of a mirage pattern for gaugino masses [23]. However, as we will see below, in this limit the scalars become tachyonic and therefore, the exact mirage pattern is disfavored. An exact numerical computation confirms the above result giving

$$M_a \approx \left[-0.03156\,\eta + \alpha_{GUT}\left(0.034086\left(3C_a - \sum_\alpha C_a^\alpha\right) + 0.07926\,(1-c)\sum_\alpha C_a^\alpha\right)\right] \times m_{3/2}\,. \tag{146}$$

Substituting the MSSM casimirs (141) into (146) we then obtain

$$\begin{aligned} M_1 &\approx (-0.03156\,\eta + \alpha_{GUT}\,(-0.22497 + 0.52313\,(1-c))) \times m_{3/2} \\ M_2 &\approx (-0.03156\,\eta + \alpha_{GUT}\,(-0.03409 + 0.55483\,(1-c))) \times m_{3/2} \\ M_2 &\approx (-0.03156\,\eta + \alpha_{GUT}\,(0.10226 + 0.47557\,(1-c))) \times m_{3/2}\,. \end{aligned} \tag{147}$$

The form of (147) allows us to see explicitely that for $c = 0$ the Konishi anomaly contribution is larger than the contribution from the conformal anomaly by a factor of a few, which is what made the gaugino mass spectrum in [2] very different from other known patterns. However, as we will see below, suppressing the scalar masses relative to the gravitino mass (which is the generic case) will automatically result in a large suppression of the Konishi anomaly.

E. Scalars

When the visible sector Kahler metric is flavor diagonal, the masses for the canonically normalized scalars are [20]

$$m^2_{\alpha\bar{\beta}} = \left(m^2_{3/2} + V_0 - e^K F^i F^{\bar{j}}\,\partial_i\partial_{\bar{j}}\,\ln\tilde{K}_\alpha\right)\delta_{\alpha\bar{\beta}}\,. \tag{148}$$

Using the definition in (148) together with (59), we obtain the following result in the leading order

$$m_\alpha \approx (1-c)^{1/2}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{3V_X}\right)^{1/2} m_{3/2}\,. \tag{149}$$

After setting $Q-P=3$, we tune the tree-level vacuum energy by imposing the constraint (112) and use (113) to obtain from (149)

$$m_\alpha \approx (1-c)^{1/2} 0.94\, m_{3/2}\,. \tag{150}$$

A numerical computation in this case gives

$$m_\alpha \approx (1-c)^{1/2} 0.998\, m_{3/2} \approx (1-c)^{1/2}\, m_{3/2}\,. \tag{151}$$

Again, for $c = 0$ we recover the old result in [1] where all the scalars have a flavor-universal mass equal to the gravitino mass.

Furthermore, the anomaly contributions to the scalar mass squareds are suppressed relative to the gravitino mass and since we wish to consider generic $\mathcal{O}(1)$ values of $(1-c)$ we will neglect such contributions. Concretely we are going to consider only those values of $0 < c < 1$ which give

$$\frac{1}{16\pi^2} << \frac{m_\alpha}{m_{3/2}}\,, \tag{152}$$

such that the anomaly mediated contributions to the scalar masses can be safely neglected. However, one can certainly extend our model and include such contributions. Once again, the result above is completely independent of the details of V_X and therefore holds for any G_2 manifold that solves the system (80) with $\tilde{a}_i > 0$ such that the Kahler metric at the minimum is positive definite.

F. Trilinear couplings

Because the Kahler metric for the matter fields is diagonal, the normalized trilinear couplings are proportional to the physical (normalized) Yukawa couplings $Y_{\alpha\beta\gamma}$ and are given by [20]

$$A_{\alpha\beta\gamma} = Y_{\alpha\beta\gamma}\left(e^{K/2}F^m\left(K_m + \partial_m \ln(Y'_{\alpha\beta\gamma}) - \partial_m \ln(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)\right)\right)\,, \tag{153}$$

where $\{\alpha\,,\beta\,,\gamma\}$ label visible sector matter fields and $Y'_{\alpha\beta\gamma}$ are the unnormalized Yukawas. The Yukawa couplings $Y'_{\alpha\beta\gamma}$ arise from the membrane instantons wrapping associative cycles $Q^{\alpha\beta\gamma}$ which connect isolated singularities with the corresponding matter multiplets. They are given by

$$Y'_{\alpha\beta\gamma} = C_{\alpha\beta\gamma}e^{i2\pi\sum_i m_i^{\alpha\beta\gamma}z_i}\,. \tag{154}$$

The integer combination of the moduli $V_{Q^{\alpha\beta\gamma}} = \sum_i m_i^{\alpha\beta\gamma}s_i$ gives the volume of the associative cycle $Q^{\alpha\beta\gamma}$ connecting codimension seven singularities α, β and γ where the chiral multiplets are localized. The coefficients $C_{\alpha\beta\gamma}$ are constants. Because the visible sector Kahler metric is flavor diagonal, the relation between the absolute values of the physical and unnormalized Yukawa couplings is simply a rescaling by $e^{K/2}\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2}$

$$|Y_{\alpha\beta\gamma}| = e^{K/2}\left|Y'_{\alpha\beta\gamma}\right|\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2} = e^{K/2}\left|C_{\alpha\beta\gamma}\right|e^{-2\pi\sum_i m_i^{\alpha\beta\gamma}s_i}\left(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma\right)^{-1/2} \sim \mathcal{O}(1)e^{-2\pi\sum_i m_i^{\alpha\beta\gamma}s_i}\,, \tag{155}$$

where we used explicit expressions for K and $\tilde{K}_\alpha$ to estimate the final expression. Using (133) and (154) we can compute the contribution

$$e^{K/2}F^m\partial_m \ln(Y'_{\alpha\beta\gamma}) = \frac{4\pi}{P_{eff}}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)V_{Q^{\alpha\beta\gamma}}m_{3/2}\,. \tag{156}$$

Similarly, evaluating the third term in (153) using (58) and (143) gives

$$e^{K/2}F^m\partial_m \ln(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma) = \left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(c\frac{\phi_0^2}{V_X} + \frac{7}{P_{eff}}\right)m_{3/2}\,. \tag{157}$$

Again, in the above expressions we dropped the overall phase factor $e^{-i\gamma_W}$. Using the definition (153) along with (143), (156) and (157) we obtain the following expression for the trilinear couplings at tree-level

$$A_{\alpha\beta\gamma} = Y_{\alpha\beta\gamma}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left((1-c)\frac{\phi_0^2}{V_X} + \frac{4\pi}{P_{eff}}V_{Q^{\alpha\beta\gamma}}\right)m_{3/2}\,, \tag{158}$$

which gets reduced to the result in [1] when $c = 0$. Once again, the detailed structure of the volume V_X played absolutely no role in our ability to obtain the above expression for the tree-level trilinear couplings. The actual volumes of three cycles $V_{Q^{\alpha\beta\gamma}}$ do depend on the microscopic properties of G_2 manifolds and in our general framework these parameters remain undetermined. However, below we will present a good argument for dropping such volume contributions completely when the third generation trilinear couplings are computed.

Again, for $Q - P = 3$ when the tree-level vacuum energy is tuned we obtain for the reduced trilinears

$$\tilde{A}_{\alpha\beta\gamma} = \left(1.41(1-c) + 0.386 \times V_{Q^{\alpha\beta\gamma}}\right) m_{3/2}\,. \tag{159}$$

From the corresponding numerical calculation we obtain the following result

$$\tilde{A}_{\alpha\beta\gamma} = \left(1.494(1-c) + 0.3966 \times V_{Q^{\alpha\beta\gamma}}\right) m_{3/2}\,. \tag{160}$$

Since the physical Yukawa couplings for the third generation fermions are much larger than the first two generation Yukawas, one can typically neglect the trilinears for the first and second generations. Moreover, the large size of the third generation Yukawas implies that the volumes of the three-cycles of the corresponding membrane instantons are very small. In fact, because the top Yukawa is of order one, one can assume that the point p_1 supporting the up-type Higgs $\mathbf{5}$ of $SU(5)$ coincides with the point p_2 supporting the third generation $\mathbf{10}$, so that the coupling $H_u\mathbf{10_3 10_3}$ has no exponential supression [5], [10]. At the same time the point p_3 supporting the down-type $\bar{\mathbf{5}}$ Higgs and the point p_4 supporting the third generation matter $\bar{\mathbf{5}}$ are distinct but still close to $p_2\,, p_1$ so that the coupling of $H_d\bar{\mathbf{5}}_\mathbf{3}\mathbf{10_3}$ which accounts for the bottom(tau) Yukawa is slightly smaller than the top Yukawa at the GUT scale. These considerations completely justify dropping the corresponding $V_{Q^{\alpha\beta\gamma}}$ terms for the third generation trilinears which then become simplified

$$\tilde{A}_t = \tilde{A}_b = \tilde{A}_\tau \approx 1.494(1-c)\, m_{3/2}\,. \tag{161}$$

For generic values of c the trilinears are of the same order as the gravitino mass. In the limit $c \to 1$, the reduced trilinear couplings at tree-level become suppressed relative to the gravitino mass. Note that as c approaches one, the supression of the trilinear couplings above is much stronger than that of the scalars. In this case, the anomaly mediated contributions may become comparable to the tree-level ones and therefore must be taken into account. General expressions given in [24] can be simplified in the nearly sequestered limit as

$$\tilde{A}^{AM}_a = -\frac{1}{16\pi^2}\gamma_a\left(e^{K/2}W^* - \frac{1}{3}e^{K/2}F^n K_n\right) + \frac{(1-c)}{16\pi^2}X_a\, m_{3/2}\,, \tag{162}$$

where the last term denotes the unknown contributions vanishing in the sequestered limit. Note that such terms are suppressed compared to the tree-level piece (161) due to the loop factor. As long as $(1-c)$ is small enough, they become subleading and we will drop them in further analysis. Using (143) and substituting the corresponding MSSM expressions for γ_as, where we set $g_1 = g_2 = g_3 = g_{GUT}$, we obtain the following expressions for the anomaly mediated contributions to the reduced trilinear couplings

$$\tilde{A}^{AM}_t \approx -\frac{1}{16\pi^2}\left(-\frac{46}{5}g^2_{GUT} + 6Y_t^2 + Y_b^2\right)\left(1 - \frac{1}{3}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X} + \frac{7}{P_{eff}}\right)\right) m_{3/2} \tag{163}$$
$$\tilde{A}^{AM}_b \approx -\frac{1}{16\pi^2}\left(-\frac{44}{5}g^2_{GUT} + Y_t^2 + 6Y_b^2 + Y_\tau^2\right)\left(1 - \frac{1}{3}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X} + \frac{7}{P_{eff}}\right)\right) m_{3/2}$$
$$\tilde{A}^{AM}_\tau \approx -\frac{1}{16\pi^2}\left(-\frac{24}{5}g^2_{GUT} + 3Y_b^2 + 4Y_\tau^2\right)\left(1 - \frac{1}{3}\left(1 + \frac{2V_X}{(Q-P)\phi_0^2}\right)\left(\frac{\phi_0^2}{V_X} + \frac{7}{P_{eff}}\right)\right) m_{3/2}\,.$$

When we set $Q-P=3$, tune the tree-level vacuum energy by imposing the constraint (112), use (113) and combine the above formula with the tree-level contribution (160), we obtain

$$\begin{aligned}
\tilde{A}_t &\approx 1.41(1-c)m_{3/2} - 0.0029\left(-\frac{46}{5}g^2_{GUT} + 6Y_t^2 + Y_b^2\right)m_{3/2} \\
\tilde{A}_b &\approx 1.41(1-c)m_{3/2} - 0.0029\left(-\frac{44}{5}g^2_{GUT} + Y_t^2 + 6Y_b^2 + Y_\tau^2\right)m_{3/2} \\
\tilde{A}_\tau &\approx 1.41(1-c)m_{3/2} - 0.0029\left(-\frac{24}{5}g^2_{GUT} + 3Y_b^2 + 4Y_\tau^2\right)m_{3/2}\,.
\end{aligned} \tag{164}$$

Numerical computations give the following expressions for the total reduced trilinears

$$\begin{aligned}
\tilde{A}_t &\approx 1.494(1-c)m_{3/2} - 0.0027\left(-\frac{46}{5}g^2_{GUT} + 6Y_t^2 + Y_b^2\right)m_{3/2} \\
\tilde{A}_b &\approx 1.494(1-c)m_{3/2} - 0.0027\left(-\frac{44}{5}g^2_{GUT} + Y_t^2 + 6Y_b^2 + Y_\tau^2\right)m_{3/2} \\
\tilde{A}_\tau &\approx 1.494(1-c)m_{3/2} - 0.0027\left(-\frac{24}{5}g^2_{GUT} + 3Y_b^2 + 4Y_\tau^2\right)m_{3/2}\,,
\end{aligned} \tag{165}$$

which demonstrate a fairly high accuracy of the analytically derived result in (164).

G. μ and $B\mu$ -terms

The full hidden sector plus visible sector Kahler potential and superpotential can be written in the following general form

$$\begin{aligned}
K_{\text{total}} &= K\left(s_i,\phi,\bar{\phi}\right) + \tilde{K}_{\alpha\bar{\beta}}(s_i,\phi,\bar{\phi})Q^\alpha\bar{Q}^{\bar{\beta}} + Z_{\alpha\beta}(s_i,\phi,\bar{\phi})Q^\alpha Q^\beta + \,c.\,c. \\
\hat{W} &= W_{np} + \mu' Q^\alpha Q^\beta + Y'_{\alpha\beta\gamma}Q^\alpha Q^\beta Q^\gamma + \,\ldots.
\end{aligned} \tag{166}$$

Here, ϕ denote the hidden sector matter fields while Q^α are visible sector chiral matter fields where $\tilde{K}_{\alpha\bar{\beta}}(s_i,\phi,\bar{\phi})$ is the visible sector Kahler metric and $Y'_{\alpha\beta\gamma}$ are the corresponding unnormalized Yukawa couplings. It can be shown that the supersymmetric mass parameter μ' can be forbidden by requiring certain discrete symmetries which are also used in order to solve the problem of doublet-triplet splitting [25]. Hence, in our analisys we will rely on the Giudice-Masiero mechanism [26] in generating effective μ and $B\mu$ terms where the bilinear coefficient $Z_{\alpha\beta}(s_i,\phi,\bar{\phi})$ in (166) plays a key role. The general expressions for the normalized μ and $B\mu$ are given by [20]

$$\begin{aligned}
\mu &= \left(\frac{W^\star_{np}}{|W_{np}|}e^{K/2}\mu' + m_{3/2}Z - e^{K/2}F^{\bar{m}}\partial_{\bar{m}}Z\right)(\tilde{K}_{H_u}\tilde{K}_{H_d})^{-1/2} \\
B\mu &= [\frac{W^\star_{np}}{|W_{np}|}e^{K/2}\mu'(e^{K/2}F^m\left[K_m + \partial_m\ln\mu' - \partial_m\ln(\tilde{K}_{H_u}\tilde{K}_{H_d})\right] - m_{3/2}) \\
&+ (2m^2_{3/2} + V_0)\,Z - m_{3/2}e^{K/2}F^{\bar{m}}\partial_{\bar{m}}Z + m_{3/2}e^{K/2}F^m(\partial_m Z - Z\partial_m\ln(\tilde{K}_{H_u}\tilde{K}_{H_d})) \\
&- e^K F^{\bar{m}}F^n(\partial_{\bar{m}}\partial_n Z - \partial_{\bar{m}}Z\partial_n\ln(\tilde{K}_{H_u}\tilde{K}_{H_d}))](\tilde{K}_{H_u}\tilde{K}_{H_d})^{-1/2}\,.
\end{aligned} \tag{167}$$

where we can set $\mu'=0$. Unfortinately, at this point we do not have a reliable way to compute the Higgs bilinear $Z_{\alpha\beta}(s_i,\phi,\bar{\phi})$ for G_2 compactifications. Therefore, in our analysis we will parametrize the μ and

$B\mu$ terms as follows

$$\begin{aligned} \mu &= Z^1_{eff}\, m_{3/2} \\ B\mu &= Z^2_{eff}\, m^2_{3/2}\,, \end{aligned} \tag{168}$$

and treat Z^1_{eff} and Z^2_{eff} as phenomenological parameters. Naturally, we expect that $Z^{1,2}_{eff} \sim \mathcal{O}(1)$ and, as we will see in the next section, tuning μ parameter in order to get the correct value of the Z - boson mass boils down to tuning the values of $Z^{1,2}_{eff}$.

VI. ELECTROWEAK SCALE SPECTRUM

In order to obtain the corresponding MSSM spectrum at the electroweak scale we need to RG-evolve all the masses and couplings from the GUT scale down to the electroweak scale. This procedure was described in great detail in [2]. Here we will only highlight a few important points and give the final results.

As we have seen in the previous section, at the GUT scale, the gaugino masses are non-universal and highly suppressed relative to the gravitino mass. On the other hand, unless c is very close to one, the scalars, trilinear couplings and the μ-term are all of order $m_{3/2}$. Hence, we can define a scale m_s at which all the heavy states decouple and the effective theory below that scale is the Standard Model plus gauginos. More specifically, we choose the decoupling scale m_s to be the geometric mean of the stop masses

$$m_s = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}\,. \tag{169}$$

This is ok as long as the mass differences between the lightest stop and the other heavy states is not too large. Then, the running is done at one loop in two stages with tree-level matching at the scale m_s.

A. Gauginos

As one notices from (147), due to the anomaly mediated contribution, the gaugino masses are sensitive to the value of α_{GUT}. However, the value of α_{GUT} is only determined once we know the exact spectrum and run the gauge coupling up to the GUT scale. Therefore, there is a feedback mechanism which allows us to completely fix the gaugino masses by imposing the gauge coupling unification. In practice, we first pick an initial value of $\alpha_{GUT} \sim 1/25$, compute the gaugino masses, scalar masses, trilinears, etc. at the GUT scale and run them down to the electroweak scale where we compute the spectrum. We then run the gauge couplings up using two-loop RGEs to check if they unify at the same value of α_{GUT} as we chose to compute the gaugino masses. If there is disagreement, we change the value of α_{GUT} by a small increment and repeat the steps until there is a match. In addition, parameter η which appears inside the gaugino masses and was defined in (139) can be safely set to one. This is because as one varies the integers w and q inside (137) over a reasoble range, the torsion, unless specifically tuned, is so small that that the KK threshold corrections can be neglected.

Since $M_{Higgsino} \sim \mu \sim \mathcal{O}(m_{3/2})$, there is a substantial threshold contribution from the Higgs-Higgsino loops which has to be taken into account when computing bino (M_1) and wino (M_2) masses [2], [27, 28, 29]:

$$\Delta M_{1\,,2} \approx -\frac{\alpha_{1\,,2}}{4\pi}\frac{\mu\sin(2\beta)}{\left(1-\frac{\mu^2}{m_A^2}\right)}\ln\frac{\mu^2}{m_A^2} \approx \frac{\alpha_{1\,,2}}{4\pi}\mu = \frac{\alpha_{1\,,2}}{4\pi}Z^1_{eff}m_{3/2}\,. \tag{170}$$

In the above expression we expanded the logarithm using $\frac{\mu^2}{m_A^2} \sim 1$ and used $\tan\beta \sim \mathcal{O}(1)$. The latter is especially true when $2\,Z^1_{eff} \approx Z^2_{eff}$. We also relied on the fact that the supersymmetric μ-term almost does not change with the RG evolution so one can use (168). Since $m_{3/2} \sim \mathcal{O}(10)$TeV, the above correction to M_2 can be as large as a few hunrded GeV. It turns out that for $0 \le c \lesssim 0.05$ and $0.8 \lesssim \mu/m_{3/2}$ this correction is large enough to completely alter the nature of the LSP when $\mu > 0$. In particular, from the plots in Fig 1 where we picked $c = 0$ - left and $c = 0.05$ - right, there is a window for large values of $\mu/m_{3/2}$ such that the LSP is Wino-like. Furthermore, as one can see from the plots, there exists a small range of values where M_1 and M_2 become nearly degenerate. This is certainly an intriguing possibility which may provide for a well-tempered neutralino candidate [28]. Note that in the Wino-like LSP case, the lightest chargino and neutralino are degenerate at tree-level, i.e. $\widetilde{\chi}^0_1 = \widetilde{\chi}^\pm_1 = M_2$. However as we take into account the 1-loop contribution from the gauge bosons [29], this degeneracy is removed, as is seen from the corresponding entries in Table II. Such splitting was discussed in detail for the pure anomaly mediation scenario in [30, 31] and is given by

$$\Delta M_{1-loop} = \frac{\alpha_2 M_2}{4\pi}\left(f\left(\frac{m_W}{M_2}\right) - c_W^2 f\left(\frac{m_Z}{M_2}\right) - s_W^2 f\left(0\right)\right), \tag{171}$$

where $f(a) \equiv \int_0^1 dx(2+2x)\ln[x^2+(1+x)a^2]$. Typically we obtain $160\,\mathrm{MeV} < \Delta M_{1-loop} < 200\,\mathrm{MeV}$. Because of this, the lightest charginos are quazistable and decay into LSP plus soft pions or soft leptons. In the collider context such decays would take place well inside the detector leaving short charged tracks.

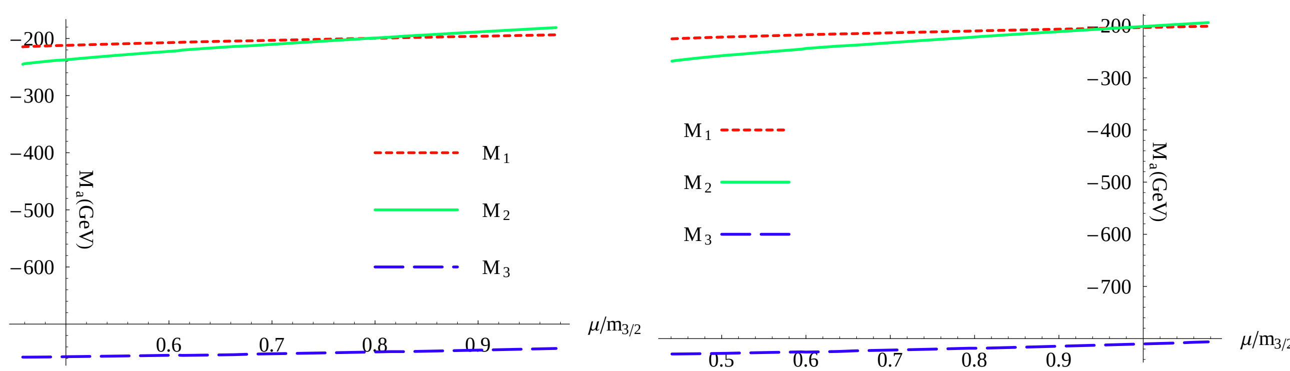

FIG. 1: Gaugino masses at the low scale as functions of $\mu/m_{3/2}$ when the gravitino mass is fixed at $m_{3/2} \approx 23\,\mathrm{TeV}$. Left plot: $c = 0$. Right plot: $c = 0.05$. In the above computation, the tree-level cosmological constant was tuned to zero, the gauge coupling unification constraint was satisfied and corrections in (170) were included with $\mu > 0$. Here the KK threshold correction to the visible sector gauge kinetic function was neglected, i.e. the Ray-Singer torsion invariant $\mathcal{T}_\omega$ in (139) is set to zero.

In addition to (170), the EW threshold corrections from gaugino-gauge-boson loops must also be included, especially for the gluino

$$\Delta M_3^{\mathrm{rad}} = \frac{3\alpha_3}{4\pi}\left(3\ln\left(\frac{M_{\mathrm{EW}}^2}{M_3^2}\right) + 5\right) M_3\,. \tag{172}$$

Unfortunately, we have no technical handle on the size of the parameter c, though we expect it to be small and hence a Wino LSP. As we increase the value of parameter c, the LSP quickly becomes Bino-like. There are two reasons for this effect. First, the ratio M_2/M_1 at the GUT scale grows as c is increased from zero to one. At the same time, the scalars and higgsinos become lighter relative to the gravitino mass. In particular, for a fixed $m_{3/2}$ the lower bound on the Higgs mass forces us to consider

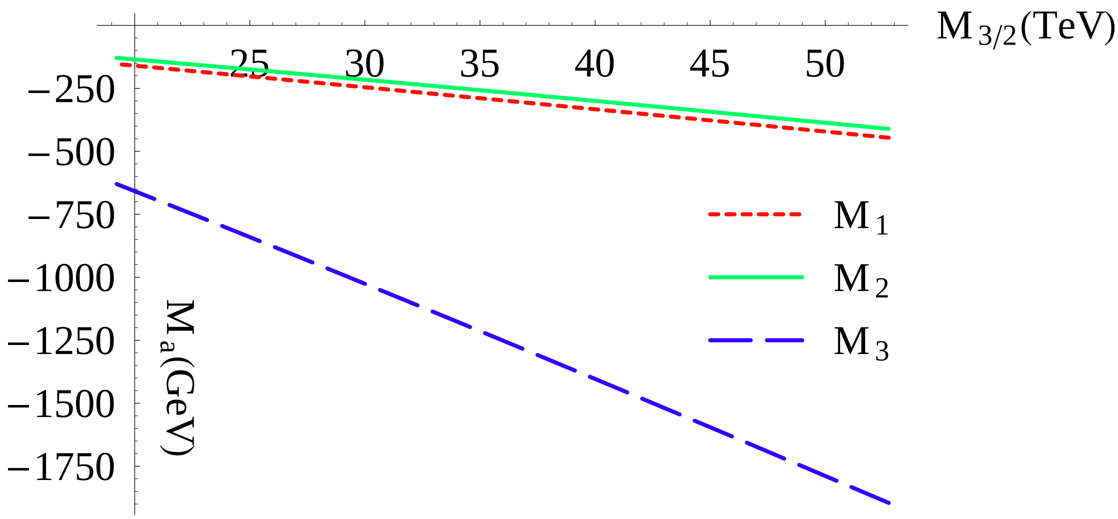

FIG. 2: Gaugino masses at the low scale as functions of $m_{3/2}$ when the tree-level cosmological constant is tuned to zero and the gauge coupling unification constraint is satisfied. The corrections in (170-172) were included with $\mu > 0$. Here parameter $c = 0$ and the KK threshold correction to the visible sector gauge kinetic function was neglected.

somewhat larger values of $\tan\beta$ which in turn leads to smaller values of μ thus significantly reducing the contributions (170) from higgsinos. Recall that it was primarily due to this contribution from heavy higgsinos for positive μ that the LSP could become Wino-like. Thus, due to the increase of M_2/M_1 at the GUT scale and the decrease in the higgsino mass, the Wino-like LSP case becomes rapidly excluded as we increase the value of c. Benchmarks 3-8 in Table II demonstrate that for generic values of c the LSP is always Bino-like.

Of course, pure Bino LSP is almost certainly excluded by the cosmological considerations [28]. Namely, because binos do not annihilate efficiently, the dark matter relic density becomes unacceptably large. However, this problem can be avoided when the higgsinos, which annihilate efficiently, are light enough to mix with gauginos. If the higgsino component of the LSP is significant, it can easily reduce the relic density to acceptable levels by increasing the annihilation crossection of the LSPs. It turns out that for generic values of c, $0 \leq c < 1$, the higgsinos are always much heavier than the gauginos. This is because at the decoupling scale m_s, the μ^2-term must be of the same order of magnitude as $m^2_{H_u}$ to give a correct value of the Z-boson mass, and since for typical values of c we get $|m^2_{H_u}| >> M^2_{1,2}$, the higgsinos do not mix with gauginos.

The only way to make the higgsinos light is to consider values of c very close to one. In this limit both scalars and higgsinos also become highly suppressed relative to the gravitino mass. As we will see in the analysis of this limit, the higgsinos can become as light as the gauginos and for certain values of $\tan\beta$ can give a well-tempered LSP candidate.

B. Squarks and sleptons

Recall that at the GUT scale all the squarks and sleptons have a universal mass (151), which for generic values of c ($0 \leq c < 1$) is smaller but nevertheless typically of the same order of magnitude as the gravitino mass. However, as we evolve these down to the electroweak scale, the third generation scalars become lighter whereas the first and second generation scalars experience a slight increase in their masses. The latter increase is due to the contributions from the gauginos which are rather light which explains the smallness of the effect. On the other hand, since the third generation Yukawa couplings are large, the stops, sbottoms, and staus are affected through the corresponding trilinear couplings (165), which are of $\mathcal{O}(m_{3/2})$. As one can see from Table II, this effect is especially dramatic for the lightest stop $\tilde{t}_1$. Yet, it is still much heavier than the gauginos and is effectively decoupled from the spectrum at the electroweak scale.

However, since gluinos can be pair produced at the LHC via gluon fusion, the gluinos (which have to decay via a quark-squark pair) have a sizeable branching fraction into top-stop – precisely because the stop is the lightest squark. This leads to "multi-top events" at the LHC.

C. Radiative electroweak symmetry breaking

The existence of the electroweak symmetry breaking (EWSB) in the effective theory below the decoupling scale m_s is determined by whether there exists a negative eigenvalue in the Higgs mass matrix

$$\begin{pmatrix} m^2_{H_u} + \mu^2 & -B\mu \\ -B\mu & m^2_{H_d} + \mu^2 \end{pmatrix} \tag{173}$$

at the scale where the scalars decouple [32]. Recall that at the GUT scale, $m^2_{H_u} = m^2_{H_d} = (1-c)\, m^2_{3/2}$ whereas $\mu = Z^1_{eff} m_{3/2}$ and $B\mu = Z^2_{eff} m_{3/2}$. It is well known that the positive contribution into the running of the up Higgs mass parameter squared from the stop is crucial for radiative EWSB as it drives $m^2_{H_u}$ negative. It turns out that for a fixed value of the gravitino mass $m_{3/2}$, as we vary parameter c there exists a narrow range of values $Z^{1,2}_{eff}$ for which the matrix (173) has a negative eigenvalue above the decoupling scale m_s defined in (169).

However, unless we tune $Z^2_{eff} \lesssim 2\, Z^1_{eff}$ as in benchmark seven of Table II, all the entries in the above matrix are $\mathcal{O}(m^2_{3/2})$. Therefore, both the lightest Higgs mass and the Z-boson mass naturally come out to be of $\mathcal{O}(m_{3/2})$. For the same reason, there is little mixing between the two Higgs doublets and naturally $\tan\beta \sim \mathcal{O}(1)$ is predicted in our framework. In practice, parameters $Z^{1,2}_{eff}$ must be tuned in such a way that the corresponding eigenvalue turns negative right at the decoupling scale [32] so that $m_Z \approx 91\,\mathrm{GeV}$. This fine tuning is a manifestation of the so-called *little hierarchy problem* - the hierarchy between the electroweak scale $M_{EW} \sim \mathcal{O}(100)\, GeV$ and the scale where the scalars decouple $m_s \sim \mathcal{O}(10)\,\mathrm{TeV}$. Once M_Z is tuned, the Standard Model Higgs mass turns out to be $m_h < 140\,\mathrm{GeV}$.

D. Approximately sequestered limit

For completeness, in this subsection we are going to examine the spectrum in the limit limit when parameter c approaches one, though this is presumably not a case which really originates from a "typical G_2-manifold". Since the scalars will now be light, the full MSSM running must be implemented from the GUT scale to the electroweak scale. To generate the sparticle spectrum at the electroweak scale we utilize the SOFTSUSY package [33] with the GUT scale soft terms specific to our model. Before we move

parameter	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8
$m_{3/2}$	23170	23170	23170	23170	37750	23170	23170	23170
c	0	0	1/3	1/3	1/3	1/2	1/2	1/2
Z^1_{eff}	1.375	0.870	0.865	0.515	1.001	0.706	0.224	-0.624
Z^2_{eff}	2.612	1.305	1.125	0.515	1.401	0.748	0.089	-0.624
α^{-1}_{unif}	26.2	26.17	26.27	26.20	26.67	26.25	26.09	26.28
$M_{\rm GUT}$	1.76×10^{16}	1.75×10^{16}	1.52×10^{16}	1.56×10^{16}	1.29×10^{16}	1.47×10^{16}	1.56×10^{16}	1.41×10^{16}
$\tan\beta$	1.58	2.11	1.80	2.77	1.64	1.85	17.28	2.01
μ	26878	19204	17904	11675	32660	14670	5036	-13286
M_1	187.7	198.4	270.9	279.8	445.8	309.9	325.3	342.4
M_2	163.0	195.8	351.7	379.5	579.1	438.6	487.6	536.9
M_3	734.8	748.2	1197.5	1216.4	1998.6	1417.3	1451.8	1419.3
$m_{\tilde{g}}$	727.6	738.7	1101.8	1115.4	1718.4	1273.02	1296.27	1274.84
$m_{\tilde{\chi}^0_1}$	163.22	196.02	270.95	279.87	445.81	310.01	325.31	342.27
$m_{\tilde{\chi}^0_2}$	187.72	198.52	351.97	379.86	579.26	438.96	487.72	536.54
$m_{\tilde{\chi}^\pm_1}$	163.38	196.20	352.14	380.03	579.43	439.13	487.90	536.71
$m_{\tilde{u}_L}$	23174	23174	18930	18930	30842	16404	16404	16404
$m_{\tilde{u}_R}$	23173	23173	18929	18929	30839	16401	16401	16401
$m_{\tilde{t}_1}$	5847	8263	6771	8456	10400	6368	8576	6734
$m_{\tilde{t}_2}$	16882	17369	14202	14631	22995	12431	12235	12526
$m_{\tilde{b}_1}$	16882	17369	14202	14631	22995	12431	12235	12526
$m_{\tilde{b}_2}$	23147	23131	18905	18881	30807	16381	15023	16378
$m_{\tilde{e}_L}$	23171	23171	18921	18921	30827	18388	16388	16388
$m_{\tilde{e}_R}$	23171	23171	18920	18920	30825	16386	16386	16386
$m_{\tilde{\tau}_1}$	23159	23151	18909	18899	30811	16377	15814	16376
$m_{\tilde{\tau}_2}$	23165	23161	18915	18910	30820	16383	16104	16383
m_h	112.0	119.7	115.2	126.6	115.2	115.5	138.5	118.1
m_A	41980	33240	29781	23565	52595	24974	14849	23525
$\tilde{A}_t$	5904	8875	5051	6682	7504	4039	5493	4348
$\tilde{A}_b$	1157	1492	891	1292	1315	706	5169	749
$\tilde{A}_\tau$	674	855	502	712	763	388	2837	411

TABLE II: Low scale spectra for eight benchmark G_2-MSSM models. All masses are in GeV. The top mass is taken to be 172.4 GeV. The spectra are largely determined by four parameters $m_{3/2}$, c, Z^1_{eff} and Z^2_{eff}. Benchmark 1 is experimentally excluded by the lower bound on the Higgs mass. The KK-threshold corrections to the gaugino masses were neglected.

to computing the MSSM spectrum at the electroweak scale, we would like to point out a useful relation between the soft terms we have computed. Namely, we can use the expression for the scalar masses (151) to express parameter c as

$$c = 1 - \left(\frac{m_\alpha}{m_{3/2}}\right)^2 . \tag{174}$$

We can then use (174) to recast the gaugino masses (147) and reduced trilinears (165) as follows

$$M_1 \approx \left(-0.03156\,\eta + \alpha_{GUT}\left(-0.22497 + 0.52313\left(\frac{m_\alpha}{m_{3/2}}\right)^2\right)\right) \times m_{3/2} \tag{175}$$

$$M_2 \approx \left(-0.03156\,\eta + \alpha_{GUT}\left(-0.03409 + 0.55483\left(\frac{m_\alpha}{m_{3/2}}\right)^2\right)\right) \times m_{3/2}$$

$$M_2 \approx \left(-0.03156\,\eta + \alpha_{GUT}\left(0.10226 + 0.47557\left(\frac{m_\alpha}{m_{3/2}}\right)^2\right)\right) \times m_{3/2}\,,$$

$$\tilde{A}_t \approx 1.494\left(\frac{m_\alpha}{m_{3/2}}\right)^2 m_{3/2} - 0.0027\left(-\frac{46}{5}g^2_{GUT} + 6Y_t^2 + Y_b^2\right) m_{3/2} \tag{176}$$

$$\tilde{A}_b \approx 1.494\left(\frac{m_\alpha}{m_{3/2}}\right)^2 m_{3/2} - 0.0027\left(-\frac{44}{5}g^2_{GUT} + Y_t^2 + 6Y_b^2 + Y_\tau^2\right) m_{3/2}$$

$$\tilde{A}_\tau \approx 1.494\left(\frac{m_\alpha}{m_{3/2}}\right)^2 m_{3/2} - 0.0027\left(-\frac{24}{5}g^2_{GUT} + 3Y_b^2 + 4Y_\tau^2\right) m_{3/2}\,.$$

From the above expressions it is clear that as the value of c approaches one, the suppression of the scalars at the GUT scale relative to the gravitino mass results in a much larger suppression of both the Konishi anomaly contribution inside the gaugino masses and the tree-level term in the reduced trilinear couplings. Such a non-trivial interplay among the soft terms is very remarkable and leads to some very interesting MSSM spectra unique to our model.

For convenience, instead of parameter c we choose to use the universal scalar mass m_α as one of the inputs. Because we cannot yet compute the μ and $B\mu$ terms from first principles, we use $\tan\beta$ as an input and tune μ to get the correct value of the Z boson mass. Although α_{GUT} (g^2_{GUT}) appears as a parameter, it is actually fixed at the value where the gauge couplings unify $\alpha_{GUT} \approx 1/25$. The GUT scale physical Yukawa couplings in (176) are fixed once $\tan\beta$ is chosen. In total, we have five relevant high scale input parameters:

$$m_{3/2}\,,\ m_\alpha\,,\ \eta\,,\ \mu\,,\ \tan\beta\,. \tag{177}$$

However, as was mentioned earlier, parameter η which appears inside the gaugino masses can be safely set to one. As we vary the ratio $m_\alpha/m_{3/2}$, we need to make sure that we are in the regime where the anomaly mediated contributions to the scalar masses can still be neglected.

Table III contains six benchmark points generated by the SOFTSUSY. To compute the thermal relic density we used SOFTSUSY generated output as input for the micrOMEGAs program [34]. Note that the first benchmark point is experimentally excluded by the lower bound on the Higgs mass. As one can see from the entries for M_1 and M_2, the LSP is mostly Bino. Thus, for a generic value of $\tan\beta$ the thermal component of the relic density turns out to be much larger than the observed value $\Omega_c h^2 \approx 0.1099\pm0.0062$ [35]. However, for large values of $\tan\beta$ there is a window where the Higgsino component of the LSP can be large enough to reduce the relic density. Hence, for each set of m_α and $m_{3/2}$ we can tune $\tan\beta$ so that the value of $\Omega_c h^2$ is compatible with experiment. For example, Points 2 and 3 with $m_\alpha = 800\,\text{GeV}$ and $m_{3/2} = 16\,\text{TeV}$ both satisfy the experimental bound on the relic density but have different values of $\tan\beta$, i.e $\tan\beta = 43$ and $\tan\beta = 38$. In this case, if $\tan\beta$ is within a window $38 \lesssim \tan\beta \lesssim 43$, the thermal component of the relic density is too low and one may have to appeal to non-thermal mechanisms [3]. Note that the $SU(5)$ GUT scale relation $Y_b \approx Y_\tau$ automatically holds in our construction. However,

parameter	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6
$m_{3/2}$	8000	16000	16000	16000	30000	50000
m_α	800	800	800	1200	2000	2000
$\tan\beta$	42.00	43.00	38.00	42.80	43.10	37.80
$M_{\rm GUT}$	2.55×10^{16}	2.0×10^{16}	2.0×10^{16}	2.02×10^{16}	1.66×10^{16}	1.39×10^{16}
μ	297.2	532.9	538.1	516.7	847.3	1411.2
$\Omega_c h^2$	0.114	0.115	0.118	0.116	0.112	0.115
M_1	134.1	274.7	274.5	275.6	530.2	898.4
M_2	203.9	413.4	412.8	413.5	784.0	1316.1
M_3	505.2	980.8	979.4	970.2	1746.2	2840.4
$m_{\tilde g}$	574.2	1053.2	1053.8	1072.5	1909.7	3008.0
$m_{\tilde\chi^0_1}$	130.2	270.8	270.7	272.1	525.8	891.1
$m_{\tilde\chi^0_2}$	193.8	408.3	409.3	408.8	779.4	1327.2
$m_{\tilde\chi^0_3}$	308.1	539.4	545.2	524.3	854.9	1419.4
$m_{\tilde\chi^0_4}$	332.0	562.7	567.5	551.0	896.4	1456.2
$m_{\tilde\chi^\pm_1}$	194.6	412.1	412.9	411.6	781.5	1333.3
$m_{\tilde\chi^\pm_2}$	330.6	558.9	564.3	547.6	891.1	1450.6
$m_{\tilde d_L}, m_{\tilde s_L}$	924.4	1223.8	1223.3	1499.7	2571.4	3326.5
$m_{\tilde u_L}, m_{\tilde c_L}$	920.2	1219.3	1218.6	1495.7	2566.3	3318.5
$m_{\tilde b_1}$	667.1	929.0	974.8	1125.9	1980.7	2756.8
$m_{\tilde t_1}$	577.6	838.1	840.9	986.4	1737.4	2379.8
$m_{\tilde e_L}, m_{\tilde\mu_L}$	818.0	875.5	875.3	1248.7	2101.1	2277.5
$m_{\tilde\nu_{e_L}}, m_{\tilde\nu_{\mu_L}}$	814.0	872.0	871.8	1246.2	2100.1	2277.3
$m_{\tilde\tau_1}$	669.0	688.9	722.5	1011.6	1699.8	1868.5
$m_{\tilde\nu_{\tau_L}}$	749.0	807.9	821.8	1148.5	1939.2	2158.3
$m_{\tilde d_R}, m_{\tilde s_R}$	912.8	1184.9	1184.5	1470.5	2512.0	3192.1
$m_{\tilde u_R}, m_{\tilde c_R}$	914.9	1194.5	1193.9	1477.8	2527.2	3225.6
$m_{\tilde b_2}$	733.7	992.0	1032.1	1193.2	2060.8	2813.1
$m_{\tilde t_2}$	714.8	1003.1	1026.6	1164.8	2004.4	2781.8
$m_{\tilde e_R}, m_{\tilde\mu_R}$	809.2	835.9	836.0	1223.4	2048.4	2134.5
$m_{\tilde\tau_2}$	755.4	816.3	829.7	1152.5	1940.2	2159.6
m_{h_0}	111.4	115.0	115.1	115.7	119.3	121.7
m_{H_0}	426.5	444.6	637.3	653.3	1140.7	1774.5
m_{A_0}	426.4	444.6	637.2	653.2	1140.6	1774.5
$m_{H^\pm}$	435.4	452.6	642.4	658.4	1143.4	1775.8
$\tilde A_t$	445.5	810.5	830.3	829.5	1485.9	2397.4
$\tilde A_b$	561.2	954.2	1079.2	1014.4	1811.7	3112.4
$\tilde A_\tau$	159.9	192.7	272.2	257.2	473.0	864.9

TABLE III: Low scale spectra for six benchmark G_2-MSSM models with approximate sequestering generated by SOFTSUSY package. All masses are in GeV. The top mass was taken to be $m_t = 172.3\,\text{GeV}$. The thermal relic density was computed by combining the output from SOFTSUSY with micrOMEGAs software. The spectra are largely determined by three parameters m_α, $m_{3/2}$ and $\tan\beta$. The Kaluza-Klein threshold correstions to the gaugino masses have been neglected. For the above spectra, the gauge couplings unify at the value of $\alpha_{GUT} \approx 1/25$.

for large $\tan\beta$ the values of all third generation Yukawa couplings become comparable. Namely, for the benchmarks in Table III, we obtain

$$0.524 \lesssim Y_t \lesssim 0.547\,, \quad 0.28 \lesssim Y_b \approx Y_\tau \lesssim 0.39\,. \tag{178}$$

VII. TUNING THE COSMOLOGICAL CONSTANT

Here we collect a few remarks which serve to clarify the tuning of the vacuum cosmological constant. Let us first identify what it is that is tuned in practice, i.e. which adjustable paramenters influence the value of P_{eff}. From its definition in (92) we see that the normalization coefficients of the superpotential A_1 and A_2 are likely candidates. Recalling the definitions (41) we can express the ratio inside P_{eff}

$$\frac{QA_1}{PA_2} = \frac{C_1}{C_2} = e^{\frac{S_2'}{2Q} - \frac{S_1'}{2P}}\,. \tag{179}$$

In the above expression quantities $S'_{1,2}$ represent threshold corrections to the tree-level gauge kinetic function f of the hidden sectors. Such corrections may, for example, come from massive Kaluza-Klein harmonics that live on the three cycles supporting the hidden sector gauge groups. Witten and Friedmann [10] explicitly computed corrections of this type for the visible sector $SU(5)$ GUT model and found that they are given in terms of certain topological invariants - Ray-Singer torsion classes (137). Because such topological quantities are functions of integers only, the ratio (179) is also a discrete function. Therefore, tuning P_{eff} to the desired value is discrete in nature, but is such a discretuum dense enough to account for the observed value of the cosmological constant? The answer is probably not. We may be able to tune P_{eff} to a value very close to the one in (112), maybe even up to a few digits after the decimal but it is very doubtful that such tuning is fine enough to produce the tiny value of the vacuum energy which is observed. However, there is one more important fact which so far has not been included into our discussion of the tuning. Recall that the superpotential we are assuming in our computations only represents the first two leading exponential terms which dominate over the rest if P and Q are larger by at least of factor of a few than the corresponding parameters in the remaining gaugino condensates. Including the rest of the terms which are exponentially suppressed relative to the first two will certainly alter the tree-level vacuum energy by some small amount while keeping the moduli vevs almost intact. All those extra terms are also functions of various integers but since their contributions to the tree-level cosmological constant are exponentially small, one can certaily imagine that some of them are so suppressed that varying the integers inside them will do the job of tuning the cosmological constant to the correct tiny value. For example, one can consider a membrane instanton wrapping the visible sector three-cycle $\mathcal{Q}^{SM}$ with a volume $V_{\mathcal{Q}^{SM}} \approx 1/\alpha_{GUT} \approx 25$. In this case, the scalar potential at the minimum receives a possible contribution

$$\delta V_0 \sim e^K |\delta W|^2 \sim \frac{m_{pl}^4}{64\pi V_X^3}\left(e^{-2\pi V_{\mathcal{Q}^{SM}} + 5\mathcal{T}_\omega}\right)^2 = m_{pl}^4 \sin^{20}(5\pi w/q)\frac{2\times 10^{-133}}{V_X^3}\,, \tag{180}$$

where $\mathcal{T}_\omega$ is the corresponding Ray-Singer torsion in (137). It is clear that contributions such as (180), where the integers w and q provide the necessary discrete dials, are definitely small enough to provide for the degree of fine tuning needed to get the experimentally observed value of the vacuum energy.

In fact, we can split the question of tuning into two categories. First, is it the coarse tuning of the integers inside $S'_{1,2}$ in (179) which kills most of the negative vacuum energy. Second is the fine discrete tuning due to the integers contained in the remaining gaugino condensates and membrane instanton terms inside W. What is more important is that once the coarse tuning is done, the quantities relevant for particle physics stay insensitive to any further fine tuning.

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