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Baryon Triality and Neutrino Masses from an Anomalous Flavor $U(1)$

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Abstract

We construct a concise $U(1)_X$ Froggatt-Nielsen model in which baryon triality, a discrete gauge $\mathbb{Z}_3$-symmetry, arises from $U(1)_X$ breaking. The proton is thus stable, however, R-parity is violated. With the proper choice of $U(1)_X$ charges we can obtain neutrino masses and mixings consistent with an explanation of the atmospheric and solar neutrino anomalies in terms of neutrino oscillations, with no right-handed neutrinos required. The only mass scale apart from M_{grav} is m_{soft}.

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1 Introduction

The Standard Model of particle physics (SM) has 19 free parameters, excluding the neutrino sector. 13 of these parameters arise through the trilinear Higgs couplings: nine masses and three mixing angles, among which one finds a hierarchy, and one phase. The Froggatt-Nielsen (FN) scenario is an elegant mechanism, which nicely explains these hierarchies in terms of a spontaneously broken gauge symmetry [1]. In the supersymmetric SM[1] (SSM), the renormalizable superpotential has 48 additional unknown new complex parameters. If we also consider non-renormalizable operators with mass-dimensionality five and six, there are correspondingly more. The SSM as such is inconsistent with the experimental lower bound on the proton lifetime and requires an additional symmetry beyond the SM gauge symmetry. From the low-energy point of view, the simplest possibility is to impose a new discrete symmetry. However, global discrete symmetries are inconsistent with quantum gravity [3]. It is possible to embed the discrete symmetry in a gauge symmetry, which is spontaneously broken at a high energy scale [3, 4]. In this case, when requiring the original gauge symmetry to be anomaly-free, and demanding a viable low-energy superpotential, only three consistent discrete symmetries remain: matter parity[2], baryon triality and proton hexality[3] [2, 5, 6]. In Ref. [7] a realistic supersymmetric FN model was constructed which conserved matter parity to all orders. It is the purpose of this paper to construct a realistic FN model with a low-energy baryon-triality symmetry to all orders.

As stated above, in the SM the FN mechanism explains the hierarchy of low-energy quark and charged lepton masses as well as the CKM mixings. But there is more to FN than just this:

- It was soon realized that the FN idea can be nicely combined with the SSM. It is then equally well applicable to the other superpotential coupling constants which arise, like the trilinear R-parity violating and/or higher dimensional operators, see for example Ref. [8].
- Furthermore in gravity mediated supersymmetry breaking, the FN model can go hand in hand with the Giudice-Masiero/Kim-Nilles (GM/KN) mechanism [9, 10].

[1]For a discussion of the difference between the minimal SSM (MSSM) and the SSM we refer to Ref. [2].

[2]Matter parity, [*cf.* Eq. (2.11)] acts on the superfields, while R-parity, $R_p \equiv (-1)^{2S+3B+L}$, is defined on the components of the superfields. Mostly this difference is irrelevant, because both symmetries allow and forbid exactly the same operators in the Lagrangian. So the two terms are often used synonymously.

[3]Proton hexality is a discrete gauge anomaly free $\mathbb{Z}_6$-symmetry and has recently been introduced in Ref. [2]. It is defined by the transformation in Eq. (2.13).

This allows for the natural generation of a μ-term and the other dimensionful bilinears without having to introduce their corresponding mass scales by hand.

- In addition, if the family symmetry is a $U(1)_X$, the FN scenario can plausibly be conjectured to come from an underlying string theory. The scale of spontaneous $U(1)_X$ breaking is then naturally just below $M_{\mathrm{grav}} = 2.4\times 10^{18}$ GeV [11, 12, 13, 14],[4] if $U(1)_X$ is anomalous in the sense of Green and Schwarz (GS) [15].

Our aim here is to build a supersymmetric FN model with the above-mentioned characteristics, *i.e.* with a hierarchical prediction for all allowed superpotential couplings and with the correct $U(1)_X$ breaking scale. We shall employ a single[5] $U(1)_X$ breaking chiral superfield A, the so-called flavon. We refer to Refs. [7, 8] and references therein for a review of the details on these models. In addition, we aim for some novel features:

- To have proton longevity, we want as a $U(1)_X$-remnant the discrete gauge $\mathbb{Z}_3$-symmetry baryon triality [5, 6], defined by the transformation in Eq. (2.12), to arise by virtue of the X-charges.

- To have the particle content as minimal as possible, we do not introduce right-handed neutrinos, instead we get the phenomenologically viable neutrino masses and mixings from matter parity violating but nevertheless baryon triality conserving coupling constants. Apart from minimality, there is a practical reason not to introduce right-handed neutrinos in this case: We do not want linear superpotential terms like $\overline{N^i}$, in order for the right-handed sneutrinos not to acquire a VEV to eat up this tree-level tadpole term, as this would constitute further flavon fields.

These two points are complementary to the model presented in Ref. [7], where compactness was also the guiding principle. There, the $U(1)_X$ gauge charges automatically lead to conserved matter parity at all orders and non-vanishing neutrino masses required the introduction of right-handed neutrinos. It was however assumed that one of the right-handed neutrino superfields mimics the flavon superfield. Inspired by *three* generations

[4]That the Green-Schwarz mechanism can soak up the anomalies requires a modified dilatonic Kähler potential, which induces a $U(1)_X$ Fayet-Iliopoulos term. This contribution to the D-term is then responsible for $U(1)_X$ to be broken, even in the case of having just one flavon. For more details see [7].

[5]There are models with more than one flavon, see *e.g.* Ref. [16]. However, in the context of obtaining a residual discrete symmetry through the spontaneous breaking of a $U(1)$, several flavon fields would unnecessarily complicate our business. We therefore choose the simplest possibility of only one flavon. Furthermore, as pointed out in Ref. [17], if one works with a vectorlike pair of flavons, the D-flat direction is spoiled, "leading to large hierarchy among the vacuum expectation values" (just before their Subsection 2.1).

of SSM superfields, we introduced *three* generations of SSM singlets, two of them being right-handed neutrinos and one constituting the flavon. Having effectively only two right-handed neutrinos then results in a massless lightest neutrino. The proton had a sufficiently long lifetime by suppressing the matter parity allowed, but baryon triality forbidden, operators $QQQL$ and $\overline{UUDE}$. However, the predicted lifetime is not too far from the current experimental bound [18, 19]. In addition, the choice of two right-handed neutrinos may seem a bit arbitrary. Without matter parity, the lepton doublets and the down-type Higgs doublet mix, hence effectively one could also advocate *four* generations of SSM singlets.

Here, compared to [7], we stabilize the proton completely, and simultaneously we are even more compact concerning the ingredients. However, as baryon triality allows for matter parity violation, the lightest supersymmetric particle decays. Therefore it does not provide a natural solution to the origin of dark matter. In our model, we thus have to stick to other possible candidates such as *e.g.* axions or axinos [20, 21, 22].

Before going on we state the renormalizable superpotential of the SSM to fix our notation:

$$\begin{aligned} W &= G^{(E)}_{ij}\, H^D L^i \overline{E^j} + G^{(D)}_{ij}\, H^D Q^i \overline{D^j} + G^{(U)}_{ij}\, H^U Q^i \overline{U^j} + \mu_0\, H^D H^U \\ &\quad + \frac{1}{2}\lambda_{ijk}\, L^i L^j \overline{E^k} + \lambda'_{ijk}\, L^i Q^j \overline{D^k} + \frac{1}{2}\lambda''_{ijk}\, \overline{U^i}\,\overline{D^j}\,\overline{D^k} + \mu_i\, L^i H^U\,. \end{aligned} \tag{1.1}$$

$SU(3)_C$ and $SU(2)_W$ indices are suppressed, i, j, k are generation indices, Q^k, $\overline{U^k}$, $\overline{D^k}$, L^k, $\overline{E^k}$, H^D, H^U denote superfields: quark doublets, u-type antiquark singlets, d-type antiquark singlets, lepton doublets, right-handed antielectron singlets and two Higgs doublets, respectively. μ_0, μ_i are the dimensionful bilinear parameters. The $G^{(E)}_{ij}$, $G^{(D)}_{ij}$, $G^{(U)}_{ij}$ and λ_{ijk}, λ'_{ijk}, λ''_{ijk} are coupling constants. Denoting the scalar component of a superfield by a tilde, the soft supersymmetry breaking Lagrangian density is (see, *e.g.*, Ref. [23])

$$\begin{aligned} -\mathcal{L}_{\text{soft}} &= [\boldsymbol{M}^{\boldsymbol{2}}_{\tilde{\boldsymbol{Q}}}]_{ij}\widetilde{Q^i}^\dagger\widetilde{Q^j} + [\boldsymbol{M}^{\boldsymbol{2}}_{\tilde{\boldsymbol{U}}}]_{ij}\widetilde{\overline{U^i}}\,\widetilde{\overline{U^j}}^* + [\boldsymbol{M}^{\boldsymbol{2}}_{\tilde{\boldsymbol{D}}}]_{ij}\widetilde{\overline{D^i}}\,\widetilde{\overline{D^j}}^* + [\boldsymbol{M}^{\boldsymbol{2}}_{\tilde{\boldsymbol{E}}}]_{ij}\widetilde{\overline{E^i}}\,\widetilde{\overline{E^j}}^* \\ &\quad + [\boldsymbol{M}^{\boldsymbol{2}}_{\tilde{\boldsymbol{L}}}]_{\alpha\beta}\widetilde{L^\alpha}^\dagger\widetilde{L^\beta} + M^2_{H^U}\widetilde{H^U}^\dagger\widetilde{H^U} + \left(b_\alpha\,\widetilde{L^\alpha}\widetilde{H^U} + \text{h.c.}\right) \\ &\quad + \left(\frac{1}{2}[\boldsymbol{A}_{\boldsymbol{E}}]_{\alpha\beta k}\,\lambda_{\alpha\beta k}\,\widetilde{L^\alpha}\widetilde{L^\beta}\widetilde{\overline{E^k}}^* + [\boldsymbol{A}_{\boldsymbol{D}}]_{\alpha jk}\,\lambda'_{\alpha jk}\,\widetilde{L^\alpha}\widetilde{Q^j}\widetilde{\overline{D^k}}^*\right) + \text{h.c.} \\ &\quad + \left([\boldsymbol{A}_{\boldsymbol{U}}]_{ij}\,G^{(U)}_{ij}\,\widetilde{H^U}\widetilde{Q^i}\widetilde{\overline{U^j}}^* + [\boldsymbol{A}_{\boldsymbol{UDD}}]_{ijk}\,\lambda''_{ijk}\,\widetilde{\overline{U^i}}^*\widetilde{\overline{D^j}}^*\widetilde{\overline{D^k}}^*\right) + \text{h.c.} \\ &\quad + \frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}^a\tilde{W}^a + M_3\tilde{G}^b\tilde{G}^b\right) + \text{h.c.} \end{aligned} \tag{1.2}$$

The supersymmetry breaking bino, wino, and gluino mass terms are given in the last line, with $a = 1, 2, 3$ and $b = 1, ..., 8$. Note that we have applied the compact notation

$L^\alpha = (L^0 \equiv H^D,\, L^i)$, $\alpha = 0, 1, 2, 3$, since the lepton doublets L^i and the down-type Higgs doublet H^D have identical quantum numbers.

In supersymmetric theories, the presence of $\mathcal{L}_{\text{soft}}$ is a potential source for flavor changing neutral currents (FCNCs). Even worse, there is one unresolved drawback to mention which is generic to FN models employing the GM/KN mechanism (for details see Ref. [7]), inducing non-universal and $U(1)_X$-charge dependent contributions to the sparticle soft squared masses. This potentially causes problems with low-energy FCNCs, and is common to all FN models. In order to suppress these FCNCs, it is necessary to theoretically provide for a specific structure of the soft supersymmetry breaking terms (*e.g.* an approximate alignment of the quark and squark mass matrices, *cf.* Refs. [24, 25]). It is beyond the scope of this work to detail the subtleties of such issues. We simply expect (or hope?) that the problem of low-energy FCNCs will be solved together with an as yet non-existing proper model of supersymmetry breaking.

Our paper is structured as follows: In Section 2 we review the importance of discrete gauge symmetries; then we show how to obtain matter parity, baryon triality and proton hexality from a Froggatt-Nielsen $U(1)_X$. The next three sections are the core of this paper. Section 3 discusses our choice of basis. Section 4 combines the requirements of anomaly cancellation, the phenomenological constraints on the X-charges of quarks and charged leptons and the conditions of achieving baryon triality. Sect. 5 then deals with the neutrino sector in detail. In Sect. 6 we finally arrive at six distinct sets of X-charges, so that the quintessential result is given by Table 5. Sect. 7 concludes. The Apps. A, B+C, D, E, F complement Sect. 2 (comparing baryon *parity* and baryon *triality*), Sect. 3 (remarks on supersymmetric zeros + eliminating sneutrino VEVs), Sect. 5.2 (analyzing the symmetries in the $LL\overline{E}$ loop contribution to the neutrino mass matrix), Sect. 5.4 (diagonalization of the neutrino mass matrix) and Sect. 6 (an explicit example of how a set of X-charges produces low-energy physics), respectively.

2 Low-Energy Discrete Symmetries from Froggatt-Nielsen Charges

2.1 Discrete Gauge Symmetries

The SM Lagrangian is invariant under Poincaré transformations as well as $SU(3)_C \times SU(2)_W \times U(1)_Y$ local gauge transformations. The most general supersymmetric SM Lagrangian with one additional Higgs doublet leads to unobserved exotic processes, in particular rapid proton decay, inconsistent with the experimental bounds [26]. In the low-energy effective Lagrangian, this problem is resolved by introducing a global discrete multiplicative symmetry, which prohibits a subset of the superpotential interactions. Prominent examples are matter parity, M_p, (or equivalently R-parity), baryon triality, B_3, as well as the recently introduced proton hexality, P_6 [2]. There is however a problem when embedding a global discrete symmetry in a unified theory at the gravitational scale, such as string theory: it is typically violated by quantum gravity effects (worm holes) [3]. This is avoided if the discrete symmetry of the low-energy effective Lagrangian is the remnant of a local gauge symmetry, G, which is spontaneously broken, in our case at or near the gravitational scale. Since G is unaffected by quantum gravity effects, after the spontaneous breakdown of G also the residual discrete symmetry remains intact [3, 4, 27]. Such a discrete symmetry is denoted a "discrete *gauge* symmetry" (DGS) [28]. In the following, we only treat Abelian DGSs, originating in an Abelian local gauge group $G \equiv U(1)_X$. For examples see [2, 29].

In order to obtain a consistent quantum field theory, we demand that the underlying local gauge theory G is anomaly-free. In general, we include the possibility that the anomalies of the original gauge symmetry are canceled by the Green-Schwarz mechanism (GS) [15]. Thus either

1) the low-energy DGS is a remnant of an anomaly-free local gauge symmetry, in which case the DGS is anomaly-free in the sense of Ibáñez and Ross [5], or

2) the DGS is a remnant of a local gauge symmetry whose anomalies are canceled by the GS mechanism. In this case the DGS can be either

  a) anomaly-free in the sense of Ibáñez and Ross or

  b) GS-anomalous, *i.e.* the DGS anomalies are canceled via a discrete version of the GS mechanism [30].[6]

The model we construct in this paper belongs to class *2a)*, *i.e.* the $U(1)_X$ gauge anomalies are canceled by the GS mechanism; however, the low-energy DGS satisfies the anomaly cancellation conditions of Ibáñez and Ross, without the GS mechanism.

In Ref. [6], the family-independent $\mathbb{Z}_2$ and $\mathbb{Z}_3$ DGSs were determined, which are anomaly-free without invoking the existence of extra light particles besides those of the SSM and, in addition, which leave the MSSM superpotential (and possibly more) invariant. This investigation was generalized to $\mathbb{Z}_N$ DGSs with arbitrary N in Ref. [2]. Taking

[6]We emphasize that not every discrete symmetry is an anomaly-free DGS, *e.g.* baryon parity, B_p, see also App. A.

into account the need for neutrino masses, only three family-independent DGSs survive: matter parity [Eq. (2.11)], baryon triality [Eq. (2.12)] and proton hexality [Eq. (2.13)], which we discuss in more detail below.

In Ref. [7], several of the authors constructed a phenomenologically viable FN model with a low-energy matter parity DGS. In the following subsection, we wish to first derive the necessary and sufficient conditions on the MSSM X-charges for baryon triality and proton hexality to arise as a DGS from a family-dependent local gauge symmetry. Afterwards, in Sect. 5 we construct phenomenologically viable FN models with a low-energy *baryon triality* DGS. We postpone the construction of such a model with a low-energy proton hexality to Ref. [31].

2.2 Baryon Triality Arising from $U(1)_X$

Consider a general product of MSSM left-chiral superfields $\Phi^a \in \{Q^k, \overline{U^k}, \overline{D^k}, L^k, \overline{E^k}, H^D, H^U\}$ and their charge conjugates $\overline{\Phi^a}$,

$$R \equiv \prod_{a,b} (\Phi^a)^{\alpha_a} \, (\overline{\Phi^b})^{\overline{\alpha_b}} \, . \tag{2.1}$$

In general, such an operator can appear in the Kähler potential or, if the $\overline{\alpha_b}$ vanish, in the superpotential. Imposing a discrete symmetry forbids some of these SM-invariant operators. We now wish to obtain a specific low-energy discrete symmetry by an appropriate $U(1)_X$ gauge charge assignment. We fix the gauge charge normalization such that the flavon superfield A has $U(1)_X$ charge $X_A = -1$. It is then obvious that only those operators with an *integer* overall X-charge, X_{total}, are allowed after the breaking of $U(1)_X$. We obtain further constraints on X_{total} by requiring $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariance of the given operator, as well as by demanding that the renormalizable MSSM superpotential operators are necessarily allowed. We thus have the conditions on X_{total} for an operator to be allowed or forbidden. We then make the connection with the corresponding discrete symmetry, originating from the MSSM X-charges, stating the necessary and sufficient conditions thereof.

We shall denote the "combined" multiplicity of each superfield in a given operator by $n_{\Phi^a} \equiv \alpha_a - \overline{\alpha_a}$. Thus for example the term $Q^1\overline{Q^2 U^1 D^1 D^2}$ has $n_{Q^1} = 1, n_{Q^2} = -1$, $n_{\overline{U^1}} = n_{\overline{D^1}} = n_{\overline{D^2}} = 1$. The total X-charge of a general product, R, of superfields Φ^a, $\overline{\Phi^b}$

can then be expressed as

$$
\begin{aligned}
X_{\rm total} \;=\;& n_{H^D}\; X_{H^D} + n_{H^U}\; X_{H^U} \;+\; \sum_i n_{Q^i}\; X_{Q^i} \;+\; \sum_i n_{\overline{D^i}}\; X_{\overline{D^i}} \\
&+\; \sum_i n_{\overline{U^i}}\; X_{\overline{U^i}} \;+\; \sum_i\; n_{L^i}\; X_{L^i} \;+\; \sum_i n_{\overline{E^i}}\; X_{\overline{E^i}}.
\end{aligned}
\tag{2.2}
$$

The coefficients $n_{...}$ and charges $X_{...}$ above are not all mutually independent:

- Since each product R should be $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariant, the $n_{...}$ are subject to the conditions (for the first equation see for example Chapter 10 in Ref. [32])

$$
\sum_i n_{Q^i} - \sum_i n_{\overline{D^i}} - \sum_i n_{\overline{U^i}} \;=\; 3\mathcal{C}, \tag{2.3}
$$

$$
n_{H^D} + n_{H^U} + \sum_i n_{Q^i} + \sum_i n_{L^i} \;=\; 2\mathcal{W}, \tag{2.4}
$$

$$
\begin{aligned}
&Y_{H^D}\; n_{H^D} + \; Y_{H^U}\; n_{H^U} \;+\; Y_Q \sum_i n_{Q^i} \;+\; Y_{\overline{D}} \sum_i n_{\overline{D^i}} \\
&+ Y_{\overline{U}} \sum_i n_{\overline{U^i}} \;+\; Y_L \sum_i n_{L^i} \;+\; Y_{\overline{E}} \sum_i n_{\overline{E^i}} \;=\; 0.
\end{aligned}
\tag{2.5}
$$

  Here $\mathcal{C}$ is an integer, $\mathcal{W}$ is an integer which is non-negative for terms in the superpotential. $Y_{...}$ denotes the hypercharge of the corresponding field. For the MSSM fields we have: $Y_{H^D} = -3Y_Q$, $Y_{H^U} = 3Y_Q$; $Y_L = -3Y_Q$, $Y_{\overline{E}} = 6Y_Q$; $Y_{\overline{U}} = -4Y_Q$, $Y_{\overline{D}} = 2Y_Q$. Solving Eqs. (2.3)-(2.5) for $n_{Q^1}, n_{\overline{D^1}}$, and $n_{\overline{E^1}}$ we obtain

$$
n_{Q^1} \;=\; 2\mathcal{W} - (n_{H^D} + n_{H^U}) - (n_{Q^2} + n_{Q^3}) - \sum_i n_{L^i}\,, \tag{2.6}
$$

$$
n_{\overline{D^1}} \;=\; -3\mathcal{C} + 2\mathcal{W} - (n_{H^D} + n_{H^U}) - (n_{\overline{D^2}} + n_{\overline{D^3}}) - \sum_i n_{L^i} - \sum_i n_{\overline{U^i}}\,, \tag{2.7}
$$

$$
n_{\overline{E^1}} \;=\; \mathcal{C} - \;\mathcal{W} + \; n_{H^D} \qquad - \;(n_{\overline{E^2}} + n_{\overline{E^3}}) + \sum_i n_{L^i} + \sum_i n_{\overline{U^i}}. \tag{2.8}
$$

- Since we assume that after the breaking of $U(1)_X$ all renormalizable MSSM superpotential operators are allowed, the corresponding gauge invariant products R must have non-fractional powers of the flavon superfield A, *i.e.* we require

  1. The renormalizable superpotential terms $H^U Q^i \overline{U^j}$, $H^D Q^i \overline{D^j}$, $H^D L^i \overline{E^j}$, and $H^D H^U$ have an overall integer X-charge.

This corresponds to the conditions

$$\begin{aligned}
X_{H^U} + X_{Q^1} + X_{\overline{U^1}} &= \text{integer}, \\
X_{H^D} + X_{Q^1} + X_{\overline{D^1}} &= \text{integer}, \\
X_{H^D} + X_{L^1} + X_{\overline{E^1}} &= \text{integer},
\end{aligned}$$

$$\begin{aligned}
X_{Q^{2,3}} - X_{Q^1} &= \text{integer}, \\
X_{L^{2,3}} - X_{L^1} &= \text{integer}, \\
X_{\overline{D^{2,3}}} - X_{\overline{D^1}} &= \text{integer}, \\
X_{\overline{U^{2,3}}} - X_{\overline{U^1}} &= \text{integer}, \\
X_{\overline{E^{2,3}}} - X_{\overline{E^1}} &= \text{integer}, \\
X_{H^D} + X_{H^U} &= \text{integer}.
\end{aligned} \tag{2.9}$$

We leave it open at the moment which other gauge invariant terms shall also have an overall integer X-charge.

With the help of Eq. (2.9), we can express all X-charges in terms of X_{L^1}, X_{Q^1}, X_{H^D}, and unknown integers. Inserting this and Eqs. (2.6)-(2.8) in Eq. (2.2) we get for the total X-charge

$$\begin{aligned}
X_{\text{total}} &= \mathcal{C} \cdot \left[3X_{Q^1} + X_{L^1} + 2(X_{H^D} - X_{L^1})\right] \\
&\quad + \left(n_{H^D} \; - \; \mathcal{W} \; + \; \sum_i n_{\overline{U^i}}\right) \cdot (X_{H^D} - X_{L^1}) \; + \; \text{integer}.
\end{aligned} \tag{2.10}$$

If we now require *no* remnant DGS at low energy whatsoever, *i.e.* if *all* renormalizable and non-renormalizable terms which are $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariant are allowed, then we must have an overall integer X-charge, and thus $X_{H^D} - X_{L^1}$ and $3X_{Q^1} + X_{L^1}$ must be integers. However, we wish to determine the necessary constraints on the gauged X-charges in order to obtain a remnant discrete matter parity, baryon triality, or proton hexality DGS arising from the $U(1)_X$. Our treatment here does not rely on the absence or cancellation of anomalies and is thus equally applicable to $\mathbb{Z}_N$-symmetries other than M_p, B_3 and P_6, *e.g.* the $\mathbb{Z}_2$-symmetry baryon parity, B_p. (For the definition of baryon parity and an investigation of its phenomenological difference with respect to baryon triality see App. A.)

Under the respective DGSs, the MSSM left-chiral superfields transform as follows

- Matter parity[7] (M_p)

$$\begin{array}{rcrl} \Big\{H^D, H^U\Big\} & \longrightarrow & & \Big\{H^D, H^U\Big\}, \\ \Big\{Q^i, \overline{U^i}, \overline{D^i}, L^i, \overline{E^i}\Big\} & \longrightarrow & e^{2\pi \mathrm{i}/2} & \Big\{Q^i, \overline{U^i}, \overline{D^i}, L^i, \overline{E^i}\Big\}, \end{array} \tag{2.11}$$

- Baryon triality (B_3)

$$\begin{array}{rcrl} Q^i & \longrightarrow & & Q^i, \\ \Big\{H^U, \overline{D^i}\Big\} & \longrightarrow & e^{2\pi \mathrm{i}/3} & \Big\{H^U, \overline{D^i}\Big\}, \\ \Big\{H^D, \overline{U^i}, L^i, \overline{E^i}\Big\} & \longrightarrow & e^{4\pi \mathrm{i}/3} & \Big\{H^D, \overline{U^i}, L^i, \overline{E^i}\Big\}, \end{array} \tag{2.12}$$

- Proton hexality (P_6), *cf.* Ref. [2],

$$\begin{array}{rcrl} Q^i & \longrightarrow & & Q^i, \\ \Big\{H^D, \overline{U^i}, \overline{E^i}\Big\} & \longrightarrow & e^{2\pi \mathrm{i}/6} & \Big\{H^D, \overline{U^i}, \overline{E^i}\Big\}, \\ L^i & \longrightarrow & e^{8\pi \mathrm{i}/6} & L^i, \\ \Big\{H^U, \overline{D^i}\Big\} & \longrightarrow & e^{10\pi \mathrm{i}/6} & \Big\{H^U, \overline{D^i}\Big\}. \end{array} \tag{2.13}$$

(None of these three symmetries has a domain wall problem, since the discrete charges of the two Higgs superfields are opposite to each other, for details see Ref. [2].) In other words, under M_p, B_3, and P_6 transformations a general product of MSSM superfields is respectively multiplied by

- $$\left(e^{2\pi \mathrm{i}/2}\right)^{\sum_i n_{Q^i} \, + \, \sum_i n_{\overline{U^i}} \, + \, \sum_i n_{\overline{D^i}} \, + \, \sum_i n_{L^i} \, + \, \sum_i n_{\overline{E^i}}}, \tag{2.14}$$

- $$\left(e^{2\pi \mathrm{i}/3}\right)^{n_{H^U} \, + \, \sum_i n_{\overline{D^i}} \, + \, 2\, n_{H^D} \, + \, 2\sum_i n_{\overline{U^i}} \, + \, 2\sum_i n_{L^i} \, + \, 2\sum_i n_{\overline{E^i}}}, \tag{2.15}$$

- $$\left(e^{2\pi \mathrm{i}/6}\right)^{n_{H^D} \, + \, \sum_i n_{\overline{U^i}} \, + \, \sum_i n_{\overline{E^i}} \, + \, 4\sum_i n_{L^i} \, + \, 5\, n_{H^U} \, + \, 5\sum_i n_{\overline{D^i}}}. \tag{2.16}$$

[7] An alternative but at low-energies physically equivalent definition of matter parity is

$$\begin{array}{rcrl} \Big\{Q^i, L^i\Big\} & \longrightarrow & & \Big\{Q^i, L^i\Big\}, \\ \Big\{\overline{U^i}, \overline{D^i}, \overline{E^i}, H^D, H^U\Big\} & \longrightarrow & e^{2\pi \mathrm{i}/2} & \Big\{\overline{U^i}, \overline{D^i}, \overline{E^i}, H^D, H^U\Big\}. \end{array}$$

See Ref. [2] for details. With this definition it is easy to see that proton hexality is the direct product of matter parity and baryon triality.

Thus in turn we may write for $[M_p/B_3/P_6]$

- $$\sum_i n_{Q^i} \;+\; \sum_i n_{\overline{U^i}} \;+\; \sum_i n_{\overline{D^i}} \;+\; \sum_i n_{L^i} \;+\; \sum_i n_{\overline{E^i}} \;=\; 2\mathcal{I}_M \;+\; \iota_M,$$
- $$n_{H^U} \;+\; \sum_i n_{\overline{D^i}} \;+\; 2\,n_{H^D} \;+\; 2\sum_i n_{\overline{U^i}} \;+\; 2\sum_i n_{L^i} \;+\; 2\sum_i n_{\overline{E^i}} \;=\; 3\mathcal{I}_B \;+\; \iota_B,$$
- $$n_{H^D} \;+\; \sum_i n_{\overline{U^i}} \;+\; \sum_i n_{\overline{E^i}} \;+\; 4\,\sum_i n_{L^i} \;+\; 5\,n_{H^U} \;+\; 5\,\sum_i n_{\overline{D^i}} \;=\; 6\mathcal{I}_P \;+\; \iota_P\,. \tag{2.17}$$

$\mathcal{I}_M$, $\mathcal{I}_B$, and $\mathcal{I}_P$ are integers; ι_M is 0 or 1 if matter parity is conserved or broken, ι_B is 0 or $1,2$ if baryon triality is conserved or broken, and ι_P is 0 or $1,\dots,5$ if proton hexality is conserved or broken. With Eqs. (2.6)-(2.8) we get from Eq. (2.17) that

$$M_p: \quad n_{H^D} \;=\; 3\mathcal{W} - \iota_M - 2(\mathcal{C} + \mathcal{I}_M + n_{H^U}) + \sum_i n_{\overline{U^i}}, \tag{2.18}$$

$$B_3: \qquad \mathcal{C} \;=\; 3\Big(-\mathcal{I}_B + n_{H^D} + \sum_i n_{L^i} + \sum_i n_{\overline{U^i}}\Big) - \iota_B, \tag{2.19}$$

$$P_6: \quad n_{H^D} \;=\; 3\mathcal{W} - \sum_i n_{\overline{U^i}} - \frac{14\,\mathcal{C} + 6\mathcal{I}_P + \iota_P}{3}, \tag{2.20}$$

respectively. We now require

1′. *All* $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariant terms which conserve the discrete symmetry $[M_p/B_3/P_6]$ each have an overall integer X-charge. This requirement is a generalization of Point 1. above Eq. (2.9).

2. All $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariant terms which do not conserve the discrete symmetry $[M_p/B_3/P_6]$ each have an overall fractional X-charge. It follows that all superfield operators which violate $[M_p/B_3/P_6]$ are forbidden even after the spontaneous breaking of $U(1)_X$. $[M_p/B_3/P_6]$ is thus conserved exactly, *i.e.* to all orders.

For any $SU(3)_C \times SU(2)_W \times U(1)_Y$ invariant operator R [*cf.* Eq. (2.1)], which violates $[M_p/B_3/P_6]$ one has respectively that $[R^2/R^3/R^6]$ conserves $[M_p/B_3/P_6]$. From Point 1′. above, we find that the X-charge of each of the latter operators, namely $[2 \cdot X_{\text{total}}(R)/3 \cdot X_{\text{total}}(R)/6 \cdot X_{\text{total}}(R)]$ respectively, is integer. Point 2. demands that $X_{\text{total}}(R)$ is fractional. *It follows that all superfield operators which violate* $[M_p/B_3/P_6]$ *have an* X_{total} *of the form* $[\frac{1}{2} + \textit{integer}/\frac{1 \text{ or } 2}{3} + \textit{integer}/\frac{1,2,3,4 \text{ or } 5}{6} + \textit{integer}]$. Bearing this in mind, we plug Eqs. (2.18) - (2.20) into Eq. (2.10) to eliminate $[n_{H^D}/\mathcal{C}/n_{H^D}]$, respectively.

- We first treat M_p. In this case

$$\begin{aligned} X_{\text{total}} &= \mathcal{C} \cdot (3X_{Q^1} + X_{L^1}) \\ &\quad + \Big[2\Big(\mathcal{W} - \mathcal{I}_M - n_{H^U} + \sum_i n_{\overline{U^i}}\Big) - \iota_M\Big] \cdot (X_{H^D} - X_{L^1}) \;+\; \text{integer.} \end{aligned} \tag{2.21}$$

Now consider an operator R forbidden by M_p, *i.e.* with $\iota_M = 1$. X_{total} must then be $\frac{1}{2}$+integer. Choosing a forbidden operator where $\mathcal{C} = 0$, and $\mathcal{W} = \mathcal{I}_M + n_{H^U} - \sum_i n_{\overline{U^i}}$, we obtain the condition

$$X_{H^D} \;-\; X_{L^1} \;\stackrel{!}{=}\; -\frac{1}{2} \;+\; \text{integer.} \tag{2.22}$$

We insert this into the expression for X_{total}:

$$X_{\text{total}} \;=\; \mathcal{C} \cdot (3X_{Q^1} + X_{L^1}) + \frac{\iota_M}{2} + \;\text{integer}\,. \tag{2.23}$$

Now for the terms which are allowed by M_p, *i.e.* which have $\iota_M = 0$ (and thus X_{total} is integer). Here we get another condition on the X-charges of the MSSM superfields when choosing an operator for which $\mathcal{C} = 1$,

$$3X_{Q^1} \;+\; X_{L^1} \;\stackrel{!}{=}\; \text{integer}\,. \tag{2.24}$$

To check consistency, we plug Eqs. (2.22) and (2.24) into Eq. (2.21); we thus find that

$$X_{\text{total}} \;=\; \frac{\iota_M}{2} \;+\; \text{integer}\,. \tag{2.25}$$

In Refs. [7, 18, 19] the implications of Eqs. (2.9), (2.22), and (2.24) in combination with a viable phenomenology were studied in detail.

- Next we treat B_3. We get

$$\begin{aligned} X_{\text{total}} = &\Big[3\Big(n_{H^D} - \mathcal{I}_B + \sum_i n_{L^i} + \sum_i n_{\overline{U^i}}\Big) - \iota_B\Big] \cdot \Big[3X_{Q^1} + X_{L^1} + 2(X_{H^D} - X_{L^1})\Big] \\ &+ \Big(n_{H^D} \;-\; \mathcal{W} \;+\; \sum_i n_{\overline{U^i}}\Big) \cdot (X_{H^D} - X_{L^1}) \;+\; \text{integer.} \end{aligned} \tag{2.26}$$

Considering an allowed operator, *i.e.* with $\iota_B = 0$ (thus X_{total} is integer) and for which also $\mathcal{I}_B = n_{H^D} + \sum_i n_{L^i} + \sum_i n_{\overline{U^i}}$ we arrive at

$$X_{\text{total}} \;=\; \Big(n_{H^D} - \mathcal{W} + \sum_i n_{\overline{U^i}}\Big) \cdot (X_{H^D} - X_{L^1}) \;+\; \text{integer}\,. \tag{2.27}$$

If we furthermore require that $\mathcal{W} = n_{H^D} + \sum_i n_{\overline{U^i}} + 1$ we obtain the condition

$$X_{H^D} \ - \ X_{L^1} \ \stackrel{!}{=} \ \text{integer} \tag{2.28}$$

[to be compared with Eq. (2.22)]. We insert this into the expression for X_{total}, getting

$$X_{\text{total}} = \Big[3\big(n_{H^D} - \mathcal{I}_B + \sum_i n_{L^i} + \sum_i n_{\overline{U^i}}\big) - \iota_B\Big] \cdot (3X_{Q^1} + X_{L^1}) \ + \ \text{integer}. \tag{2.29}$$

Now considering a forbidden operator for which $\mathcal{I}_B = n_{H^D} + \sum_i n_{L^i} + \sum_i n_{\overline{U^i}}$, we arrive at

$$X_{\text{total}} \ = \ - \ \iota_B \cdot (3X_{Q^1} + X_{L^1}) \ + \ \text{integer}\,. \tag{2.30}$$

Setting $\iota_B = 1$ (thus X_{total} being $\frac{1 \text{ or } 2}{3}$+integer) we get

$$3X_{Q^1} + X_{L^1} \ \stackrel{!}{=} \ -\frac{b}{3} + \ \text{integer}\,, \tag{2.31}$$

with $b \in \{1, 2\}$ [to be compared with Eq. (2.24)]. This is compatible with $\iota_B = 2$ also requiring X_{total} not to be an integer. To check consistency, we plug Eqs. (2.28) and (2.31) into Eq. (2.26); this gives

$$X_{\text{total}} \ = \ \frac{b \cdot \iota_B}{3} \ + \ \text{integer}\,. \tag{2.32}$$

- For P_6 we find that

$$\begin{aligned} X_{\text{total}} \ &= \ \mathcal{C} \cdot (3X_{Q^1} + X_{L^1}) \\ &\quad + \ \left[2(\mathcal{W} - \mathcal{C} - \mathcal{I}_P) - \frac{2\mathcal{C} + \iota_P}{3}\right] \cdot (X_{H^D} - X_{L^1}) \ + \ \text{integer}\,. \end{aligned} \tag{2.33}$$

Before continuing it is important to point out that $\frac{2\mathcal{C}+\iota_P}{3}$ has to be an integer due to Eq. (2.20). Hence, when deriving the conditions on the X-charges for P_6 conservation, we are restricted to consider only those operators for which $2\mathcal{C} + \iota_P$ is a multiple of three. Thus we have that $\iota_P = 0, 3/1, 4/2, 5$ requires $\mathcal{C} = 0/1/2 \bmod 3$, respectively. Defining $\mathcal{J} \equiv \frac{2\mathcal{C}+\iota_P}{3} \in \mathbb{Z}$, we have

$$2\mathcal{C} \ = \ -\iota_P + 3\mathcal{J}\,. \tag{2.34}$$

Returning to Eq. (2.33), consider $\iota_P = 3$ (already the square of such an operator is P_6 invariant, therefore we have $X_{\text{total}} = \frac{1}{2} + \text{integer}$ in this case) and $\mathcal{C} = \mathcal{W} = \mathcal{I}_P = 0$; this leads to the condition[8]

$$X_{H^D} - X_{L^1} \stackrel{!}{=} -\frac{1}{2} + \text{integer}\,, \tag{2.35}$$

[to be compared with Eq. (2.22)]. Inserting this into X_{total} of Eq. (2.33) we get

$$X_{\text{total}} = \mathcal{C} \cdot (3X_{Q^1} + X_{L^1}) + \frac{2\,\mathcal{C} + \iota_P}{6} + \text{integer}\,. \tag{2.36}$$

Next consider $\iota_P = \mathcal{C} = 1$. For $\iota_P = 1$ we need $X_{\text{total}} = \frac{p}{6} + \text{integer}$, with $p = 1, 5$. $p = 2, 3, 4$ are not allowed as these have common prime factors with 6. This would lead to a term in the Lagrangian whose square or cube is P_6 invariant contrary to the assumption that ι_P is 1. This way the following condition is obtained

$$\begin{aligned} 3X_{Q^1} + X_{L^1} &\stackrel{!}{=} -\frac{3-p}{6} + \text{integer} \\ &\equiv -\frac{\widetilde{p}}{3} + \text{integer}, \end{aligned} \tag{2.37}$$

with $\widetilde{p} = \pm 1$ [to be compared with Eq. (2.24)]. Plugging Eqs. (2.35) and (2.37) into Eq. (2.33) we get

$$X_{\text{total}} = \frac{\iota_P}{6} + (1 - \widetilde{p}) \cdot \frac{2\mathcal{C}}{6} + \text{integer}\,. \tag{2.38}$$

Recalling the condition in Eq. (2.34), we can rewrite this as

$$\begin{aligned} X_{\text{total}} &= \frac{\iota_P}{6} + (1 - \widetilde{p}) \cdot \frac{-\iota_P}{6} + (1 - \widetilde{p}) \cdot \frac{3\mathcal{J}}{6} + \text{integer} \\ &= \frac{\widetilde{p} \cdot \iota_P}{6} + (1 - \widetilde{p}) \cdot \frac{\mathcal{J}}{2} + \text{integer}\,. \end{aligned} \tag{2.39}$$

As $(1 - \widetilde{p})$ is always an even number, we finally arrive at

$$X_{\text{total}} = \frac{\widetilde{p} \cdot \iota_P}{6} + \text{integer}\,. \tag{2.40}$$

[8]This condition has already been stated in Ref. [2], below Eq. (6.9). But in its PRD-version we unfortunately made a typo by including a wrong factor of "3", which we have however corrected in the newest arXiv-version.

As a summary, in addition to Eq. (2.9), depending on the desired remnant low-energy discrete symmetry, we need to impose the following conditions on the X-charges:

$$X_{H^D} - X_{L_1} \;=\; \begin{cases} \text{integer} \\ \text{integer} - m/2 \\ \text{integer} \\ \text{integer} \\ \text{integer} - 1/2 \end{cases} , \qquad 3X_{Q^1} + X_{L^1} \;=\; \begin{cases} \text{integer} \\ \text{integer} \\ \text{integer} - b'/2 \\ \text{integer} - b/3 \\ \text{integer} - \widetilde{p}/3 \end{cases} ,$$

$$\Longrightarrow \qquad X_{\text{total}} \;=\; \begin{cases} \text{integer} \\ \text{integer} + m \cdot \iota_M/2, \\ \text{integer} + \, b' \cdot \iota'_B/2 \\ \text{integer} + \, b \cdot \iota_B/3 \\ \text{integer} + \, \widetilde{p} \cdot \iota_P/6 \end{cases} , \tag{2.41}$$

with $m, b' = 1,\ b \in \{1,2\}$, and $\widetilde{p} \in \{-1,1\}$. We also have $\iota_M, \iota'_B \in \{0,1\},\ \iota_B \in \{0,1,2\}$, and $\iota_P \in \{0,1,2,3,4,5\}$. The five cases in Eq. (2.41) correspond to having *all* terms, only M_p terms, only B_p terms (see App. A), only B_3 terms, or only P_6 terms allowed by virtue of the X-charges, respectively. Note that in Ref. [7] it was shown that Eq. (2.9) together with the coefficients $\mathcal{A}_{CCX}$ and $\mathcal{A}_{WWX}$ of the $SU(3)_C$-$SU(3)_C$-$U(1)_X$ and $SU(2)_W$-$SU(2)_W$-$U(1)_X$ anomalies and the condition of Green-Schwarz anomaly cancellation require

$$3X_{Q^1} + X_{L^1} = \frac{\text{integer}}{\mathcal{N}_g}, \tag{2.42}$$

where $\mathcal{N}_g$ symbolizes the number of generations. With $\mathcal{N}_g = 3$ all possibilities listed above except the anomalous B_p are compatible with Eq. (2.42).

3 Sequence of Basis Transformations

The Froggatt-Nielsen charges determine the structure of the theory just below the gravitational scale M_{grav}. The low-energy theory emerges after the successive breakdown of the $U(1)_X$ gauge symmetry, supersymmetry, and then $SU(2)_W \times U(1)_Y$. The hierarchy of the fermion mass spectrum is given in terms of powers of the ratio $\epsilon \equiv \frac{\langle A \rangle}{M_{\text{grav}}}$ of the vacuum expectation value (VEV) of the $U(1)_X$ flavon field, A, and the gravitational scale. Within a string-embedded FN framework this expansion parameter originates in the Dine-Seiberg-Wen-Witten mechanism [11, 12, 13, 14], leading to a value of about $\epsilon \sim 0.2$ (see *e.g.* Ref. [7]). Neglecting $\mathcal{O}(1)$ renormalization flow effects and imposing that B_3 arises from the X-charges as described in Section 2, we obtain a Lagrangian which is B_3 invariant but M_p violating ($\not{M}_p$). The resulting kinetic terms obtained from the Kähler potential are non-canonical, *i.e.* they are not diagonal in generation space and not properly normalized. Furthermore, the sneutrino vacuum expectation values are in general non-zero. It is usually more convenient to formulate $\not{M}_p$ theories where the neutrino masses are induced radiatively in a basis with vanishing sneutrino VEVs and the down-type fields rotated to their mass bases [23].[9] Therefore, we apply the sequence of basis transformations, depicted in the diagram below, and study its effects on the FN-generated coupling constants. The numbers in brackets refer to the explanations of each step, below.

Type of Basis	Redefinition of Chiral Fields
Froggatt-Nielsen basis (denoted by the subscript FN)	
$\downarrow$	(1) Non-unitary transformation of Q^i, $\overline{D^i}$, $\overline{U^i}$, L^α, $\overline{E^i}$, H^D, H^U
Basis with canonical Kähler potential	
$\downarrow$	(2) Unitary transformation of L^α
Basis without sneutrino VEVs	
$\downarrow$	(3) Unitary transformation of Q^i, $\overline{D^i}$, L^i, and $\overline{E^i}$
Mass basis of down-type quarks and charged leptons	

In the third step we only rotate the L^i, not the L^α. The transformations of the $\overline{U^i}$ do not affect any of the terms we are interested in, so that we do not further consider them. After the above transformations, we again find an FN structure for the coupling constants in the new basis. Working backwards, it is then possible to deduce phenomenologically viable X-charge assignments from the experimentally observed masses and mixings of quarks and leptons:

[9] For a pedagogical review of the different contributions to the neutrino masses, expressed in a basis independent way, see Ref. [33].

1. *Canonicalization of the Kähler potential (CK):* The Kähler potential for n species of superfields $\Phi^i_{\rm FN}$ ($i = 1, ..., n$) with equal gauge quantum numbers in the FN basis, *i.e.* which can mix, is canonicalized by the $n \times n$ *non-unitary* matrix $\boldsymbol{C^{(\Phi)}}$, with the texture (see *e.g.* Ref. [7])

$$C^{(\Phi)}{}_{ij} \quad \sim \quad \epsilon^{|X_{\Phi^i} - X_{\Phi^j}|} . \tag{3.1}$$

In terms of the canonicalized superfields $\Phi^i \equiv C^{(\Phi)}{}_{ij}\ \Phi^j_{\rm FN}$, the kinetic operators are given in their standard diagonal and normalized form. The interaction coupling constants $c_{{\rm FN}\, i}$ also change correspondingly through the basis transformation, *e.g.* for a trilinear interaction of superfields Φ^i, Ψ^j and Θ^k

$$c_{{\rm FN}\, ijk}\ \Phi^i_{\rm FN} \Psi^j_{\rm FN} \Theta^k_{\rm FN} \quad = \quad c_{ijk}\ \Phi^i \Psi^j \Theta^k , \tag{3.2}$$

with

$$c_{ijk} \quad \equiv \quad [\boldsymbol{C^{(\Phi)}}^{-1}]_{i'i}\, [\boldsymbol{C^{(\Psi)}}^{-1}]_{j'j}\, [\boldsymbol{C^{(\Theta)}}^{-1}]_{k'k}\ c_{{\rm FN}\, i'j'k'} . \tag{3.3}$$

Note that each index transforms separately. In the following, while discussing the general FN power structure, we focus on one index for notational simplicity, *i.e.* we suppress additional indices that might be attached to the coupling constants

$$c_i \quad \equiv \quad [\boldsymbol{C^{(\Phi)}}^{-1}]_{ji}\ c_{{\rm FN}\, j} . \tag{3.4}$$

The generalization to n indices is trivial. Considering superpotential couplings which are free of supersymmetric zeros,[10] we have $c_{{\rm FN}\, i} \propto \epsilon^{X_{\Phi^i}}$. Under the above transformations, we obtain [17]

$$c_i \quad \propto \quad \epsilon^{|X_{\Phi^j} - X_{\Phi^i}|}\ \epsilon^{X_{\Phi^j}} \quad \sim \quad \epsilon^{X_{\Phi^i}} . \tag{3.5}$$

Coupling constants which are not generated by FN alone but involve a combination of FN and Giudice-Masiero/Kim-Nilles mechanism (see *e.g.* Ref. [7]) are treated slightly differently:

- Later we will *e.g.* assume that the bilinear superpotential terms $\mu_\alpha L^\alpha H^U$ are due to the GM/KN mechanism. Therefore the corresponding coupling constants have the X-charge dependence $c_{{\rm FN}\, i} \propto \epsilon^{-X_{\Phi^i}}$. In this case, the canonicalization of the Kähler potential yields

$$c_i \quad \propto \quad \epsilon^{|X_{\Phi^j} - X_{\Phi^i}|}\ \epsilon^{-X_{\Phi^j}} \quad \sim \quad \epsilon^{-X_{\Phi^i}} . \tag{3.6}$$

[10] The problems connected with having supersymmetric zeros in the Yukawa mass matrices are discussed in App. B.

- We also deal with the case where on the one hand the MSSM operators $H^D L^i \overline{E^j}$, $H^D Q^i \overline{D^j}$ are required to have overall positive integer X-charges, whereas the corresponding $\not{M}_p$-operators with $L^0 \equiv H^D \to L^i$ $(i = 1,2,3)$ replaced, *i.e.* $L^i L^j \overline{E^k}$, $L^i Q^j \overline{D^k}$, have overall negative integer X-charges. This assumption implies $X_{L^0} > X_{L^i}$. Due to the GM/KN mechanism, the supersymmetric zeros of the coupling constants with negative overall X-charge are actually not zero. However, for trilinear couplings the resulting terms are suppressed by a factor of $\mathcal{O}(\frac{m_{\text{soft}}}{M_{\text{grav}}})$ and therefore effectively absent; *e.g.*

$$X_{H^D} + X_{Q^2} + X_{\overline{D^2}} = 2 \quad \xrightarrow{\text{pure FN}} \quad \epsilon^2, \tag{3.7}$$

$$\text{but} \qquad X_{L^1} + X_{Q^2} + X_{\overline{D^2}} = -3 \quad \xrightarrow{\text{GM/KM}} \quad \frac{m_{\text{soft}}}{M_{\text{grav}}} \cdot \epsilon^3 \ll \epsilon^3\,, \tag{3.8}$$

where on the right we show the power of ϵ of the corresponding coupling. So for the coupling constants we effectively have $c_{\text{FN}\,\alpha} \propto (\epsilon^{X_{L^0}}, 0, 0, 0)_\alpha$. But thanks to the canonicalization of the kinetic terms, these "quasi supersymmetric zeros" are filled in so that

$$c_{\alpha} = [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{0\alpha}\, c_{\text{FN}\,0} \;\;\propto\;\; \epsilon^{|X_{L^0} - X_{L^\alpha}|}\, \epsilon^{X_{L^0}} \;\;\sim\;\; \epsilon^{2X_{L^0} - X_{L^\alpha}}. \tag{3.9}$$

We can apply a similar consideration to operators which contain $\epsilon^{ab} L^\alpha_a L^\beta_b$, where $a, b \in \{1,2\}$ are $SU(2)$ doublet indices. As the symmetric part of the corresponding coupling constant $c_{\text{FN}\,\alpha\beta}$ cancels automatically, it can be taken antisymmetric without loss of generality. Now when constructing a viable model, we choose the X-charges such that the terms $\epsilon^{ab} L^i_a L^j_b$, with $i, j = 1,2,3$, are forbidden by a negative integer total X-charge, whereas $\epsilon^{ab} L^i_a L^0_b$ and $\epsilon^{ab} L^0_a L^j_b$ are allowed. In this special case we find[11]

$$\begin{aligned} c_{\alpha\beta} \;&=\; [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{0\alpha}\, [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{j\beta}\, c_{\text{FN}\,0j} \;-\; (\alpha \leftrightarrow \beta) \\ &\propto\; \epsilon^{2X_{L^0} - X_{L^\alpha} + X_{L^\beta}} \;-\; (\alpha \leftrightarrow \beta). \end{aligned} \tag{3.10}$$

[11]This can be seen as follows: $c_{\alpha\beta} = [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{\alpha'\alpha}[\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{\beta'\beta}\, c_{\text{FN}\,\alpha'\beta'}$. The assumption $c_{\text{FN}\,ij} = 0$ together with the condition of antisymmetry, $c_{\text{FN}\,\alpha 0} = -c_{\text{FN}\,0\alpha}$, leads to

$$\begin{aligned} c_{\alpha\beta} \;&=\; [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{0\alpha}[\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{j\beta} \cdot c_{\text{FN}0j} + [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{i\alpha}[\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{0\beta} \cdot c_{\text{FN}i0}\,, \\ &=\; [\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{0\alpha}[\boldsymbol{C}^{(\boldsymbol{L})^{-1}}]_{j\beta} \cdot c_{\text{FN}\,0j} - (\alpha \leftrightarrow \beta). \end{aligned}$$

The ϵ-structure is then given by

$$\begin{aligned} c_{\alpha\beta} \;&\propto\; \epsilon^{|X_{L^0} - X_{L^\alpha}|}\, \epsilon^{|X_{L^j} - X_{L^\beta}|}\, \epsilon^{X_{L^0} + X_{L^j}} - (\alpha \leftrightarrow \beta) \\ &\propto\; \epsilon^{2X_{L^0} - X_{L^\alpha} + X_{L^\beta}} - (\alpha \leftrightarrow \beta)\,. \end{aligned}$$

After the canonicalization of the Kähler potential, all superpotential coupling constants of the fields Φ^i will therefore include a factor of either $\epsilon^{X_{\Phi^i}}$ or $\epsilon^{-X_{\Phi^i}}$.

2. *Rotating away the sneutrino VEVs:* Next we perform a unitary transformation on the superfields L^α $(\alpha = 0, 1, 2, 3)$ in order to eliminate the sneutrino VEVs. The four vacuum expectation values υ_α of the scalar component fields in L^α are determined by the minimization conditions for the neutral scalar potential. If we make the well-motivated [34] assumption of an FN structure in the soft supersymmetry breaking terms (for details see App. C), we find

$$\upsilon_\alpha \quad \propto \quad \epsilon^{-X_{L^\alpha}}. \tag{3.11}$$

We eliminate the sneutrino VEVs υ_i $(i = 1, 2, 3)$ by the unitary matrix which in Ref. [8] was used to rotate away the bilinear superpotential terms $L^i H^U$. In our case it has the texture[12]

$$\boldsymbol{U}^{\mathbf{VEVs}} \quad \sim \quad \begin{pmatrix} 1 & \epsilon^{X_{L^0} - X_{L^j}} \\ \epsilon^{X_{L^0} - X_{L^i}} & \delta_{ij} + \epsilon^{2X_{L^0} - X_{L^i} - X_{L^j}} \end{pmatrix}. \tag{3.12}$$

Accordingly, all coupling constants involving L^α also have to be transformed. However, as $[\boldsymbol{U}^{\mathbf{VEVs}\dagger}]_{\beta\alpha}\, \epsilon^{\pm X_{L^\beta}} \;\sim\; \epsilon^{\pm X_{L^\alpha}}$, their ϵ-structure remains unchanged.

3. *Rotation of the quarks and charged leptons into their mass bases:* In a third step, the down-type quark[13] and charged lepton mass matrices are diagonalized by the unitary transformations $\boldsymbol{U^{(Q)}}$, $\boldsymbol{U^{(\overline{D})}}$, $\boldsymbol{U^{(L)}}$, and $\boldsymbol{U^{(\overline{E})}}$ of the corresponding superfields. Their ϵ-power structure is given by, see also Ref. [35],

$$\begin{aligned} U^{(Q)}{}_{ij} &\sim \epsilon^{|X_{Q^i} - X_{Q^j}|}, \qquad U^{(\overline{D})}{}_{ij} \sim \epsilon^{|X_{\overline{D^i}} - X_{\overline{D^j}}|}, \\ U^{(L)}{}_{ij} &\sim \epsilon^{|X_{L^i} - X_{L^j}|}, \qquad U^{(\overline{E})}{}_{ij} \sim \epsilon^{|X_{\overline{E^i}} - X_{\overline{E^j}}|}. \end{aligned} \tag{3.13}$$

[12] Replacing $\mu \to \upsilon_0$ and $K_i \to \upsilon_i$ in Eq. (4.10) of Ref. [8], we have $K = \sqrt{\upsilon_i^* \upsilon_i}$ and $\mathcal{M} = \sqrt{\upsilon_\alpha^* \upsilon_\alpha}$. For the matrix we then have

$$\begin{aligned} U^{\mathrm{VEVs}}{}_{0j} &= \frac{|\upsilon_0|}{\mathcal{M}} \cdot \frac{\upsilon_j^*}{\upsilon_0^*} \sim \epsilon^{X_{L^0} - X_{L^j}}, \quad U^{\mathrm{VEVs}}{}_{i0} = -\frac{|\upsilon_0|}{\mathcal{M}} \cdot \frac{\upsilon_i}{\upsilon_0} \sim \epsilon^{X_{L^0} - X_{L^i}}, \\ U^{\mathrm{VEVs}}{}_{ij} &= \delta_{ij} + \frac{\upsilon_i \upsilon_j^*}{K^2} \cdot \left(\frac{|\upsilon_0|}{\mathcal{M}} - 1 \right) \approx \delta_{ij} - \frac{\upsilon_i \upsilon_j^*}{2|\upsilon_0|^2} \sim \delta_{ij} + \epsilon^{2X_{L^0} - X_{L^i} - X_{L^j}}. \end{aligned}$$

In the penultimate step we applied the approximation $K \ll \mathcal{M} \approx |\upsilon_0|$.

[13] As we apply the basis transformations equally on both components of the $SU(2)_W$ superfield doublets Q^i, we can diagonalize *either* the up- *or* the down-type quark mass matrix. The latter is more appropriate for our purpose because, in the context of radiatively generated neutrino masses, only down-type loops contribute to the neutrino mass matrix and the computations are simpler in this basis [23]. After $SU(2)_W \times U(1)_Y$ breaking, we rotate the left- and right-handed up-type quark superfields U_L and $\overline{U}$ into their mass basis.

Here we have to assume a decreasing X-charge for increasing generation index.[14] The transformations of Eq. (3.13) diagonalize the down-type mass matrices. However, they do not alter the ϵ-structure of the up-type Yukawa couplings and other renormalizable or non-renormalizable coupling constants, with one significant exception. The ϵ-structure of the down-type Yukawa mass matrices is obviously changed drastically in its off-diagonal entries by the transition to the mass basis. Through our transformations it is possible that supersymmetric zeros are filled proportional to the down-type mass matrices. Diagonalizing the down-type mass matrix then also diagonalizes these coupling constants in the corresponding two indices. In our model, we encounter such a proportionality when generating interactions as in Eq. (3.9), for example the trilinear $\not{M}_p$ terms $\lambda'_{ijk}\, L^i Q^j \overline{D^k}$. Together with the mass terms $G^{(D)}_{jk}\, H^D Q^j \overline{D^k} \;\equiv\; \lambda'_{0jk}\, L^0 Q^j \overline{D^k}$ for the down-type quarks we find

$$\lambda'_{ijk} \;=\; \frac{[\boldsymbol{C}^{(\boldsymbol{L})^{-1}} \cdot \boldsymbol{U}^{\mathbf{VEVs}^\dagger}]_{0l}}{[\boldsymbol{C}^{(\boldsymbol{L})^{-1}} \cdot \boldsymbol{U}^{\mathbf{VEVs}^\dagger}]_{00}}\, [\boldsymbol{U}^{(\boldsymbol{L})^\dagger}]_{li}\, \lambda'_{0jk}\,, \tag{3.14}$$

with the coupling constants $\lambda'_{\alpha jk}$ now given in the basis of diagonal down-type mass matrices. Analogously, we have for the superpotential terms $\frac{1}{2}\,\lambda_{ijk}\, L^i L^j \overline{E^k}$, together with $\lambda_{0jk}\, L^0 L^j \overline{E^k}$,

$$\lambda_{ijk} \;=\; \frac{[\boldsymbol{C}^{(\boldsymbol{L})^{-1}} \cdot \boldsymbol{U}^{\mathbf{VEVs}^\dagger}]_{0l}}{[\boldsymbol{C}^{(\boldsymbol{L})^{-1}} \cdot \boldsymbol{U}^{\mathbf{VEVs}^\dagger}]_{00}}\, [\boldsymbol{U}^{(\boldsymbol{L})^\dagger}]_{li}\, \lambda_{0jk} \;-\; (i \leftrightarrow j). \tag{3.15}$$

Here we have neglected the second antisymmetrizing contribution of Eq. (3.10) when expressing λ_{0jk} in terms of $\lambda_{\mathrm{FN}\,0j'k'}$ as it is suppressed by a factor of $\epsilon^{2\,(X_{L^0}-X_{L^j})}$. Hence, both types of trilinear $\not{M}_p$ coupling constants are proportional to the corresponding Yukawa mass matrices,[15] which are diagonal in our basis.

Table 1 summarizes the FN structure of some important superpotential coupling constants at different steps in the sequence of basis transformations. We omitted the up-type quark Yukawa coupling constants in Table 1, as they have the standard FN structure which does not change under the sequence of basis transformations.

[14] The diagonalization matrices $\boldsymbol{U}^{(\cdots)}$ have the structure of Eq. (3.13) only if the X-charges of the left- and the right-chiral superfields are ordered in the same way. Demanding further that the third generation is the heaviest and the first the lightest, we are restricted to decreasing X-charge for increasing generation index.

[15] In Ref. [36] quite generally models for radiatively generated neutrino masses are studied in which as it so happens *1)* baryon triality is (accidentally) conserved and *2)* the trilinear $\not{M}_p$ coupling constants are proportional to the mass matrices of the down-type quarks and charged leptons. Our models belong to this category, with both *1)* and *2)* arising by virtue of the X-charges. The 5^{th} charge assignment in Table 5 is presented in Ref. [36], as an example.

	$\frac{\mu_\alpha}{m_{3/2}}$	$\lambda'_{\alpha jk}$	$\lambda_{\alpha\beta k}$
(0)	$\epsilon^{-X_{L^\alpha}-X_{H^U}}$	only $\lambda'_{0jk} \sim \epsilon^{X_{L^0}+X_{Q^j}+X_{\overline{D^k}}}$	only $\lambda_{0jk} \sim \epsilon^{X_{L^0}+X_{L^j}+X_{\overline{E^k}}}$
(1)	$\epsilon^{-X_{L^\alpha}-X_{H^U}}$	$\epsilon^{2X_{L^0}-X_{L^\alpha}+X_{Q^j}+X_{\overline{D^k}}}$	$\epsilon^{2X_{L^0}-X_{L^\alpha}+X_{L^\beta}+X_{\overline{E^k}}} - (\alpha \leftrightarrow \beta)$
(2)	$\epsilon^{-X_{L^\alpha}-X_{H^U}}$	$\lambda'_{0jk} \sim \delta_{jk}\ \epsilon^{X_{L^0}+X_{Q^k}+X_{\overline{D^k}}}$, $\lambda'_{ijk} \sim \epsilon^{X_{L^0}-X_{L^i}}\lambda'_{0jk}$,	$\lambda_{0jk} \sim \delta_{jk}\ \epsilon^{X_{L^0}+X_{L^k}+X_{\overline{E^k}}}$, $\lambda_{ijk} \sim \epsilon^{X_{L^0}-X_{L^i}}\lambda_{0jk} - (i \leftrightarrow j)$

Table 1: FN structure of superpotential couplings at various stages of the basis transformations: Before (0) and after (1) the canonicalization of the Kähler potential, and finally in the mass basis of the down-type quarks and charged leptons (2).

We now state a first set of constraints on the X-charges, required for our model, which we shall make more quantitative in the subsequent sections:

- By choosing positive integer X-charges for all trilinear MSSM interactions we avoid troubles in the fermionic mass spectrum associated with supersymmetric zeros (see App. B).

- The generalized μ-problem ($\mu_\alpha\, L^\alpha H^D$) is solved by the Giudice-Masiero/Kim-Nilles mechanism.

- In order to avoid too heavy neutrino masses [37], we require $\frac{\mu_i}{\mu_0} \sim \epsilon^{X_{L^0}-X_{L^i}} < 1$, *i.e.* $X_{L^0} > X_{L^i}$.

- If the trilinear $\not{M}_p$ interactions are only suppressed by powers of ϵ comparable to the trilinear MSSM terms, they are in disagreement with the experimental bounds [38, 7]. Therefore we choose to generate trilinear $\not{M}_p$ terms by the canonicalization of the Kähler potential as described above, *cf.* Eq. (3.9). This mechanism has also been employed in[16] Ref. [39]. The term $\overline{UDD}$ is forbidden altogether by B_3.

In the following we study phenomenological constraints on the X-charges arising from the fermionic mass spectrum. In our basis, the down-type mass matrices are diagonal. Therefore the CKM matrix is obtained solely from the diagonalization of the up-type quark mass matrix, and exhibits the ϵ-structure ${U^{\mathrm{CKM}}}_{ij} \sim \epsilon^{|X_{Q^i}-X_{Q^j}|}$. Furthermore, we need to specify and diagonalize the neutrino mass matrix.

[16] The authors of Ref. [39] construct their model such that $X_{\overline{U^i D^j D^k}} < 0$, so that it is GM/KN-suppressed if $X_{\overline{U^i D^j D^k}}$ is integer. However, with $X_{L^i} + X_{H^U}$ required to be integer and working with $\Delta^L_{21} = \Delta^L_{31} = 0$, $z = 1$, their proposed X-charge assignment in fact also accidentally generates B_3, so that $\overline{U^i D^j D^k}$ is not only highly suppressed but absent altogether.

4 Non-Neutrino Constraints on the X-Charges

In the previous section, we translated our model from the scale of $U(1)_X$ breaking down to the electroweak scale. The FN charges of the MSSM superfields are now directly connected to the low-energy fermionic mass spectrum. For our model, we require the X-charges to reproduce phenomenologically acceptable quark masses and mixings as well as charged lepton masses. Furthermore, we demand the GS anomaly cancellation conditions [40, 7] to be satisfied (apart from the constraints listed at the end of the previous section, which we will not implement right away but later). We then find that all 17 X-charges can be expressed in terms of only six real numbers (see Table 1 in Ref. [7]):

$$\begin{aligned} & x = 0,1,2,3\ , \qquad && y = -1,0,1\ , \qquad && z = 0,1\ , \\ & \Delta^L_{31} \equiv X_{L^3} - X_{L^1}\ , \qquad && \Delta^L_{21} \equiv X_{L^2} - X_{L^1}\ , \qquad && X_{L^1}\ . \end{aligned} \tag{4.1}$$

Δ^L_{31} and Δ^L_{21} are necessarily integer whereas X_{L^1} is arbitrary.[17] For phenomenological reasons x, y, z can only take on the shown integer values. As we choose to generate the μ-term via the GM/KN mechanism, we take $z \equiv -X_{H^U} - X_{H^D} = 1$ throughout this article. x is related to the ratio of the Higgs VEVs by $\epsilon^x \sim \frac{m_b}{m_t} \tan\beta$, with $\tan\beta = \left|\frac{\upsilon_u}{\upsilon_0}\right|$. Recall, the sneutrino VEVs are rotated away, so $|\upsilon_0| = |\upsilon_d| \equiv \sqrt{\upsilon^*_\alpha\ \upsilon_\alpha}$. y parameterizes all phenomenologically viable CKM matrices. Our preferred choice is $y = 0$, resulting in ${U^{\mathrm{CKM}}}_{12} \sim \epsilon$, ${U^{\mathrm{CKM}}}_{13} \sim \epsilon^3$, and ${U^{\mathrm{CKM}}}_{23} \sim \epsilon^2$, see Ref. [7].

Assuming a string-embedded FN framework, the parameter ϵ originates solely in the Dine-Seiberg-Wen-Witten mechanism [11, 12, 13, 14]. Thus it is a derived quantity which depends on x and z (for details see Ref. [7]). Taking $z = 1$ and $x = 0, 1, 2, 3$ we get ϵ within the interval $\epsilon \in [0.186, 0.222]$.[18]

Our goal is to construct a conserved B_3 model. As discussed in detail in Sect. 2, this leads to additional constraints, which are best expressed in terms of the new parameters Δ^H, ζ

$$\Delta^H \equiv X_{L^1} - X_{L^0}\ , \qquad 3\zeta + b \equiv \Delta^L_{21} + \Delta^L_{31} - z\,. \tag{4.2}$$

Here the parameter $b = 1, 2$ is as introduced in Eq. (2.31). Rewriting Eqs. (2.28) and (2.31), the latter by making use of Table 1 in Ref. [7], we see that demanding B_3 conser-

[17] Δ^L_{31} and Δ^L_{21} actually do not have to be integers, if it weren't for the sake of Eq. (2.9). Furthermore, with Δ^L_{31} and Δ^L_{21} being fractional Eq. (3.1) would not hold.

[18] The parameterization of the mass ratios of the SM fermions in terms of ϵ is based on $\epsilon = 0.22$, so that working with other values for ϵ is strictly speaking slightly inconsistent.

vation is equivalent to demanding[19]

$$\Delta^H, \zeta \in \mathbb{Z}. \tag{4.3}$$

We now replace X_{L^1} and Δ^L_{21} in favor of Δ^H, ζ, and $b = 1, 2$ and we arrive at the constrained X-charges of Table 2. This is the equivalent of Table 2 in Ref. [7] for the case of B_3 instead of M_p. Note that the parameters ζ and b appear in Table 2 only in the combination $3\zeta + b$.

Phenomenologically, the conservation of B_3 renders the proton stable. For the proton to decay we need a baryon-number violating operator. This in turn requires the parameter $\mathcal{C}$ in Eq. (2.3) to be non-zero. On the other hand, $\mathcal{C}$ must be an integer multiple of three in the case of B_3 conservation [see Eq. (A.8) in App. A, with $\iota_B = 0$]. Hence, only operators with $|\mathcal{C}| = 3, 6, 9, ...$ are B_3 conserving and baryon-number violating. Comparing with Eq. (2.3) we see that at least nine quark (or antiquark) superfields are needed. Such a superpotential term, however, is suppressed by a factor of $\frac{1}{M^6_{\mathrm{grav}}}$ and thus negligible.

Our baryon triality conserving model is not compatible with grand unified theories (GUTs). Unlike in the M_p conserving model in Ref. [7], it is impossible to choose the parameters $x, y, z, \Delta^H, \zeta, b, \Delta^L_{31}$ such that the X-charges of Table 2 are $SU(5)$ invariant. This should be obvious, since after symmetry breaking the trilinear GUT superpotential term $\overline{\mathbf{5}}\,\overline{\mathbf{5}}\,\mathbf{10}$ produces $LL\overline{E}$, $LQ\overline{D}$ (both B_3 conserving), and $\overline{U}\overline{D}\overline{D}$ (B_3 violating). $SU(5)$ invariance requires $y = 1$ and $z = \Delta^L_{21} = \Delta^L_{31} = 0$. However, the latter is not compatible with the second condition in Eq. (4.2) with $\zeta \in \mathbb{Z}$. For a review of models where horizontal symmetries are combined with unification see Ref. [41].

5 The Neutrino Sector

5.1 Experimental Results

We now include the experimental constraints from the neutrino sector, in particular from the solar [42, 43], atmospheric [44], reactor [45], and accelerator [46] neutrino experiments.[20] We first need to translate the data into a form, such that we can compare it to our FN-models. Then we can use this to further constrain the X-charges.

[19] The corresponding conditions for conserved M_p are

$$\Delta^H \equiv X_{L^1} - X_{L^0} - \tfrac{1}{2} \in \mathbb{Z}, \qquad \zeta \equiv \frac{1}{3}(\Delta^L_{21} + \Delta^L_{31} - z) \in \mathbb{Z}.$$

[20] We do not include the result of the LSND experiment [47].

$$
\begin{array}{rcl}
X_{H^D} &=& \frac{1}{5\ (6+x+z)}\Big(6y + x\ (2x+12+z-2\Delta^H) \\
&& -z\ (4+3\Delta^H) - 2\ (3+6\Delta^H-\Delta^L_{31}) - \frac{2}{3}\ (6+x+z)(3\zeta+b)\Big) \\
X_{H^U} &=& -z - \ X_{H^D} \\
X_{Q^1} &=& \frac{1}{3}\Big(10 - X_{H^D} + x + 2y + z - \Delta^H - \frac{1}{3}(3\zeta+b)\Big) \\
X_{Q^2} &=& X_{Q^1} - 1 - y \\
X_{Q^3} &=& X_{Q^1} - 3 - y \\
X_{\overline{U^1}} &=& X_{H^D} - X_{Q^1} + 8 + z \\
X_{\overline{U^2}} &=& X_{\overline{U^1}} - 3 + y \\
X_{\overline{U^3}} &=& X_{\overline{U^1}} - 5 + y \\
X_{\overline{D^1}} &=& -X_{H^D} - X_{Q^1} + 4 + x \\
X_{\overline{D^2}} &=& X_{\overline{D^1}} - 1 + y \\
X_{\overline{D^3}} &=& X_{\overline{D^1}} - 1 + y \\
X_{L^1} &=& X_{H^D} + \Delta^H \\
X_{L^2} &=& X_{L^1} - \Delta^L_{31} + z + (3\zeta+b) \\
X_{L^3} &=& X_{L^1} + \Delta^L_{31} \\
X_{\overline{E^1}} &=& -X_{H^D} + 4 - X_{L^1} + x + z \\
X_{\overline{E^2}} &=& X_{\overline{E^1}} - 2 - 2z + \Delta^L_{31} - (3\zeta+b) \\
X_{\overline{E^3}} &=& X_{\overline{E^1}} - 4 - \ \ z - \Delta^L_{31}
\end{array}
$$

Table 2: The constrained X-charges with an acceptable low-energy phenomenology of quark and charged lepton masses and quark mixing. In addition, the GS anomaly cancellation conditions are satisfied and conservation of B_3 is imposed. x, y, z and b are integers specified in Eqs. (2.31) and (4.1). Δ^H, Δ^L_{31}, and ζ are integers as well but as yet unconstrained. $SU(5)$ invariance would require $y = 1$ and $z = \Delta^L_{21} = \Delta^L_{31} = 0$, but the latter is not compatible with the second condition in Eq. (4.2).

In our B_3 conserving model, there are no right-handed neutrinos. Hence in our phenomenological analysis of the data we only consider Majorana mass terms for the left-handed neutrinos with a symmetric mass matrix $\boldsymbol{M}^{(\boldsymbol{\nu})}$ in the current eigenstate basis

$(\nu_e, \nu_\mu, \nu_\tau)$. The Takagi factorization of the matrix $\boldsymbol{M^{(\nu)}}$ is given by

$$\boldsymbol{M}^{(\boldsymbol{\nu})}_{\mathbf{diag}} \;=\; \boldsymbol{U}^{(\boldsymbol{\nu})^*} \cdot \boldsymbol{M}^{(\boldsymbol{\nu})} \cdot \boldsymbol{U}^{(\boldsymbol{\nu})^\dagger}, \tag{5.1}$$

where $\boldsymbol{U}^{(\boldsymbol{\nu})}$ is a unitary matrix and $\boldsymbol{M}^{(\boldsymbol{\nu})}_{\mathbf{diag}} = \mathbf{diag}(m_1, m_2, m_3)$. The neutrino masses m_1, m_2, m_3 are the singular values of $\boldsymbol{M}^{(\boldsymbol{\nu})}$ (not the eigenvalues). They are most easily computed as the positive square roots of the eigenvalues of $\boldsymbol{M}^{(\boldsymbol{\nu})^\dagger}\boldsymbol{M}^{(\boldsymbol{\nu})}$. At this stage, we choose not to make any statement on the relative size of the three masses. The corresponding singular vectors are denoted (ν_1, ν_2, ν_3).

In order to determine the connection between the structure of the original mass matrix $\boldsymbol{M}^{(\boldsymbol{\nu})}$ and the ordering of the masses, we will need to fix the lepton mixing matrix. Experimentally, we have access to the Maki-Nakagawa-Sakata (MNS) matrix [48], which is the product of the left-handed charged lepton mixing matrix $\boldsymbol{U}^{(\boldsymbol{E_L})}$ and $\boldsymbol{U}^{(\boldsymbol{\nu})^\dagger}$. As we are working in the basis with a diagonal charged lepton mass matrix, we have $\boldsymbol{U}^{(\boldsymbol{E_L})} = \mathbb{1}_3$ and thus

$$\boldsymbol{U}^{\mathbf{MNS}} \;\equiv\; \boldsymbol{U}^{(\boldsymbol{E_L})} \cdot \boldsymbol{U}^{(\boldsymbol{\nu})^\dagger} = \boldsymbol{U}^{(\boldsymbol{\nu})^\dagger}. \tag{5.2}$$

In the standard parameterization [49], $\boldsymbol{U}^{\mathbf{MNS}}$ is given by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{\mathrm{i}\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{-\mathrm{i}\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{-\mathrm{i}\alpha_1/2} & 0 & 0 \\ 0 & e^{-\mathrm{i}\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5.3}$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. Here $i, j = 1, 2, 3$ are generation indices and δ and α_1, α_2 are the CP-violating Dirac and Majorana phases, respectively. A global three-generation neutrino oscillation fit assuming CP conservation (*i.e.* $\delta = \alpha_1 = \alpha_2 = 0$) yields the 3σ CL allowed ranges [50, 51]

$$\begin{aligned} \Delta m^2_{21} \;\equiv\; m_2^2 - m_1^2 \;&= 8.2^{+1.1}_{-0.9} \times 10^{-5}\,\mathrm{eV}^2, \qquad && \tan^2\theta_{12} = 0.39^{+0.21}_{-0.11}\,, \\ |\Delta m^2_{32}| \equiv |m_3^2 - m_2^2| &= 2.2^{+1.4}_{-0.6} \times 10^{-3}\,\mathrm{eV}^2, && \tan^2\theta_{23} = \;1.0\,^{+1.1}_{-0.5}\,, \\ & && \sin^2\,\theta_{13} \leq 0.041\,. \end{aligned} \tag{5.4}$$

The neutrino mass eigenstates are conventionally labeled such that the solar neutrino problem is predominantly solved by $\nu_1 \leftrightarrow \nu_2$ oscillations and the atmospheric neutrino problem by $\nu_2 \leftrightarrow \nu_3$ oscillations. Furthermore ν_1 is defined as the neutrino which is predominantly ν_e. In this convention the sign of Δm^2_{12} is known to be positive from the solar neutrino data [43], whereas the sign of Δm^2_{23} is unknown. There are then two possible orderings of the masses [52], either $m_1 < m_2 < m_3$ or $m_3 < m_1 < m_2$. Taking into account the magnitudes of Δm_{21}^2 and Δm_{32}^2 we have the following possible solutions

$$
\begin{aligned}
& m_1 < m_2 \ll m_3, && \text{hierarchical,} \\
& m_3 \ll m_1 < m_2, && \text{inverted hierarchical,} \\
\sqrt{|m_i^2 - m_j^2|} \ll \ & m_1 \approx m_2 \approx m_3, && \text{degenerate,}
\end{aligned}
\tag{5.5}
$$

where in the degenerate case $i, j = 1, 2, 3$ can take on all values and we again have the two possible mass orderings. We discuss these solutions and their individual implications in the context of our FN scenario in Sects. 5.3 and 5.4.

The mixing angles together with their uncertainties can be translated [51] into allowed ranges for the entries of the MNS matrix in terms of the FN-parameter ϵ. For the mixing angles θ_{12} and θ_{23}, we assume Gaussian errors in their measured values.[21] Furthermore, assuming flat distributions for the unmeasured quantities $\theta_{13} \in [0^\circ, 11.7^\circ]$ and the Dirac phase $\delta \in [0, 2\pi]$, we calculate the scatter of the absolute values of the MNS matrix elements. Figure 1 shows the powers in $\epsilon = 0.2$ of the $(1,2)$-element for an ensemble of 3000 sets of mixing parameters obeying the upper assumed statistics. From this we deduce an FN ϵ-structure (by definition the exponents must be integer) of ϵ^0 or ϵ^1 for the (1,2)-element.

We employ a similar analysis of the other matrix elements. Due to the unknown $\mathcal{O}(1)$ coefficients in FN models, we allow all (integer) powers in ϵ within about ± 1 of the center of the scattering region. We then obtain the experimentally acceptable ϵ-structure for the MNS matrix

$$
\boldsymbol{U}_{\mathbf{exp}}^{\mathbf{MNS}} \sim \begin{pmatrix} \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1,2,\ldots} \\ \epsilon^{0,1,2} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1,2} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}, \tag{5.6}
$$

where multiple possibilities for the exponents are separated by commas. The dots in the (1,3)-entry of Eq. (5.6) indicate that arbitrarily high integer exponents are experimentally allowed. Requiring an FN structure in the neutrino mass matrix however excludes values beyond 2. It should be mentioned again that this calculation is done for $\epsilon = 0.2$. Varying ϵ within the interval $[0.18, 0.22]$ does not alter the allowed exponents in Eq. (5.6).

[21] Disregarding systematic effects, measured quantities follow a Gaussian distribution. Derived quantities such as the mixing angles θ_{ij} might however show a distorted statistical spread. Taking the central values of $\tan^2\theta_{ij}$ plus their 3σ CL limits and translating these into corresponding angles θ_{ij}, we found approximately symmetrical distributions for the mixing angles. Thus we are led to our simplifying assumption of Gaussian errors.

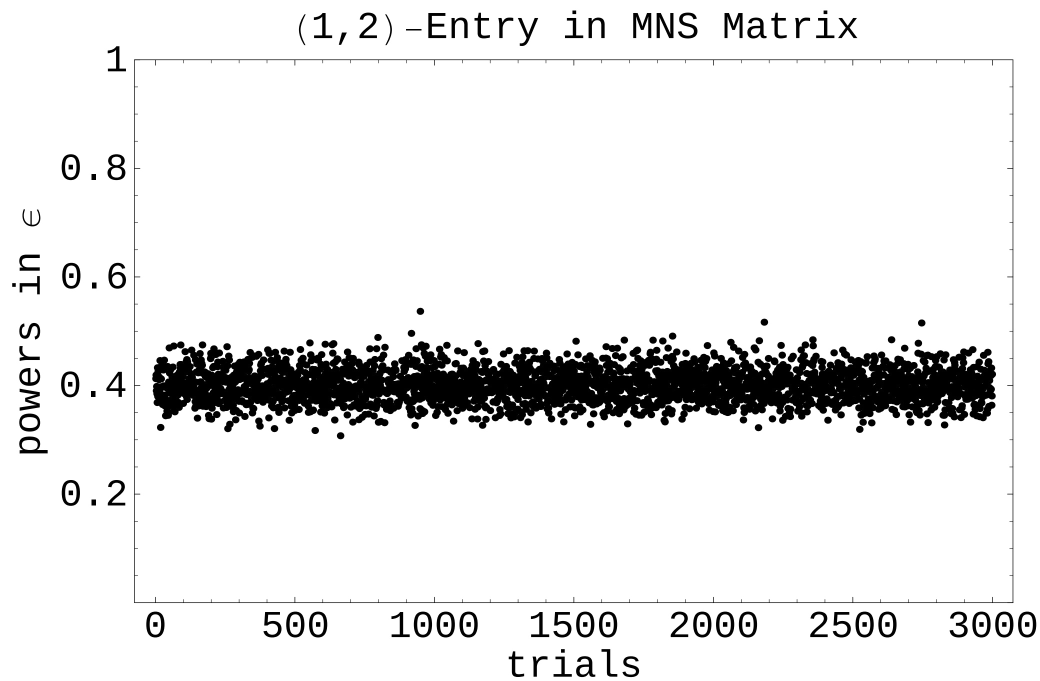

Figure 1: The powers in ϵ of the $(1,2)$-element for an ensemble of 3000 sets of mixing parameters obeying Gaussian statistics for θ_{12} and θ_{23}, whereas $\theta_{13} \in [0°, 11.7°]$ and $\delta \in [0, 2\pi]$ are taken from an equal distribution.

5.2 The Neutrino Mass Matrix

In order to make use of the experimental information about the neutrino sector, we need to specify the origin of the neutrino masses. It has already been pointed out that B_3 invariance allows for lepton-number violating $\not{M}_p$ interactions. Due to the bilinear terms $\mu_i L^i H^U$ the neutrinos mix with the neutralinos, which leads to *one* massive neutrino at tree level[22] [53]. The inferred measured mass squared differences Δm^2_{21} and Δm^2_{31} require at least *two* massive neutrinos. Therfore we must consider higher order contributions to the neutrino mass matrix. In the following, we concentrate on the effects of quark-squark and charged lepton-slepton loop corrections [23, 54, 55] due to the operators $LQ\overline{D}$ and $LL\overline{E}$, respectively. (For a thorough analysis of all one-loop contributions to the neutrino mass matrix in the lepton-number violating but B_3 conserving MSSM see Ref. [56].) The resulting effective neutrino mass matrix in the flavor basis is given by

$$\boldsymbol{M}^{(\nu)} \;=\; \boldsymbol{M}^{(\nu)}_{\textbf{tree}} + \boldsymbol{M}^{(\nu)}_{\boldsymbol{\lambda}'\textbf{-loop}} + \boldsymbol{M}^{(\nu)}_{\boldsymbol{\lambda}\textbf{-loop}}\,, \tag{5.7}$$

[22] Strictly speaking, the distinction between neutrino and neutralino mass eigenstates is no longer appropriate. However, due to stringent experimental constraints on the neutrino masses: $\mu_i/M_W \ll 1$ and the mixing between neutralinos and neutrinos is small [37].

with

$$\left(\boldsymbol{M}^{(\boldsymbol{\nu})}_{\textbf{tree}}\right)_{ij} = \frac{m_Z^2\ M_{\tilde{\gamma}}\ \mu_0\ \cos^2\beta}{m_Z^2\ M_{\tilde{\gamma}}\ \sin 2\beta\ -\ M_1\ M_2\ \mu_0} \cdot \frac{\mu_i\ \mu_j}{\mu_0^2}, \tag{5.8}$$

$$\left(\boldsymbol{M}^{(\boldsymbol{\nu})}_{\boldsymbol{\lambda'}\textbf{-loop}}\right)_{ij} \simeq \frac{3}{32\pi^2}\sum_{k,n}(\lambda'_{ikn}\ \lambda'_{jnk} + \lambda'_{jkn}\ \lambda'_{ink})\ m_k^d \sin 2\phi_n^{(\tilde{d})}\ \ln\left[\left(\frac{m^{\tilde{d}}_{n_1}}{m^{\tilde{d}}_{n_2}}\right)^2\right], \tag{5.9}$$

$$\left(\boldsymbol{M}^{(\boldsymbol{\nu})}_{\boldsymbol{\lambda}\textbf{-loop}}\right)_{ij} \simeq \frac{1}{32\pi^2}\sum_{k,n}(\ \lambda_{ikn}\ \lambda_{jnk}\ +\ \lambda_{jkn}\ \lambda_{ink}\ )\ m_k^e \sin 2\phi_n^{(\tilde{e})}\ \ln\left[\left(\frac{m^{\tilde{e}}_{n_1}}{m^{\tilde{e}}_{n_2}}\right)^2\right]. \tag{5.10}$$

Here m_Z is the Z-boson mass and $m_k^{d/e}$ denote the masses of the down-type quarks/charged leptons of generation $k = 1, 2, 3$. M_1 and M_2 are the soft supersymmetry breaking gaugino mass parameters, which, together with the weak mixing angle θ_W, define the photino mass parameter $M_{\tilde{\gamma}} = M_1 \cos^2\theta_W + M_2 \sin^2\theta_W$. In addition, we have the down-type squark/charged slepton masses $m^{\tilde{d}/\tilde{e}}_{n_i}$ of generation n. $i = 1, 2$ labels the two sfermion mass eigenstates in each generation n. The $\phi_n^{(\tilde{d},\tilde{e})}$ are the mixing angles in the sfermion sector for generation n. Explicitly (no summation over repeated indices),

$$\tan 2\phi_n^{(\tilde{d})} = \frac{2m_n^d\Big|[\boldsymbol{A_D}]_{0nn} - \mu_0^*\tan\beta\Big|}{[\boldsymbol{M}^{\boldsymbol{2}}_{\widetilde{\boldsymbol{Q}}}]_{nn} - [\boldsymbol{M}^{\boldsymbol{2}}_{\widetilde{\boldsymbol{D}}}]_{nn} - \frac{1}{24}(g_Y^2 - 3g_W^2)(v_u^2 - v_d^2)}, \tag{5.11}$$

$$\tan 2\phi_n^{(\tilde{e})} = \frac{2m_n^e\Big|[\boldsymbol{A_E}]_{0nn} - \mu_0^*\tan\beta\Big|}{[\boldsymbol{M}^{\boldsymbol{2}}_{\widetilde{\boldsymbol{L}}}]_{nn} - [\boldsymbol{M}^{\boldsymbol{2}}_{\widetilde{\boldsymbol{E}}}]_{nn} - \frac{1}{8}(3g_Y^2 - g_W^2)(v_u^2 - v_d^2)}, \tag{5.12}$$

where the $\boldsymbol{A_{D,E}}$ are the coefficients of the soft supersymmetry breaking trilinear scalar interactions $[\boldsymbol{A_D}]_{\alpha jk}\ \lambda'_{\alpha jk}\ \widetilde{L^\alpha}\widetilde{Q^j}\widetilde{D^k}^*$ and $\frac{1}{2}[\boldsymbol{A_E}]_{\alpha\beta k}\ \lambda_{\alpha\beta k}\ \widetilde{L^\alpha}\widetilde{L^\beta}\widetilde{E^k}^*$. Here $\widetilde{L^\alpha}$ refers to the scalar component of the chiral superfield L^α. The $[\boldsymbol{M}^{\boldsymbol{2}}_{\widetilde{\dots}}]_{ij}$ are the soft scalar masses squared. g_Y, g_W are the $U(1)_Y$ and $SU(2)_W$ gauge couplings, respectively.

Assuming all soft supersymmetry breaking mass parameters are $\mathcal{O}(m_{3/2}) > 100\,\text{GeV}$, and excluding accidental cancellations, the denominators of Eqs. (5.11) and (5.12) are of order $m_{3/2}^2$. For the numerators we get $2\,m_n^{d/e} m_{3/2} \cdot \mathcal{O}(1 + \epsilon\tan\beta)$. Taking into account the lower limit for $m_{3/2}$ of about $500\,\text{GeV}$, which originates from the combination of the experimental lower bound on $\mu_0 \geq 100\,\text{GeV}$ and its ϵ-structure $\mu_0 \sim m_{3/2}\cdot\epsilon$ in our model (see also App. C), we conclude that even for large $\tan\beta \lesssim 50$ the left-right mixing in (one generation of) the down squark and charged slepton sectors is small. Thus the sines in Eqs. (5.9) and (5.10) can be approximated by tangents. Furthermore, the logarithms become $\mathcal{O}(1)$ coefficients if the sfermion masses are non-degenerate but not too different either, *i.e.* $\mathcal{O}(1) \lesssim \Big([m^{\tilde{d}/\tilde{e}}_{n_i}]^2 - [m^{\tilde{d}/\tilde{e}}_{n_j}]^2\Big)\Big/[m^{\tilde{d}/\tilde{e}}_{n_j}]^2 \lesssim \mathcal{O}(10)$, where $[m^{\tilde{d}/\tilde{e}}_{n_i}] > [m^{\tilde{d}/\tilde{e}}_{n_j}]$. (Once again, i and j label the two mass eigenstates of a particular generation n.) We consider these assumptions natural and apply the corresponding simplifications to Eqs. (5.9) and (5.10). Inserting the $\not{M}_p$ parameters of the last line in Table 1, using the phenomenological constraints of Table 2, and keeping only leading terms, we obtain the FN structure of the tree and the loop contributions to the neutrino mass matrix

$$
\begin{aligned}
M^{(\nu)}_{\text{tree}\,ij} &\sim \frac{m_Z^2\ M_{\tilde{\gamma}}\ \mu_0\ \cos^2\beta}{m_Z^2\ M_{\tilde{\gamma}}\ \sin 2\beta\ -\ M_1\ M_2\ \mu_0} \cdot \epsilon^{-2\Delta^H-\Delta^L_{i1}-\Delta^L_{j1}}, \qquad (5.13)\\
M^{(\nu)}_{\lambda'\text{-loop}\,ij} &\sim \frac{3}{8\pi^2}\frac{m_b^2\left|[\boldsymbol{A_D}]_{033}-\mu_0^*\tan\beta\right|\epsilon^{2x}}{[\boldsymbol{M^2_{\tilde{Q}}}]_{33}-[\boldsymbol{M^2_{\tilde{D}}}]_{33}-\frac{1}{24}(g_Y^2-3g_W^2)(\upsilon_u^2-\upsilon_d^2)} \cdot \epsilon^{-2\Delta^H-\Delta^L_{i1}-\Delta^L_{j1}},\\
M^{(\nu)}_{\lambda\text{-loop}\,ij} &\sim \frac{1}{8\pi^2}\frac{m_\tau^2\left|[\boldsymbol{A_E}]_{033}-\mu_0^*\tan\beta\right|\epsilon^{2x}}{[\boldsymbol{M^2_{\tilde{L}}}]_{33}-[\boldsymbol{M^2_{\tilde{E}}}]_{33}-\frac{1}{8}(3g_Y^2-g_W^2)(\upsilon_u^2-\upsilon_d^2)} \cdot \epsilon^{-2\Delta^H-\Delta^L_{i1}-\Delta^L_{j1}} \cdot f_{ij}\,.
\end{aligned}
$$

Here we have replaced m_3^d by m_b and m_3^e by m_τ. The factors $f_{ij}=f_{ji}$ in the last term take care of λ_{ikn}'s direct dependence on the charged lepton mass matrix and its antisymmetry under interchange of the first two indices. Depending on i and j the tau-stau loop may be forbidden by symmetry and thus does not give the leading contribution. For $i,j=1,2$ we find $f_{ij}\sim 1$, whereas $f_{23}\sim\epsilon^4$ and $f_{13}\sim f_{33}\sim\epsilon^8$. See App. D for details.

Some remarks are in order at this point. Compared to the quark-squark loop, the charged lepton-slepton loop does not contribute significantly to the neutrino mass matrix. Therefore we neglect it in our following discussion. There is a further source of neutrino masses: The non-renormalizable but B_3 conserving superpotential term $L^iH^UL^jH^U$. In our model, this effective term is generated via the GM/KN mechanism and thus suppressed by a factor of $\frac{m_{3/2}}{M^2_{\text{grav}}}\epsilon^{2z-2\Delta^H-\Delta^L_{i1}-\Delta^L_{j1}}$. Inserting the Higgs VEV υ_u for H^U we find that the resulting neutrino mass scale is negligibly small compared to the tree level contribution in Eq. (5.8). The ratio of the two is of the order $\frac{m^2_{3/2}}{M^2_{\text{grav}}}(1+\tan^2\beta)$. So even for large $\tan\beta$ it can be safely discarded. Similarly, we find that the quark-squark loop contribution of Eq. (5.9) is significantly larger than the mass scale of the non-renormalizable operators $L^iH^UL^jH^U$.

5.3 Constraints from Neutrino Masses

In our model, we obtain one massive neutrino at tree level. A second non-zero mass is supplied by the quark-squark loop. Notice that except for an overall relative factor the ϵ-structure of the tree-level and one-loop matrices is exactly the same. However, they are not aligned in the sense that one matrix is a (real or complex) multiple of the other. The μ_i and the λ'_{i33} have a completely different origin, *i.e.* the $\mathcal{O}(1)$ coefficients are in general different. Adding the two terms, we therefore expect not one but two non-zero masses. One neutrino remains massless since $\boldsymbol{M}^{(\boldsymbol{\nu})}_{\textbf{tree}}$ and $\boldsymbol{M}^{(\boldsymbol{\nu})}_{\boldsymbol{\lambda}'\textbf{-loop}}$ are both rank one matrices.[23] Hence a degenerate neutrino scenario is excluded. Notice that this remains true even if we include the charged lepton-slepton loop contribution: The resulting third non-zero mass is smaller by a factor of $\frac{m_\tau^2}{3m_b^2} \approx \frac{1}{15}$ compared to the quark-squark loop mass. This is inconsistent with the degenerate neutrino mass solution of Eq. 5.5.

In order to see whether our model is compatible with the hierarchical or the inverse-hierarchical neutrino solutions of Eq. 5.5, we calculate the relative factor $\frac{m^{\text{tree}}}{m^{\text{loop}}}$ between the overall scales of the tree and the loop mass matrix. This factor must not come out larger than the experimental ratio of the atmospheric and solar neutrino mass scales, which is approximately 5. First, we do a rough estimate for $\tan\beta \lesssim 2$, that is $x = 2, 3$ (thus $\cos\beta \gtrsim 0.5$), where we assume all soft breaking parameters, even the gaugino masses M_1 and M_2, to be of the same order $\mathcal{O}(m_{3/2})$. Neglecting the first term in the denominator of the tree level as well as the second term in the numerator of the loop level overall mass scale, we arrive at $\frac{m^{\text{tree}}}{m^{\text{loop}}} \sim \frac{8\pi^2}{3} \cos^2\beta \, \frac{m_Z^2}{m_b^2} \, \epsilon^{-2x}$. This is much too large for $x \geq 2$, so we are restricted to the cases with $x = 0, 1$, *i.e.* $\tan\beta \gtrsim 8$. We can then approximate $\cos\beta$ by $\cot\beta \sim \epsilon^{-x} \frac{m_b}{m_t}$. Neglecting again the first term in the denominator of the tree level overall mass scale, we get

$$\frac{m^{\text{tree}}}{m^{\text{loop}}} \;\sim\; \epsilon^{-4x} \; \frac{8\pi^2 \; m_Z^2}{3m_t^2} \cdot \frac{M_{\tilde{\gamma}}}{M_1 \; M_2} \cdot \frac{[\boldsymbol{M}^{\boldsymbol{2}}_{\boldsymbol{\tilde{Q}}}]_{33} - [\boldsymbol{M}^{\boldsymbol{2}}_{\boldsymbol{\tilde{D}}}]_{33} - \frac{1}{24}(g_Y^2 - 3g_W^2)(\upsilon_u^2 - \upsilon_d^2)}{\left| [\boldsymbol{A_D}]_{033} - \mu_0^* \;\; \epsilon^x \;\; \frac{m_t}{m_b} \right|} \,. \tag{5.14}$$

Note that we have replaced $\tan\beta$ in the denominator of the last factor. The second factor, $\frac{8\pi^2 \; m_Z^2}{3m_t^2} \approx 7$. Taking $x = 1$ requires the product of the last two factors to yield a tiny fraction of their natural value of about 1. Such a scenario, where there is either fine tuning in the scalar masses or the gaugino masses are about 1000 times larger than the scalar masses is very unnatural. We therefore reject this case and focus on $x = 0$. This together with $z = 1$ numerically determines the expansion parameter $\epsilon \equiv \frac{\langle A \rangle}{M_{\text{grav}}} = 0.186$, see Ref. [7].[24] Notice that with $x = 0$ it seems reasonable to assume that the denominator of the last term in Eq. (5.14) is now dominated by the second term. Taking the gaugino masses at a common scale $M_{1/2}$, the scalar mass parameters all of $\mathcal{O}(m_{3/2})$, we can

[23] For the loop contribution this statement relies on the fact that the $\not{M}_p$ coupling constants are generated via the canonicalization of the Kähler potential and thus proportional to the down-type quark mass matrix, *cf.* Eq. (3.14).

[24] The highest exponent of this expansion parameter occurs in the up-type quark mass matrix. Since $\left(\frac{0.186}{0.22}\right)^8 \approx \frac{1}{4}$, it is unproblematic to work with $\epsilon = 0.186$.

simplify Eq. (5.14)

$$\frac{m^{\rm tree}}{m^{\rm loop}} \;\sim\; \frac{8\pi^2 m_Z^2 m_b}{3m_t^3}\cdot\epsilon^{-z}\cdot\frac{m_{3/2}}{M_{1/2}} \;\sim\; \mathcal{O}(1)\,\frac{m_{3/2}}{M_{1/2}}\,. \tag{5.15}$$

Thus if we choose supersymmetric parameter points for which the scalar quark masses are bigger than the gaugino mass parameters by factors of about two to five ($m_{3/2} \approx 5M_{1/2}$) [57, 58], we can accommodate a hierarchical neutrino mass scenario. The tree level contribution then provides for one relatively heavy neutrino while the second neutrino remains light. On the other hand, an inverse hierarchy is possible just as well. Then the tree and the quark-squark loop mass matrices must have the same order of magnitude, thus generating two relatively heavy neutrinos while the third neutrino remains light. Due to our ignorance of the soft breaking sector and the arbitrariness of all $\mathcal{O}(1)$ coefficients, our B_3-conserving FN models allow both, the hierarchical and the inverse-hierarchical neutrino scenario.

In both cases however, the mass of the heaviest neutrino is given by the atmospheric neutrino mass scale $\sqrt{|\Delta m_{32}^2|}$. Thus the integer parameter Δ^H can be determined. Equating the eigenvalue of the tree level neutrino mass matrix, which is proportional to $\sum_i \frac{\mu_i^2}{\mu_0^2}$, with $\sqrt{|\Delta m_{32}^2|}$ and putting $M_{1/2} = \mathcal{O}(m_{3/2})$ yields

$$-\,2\Delta^H \;\sim\; \frac{1}{\ln\epsilon}\cdot\ln\frac{m_t^2\, m_{3/2}\sqrt{|\Delta m_{32}^2|}}{m_b^2\, m_Z^2}\,. \tag{5.16}$$

Here we made use of the ordering $X_{L^3} \leq X_{L^2} \leq X_{L^1}$, so that $\sum_i \epsilon^{-2\Delta_{i1}^L} \sim 1$. Inserting $\epsilon = 0.186$, $m_t = 175\,\text{GeV}$, $m_b = 4.2\,\text{GeV}$, $m_Z = 91.2\,\text{GeV}$, $\sqrt{|\Delta m_{32}^2|} = 0.047\,\text{eV}$, and $1000\,\text{GeV} \geq m_{3/2} \geq 100\,\text{GeV}$ we obtain

$$-\,2\Delta^H \;\in\; [\,11.0\,,\,12.3\,]\,. \tag{5.17}$$

Here the lower bound corresponds to $m_{3/2} = 1000\,\text{GeV}$ and the upper one to $m_{3/2} = 100\,\text{GeV}$. Since Δ^H is integer, we end up with the single option

$$\Delta^H \;=\; -\,6\,. \tag{5.18}$$

At the end of App. C, we argue that the sequence of basis transformations in Sect. 3 generates $\not{M}_p$ coupling constants which are to some extent larger than expected. Taking this feature into account, the interval in Eq. (5.17) is shifted slightly to higher values. For $\mu_i \sim \epsilon^{-0.5}\cdot m_{3/2}\,\epsilon^{-X_{L^i}-X_{H^U}}$, where the first factor quantifies such a systematic effect, this shift is about one unit. So the solution given in Eq. (5.18) remains stable.

5.4 Constraints from Neutrino Mixing

We now turn to the conditions on the X-charges imposed by the MNS matrix. The effective neutrino mass matrix of Eq. (5.7) is diagonalized by the unitary transformation ${U^{(\nu')}}_{ij} \sim \epsilon^{|X_{L^i}-X_{L^j}|}$. This transforms the current eigenstate basis into the mass eigenstate basis (ν_1', ν_2', ν_3') of $\boldsymbol{M}^{(\boldsymbol{\nu})\dagger}\boldsymbol{M}^{(\boldsymbol{\nu})}$. In the latter basis we denote the diagonal entries of the mass matrix as (m_1', m_2', m_3'), with relative values $m_3' \ll m_2' \lesssim m_1'$ (for details see App. E). It is important to note that $\boldsymbol{U}^{(\boldsymbol{\nu}')}$ is different from $\boldsymbol{U}^{(\boldsymbol{\nu})}$ defined in Eq. (5.1). In order to compare with the possible solutions in Eq. (5.5) and the data of Eq. (5.4) it is more convenient to reorder the basis (ν_1', ν_2', ν_3') into a new basis (ν_1, ν_2, ν_3), with the corresponding masses (m_1, m_2, m_3) in the order of the hierarchical or inverted hierarchical solution. We can then fix the mixing angles so that (ν_1, ν_2) solve the solar neutrino problem. We summarize the bases choices in the following table

Mass Ordering		Hierarchy	Inverse Hierarchy
Heaviest	ν_1', m_1'	ν_3, m_3	ν_2, m_2
Medium	ν_2', m_2'	ν_2, m_2	ν_1, m_1
Lightest	ν_3', m_3'	ν_1, m_1	ν_3, m_3

Table 3: Options for the mass ordering of the neutrinos.

For the hierarchical scenario, m_1 must be the lightest and m_3 the heaviest neutrino mass. We must therefore exchange the first and third states in the primed basis to obtain the relevant unprimed basis. The new diagonalization matrix is then given by

$$\boldsymbol{U}^{(\boldsymbol{\nu},\,\mathbf{h.})} \equiv \boldsymbol{T}^{(\mathbf{h.})} \cdot \boldsymbol{U}^{(\boldsymbol{\nu}')}, \tag{5.19}$$

where

$$\boldsymbol{T}^{(\mathbf{h.})} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{5.20}$$

Here the superscript **h.** refers to the hierarchical solution. Combining Eqs. (5.2), (5.6), (5.19), and (5.20) we get

$$\begin{aligned} \epsilon^{|L^i-L^j|} \sim \left[\boldsymbol{U}^{(\boldsymbol{\nu}')}\right]_{ij} &= \left[\boldsymbol{T}^{(\mathbf{h.})} \cdot \boldsymbol{U}^{(\boldsymbol{\nu},\,\mathbf{h.})}\right]_{ij} \qquad (5.21) \\ &= \left[\boldsymbol{T}^{(\mathbf{h.})} \cdot \boldsymbol{U}^{\mathbf{MNS}^\dagger}\right]_{ij} \sim \begin{pmatrix} \epsilon^{0,1,2,\ldots} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1,2} & \epsilon^{0,1,2} \end{pmatrix}_{ij}, \end{aligned}$$

Δ^L_{31}	Δ^L_{21}	Hierarchy	Inverse Hierarchy	Conservation of B_3
-1	-1	yes	no	no
-1	0	yes	yes	yes
0	0	yes	yes	yes

Table 4: All combinations of Δ^L_{i1} which are compatible with the experimental MNS matrix for the hierarchical and the inverse-hierarchical neutrino scenario. In addition, the condition of B_3 conservation on the X-charges as stated in Eq. (4.2) is checked.

which restricts $\Delta^L_{i1} \equiv X_{L^i} - X_{L^1}$, for $i = 2, 3$. All acceptable combinations which also comply with the ordering $\Delta^L_{31} \leq \Delta^L_{21} \leq 0$ are listed in Table 4. As we impose conservation of B_3, the second condition in Eq. (4.2) has to be satisfied, *i.e.* $\Delta^L_{31} + \Delta^L_{21} - z \neq 0 \bmod 3$. The last column of Table 4 shows which cases are compatible with B_3 conservation for $z = 1$.

In the case of an inverse hierarchy, $m_3 \ll m_1 \lesssim m_2$, we need to interchange the first two states of the primed basis to obtain the relevant unprimed basis. We have

$$\boldsymbol{T}^{(\mathbf{i.h.})} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{5.22}$$

With this we find

$$\begin{aligned} \epsilon^{|L^i - L^j|} \sim \left[\boldsymbol{U}^{(\boldsymbol{\nu}')}\right]_{ij} &= \left[\boldsymbol{T}^{(\mathbf{i.h.})} \cdot \boldsymbol{U}^{(\boldsymbol{\nu},\,\mathbf{i.h.})}\right]_{ij} \\ &= \left[\boldsymbol{T}^{(\mathbf{i.h.})} \cdot \boldsymbol{U}^{\mathbf{MNS}^\dagger}\right]_{ij} \sim \begin{pmatrix} \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1,2} & \epsilon^{0,1,2} \\ \epsilon^{0,1,2,\ldots} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}_{ij} , \end{aligned} \tag{5.23}$$

for the inverse-hierarchical neutrino scenario. Again, the allowed Δ^L_{i1} are given in Table 4. In addition to the constraints arising from the experimental MNS matrix, we now have to ensure that the ratio $\frac{m_1'}{m_2'}$ of the two heavy masses is of order one. As shown in App. E, m_2' is not only determined by the scale of the second largest contribution to the neutrino mass matrix, but it is additionally suppressed by a factor of $\epsilon^{-2\Delta^L_{21}}$, *cf.* Eq. (E.14). For this reason $\Delta^L_{21} = -1$ is forbidden in the case of an inverse hierarchy.

6 Viable X-Charge Assignments

In summary, we have fixed almost all parameters determining the FN charges by imposing conservation of B_3, requiring GS anomaly cancellation and finally taking into account the phenomenological constraints of the low-energy fermionic mass spectrum, including the neutrinos. Starting with Table 2, we need $z = 1$ if the bilinear superpotential terms are to be generated via the Giudice-Masiero/Kim-Nilles mechanism. Then we have $x = 0$ due to the upper limit on $\frac{m^{\text{tree}}}{m^{\text{loop}}}$, which is given by the ratio of the atmospheric and the solar neutrino mass scales. Δ^H is fixed through the absolute neutrino mass scale. As degenerate neutrinos are excluded, this corresponds to the atmospheric mass scale. Hence we find $\Delta^H = -6$. Finally, the constraints coming from the MNS mixing matrix together with the requirement of B_3 conservation yield $\Delta^L_{21} = 0$ and $\Delta^L_{31} = -1, 0$ (see Table 4). So, in the end we are left with only the choice of

$$y = -1, 0, 1 , \qquad \text{and} \qquad 3\zeta + b \equiv \Delta^L_{31} + \Delta^L_{21} - z = \Delta^L_{31} - 1 = -2, -1 . \tag{6.1}$$

This leads to six sets of viable X-charge assignments displayed in Table 5. All sets are compatible with either a hierarchical or an inverse-hierarchical neutrino scenario, depending on the ratio $\frac{m^{\text{tree}}}{m^{\text{loop}}}$ [*cf.* Eq. (5.15)] and unknown $\mathcal{O}(1)$ coefficients in $\boldsymbol{M^{(\nu)}}$. Taking the smallness of the $(1, 3)$-element of the MNS matrix in Eq. (5.6) [corresponding to the $(1, 1)$-entry of Eq. (5.21) in the hierarchical, and the $(3, 1)$-entry of Eq. (5.23) in the inverse-hierarchical case] as a crucial criterion, we prefer the inverse-hierarchical cases with $\Delta^L_{31} = -1$. It is only there, that the FN prediction for this entry is of $\mathcal{O}(\epsilon)$. In all other cases we have to assume an unattractively small "$\mathcal{O}(1)$ coefficient". Remarkably, there exists one set where all FN charges are multiples of one third. This salient charge assignment is obtained for $3\zeta + b = -2$ (or equivalently $\Delta^L_{31} = -1$) and $y = 1$. However, as $y \neq 0$ the CKM matrix is not optimal (*cf.* App. F), but nonetheless acceptable due to the possibility of mildly adjusting the unknown $\mathcal{O}(1)$ coefficients.

All other sets contain highly fractional X-charges (just like the sets in Ref. [7]) and are thus "esthetically disfavored". However, requiring that the FN scenario is in agreement with the very tight experimental bounds on exotic processes usually leads to highly fractional X-charge assignments [7, 8]. Thus the six models presented in this section are so-to-speak in good company; our ignorance of the origin of the $U(1)_X$ gauge symmetry does not allow us to exclude models just because of unpleasant X-charges. Due to our experience with hypercharge, it is natural to hope for "nice", *i.e.* not too fractional, X-charges. But actually, in string models (*e.g.* [59]) the anomalous $U(1)$-charges can very well be highly fractional [60]. It is therefore not clear at all whether one should expect "nice" charges or not.

In the manner of Ref. [8], we checked that the $\not{M}_p$ coupling constants which are produced by the six sets of X-charges are all in agreement with the very tight experimental bounds [38], unless there is an unnatural adding-up among the $\mathcal{O}(1)$ coefficients. In

Input			**Output**							
$\boldsymbol{\Delta^L_{31}}$	$\boldsymbol{3\zeta + b}$	$\boldsymbol{y}$	$\boldsymbol{X_{H^D}}$	$\boldsymbol{X_{H^U}}$	$\boldsymbol{i}$	$\boldsymbol{X_{Q^i}}$	$\boldsymbol{X_{\overline{U^i}}}$	$\boldsymbol{X_{\overline{D^i}}}$	$\boldsymbol{X_{L^i}}$	$\boldsymbol{X_{\overline{E^i}}}$
-1	-2	-1	$\frac{244}{105}$	$-\frac{349}{105}$	1	$\frac{467}{105}$	$\frac{722}{105}$	$-\frac{97}{35}$	$-\frac{386}{105}$	$\frac{667}{105}$
					2	$\frac{467}{105}$	$\frac{302}{105}$	$-\frac{167}{35}$	$-\frac{386}{105}$	$\frac{352}{105}$
					3	$\frac{257}{105}$	$\frac{92}{105}$	$-\frac{167}{35}$	$-\frac{491}{105}$	$\frac{247}{105}$
-1	-2	0	$\frac{262}{105}$	$-\frac{367}{105}$	1	$\frac{177}{35}$	$\frac{676}{105}$	$-\frac{373}{105}$	$-\frac{368}{105}$	$\frac{631}{105}$
					2	$\frac{142}{35}$	$\frac{361}{105}$	$-\frac{478}{105}$	$-\frac{368}{105}$	$\frac{316}{105}$
					3	$\frac{72}{35}$	$\frac{151}{105}$	$-\frac{478}{105}$	$-\frac{473}{105}$	$\frac{211}{105}$
-1	-2	1	$\frac{8}{3}$	$-\frac{11}{3}$	1	$\frac{17}{3}$	6	$-\frac{13}{3}$	$-\frac{10}{3}$	$\frac{17}{3}$
					2	$\frac{11}{3}$	4	$-\frac{13}{3}$	$-\frac{10}{3}$	$\frac{8}{3}$
					3	$\frac{5}{3}$	2	$-\frac{13}{3}$	$-\frac{13}{3}$	$\frac{5}{3}$
0	-1	-1	$\frac{236}{105}$	$-\frac{341}{105}$	1	$\frac{458}{105}$	$\frac{241}{35}$	$-\frac{274}{105}$	$-\frac{394}{105}$	$\frac{683}{105}$
					2	$\frac{458}{105}$	$\frac{101}{35}$	$-\frac{484}{105}$	$-\frac{394}{105}$	$\frac{368}{105}$
					3	$\frac{248}{105}$	$\frac{31}{35}$	$-\frac{484}{105}$	$-\frac{394}{105}$	$\frac{158}{105}$
0	-1	0	$\frac{254}{105}$	$-\frac{359}{105}$	1	$\frac{174}{35}$	$\frac{677}{105}$	$-\frac{356}{105}$	$-\frac{376}{105}$	$\frac{647}{105}$
					2	$\frac{139}{35}$	$\frac{362}{105}$	$-\frac{461}{105}$	$-\frac{376}{105}$	$\frac{332}{105}$
					3	$\frac{69}{35}$	$\frac{152}{105}$	$-\frac{461}{105}$	$-\frac{376}{105}$	$\frac{122}{105}$
0	-1	1	$\frac{272}{105}$	$-\frac{377}{105}$	1	$\frac{586}{105}$	$\frac{631}{105}$	$-\frac{146}{35}$	$-\frac{358}{105}$	$\frac{611}{105}$
					2	$\frac{376}{105}$	$\frac{421}{105}$	$-\frac{146}{35}$	$-\frac{358}{105}$	$\frac{296}{105}$
					3	$\frac{166}{105}$	$\frac{211}{105}$	$-\frac{146}{35}$	$-\frac{358}{105}$	$\frac{86}{105}$

Table 5: All six sets of viable X-charge assignments, where $z = 1$, $x = 0$ (*i.e.* large $\tan\beta$), and $\Delta^H = -6$. The other input parameters of Table 2, namely Δ^L_{31}, $3\zeta + b$, and y, differentiate between the various possible scenarios. All of them are compatible with hierarchical and inverse-hierarchical neutrino masses, depending on the ratio $\frac{m^{\text{tree}}}{m^{\text{loop}}}$ and unknown $\mathcal{O}(1)$ coefficients in $\boldsymbol{M^{(\nu)}}$. The former depends on the parameters of supersymmetry breaking. Here we assume gravity mediation so that all soft breaking mass parameters are of $\mathcal{O}(m_{3/2})$, with $m_{3/2} \in [100\,\text{GeV}, 1000\,\text{GeV}]$. In order to determine the structure of the sneutrino VEVs, we have assumed an FN structure for b_α and $[\boldsymbol{M^2_{\widetilde{L}}}]_{\alpha\beta}$, see App. C.

App. F, we give an explicit example of how the physics at the high energy scale, constrained by the third X-charge assignment in Table 5, boils down to a viable low-energy phenomenology.

Finally, one could raise the question: Is it possible to construct a scenario where no hidden sector fields are needed to cancel the $\mathcal{A}_{CCX}$ and $\mathcal{A}_{GGX}$ anomalies? Explicitly [7]

$$\begin{aligned} \mathcal{A}_{GGX} &= 2X_{H^U} + 2X_{H^D} + \sum_i \Big(6X_{Q^i} + 3X_{\overline{U^i}} + 3X_{\overline{D^i}} + 2X_{L^i} + X_{\overline{E^i}} \Big) \\ &\qquad + X_A + \mathcal{A}_{GGX}^{\text{hidden sector}}, \end{aligned} \tag{6.2}$$

$$\mathcal{A}_{CCX} = \frac{1}{2}\Big[\sum_i \Big(2\ X_{Q^i} + X_{\overline{U^i}} + X_{\overline{D^i}} \Big) \Big]. \tag{6.3}$$

Inserting the relations of Table 2 with $z = 1$ and $x = 0$ yields

$$\mathcal{A}_{GGX} = (3\zeta + b) + 3\Delta^H + 68 + \mathcal{A}_{GGX}^{\text{hidden sector}} \qquad \text{and} \qquad \mathcal{A}_{CCX} = \frac{21}{2}. \tag{6.4}$$

Anomaly cancellation à la Green-Schwarz requires [7] $\frac{\mathcal{A}_{CCX}}{k_C} = \frac{\mathcal{A}_{GGX}}{24}$, where k_C is the *positive integer* Kač-Moody level of $SU(3)_C$. Assuming that the hidden sector fields are uncharged under $U(1)_X$, *i.e.* $\mathcal{A}_{GGX}^{\text{hidden sector}} = 0$, we arrive at the condition

$$\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7}{k_C} = (3\zeta + b) + 3\Delta^H + 68\,. \tag{6.5}$$

As both sides of this equation have to be integer, k_C is restricted to a product of a subset of the primes in the numerator on the left. With $3\zeta + b = -2, -1$ and $\Delta^H = -6$ the right-hand side of Eq. (6.5) is either 48 or 49. This is not attainable with the left-hand side, even if we allow a variation of ± 1 in Δ^H. This shows that $U(1)_X$-charged hidden sector fields are necessary to cancel the gravity-gravity-$U(1)_X$ anomaly.

7 Summary, Conclusion and Outlook

We have constructed a minimalist and compact $U(1)_X$ Froggatt-Nielsen scenario with the MSSM particle content plus one additional flavon field A. Furthermore, our model exhibits only two mass scales, M_{grav} and $m_{3/2}$. A discrete symmetry is needed to ensure a long-lived proton. Without enlarging the (low-energy) fermionic particle content of the MSSM and excluding a GS mechanism, there are only three discrete symmetries [5, 6, 2] which, besides allowing for neutrino masses, can originate from an anomaly-free gauge symmetry and thus do not experience violation by quantum gravity effects. These salient discrete symmetries are M_p, B_3, and P_6. Following the philosophy of [7], where M_p is a remnant of the continuous $U(1)_X$ symmetry, we have examined the case with B_3 being generated by virtue of the X-charge assignment. This $\mathbb{Z}_3$-symmetry has some attractive features: First, it phenomenologically stabilizes the proton. Second, it allows bilinear and trilinear $\not{M}_p$ coupling constants, so that neutrino masses are possible at the renormalizable level without the need to introduce right-handed neutrinos. Imposing the restrictions of the measured fermionic mass spectrum and the GS anomaly cancellation conditions we arrive at six phenomenologically viable sets of X-charges presented in Table 5. All of them feature large $\tan\beta$ ($\gtrsim$ 40). Our ignorance about the details of the soft supersymmetry breaking parameters does not allow us to distinguish between models of normal and inverse neutrino mass hierarchy. However, taking the smallness of ${U^{\mathrm{MNS}}}_{13}$ as a crucial criterion, we should prefer the first three cases (*i.e.* those with $\Delta^L_{31} = -1$) of Table 5 and an inverse hierarchy. Doing so, our model *predicts* inverse-hierarchical neutrino masses. Of all six possibilities, we find the third X-charge assignment of Table 5 the most pleasing: All X-charges are integer multiples of one third in this case. However, the other five models (with more fractional X-charges) are phenomenologically possible just as well.

In constructing viable models of the fermionic mass spectrum we have been guided by the principle of minimality and compactness. With only the $U(1)_X$ symmetry and two mass scales at hand, we had to exclude the choice $z = 0$ right from the beginning as it does not satisfactorily explain the origin of the μ-parameter. However, the quest for a dark matter candidate requires us to introduce at least one additional particle like *e.g.* the axion, which in turn would suggest the existence of a new global $U(1)$ symmetry [61]. Also superstring models often predict more than just one $U(1)$. So it is tempting to assume that the μ-term is originally forbidden by such a symmetry. Effectively it may then be generated via some mechanism (other than GM) at the phenomenologically needed mass scale [62, 63, 64, 65, 66]. In that case, the possibility of $z = 0$ should be considered and investigated seriously.

Allowing for supersymmetric zeros in the leptonic mass matrices could be another direction of further study. In this paper, we have excluded the existence of supersymmetric zeros in all Yukawa mass matrices as we wanted to evade difficulties like those encountered with the CKM matrix. However, maybe they are a blessing for the MNS matrix. It would be interesting to examine if a small θ_{13} can naturally arise from supersymmetric zeros appearing in the charged lepton and/or neutrino mass matrix, see *e.g.* [67].

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Appendix

A Baryon Parity vs. Baryon Triality

In this appendix we would like to emphasize the differences between baryon parity and baryon triality. Baryon parity, B_p, is defined on the MSSM chiral superfields by

$$\begin{aligned} \{H^D, H^U, L^i, \overline{E^i}\} &\longrightarrow \qquad \{H^D, H^U, L^i, \overline{E^i}\}\,, \\ \{Q^i, \overline{U^i}, \overline{D^i}\} &\longrightarrow e^{2\pi \mathrm{i}/2}\, \{Q^i, \overline{U^i}, \overline{D^i}\}\,. \end{aligned} \tag{A.1}$$

The total B_p-charge of an operator R [*cf.* Eq. (2.1)] can then be written as

$$\sum_i n_{Q^i} \;+\; \sum_i n_{\overline{U^i}} \;+\; \sum_i n_{\overline{D^i}} \;=\; 2\,\mathcal{I}'_B + \;\iota'_B\,. \tag{A.2}$$

Here $\mathcal{I}'_B$ is an integer, which can differ for each operator. ι'_B is fixed for all operators and is 0 or 1 if B_p is conserved or broken, respectively. In order to achieve B_p by virtue of the X-charges, we need Eq. (2.9) as well as

$$X_{H^D} - X_{L^1} = \text{ integer}, \qquad \text{and} \qquad 3X_{Q^1} + X_{L^1} = \text{integer} - \frac{b'}{2}\,, \tag{A.3}$$

with $b' = 1$. Plugging this into Eq. (2.10), we get

$$X_{\text{total}} = \text{integer} - \mathcal{C} \cdot \frac{b'}{2}\,. \tag{A.4}$$

In analogy to Eq. (2.18) we have $\iota'_B + \mathcal{C} = 2\left(\sum_i n_{Q^i} - \mathcal{C} - \mathcal{I}'_B\right)$ [which is obtained by adding Eqs. (2.3) and (A.2)], leading to $\mathcal{C} = 2 \cdot \text{integer} - \iota'_B$, and thus the final condition

$$X_{\text{total}} = \text{integer} + \frac{b' \cdot \iota'_B}{2}\,. \tag{A.5}$$

The first main difference between B_p and B_3 is that the former is an anomalous discrete gauge symmetry whereas the latter is anomaly free [5, 6, 2]. The next question we would like to address is what is the lowest dimensional operator of MSSM chiral superfields which is allowed by B_p but forbidden by B_3. It is easy to see that for the renormalizable interactions the two symmetries act identically. This equality persists for operators which are the product of four superfields. For a more systematic approach consider that, since B_p is a $\mathbb{Z}_2$-symmetry, Eq. (A.2) can be recast as

$$\sum_i n_{Q^i} \;-\; \sum_i n_{\overline{U^i}} \;-\; \sum_i n_{\overline{D^i}} \;=\; 2\,\mathcal{I}'_B + \;\iota'_B\,. \tag{A.6}$$

This is to be compared with Eq. (2.3), which leads to

$$3\mathcal{C} \;=\; 2\,\mathcal{I}'_B + \;\iota'_B\,. \tag{A.7}$$

Solving Eqs. (2.3)-(2.5) for $\sum_i n_{Q^i}$, $\sum_i n_{\overline{D^i}}$, and $\sum_i n_{\overline{U^i}}$ and plugging the result into the second line of Eq. (2.17) we get

$$3\mathcal{W} \;+\; 3\sum_i n_{\overline{E^i}} \;-\; 4\,\mathcal{C} \;=\; 3\mathcal{I}_B \;+\; \iota_B\,. \tag{A.8}$$

Eqs. (A.7) and (A.8) imply that B_p is conserved if $\mathcal{C}$ is an integer multiple of two, whereas B_3 is conserved if $4\,\mathcal{C}$ is an integer multiple of three. Thus the smallest value for which baryon parity is conserved and baryon triality is violated is $|\mathcal{C}| = 2$. This in turn implies that the relevant operator contains at least six quark superfields [see Eq. (2.3)]. An example is

$$\varepsilon^{abc}\varepsilon^{def}\,\overline{U^i{}_a D^j{}_b D^k{}_c U^\ell{}_d D^m{}_e D^n{}_f}\,, \tag{A.9}$$

where $a, \ldots, f$ are $SU(3)_C$ indices and $i, \ldots, n$ are generation indices. Such a non-renormalizable operator is highly suppressed so that the effective low-energy phenomenology is identical for B_p and B_3.

B Supersymmetric Zeros in Mass Matrices

The structure of the CKM matrix depends on the overall X-charges of the operators in the quark mass matrices. Due to holomorphy of the superpotential, terms with negative

X-charge are forbidden. These supersymmetric zeros are filled in by the canonicalization of the Kähler potential [17]. Naively, the resulting mass matrices might suggest a CKM matrix consistent with the experimentally measured quark mixing. However, if one allows for supersymmetric zeros then things are more involved since the matrix canonicalizing the kinetic terms of the quark doublet Q affects both, the up- *and* the down-type quark mass matrices. Diagonalizing these, we therefore encounter cancellations in the CKM matrix (which is a product of the two left-handed diagonalization matrices). These cancellations spoil the naively expected nice results. Espinosa and Ibarra [68, 69] have investigated the influence of supersymmetric zeros on the CKM matrix. In the following we illustrate such a situation explicitly for *two* generations of quarks and the trilinear superpotential terms

$$W_3 = \left(\boldsymbol{G}^{(\boldsymbol{D})}_{\mathbf{FN}}\right)_{ij} H^D Q^i \overline{D^j} + \left(\boldsymbol{G}^{(\boldsymbol{U})}_{\mathbf{FN}}\right)_{ij} H^U Q^i \overline{U^j} \,. \tag{B.1}$$

Here the subscript "FN" refers to the fact that the Kähler potential is not canonicalized yet at this point. Now consider an X-charge assignment with

$$\begin{aligned} X_{Q^2} &= X_{Q^1} + 1, \qquad X_{\overline{U^2}} = X_{\overline{U^1}} - 5, \qquad X_{\overline{D^2}} = X_{\overline{D^1}} - 3, \\ X_{H^U} &= 4 - X_{Q^1} - X_{\overline{U^1}}, \qquad X_{H^D} = 2 - X_{Q^1} - X_{\overline{D^1}} \,. \end{aligned} \tag{B.2}$$

Then in the Froggatt-Nielsen scenario the Yukawa couplings come out to be

$$\boldsymbol{G}^{(\boldsymbol{U})}_{\mathbf{FN}} \sim \begin{pmatrix} \epsilon^4 & 0 \\ \epsilon^5 & 1 \end{pmatrix}, \qquad \boldsymbol{G}^{(\boldsymbol{D})}_{\mathbf{FN}} \sim \begin{pmatrix} \epsilon^2 & 0 \\ \epsilon^3 & 1 \end{pmatrix}, \tag{B.3}$$

where the $(1,2)$-elements are supersymmetric zeros. The Kähler potential has to be canonicalized by matrices of the form

$$\boldsymbol{C}^{(\boldsymbol{Q})^{-1}} \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}, \quad \boldsymbol{C}^{(\overline{\boldsymbol{U}})^{-1}} \sim \begin{pmatrix} 1 & \epsilon^5 \\ \epsilon^5 & 1 \end{pmatrix}, \quad \boldsymbol{C}^{(\overline{\boldsymbol{D}})^{-1}} \sim \begin{pmatrix} 1 & \epsilon^3 \\ \epsilon^3 & 1 \end{pmatrix}. \tag{B.4}$$

These transformations change the Yukawa matrices to

$$\boldsymbol{G}^{(\boldsymbol{U})} \sim \begin{pmatrix} \epsilon^4 & \epsilon \\ \epsilon^5 & 1 \end{pmatrix}, \qquad \boldsymbol{G}^{(\boldsymbol{D})} \sim \begin{pmatrix} \epsilon^2 & \epsilon \\ \epsilon^3 & 1 \end{pmatrix}. \tag{B.5}$$

These are diagonalized by unitary matrices with the textures[25]

$$\boldsymbol{U}^{(\boldsymbol{U_L})} \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}, \qquad \boldsymbol{U}^{(\overline{\boldsymbol{U}})} \sim \begin{pmatrix} 1 & \epsilon^5 \\ \epsilon^5 & 1 \end{pmatrix},$$
$$\boldsymbol{U}^{(\boldsymbol{D_L})} \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}, \qquad \boldsymbol{U}^{(\overline{\boldsymbol{D}})} \sim \begin{pmatrix} 1 & \epsilon^3 \\ \epsilon^3 & 1 \end{pmatrix}. \tag{B.6}$$

If we naively neglect possible cancellations between $\boldsymbol{U}^{(\boldsymbol{U_L})}$ and $\boldsymbol{U}^{(\boldsymbol{D_L})}$ we get

$$\boldsymbol{U}^{\mathbf{CKM}} \equiv \boldsymbol{U}^{(\boldsymbol{U_L})} \cdot \boldsymbol{U}^{(\boldsymbol{D_L})\dagger} \ \sim \ \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}, \tag{B.7}$$

which agrees well with the data. However, in order to show that this line of reasoning is too simple, we numerically calculated the CKM matrix for an ensemble of 3000 *Mathematica*© -randomly generated sets of complex $\mathcal{O}(1)$ coefficients which remain undetermined by any FN model and appear in both, the FN-generated Yukawa matrices and the kinetic (Hermitian) Kähler potential terms. Figure 2 shows the powers in $\epsilon = 0.2$ of the off-diagonal element in the CKM matrix. The corresponding quark mass ratios $\frac{m_u}{m_c}$ and $\frac{m_d}{m_s}$ are depicted in Figure 3.

Obviously, with supersymmetric zeros the naive result of Eq. (B.7) is in gross disagreement with the numerically calculated CKM matrix, where instead of ϵ we have $\epsilon^{4,5,\,\text{or}\,6}$. Cancellations between $\boldsymbol{U}^{(\boldsymbol{U_L})}$ and $\boldsymbol{U}^{(\boldsymbol{D_L})}$ render the off-diagonal entries from $\mathcal{O}(\epsilon)$ to $\mathcal{O}(\epsilon^5)$. However, the quark mass ratios come out correct in the naive calculation, *i.e.* $\frac{m_u}{m_c} \sim \epsilon^4$ and $\frac{m_d}{m_s} \sim \epsilon^2$.

C The Structure of the Sneutrino VEVs

In $\not{M}_p$ theories, the five neutral scalars $\tilde{\nu}_\alpha$, h_0^U mix and we have the following minimization conditions for the scalar potential [23]

$$\Big(M^2_{H^U} + \mu^*_\alpha \; \mu_\alpha + \frac{g_W^2 + g_Y^2}{8} \left(|\upsilon_u|^2 - |\upsilon_d|^2 \right) \Big) \; \upsilon_u \; - \; b^*_\alpha \upsilon^*_\alpha \ \stackrel{!}{=} \ 0\,, \tag{C.1}$$

$$\Big([\boldsymbol{M}^{\boldsymbol{2}}_{\tilde{\boldsymbol{L}}}]_{\alpha\beta} + \mu^*_\alpha \; \mu_\beta + \frac{g_W^2 + g_Y^2}{8} \left(|\upsilon_d|^2 - |\upsilon_u|^2 \right) \delta_{\alpha\beta} \Big) \; \upsilon_\beta \; - b^*_\alpha \upsilon^*_u \ \stackrel{!}{=} \ 0\,, \tag{C.2}$$

---

[25] Note that most general unitary matrix with texture $\begin{pmatrix} 1 & \epsilon^a \\ \epsilon^a & 1 \end{pmatrix}$ is

$$\boldsymbol{U} = \begin{pmatrix} \xi & 0 \\ 0 & \tilde{\xi} \end{pmatrix} \cdot \begin{pmatrix} 1 & -\chi^* \epsilon^a \\ \chi \epsilon^a & 1 \end{pmatrix}, \qquad \text{with} \qquad |\xi|^{-1} = |\tilde{\xi}|^{-1} = \sqrt{1 + |\chi|^2 \ \epsilon^{2a}}\,,$$

and $a \in \mathbb{N}$. Applying this form to calculate the off-diagonal elements of $\boldsymbol{U}^{(\boldsymbol{U_L})^*} \cdot \boldsymbol{G}^{(\boldsymbol{U})} \cdot \boldsymbol{U}^{(\overline{\boldsymbol{U}})^\dagger}$ we readily find that $\boldsymbol{G}^{(\boldsymbol{U})}$ is diagonalized if χ is of $\mathcal{O}(1)$. The same holds for $\boldsymbol{G}^{(\boldsymbol{D})}$. With our choice of X-charges the ratios of the quark masses are $\frac{m_u}{m_c} \sim \epsilon^4$ and $\frac{m_d}{m_s} \sim \epsilon^2$, respectively.

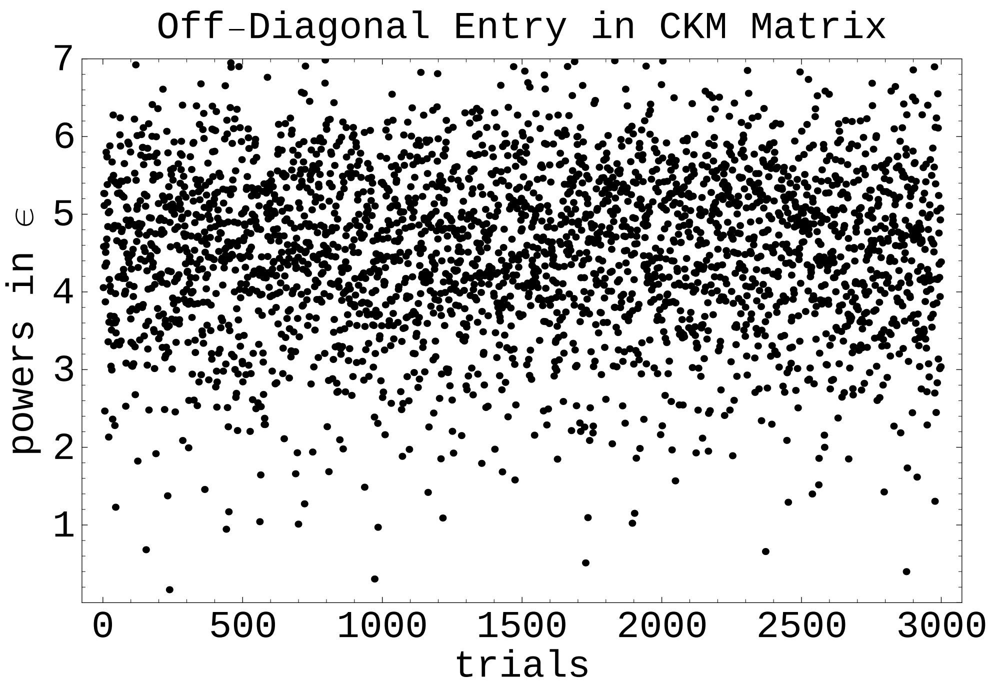

Figure 2: The powers in ϵ of the off-diagonal entry in the two generation CKM matrix for 3000 sets of randomly generated complex $\mathcal{O}(1)$ coefficients in the FN-generated Yukawa matrices and the kinetic Kähler potential terms.

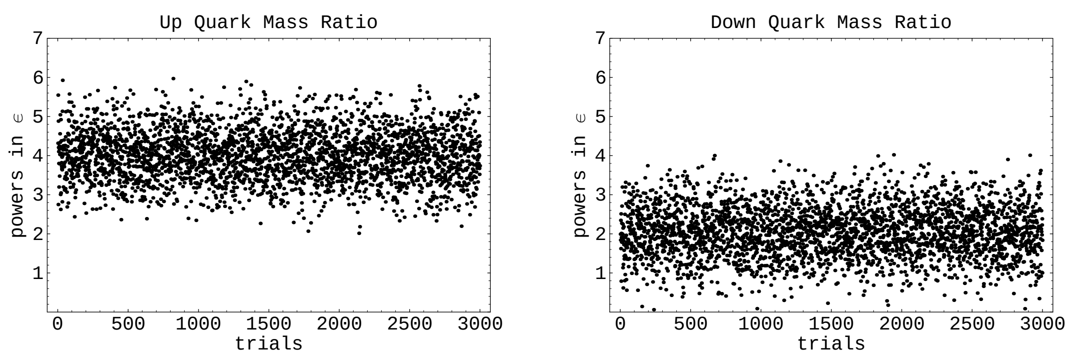

Figure 3: The powers in ϵ of the ratios $\frac{m_u}{m_c}$ and $\frac{m_d}{m_s}$.

with $|\upsilon_d| \equiv \sqrt{\upsilon_\alpha^* \, \upsilon_\alpha}$. M_{H^U} is the soft supersymmetry breaking mass for the Higgs scalar H^U and b_α is the soft supersymmetry breaking bilinear mass parameter of the term $b_\alpha \widetilde{L^\alpha} H^U$. In an explicit $\not{M}_p$ mSUGRA model, the complete scalar potential was numerically minimized in Ref. [37]. However within the context of our FN models, we have no prediction for the soft supersymmetry breaking parameters. We shall thus make the assumption that the hidden and observable sector superpotentials separate and we get the alignment of the soft supersymmetry breaking bilinear operator [37]

$$b_\alpha \;\propto\; \mu_\alpha\,, \tag{C.3}$$

at the unification scale. In the context of our FN model, this implies

$$b_\alpha \;\sim\; B\, m_{3/2}\, \epsilon^{-X_{L^\alpha}-X_{H^U}}\,. \tag{C.4}$$

B is a soft supersymmetry breaking mass parameter of $\mathcal{O}(m_{3/2})$. The other crucial ingredient is the structure of the soft supersymmetry breaking slepton mass squared $[\boldsymbol{M^2_{\widetilde{L}}}]_{\boldsymbol{\alpha\beta}}$. For simplicity we take

$$[\boldsymbol{M^2_{\widetilde{L}}}]_{\alpha\beta} \;\sim\; m^2_{3/2}\, \epsilon^{|X_{L^\alpha}-X_{L^\beta}|}\,. \tag{C.5}$$

This might originate either directly from an FN structure of the corresponding parent terms or via the CK transformation of a *diagonal* $[\boldsymbol{M^2_{\widetilde{L}}}]_{\alpha\beta}$ whose eigenvalues are all of the same order but *not equal*. Soft supersymmetry breaking parameters with the structure of Eqs. (C.4) and (C.5) have also been considered in Ref. [34].

Given Eqs. (C.4) and (C.5), we can now solve the minimization conditions Eqs. (C.1) and (C.2) and obtain an FN structure for the VEVs. To see this, we first simplify Eq. (C.2) by the observation that $\frac{g_W^2+g_Y^2}{8}(|\upsilon_u|^2-|\upsilon_d|^2) < \frac{1}{10}\,(246\text{GeV})^2$. On the other hand, the lower bound on the chargino production cross section from LEP implies $\mu_0 \geq 100\,\text{GeV}$ [70]. In our case this translates into a lower bound on $m_{3/2}$. As we assume a GM/KN-generated $\mu_0 \sim m_{3/2}\,\epsilon^{-X_{L^0}-X_{H^U}} \sim m_{3/2}\,\epsilon$, the lowest allowed value for $m_{3/2}$ is about 500 GeV. Putting this together we have

$$\frac{g_W^2+g_Y^2}{8}\,(|\upsilon_u|^2-|\upsilon_d|^2) \;\ll\; (246\,\text{GeV})^2 \;\lesssim\; m^2_{3/2} \;\sim\; [\boldsymbol{M^2_{\widetilde{L}}}]_{\alpha\alpha}\,, \tag{C.6}$$

so that the cubic term of Eq. (C.2) is negligible. Applying this approximation, we are left with a set of linear equations in the VEVs, which is solved with the *ansatz*

$$\begin{pmatrix} \upsilon_u \\ \\ \upsilon_\alpha \end{pmatrix} \;=\; \begin{pmatrix} N_u\,\frac{m_{3/2}}{B} \\ \\ N_\alpha\,\epsilon^{-X_{L^\alpha}-X_{H^U}} \end{pmatrix}, \tag{C.7}$$

if the coefficients N_u and N_α are of the same order. This qualitative statement relies on the assumption commonly made in FN models that the sum of several complex numbers with absolute value of $\mathcal{O}(1)$ is again of $\mathcal{O}(1)$. The overall scale of the VEVs is determined by the normalization requirement $\sqrt{|\upsilon_u|^2+|\upsilon_d|^2} = 246$ GeV. Eq. (C.1) does not constrain the sneutrino VEVs as the relevant terms are negligibly small, *i.e.* $\frac{b_i\ \upsilon_i}{b_0\ \upsilon_0} \sim \epsilon^{2\,(X_{L^0}-X_{L^i})}$. Hence, we finally end up with

$$\upsilon_\alpha \quad \propto \quad \epsilon^{-X_{L^\alpha}}. \tag{C.8}$$

The apparent alignment of υ_α and μ_α is with respect to the power of ϵ and not exact. Both sets of $\mathcal{O}(1)$ coefficients differ from each other due to the VEVs' dependence on $[\boldsymbol{M^2_{\tilde{L}}}]_{\alpha\beta}$. Therefore, excluding artificial exact alignment, the υ_i and μ_i $(i = 1, 2, 3)$ cannot be rotated away simultaneously. This is important in order to obtain a massive neutrino through mixing with the neutralinos.

Again, we have checked these results numerically. Except for a slight tendency to have less ϵ-suppression in the υ_i $(i = 1, 2, 3)$ we found agreement with Eq. (C.8). The systematic effect of bigger υ_i is caused by the two-dimensional random walk. Changing to a basis without sneutrino VEVs, this feature passes on to other coupling constants with the generation structure $\epsilon^{-X_{L^\alpha}}$. We take account of this by preferring higher $\mathcal{O}(1)$ coefficients for those coupling constants which are proportional to $\epsilon^{-X_{L^i}}$, namely μ_i, λ'_{ijk}, and λ_{ijk}. Coupling constants which – after the canonicalization of the Kähler potential – have the structure $\epsilon^{+X_{L^\alpha}}$ are affected differently by the $\boldsymbol{U}^{\mathbf{VEVs}}$ transformation. Their $\alpha = 0$ component gets somewhat enlarged. Thus the λ_{0jk} remain unchanged.

D Symmetries in the $\boldsymbol{\lambda}$-Loop Contribution to the Neutrino Mass Matrix

The contribution of the charged lepton-slepton loop to the neutrino mass matrix is given in Eq. (5.10). In our model, the $\not{M}_p$ parameters λ_{ikn} are generated out of the charged lepton mass matrix via the canonicalization of the Kähler potential. This mechanism leads to Eq. (3.15), which can be written as

$$\lambda_{ikn} \quad = \quad c_i\,\lambda_{0kn} - (i \leftrightarrow k)\,, \tag{D.1}$$

with c_i being some coefficient. Remember that we are working in the charged lepton mass eigenstate basis, *i.e.* $\lambda_{0kn} = \delta_{kn}\,\lambda_{0kn}$. Using this structure of λ_{ikn} we can calculate the first term of the mass matrix $\boldsymbol{M^{(\nu)}_{\lambda\text{-loop}}}$ in Eq. (5.10) symbolically

$$\begin{aligned}\sum_{k,n} \lambda_{ikn}\,\lambda_{jnk}\,F_k^{(1)}F_n^{(2)} \quad = \quad c_i\,c_j\,\Big(&\lambda_{011}^2\,F_1^{(1)}F_1^{(2)} + \lambda_{022}^2\,F_2^{(1)}F_2^{(2)} \\ &+ \lambda_{033}^2\,F_3^{(1)}F_3^{(2)} + \lambda_{0ii}\,\lambda_{0jj}\,F_j^{(1)}F_i^{(2)} \\ &- \lambda_{0ii}^2\,F_i^{(1)}F_i^{(2)} - \lambda_{0jj}^2\,F_j^{(1)}F_j^{(2)}\Big).\end{aligned} \tag{D.2}$$

Here $F_k^{(1,2)}$ are functions of the charged lepton masses as well as the charged slepton masses and mixing angles. For the purposes of this appendix it suffices to know the ratios $F_k^{(1)}/F_3^{(1)}$ and $F_k^{(2)}/F_3^{(2)}$. Eq. (5.10) together with the simplifications made below yields

$$\frac{F_k^{(1)}}{F_3^{(1)}} \quad = \quad \frac{m_k^{(e)}}{m_3^{(e)}} \quad \sim \quad \frac{F_k^{(2)}}{F_3^{(2)}}\,. \tag{D.3}$$

Depending on i and j, we can encounter exact cancellations of seemingly dominating terms in Eq. (D.2). Applying the FN structure of the charged lepton masses, $m_e : m_\mu : m_\tau \sim \epsilon^{4+z} : \epsilon^2 : 1$, and keeping only the leading (non-zero) terms, we obtain

$$\sum_{kn} \lambda_{ikn}\,\lambda_{jnk}\,F_k^{(1)}F_n^{(2)} \quad = \quad c_i\,c_j\,\lambda_{033}^2\,F_3^{(1)}F_3^{(2)}\,f_{ij}\ , \tag{D.4}$$

with $f_{ij} \sim 1$ for $i,j = 1,2$, $f_{23} \sim f_{32} \sim \epsilon^4$ and $f_{13} \sim f_{31} \sim f_{33} \sim \epsilon^8$. Adding the second term of Eq. (5.10) symmetrizes the mass matrix $\boldsymbol{M}^{(\boldsymbol{\nu})}_{\boldsymbol{\lambda}\textbf{-loop}}$. As our result in Eq. (D.4) is already symmetric in i and j concerning the magnitudes, we simply get a factor of two.

This shows that due to the λ_{ikn}'s direct dependence on the charged lepton mass matrix and its antisymmetry under interchange of the first two indices, the $(i,3)$- and the $(3,i)$-elements $(i = 1,2,3)$ of $\boldsymbol{M}^{(\boldsymbol{\nu})}_{\boldsymbol{\lambda}\textbf{-loop}}$ are highly suppressed.

E Diagonalization of the Neutrino Mass Matrix

The effective neutrino mass matrix $\boldsymbol{M}^{(\boldsymbol{\nu})}$ is diagonalized by the unitary matrix $\boldsymbol{U}^{(\boldsymbol{\nu})}$ defined in Eq. (5.1). In Sect. 5.2, we have seen that the mass matrix has an ϵ-structure

$${M^{(\nu)}}_{ij} \quad \propto \quad \epsilon^{-X_{L_i}-X_{L_j}}\,. \tag{E.1}$$

Since $X_{L_1} \geq X_{L_2} \geq X_{L_3}$, we wish to find the unitary matrix $\boldsymbol{U}^{(\boldsymbol{\nu}')}$ such that $\boldsymbol{M}^{(\boldsymbol{\nu}')}_{\textbf{diag}} = \textbf{diag}(m_1', m_2', m_3')$ with the mass ordering $m_1' \gtrsim m_2' \gtrsim m_3'$. It is given as [35]

$${U^{(\nu')}}_{ij} \quad \sim \quad \epsilon^{|X_{L_i}-X_{L_j}|}, \tag{E.2}$$

as in the case of the down-type fermions, see Eq. (3.13). Unfortunately the two main contributions to the neutrino mass matrix, Eqs. (5.8) and (5.9), are both matrices of rank one. They thus show an additional symmetry, which obscures the validity of Eq. (E.2) in the model we consider. We wish to examine this problem in more detail in this appendix. We focus on the case with only the tree level and the quark-squark loop contributing to the neutrino mass matrix, *i.e.* with two independent contributions leading to two massive neutrinos. The mass matrix can then be written in the form

$$M^{(\nu)}{}_{ij} \;\; = \;\; A\; a_i\; a_j + B\; b_i\; b_j, \tag{E.3}$$

where the upper case letters define the overall mass scale of each term and the lower case letters give the generation structure $a_i \sim b_i \sim \epsilon^{X_{L^1}-X_{L^i}}$. a_i, b_i include in general different factors of order 1. Since $X_{L^1} \geq X_{L^2} \geq X_{L^3}$, we have $|a_1| \gtrsim |a_2| \gtrsim |a_3|$ and $|b_1| \gtrsim |b_2| \gtrsim |b_3|$. In addition we take $A \gtrsim B$.[26]

Notice that there are six degrees of freedom in Eq. (E.3). So at first glance Eq. (E.2) seems applicable. However, the mass matrix $\boldsymbol{M^{(\nu)}}$ in Eq. (E.3) exhibits an additional symmetry. It is of rank two, thus leaving one neutrino massless. Therefore it is not obvious at all that Eq. (E.2) correctly describes the structure of the diagonalization matrix. We need to have a closer look at Eq. (E.3). In analogy to Eq. (3.12), we perform a unitary transformation which rotates away a_2 and a_3. Thus with

$$\boldsymbol{V}^* \;\; \sim \;\; \begin{pmatrix} 1 & \epsilon^{X_{L^1}-X_{L^2}} & \epsilon^{X_{L^1}-X_{L^3}} \\ \epsilon^{X_{L^1}-X_{L^2}} & 1 & \epsilon^{2X_{L^1}-X_{L^2}-X_{L^3}} \\ \epsilon^{X_{L^1}-X_{L^3}} & \epsilon^{2X_{L^1}-X_{L^2}-X_{L^3}} & 1 \end{pmatrix}, \tag{E.4}$$

and $V^*_{ij}\, a_j \equiv a'_i = \delta_{1i}\, a'_i$ and $V^*_{ij}\, b_j \equiv b'_i \sim \epsilon^{X_{L^1}-X_{L^i}}$ we have

$$\boldsymbol{M^{(\nu)}}' \equiv \boldsymbol{V}^* \cdot \boldsymbol{M^{(\nu)}} \cdot \boldsymbol{V}^\dagger = B \begin{pmatrix} \frac{A}{B}\; {a'_1}^2 + {b'_1}^2 & b'_1 b'_2 & b'_1 b'_3 \\ b'_2 b'_1 & b'_2 b'_2 & b'_2 b'_3 \\ b'_3 b'_1 & b'_3 b'_2 & b'_3 b'_3 \end{pmatrix}. \tag{E.5}$$

In the next step, we want to find the unitary matrix $\boldsymbol{W}$ which finally diagonalizes $\boldsymbol{M^{(\nu)}}'$, with the ordered mass singular values. For this we consider

$$M^{(\nu)'}{}_{ij}\, W^\dagger_{jk} \;\; = \;\; W^T_{ik}\, m'_k\,, \tag{E.6}$$

where $m'_1 \geq m'_2$ and $m'_3 = 0$. For $k = 3$ we find that

$$W^\dagger_{j3} \;\; \sim \;\; \frac{1}{b'_2} \begin{pmatrix} 0 \\ -b'_3 \\ b'_2 \end{pmatrix}_j \;\; \sim \;\; \begin{pmatrix} 0 \\ \epsilon^{X_{L^2}-X_{L^3}} \\ 1 \end{pmatrix}_j \tag{E.7}$$

[26] We do not specify which term is tree and which is loop level in order to stay general as long as possible. So the treatment in this appendix is valid also for $\frac{m^{\text{tree}}}{m^{\text{loop}}} \lesssim 1$.

satisfies Eq. (E.6). Now consider $k = 1$. For $i = 1, 2, 3$ we get the following order of magnitude relations

$$\begin{aligned}
\left[\frac{A}{B}\,\mathcal{O}(1) + \mathcal{O}(1)\right] W^\dagger_{11} + \mathcal{O}(\epsilon^{X_{L^1}-X_{L^2}}) W^\dagger_{21} + \mathcal{O}(\epsilon^{X_{L^1}-X_{L^3}}) W^\dagger_{31} &= \frac{m_1'}{B} W^T_{11}, &\text{(E.8)}\\
\mathcal{O}(\epsilon^{X_{L^1}-X_{L^2}}) W^\dagger_{11} + \mathcal{O}(\epsilon^{2(X_{L^1}-X_{L^2})}) W^\dagger_{21} + \mathcal{O}(\epsilon^{2X_{L^1}-X_{L^2}-X_{L^3}}) W^\dagger_{31} &= \frac{m_1'}{B} W^T_{21}, &\text{(E.9)}\\
\mathcal{O}(\epsilon^{X_{L^1}-X_{L^3}}) W^\dagger_{11} + \mathcal{O}(\epsilon^{2X_{L^1}-X_{L^2}-X_{L^3}}) W^\dagger_{21} + \mathcal{O}(\epsilon^{2(X_{L^1}-X_{L^3})}) W^\dagger_{31} &= \frac{m_1'}{B} W^T_{31}. &\text{(E.10)}
\end{aligned}$$

Assuming no accidental cancellations among $\mathcal{O}(1)$ coefficients and keeping only leading terms[27], we can determine the magnitude of $W^\dagger_{21}$ and $W^\dagger_{31}$ from Eqs. (E.9), (E.10):

$$W^\dagger_{21} \sim \frac{B}{m_1'}\,\epsilon^{X_{L^1}-X_{L^2}}\, W^T_{11}\,, \qquad \text{and} \qquad W^\dagger_{31} \sim \frac{B}{m_1'}\,\epsilon^{X_{L^1}-X_{L^3}}\, W^T_{11}. \tag{E.11}$$

Eq. (E.8) does not contain any information on the ϵ-structure of $W^\dagger_{j1}$. It simply states that m_1' is of $\mathcal{O}(A)$, the scale of the leading contribution to the neutrino mass matrix. By means of normalization arguments we conclude that

$$W^\dagger_{j1} \;\sim\; \begin{pmatrix} 1 \\ \frac{B}{A}\,\epsilon^{X_{L^1}-X_{L^2}} \\ \frac{B}{A}\,\epsilon^{X_{L^1}-X_{L^3}} \end{pmatrix}_j . \tag{E.12}$$

The remaining (second) column of the unitary matrix $\boldsymbol{W}^\dagger$ is obtained by finding the normalized vector which is orthogonal to $W^\dagger_{j3}$ and $W^\dagger_{j1}$. We get

$$W^\dagger_{j2} \;\sim\; \begin{pmatrix} \frac{B}{A}\,\epsilon^{X_{L^1}-X_{L^2}} \\ 1 \\ \epsilon^{X_{L^2}-X_{L^3}} \end{pmatrix}_j . \tag{E.13}$$

Inserting this into Eq. (E.6), we find the magnitude of the second neutrino mass

$$m_2' \;\sim\; B \cdot \epsilon^{2(X_{L^1}-X_{L^2})}\,. \tag{E.14}$$

Notice that m_2' is not simply given by the scale B of the second largest contribution to the neutrino mass matrix but it is additionally suppressed by a factor of $\epsilon^{2(X_{L^1}-X_{L^2})}$. This is of course only relevant if $X_{L^1} > X_{L^2}$.

[27]Notice that $\frac{m_1'}{B} \geq 1$. Hence, considering for example the last equation, we can neglect the third term of the LHS compared to the RHS.

We finally arrive at the diagonalization matrix $\boldsymbol{U}^{(\boldsymbol{\nu}')}$ with the eigenvalues in the order $m_1' \gtrsim m_2' > m_3' = 0$:

$$U^{(\nu')}{}_{ij} \;\; \equiv \;\; W_{ik}\, V_{kj} \;\sim\; \epsilon^{|X_{L^i} - X_{L^j}|} \,. \tag{E.15}$$

The dependence on the factor $\frac{B}{A}$ which appears in $\boldsymbol{W}$ drops out in leading order.

F A Top-down Example

Here we consider the X-charge assignment of Table 5 with $\Delta^L_{31} = -1$, $3\zeta + b = -2$, and $y = 1$. This is our preferred scenario since the resulting X-charges are all multiples of one third, *i.e.* they are not highly fractional. With this choice we obtain the following Yukawa matrices for the superpotential terms $H^D Q^i \overline{D^j}$, $H^D L^i \overline{E^j}$, and $H^U Q^i \overline{U^j}$ without supersymmetric zeros:

$$\boldsymbol{G}^{(\boldsymbol{D})}_{\mathbf{FN}} \sim \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} , \quad \boldsymbol{G}^{(\boldsymbol{E})}_{\mathbf{FN}} \sim \begin{pmatrix} \epsilon^5 & \epsilon^2 & \epsilon \\ \epsilon^5 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix} , \quad \boldsymbol{G}^{(\boldsymbol{U})}_{\mathbf{FN}} \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} . \tag{F.1}$$

The trilinear $\not{M}_p$ terms $L^i Q^j \overline{D^k}$ and $L^i L^j \overline{E^k}$ are disallowed due to negative integer overall X-charges: $X_{L^1} + X_{Q^1} + X_{\overline{D^1}} = -2$ and $X_{L^1} + X_{L^2} + X_{\overline{E^1}} = -1$, respectively. For higher generational indices we obtain even smaller total X-charges. Analogously, we have for the bilinear terms $L^\alpha H^U$: $X_{L^0} + X_{H^U} = -1$ (corresponding to the statement $z = 1$) and even smaller for the three lepton doublets L^i due to $X_{L^i} < X_{L^0}$. The B_3 violating terms $\overline{U^i D^j D^k}$ are forbidden by non-integer overall X-charge, $X_{\overline{U^1}} + X_{\overline{D^1}} + X_{\overline{D^2}} = -\frac{8}{3}$.

The GM/KN mechanism, however, reintroduces the terms disallowed by *negative integer* overall X-charge in the effective superpotential. Thus we get

$$\frac{m_{3/2}}{M_{\text{grav}}}\, \epsilon^{-(X_{L^i} + X_{Q^j} + X_{\overline{D^k}})}\; L^i Q^j \overline{D^k}, \qquad \frac{m_{3/2}}{M_{\text{grav}}}\, \epsilon^{-(X_{L^i} + X_{L^j} + X_{\overline{E^k}})}\; L^i L^j \overline{E^k}, \tag{F.2}$$

and

$$\mu_{\text{FN}\,\alpha}\; L^\alpha H^U, \qquad \text{with} \qquad \mu_{\text{FN}\,\alpha} \sim\; m_{3/2}\; \epsilon^{-(X_{L^\alpha} + X_{H^U})}. \tag{F.3}$$

Since $\frac{m_{3/2}}{M_{\text{grav}}} = \mathcal{O}(\frac{10^3\,\text{GeV}}{10^{18}\,\text{GeV}}) = \mathcal{O}(10^{-15})$ the GM/KN-generated trilinear terms are negligibly small. In contrast, the bilinear terms including the MSSM μ-term are phenomenologically the correct order of magnitude. Of course, care has to be taken for sufficient ϵ-suppression of the $\not{M}_p$ bilinears. In the scenario considered here, we have

$$\mu_{\text{FN}\,\alpha} \;\; \sim \;\; m_{3/2} \begin{pmatrix} \epsilon \\ \epsilon^7 \\ \epsilon^7 \\ \epsilon^8 \end{pmatrix}_\alpha . \tag{F.4}$$

Next, we canonicalize the Kähler potential. The only CK transformation that changes the ϵ-structure of the above coupling constants is the one connected to the superfields L^α [thus *e.g* $\boldsymbol{G^{(U,D)}} \sim \boldsymbol{G^{(U,D)}_{\mathbf{FN}}}$ concerning the CK transformations of $Q^i, \overline{U^i}, \overline{D^i}, H^U, H^D$]. The corresponding transformation matrix takes the form [see Eq. (3.1)]

$$\boldsymbol{C^{(L)}} \quad \sim \quad \begin{pmatrix} 1 & \epsilon^6 & \epsilon^6 & \epsilon^7 \\ \epsilon^6 & 1 & 1 & \epsilon \\ \epsilon^6 & 1 & 1 & \epsilon \\ \epsilon^7 & \epsilon & \epsilon & 1 \end{pmatrix}. \tag{F.5}$$

The $\not{M}_p$ coupling constants λ'_{ijk} of the trilinear terms $L^iQ^j\overline{D^k}$ are now generated from $\lambda'_{\mathrm{FN}\,0jk} \equiv G^{(D)}_{\mathrm{FN}\ jk}$ as shown in Eq. (3.9):

$$\lambda'_{\alpha jk} \quad = \quad [\boldsymbol{C^{(L)}}^{-1}]_{0\alpha}\ G^{(D)}_{\mathrm{FN}\ jk} \quad \sim \quad \begin{pmatrix} 1 \\ \epsilon^6 \\ \epsilon^6 \\ \epsilon^7 \end{pmatrix}_\alpha G^{(D)}_{\mathrm{FN}\ jk}\,. \tag{F.6}$$

Likewise, the $\not{M}_p$ coupling constants λ_{ijk} of the trilinear terms $L^iL^j\overline{E^k}$ are generated from $\boldsymbol{G^{(E)}_{\mathbf{FN}}}$. An additional antisymmetrizing term accounts for the antisymmetry of λ_{ijk} in the first two indices, see Eq. (3.10). The ϵ-structure of the bilinear coupling constants μ_α is not affected by the CK-transformation:

$$\mu_\alpha \;=\; [\boldsymbol{C^{(L)}}^{-1}]_{\beta\alpha}\ \mu_{\mathrm{FN}\,\beta} \;\sim\; m_{3/2} \begin{pmatrix} 1 & \epsilon^6 & \epsilon^6 & \epsilon^7 \\ \epsilon^6 & 1 & 1 & \epsilon \\ \epsilon^6 & 1 & 1 & \epsilon \\ \epsilon^7 & \epsilon & \epsilon & 1 \end{pmatrix}_{\beta\alpha} \cdot \begin{pmatrix} \epsilon \\ \epsilon^7 \\ \epsilon^7 \\ \epsilon^8 \end{pmatrix}_\beta \;\sim\; m_{3/2} \begin{pmatrix} \epsilon \\ \epsilon^7 \\ \epsilon^7 \\ \epsilon^8 \end{pmatrix}_\alpha. \tag{F.7}$$

Neglecting renormalization flow effects, we now rotate away the sneutrino VEVs [23]. To leading order in ϵ the necessary unitary transformation is given in Eq. (3.12). For our X-charge assignment it reads

$$\boldsymbol{U^{\mathbf{VEVs}}} \quad \sim \quad \begin{pmatrix} 1 & \epsilon^6 & \epsilon^6 & \epsilon^7 \\ \epsilon^6 & 1 & \epsilon^{12} & \epsilon^{13} \\ \epsilon^6 & \epsilon^{12} & 1 & \epsilon^{13} \\ \epsilon^7 & \epsilon^{13} & \epsilon^{13} & 1 \end{pmatrix}. \tag{F.8}$$

It is easy to see that this transformation does not change the coupling constants $\lambda'_{\alpha jk}$, $\lambda_{\alpha\beta k}$, and μ_α in their flavor structure.

Having generated the above $\not{M}_p$ couplings via the GM/KN mechanism and the subsequent CK transformation, it is possible to have neutrino masses without introducing right-handed neutrinos. Assuming[28] $\frac{m^{\rm tree}}{m^{\rm loop}} \gtrsim 1$, we obtain an effective Majorana neutrino mass matrix of the structure [*cf.* Eq. (5.8) for $x = 0$ which corresponds to $\tan\beta \gtrsim 40$, thus $\cos^2\beta \approx \frac{1}{\tan^2\beta} \sim \frac{m_b^2}{m_t^2}$ and $\sin 2\beta = 2\,\sin\beta\cos\beta \approx \frac{2}{\tan\beta} \ll 1$]

$$\boldsymbol{M^{(\nu)}} \;\sim\; \frac{m_Z^2\; m_b^2}{m_t^2} \cdot \frac{M_{\tilde\gamma}}{M_1\; M_2} \cdot \epsilon^{12} \cdot \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & \epsilon^2 \end{pmatrix}.$$

Differentiating between hierarchical and inverse-hierarchical neutrino scenarios (*cf.*Sect.5.4) we arrive at an MNS mixing matrix with either

$$\boldsymbol{U}^{\mathbf{MNS}}_{\mathbf{(h.)}} = \boldsymbol{U}^{(\boldsymbol{\nu}')^\dagger} \cdot \boldsymbol{T}^{\mathbf{(h.)}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \cdot \boldsymbol{T}^{\mathbf{(h.)}} \sim \begin{pmatrix} \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \\ 1 & \epsilon & \epsilon \end{pmatrix}, \tag{F.9}$$

or

$$\boldsymbol{U}^{\mathbf{MNS}}_{\mathbf{(i.h.)}} = \boldsymbol{U}^{(\boldsymbol{\nu}')^\dagger} \cdot \boldsymbol{T}^{\mathbf{(i.h.)}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \cdot \boldsymbol{T}^{\mathbf{(i.h.)}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}. \tag{F.10}$$

Both scenarios are compatible with Eq. (5.6). However, due to the smallness of the $(1,3)$-element in the experimentally measured MNS matrix, leptonic mixing would suggest an inverse hierarchy. Then, consistency with the neutrino mass single values (see Sect. 5.3) would require two masses of similar magnitude, thus $\frac{m^{\rm tree}}{m^{\rm loop}} \sim \mathcal{O}(1)$.

As for the CKM matrix we refer to Ref. [7] and state the result for the sake of completeness:

$$\boldsymbol{U}^{\mathbf{CKM}} \;\sim\; \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}. \tag{F.11}$$

The price we have to pay for nice, *i.e.* not too fractional, X-charges is a not-so-nice CKM matrix [*e.g.* the (1,2)-element is $\mathcal{O}(\epsilon^2)$ and not $\mathcal{O}(\epsilon)$].

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