Light-Front Holography and AdS/QCD Correspondence

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Abstract

Light-Front Holography is a remarkable consequence of the correspondence between string theory in AdS space and conformal field theories in physical-space time. It allows string modes $\Phi(z)$ in the AdS fifth dimension to be precisely mapped to the light-front wavefunctions of hadrons in terms of a specific light-front impact variable ζ which measures the separation of the quark and gluonic constituents within the hadron. This mapping was originally obtained by matching the exact expression for electromagnetic current matrix elements in AdS space with the corresponding exact expression for the current matrix element using light-front theory in physical space-time. More recently we have shown that one obtains the identical holographic mapping using matrix elements of the energy-momentum tensor, thus providing an important consistency test and verification of holographic mapping from AdS to physical observables defined on the light-front. The resulting light-front Schrodinger equations predicted from AdS/QCD give a good representation of the observed meson and baryon spectra and give excellent phenomenological predictions for amplitudes such as electromagnetic form factors and decay constants.

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1   Introduction

One of the most challenging problems in strong interaction dynamics is to understand the composition of mesons and baryons in terms of the fundamental quark and gluon degrees of freedom of the QCD Lagrangian. Because of the strong-coupling of QCD in the infrared domain, it has been difficult to find analytic solutions for the wavefunctions of hadrons or to make precise predictions for hadronic properties outside of the perturbative regime. Thus an important theoretical goal is to find an initial approximation to bound-state problems in QCD which is analytically tractable and which can be systematically improved. Recently the AdS/CFT correspondence [1] between string states in anti–de Sitter (AdS) space and conformal field theories in physical space-time, modified for color confinement, has led to a semiclassical model for strongly-coupled QCD which provides analytical insights into its inherently non-perturbative nature, including hadronic spectra, decay constants, and wavefunctions. As we have recently shown [2, 3], there is a remarkable mapping between the AdS description of hadrons and the Hamiltonian formulation of QCD in physical space-time quantized on the light front.

The natural extension of a wavefunction for relativistic quantum field theories such as QCD is the light-front wavefunction $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ which specifies the n quark and gluon constituents of a hadron's Fock state as a function of the light-cone fractions $x_i = k^+/P^+ = (k^0 + k^z)/(P^0 + P^z)$ transverse momenta $\mathbf{k}_{\perp i}$ and spin projections λ_i. The light-front wavefunctions of bound states in QCD are the relativistic generalizations of the familiar Schrödinger wavefunctions of atomic physics, but they are determined at fixed light-cone time $\tau = t + z/c$—the "front form" advocated by Dirac [4]—rather than at fixed ordinary time t. The light-front wavefunctions of a hadron are independent of the momentum of the hadron, and they are thus boost invariant; Wigner transformations and Melosh rotations are not required. The light-front formalism for gauge theories in light-cone gauge is particularly useful in that there are no ghosts, and one has a direct physical interpretation of orbital angular momentum.

When a flash from a camera illuminates a scene, each object is illuminated along the light-front of the flash; i.e., at a given τ. Similarly, when a sample is illuminated by an x-ray source such as the Linac Coherent Light Source, each element of the target is struck at a given τ. In contrast, setting the initial condition using conventional instant time t requires simultaneous scattering of photons on each constituent. Thus it is natural to set boundary conditions at fixed τ and then evolve using the light-front Hamiltonian $P^- = P^0 - P^z = id/d\tau$. The invariant Hamiltonian $H_{LF} = P^+ P^- - P_\perp^2$ then has eigenvalues $\mathcal{M}^2$ where $\mathcal{M}$ is the physical mass. Its eigenfunctions are the light-front eigenstates whose Fock state projections define the light-front wavefunctions.

Light-Front Holography is an important feature of AdS/CFT; it allows string modes $\Phi(z)$ in the AdS fifth dimension to be precisely mapped to the light-front

wavefunctions of hadrons in physical space-time in terms of a specific light-front impact variable ζ which measures the separation of the quark and gluonic constituents within the hadron. The AdS/CFT correspondence implies that a strongly coupled gauge theory is equivalent to the propagation of weakly coupled strings in a higher dimensional space, where physical quantities are computed in terms of an effective gravitational theory. Thus, the AdS/CFT duality provides a gravity description in a $(d+1)$-dimensional AdS space-time in terms of a d-dimensional conformally-invariant quantum field theory at the AdS asymptotic boundary [5, 6].

QCD is a confining theory in the infrared with a mass gap Λ_{QCD} and a well-defined spectrum of color singlet states. Conformal symmetry is broken in physical QCD by quantum effects and quark masses. There are indications however, both from theory and phenomenology, that the QCD coupling is slowly varying at small momentum transfer [7]. In particular, a new extraction of the effective strong coupling constant $\alpha_{s,g_1}(Q^2)$ from CLAS spin structure function data in an extended Q^2 region using the Bjorken sum $\Gamma_1^{p-n}(Q^2)$ [8], indicates the lack of Q^2 dependence of α_s in the low Q^2 limit. One can understand this physically [7]: in a confining theory where gluons have an effective mass or maximal wavelength, all vacuum polarization corrections to the gluon self-energy decouple at long wavelength; thus an infrared fixed point appears to be a natural consequence of confinement [9]. Furthermore, if one considers a semi-classical approximation to QCD with massless quarks and without particle creation or absorption, then the resulting β function is zero, the coupling is constant, and the approximate theory is scale and conformal invariant [10]. One can thus use conformal symmetry as a *template*, systematically correcting for its nonzero β function as well as higher-twist effects [11].

Different values of the holographic variable z determine the scale of the invariant separation between the partonic constituents. Hard scattering processes occur in the small-z ultraviolet (UV) region of AdS space. In particular, the $Q \to \infty$ zero separation limit corresponds to the $z \to 0$ asymptotic boundary, where the QCD Lagrangian is defined. In the large-z infrared (IR) region a cut-off is introduced to truncate the regime where the AdS modes can propagate. The infrared cut-off breaks conformal invariance, allows the introduction of a scale and a spectrum of particle states. In the hard wall model [12] a cut-off is placed at a finite value $z_0 = 1/\Lambda_{\mathrm{QCD}}$ and the spectrum of states is linear in the radial and angular momentum quantum numbers: $\mathcal{M} \sim 2n + L$. In the soft wall model a smooth infrared cutoff is chosen to model confinement and reproduce the usual Regge behavior $\mathcal{M}^2 \sim n + L$ [13]. The resulting models, although *ad hoc*, provide a simple semi-classical approximation to QCD which has both constituent counting rule behavior at short distances and confinement at large distances [7].

It is thus natural, as a useful first approximation, to use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS space to the modes propagating inside AdS. The short-distance behavior of a hadronic state is characterized by its twist (dimension minus spin) $\tau = \Delta - \sigma$, where σ is the sum over

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the constituent's spin $\sigma = \sum_{i=1}^{n} \sigma_i$. Twist is also equal to the number of partons $\tau = n$. Under conformal transformations the interpolating operators transform according to their twist, and consequently the AdS isometries map the twist scaling dimensions into the AdS modes [14].

The eigenvalues of normalizable modes in AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for the distribution of quarks and gluons at a given scale. There are also non-normalizable modes which are related to external currents: they propagate into the AdS interior and couple to boundary QCD interpolating operators [5, 6]. Following this simplified "bottom up" approach, a limited set of operators is introduced to construct phenomenological viable five-dimensional dual holographic models [15, 16, 17, 18].

An important feature of light-front quantization is the fact that it provides exact formulas for current matrix elements as a sum of bilinear forms which can be mapped into their AdS/CFT counterparts in the semi-classical approximation. The AdS metric written in terms of light front coordinates $x^{\pm} = x^0 \pm x^3$ is

$$ds^2 = \frac{R^2}{z^2} \left(dx^+ dx^- - d\mathbf{x}_{\perp}^2 - dz^2 \right). \tag{1}$$

At fixed light-front time $x^+ = 0$, the metric depends only on the transverse $\mathbf{x}_{\perp}$ and the holographic variable z. Thus we can find an exact correspondence between the fifth-dimensional coordinate of anti-de Sitter space z and a specific impact variable ζ in the light-front formalism. The new variable ζ measures the separation of the constituents within the hadron in ordinary space-time. The amplitude $\Phi(z)$ describing the hadronic state in AdS_5 can then be precisely mapped to the valence light-front wavefunctions $\psi_{n/H}$ of hadrons in physical space-time [2, 3], thus providing a relativistic description of hadrons in QCD at the amplitude level.

The correspondence of AdS amplitudes to the QCD wavefunctions in light-front coordinates was carried out in [2, 3] by comparing the expressions for the electromagnetic matrix elements in QCD and AdS for any value of the momentum transfer q^2. More recently we have shown that one obtains the identical holographic mapping using the matrix elements of the energy-momentum tensor [19]. To prove this, we show that there exists a correspondence between the matrix elements of the energy-momentum tensor of the fundamental hadronic constituents in QCD with the transition amplitudes describing the interaction of string modes in anti-de Sitter space with an external graviton field which propagates in the AdS interior. The proof is outlined below.

The matrix elements of the energy-momentum tensor $\Theta^{\mu\nu}$ of each constituent define the gravitational form factor of a composite hadron. One can also use gravitational matrix elements to obtain the holographic mapping of the AdS mode wavefunctions $\Phi(z)$ in AdS space to the light-front wavefunctions $\psi_{n/H}$ in physical $3+1$ space-time defined at fixed light-cone time $\tau = t + z/c$. We find the identical holographic mapping from $z \to \zeta$ as in the electromagnetic case. The agreement of the

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results for electromagnetic and gravitational hadronic transition amplitudes provides an important consistency test and verification of holographic mapping from AdS to physical observables defined on the light-front. It is indeed remarkable that such a correspondence exists, since strings describe extended objects coupled to an electromagnetic field distributed in the AdS interior, whereas QCD degrees of freedom are pointlike particles with individual local couplings to the electromagnetic current. However, as we have recently shown [2, 3, 19], a precise mapping of AdS modes to hadronic light-front wavefunctions can be found in the strongly coupled semiclassical approximation to QCD.

2 The Light-Front Fock Representation

The light-front expansion of any hadronic system is constructed by quantizing QCD at fixed light-cone time [4] $\tau = t + z/c$. In terms of the hadron four-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, the light-cone Lorentz invariant Hamiltonian for the composite system, $H_{LF}^{QCD} = P^- P^+ - \mathbf{P}_\perp^2$, has eigenvalues given in terms of the eigenmass $\mathcal{M}$ squared corresponding to the mass spectrum of the color-singlet states in QCD [20] $H_{LF}|\psi_H\rangle = \mathcal{M}_H^2|\psi_H\rangle$. Each hadronic eigenstate $|\psi_H\rangle$ can be expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\psi_H(P^+, \mathbf{P}_\perp, S_z)\right\rangle = \sum_{n,\lambda_i} \prod_{i=1}^n \int \frac{dx_i}{\sqrt{x_i}} \frac{d^2\mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right)$$
$$\times \psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i)\left|n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\right\rangle, \quad (2)$$

where the sum begins with the valence state; e.g., $n \geq 3$ for baryons. The coefficients of the Fock expansion $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \left\langle n : x_i, \mathbf{k}_{\perp i}, \lambda_i | \psi_H \right\rangle$, are independent of the total momentum P^+ and $\mathbf{P}_\perp$ of the hadron and depend only on the relative partonic coordinates, the longitudinal momentum fraction $x_i = k_i^+/P^+$, the relative transverse momentum $\mathbf{k}_{\perp i}$ and λ_i, the projection of the constituent's spin along the z direction. Momentum conservation requires $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \mathbf{k}_{\perp i} = 0$. In addition, each light front wavefunction $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ obeys the angular momentum sum rule [21] $J^z = \sum_{i=1}^n S_i^z + \sum_{i=1}^{n-1} L_i^z$, where $S_i^z = \lambda_i$ and the $n-1$ orbital angular momenta have the operator form $L_i^z = -i\left(\frac{\partial}{\partial k_i^x} k_i^y - \frac{\partial}{\partial k_i^y} k_i^x\right)$. It should be emphasized that the assignment of quark and gluon spin and orbital angular momentum of a hadron is a gauge-dependent concept. The LF framework in light-cone gauge $A^+ = 0$ provides a physical definition since there are no gauge field ghosts and the gluon has spin-projection $J^z = \pm 1$; moreover, it is frame-independent.

3 Light-Front Holography

Light-Front Holography can be derived by observing the correspondence between matrix elements obtained in AdS/CFT with the corresponding formula using the LF representation. We will outline the basic elements of the derivation in this section. For simplicity we discuss the specific mapping for two-parton hadronic state. The n-parton case is derived [19] using an effective single particle density [22].

We first consider the light-front electromagnetic form factor in impact space [2, 3]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\Big(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\Big) \Big|\tilde\psi_n(x_j, \mathbf{b}_{\perp j})\Big|^2, \qquad (3)$$

written as a sum of overlap of light-front wave functions of the $j = 1, 2, \cdots, n-1$ spectator constituents. We have included explicitly in (3) the contribution from each active constituent q with charge e_q. The formula is exact if the sum is over all Fock states n.

For definiteness we shall consider a two-quark π^+ valence Fock state $|u\bar d\rangle$ with charges $e_u = \frac{2}{3}$ and $e_{\bar d} = \frac{1}{3}$. For $n = 2$, there are two terms which contribute to the q-sum in (3). Exchanging $x \leftrightarrow 1 - x$ in the second integral we find ($e_u + e_{\bar d} = 1$)

$$
\begin{aligned}
F_{\pi^+}(q^2) &= \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp(1-x)} \Big|\tilde\psi_{u\bar d/\pi}(x, \mathbf{b}_\perp)\Big|^2 \\
&= 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta\, J_0\Big(\zeta q\sqrt{\frac{1-x}{x}}\Big) \Big|\tilde\psi_{u\bar d/\pi}(x, \zeta)\Big|^2, \qquad (4)
\end{aligned}
$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$ and $F_\pi^+(q=0) = 1$. Notice that by performing an identical calculation for the π^0 meson the result is $F_{\pi^0}(q^2) = 0$ for any q, as expected from C-charge conjugation invariance.

We now compare this result with the electromagnetic form-factor in AdS space:

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, J(Q^2, z)|\Phi(z)|^2, \qquad (5)$$

where $J(Q^2, z) = zQK_1(zQ)$. Using the integral representation of $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx\, J_0\Big(\zeta Q\sqrt{\frac{1-x}{x}}\Big), \qquad (6)$$

we can write the AdS electromagnetic form-factor as

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3}\, J_0\Big(zQ\sqrt{\frac{1-x}{x}}\Big) |\Phi(z)|^2. \qquad (7)$$

Comparing with the light-front QCD form factor (4) for arbitrary values of Q

$$|\tilde{\psi}(x,\zeta)|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi(\zeta)|^2}{\zeta^4}, \qquad (8)$$

where we identify the transverse light-front variable ζ, $0 \leq \zeta \leq \Lambda_{\text{QCD}}$, with the holographic variable z.

Matrix elements of the energy-momentum tensor $\Theta^{\mu\nu}$ which define the gravitational form factors play an important role in hadron physics. Since one can define $\Theta^{\mu\nu}$ for each parton, one can identify the momentum fraction and contribution to the orbital angular momentum of each quark flavor and gluon of a hadron. For example, the spin-flip form factor $B(q^2)$ which is the analog of the Pauli form factor $F_2(Q^2)$ of a nucleon provides a measure of the orbital angular momentum carried by each quark and gluon constituent of a hadron at $q^2 = 0$. Similarly, the spin-conserving form factor $A(q^2)$, the analog of the Dirac form factor $F_1(q^2)$, allows one to measure the momentum fractions carried by each constituent. This is the underlying physics of Ji's sum rule [23]: $\langle J^z \rangle = \frac{1}{2}[A(0) + B(0)]$, which has prompted much of the current interest in the generalized parton distributions (GPDs) measured in deeply virtual Compton scattering. Measurements of the GDP's are of particular relevance for determining the distribution of partons in the transverse impact plane, and thus could be confronted with AdS/QCD predictions which follow from the mapping of AdS modes to the transverse impact representation [2].

An important constraint is $B(0) = \sum_i B_i(0) = 0$; i.e. the anomalous gravitomagnetic moment of a hadron vanishes when summed over all the constituents i. This was originally derived from the equivalence principle of gravity [24]. The explicit verification of these relations, Fock state by Fock state, can be obtained in the light-front quantization of QCD in light-cone gauge [21]. Physically $B(0) = 0$ corresponds to the fact that the sum of the n orbital angular momenta L in an n-parton Fock state must vanish since there are only $n-1$ independent orbital angular momenta.

Gravitational form factors can also be computed in AdS/QCD from the overlap integral of hadronic string modes propagating in AdS space with a graviton field $h_{\mu\nu}$ which acts as a source and probes the AdS interior. This has been done very recently for the gravitational form factors of mesons by Abidin and Carlson [25], thus providing restrictions on the GPDs.

The light-front expression for the helicity-conserving gravitational form factor in impact space is [19]

$$A(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_f x_f \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) \left|\tilde{\psi}_n(x_j, \mathbf{b}_{\perp j})\right|^2, \qquad (9)$$

which includes the contribution of each struck parton with longitudinal momentum x_f and corresponds to a change of transverse momentum $x_j\mathbf{q}$ for each of the $j =$

$1, 2, \cdots, n - 1$ spectators. For $n = 2$, there are two terms which contribute to the f-sum in (9). Exchanging $x \leftrightarrow 1 - x$ in the second integral we find

$$
\begin{aligned}
A_\pi(q^2) &= 2 \int_0^1 x \, dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left| \tilde{\psi}(x, \mathbf{b}_\perp) \right|^2 \\
&= 4\pi \int_0^1 \frac{dx}{(1-x)} \int \zeta d\zeta \, J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) |\tilde{\psi}(x, \zeta)|^2,
\end{aligned}
\tag{10}
$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$. It is simple to prove that if ψ is a symmetric function of x and $(1-x)$ then

$$
\int_0^1 x \, dx \int d^2\mathbf{b}_\perp |\tilde{\psi}(x, \mathbf{b}_\perp)|^2 = \frac{1}{2},
\tag{11}
$$

and thus $A_\pi(q^2)$ satisfy the sum rule $A_\pi(q{=}0) = 1$.

We now consider the expression for the hadronic gravitational form factor in AdS space

$$
A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,
\tag{12}
$$

where $H(Q^2, z) = \frac{1}{2}Q^2 z^2 K_2(zQ)$. The hadronic form factor is normalized to one at $Q = 0$, $A(0) = 1$. Using the integral representation of $H(Q^2, z)$

$$
H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right),
\tag{13}
$$

we can write the AdS gravitational form factor

$$
A(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right) |\Phi(z)|^2.
\tag{14}
$$

Comparing with the QCD gravitational form factor (10) we find an identical relation between the light-front wave function $\tilde{\psi}(x, \zeta)$ and the AdS wavefunction $\Phi(z)$ in Eq. (8) obtained from the mapping of the pion electromagnetic transition amplitude.

4 Holographic Light-Front Hamiltonian and Schrödinger Equation

The above analysis provides an exact correspondence between the holographic variable z and an impact variable ζ which measures the transverse separation of the constituents within a hadron, we can identify $\zeta = z$. The mapping of z from AdS space to ζ in light-front frame allows the equations of motion in AdS space to be recast in the form of a light-front Hamiltonian equation [20]

$$
H_{LF} |\phi\rangle = \mathcal{M}^2 |\phi\rangle,
\tag{15}
$$

a remarkable result which allows the discussion of the AdS/CFT solutions in terms of light-front equations in physical 3+1 space time. By substituting $\phi(\zeta) = \zeta^{-3/2}\Phi(\zeta)$ in the AdS scalar wave equation we find an effective Schrödinger equation as a function of the weighted impact variable ζ [2, 3]

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \tag{16}$$

with the conformal potential $V(\zeta) \rightarrow -(1-4L^2)/4\zeta^2$, an effective two-particle light-front radial equation for mesons. Its eigenmodes determine the hadronic mass spectrum. We have written above $(\mu R)^2 = -4 + L^2$. The holographic hadronic light-front wave functions $\phi(\zeta) = \langle\zeta|\phi\rangle$ are normalized according to

$$\langle\phi|\phi\rangle = \int d\zeta\,|\langle\zeta|\phi\rangle|^2 = 1, \tag{17}$$

and represent the probability amplitude to find n-partons at transverse impact separation $\zeta = z$. Its eigenvalues are determined by the boundary conditions at $\phi(z = 1/\Lambda_{\text{QCD}}) = 0$ and are given in terms of the roots of Bessel functions: $\mathcal{M}_{L,k} = \beta_{L,k}\Lambda_{\text{QCD}}$. The normalizable modes are

$$\phi_{L,k}(\zeta) = \frac{\sqrt{2}\Lambda_{\text{QCD}}}{J_{1+L}(\beta_{L,k})}\sqrt{\zeta}J_L(\zeta\beta_{L,k}\Lambda_{\text{QCD}})\,\theta\big(\zeta \leq \Lambda_{\text{QCD}}^{-1}\big). \tag{18}$$

The lowest stable state $L = 0$ is determined by the Breitenlohner-Freedman bound [26]. Higher excitations are matched to the small z asymptotic behavior of each string mode to the corresponding conformal dimension of the boundary operators of each hadronic state. The effective wave equation (16) is a relativistic light-front equation defined at $x^+ = 0$. The AdS metric ds^2 (1) is invariant if $\mathbf{x}_\perp^2 \rightarrow \lambda^2\mathbf{x}_\perp^2$ and $z \rightarrow \lambda z$ at equal light-front time $x^+ = 0$. The Casimir operator for the rotation group $SO(2)$ in the transverse light-front plane is L^2. This shows the natural holographic connection to the light front.

For higher spin-S bosonic modes [27], we can also recast the AdS wave equation describing a p-form field Φ_P $(P = S)$ into its light-front form (15). Using the substitution $\phi_S(\zeta) = \zeta^{-3/2+S}\Phi_S(\zeta)$, $\zeta = z$, we find a light-front Schrödinger equation identical to (16) with $\phi \rightarrow \phi_S$, provided that $(\mu R)^2 = -(2-S)^2 + L^2$. Stable solutions satisfy a generalized Breitenlohner-Freedman bound $(\mu R)^2 \geq -(d-2S)^2/4$, and thus the lowest stable state has scaling dimensions $\Delta = 2$, independent of S. The fundamental light-front equation of AdS/CFT has the appearance of a Schrödinger equation, but it is relativistic, covariant, and analytically tractable.

The pseudoscalar meson interpolating operator $\mathcal{O}_{2+L} = \bar{q}\gamma_5 D_{\{\ell_1}\cdots D_{\ell_m\}}q$, written in terms of the symmetrized product of covariant derivatives D with total internal space-time orbital momentum $L = \sum_{i=1}^m \ell_i$, is a twist-two, dimension $3+L$ operator with scaling behavior determined by its twist-dimension $2+L$. Likewise the vector-meson operator $\mathcal{O}_{2+L}^\mu = \bar{q}\gamma^\mu D_{\{\ell_1}\cdots D_{\ell_m\}}q$ has scaling dimension $2+L$. The scaling

behavior of the scalar and vector AdS modes is precisely the scaling required to match the scaling dimension of the local pseudoscalar and vector-meson interpolating operators. The spectral predictions for the hard wall model for both light meson and baryon states is compared with experimental data in [7].

A closed form of the light-front wavefunctions $\tilde{\psi}(x, \mathbf{b}_\perp)$ follows from (8)

$$
\tilde{\psi}_{L,k}(x, \mathbf{b}_\perp) = \frac{\Lambda_{\mathrm{QCD}}}{\sqrt{\pi} J_{1+L}(\beta_{L,k})} \sqrt{x(1-x)}
$$
$$
\times J_L\left(\sqrt{x(1-x)} \, |\mathbf{b}_\perp| \beta_{L,k} \Lambda_{\mathrm{QCD}}\right) \theta\left(\mathbf{b}_\perp^2 \leq \frac{\Lambda_{\mathrm{QCD}}^{-2}}{x(1-x)}\right). \quad (19)
$$

The resulting wavefunction displays confinement at large interquark separation and conformal symmetry at short distances, reproducing dimensional counting rules for hard exclusive amplitudes.

5 Conclusions

Light-Front Holography is one of the most remarkable features of AdS/CFT. It allows one to project the functional dependence of the wavefunction $\Phi(z)$ computed in the AdS fifth dimension to the hadronic frame-independent light-front wavefunction $\psi(x_i, \mathbf{b}_{\perp i})$ in $3+1$ physical space-time. The variable z maps to $\zeta(x_i, \mathbf{b}_{\perp i})$. As we have discussed, this correspondence is a consequence of the fact that the metric ds^2 for AdS_5 at fixed light-front time τ is invariant under the simultaneous scale change $\mathbf{x}_\perp^2 \rightarrow \lambda^2 \mathbf{x}_\perp^2$ in transverse space and $z^2 \rightarrow \lambda^2 z^2$. The transverse coordinate ζ is closely related to the invariant mass squared of the constituents in the LFWF and its off-shellness in the light-front kinetic energy, and it is thus the natural variable to characterize the hadronic wavefunction. In fact ζ is the only variable to appear in the light-front Schrödinger equations predicted from AdS/QCD. These equations for both meson and baryons give a good representation of the observed hadronic spectrum, especially in the case of the soft wall model. The resulting LFWFs also have excellent phenomenological features, including predictions for the electromagnetic form factors and decay constants.

It is interesting to note that the form of the nonperturbative pion distribution amplitude $\phi_\pi(x)$ obtained from integrating the $q\bar{q}$ valence LFWF $\psi(x, \mathbf{k}_\perp)$ over $\mathbf{k}_\perp$, has a quite different x-behavior than the asymptotic distribution amplitude predicted from the PQCD evolution [28] of the pion distribution amplitude. The AdS prediction $\phi_\pi(x) = \sqrt{3} f_\pi \sqrt{x(1-x)}$ has a broader distribution than expected from solving the Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation in perturbative QCD. This observation appears to be consistent with the results of the Fermilab diffractive dijet experiment [29], the moments obtained from lattice QCD [7] and pion form factor data [30].

Nonzero quark masses are naturally incorporated into the AdS predictions [7] by including them explicitly in the LF kinetic energy $\sum_i \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i}$. Given the nonperturbative LFWFs one can predict many interesting phenomenological quantities such as heavy quark decays, generalized parton distributions and parton structure functions. The AdS/QCD model is semi-classical and thus only predicts the lowest valence Fock state structure of the hadron LFWF. In principle, the model can be systematically improved by diagonalizing the full QCD light-front Hamiltonian on the AdS/QCD basis.

Another interesting application is hadronization at the amplitude level. In this case one uses light-front time-ordered perturbation theory for the QCD light-front Hamiltonian to generate the off-shell quark and gluon T-matrix helicity amplitudes such as $e^+ e^- \to \gamma^* \to X$. The amplitude can be renormalized using the "alternate denominator" method [31]. If at any stage a set of color-singlet partons has light-front kinetic energy $\sum_i \mathbf{k}_{\perp i}^2 / x_i < \Lambda_{QCD}^2$, then one coalesces the virtual partons into a hadron state using the AdS/QCD LFWFs. A similar approach was used to predict antihydrogen formation from virtual positron–antiproton states produced in $\bar{p}A$ collisions [32].

The hard wall AdS/QCD model resembles bag models where a boundary condition is introduced to implement confinement. However, unlike traditional bag models, the AdS/QCD is frame-independent. An important property of bag models is the dominance of quark interchange as the underlying dynamics of large-angle elastic scattering, This agrees with the survey of two-hadron exclusive reactions [33]. In addition the AdS/QCD model implies a maximal wavelength for confined quarks and gluons and thus a finite IR fixed point for the QCD coupling. It is also consistent with the picture that QCD condensates only appear within the hadronic boundary [34].

Our analysis also allows one to predict the individual quark and gluon contributions to the gravitational form factors $A(q^2)$ and $B(q^2)$. Thus we can immediately predict the momentum fractions for quarks q and gluons g, $A_{q,g}(0) = \langle x_{q,g} \rangle$, and orbital angular momenta $B_{q,g}(0) = \langle L_{q,g} \rangle$ carried by each quark flavor and gluon in the hadron with sum rules $\sum_{q,g} A_{q,g}(0) = A(0) = 1$ and $\sum_{q,g} B_{q,g}(0) = B(0) = 0$. The last sum rule corresponds to the vanishing of the anomalous gravitational moment which is true Fock state by Fock state [21] in light-front theory.

The holographic mapping from $\Phi(z)$ to the light-front wave functions of relativistic composite systems provides a new tool for extending the AdS/CFT correspondence to theories such as QCD which are not conformally invariant. The mathematical consistency of light-front holography for both the electromagnetic and gravitational hadronic transition matrix elements demonstrates that the mapping between the AdS holographic variable z and the transverse light-front variable ζ, which is a function of the multi-dimensional coordinates of the partons in a given light-front Fock state $x_i, \mathbf{b}_{\perp i}$ at fixed light-front time τ, is a general principle.

Acknowledgments

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