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# Luminosity Lifetime*†

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### 3.4.1 Luminosity Lifetime

<u>Symmetric Collider</u>

In a symmetric or "energy transparent" relativistic collider, the luminosity is given by

$$L = \frac{N^2 f_c}{4\pi \sigma_x^* \sigma_y^*} \tag{1}$$

where $N$ is the number of electrons or positrons per bunch, $\sigma_x^*$ ($\sigma_y^*$) is the horizontal (vertical) rms beam size at the interaction point (IP[1]), and $f_c$ is the collision frequency[2]. If the beam sizes remain constant as the luminosity decreases, then the time dependence of luminosity is contained entirely in the time dependence of the beam currents, i.e., $N \varnothing N(t)$ , and we can rewrite Eq. (1) as

$$L(t) = \frac{N^2(t) f_c}{4\pi \sigma_x^* \sigma_y^*} \tag{2}$$

There are two distinct categories for luminosity loss. In the first category are loss processes due to collisions between the two beams, that is, processes associated directly with the luminosity. In the second category (see below) are single-beam loss processes. The processes in the first category relevant to a high-energy collider are Bhabha scattering ($e^+e^- \varnothing e^+e^-$) and "radiative" Bhabha scattering ($e^+e^- \varnothing e^+e^-\gamma$). In the first process, a beam particle is lost if its angular deflection is beyond the ring's transverse acceptance; in the second process, loss occurs if the beam particle's momentum change is outside the longitudinal acceptance of the ring (typically determined by the RF bucket height). Useful expressions for the loss cross sections for both Bhabha and radiative Bhabha scattering can be found in [1].

For a symmetric collider with a loss cross section $\sigma$, the particle loss rate is

$$\frac{dN}{dt}(t) = -\sigma L(t) = -\sigma k N^2 \tag{3}$$

where, from Eq. (2), we define $k = L_0 / N_0^2$ and the subscript zero denotes $t = 0$, that is, $L_0 + L(0)$ and $N_0 + N(0)$. Solving Eq. (3) yields [1]

$$N(t) = N_0 \left[ \frac{1}{1 + (\sigma L_0 / N_0) t} \right] \tag{4}$$

and

$$L(t) = L_0 \left[ \frac{1}{1 + (\sigma L_0 / N_0) t} \right]^2 \tag{5}$$

The time for the luminosity to decay to a fraction $f$ of its initial value is

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[1] Quantities evaluated at the IP are denoted here by appending an asterisk to the symbol.

[2] For a circular collider with bunch spacing $s_B$, the collision frequency is $f_c = \beta c / s_B$.

$$t_f = \frac{N_0}{\sigma L_0}\left(\frac{1}{\sqrt{f}} - 1\right) \tag{6}$$

Asymmetric Collider

Generalizing to a two-ring asymmetric collider, Eq. (2) becomes

$$L(t) = \frac{N_-(t)\,N_+(t)\,f_c}{2\pi\sqrt{\left(\sigma_{x,+}^{*2} + \sigma_{x,-}^{*2}\right)\left(\sigma_{y,+}^{*2} + \sigma_{y,-}^{*2}\right)}} \tag{7}$$

The particle loss is now represented by two coupled equations [1]

$$\frac{dN_+}{dt}(t) = -\sigma_+ k\, N_+ N_- \qquad \frac{dN_-}{dt}(t) = -\sigma_- k\, N_+ N_- \tag{8}$$

where, in an obvious notation, $k = L_0/N_{0+}N_{0-}$ and we no longer assume that the loss cross sections for the two beams are identical.

As given in [1], the solution to Eq. (8) is

$$N_+(t) = N_{0+}\left(\frac{1-r}{e^{Gt} - r}\right) \qquad N_-(t) = N_{0-}\left(\frac{1-r}{1-re^{-Gt}}\right) \tag{9}$$

where

$$G = L_0\left(\frac{\sigma_+}{N_{0+}} - \frac{\sigma_-}{N_{0-}}\right) \tag{10}$$

and

$$r = \frac{N_{0+}\,\sigma_-}{N_{0-}\,\sigma_+} \tag{11}$$

The time dependence of the luminosity becomes

$$L(t) = L_0 e^{Gt}\left(\frac{1-r}{e^{Gt}-r}\right)^2 \tag{12}$$

and the time for the luminosity to decay to a fraction $f$ of its initial value is

$$t_f = \frac{1}{G}\ln\left\{\frac{1}{2f}\left[(1-r)^2 + 2fr + (1-r)\sqrt{(1-r)^2 + 4fr}\right]\right\} \tag{13}$$

Other Loss Processes

In addition to the category of loss mechanisms directly related to the luminosity, a full treatment of luminosity lifetime must also take into account the single-beam losses that can independently decrease $N_+$ and $N_-$. These include gas scattering (both elastic and inelastic i.e., gas bremsstrahlung) and Touschek scattering, an intrabeam scattering process that exchanges momentum between transverse and longitudinal coordinates. Each of these topics is treated elsewhere in this Handbook. In a simple model, the gas scattering rate is independent of beam intensity, leading to a simple exponential decay. However, in a typical ring the residual gas pressure, and thus the decay rate, includes a term proportional to the beam intensity, leading to non-exponential decay. The decay rate for Touschek scattering also depends on the beam intensity [2], so the resultant lifetime is not exponential. For typical collider parameters, it is the beam-gas bremsstrahlung that is the most important of the single-beam loss mechanisms.

References

[1] F. C. Porter, *Nucl. Instr. Meth.* A302, 209 (1991), and references contained therein.

[2] M. S. Zisman, J. Bisognano, and S. Chattopadhyay, *Part. Accel.* 23, 289 (1988); M. S. Zisman, J. Bisognano, and S. Chattopadhyay, *ZAP User's Manual*, LBL-21270; UC-28, December 1986, unpublished.