Statistical Studies on Sequential Probability Ratio Test for Radiation Detection

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Abstract *- A Sequential Probability Ratio Test (SPRT) algorithm helps to increase the reliability and speed of radiation detection. This algorithm is further improved to reduce spatial gap and false alarm. SPRT, using Last-in-First-Elected-Last-Out (LIFELO) technique, reduces the error between the radiation measured and resultant alarm. Statistical analysis determines the reduction of spatial error and false alarm.*

Keywords: Background, Gamma Count, Samples, Source

1 Introduction

Reliable and fast estimation of the local relative radiation level, with respect to that of the regional natural background, under prescribed confidence levels, are highly desirable [1]. When taking radiation measurements, exposure to regions must be reduced. Precise location of the alarm with respect to the radiation strength as well as accurate interpretation of the source and background region helps reduce exposure to radiation hazards. Conventional SPRT algorithm is implemented in a FIFO (First-In-First-Out) scheme. In an attempt to reduce the spatial uncertainties, Yuan and Kernan [2] proposed a LIFELO (Last-In-First-Elected-Last-Out) for handling temporal-spatial survey data. In this paper, two algorithms are researched and implemented to compare merits and defects. Algorithm FIFO uses oldest data points of the sample to make a decision, but algorithm LIFELO uses youngest data points of the sample to make a decision. Compared to FIFO, LIFELO decreases spatial error and false alarm.

The rest of the paper is organized as follows. Section 2 presents the FIFO algorithm and the newly proposed algorithm is presented. Section 3 looks into the experimental data obtained from the LIFELO algorithm. Section 4 includes an intriguing analysis and discussion of the two algorithms. Section 5 concludes the paper.

2 Algorithm

2.1 First-In-First-Out (FIFO)

FIFO generates a conclusion about the location being source or background based on the first several data points of each sample. Let $\{x_1, x_2, \ldots, x_{11}\}$ be the measurements of the same sample. FIFO starts from x_1 to compute LLR (Log Likelihood Ratio [LLR] for the sample). The formulation and computational details are based on [1].

According to [1], the null Hypothesis (H0) is used to represent possibility that the sample was from background region, and the Alternative Hypothesis (H1) is used to represent possibility that the sample was from source region. Test strengths α and β denote false positive and false negative errors respectively. False positive means that H1 is accepted while H0 is true; likewise, false negative means that H0 is accepted while H1 is true.

Parameters are set to specify the minimum or maximum number of measurements required to make a decision about the location being source or background. The minimum (maximum) number of required measurements are denoted by imin (imax). Clearly, parameters imin and imax for the source must be smaller than those for the background to keep the exposure time to a minimum.

The process to compute LLR of the sample is an iteration of numerical analysis. First compute mean and standard deviation based on data point x_1, then take in one more data point into consideration in each loop. When the minimum data points are processed, LLR can be generated. If the current result is less than background threshold, a decision that the sample is in background area can be made. Likewise, if the current result is greater than source threshold, a decision that the sample is in source area can be made. The dilemma occurs when dealing with results in between background and source thresholds. In this situation, more data points are needed to make a decision. If the maximum data points have been used and the current result is still in between background and source thresholds, an approximate decision is made. Mean of the thresholds of background and source is used to help make a decision. If the current result is less than this mean, the sample is in background area; otherwise, the sample is in source area.

Another important parameter is the overlapping factor denoted by ilap used to treat the sampling sequence as a continuous sequence. Meaning of ilap is shown in Fig. 1.

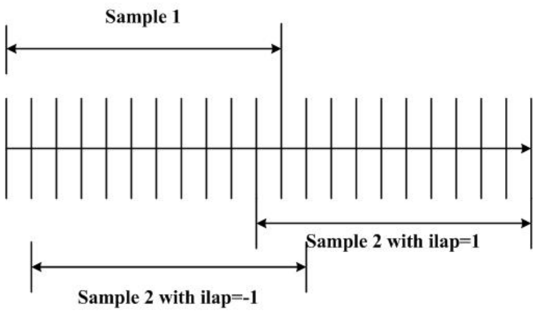

Figure 1: Meaning of parameter ilap

In Figure 1, if ilap is negative, the distance of the beginning of one sample to the beginning of the next is absolute value of ilap. If ilap is positive, the distance of the beginning of one sample to the beginning of the next is n-ilap, where n is data points of each sample. In this experiment, n is 11. With this argument, sample data is used efficiently, and different samples have same pace from data points. This experiment uses ilap as -1, meaning current sample data points start from second point of last sample.

FIFO Algorithm:
Input:
background data;
source data;
error ratio to accept H1 when H0 is true (α);
error ratio to accept H0 when H1 is true (β);
hypotheses: normal or Poisson distribution.

Output:
alarm level;
number of data points used to make decision.

Process:
compute mean and standard deviation of background;
compute constants A, B according to α and β ;
choose formulas according to hypotheses combinations.

Loop:
Start *from the oldest to youngest* data point of the sample, take one data point, compute mean and standard deviation;
Compute LLR based on the data point of the sample;
Take another data point, continue the loop.

When LLR is lower than A or greater than B, or all data points of the sample have been computed, stop.

If no decision has been made when all data points of the sample have been computed, an approximate decision is made after comparing LLR and (A+B)/2.

2.2 Last-In-First-Out (LIFELO)

An improvement to the FIFO algorithm is the LIFELO algorithm. This algorithm takes the current sample point to start the SPRT calculation process. Now instead of taking new sample point to help compute the SPRT, the algorithm will take the previous sample points collected previously in the negative x-direction. The changes implemented to the algorithm to perform the SPRT in the negative x-direction are as follows. Create a stack of size 11, the maximum number of samples needed to conclude the alarm level of a sample. Push data points of the sample to stack, then pop each data point to compute.

The reason for stacking is that instead of working from the oldest data point to youngest, stacking will move from the current data point being calculated to the oldest available in the stack. The rest of the calculation process is the same as the previous algorithm. Once decision for the current sample has been made, the stack is emptied and pushed in data points of a new sample. The last change is instead of taking the data from the original whole *dataIn* vector, data is only taken from the stack. For example, whenever a data point (*xin*) is needed, the following code occurs.

LIFELO Algorithm:

The main process of this algorithm is similar to previous one; only difference is to use data points in a different direction, from the youngest to the oldest data points of the sample.
Input:
background data;
source data;
error ratio to accept H1 when H0 is true (α);
error ratio to accept H0 when H1 is true (β);
hypotheses: normal or Poisson distribution.

Output:
alarm level;
number of data points used to make decision.

Process:
Start *from the youngest to oldest* data point of the sample, take one data point, compute mean and standard deviation of background;
compute constants A, B according to α and β ;
choose formulas according to hypotheses combinations.

Loop:

Take one data point of the sample, compute mean and standard deviation;
Compute LLR based on the data point of the sample;
Take another data point, continue the loop.

When LLR is lower than A or greater than B, or all data points of the sample have been computed, stop. If no decision has been made when all data points of the sample have been computed, an approximate decision is made after comparing LLR and (A+B)/2.

Two algorithms' flowcharts are included in appendix.

3 Experimental Data

The data points taken to analyze the behavior of both the FIFO and LIFELO algorithms correspond to 5 different regions. The 35 samples are taken from regions that exhibit constant background and source, transition from background to source, and vice versa. Figure 2 shows the Gamma Count strength for the sample points selected. The output alarm for each sample is shown in Figure 3. The spatial gap with respect to output alarm for the FIFO algorithm is shown in Figure 4.

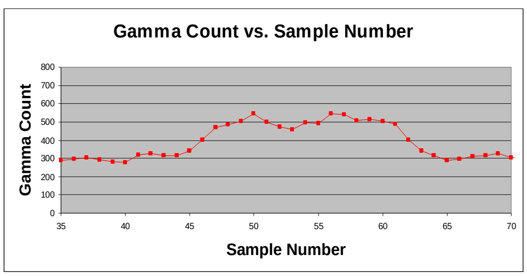

Figure 2: Gamma Count for sample 35 until 70

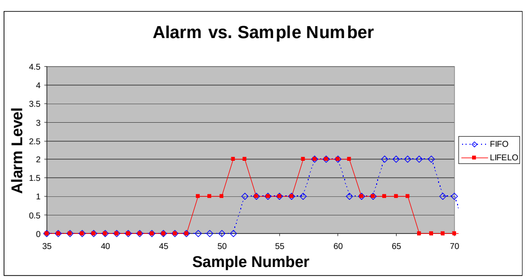

Figure 3: Output Alarm for FIFO and LIFELO

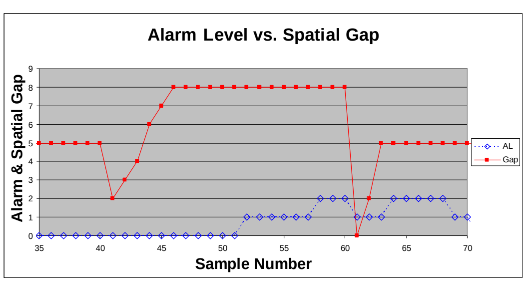

Figure 4: Spatial Gap for FIFO Algorithm

4 Analysis

To further analyze the result in detail and show the advantages and disadvantages of the FIFO and LIFELO schemes, a cluster of sample points are taken in such a way that there is a change from the background to the source region and vice versa. If there are no changes in the alarm output (such that it is always background or source) a detailed comparison between the two schemes cannot be made. The results of the two schemes are shown below.

Two major points of consideration when analyzing the data are false alarm and spatial gap. False alarm occurs when there is an inconsistency between the gamma count and the resultant alarm output level. Spatial gap is the distance or number of samples between the sample being computed and the resultant output. For example, when computing for the 11^{th} sample point, a resultant alarm is produced at the 6^{th} sample, translating to a spatial gap of five samples.

In Region 1, both algorithms generate an alarm level (AL) of 0. There are no false alarms in this region since all the samples are background. But the FIFO algorithm generates the alarm five sample points away from the youngest sample. This algorithm exhibits spatial error of five samples.

In Region 2, the false alarm level generated by the LIFELO algorithm will be higher than that rendered by the FIFO algorithm. The LIFELO algorithm takes the youngest sample being measured and moves towards the oldest sample. If the youngest sample is a source then the algorithm will require two additional samples to help decide the alarm level and consequently generate alarm of 1 or greater. On the other hand, FIFO algorithm takes the oldest sample available and moves towards the youngest sample. If the youngest sample is source and the oldest sample is background, the computation begins from the background sample to the source. The computation may generate alarm before reaching the youngest sample, therefore giving a false alarm reading for the sample measured. The spatial error in this region varies. Spatial error is minimal at the beginning of the transition region and increases as the region approaches source.

In Region 3, FIFO algorithm generates false alarm. The alarm level generated by LIFELO algorithm will be higher in the initial part of this region. This occurs because FIFO algorithm still uses sample from the background and the transition region to deduce the alarm level for the youngest sample. On the other hand, the FIFO algorithm uses only three samples to generate the alarm level and therefore having a spatial gap of eight samples.

Table 1: FIFO & LIFELO Results

	K	GC	AL (FIFO)	AL (LIFELO)	Spatial Gap(FIFO)
Region 1	35	290	0	0	5
	36	298	0	0	5
	37	303	0	0	5
	38	293	0	0	5
	39	281	0	0	5
	40	279	0	0	5
Region 2	41	320	0	0	2
	42	328	0	0	3
	43	314	0	0	4
	44	316	0	0	6
	45	346	0	0	7
	46	401	0	0	8
	47	468	0	0	8
Region 3	48	484	1	0	8
	49	503	1	0	8
	50	543	1	0	8
	51	500	2	0	8
	52	475	2	1	8
	53	457	1	1	8
	54	494	1	1	8
	55	492	1	1	8
	56	543	1	1	8
	57	541	2	1	8
	58	508	2	2	8
	59	516	2	2	8
	60	504	2	2	8
Region 4	61	487	2	1	0
	62	402	1	1	2
	63	340	1	1	5
	64	317	1	2	5
	65	284	1	2	5
Region 5	66	298	1	2	5
	67	310	0	2	5
	68	315	0	2	5
	69	327	0	1	5
	70	306	0	1	5

For Region 4, FIFO algorithm generates false alarm. The reason for this occurrence is similar to Region 2, but now the youngest sample is background and the oldest sample is source. For example, the algorithm begins computation using source samples and moves towards background samples. The resultant alarm may be generated before reaching the youngest sample. Therefore the strength of the FIFO alarm is higher than the LIFELO alarm in this region. The LIFELO algorithm uses background and weak source samples to begin the alarm computation. The spatial error behaves similar to Region 2, being minimal at the beginning and increasing as the region changes to background.

Region 5 is similar to Region 1, having spatial error of five samples.

Table 2: Sample Regions

Region No.	Region Type	Transition	Sample No.
1	Background	No	35-40
2	Background to Source	Yes	41-47
3	Source	No	48-60
4	Source to Background	Yes	61-65
5	Background	No	66-70

5 Conclusion

We have shed some light on the improvement to the SPRT radiation detection and compared with the existing algorithm. The spatial error and the false alarm have been detected and reduced.

The LIFELO algorithm is spatial error free while the FIFO algorithm has spatial error depending on the region of the sample. In the source region the spatial error is high: nearly eight samples; in the background region the spatial error is five samples. The spatial error is the lowest in the beginning of both the transition region, from background to source and source to background.

FIFO algorithm generates false alarm, and the alarm output is shifted to the right (alarm occurring after the LIFELO alarm has occurred). For the FIFO algorithm, false negative reading occurs in Region 2 and is potentially dangerous for technicians taking the readings. Alarm reading in Region 4 is false positive. Comparison between the two algorithms clearly indicates that the LIFELO algorithm outperforms the FIFO algorithm and will be used for future work.

Particular interest for further research is to investigate and develop further efficient algorithms for dynamic background level. Methods to help reduce discrepancy in LIFELO algorithm in extreme cases are to be investigated.

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Appendix: Flowcharts of algorithm FIFO and LIFELO. Solid line represents FIFO, and dashed line represents LIFELO.