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Cosmological Constant as a Manifestation of the Hierarchy

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There has been the suggestion that the cosmological constant as implied by the dark energy is related to the well-known hierarchy between the Planck scale, $M_{\rm Pl}$, and the Standard Model scale, $M_{\rm SM}$. Here we further propose that the same framework that addresses this hierarchy problem must also address the smallness problem of the cosmological constant. Specifically, we investigate the minimal supersymmetric (SUSY) extension of the Randall-Sundrum model where SUSY-breaking is induced on the TeV brane and transmitted into the bulk. We show that the Casimir energy density of the system indeed conforms with the observed dark energy scale.

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The accelerating expansion of the present universe was discovered in 1998 [1, 2] and further confirmed by type Ia supernova (SN Ia) distance measurement [3, 4] and other observations [5, 6]. This cosmic acceleration may be driven by anti-gravity (repulsive gravity) generated by some energy source, generally referred to as dark energy. By far positive cosmological constant (CC) is the simplest realization of dark energy, which has become more favored by the recent observations [3, 4, 5, 6]. If the dark energy is indeed a cosmological constant which never changes in space and time, then it must be a fundamental property of the spacetime. This would then introduce a new energy scale, $M_{\rm CC} \simeq \rho_{\rm DE}^{1/4} \sim 10^{-3}\,{\rm eV}$, which is 15 orders of magnitude smaller than the Standard Model scale, $M_{\rm SM} \sim$ TeV. Why is this energy gap so huge?

There has been another well-known hierarchy problem in physics, i.e., the existence of a huge gap between the Standard Model scale and the Planck scale of quantum gravity at $M_{\rm Pl} \sim 10^{19}\,{\rm GeV}$ by a factor $\sim 10^{16}$. The surprising numerical coincidence between these two energy gaps prompts us to the wonder: Are these two hierarchy problems related?

The idea that these two hierarchies are actually related is not new. Arkani-Hamed et al. [7] first invoked it to address the cosmic coincidence problems. Various authors employed it under different guises of "cosmological constant seesaw relation" [8]. Recently one of us introduced yet another variation of the theme [9]. If one equates the two energy gaps as

$$M_{\rm CC} \simeq \frac{M_{\rm SM}}{M_{\rm pl}} M_{\rm SM} = \left(\frac{M_{\rm SM}}{M_{\rm Pl}}\right)^2 M_{\rm Pl} \equiv \alpha_G^2 M_{\rm Pl}, \qquad (1)$$

then it suggests that the underlying mechanism which induces the CC must be resulted from a double suppression by the same hierarchy factor descended from the Planck scale to the SM scale.

We note that such a situation is not unique in physics. For example, in atomic physics the hydrogen ground state energy is suppressed from the electron rest mass, m_e, an energy scale which enters naturally into the Schrödinger equation, by two powers of the fine structure constant, $\alpha = e^2/\hbar c$, due to the presence of the coupling constant, e, in the Coulomb potential. Analogous to that, the Planck-SM hierarchy ratio can be viewed as a 'gravity fine structure constant', α_G. Various physical energy scales in the system would then be associated with the Planck mass through different powers of α_G.

Guided by this philosophy, we construct a model for CC by exploiting the Randall-Sundrum (RS1) [10] geometry that addresses the SM-Planck hierarchy problem as our framework. To accomplish our goal we find it necessary to extend the RS1 model to incorporate minimal supersymmetry (SUSY). SUSY guarantees the perfect cancellation of the vacuum energy between super-partners from the outset. It therefore serves as a natural foundation in solving the smallness problem of CC. Since SUSY must be broken in our 4d world, we device its breaking via a Higgs field on the TeV brane, which is then transmitted to the bulk through its coupling to gravitino. Aside from this rather natural and minimal extension, we follow the original RS1 scenario where the gravity sector lives in the bulk while the standard model fields are confined on the TeV brane. In the brane scenario where extra dimensions are compactified the existence of the 4d Casimir energy on the brane induced by the bulk field is inevitable. Such a vacuum energy is a natural candidate for CC. Our task is to demonstrate that the Casimir energy in our setup scales generically as $\alpha_G^2 M_{\rm Pl}$.

Casimir effect has been considered as a possible origin for the dark energy by many authors [11, 12, 13, 14, 15, 16, 17]. It is known that the conventional Casimir energy

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in the ordinary 3+1 dimensional spacetime cannot provide repulsive gravity necessary for dark energy. On the other hand, Casimir energy on a 3-brane imbedded in a higher-dimensional world with suitable boundary conditions can in principle give rise to a positive cosmological constant. The general expression for the 4d Casimir energy density on the 3-brane contributed from a bulk field is given by

$$\rho_{\rm C}^{(4)} = \left[\frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\sum_n \sqrt{p^2+m_n^2}\right]_{\rm ren} , \qquad (2)$$

where p is the momentum in the ordinary 3-space, m_n the mass of the nth Kaluza-Klein (KK) mode, and the subscript "ren" stands for renormalization. For a massless field propagating in a flat bulk, the KK mass scales as $m_n \sim n/a$, and the resulting Casimir energy density on the brane scales as

$$\rho_{\rm C}^{(4)} \sim a^{-4} , \qquad (3)$$

where a is the extra dimension size. As summarized by Milton [11], for $\rho_{\rm C}^{(4)}$ to conform with dark energy the required extra dimension scale would have to be large.

Casimir energy in the RS1 geometry has been investigated by several authors [18, 19, 20]. In the supersymmetric brane-world, the contributions to the Casimir energy on the brane from the bulk field superpartners cancel each other perfectly. When SUSY is broken on the brane, its modification of the KK mass spectrum becomes the primary source of Casimir energy. Assuming that the gravitino KK mass spectrum is modified from m_n^2 to $m_n^2+\delta m_n^2$ while the graviton mass spectrum remains unchanged, then the net 4d Casimir energy density on the brane is

$$\Delta\rho_{\rm C}^{(4)} = \rho_{\rm C}^{(4)}\Big|_{\delta m_n^2} - \rho_{\rm C}^{(4)}\Big|_{\delta m_n^2=0\ \forall\ n} , \qquad (4)$$

where $\rho_{\rm C}^{(4)}|_{\delta m_n^2}$ denotes that in Eq. (2) with m_n^2 therein replaced by $m_n^2+\delta m_n^2$. Note that the term "$-\rho_{\rm C}^{(4)}|_{\delta m_n^2=0}$" is exactly the contribution from the SUSY partner. Generally speaking, when the SUSY-breaking induced KK mass-square shift is much smaller than the energy gap, i.e., $\delta m_n^2 \ll (m_n - m_{n-1})^2 \equiv \Delta m_n^2$, and if both are insensitive to n, then it can be shown that the net 4d Casimir energy density on the brane scales as

$$\Delta\rho_{\rm C}^{(4)} \sim \Delta m_n^2 \delta m_n^2 . \qquad (5)$$

We emphasize that this scaling for the brane Casimir energy under SUSY-breaking is generic, and is essential for attaining the desired CC scale. Relevant to our consideration, it has been shown that in the RS1 geometry the graviton and gravitino KK mass spectra on the TeV brane scales as $\Delta m_n \sim \alpha_G M_{\rm Pl}$ [21, 22], which is not surprising. What we are obliged to demonstrate is that the graviton-gravitino KK nonzero mode mass-square difference induced by our SUSY-breaking mechanism is compelled to be as small as $\delta m_n^2 \sim (\alpha_G^3 M_{\rm Pl})^2$. We will show that such an extremely small value can emerge very naturally in our setup.

The Randall-Sundrum RS1 model invokes the following metric:

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + a^2 dy^2 , \qquad (6)$$

where $\sigma = ka|y|$, $\mu,\nu = 0,1,2,3$, $-\pi \le y \le \pi$, and a is the radius and k the curvature of the orbifold $\mathcal{S}^1/\mathcal{Z}_2$ in the compactified 5th dimension y. The hidden, or Planck, brane locates at $y=0$ while the visible, or TeV, brane locates at $y=\pi$. As is well-known, the Planck-SM hierarchy is bridged if $ka \sim \mathcal{O}(10)$ so that the mass scale at $y=\pi$ is suppressed by the warp factor $e^{-\pi ka} = \alpha_G$. It is customary to take $k \sim M_{\rm Pl}$. So in the RS1 model the extra dimension size a is only about 10 times the Planck length.

Supersymmetry in a slice of AdS spacetime has been investigated by various authors [21, 22, 23]. The complete supergravity action for our configuration would include graviton, gravitino and graviphoton. But in our construction the graviton KK masses would remain unchanged at the tree level in our SUSY-breaking mechanism. Therefore it suffices our purpose that we concentrate on the gravitino kinetic and mass terms only, which are given by [21]:

$$\begin{aligned} S &= S_5 + S_0 + S_\pi , \\ S_5 &= \int d^4x \int dy \sqrt{-g}\Big[-\frac{1}{2}M_5^3\Big(\mathcal{R} + i\bar{\Psi}_M^i \gamma^{MNP} D_N \Psi_P^i \\ &\quad -i\frac{3}{2}\sigma' \bar{\Psi}_M^i \gamma^{MN}(\sigma_3)^{ij}\Psi_N^j\Big) - \Lambda\Big] , \\ S_{0,\pi} &= \int d^4x \sqrt{-g_4}\Big[\mathcal{L}_{0,\pi} - \Lambda_{0,\pi}\Big] . \end{aligned} \qquad (7)$$

Here $M,N,P = (\mu,5)$, $g = \det(g_{MN})$, γ^{MNP} is the antisymmetric product of gamma matrices, and $\sigma' \equiv d\sigma/dy$. Note that under our metric convention, $\sqrt{-g} = ae^{-4\sigma}$. $\mathcal{R}$ is the 5d Ricci scalar, Ψ_M^i $(i=1,2)$ the two symplectic Majorana gravitinos, and Λ is the bulk cosmological constant. M_5 is the 5d Planck mass, which is related to the 4d Planck mass by $M_{\rm Pl}^2 = (1-\alpha_G)M_5^3/k$.

The RS1 metric solution of Eq. (6) to the 5d Einstein equations is valid provided that the bulk and boundary cosmological constants are related by $\Lambda = -6M_5^3k^2 \simeq -6M_{\rm Pl}^2k^3$ and $\Lambda_0 = -\Lambda_\pi = -\Lambda/k$. With $k \sim M_{\rm Pl}$, we have $\Lambda_\pi \simeq M_{\rm Pl}^4$. This is associated with the "old" cosmological constant problem [24] where the field-theoretical argument would result in the vacuum energy which is 123 orders of magnitude too large. The resolution of this problem is beyond our ability here. We merely assume that this problem would be resolved in the future. (For a review of current attempts and possible approaches to solving this problem, see [25].) With the assumption that such an unphysical value for the vacuum energy can be removed, our effort is to address the new cosmological constant problem of its nonzero but extremely small energy scale.

The gravitino supersymmetry transformation is given by

$$\delta\Psi^i_M = D_M\eta^i + \frac{\sigma'}{2}\gamma_M(\sigma_3)^{ij}\eta^j\,, \tag{8}$$

where the symplectic Majorana spinor $\eta^i (i = 1, 2)$ is the 5d supersymmetry parameter. The equation of motion, i.e., the Rarita-Schwinger equation, for the bulk gravitino in the AdS background is

$$\gamma^{NMP}D_N\Psi_P - \frac{3}{2}\sigma'\gamma^{MP}\Psi_P = 0\,. \tag{9}$$

The KK decomposition and the associated eigenmodes for bosons and fermions in the RS1 geometry have been well studied in recent years [21, 22, 26, 27]. Goldberger and Wise [26] first studied the behavior of bulk scalar field in the RS1 model. Flachi et al. [27] investigated that for the bulk fermion field. Gherghetta and Pomarol (GP1) [21] extended the study to different supermultiplets in the bulk. The bulk gravitino field in the RS1 AdS geometry was studied in detail in a second paper by Gherghetta and Pomarol (GP2) [22]. Here we briefly summarize those results relevant to our discussion. The 5d fields are decomposed as

$$\Psi_{\mu,L,R}(x^\mu, y) = \frac{1}{\sqrt{2\pi a}}\sum_n \Psi^{(n)}_{\mu,L,R}(x^\mu) f^{(n)}_{L,R}(y)\,, \tag{10}$$

where $\Psi_{\mu,L}(\Psi_{\mu,R})$ are defined even (odd) under the $\mathcal{Z}_2$-parity. $\Psi_{5,L,R}$ and $\eta_{L,R}$ follow the similar KK decomposition. GP2 solved the equation of motion and found the y-dependent KK eigenfunction as

$$f^{(n)}_L = \frac{1}{N_n}e^{3\sigma/2}\Big[J_2\Big(\frac{m_n}{k}e^\sigma\Big) + bY_2\Big(\frac{m_n}{k}e^\sigma\Big)\Big], \tag{11}$$

$$f^{(n)}_R = \frac{\sigma'}{kN_n}e^{3\sigma/2}\Big[J_1\Big(\frac{m_n}{k}e^\sigma\Big) + bY_1\Big(\frac{m_n}{k}e^\sigma\Big)\Big], \tag{12}$$

where m_n is the 4d mass for the nth mode and $b = -J_1(m_n/k)/Y_1(m_n/k)$ satisfies the boundary condition: $b(m_n) = b(\alpha_G^{-1}m_n)$. In the limit where $m_n \ll k$ and $ka \gg 1$ the 4d KK masses for $n = 1, 2, ...,$ which are identical for both even and odd modes, are found to be

$$m_n \simeq \alpha_G\Big(n + \frac{1}{4}\Big)\pi k\,. \tag{13}$$

Note that the KK mass spectrum energy gap between adjacent modes is independent of n, and $\Delta m_n \sim \pi\alpha_G M_{\rm Pl} \sim$ TeV, as mentioned earlier.

Now we introduce the following action as a perturbation to break SUSY:

$$\begin{aligned} S_{\Phi\Psi} &= \int d^4x \int \frac{dy}{a}\delta(y-\pi)\sqrt{-g}g_5\Phi(x)\bar\Psi(x,y)\Psi(x,y) \\ &= \sum_{n=0}^{\infty}\int \frac{dy}{a}\delta(y-\pi)\sqrt{-g}\frac{1}{2\pi a}[f^{(n)}]^2 \\ &\quad\times\int d^4x g_5\Phi(x)\bar\Psi^{(n)}(x)\Psi^{(n)}(x) \\ &\equiv \sum_{n=0}^{\infty}\int d^4x\delta m_n\bar\Psi^{(n)}(x)\Psi^{(n)}(x)\,, \end{aligned} \tag{14}$$

where Φ is the fundamental Higgs field on the brane, $f^{(n)} = f^{(n)}_L + f^{(n)}_R$ and g_5 the 5d Higgs-gravitino Yukawa coupling. The gravitino mass-square shift for the nth KK mode is thus

$$\delta m_n^2 = \left(g_5\langle\Phi\rangle\int\frac{dy}{a}\delta(y-\pi)\sqrt{-g}\frac{1}{2\pi a}[f^{(n)}]^2\right)^2\,, \tag{15}$$

where $\langle\Phi\rangle \sim M_{\rm Pl}$ is the vacuum expectation value (vev) of the fundamental Higgs field [*cf.* Eq.(17) in RS1]. As demonstrated in RS1, the physical mass scales on the TeV brane are set, however, by the symmetry breaking scale, $\alpha_G M_{\rm Pl}$, instead. The fundamental 5d coupling g_5 has the dimensionality of 1/mass and thus $g_5 \sim 1/M_{\rm Pl}$. Therefore $g_5\langle\Phi\rangle \sim 1$, and the suppression of the SUSY-breaking gravitino mass shift is due solely to the extreme smallness of the y-integral, which represents the probability of finding the nth KK mode of gravitino on the TeV brane.

We note that for $n \ge 1$ the argument of the Bessel functions in Eqs. (11) and (12) is $\alpha_G^{-1}m_n/k \simeq (n+1/4)\pi \gg 1$. Accordingly the values of the Bessel functions are either $\simeq 0$ or $\simeq \pm\sqrt{2/n\pi^2}$. The normalization constant for the nth mode can be determined from Eqs. (10)–(12) (via the normalization condition for $\Psi_{\mu,L,R}$ [21, 22]), and it can be shown that

$$N_n \simeq \frac{\alpha_G^{-1}}{\sqrt{2\pi ka}}J_2(\alpha_G^{-1}m_n/k) \simeq \frac{\alpha_G^{-1}}{\sqrt{n\pi^3 ka}}\,. \tag{16}$$

It is interesting to note that in this limit the KK gravitino wavefunction on the TeV brane is independent of n:

$$f^{(n)}(y=\pi) \simeq \sqrt{2\pi ka}\,\alpha_G^{-1/2}\,. \tag{17}$$

Collecting all the σ-dependences, we find that the y-integral of the SUSY-breaking action scales as

$$\frac{1}{2\pi a}\int\frac{dy}{a}\delta(y-\pi)\sqrt{-g}[f^{(n)}]^2 \simeq k\alpha_G^3\,. \tag{18}$$

Putting these together, we obtain the SUSY-breaking induced KK gravitino mass-square shift for $n \ge 1$ modes:

$$\delta m_n^2 \sim \left(\alpha_G^3 M_{\rm Pl}\right)^2\,. \tag{19}$$

As for the $n = 0$ mode, its wavefunction localizes on the Planck brane instead, with $f^{(0)}_L(y=\pi) \propto e^{-\sigma/2} = \alpha_G^{1/2}$ [22]. This results in the mass shift $\delta m_0 \simeq \alpha_G^5 M_{\rm Pl}$, which is totally negligible.

Now we prove the generic Casimir energy scaling in Eq. (5). Employing the Jacobi θ function and the reflection formula,

$$\theta(z;x) = \sum_{n=-\infty}^{\infty}e^{-\pi n^2 x}e^{2\pi nz} = \frac{1}{\sqrt{x}}e^{\pi z^2/x}\theta\left(\frac{z}{ix};\frac{1}{x}\right) \tag{20}$$

for the regularization, one can obtain

$$\rho^{(4)}_{\rm C}(\mu^2) \sim -\frac{\Delta m_n^4}{2^5\pi^{5/2}}\sum_{n=-\infty}^{\infty}{}'\int_0^\infty\frac{(-1)^n y^{3/2}dy}{\exp[\mu^2/y + 4n^2 y]}\,, \tag{21}$$

where $\mu^2 \equiv \delta m_n^2/\Delta m_n^2$ is assumed to be insensitive to n, and the prime denotes the summation without the $n = 0$ term.

In the case where $\mu^2 \ll 1$,

$$\rho_{\rm C}^{(4)}(\mu^2) = \rho_{\rm C}^{(4)}\Big|_{\mu^2=0} + \frac{d\rho_{\rm C}^{(4)}}{d\mu^2}\Bigg|_{\mu^2=0} \mu^2 + \mathcal{O}(\mu^4)\,, \qquad (22)$$

where

$$\frac{d\rho_{\rm C}^{(4)}}{d\mu^2}\Bigg|_{\mu^2=0} \sim \frac{\Delta m_n^4}{2^8\pi^{5/2}} \sum_{n=-\infty}^{\infty}{}' \frac{(-1)^n}{n^3}\,. \qquad (23)$$

Thus,

$$\Delta\rho_{\rm C}^{(4)} \simeq \frac{d\rho_{\rm C}^{(4)}}{d\mu^2}\Bigg|_{\mu^2=0} \mu^2 \sim \Delta m_n^2 \delta m_n^2\,. \qquad (24)$$

Consequently for our case

$$\Delta\rho_{\rm C}^{(4)} \sim \alpha_G^8 M_{\rm Pl}^4 \sim \left[\left(\frac{M_{\rm SM}}{M_{\rm Pl}}\right)^2 M_{\rm Pl}\right]^4 \sim M_{\rm CC}^4\,. \qquad (25)$$

Recent observational evidence indicates that the dark energy may actually be the cosmological constant. The surprising numerical coincidence between the Planck-SM hierarchy and the SM-CC hierarchy suggests a deeper connection between the two. In this paper we explored the possibility of addressing these two hierarchies within a single framework. Invoking the minimal SUSY extension to RS1 model and SUSY-breaking on the TeV brane, which is transmitted to the bulk through the Higgs-gravitino coupling, we demonstrated that the 4d Casimir energy on the brane indeed scales as $\alpha_G^2 M_{\rm Pl}$, just right for the dark energy. While the old cosmological constant problem is yet to be addressed, it is remarkable that our model seems able to solve the new CC problem rather naturally.

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