Closed-String Tachyon Condensation and the Worldsheet Super-Higgs Effect*

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Alternative gauge choices for worldsheet supersymmetry can elucidate dynamical phenomena obscured in the usual superconformal gauge. In the particular example of the tachyonic E_8 heterotic string, we use a judicious gauge choice to show that the process of closed-string tachyon condensation can be understood in terms of a worldsheet super-Higgs effect. The worldsheet gravitino assimilates the goldstino and becomes a dynamical propagating field. Conformal, but not superconformal, invariance is maintained throughout.

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In theories with local worldsheet supersymmetry, it is traditional to work in superconformal gauge, sometimes supplemented by a subsidiary gauge-fixing condition such as light-cone gauge. This superconformal gauge choice has been successful in a large class of static backgrounds, but it may obscure aspects of more complicated systems.

String theory has now reached the stage which requires answers to a new class of dynamical questions, involving backgrounds far from equilibrium and with substantial time dependence. Such questions are particularly pressing in the cosmological setting. This challenge is likely to require new techniques.

We propose using a meaningful alternative to superconformal gauge; this previously unexplored tool can lead to new insights into string dynamics. To illustrate this phenomenon, we focus on a particular example which epitomizes time-dependent processes: The problem of closed-string tachyon condensation. While understanding the dynamics of its open-string cousins has improved our picture of brane decay into the vacuum (or to lower-dimensional stable branes), the closed-string tachyon is related to the possible decay of spacetime itself. This makes the problem difficult, and highlights the current limitations in our undestanding of string theory far from equilibrium.

In this letter, we demonstrate the use of alternative worldsheet gauge choices in the specific example of a ten-dimensional heterotic string model with E_8 gauge symmetry and a singlet tachyon. This little-studied model belongs to the early classification of modular invariant string theories in ten dimensions [1], but its *raison d'être* has been hitherto obscure. In a more detailed companion paper [2], we argue that this E_8 heterotic string describes the fate of an unstable configuration in heterotic M-theory with the two E_8 boundaries breaking complementary sets of sixteen supercharges [3]. This configuration suffers from the "decay to nothing" instability (at least in supergravity), due to an instanton connecting the two boundaries by a throat. Since there is an attractive Casimir force between the two E_8 boundaries, the fate of the "decay-to-nothing" instability should be addressed at weak string coupling where it turns into a perturbative tachyonic instability. The unique tachyonic E_8 string is the only promising candidate for describing this decay.

In the present work we put this motivation aside, and study the tachyon decay in the weakly coupled heterotic E_8 model as an independently interesting problem. The worldsheet theory of the heterotic E_8 string consists of $(0,1)$ chiral supergravity, coupled to matter supermultiplets. In the fermionic representation, the worldsheet action of the E_8 string is locally identical to that of its supersymmetric cousins,

$$S_0 = \frac{1}{4\pi\alpha'}\int d^2\sigma e\left\{(h^{mn}\partial_m X^\mu\partial_n X^\nu + \frac{i}{2}\psi^\mu\gamma^m D_m\psi^\nu + i\chi_m\gamma^n\gamma^m\partial_n X^\mu\psi^\nu)\eta_{\mu\nu} + \frac{i}{2}\lambda^A\gamma^m D_m\lambda^A + F^AF^A\right\}. \quad (1)$$

The matter fermions are chiral, with $\Gamma\lambda=\lambda$, $\Gamma\psi=-\psi$ where $\Gamma\equiv\gamma^0\gamma^1$, and the gravitino satisfies $\Gamma\chi_m=\chi_m$. We are in Minkowski signature, with $h_{mn}=e_m{}^a e_n{}^b\eta_{ab}$.

This tachyonic E_8 model is distinguished by its GSO projection: It is the only one in which no two fermions among λ^A carry always the same spin structure. It turns out that 31 of the λ^As give rise to the level-two E_8 current algebra, of left-moving central charge $c_L=31/2$. We denote by A' the index that runs over those first 31 values of A. The remaining $1/2$ of central charge is supplied by the remaining fermion $\lambda^{32}\equiv\lambda$. The spin structure of λ is always the same as that of the right-moving NSR fermions ψ^μ, a property that singles out λ uniquely. As we shall see, this "lone fermion" λ plays a central role in the process of tachyon condensation.

We wish to examine the spacetime evolution of the tachyon condensate as an on-shell process in string theory. Thus our worldsheet theory must maintain conformal invariance. Accordingly, we turn on a linear dilaton background $\Phi=V_\mu X^\mu$. Since we do not wish to change the critical dimension, the dilaton gradient V is chosen to be null. We choose spacetime lightcone coordinates

$X^{\pm} = X^0 \pm X^1$ (with the spacetime index μ splitting as $(\pm, i)$, $i = 1, \dots 8$), such that the only nonzero component of V is V_-. The action is $S_0 + S_V$, where

$$S_V = -\frac{1}{4\pi}\int d^2\sigma e\, V_- \left(X^- R + 2i\chi_m\gamma^n\gamma^m D_n\psi^-\right). \quad (2)$$

This is invariant under local supersymmetry transformations, given by

$$\begin{aligned} \delta e_m{}^a &= 2i\epsilon\gamma^a\chi_m, \quad \delta\chi_m = D_m\epsilon \\ \delta X^\mu &= i\epsilon\psi^\mu, \quad \delta\psi^\mu = \gamma^m\partial_m X^\mu\epsilon + \alpha' V^\mu\gamma^m D_m\epsilon, \\ \delta\lambda^A &= F^A\epsilon, \quad \delta F^A = i\epsilon\gamma^m D_m\lambda^A. \end{aligned} \quad (3)$$

At this stage, the auxiliary fields F^A are usually integrated out; we indeed set $F^{A'} = 0$, but keep $F \equiv F^{32}$, as it plays a rather notrivial role in the dynamics of the tachyon condensation. Indeed, the tachyon vertex operator with spacetime momentum p_μ is related to the lone fermion supermultiplet, and given (in picture 0) by

$$\mathcal{V} = (F + \lambda\, p_\mu\psi^\mu)\exp(ip_\mu X^\mu). \quad (4)$$

We now turn on a condensate of the tachyon in the form of worldsheet superpotential

$$S_W = -\frac{\mu}{\pi\alpha'}\int d^2\sigma\left\{F\mathcal{T}(X) - i\lambda\psi^\mu\partial_\mu\mathcal{T}(X)\right\}, \quad (5)$$

and choose $\mathcal{T}(X)$ such that (5) is an exactly marginal deformation in conformal gauge. This requires

$$\mathcal{T}(X) = \exp(k_+X^+) \quad \text{and} \quad -k^2 + 2V\cdot k = \frac{2}{\alpha'}. \quad (6)$$

The real constant $k_+ > 0$ is thus related to the dilaton gradient via

$$V_- k_+ = -\frac{1}{2\alpha'}. \quad (7)$$

These conditions arise at the first and second order of conformal perturbation theory in the dimensionless μ.

As the next step, we need to choose a gauge. In critical string theories with worldsheet supersymmetry, superconformal gauge – in which $e_m{}^a = \delta_m{}^a$ and the gravitino χ_m is set to zero – is virtually always chosen. The residual superconformal symmetry of this gauge is sometimes further fixed by going additionally to lightcone gauge; however, this extra gauge fixing is supplemental to the conformal gauge and not an alternative to it.

To motivate a meaningful alternative gauge, consider the transformation properties of the lone fermion λ in (3). In the presence of the tachyon condensate, F develops a vacuum expectation value, and can be integrated out using its algebraic equation of motion,

$$F = 2\mu\exp(k_+X^+). \quad (8)$$

In the rest of this letter, we use F as a shorthand for the composite operator (8).

In the presence of the condensate, the supersymmetry transformation $\delta\lambda = F\epsilon$ indicates that λ may play the role of a goldstino for spontaneously broken worldsheet supersymmetry. Since this symmetry is local, this in turn suggests that a super-Higgs mechanism takes place, in which the gravitino develops a propagating degree of freedom. As a result, it may be wise not to set the gravitino to zero by gauge choice, and an alternative to the superconformal gauge is required.

In higher dimensions, the Higgs mechanism is usually associated with the gauge field becoming massive. In our case, no mass scale should be generated if the worldsheet theory is to stay conformal. In our view, the more general signature of the Higgs mechanism is that the gauge field acquires more physical polarizations. In four spacetime dimensions, this happens if a massless gauge field develops a mass. The worldsheet situation is different. In the absence of tachyon condensation, the worldsheet gravitino imposes a constraint, setting the supercurrent to zero on physical states; it effectively carries "minus one" polarization. When the tachyon condensate develops, the worldsheet gravitino will acquire a propagating polarization: The net number of polarizations increases, but – unlike in four dimensions – no mass is acquired.

Our alternative gauge for worldsheet supersymmetry still involves fixing worldsheet diffeomorphisms by the conventional conformal gauge,

$$e_m{}^a = \delta_m{}^a. \quad (9)$$

This is consistent with our expectation of worldsheet conformal invariance. From now on, our analysis continues in this gauge; and we use worldsheet lightcone coordinates $\sigma^\pm = \tau \pm \sigma$. In these coordinates, the only nonzero components of the fermi fields are χ_{++}, ψ^μ_-, and λ^A_+.

Since the gravitino is expected to gain a physical polarization at the expense of the goldstino λ, the first instinct for a better gauge might be to set $\lambda = 0$. The equation of motion that follows from varying λ would then be imposed as a constraint on physical states. In our case, this would imply

$$\mu\psi^+_-\exp(k_+X^+) = 0, \quad (10)$$

which can be solved by setting $\psi^+_- = 0$. This would in turn require that the equation of motion obtained from varying ψ^+_- becomes a new constraint.

The benefit of such an alternative gauge choice is that the gravitino is not artificially set to zero, and is free to develop its own dynamics. Once we set $\psi^+_- = 0$ in the full action, as tenatively suggested by our first look at this alternative gauge above, the gravitino couples to the rest of the system in a relatively simple way, via the terms

$$\chi_{++}\psi^-_-\partial_-X^+ + 2\alpha' V_-\chi_{++}\partial_-\psi^-_- - 2\chi_{++}\psi^i\partial_-X^i \quad (11)$$

in the action. This is further simplified by rescaling

$$\widetilde{\chi}_{++} = F\chi_{++}, \qquad \widetilde{\psi}^-_- = \frac{\psi^-_-}{F}. \quad (12)$$

This rescaling changes the conformal weights of these fields from $(1,-1/2)$ and $(0,1/2)$ to $(3/2,0)$ and $(-1/2,0)$ respectively, and transforms (11) to

$$2\alpha' V_- \widetilde{\chi}_{++}\partial_- \widetilde{\psi}_-^- - \frac{2}{F}\widetilde{\chi}_{++}\psi^i\partial_- X^i. \qquad (13)$$

Thus, $\widetilde{\chi}_{++}$ and $\widetilde{\psi}_-^-$ appear to form a left-moving first-order system of spin 3/2 and $-1/2$ and central charge $c_L = -11$, whose coupling to the transverse degrees of freedom X^i, ψ^i becomes arbitrarily weak at late X^+. This left-moving first-order system is our dynamical gravitino sector!

In the precise implementation of this alternative gauge, a slight modification of the process outlined above is useful. Thus, as our gauge choice, we choose to supplement (9) by setting

$$\psi_-^+ = 0, \qquad (14)$$

instead of setting $\lambda_+ = 0$ (or $\chi_{++} = 0$ as in superconformal gauge). As we will see, this accomplishes all the expected benefits outlined above, and leads to other simplifications.

The Faddeev-Popov superdeterminant $\Delta_{\rm FP}$ for our gauge choice factorizes, $\Delta_{\rm FP} = J_{bc}/J_{\psi_-^+\epsilon}$. Here J_{bc} is the determinant associated with the conformal gauge for worldsheet diffeomorphisms, traditionally realized by the path integral over a bc ghost system of spin 2 and central charge $c_L = c_R = -26$. $J_{\psi_-^+\epsilon}$ is the determinant of the operator associated with the change of variables in the fermionic sector, from ψ_-^+ to ϵ, as determined from the supersymmetry variation of the gauge-fixing condition,

$$\begin{aligned}\delta\psi^+ &= \gamma^m\partial_m X^+\epsilon - 2\alpha' V_-\gamma^m D_m\epsilon\\ &= \frac{4\alpha' V_-}{F}D_-(F\epsilon). \end{aligned}\qquad (15)$$

Hence, $J_{\psi_-^+\epsilon}$ is the determinant of

$$D_F \equiv \frac{1}{F}D_- F, \qquad (16)$$

acting on fields of spin $j = 1/2$. When properly regularized and renormalized, this determinant will depend not only on the Liouville field ϕ but also on X^+ due to the explicit appearance of F in (15). This determinant can be explicitly evaluated using formulas found for example in [4] (see [2] for details). It is useful to consider the slight generalization of D_F given by (16) as acting on fields of spin j; the determinant of the corresponding Laplacian is

$$\begin{aligned}&\frac{1}{2}\log\det(D_F^\dagger D_F)\\ &= -\frac{1}{48\pi}\int d^2\sigma\,\widehat{e}\Big\{[3(2j-1)^2-1]\widehat{h}^{mn}\partial_m\phi\partial_n\phi\\ &\quad +12(2j-1)k_+\widehat{h}^{mn}\partial_m\phi\partial_n X^+\\ &\quad + 12k_+^2\widehat{h}^{mn}\partial_m X^+\partial_n X^+\Big\}, \end{aligned}\qquad (17)$$

where $h_{mn} = e^{2\phi}\widehat{h}_{mn}$, and $\widehat{h}_{mn}$ is a fiducial metric.

At $j = 1/2$, the first term can be realized by introducing a bosonic ghost-antighost pair, $\widetilde{\beta}$, $\widetilde{\gamma}$. These are standard left-moving first-order fields of conformal weight $(1/2,0)$. The second term in (17) vanishes for $j = 1/2$, and the third, X^+ dependent term is a one-loop contribution to the action of the $X^\pm$ system. Note that the ghosts $\widetilde{\beta}$, $\widetilde{\gamma}$ associated with our gauge choice are purely *left-moving*, and contribute $c_L = -1$ to the *bosonic* sector on the worldsheet. This is to be contrasted with conventional superconformal ghosts in superconformal gauge, which are *right-moving* and contribute $c_R = 11$ to the supersymmetric sector instead.

In our alternative gauge, the worldsheet theory thus looks very different than in superconformal gauge. We have gained a left-moving propagating massless gravitino of spin 3/2, which (together with its conjugate partner $\widetilde{\psi}_-^-$ of spin $-1/2$) contributes central charge $c_L = -11$. We have also gained left-moving bosonic ghosts of spin 1/2, contributing an additional -1 to c_L; altogether we have lost twelve units of left-moving central charge. Similarly, in the right-moving sector we have lost the $\psi_-^\pm$ pair of fermions, worth $c_R = 1$: ψ_-^+ is zero by our gauge choice, and ψ_-^- has turned into a left-moving field. In addition, no conventional right-moving superconformal ghosts β,γ of central charge $c_R = 11$ arise, leading to the same apparent total discrepancy of 12 units of central charge as in the left-moving sector.

Does this mean that the theory lost its conformal invariance? The answer is no: A careful analysis of the changes of variables (*i.e.*, the gauge fixing and the X^+ dependent rescaling of fields) reveals one-loop corrections due to the measure factors that precisely shift the dilaton from its originally null direction, making up for the missing twelve units of central charge and restoring quantum conformal invariance. The one-loop correction from the Faddeev-Popov determinant to the effective action has been calculated above, and is given by

$$-\frac{1}{4\pi}\int d^2\sigma\,\widehat{e}\,k_+^2\widehat{h}^{mn}\partial_m X^+\partial_n X^+. \qquad (18)$$

Another similar contribution comes from the rescaling of the fields in the gravitino sector (12). This change has led to a canonical kinetic term in the classical action, but it did not produce the canonical measure for such first-order fields: The original measure of the χ_{++} and ψ_-^+ fields maps to a measure with an explicit X^+ dependence for the rescaled fields. We wish to replace this X^+ dependent measure by the canonical one. This generates a conversion factor, which can be evaluated by comparing the Gaussian part of the path integral in the original χ_{++}, ψ_-^- variables, to the path integral in the new variables $\widetilde{\chi}_{++}, \widetilde{\psi}_-^-$ *with respect to the canonical measure independent of* X^+. The former is obtained from (17) for

$j = 3/2$, and is given by

$$\log \int \mathcal{D}\chi_{++}\, \mathcal{D}\psi_-^-\, e^{-S_2(\chi_{++},\psi_-^+,X^+)}$$
$$= -\frac{1}{48\pi}\int d^2\sigma\, \widehat{e}\left\{11\widehat{h}^{mn}\partial_m\phi\partial_n\phi \right. \quad (19)$$
$$\left. + 24k_+\widehat{h}^{mn}\partial_m\phi\partial_n X^+ + 12k_+^2\widehat{h}^{mn}\partial_m X^+\partial_n X^+\right\},$$

where S_2 is the part of the action bilinear in χ_{++} and ψ_-^-. The integral over $\widetilde{\chi}_{++}$ and $\widetilde{\psi}_-^-$ (with the measure independent of X^+) reproduces correctly the Weyl anomaly represented by the first, Liouville-dependent term; this confirms that these fields represent a left-moving first-order system with $c_L = -11$. The remaining two terms in (19) are the conversion factor due to the replacement of the measure in the gravitino sector by the canonical one. The third term cancels the X^+ dependent contribution (18) of the Faddeev-Popov determinant. The second one then shifts the linear dilaton term in the action (which we again write in Minkowski signature), by

$$\Delta S = -\frac{1}{4\pi}\int d^2\sigma\, e\, k_+ X^+ R. \quad (20)$$

Here we have integrated the $\partial\phi\partial X^+$ term in (19) by parts, and used the expression $eR = -2\widehat{e}\widehat{\nabla}^2\phi$ for the scalar curvature in conformal gauge. The one-loop correction (20) makes the dilaton spacelike, and provides the missing $c = 12$ units of the central charge. The worldsheet theory is conformal at the quantum level.

The proper implementation of the BRST quantization involves the BRST partner B of the antighost $\widetilde{\beta}$. The integration over B produces a delta function which imposes the gauge fixing condition. Once B is integrated out, it needs to be eliminated from the BRST transformation rule for $\widetilde{\beta}$. The new BRST variation of $\widetilde{\beta}$ gives essentially the equation of motion resulting from the variation of ψ_-^+, and must vanish on physical states. This constraint has a clear intuitive meaning: It takes the form $\sim \mu\lambda_+ \exp(k_+X^+) = \ldots$, where the λ_+ term comes from the variation of the superpotential, and "…" are the contributions from all the other terms, none of which depends on λ_+. This simply means that the goldstino λ_+ is not a separate dynamical field, but can be solved for in terms of the other fields, including the now-dynamical gravitino. This is how the super-Higgs mechanism is implemented in this gauge.

Note also that our gauge fixing condition leaves no residual supersymmetry at finite μ. Thus, in our alternative gauge, the gauge-fixed theory is not superconformal, even though manifest conformal invariance is maintained.

The picture of the heterotic E_8 model in our alternative gauge can be easily extended to other models with worldsheet supersymmetry. For example, in Type 0 string theory in the presence of null linear dilaton $\Phi = V_- X^-$ and an exponential tachyon condensate along X^+ (studied extensively in [5]), we can choose a gauge by setting both ψ_-^+ and ψ_+^+ (the left-moving and right-moving superpartners of X^+) to zero. Compared to the heterotic model, we expect the entire gauge-fixing procedure to be doubled, resulting in spin 1/2 bosonic ghosts $\widetilde{\beta}$, $\widetilde{\gamma}$ and a propagating gravitino sector $\widetilde{\chi}$, $\widetilde{\psi}^-$ in the left-moving as well as right-moving sector. Before the one-loop effects of the Faddeev-Popov determinant and of the rescaling of the gravitino sector are incorporated, the apparent discrepancy in the central charge is -24 for each movers. The one-loop determinants then shift the dilaton by twice the heterotic amount, adjusting the central charge by 24 and restoring exact conformal invariance at the quantum level. In the process, the worldsheet gravitino has again been liberated, leading to a free fermion system of $c = -11$. This elucidates some of the results of [5] (as well as the related construction of [6]), in which a fermionic $c = -11$ system was found in superconformal gauge. Our gauge shows that this system shoud be interpreted as the dynamical worldsheet gravitino.

The two examples of tachyon condensation considered above suggest that alternative worldsheet gauge choices may illuminate a more general class of string backgrounds out of equilibrium. Yet, the physics of observables in any given background should be independent of the choice of gauge, suggesting a new type of worldsheet duality in string theory: A duality between different gauge choices of the same worldsheet theory. This clearly extends the class of perturbative string dualities encountered so far: Instead of representing one CFT in two dual ways, we can now look at one string background in two gauges, in which they are not even described by the same CFT. This feature appears to be conceptually new, and we hope that it will be useful in understanding string backgrounds with substantial time dependence, beyond the example of heterotic tachyon condensation studied here.

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