LBL-12107 c.2
Preprint

# Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA



## Accelerator & Fusion Research Division

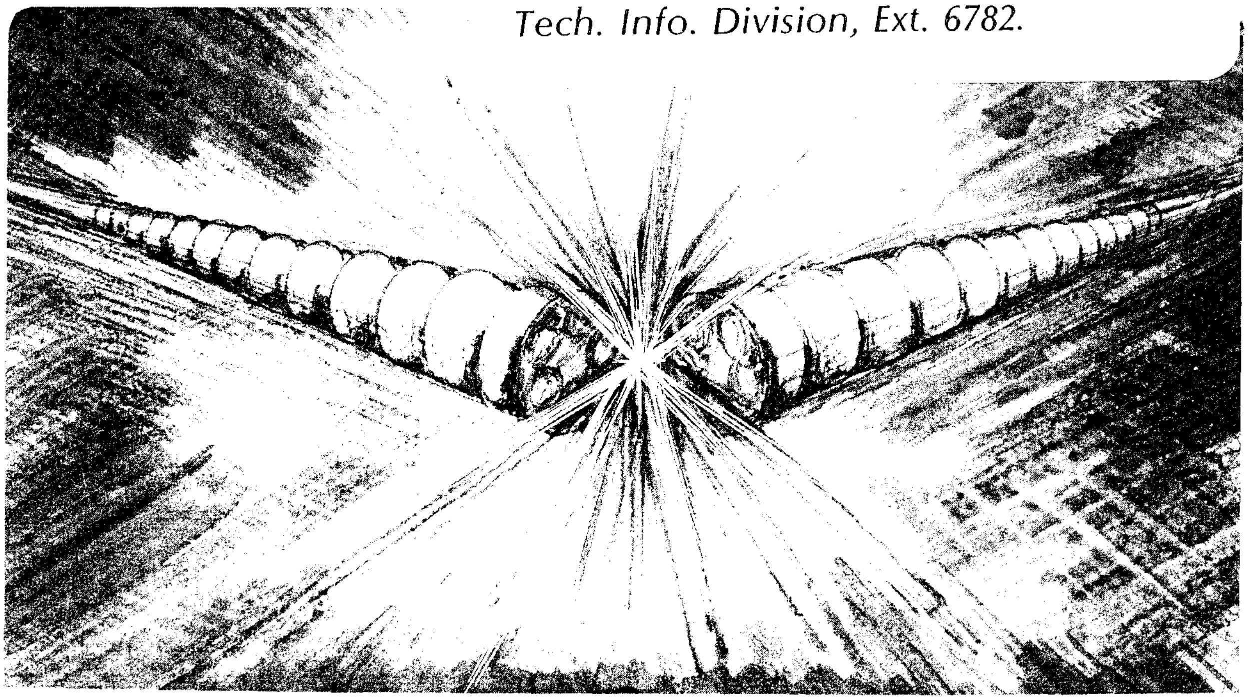
To be submitted for publication

THE LONGITUDINAL STABILITY OF INTENSE NON-RELATIVISTIC PARTICLE BUNCHES IN RESISTIVE STRUCTURES

Paul J. Channell, Andrew M. Sessler and
Jonathan S. Wurtele

February 1981







Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

LBL-12107

To be submitted for publication

# THE LONGITUDINAL STABILITY OF INTENSE NON-RELATIVISTIC PARTICLE BUNCHES IN RESISTIVE STRUCTURES

Paul J. Channell, Andrew M. Sessler and Jonathan S. Wurtele

February 1981

Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

*This work was supported by the Director, Office of Energy Research, Office of Inertial Fusion, Research Division of the U. S. Department of Energy under Contract No. W-7405-ENG-48.

# The Longitudinal Stability of Intense Non-Relativistic Particle Bunches in Resistive Structures

Paul J. Channell
Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545

and

Andrew M. Sessler and Jonathan S. Wurtele
Lawrence Berkeley Laboratory, Berkeley, California 94720

February 12, 1981

ABSTRACT

The longitudinal stability of intense particle bunches is investigated theoretically in the limit of small wall resistivity compared to total reactance. It is shown that both in the absence of resistivity and to lowest order in the resistance that an intense bunch is stable aganist longitudinal collective modes. An expression is derived for the lowest order instability rate. Application of these results are made to drivers for heavy ion inertial fusion.

PACS numbers: 41.70 +t, 29.15 Dt, 52.35 Py, 52.60 +h

Heavy ion fusion is envisioned as having for a driver either an rf linac with storage rings or an induction linac.[1] In the rf linac approach the major current multiplication, so as to reach the requisite power level, is done in the storage rings. The induction linac, on the other hand, must accelerate significant currents directly to the target. Either approach has difficulties (such as the manipulation of beams in and out of storage rings in the rf linac approach), but common to both methods is the need for the stability of intense bunches of particles. Much effort has been devoted to this subject.[1,2]

For a bunch in an induction linac an estimate can be obtained by employing the analysis which has been developed for circular machines and modifying it for a linear structure.[3] Firstly, one notes that one is "below transition" or in a positive mass regime so that only in the presence of resistivity is there instability. One finds, for above threshold, that the e-folding length, $\lambda$, is given by:

$$\lambda^{-1} = (R\ /Z_0) \left[ \frac{4\pi^2 q^2}{(1+2\,\ell n\ (b/a))} \frac{M_p}{M} \cdot \frac{N}{L} \cdot r_p \right]^{1/2} \qquad (1)$$

where,

$Z = R + iX$ = the impedence per unit length

$N/L$ = line density of ions

$r_p$ = classical proton radius

$Z_o$ = free-space impedence (or 377 ohms)

$q$ = degree of ionization of the ions

$M/M_p$ = mass of the ions in units of the proton mass

Putting in R = 200 ohms/meter, q = 2, $M_p/M = 1/200$, $N/L = 10^{15}/20$ meters, b/a = 1.5. Eq. (1) yields a length, $\lambda$, of 300 meters which is uncomfortably short for a linac of the length required.

For a storage ring a similar method may be employed and yields growth times which are also uncomfortably short.[4] The use of unbunched-beam theory for bunches has been observed to be valid on a variety of storage rings (see Ref. 4), while theoretical analysis has shown the unbunched-beam theory to be valid under certain circumstances.[5]

It is the purpose of this communication to report on a theoretical analysis which directly applies to the storage ring or the induction linac of heavy ion fusion. We show that to lowest order there is, for any finite bunch, no net resistive instability so that the situation is very much better than estimated above. We also allow in our formalism for an arbitrary impedance of the structure which, at least for the induction linac, is an important effect. Our work is a generalization of that of Kwang Je Kim who first showed no instability for a finite bunch of uniform charge, with a step-function distribution in momentum, and the impedance of a uniform structure.[6] Removing the special assumptions of Kim is important for it allows us to conclude that either in a practical linear induction accelerator or in a realistic storage ring intense particle bunches will not be subject to significant longitudinal instability and, hence, from this very important theoretical point of view heavy ion fusion is a viable and interesting possibility.

The ions, which are collisionless, are described by the non-relativistic non-linear Vlasov equation

$$\left(\frac{\partial}{\partial t} + v\,\frac{\partial}{\partial z} + \frac{qe}{M}\,E\,\frac{\partial}{\partial v}\right) f(z,\ v,\ t) = 0\ , \tag{2}$$

where z is the longitudinal coordinate, v is the velocity associated with the z coordinate, e is the proton charge, t is the time and the ion distribution function is the unknown f. The longitudinal electric field consists of an applied field, $E_A$, and a functional $E_s(n)$ of the line charge density n(z,t) where

$$n(z,\ t) = \int f(z,\ v,\ t)dv\ . \tag{3}$$

We may take moments of the Vlasov equation and close the heirarchy by noting that in our applications the particle thermal velocity is small compared to the collective motion density wave phase velocity. Stoping after two moments we obtain fluid equations which can be combined to yield:

$$\frac{\partial^2 n(z,t)}{\partial t^2} + \frac{qe}{M}\ \frac{\partial}{\partial z}\Big(En(z,t)\Big) = 0 \tag{4}$$

This equation may be linearized about an equilibrium charge distribution $n_o$ (z) which can, in this approximation, be arbitrary; ie, we can choose an applied field so as to make any $n_0$ (z) stationary. Introduce space and time Fourier transforms by means of

$$\tilde{n}_1(k,\omega) = \int_{-\infty}^{\infty} dt \int_{-L/2}^{L/2} dz n_1(z,t) e^{i(kz - \omega t)}, \tag{5}$$

where $n_1$ is the pertrubed density. We thus obtain

$$-\omega^2 n_1(k,\omega) + \frac{qe}{M} ik \int dz e^{ikz} n_0(z)\, E_s\,(n_1) = 0 \tag{6}$$

The self electric field, $E_s$ (n) , can be related to $n_1$ by means of an impedance function

$$\tilde{E}_s\,(k,\omega) = -Z\,(k,\omega)\,\tilde{n}_1\,(k,\omega). \tag{7}$$

The entire effect of the storage ring or the induction linac is contained in the function Z (k,ω) which describes the reaction of the structure to a disturbance of laboratory frequency ω'. The laboratory frequency ω' is $\omega' \simeq kv_b + \omega$, where $v_b$ is the beam velocity and the term in ω is due to motion of the disturbance in the beam frame. To good approximation the term in ω can be neglected, and then Z (k,ω) is a function of k alone. We use this approximation in most of our work, but take the ω in to account in Z (k,ω) when we calculate to second order (Eq. (17)).

We combine Eqs. (6) and (7) and the approximation for Z (k,ω) to obtain

$$-\omega^2 \tilde{n}_1\,(k,\omega) + \frac{qe}{M}\frac{ik}{2\pi}\int dk_1\,\tilde{n}_0\,(k-k_1)\,Z\,(k_1)\,\tilde{n}_1(k_1,\omega) = 0, \tag{8}$$

where clearly $\tilde{n}_0$ (k) is related to the equilibrium density $n_0$ (z) by an equation analogous to Eq. (5).

It is easy to see that Eq. (8) yields a growth rate as given by Eq. (1) under the same circumstances. For a long wavelength disturbance, on a beam of radius b in a structure of radius a, the impedance is

$$Z\,(k) = -qe\,ik\,(1 + 2\ell n\, b/a) - qev_b R, \tag{9}$$

where $v_b$ is the beam velocity. For a uniform beam $n_o(z)$ is a constant and $n_1(z,t)$ varies sinusoidally in space and time. For a long wavelength disturbance and for a uniform beam, Eq. (8) and Eq. (9) yield Eq. (1).

For a bunched beam, however, we can show two consequences of Eq. (8); namely there is no instability if the impedance is purely imaginary (ie, purely reactive) and furthermore that there is no instability if the resistance is small. Firstly, consider the case in which there is no resistance so that we may write

$$Z(k) = i\, X(k) \tag{10}$$

where the reactance, X, is odd. We assume X is negative for positive k, in order to be in the positive mass regime, and it is under this condition that there is stability.

Multiplying Eq. (8) by $\tilde{n}_1{}^*(k)\, Z^*(k)/k$, and integrating we obtain

$$-\omega^2 \int_{-\infty}^{\infty} \left|\tilde{n}_1(k)\right|^2 \frac{Z^*(k)}{k}\, dk \; +$$
$$\frac{qei}{M2\pi} \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dk_1 \; Z^*(k)\, Z(k_1)\, \tilde{n}_1{}^*(k)\, \tilde{n}_o(k-k_1)\, \tilde{n}_1(k_1) = 0 \tag{11}$$

We use Eq. (10) and the theorem that

$$\int_{-\infty}^{\infty} \tilde{F}(k)\, \tilde{G}^*(k)\, dk = \int_{\infty} F(x)\, G^*(x)\, dx, \tag{12}$$

with

$$\tilde{F}(k) = \int_{-\infty}^{\infty} dk_1\, \tilde{n}_o(k-k_1)\, Z(k_1)\, \tilde{n}_1(k_1),$$
$$\tilde{G}^*(k) = Z^*(k)\, \tilde{n}_1{}^*(k), \tag{13}$$

to write Eq. (11) in the form:

$$-\omega^2 \int_{-\infty}^{\infty} |\tilde{n}_1(k)|^2 \left| \frac{X(k)}{k} \right| dk + \frac{qe}{M} \int_{-\infty}^{\infty} n_o(z)\, |G(z)|^2\, dz = 0. \tag{14}$$

In this form it is clear that $\omega^2$ is real and positive and, hence, there is stability.

Secondly consider the case in which

$$Z(k) = i X(k) + R(k), \tag{15}$$

and R (k) is real and symmetric and small; ie R (k) << X (k). For a non-relativistic bunch this will generally be true since the self-term in the reactance is non-negligible.[7] Employing perturbation theory we find that R (k) creates a frequency shift, $\delta\omega_n$, of the $n^{\underline{th}}$ mode frequency, $\omega_n^o$, given by

$$\delta\omega_n = \frac{qei \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk\, dk_1\, X(k)\, \tilde{n}_n^*(k)\, \tilde{n}_n(k_1)\, \tilde{n}_o(k-k_1)\, R(k_1)}{2\omega_n^o \int_{-\infty}^{\infty} 2\pi M\, dk\, |\tilde{n}_n(k)|^2\, \frac{X(k)}{k}}, \tag{16}$$

where $\tilde{n}_n(k)$ is the eigenfunction of the $n^{\underline{th}}$ mode.[8] For a symmetric unperturbed bunch, and provided the modes, $\omega_n$, are non-degenerate it is easy to show that $\delta\omega_n$ is zero.

With these results we conclude that the growth distance (or time) is greatly increased over that given by Eq. (1). Explicit calculation must employ improved equations and a particular model for $n_o(z)$ (and hence $\tilde{n}_n(k)$ and $\omega_n$).[9] This work, which will be described elsewhere, yields

$$\lambda^{-1} = \lambda^{-1}_{\text{Uniform Beam}} \left(\frac{V_p}{V_b}\right) g(n) \tag{17}$$

where $\lambda^{-1}_{\text{Uniform Beam}}$ is given by Eq. (1), g (n) is a dimensionless function of mode number, n, and

$$V_p = qe \left[\frac{n_o(1 + 2\ell n\, b/a)}{M}\right]^{1/2} . \tag{18}$$

For typical parameters the additional factor in Eq. (17), over Eq. (1), is ≃ 500.

We have shown that to lowest order there is no instability whereas Wang and Pellegrini have shown that, under certain circumstances, bunches are unstable.[5] An important difference between our work and theirs is that they, working with relativistic particles, take Z (k) to have a broad resonance and no self-term so that an expansion in R (k)/X (k) would be invalid. On the other hand, for the non-relativistic particles of heavy ion fusion an expansion in R/X -- and hence a very different conclusion -- is valid.

Although we have shown that to lowest order there is neither an absolute or a convective instability, there is still the possibility of transient spatial amplification. We have estimated this effect; using uniform beam theory, the impedance one expects in practice, and the time for a disturbance to reach a bunch end.[7] We find that less than one e-folding occurs.

Finally, we have been concerned that our results depend upon reflection of disturbances at bunch ends, which is exactly where our analysis is invalid because we have linearized about an unperturbed distribution which is, in fact, going to zero at the bunch end. We have, consequently, examined a more realistic model for the impedance than Eq. (9); namely a model in which (neglecting resistance)

$$Z\ (k) = \frac{c_1\ k}{c_2 + k^2} \quad , \tag{19}$$

where $c_1$, $c_2$ are constants. For small k this model can be matched to Eq. (9), but for short wavelengths Eq. (19) converts to a plasma oscillation in which the structure is not important. For an impedance given by Eq. (19) we are able to reduce Eq. (8) to a second order differential equation and show that a wave reflects <u>before</u> it reaches a bunch end; ie, before the linear approximation becomes invalid. This work, which will be described elsewhere, lays to rest our concern about the validity of the results reported herein.

The authors are grateful to Kwang Je Kim for helpful discussions and to Eliezer Hameiri for assistance with the mathematics. This work was supported by the Office of Inertial Confinement Fusion of the Department of Energy and performed under Contract W-7405-ENG-48.

## REFERENCES

1. D. Keefe and A. M Sessler, Proceedings of the XI International Conference on High Energy Accelerators, (Birkhauser - Verlag, Basel, 1980), p.201.
2. I. Haber, Proceedings of the Heavy Ion Fusion Workshop, Argonne National Laboratory Report ANL-79-41 (1978); G. Lambertson, L. J. Laslett and L. Smith, IEEE Trans. NS-24, 993 (1977).
3. V. K. Neil and A. M. Sessler, Rev. Sci. Instr. 36, 429 (1965).
4. D. Mohl, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980), p. 315.
5. J. M. Wang and C. Pellegrini, Proceedings of the XI International Conference on High Energy Accelerators, (Birkhauser - Verlag, Basel, 1980), p. 554.
6. Kwang Je Kim, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980), p. 187.
7. A. Faltens, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-10301 (1980), p. 182.
8. P. Morse and H. Feshbach, Methods in Theoretical Physics, Part II (McGraw-Hill, New York, 1953), p. 1015.
9. D. Neuffer, Proceedings of the Heavy Ion Fusion Workshop, Lawrence Berkeley Laboratory Report LBL-1031 (1980), p. 245.