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Supersymmetry and Superstring Phenomenology^{*†}

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Abstract

We briefly cover the early history of supersymmetry, describe the relation of SUSY quantum field theories to superstring theories and explain why they are considered a likely tool to describe the phenomenology of high energy particle theory beyond the Standard Model.

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SUPERSYMMETRY AND SUPERSTRING PHENOMENOLOGY

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1 Introduction

This paper is dedicated to the memory of Julius Wess, one of the inventors of supersymmetry (SUSY), who passed away in August 2007. Julius participated in the conference SUSY 07, that took place in Karlsruhe, and gave an invited lecture there, just a few weeks before his death. In his lecture he detailed a brief history of the discovery of SUSY quantum field theories in four space-time dimensions, and expressed his hope that the superparticles predicted by SUSY will be found in experiments at the Large Hadron Collider (LHC) at CERN that are expected to begin this summer.

In this paper we will briefly cover the early history of SUSY, describe the relation of SUSY quantum field theories to superstring theories and explain why they are considered a likely tool to describe the phenomenology of high energy particle theory beyond the Standard Model (SM). More extensive descriptions of SUSY and its applications to physics can be found in the now standard text by Wess and Bagger [?] as well as in several recently published books [?].

2 The beginning of supersymmetry

Four dimensional SUSY was discovered independently three times in the early 1970's: first in Moscow by Golfand and Likhtman, then in Kharkov, by Volkov and Akulov, and Volkov and Soroka, and finally by Julius Wess and one of us (BZ), who collaborated at CERN in Geneva and in Karlsruhe, and were totally unaware of the previous work. It is remarkable that Volkov and his collaborators did not know about the work of Golfand and Likhtman, since all of them were publishing papers in Russian in Soviet Journals. For information on the life and work of Golfand and Likhtman, we refer to the Yuri Golfand Memorial Seminar Volume [?]. This book contains an interesting graph on page 43 that shows the remarkably fast increase in the number of papers on SUSY as a function of time after the first three *preprints* that carried the names of Wess and Zumino. For information on Volkov's life and work, we refer to the Proceedings of the 1997 Volkov Memorial Seminar in Kharkov [?].

Supersymmetry is a symmetry that relates the properties of integral-spin bosons to those of half-integral spin fermions. The generators of the symmetry form what has come to be called a superalgebra, which is a super extension of the Poincaré Lie algebra of quantum field theory (Lorentz transformations and space-time translations) by fermionic generators. In a superalgebra both commutators and anticommutators occur. The early physics papers on SUSY also gave rise to renewed interest by mathematicians in the theory of superalgebras. Eventually a complete classification of simple and semisimple superalgebras was obtained, analogous to Cartan's classification of Lie algebras, and even the prefix super was adopted in mathematics. Unfortunately the Poincaré superalgebra is not semisimple, although it can be obtained by a suitable contraction; the situation is similar to that for the Poincaré Lie algebra. The general classification of superalgebras does not appear to be very useful in physics, because, unlike Lie algebras, simple or semisimple superalgebras cannot be used as internal symmetries, or so it seems.

The early work on supersymmetric field theories considered only one fermionic generator which is a Majorana spinor. The corresponding superalgebra is therefore called $N = 1$ SUSY. An important development was the study of extended ($N > 1$) SUSY and the construction of quantum field theories admitting extended SUSY. It turns out that $N = 1$ SUSY in four space-time dimensions is still the best choice for a SUSY extension of the Standard Model of elementary particle physics, because of the chirality properties of physical fermions. We describe in Section ?? a popular version of such an extension: the Minimal Supersymmetric Standard Model (MSSM).

3 Superspace

The influence of SUSY on mathematics can be seen by the great interest mathematicians have developed in the study of supermanifolds. From a physicist's point of view this began with an important paper by A. Salam and J. Strathdee [?] who introduced the concept of "superspace", a space with both commuting and anticommuting coordinates, and showed that $N = 1$ supersymmetry can be defined as a set of coordinate transformations in superspace. Ferrara, Wess and Zumino then wrote some papers using the concept of "superfields" (fields in superspace). Eventually the technique of superpropagators was developed and shown to be a useful tool for supersymmetric perturbation theory.

With the discovery of supergravity (SUGRA), it became natural to study the geometry of curved supermanifolds. Wess and Zumino realized that superRiemannian geometry had to be enlarged by the introduction of a super-vielbein and a constrained, but nonvanishing, supertorsion. They also formulated the geometry in terms of exterior superdifferential forms, not unlike those introduced

independently by Berezin.

Just as in the case of supersymmetric quantum field theories in flat space, SUGRA has remarkable properties of convergence; however it does not provide a theory that is finite or even renormalizable. $N = 1$ SUGRA admits higher N extensions. For various reasons, the highest physically acceptable value is $N = 8$, Recently, remarkable properties of $N = 8$ SUGRA in four dimensions have been discovered [?]. With clever combinations of diagrams, some of the perturbation theory divergences cancel; *these* cancellations do not seem to be understandable from the supersymmetry of the theory. Are they due to some other symmetry? Is $N = 8$ SUGRA in 4 dimensions actually finite in perturbation theory?

4 Successes and problems of the Standard Model

The Standard Model can be briefly summarized as follows:

Spin- $\frac{1}{2}$ fermions, called quarks q and leptons ℓ fall into three families with identical interactions but increasing mass (except for the neutrinos whose masses are all very small):

$$\begin{array}{ccccccc}
 SU(3) & & \longleftrightarrow & & \longleftrightarrow & & \longleftrightarrow \\
 & & \left(\begin{array}{cccc} u_1 & u_2 & u_3 & \nu_e \\ d_1 & d_2 & d_3 & e \end{array} \right) & & \left(\begin{array}{cccc} c_1 & c_2 & c_3 & \nu_\mu \\ s_1 & s_2 & s_3 & \mu \end{array} \right) & & \left(\begin{array}{cccc} t_1 & t_2 & t_3 & \nu_\tau \\ b_1 & b_2 & b_3 & \tau \end{array} \right) & & \Updownarrow \quad SU(2)
 \end{array}$$

The labels u, d, c, s, t, b for quarks q and e, μ, τ for negatively charged leptons ℓ^- and their associated neutrinos ν_ℓ distinguish among the fermion “flavors”, and the indices $i = 1, 2, 3$ for quarks of fixed flavor distinguish among three “colors”. Interactions among fermions are mediated by spin-1 gauge bosons A_μ :

$$\begin{array}{ccc}
 SU(3) & \longleftrightarrow & \\
 & & \left(\begin{array}{c} W_1 \\ W_2 \\ W_3 \end{array} \right) \quad \Updownarrow \quad SU(2) \quad B \\
 g_1, \dots, g_8 & &
 \end{array}$$

where the eight gluons g couple quarks of different color but the same flavor, and the three W 's couple fermions of different flavor. The laws of physics are invariant under local $SU(3) \otimes SU(2) \otimes U(1)$ transformations, with $SU(3)$ acting horizontally on color and $SU(2)$ acting vertically on flavor as indicated by the arrows. The vacuum configuration is not invariant under the full $SU(3) \otimes SU(2) \otimes U(1)$ group of symmetries; “spontaneous” breaking of the electroweak symmetry down to gauge invariance of electromagnetism (em)

$$SU(2) \otimes U(1) \rightarrow U(1)_{\text{em}}$$

is attributed to the vacuum value of the spin-0 “Higgs” field H

$$v = \langle H \rangle \approx 250\text{GeV}$$

which gives masses to the electrically charged W 's and one linear combination of the neutral W and the $U(1)$ gauge boson B :

$$\begin{aligned} Z &= \cos\theta_w W_3 - \sin\theta_w B, & \gamma &= \cos\theta_w B + \sin\theta_w W_3, \\ m_\gamma &= 0, & m_{W_{1,2}} &= \frac{g}{2}v = m_Z \cos\theta_w, \end{aligned}$$

where g is the $SU(2)$ coupling constant. The fermions also acquire masses through their coupling to the Higgs field which also induces a small mixing among fermions of the same electric charge but different flavors, with mixing amplitudes of order:

$$\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau \ll d \leftrightarrow s \leftrightarrow b \ll 1.$$

The absence of large flavor changing effects is an important property of the Standard Model that has to be reproduced by the underlying theory.

The SM has been tested to a high degree of accuracy, but it is not without its difficulties. Foremost among these is the large hierarchy between the electroweak symmetry breaking scale and the reduced Planck scale m_P , known as the “gauge hierarchy”:

$$m_Z \approx 90\text{GeV} \ll m_P = \sqrt{\frac{8\pi}{G_{\text{Newton}}}} \approx 2 \times 10^{18}\text{GeV},$$

which is hard to understand within the context of quantum field theory. In addition there is no understanding of the pattern of quark masses and flavor mixing, nor why the θ -parameter, that governs CP violation in the QCD vacuum, is so small: $\theta < 10^{-10}$, where QCD, or quantum chromodynamics, is the $SU(3)$ gauge theory of the Standard Model. The only potential SM candidate for dark matter, a neutrino, has been ruled out by the study of galaxy formation. On the other hand, many extensions of the SM that attempt to address the “ θ -problem” require the existence of a very light pseudoscalar, or axion, that remains a viable dark matter candidate. The issues of the cosmological constant and dark energy are totally outside the domain of the SM.

5 The particle content of the MSSM

One way to understand the gauge hierarchy alluded to above is through supersymmetry: cancellations among boson and fermion loops remove ultraviolet divergences and stabilize mass hierarchies

against large quantum corrections. This requires a doubling of the number of particles in the SM. Spin- $\frac{1}{2}$ quarks and leptons form chiral supermultiplets with spin-0 squarks \tilde{q} and sleptons $\tilde{\ell}$:

$$SU(3) \quad \begin{array}{c} \left(\begin{array}{cccc} \tilde{u}_1 & \tilde{u}_2 & \tilde{u}_3 & \tilde{\nu}_e \\ \tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 & \tilde{e} \end{array} \right) \end{array} \quad \begin{array}{c} \longleftrightarrow \\ \longleftrightarrow \\ \longleftrightarrow \end{array} \quad \begin{array}{c} \left(\begin{array}{cccc} \tilde{c}_1 & \tilde{c}_2 & \tilde{c}_3 & \tilde{\nu}_\mu \\ \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 & \tilde{\mu} \end{array} \right) \end{array} \quad \begin{array}{c} \longleftrightarrow \\ \longleftrightarrow \\ \longleftrightarrow \end{array} \quad \begin{array}{c} \left(\begin{array}{cccc} \tilde{t}_1 & \tilde{t}_2 & \tilde{t}_3 & \tilde{\nu}_\tau \\ \tilde{b}_1 & \tilde{b}_2 & \tilde{b}_3 & \tilde{\tau} \end{array} \right) \end{array} \quad \Downarrow \quad SU(2)$$

and spin-1 gauge bosons A_μ form vector supermultiplets with spin- $\frac{1}{2}$ gauginos λ :

$$SU(3) \quad \begin{array}{c} \left(\begin{array}{c} \tilde{g}_1, \dots, \tilde{g}_8 \\ \tilde{W}_1 \\ \tilde{W}_2 \\ \tilde{W}_3 \end{array} \right) \end{array} \quad \begin{array}{c} \longleftrightarrow \\ \longleftrightarrow \\ \longleftrightarrow \end{array} \quad \begin{array}{c} \left(\begin{array}{c} \tilde{W}_1 \\ \tilde{W}_2 \\ \tilde{W}_3 \end{array} \right) \end{array} \quad \Downarrow \quad SU(2) \quad \tilde{B}$$

called gluinos, Winos and Bino, respectively. There is also the Higgsino fermionic superpartner of the SM Higgs boson, but the supersymmetric extension of the the SM requires *two* Higgs chiral supermultiplets in order to avoid gauge anomalies and to generate mass terms for all charged chiral fermions as well as for all Winos. Thus the spectrum includes

$$\left(\begin{array}{cccc} H_u^+ & \tilde{H}_u^+ & H_d^0 & \tilde{H}_d^0 \\ H_u^0 & \tilde{H}_u^0 & H_d^- & \tilde{H}_d^- \end{array} \right) \quad \Downarrow \quad SU(2)$$

and their charge conjugates. While there is no experimental evidence for SUSY partners—indeed there are ever increasing lower bounds on their masses—there is a tantalizing piece of indirect evidence. In the SM, the three coupling constants of the $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory almost converge at a single point at an energy of about 10^{15} GeV, but present high precision data excludes exact unification by about 9 standard deviations. In the MSSM the running of the coupling constants is modified, most importantly by the presence of *two* Higgs *super*multiplets, which together are equivalent to six ordinary Higgs fields, and their convergence is much closer, now at an energy of about 10^{16} GeV. The higher scale of unification is welcome in view of strong limits on the proton lifetime, since theories that unify the SM gauge groups generally predict proton decay with a Fermi coupling inversely proportional to the square of the unification scale. In addition it is just two orders of magnitude below the reduced Planck scale, perhaps a hint at unification with gravity?

However the MSSM is not without problems of its own. As explained in Section ?? supersymmetry cannot be broken spontaneously, requiring the introduction of a large number of arbitrary complex parameters and no rationale for suppressing flavor and CP violating terms to the small values needed to be consistent with observation. In addition one has to impose a discrete symmetry,

called R-symmetry, to avoid fast proton decay and suppress lepton flavor violating processes such as $\mu \rightarrow 3e$. Thus one assigns $R = +1$ to the particles of the SM including H_u, H_d , and $R = -1$ to their superpartners. As a consequence superpartners must be produced in pairs, and the lightest one is stable, providing a possible dark matter candidate, as discussed in Section ??.

6 Supersymmetry in string theory

When supersymmetry is combined with general relativity, one gets gauged supersymmetry, or supergravity, which is the infinite string tension limit of string theory. At present most particle theorists view superstring theory in 10 dimensions as the most promising candidate that reconciles general relativity with quantum mechanics. There are five of these theories, and they are related to one another by two types of dualities: S-duality that relates strong to weak coupling: $\alpha \rightarrow 1/\alpha$, and T-duality that relates large to small radius of compactification: $R \rightarrow 1/R$, where $\alpha = g_s^2/4\pi$ is the fine structure constant of the gauge group(s) at the string scale, and R is a radius of compactification from dimension D to dimension $D - 1$. Figure ?? shows [?] how these dualities relate the various 10-D superstring theories to one another, and to M-theory, which lives in 11 dimensions and involves membranes. In Figure ?? the small circles, line, torus and cylinder represent the relevant compact manifolds in reducing D by one or two. The two $O(32)$ theories are S-dual to one another, while the $E_8 \otimes E_8$ weakly coupled heterotic string theory (WCHS) is perturbatively invariant [?] under T-duality when compactified to four dimensions; in this case R is the radius of the compact six dimensional manifold.

We will be specifically focusing on this theory, and T-duality will play an important role. Another image of M-theory, the “puddle diagram” of Figure ??, indicates [?] that all the known superstring theories, as well as $D = 11$ supergravity, are particular limits of M-theory. Each point in the puddle has a very large number of possible vacua, and currently there is a lot of activity in trying to count the number of type IIB vacua; the number is very large. This endeavor is related to an attempt to address the problem of the cosmological constant as mentioned in Section ?? and discussed in [?]. The Hořava-Witten (HW) scenario [?] and its inspirations have also received considerable attention. If one compactifies one dimension of the 11-dimensional M-theory, one gets the HW scenario with two 10-D branes, each having an E_8 gauge group. As the radius of this 11th dimension is shrunk to zero, the weakly coupled heterotic string scenario is recovered; it is this string theory that most naturally incorporates the standard model. In the limit of infinite string tension, the WCHS reduces to 10-D supergravity coupled to an $E_8 \otimes E_8$ Yang-Mills gauge supermultiplet. If 6 dimensions are compactified on, say, three 2-tori, that have a flat geometry, the resulting 4-D theory will have $N=4$

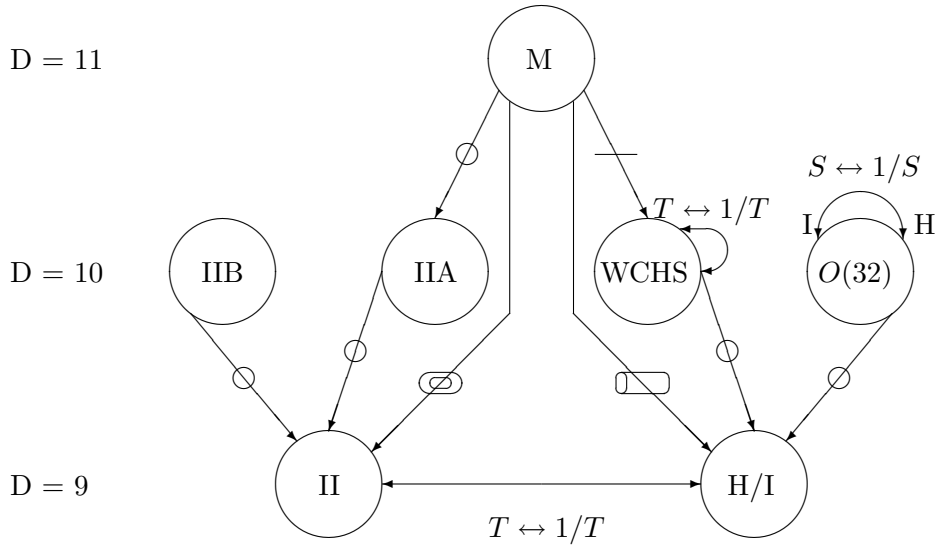


Figure 1: M-theory according to John Schwarz.

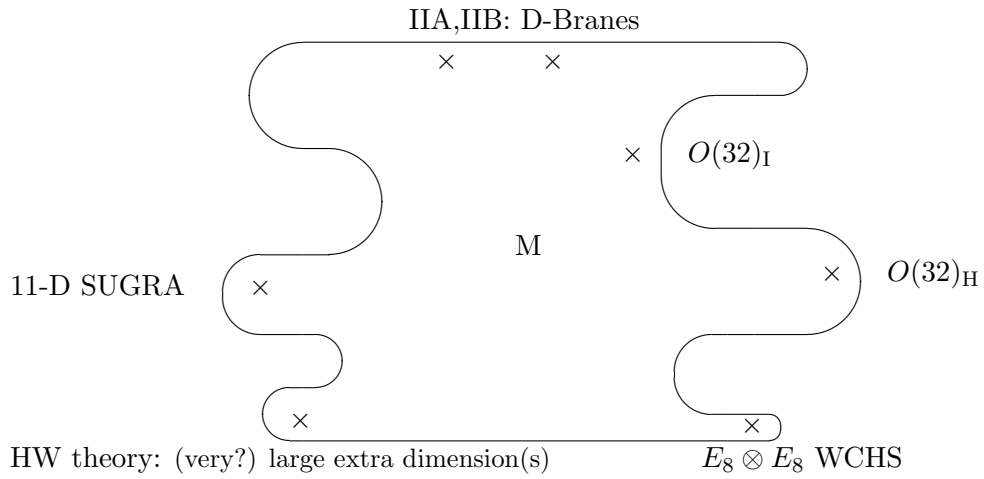


Figure 2: M-theory according to Mike Green.

supersymmetry because the 8-component spinor that generates supersymmetry transformations in 10 dimensions gives four 2-component supersymmetry generators in four dimensions. As explained in Section ?? only N=1 supersymmetry can provide a viable framework for observable particle physics. Therefore we need a curved 6-D manifold that has a nontrivial holonomy group (group of transformations under parallel transport). Calabi-Yau compactification has an $SU(3) \in SO(6)$ holonomy group that leaves only one 2-component spinor single-valued under parallel transport on the compact manifold, giving N=1 supersymmetry in four dimensions. The gauge group may be decomposed as

$$E'_8 \otimes E_8 \ni E'_8 \otimes E_6 \otimes SU(3),$$

and the equations of motion require that the space-time background curvature be balanced against a background gauge field strength in such a way that an $SU(3)$ subgroup of one E_8 is identified with the $SU(3)$ holonomy group of the Calabi-Yau manifold. Then the surviving 4-D gauge group is $E'_8 \otimes E_6$. Orbifold compactifications that mimic the Calabi-Yau case have been studied more extensively because the compact manifold is flat except at singular points with infinite curvature, and therefore it is easier to extract the low energy theory. In this case the residual gauge group in four dimensions is

$$E'_8 \otimes E_6 \otimes [G' \in SU(3)].$$

In either case, the outcome is promising because E_6 has long been recognized as the largest group that is a viable candidate for the unification of the strong and electroweak gauge groups of the Standard Model. The massless spectrum consists of single-valued fields that are invariant under the diagonal of the two broken $SU(3)$'s (or, in orbifold compactification, an appropriate subgroup thereof). The surviving degrees of freedom of the 10-D gauge supermultiplet

$$(A_M, \tilde{g})_{E'_8 \otimes E_8}, \quad M = 0, \dots, 9,$$

are the 4-D gauge supermultiplet

$$(A_\mu, \tilde{g})_{E'_8 \otimes E_6}, \quad \mu = 0, \dots, 3,$$

which is invariant under both $SU(3)$'s, and matter chiral multiplets

$$(A_m, \tilde{g})_{E_8/[E_6 \otimes SU(3)]} = (27 + \bar{27})_{E_6}, \quad m = 4, \dots, 9,$$

which transform as $(3, \bar{3}) + (\bar{3}, 3)$. These states decompose under the smaller candidate gauge unification groups $SO(10)$ and $SU(5)$ as

$$27_{E_6} = (16 + 10 + 1)_{SO(10)} = (\bar{5} + 10 + 1 + 5 + \bar{5} + 1)_{SU(5)}.$$

The $\bar{5} + 10$ of $SU(5)$ contains the quarks and leptons of the standard model. These form the 16 of $SO(10)$ together with a Standard Model singlet that may be responsible for the recently observed small neutrino masses and mixing. The $5 + \bar{5}$ contained in the 10 of $SO(10)$ includes, among other things, the two Higgs doublets needed in the supersymmetric extension of the Standard Model, as discussed in Section ???. What does not appear in the massless spectrum is a chiral multiplet transforming according to a large representation of the gauge group, such as the adjoint representation, that could include a Higgs particle whose vacuum value would break E_6 to the Standard Model. Instead this is achieved by what is known as the Hosotani mechanism or Wilson lines. If the compact manifold is not simply connected, gauge flux can be trapped around a noncontractible loop:

$$\left\langle \int dl^m A_m \right\rangle \neq 0,$$

in a manner reminiscent of the Aharonov-Bohm effect. The nonvanishing gauge flux has the same effect as an adjoint Higgs field, further breaking the symmetry to leave a 4-D gauge group:

$$(G_{\text{hid}} \in E'_8) \otimes [(SM \otimes G'') \in E_6] \otimes [G' \in SU(3)]$$

where the Standard Model gauge group is $SM = SU(3) \otimes SU(2) \otimes U(1)$. As mentioned previously, the class of vacua described here is a tiny subset of the full set of possible vacua, even within the framework of the WCHS; a recent review of the many facets of string theory and M-theory can be found in [?]. The attractiveness of Calabi-Yau-like compactification of the WCHS is that the gauge group and states of the SM emerge naturally. In addition the spectrum includes an axion that is a candidate for the “QCD axion” mentioned at the end of Section ??? and/or for dark matter. The presence of a hidden sector is also welcome, as it may provide a mechanism for spontaneous supersymmetry breaking.

7 Dark matter and dark energy in supersymmetry

Many independent lines of cosmological evidence have led to the conclusion that the vast majority of matter in the universe is “dark” in the sense that it has evaded observation based on direct interaction with electromagnetic radiation. Nonbaryonic dark matter out-masses ordinary matter by a factor of about 8. The dominant class of dark matter candidates are “Weakly Interacting Massive Particles” (WIMPs). There have been a number of suggestions for dark matter particles, but it seems that the best candidate is the lightest “neutralino” that is provided by TeV-scale SUSY.

A particle dark matter candidate must satisfy the following criteria:

- It must be “stable” in the sense that its lifetime is longer than the age of the universe.
- There must be an effective production mechanism to create the right amount in the early universe.
- It must be nonrelativistic during structure formation; in other words it must be “cold” dark matter.
- It must be weakly interacting to have escaped detection, which implies that it must be electrically neutral and colorless.

These constraints are satisfied by the neutral Higgsinos (\tilde{H}_u, \tilde{H}_d), the neutral Wino (\tilde{W}^0) and the Bino (\tilde{B}), four Majorana fermions with the same quantum numbers, that can mix to give four mass eigenstates: $\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0$, that are called neutralinos. The lightest of these is stable and a good candidate for dark matter. Other possibilities include the lightest sneutrino, which is apparently excluded by accelerator experiments, and the superpartner of the graviton, the gravitino, which would be very hard to detect. A very comprehensive review of cold dark matter can be found in [?].

In addition, it appears to be generally accepted by astrophysicists and cosmologists that the cosmological constant Λ , long believed to vanish, is actually positive but very small. As a consequence, the expansion of the universe is accelerating. If we interpret Λ as the energy density of the vacuum, dimensional arguments, as well as quantum field theory (QFT) calculations would give it a value of

$$\Lambda_P = \frac{\text{Planck mass}}{(\text{Planck length})^3} \approx 10^{94} \text{grams/cm}^3,$$

while the actual value is $\Lambda \sim 10^{-120} \Lambda_P$.

In SUSY QFT, without supergravity, the vacuum energy vanishes to all order in perturbation theory. In a generic QFT it diverges quartically, while if SUSY is broken only softly it diverges logarithmically. Still, for any reasonable cut-off, Λ comes out much larger than the measured value. The MSSM and other SUSY extension of the SM have nothing to say about this. In supergravity, the cosmological constant is no longer related to the scale of supersymmetry breaking and can take any value, but there is no simple understanding of its measured value. As mentioned in Section ??, there is considerable activity in counting vacua with the notion that there might be a probabilistic determination of the cosmological constant; this raises a number of philosophical questions that we do not wish to enter into here.

8 Breaking supersymmetry: beyond the MSSM

The absence of observed superpartners implies that supersymmetry is broken in the vacuum. Furthermore an analysis of the observed particle spectrum shows that supersymmetry cannot be spontaneously broken in the observable sector. This can simply be seen as follows [?]. The squark squared mass matrices $M_{|Q|}^2$, are 6×6 matrices in flavor space; Q is the electric charge of the squark sextuplets $(\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)$, with $Q = \frac{2}{3}$, and $(\tilde{d}_L, \tilde{s}_L, \tilde{d}_R, \tilde{b}_R)$, with $Q = -\frac{1}{3}$. The subscripts L, R , denote, respectively, squarks in chiral supermultiplets $\Phi(\tilde{q}_L, q_L)$ together with left-spinning quarks, and squarks in antichiral supermultiplets $\Phi^\dagger(\tilde{q}_R, q_R)$ together with right-spinning quarks. If supersymmetry is broken spontaneously and the vacuum conserves electric charge and color, these matrices satisfy

$$M_{\frac{2}{3}}^2 = \begin{pmatrix} m_{\frac{2}{3}}^2 + \frac{1}{2}gD_W^3 + \frac{1}{6}g'D_Y & \Delta \\ \Delta^\dagger & m_{\frac{2}{3}}^2 - \frac{2}{3}g'D_Y \end{pmatrix}, \quad M_{\frac{1}{3}}^2 = \begin{pmatrix} m_{\frac{1}{3}}^2 - \frac{1}{2}gD_W^3 + \frac{1}{6}g'D_Y & \Delta' \\ \Delta'^\dagger & m_{\frac{1}{3}}^2 + \frac{1}{3}g'D_Y \end{pmatrix},$$

where $m_{|Q|}^2$ are the 3×3 squared mass matrices of the corresponding quarks, the ‘‘D-term’’ contributions are proportional the appropriate electroweak quantum numbers of the squarks, and the off-diagonal elements Δ, Δ' do not need to be specified for our purposes. If we define a 12×12 squark squared mass matrix M^2 , and a 6×6 quark squared mass matrix m^2 , by

$$M^2 = \begin{pmatrix} M_{\frac{2}{3}}^2 & 0 \\ 0 & M_{\frac{1}{3}}^2 \end{pmatrix}, \quad m^2 = \begin{pmatrix} m_{\frac{2}{3}}^2 & 0 \\ 0 & m_{\frac{1}{3}}^2 \end{pmatrix},$$

We see immediately that $\text{Tr}M^2 = 2\text{Tr}m^2 \approx 2m_t^2 \approx (240\text{GeV})^2 = \sum_{i=1}^{12} M_q^2$, while data from the Tevatron at Fermilab puts a lower limit on the lightest squark mass at about 250GeV . However one might imagine that there is a fourth family of quarks and leptons and their superpartners (with a heavy neutrino, since a fourth light neutrino is ruled out by data from the LEP collider at CERN and by the abundance of light elements). This will not help due to the following argument[?]. Since the D-terms on the diagonal in M^2 sum to zero, either they are all zero, or at least one of them is ≤ 0 . Assume this is the case for the first one

$$\frac{1}{2}gD_3 + \frac{1}{6}g'D_Y = -a^2 \leq 0.$$

Denote by β the 3-component vector that is the eigenstate of $m_{\frac{2}{3}}^2$ with the smallest eigenvalue (i.e. the physical up quark): $m_{\frac{2}{3}}^2\beta = m_u^2$. Then

$$\bar{\beta}^\dagger M_{\frac{2}{3}}^2 \bar{\beta} = m_u^2 - a^2 \leq m_u^2, \quad \bar{\beta} = \begin{pmatrix} \beta \\ 0 \end{pmatrix}.$$

But we know from matrix theory that $\bar{\beta}^\dagger M_{\frac{2}{3}}^2 \bar{\beta}$ is greater than or equal to the smallest eigenvalue M_0^2 of $M_{\frac{2}{3}}^2$: $M_0^2 \leq m_a^2$. If one assumes instead the second D-term $-D_Y \leq 0$, one reaches the same conclusion, and taking the third or fourth D-term to be ≤ 0 , one finds $M_0'^2 \leq m_a^2$, where $M_0'^2$ is the smallest eigenvalue of $M_{\frac{1}{3}}^2$. If SUSY is spontaneously broken in the MSSM there must be a squark lighter than a few MeV, which is clearly ruled out by electron-positron annihilation data.

Therefore the only way to break supersymmetry in a supersymmetric extension of the Standard Model without reintroducing a gauge hierarchy problem is by introducing “soft” (operators of dimension three or less in the Lagrangian) supersymmetry breaking. This leads to a plethora of arbitrary parameters, and therefore to the idea that supersymmetry must be spontaneously broken in a “hidden sector” of the full theory. For example there could be a sector that interacts with ours only through gravitational strength couplings. As mentioned in Section ??, this scenario arises naturally in the context if the weakly coupled heterotic string.

For a gauge group G_a the β -function, which governs the energy-dependence of the coupling “constant” $g_a(\mu)$, is defined by

$$\mu \frac{\partial g_a(\mu)}{\partial \mu} = \beta(\mu) = -\frac{3}{2} b_a g_a^3(\mu) + O(g_a^5).$$

Suppose that the hidden sector gauge group G_{hid} contains a subgroup G_c with β -function coefficient b_c that is larger than the coefficient b_{QCD} of the SM color gauge theory, QCD. In this case hidden sector confinement and gaugino condensation

$$\langle \lambda\lambda \rangle_{\text{hid}} \neq 0 \tag{8.1}$$

will occur at a scale Λ_c exponentially larger than the scale Λ_{QCD} at which color confinement and quark condensation take place in QCD. To see how this can provide a source of supersymmetry breaking in the superstring context, we first note that four dimensional supergravity is specified by three functions of chiral superfields: the superpotential $W(Z)$ which is a holomorphic function that determines Yukawa couplings of chiral fermions to scalars, the Kähler potential $K(Z, \bar{Z})$ which is a real function that governs the kinetic energy terms for chiral fields and the holomorphic function $f(Z)$ whose vacuum value gives the gauge coupling and the angle θ that determines the vacuum configuration of the Yang-Mills fields: $\langle f(Z) \rangle = g^{-2} - i\theta/8\pi^2$. In addition we need to introduce important chiral supermultiplets, known as moduli supermultiplets, that are remnants of 10-D supergravity. The 10-D supergravity supermultiplet consists of the 10-D metric g_{MN} , an antisymmetric tensor B_{MN} , a scalar ϕ known as the dilaton, and their fermionic superpartners, the 10-D gravitino ψ_M , and a spin- $\frac{1}{2}$ fermion χ . These are all invariant under the gauge $SU(3) \in E_8$, so those that remain in the massless spectrum must also be invariant under the holonomy $SU(3) \in SO(6)$,

leaving the graviton $g_{\mu\nu}$ and the gravitino ψ_μ of 4-D supergravity, and an invariant subset of the two real scalars g_{mn} and B_{mn} and the spin- $\frac{1}{2}$ fermion ψ_m , the 4-D antisymmetric tensor $B_{\mu\nu}$, the dilaton ϕ and the single valued component of χ . These combine to form chiral supermultiplets, whose vacuum values determine the size and shape of the compact manifold, the gauge coupling constant g_s at the string scale $m_s = g_s m_P$, and the θ angle of the 4-D theory. For example, in compactifications on an orbifold M_6 that factorizes into three 2-tori T^2 : $M_6 = (T^2/G)^3$, where G is a discrete group of symmetries on the torus, there are three ‘‘Kähler moduli’’ superfields T^I with complex scalar components

$$t^I = \phi^{\frac{3}{4}} \det^{\frac{1}{2}} g_{mn}^I + i \frac{b_{12}^I}{\sqrt{2}}, \quad \langle \text{Re} t^I \rangle = R^I, \quad I = 1, 2, 3$$

where g_{mn}^I , $m, n = 1, 2$ is the 2-D metric on the I 'th 2-torus and R^I is its radius. For some orbifolds there are also ‘‘complex structure’’ moduli superfields U^I with the scalar vacuum value $\langle \text{Re} u^I \rangle$ determining the ratio of radii on the 2-torus. More generally, there can be a 3×3 matrix-valued modulus superfield T^{IJ} in the low energy theory; however most phenomenological studies assume that only the three diagonal moduli T^I are part of the massless spectrum. All compactifications of the WCHS have a ‘‘dilaton’’ chiral supermultiplet S with $f(Z) = S$; thus the vacuum value of its scalar component s determines the gauge coupling constant and the θ angle:

$$\langle s \rangle = g_s^{-2} - i\theta/8\pi^2.$$

The real part of its scalar component is given by

$$\text{Res} = \phi^{-\frac{3}{4}} \det g_{mn},$$

and the imaginary part, known as the ‘‘universal axion’’ is dual to the 2-form:

$$\partial_\mu \text{Im} s = \phi^{-\frac{3}{2}} (\det g_{mn}) \epsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma}.$$

As a consequence of the dilaton coupling to the Yang-Mills sector, gaugino condensation (??) generates an effective superpotential for S

$$W(S) \propto e^{-S/b_c}, \quad (8.2)$$

which in turn induces a gravitino mass:

$$m_{\frac{3}{2}} \propto \langle W(s) \rangle \propto e^{-s/b_c} = e^{-1/b_c g_c^2} = \Lambda_c^3$$

thereby breaking local supersymmetry. The form of the superpotential (??) led to what is called the ‘‘runaway dilaton’’ problem. At the classical level, the scalar potential is proportional to the square of the superpotential:

$$V(s) \propto e^{-2\text{Res}/b_c},$$

so the vacuum corresponds to vanishing coupling and no condensation: $V(s) \rightarrow 0$ for $\langle (\text{Res})^{-1} \rangle = g^2 \rightarrow 0$. However, the effective potential for s is constructed by anomaly matching [?]:

$$\delta\mathcal{L}_{eff}(s) \longleftrightarrow \delta\mathcal{L}_{hid}$$

under the classical symmetries of the effective QFT that are anomalous at the quantum level. One of these is T-duality itself, which is not anomalous in the underlying string theory. The symmetry is restored in the effective QFT by adding a four dimensional counterpart [?] of the Green-Schwarz counter-term [?] in 10-D supergravity. This modifies the classical dilaton Kähler potential: $K(s + \bar{s}) = -\ln(2\text{Res}) \rightarrow K(\ell) = \ln \ell$, where $\ell^{-1} = \left[2\text{Res} + b_{GS} \sum_I \ln(2\text{Re}t^I) \right]$, and introduces a second runaway direction to strong coupling: $V(\ell) \rightarrow -\infty$ for $\langle \ell \rangle = g_s^2/2 \rightarrow \infty$, where the weak coupling approximation is no longer valid. Including [?] nonperturbative string effects [?] $\sim Ae^{-c/\sqrt{\ell}}$ and/or other corrections [?] to the dilaton Kähler potential allows for dilaton stabilization at weak coupling and very small vacuum energy by adjusting, for example, the parameters A and c in a parametrization of these effects in the region around the vacuum. This mechanism for stabilizing the dilaton is known as “Kähler” stabilization. The attractive features of Kähler stabilized, T self-dual heterotic string models can be summarized as follows [?]

- In contrast with models that stabilize the dilaton using more than one gaugino condensate (and adjusting their relative β -functions), there is no difficulty in generating a positive semi-definite potential.
- The Kähler moduli are stabilized at self-dual points [?]: $T_{sd} \rightarrow T_{sd}$, with supersymmetry conserving vacuum values $\langle T_{sd} \rangle$. As a consequence, no large flavor mixing is induced by supersymmetry breaking, which arises only from the condensate and dilaton vacuum values. In contrast to the Kähler moduli, the dilaton has flavor-independent couplings to observable matter.
- The condition of (nearly) vanishing vacuum energy leads to mass hierarchies [?]:

$$m_\ell \gg m_t \sim (10 - 20)m_{\frac{3}{2}} \sim m_0 \gg m_{\frac{1}{2}}$$

The enhancement of the moduli masses m_ℓ, m_t relative to the gravitino mass $m_{\frac{3}{2}}$ avoids potential problems [?] with Big Bang nucleosynthesis, and the suppression of gaugino masses $m_{\frac{1}{2}}$ relative to scalar masses m_0 results in important quantum corrections to the former. As a result these models naturally accommodate a dark matter candidate [?].

- One hidden sector condensate is sufficient to break supersymmetry and stabilize the dilaton. As a result there is a residual R-symmetry (a continuous version of the R-parity introduced in Section ??) that guarantees a massless axion at the QFT level.

- In supergravity, this R-symmetry is protected by T-duality, allowing for a possible solution to the CP problem of QCD [?].
- T-duality provides a possible mechanism for generating R-parity in the MSSM [?].

The phenomenology of Kähler stabilized self-dual heterotic string models is reviewed in [?], which includes extensive references to, and comparisons with, other models for dilaton stabilization.

Reviews of other models for communicating SUSY breaking from the hidden sector to the observable sector can be found in summer school and conference proceedings such as SUSY 07. All of these models predict different patterns of soft SUSY breaking. Typically these patterns appear simple at some high scale, such as the Planck scale, the scale of gauge coupling unification or the scale of supersymmetry breaking itself; for example in the case of gaugino condensation described above, supersymmetry is broken at the condensation scale $\Lambda_c \sim 10^{14}\text{GeV}$. As discussed by Peskin [?], it is these high energy parameters that inform us about physics at near Planck-scale energies, that we want to determine, while it is the low energy parameters that we will measure. Just as we were able to use the renormalization group equations of quantum field theory to determine high energy values of the couplings and discover near unification in the SM and more accurate unification in the MSSM, we can use similar QFT tools to probe high energy values of soft parameters. For example, gaugino masses scale with energy the same way as the fine structure constants $\alpha_i(\mu) = g_i^2(\mu)/4\pi^2$, so if they have a common mass m at the unification scale, at the scales μ at which the masses are measured they will be in the ratio $m_1(\mu) : m_2(\mu) : m_3(\mu) = \alpha_1(\mu) : \alpha_2(\mu) : \alpha_3(\mu)$. Relations among squark and slepton masses are more complicated. Since squarks have strong QCD couplings which increase their masses as μ decreases, they are expected to be heavier than sleptons if all the sfermions have the same mass at some high scale. The most massive third family has larger couplings to the Higgs field, which has the opposite effect, making them lighter than their companions with the same gauge charges. The gaugino and Higgsino sector is even more complicated with very model-dependent mixing among the neutralinos discussed in Section ??, and among the “charginos” $\tilde{W}^\pm, H_u^\pm, H_d^\pm$. For more details on the predicted spectra for different models the reader is referred to the reviews [?] and [?].

9 Concluding remarks

We are very optimistic that the MSSM, or some extension of it, for which there are many proposals in the literature (the simplest being the nMSSM which postulates an additional scalar superfield [?]), will correctly describe the particle and superparticle spectrum that will be produced at

LHC energies. We know that our friend Julius shared this conviction. Clearly the MSSM and its extensions still have new difficulties of their own, even though they solve problems of the SM as we have attempted to explain. The history of physics teaches us that this is a situation which often occurs when an important step forward takes place in our understanding of the basic equations governing the fundamental processes of our universe.

If the superpartners are not found at the LHC, it could simply be that the accelerator energy is just not high enough and that their masses are beyond the reach of the LHC. Even if the superpartners are found at the LHC, it will be very difficult to distinguish among different models, and even among SUSY and other proposals such as “large extra space-time dimensions” [?]. One will need a higher *precision* instrument such as the International Linear Collider (ILC) to resolve these issues. Will there be the political will to build it? Unfortunately, we are not optimistic about this.

Another possibility is that the beautiful mathematics of supersymmetry will find its application to physics in a different interpretation than the one assumed so far. To explain what we mean, we refer to the history of non-Abelian gauge theories, which for a long time were thought to be only a description of vector resonances. It took the realization of their property of renormalizability *as well as* a total paradigm change, due to the development of the quark picture and of the flavor-color properties of gauge theories, which led to the very successful SM. The analogy is striking for us. As we explained, SUSY quantum gauge field theories are renormalizable and have even fewer divergences than generic gauge field theories.

Finally SUSY, through supergravity, provides a unified understanding of all forces of nature, although not a perfect one, neither at the four dimensional quantum field theory level (see however [?]), nor at the superstring level.

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