# **Optimal Coupler and Power Settings for Superconductive Linear Accelerators**\*

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#### Abstract

The modeling analysis presented in this paper addresses the question of how to achieve the highest vector sum gradient for all beam currents when individual cavities operate at different gradients due to their inherent quenching limitations. The analytical method explained here constitutes a step forward toward the operability of the International Linear Collider (ILC), Project X [8], or XFEL [7]. Unlike previously proposed methods [1, 2], this approach prevents cavities from quenching should the beam current be lower than its maximum value.

## **INTRODUCTION**

Ideally, all superconducting cavities of a linear accelerator RF station operate at the target gradient (31.5 MV/m for the ILC). Practically however, cavities show a certain disparity in their gradient performance. Based on the experience acquired at DESY XFEL [6], we know that some cavities quench or exhibit a Q drop behavior when operating above 22 MV/m while others will sustain an accelerating gradient of 34 MV/m. This disparity among cavities raises challenging issues related to conditioning and operations of linear accelerators using one klystron per RF station. More precisely, this paper presents an analytical solution to the following question: for N cavities with a given maximum gradient distribution (i.e. quenching limits), what is the highest vector sum gradient that can be maintained for the entire flat top duration with beam or in the absence of beam, while ensuring that no cavity quenches? Currently, one approach has been proposed [1, 2] to address this issue but it only guaranties maximum gradient operability when the beam current is maximum. A second approach implemented at DESY for XFEL [7] only guaranties maximum gradient without beam current. In contrast, the method presented here predicts the maximum vector sum gradient that can be reached for a given cavity distribution, independently of the beam loading.

# $Q_L$ AND $P_K$ SETTINGS

The present model consists of a low level RF (LLRF) controller driving a single klystron, providing the RF forward power to multiple cavities. The amplitude and phase of the klystron drive signal can be adjusted from the LLRF controller, the proportion of forward RF power delivered to individual cavities can be adjusted at the wave guide level

by tuning the wave guide couplers, (referred to as  $P_k$  setting) and at the cavity level by changing the loaded Q of each cavity's input coupler, (referred to as  $Q_L$  settings). Several techniques have been suggested to adjust the forward power distributed to a pair of cavities: variable tap off splitters [5], or phase shifters modulators [9]. Either of these techniques require hardware replacement and interrupting accelerator operations for an extended period of time. Hence, this tuning is considered to be set once and not changed again. Setting the external Q at the cavity input coupler is an operation which does not require shutting down the RF station. It is nonetheless a time consuming procedure and is not practical for large scale machines. Finally, the LLRF drive signal can dynamically (i.e. during the RF pulse) control and adjust the amplitude and phase of the klystron drive signal. The goal of this study is to find the optimal configuration of these parameters to achieve the highest vector sum gradient, while respecting the operational and security constains listed above.

The approach described in [4] consists of choosing  $Q_L$ and  $P_k$  settings specific to each cavity so as to match every cavity with maximum beam loading. One major issue associated with this scheme is that the individual  $Q_L$  and  $P_k$  need to be readjusted everytime the beam loading is less than maximum which can become a real operation bottleneck for large scale accelerators. Alternatively, lowering the klystron forward power for less than maximum beam current operations can prevent cavities from quenching but will significantly degrade the vector sum gradient. In the DESY approach [7], unless the vector sum gradient is lowered, some cavities will quench when the beam is on. The approach presented here addresses these issues.

#### ANALYTICAL INSIGHT

#### Flat top without beam

The voltage inside a superconducting cavity, on resonance and with on-crest beam loading can be modeled as follows [3]:

$$V_C(t) = 2R_L \left[ I_{g0} (1 - e^{-\frac{t}{\tau}}) - I_{b0} (1 - e^{-\frac{t-t_0}{\tau}}) \right]$$
(1)

where  $R_L$  is the loaded resistance of the cavity,  $I_{b0}$  is the nominal DC beam current,  $\tau = \frac{2Q_L}{\omega_0}$  is the cavity time constant and  $t_0$  is the duration of the fill time of the cavity, after which the forward power is dropped by four when no beam is present. Achieving a flat top in the absence of beam is equivalent to a null time-derivative of the cavity voltage which is obtained when  $t_0 = \tau \ln 2$ . The steady state cavity voltage is then  $V_{SS}^{nobeam} = R_L I_{g0}$ , where  $I_{g0} = 2I_{b0}$  is the

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nominal generator current. This flat top condition under no beam is solely function of  $\tau$ , (i.e indirectly function of  $Q_L$ ). So, setting all cavities to the same  $Q_L$ , and setting the fill time  $t_0 = \tau \ln 2$  will guarantee a flat top for all cavities hence for their vector sum.

#### Flat top with beam

Setting all cavities to the same  $Q_L$  will result in all  $R_L$ and all  $\tau$  being equal. The vector sum can then be simplified to the following expression:

$$V_S(t) = 2R_L(1 - e^{-\frac{t}{\tau}}) \frac{1}{N} \sum_i I_{gi} - 2R_L I_{b0}(1 - e^{-\frac{t-t_0}{\tau}})$$
(2)

The flat top for the vector sum is achieved by annulling the time derivative of  $V_S(t)$ , which is equivalent to setting  $\frac{1}{N}\sum_{i} I_{gi} = 2I_{b0}$ . Using the notation  $I_{gi} = \alpha_i I_{g0}$ , the flat tap condition for the vector sum with been becomes

top condition for the vector sum with beam becomes

$$\frac{1}{N}\sum_{i}\alpha_{i} = 1 \tag{3}$$

#### The no-quench guarantee

When the beam is on, a cavity which receives less (more) than the nominal generator current,  $\alpha_i < 1$ ,  $(\alpha_i > 1)$  sees its voltage drop (increase) during beam loading. Comparing the cavity voltage at the beginning (end) of the beam loading against the cavity quenching gradient can determine whether the quenching limit has been exceeded, as illustrated in Fig.1. From Eq.1, and by introducing the fol-



Figure 1: Quenching limits as a function of  $I_{gi}$ .

lowing notation for the flat top duration  $T_{FT} = t_1 - t_0 = \tau \ln \beta$ , the cavity voltage at the begining and at the end of the beam loading are  $V_{Ci}(t_0) = 2R_L\alpha_i I_{b0}$  and  $V_{Ci}(t_1) = 2R_L I_{b0} \left[ \alpha_i (2 - \frac{1}{\beta}) - (1 - \frac{1}{\beta}) \right]$ . For cavities operating below (above) the vector sum, the no-quenching condition is equivalent to having  $V_{Ci}(t_0) \leq V_{qi}$ ,  $(V_{Ci}(t_1) \geq V_{qi})$ , where the  $V_{qi}$ 's are the cavity quenching limits. Introducing the following notation,

$$\alpha_i^- = \frac{V_{qi}}{2R_L I_{b0}} \tag{4}$$

$$\alpha_i^+ = \left[\frac{V_{qi}}{2R_L I_{b0}} + (1 - \frac{1}{\beta})\right] \frac{\beta}{2\beta - 1} \tag{5}$$

the maximum gradient corresponding to the critical quenching case is reached when the cavities operating below (above) the vector sum have  $\alpha_i = \alpha_i^- (\alpha_i = \alpha_i^+)$ . One can note that  $\alpha_i^-$  and  $\alpha_i^+$  are monotonous in  $V_{qi}$  and that for a cavity operating at the vector sum gradient, we have  $\alpha_i^- = \alpha_i^+ = 1$ . Hence, choosing  $\alpha_i^{lim} = \min\{\alpha_i^-, \alpha_i^+\}$  for every cavity will guaranty that no cavity quenches. Normalizing all  $\alpha_i$ 's by  $\bar{\alpha} = \frac{1}{N} \sum_i \alpha_i^{lim}$  will guaranty that their arithmetic mean is unity. Assuming  $\bar{\alpha} \ge 1$ , the normalized  $\alpha_i = \frac{\alpha_i^{lim}}{\bar{\alpha}}$  now verify Eq.3, 4 and 5, resulting in a flat vector sum and no quench. With this choice of  $\alpha_i$ 's, the steady state vector sum with beam is  $V_{SS}^{beam} = 2R_L I_{b0}$ , also equal to the flat top gradient in the absence of beam. Furthermore, the critical case of  $\bar{\alpha} = 1$  corresponds to the maximum flat vector sum without a cavity quench.

#### RESULTS

In the ILC baseline design, each RF station comprises of 26 cavities in pairs. Unless specified otherwise, the simulations in this work follow the gradient distribution introduced in [4] consisting of 13 cavities with maximum gradients uniformly distributed between 22 and 34 MV/m. The amplitude plot of Fig. 2 is obtained maximum beam current (a) and no beam (b). Individual quenching gradients are indicated with dashed lines, while the vector sum gradient is shown with a thicker trace. As this result illustrates the critical case of  $\bar{\alpha} = 1$ , under maximum beam, Fig. 2(a), the cavities with a gradient below (above) the vector sum reach their quench limit at the begining (end) of the beam time. In the no-beam case, Fig. 2(b), the cavities operating below the vector sum are at their quenching limit while the cavities above do not reach their own limit. These two plots illustrate the extreme cases. Any intermediate beam current would result in all cavities running below their quenching limits at all time. The maximum vector sum gradient  $V_S$ is a function of the cavity gradient distribution and of the beam time duration. In the example of Fig. 2, a maxium vector sum of 27.1 MV/m is found. This is approximately 97% of the intrinsic limit gradient of 28 MV/m. A higher maximum vector sum gradient can be reached as the cavity spread decreases. A uniform spread of 6 MV/m centered around 28 MV/m yields a vector sum of 27.6 MV/m (or 98.6% of the limit gradient). The distribution of the cavity gradients within a given range also has an impact on  $V_S$ . A more realistic distribution than that used in the previous example would be where the larger portion of the cavities have a quenching limit close to the average of 28 MV/m, while a few "extreme" cavities would perform at the lower or higher end of the distribution. A such "gaussian-like" cavity gradient distribution for the same spread yields a vector sum typically above 98% of the limit gradient. This is illustrated in the first two rows of Table 1, showing the maximum vector sum gradient obtained for two cavity gradient distributions, spread between 22 and 34 MV/m.

A significant improvement of this present scheme with



Figure 2: Amplitude plot for 13 cavities with (a) and without (b) beam.

Table 1: Impact of the distribution of cavity gradients

	uniform	gaussian	
$V_S$	27.1	27.5	[MV/m]
$V_S/V_{lim}$	96.8	98.2	[%]
$P_{fwd}^*$	6.65	6.62	[MW]
$\check{P_{ref}^*}$	0.039	0.020	[MW]

\*power calculated for 26 cavities

respect to [1, 2] is the reduced reflected power during the beam time. For 26 cavities, the reflected power is reduced by more than 400 kW compared to that calculated with the previous scheme. The plot of Fig. 3 shows the variations of the maximum vector sum gradient as a function of  $Q_L$ . As  $Q_L$  is increased the vector sum improves until the quenching limit is reached. Setting the cavities to a higher  $Q_L$  is possible so long the overall forward power is reduced to avoid a cavity quench. However, this has a negative impact on the vector sum gradient  $V_S$  which starts to decrease once the critical  $Q_L$  value is passed. Also shown in Fig. 3 is the total reflected power  $P_{ref}$  summed for 13 cavities and over the duration of the flat top. As can be seen, the  $Q_L$  value maximizing the vector sum is also the optimal  $Q_L$  minimizing the reflected power.

### CONCLUSION

A method to calibrate N superconducting cavities with any gradient disparity is proposed in this report. This approach provides the optimal coupler and power settings and is applicable for any large scale linear accelerator where a single-klystron/multiple-cavities scheme is used. The complete analytical derivation for this method is exposed and



Figure 3: Variations of the vector sum gradient  $V_S$  and the total reflected power  $P_{ref}$  as a function of  $Q_L$ .

explained in details. For a given distribution of quenching limits, choosing the optimal  $Q_L$  for all cavities provides the highest vector sum gradient, guaranties that it will remain flat under any beam or no beam condition, and that no cavity will quench during operation. Depending on the cavity distribution, accelerating gradients over 98% of the intrinsic limit can be reached. As a by product, the reflected power is greatly reduced during the beam time. This constitues a more efficient way of accelerating the beam.

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