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Chromaticity Tracking with a Phase Modulation/Demodulation Technique in the Tevatron

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Abstract

The Tevatron chromaticity tracker (CT) has been successfully commissioned and is now operational. The basic idea behind the CT is that when the phase of the Tevatron RF is slowly modulated, the beam momentum is also modulated. This momentum modulation is coupled transversely via chromaticity to manifest as a phase modulation on the betatron tune. And so by phase demodulating the betatron tune, the chromaticity can be recovered. However, for the phase demodulation to be successful, it is critical that the betatron tune be a coherent signal that can be easily picked up by a phase detector. This is easily done because the Tevatron has a phase locked loop based tune tracker which coherently excites the beam at the the betatron tune.

Key words:
Tevatron, chromaticity tracker, chromaticity tracking, tune tracker PLL,
phase modulation, phase demodulation

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1 Introduction

The Tevatron operating at 980 GeV is still the highest energy collider in the world in 2008. Its performance in 2008 has been exceptional: both initial luminosity records (361×10^{30} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in store 6683), weekly integrated luminosity (74 pb^{-1} for the week from 08 Dec to 15 Dec of 2008), and monthly integrated luminosity (260 pb^{-1} for the month of Dec 2008) records have been set. More importantly, the ongoing Run II programme has collected over 8 fb^{-1} of integrated luminosity since 2001, and at the current rate, it has been projected that the experiments at CDF and DØ will accumulate a total of 16 fb^{-1} by the end of 2010. With this large data set, the Tevatron will be able to probe for the standard model Higgs between 145 to 185 GeV.

The record smashing performance has been made possible by improvements to the entire chain of accelerators at the Fermilab complex. For the Tevatron itself, there have been optimisation of many beam and machine parameters. One important beam parameter that has been carefully controlled is the chromaticity. Chromaticity determines both the lifetime and the stability of the protons and anti-protons in the Tevatron. If the chromaticity is too small, the beam tends to go unstable because of insufficient Landau damping. If the chromaticity is too large, the beam lifetime is poor because its tune footprint covers too many resonances. Therefore, proper setting of the chromaticity is important. Furthermore, the Tevatron is a superconducting machine which means that there are persistent currents in its dipole magnets which depend

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on its ramp and squeeze history. These persistent currents slowly decay away and chromaticity drift at the injection porch is one manifestation from this decay. Another effect from the persistent current is the "snapback" effect which in the first few seconds of the energy ramp causes the magnetic fields of the dipoles to snap back to the b_2 hysteresis curve. For stable operation, both effects must be compensated.[1]

In the traditional method for measuring chromaticity, the RF frequency is changed and the excursion of the betatron tune from its nominal position is measured from which the chromaticity can be extracted with the formula

$$\Delta Q = -\frac{\xi}{\eta}\left(\frac{\Delta f}{f_{\mathrm{RF}}}\right) \tag{1}$$

where ξ is the chromaticity, η is the slip factor, f_{RF} is the nominal RF frequency, Δf is the change in RF frequency from f_{RF} and ΔQ is the change in betatron tune from the nominal betatron tune when $\Delta f = 0$. Using this method, an experienced operator takes about 3 minutes to measure just one point, which becomes very tedious and error prone when he uses it to plot out the chromaticity drift because it requires many points.

It is obvious from equation (1) that if there is a way to continuously track the tune, a slow frequency modulation of the RF will allow the continuous measurement of chromaticity. In fact, this technique has been applied successfully at both the RHIC (Relativistic Heavy Ion Collider) and at the SPS (Super Proton Synchrotron) and will be the baseline technique for the LHC (Large Hadron Collider) because both machines have phase locked loop tune trackers (PLLTT) which can measure the betatron tunes continuously with high precision.

The Tevatron also has PLLTTs[2], but instead of using the technique described above, a method first proposed by D. McGinnis[3] has been successfully implemented and made operational in the Tevatron. McGinnis' method is to phase modulate the RF which becomes a modulation of the beam momentum. This momentum modulation is coupled transversely via chromaticity to become a phase modulation of the betatron tune. Therefore, by phase demodulating the betatron tune, the chromaticity can be recovered. One major advantage of this method is that the frequency of the phase modulation is chosen so that it lies outside the PLLTT loop bandwidth. This means that the PLLTT does not "see" the modulation and so will not track it and hence there is no danger that the PLLTT will lose lock of the betatron tune. This is unlike the traditional method where the PLLTT must track the tune motion from the RF frequency changes. The successful implementation of this method has also added to the arsenal of chromaticity tracking techniques.

2 Theory

The theory has been completely worked out in other papers[3, 4] and so only the relevant equations will be quoted here. McGinnis' method starts with the modulation of the RF frequency $\dot{\phi}_{\mathrm{RF}}(t)$ with a sinusoid, so that the phase of the RF at any time is given by

$$\phi_{\mathrm{RF}}(t) = \omega_{\mathrm{RF}} t + \Delta\phi_{\mathrm{mod}} \sin \Omega_{\mathrm{mod}} t \tag{2}$$

where ω_{RF} is the angular RF frequency when there is no modulation, $\Delta\phi_{\mathrm{mod}}$ is the amplitude of the phase modulation and Ω_{mod} is the angular RF phase modulation. From here, it can be shown that the RF phase modulation is coupled to the betatron phase $\phi_Q(t)$ by chromaticity ξ and is

$$\begin{aligned}\phi_Q(t) = \omega_{\rm rev} Q_0 t &+ \frac{\Delta\phi_{\rm mod}}{h}\left(Q_0 - \frac{\xi}{\eta}\right)\sin\Omega_{\rm mod} t \\ &+\Delta\phi_s\left(Q_0 - \frac{\xi}{\eta}\right)\sin\left(\Omega_s t + \theta_s\right) + \phi_{Q_0}\end{aligned} \tag{3}$$

where $\omega_{\rm rev}$ is the angular revolution frequency, Q_0 is the betatron tune, h is the harmonic number, η is the slip factor $\Delta\phi_s$ is the amplitude of the synchrotron phase motion, Ω_s is the angular synchrotron frequency, θ_s and ϕ_{Q_0} are the synchrotron phase and the betatron phase at $t = 0$ respectively.

If the bunch in the Tevatron can be approximated with δ-function longitudinal and transverse distributions, then it can be shown that the spectrum of the transverse signal on a pickup is an infinite number of δ-function betatron lines. And around each betatron line are the synchrotron lines, and around each synchrotron line are the forced modulation lines. See Figure 1.

When the PLLTT is locked to one of the betatron lines given by quantum number $(k,+)$ or $(k,-)$, and the phase is demodulated w.r.t. the locked betatron tune, the amplitude of the phase demodulated signal is related to the chromaticity by

$$Z_\pm = \left(k \pm q_0 \mp \frac{\xi}{\eta}\right)\frac{\Delta\phi_{\rm mod}}{h} \tag{4}$$

where $Z_\pm \in \mathbb{R}$ is the amplitude of the phase demodulated signal, $k \in \mathbb{N} \cup \{0\}$ is the betatron mode quantum number, and q_0 is the fractional betatron tune. This can be solved for ξ to yield

$$\xi_\pm = \eta\left(\pm k + q_0 \mp \frac{hZ_\pm}{\Delta\phi_{\rm mod}}\right) \tag{5}$$

In particular, for the Tevatron, the CT looks at the $(k, +)$ mode, i.e.

$$\xi_+ = \eta \left[(k + q_0) - \frac{hZ_+}{\Delta\phi_{\text{mod}}} \right] \tag{6}$$

Therefore, it is obvious from equation (6) that once the phase amplitude Z_+ and the betatron tune q_0 are measured, the chromaticity ξ is easily calculated using the Tevatron and CT parameters shown in Table 1.

In practice, the measurement of the phase oscillation w.r.t. the betatron tune is difficult to accomplish if the betatron tune is not coherent. In fact, early experiments performed by McGinnis using this technique did not yield satisfactory results because the betatron tune from the Schottky pickups is an incoherent signal and therefore the phase is not well defined. This technique has only become feasible after the Tevatron PLLTT system became operational because the PLLTT excites the beam coherently. And thus the phase can be reliably measured w.r.t. the coherently excited betatron tune.

3 Implementation

The current implementation of the CT is shown in Figure 2. The locations of all the hardware are shown in an aerial map of the entire Fermilab accelerator complex in Figure 3. At house F0, a phase modulator is connected directly to the low level RF system so that the Tevatron RF can be phase modulated. The modulation frequency Ω_{mod} of the RF has been chosen to be $(2\pi \times 23)\ \text{s}^{-1}$. This choice has been dictated by the closed loop bandwidth of the PLL tune tracker which is about 5 Hz and the range of synchrotron frequencies of the Tevatron.

At house A1, the CT module picks up and processes the phase modulated betatron signal from the Tevatron 21.4 MHz Schottky system after it has been filtered with the commutating filter (to be described in section *Commutating Filter*). At the heart of the CT module is the hardware phase detector (PD) which has been implemented around an ALTERA Cyclone FPGA. See Figure 4. The CT module has been designed to be compatible with both the modulation frequency and the betatron frequency. In the ALTERA, there are two major blocks (a complete in depth discussion of the engineering design is in reference [5]):

(1) A hardware PD which extracts out the sine and cosine of the phase w.r.t. betatron tune. The inputs to the PD are the phase modulated betatron tune from the 21.4 MHz Schottky after it has been filtered with the commutating filter and the betatron tune from the PLLTT which is the carrier frequency. These two signals are sampled at 250 kHz which is $\sim 10\times$ higher than the betatron frequency.

(2) A 32-bit NIOS II floating point processor running at 50 MHz. It takes the arctangent of the sine and cosine of the phase from the hardware PD. This is done at (16×23) Hz which yields an oscillating signal which contains the 23 Hz component. The amplitude of the 23 Hz oscillation is recovered by putting it through a Sliding Goertzel filter[4] which is a clever discrete Fourier transform. In particular, it is a 320 point Fourier transform. The amplitude of the 23 Hz component is communicated back to the control system via ethernet.

From the amplitude of the 23 Hz component, the chromaticity is calculated using equation (6) by a dæmon running on the control system.

3.1 Commutating Filter

One key to the success of the CT is the commutating filter which is a narrow band filter which has the feature that its resonant frequency can be easily changed. This allows the filter to track the betatron tune and to filter out noise outside the betatron tune.

From Figure 5, it can be seen that a commutating filter is a series of N identical capacitors C which is connected to a resistor R for a period $2\pi/N\omega_{sw}$ where ω_{sw} is the switching frequency. This creates is a series of narrow band filters whose resonant frequencies Ω_{res} are given by

$$\Omega_{\text{res}}(k) = k\left(\frac{\omega_{sw}}{N}\right) \qquad k \in \mathbb{Z} \tag{7}$$

and the bandwidth of each resonance is

$$f_{\text{bw}} = \frac{1}{\pi NRC} \tag{8}$$

When the PLLTT is locked to the betatron tune q_0, and if $N = 8$ and $\omega_{sw} = 8 \times q_0\omega_{\text{rev}}$, then the $k = 1$ resonance of the commutating filter is exactly on the betatron tune.

3.2 Limitations

The present CT implementation has the limitation that the phase detector only returns $|Z_+|$ because the phase relationship between the 23 Hz at the CT and the 23 Hz from the signal generator which is injected into the phase shifter of the LLRF (low level RF) is not measured. See Figure 2. This means that there will always be a sign ambiguity in the measured chromaticity. Fortunately, the Tevatron is set up to run with positive chromaticity only, and with this knowledge, it is obvious from equation (6) that $Z_+ \leq 0$ and so

$$\xi_+ = \eta \left[(k + Q_0) + \frac{h|Z_+|}{\Delta\phi_{\text{mod}}} \right] \geq \xi_{\text{min}} \qquad \text{for } Z_+ \leq 0 \tag{9}$$

In fact, the minimum chromaticity ξ_{min} that can be measured by the CT is when $Z_+ = 0$. In particular, using the parameters for the Tevatron and CT from Table 1, ξ_{min} can be calculated and is

$$\xi_{\text{min}} = \eta \Big(k + Q_0 \Big) \approx 0.0029 \times 448.575 = 1.3 \tag{10}$$

4 Beam Measurements

The chromaticity of uncoalesced protons with the CT has been measured as a function of the chromaticity sextupole setting CXINJ. In the Tevatron, the term “uncoalesced protons” means filling 30 adjacent RF buckets in the Tevatron with protons to give a total of 200×10^9 to 300×10^9 protons. After CXINJ has been calibrated using the traditional method by hand, the CT measured chromaticity as a function of the calibrated CXINJ settings can be plotted and is shown in Figure 6. From here, it can be seen that the CT tracks the CXINJ changes very well and the noise on the CT is < 0.1 units for $1.3 < \xi_h < 10$.

However, it should be noted that the CT returns a much smaller value of chromaticity for $\xi_h > 10$. This problem is not fully understood because the CT has been shown to measure large chromaticities on the bench with a beam simulator. Computer simulations have given some ideas about why this happens but they are not full explanations.[5]

4.1 Chromaticity Drift At Injection

The Tevatron is a superconducting machine and so the persistent currents in its dipole magnets depend on its ramp and squeeze history.[6] These persistent currents slowly decay away and one manifestation is a drift of the chromaticity at the injection porch with the following ramp history: the Tevatron was left at its collision energy at 980 GeV and low beta squeezed for 15 minutes and then unsqueezed and ramped down to its injection energy at 150 GeV.

Figure 7(a) shows the drift of the horizontal chromaticity measured both by hand with the traditional method and with the CT. It is interesting to notice that ξ_h has some dependence on beam current which is seen by both hand measurements as well as with the CT measurements. After the third injection of beam around 2500 s, ξ_h jumps from 6 units before injection to 8 units after re-injection with lower beam current.

Figure 7(b) shows the evolution of $\xi_{h,v}$ from the decay of the persistent currents at the injection porch as a function of time without repeated proton injections. It is clear from these plots that chromaticity compensation at the injection porch for this ramp history is imperfect.

5 Conclusion

The CT has been operational since 22 Oct 2008 and has been used in conjunction with the traditional method for HEP (high energy physics) shot set up. The operation of the CT has been reliable and it will replace the hand measured traditional method for HEP tune ups soon. Although the CT works well with uncoalesced protons, there are some indications that all techniques which calculate the chromaticity by using phase locking (like the ones used at RHIC and the SPS mentioned in the *Introduction*) or modulation/demodulation in this paper have some dependence on transverse emittance.[4] This observation is being resolved right now and machine studies have been requested to study the frequency response of the beam which forms the underlying basis for all the PLLTT and CT measurement techniques mentioned in this paper.

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(2) V. Ranjbar of Tech-X Corp., for insightful discussions.

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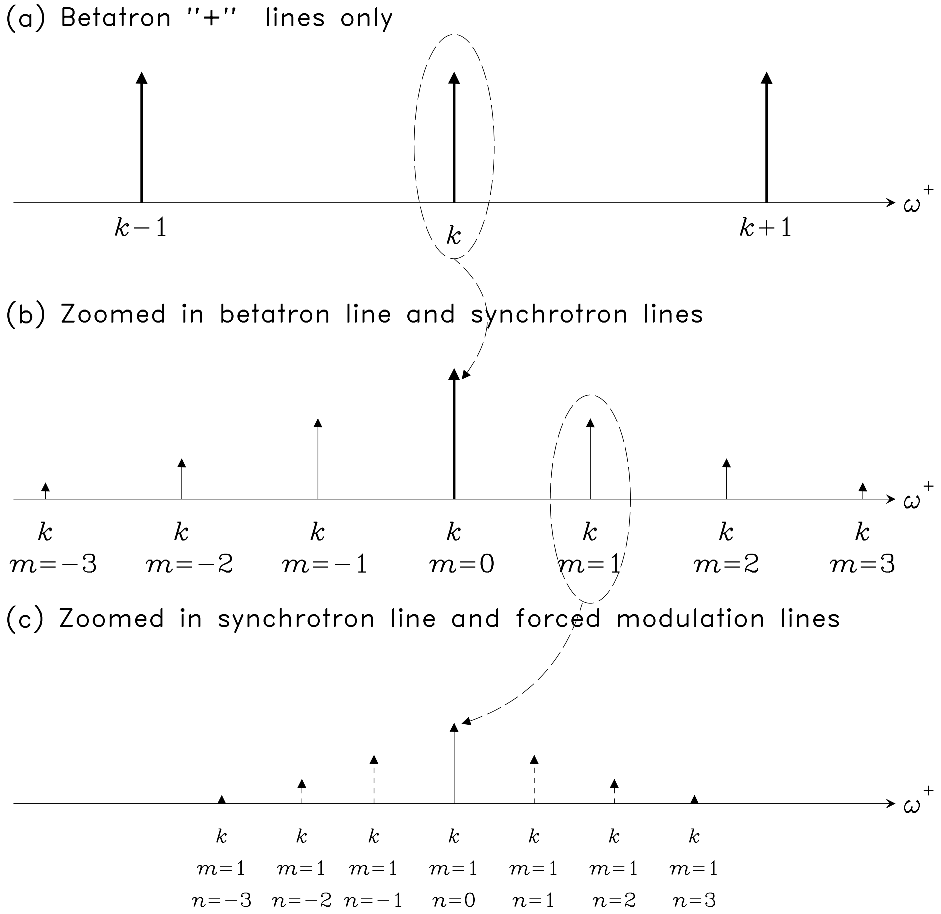

Fig. 1. (a) The $(k + q_0)$ betatron lines (b) The $(k, +)$ betatron line with its synchrotron lines (c) The $(k, m, +)$ synchrotron line and its forced modulation lines. Note that this is a representative cartoon, because k, m, n extends from $-\infty$ to $+\infty$.

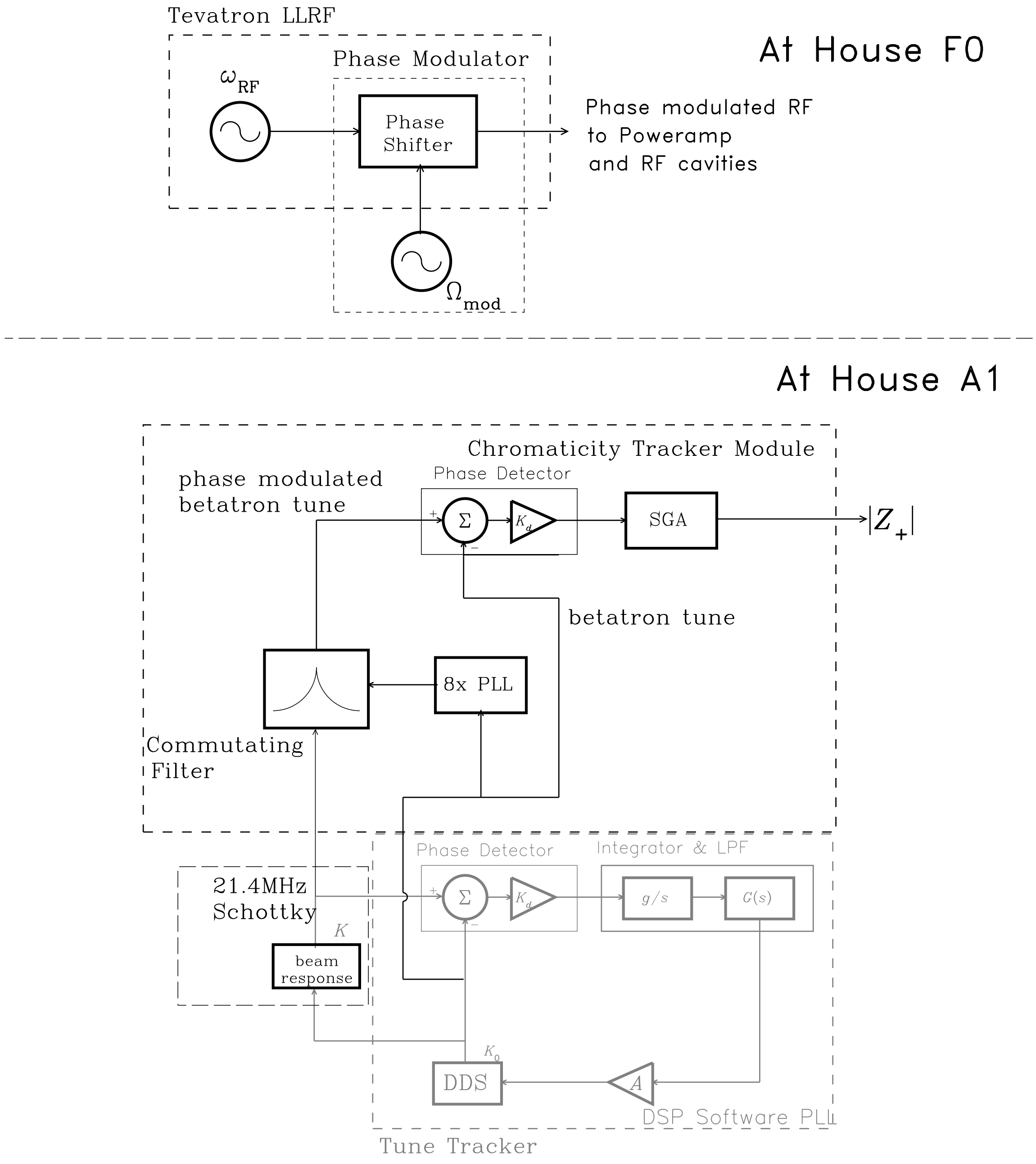

Fig. 2. The current implementation of the CT consists of a phase modulator at house F0 which phase modulates the Tevatron RF. At house A1, the betatron signal is tracked and excited by the tune tracker PLL. The phase locked signal from the direct digital synthesiser (DDS) of the tune tracker PLL is used as the reference signal for the phase detector of the CT. It is also upconverted 8× for the commutating filter.

Fig. 3. An aerial view of the Fermilab accelerator complex. The white circle marks the Tevatron which is 2000 m in diameter. The phase modulation is done at house F0 and the CT is at house A1.

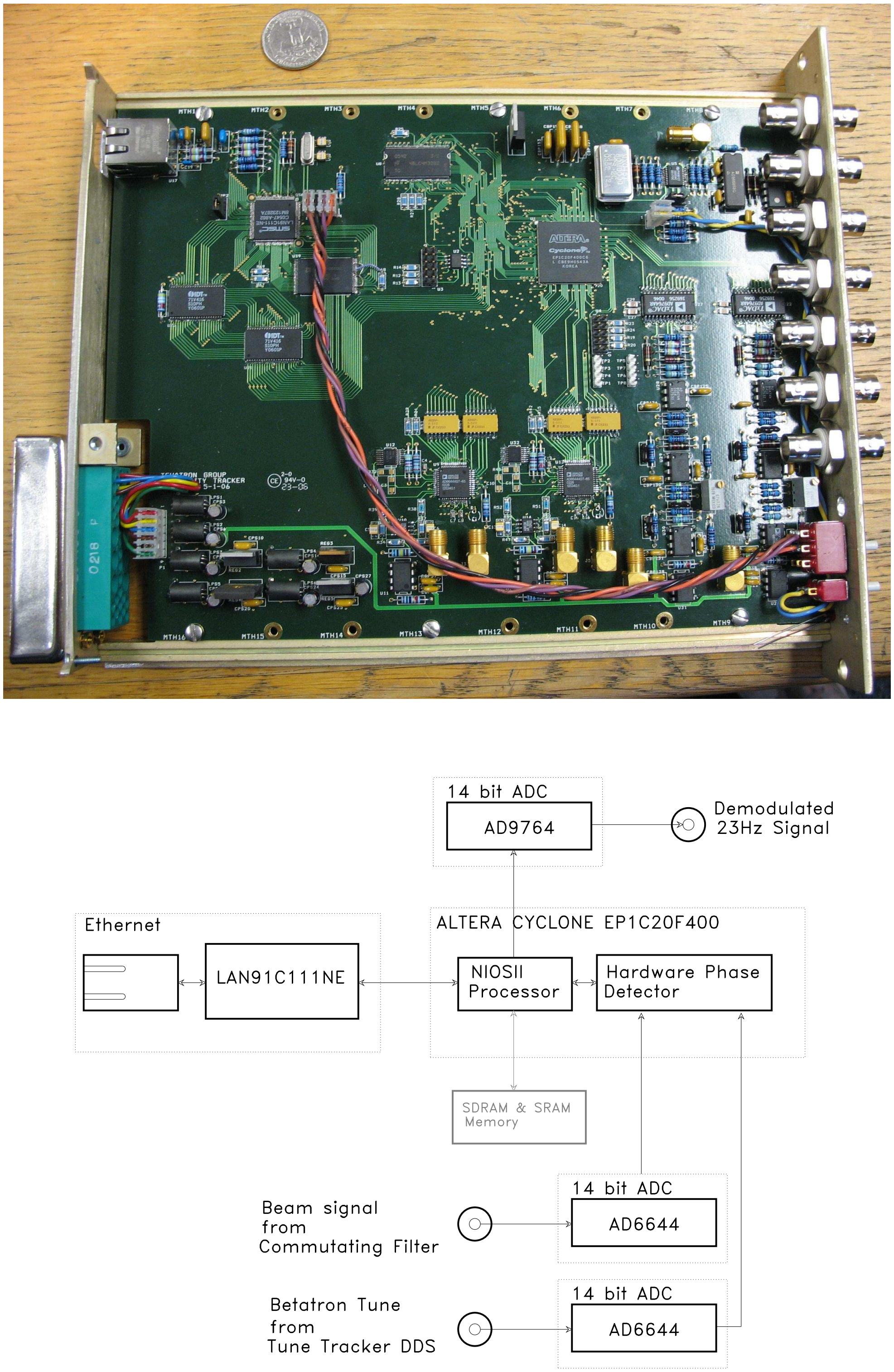

Fig. 4. The CT module lives on a NIM card. A diagram of the essential blocks of the electronics on the card is also shown here.

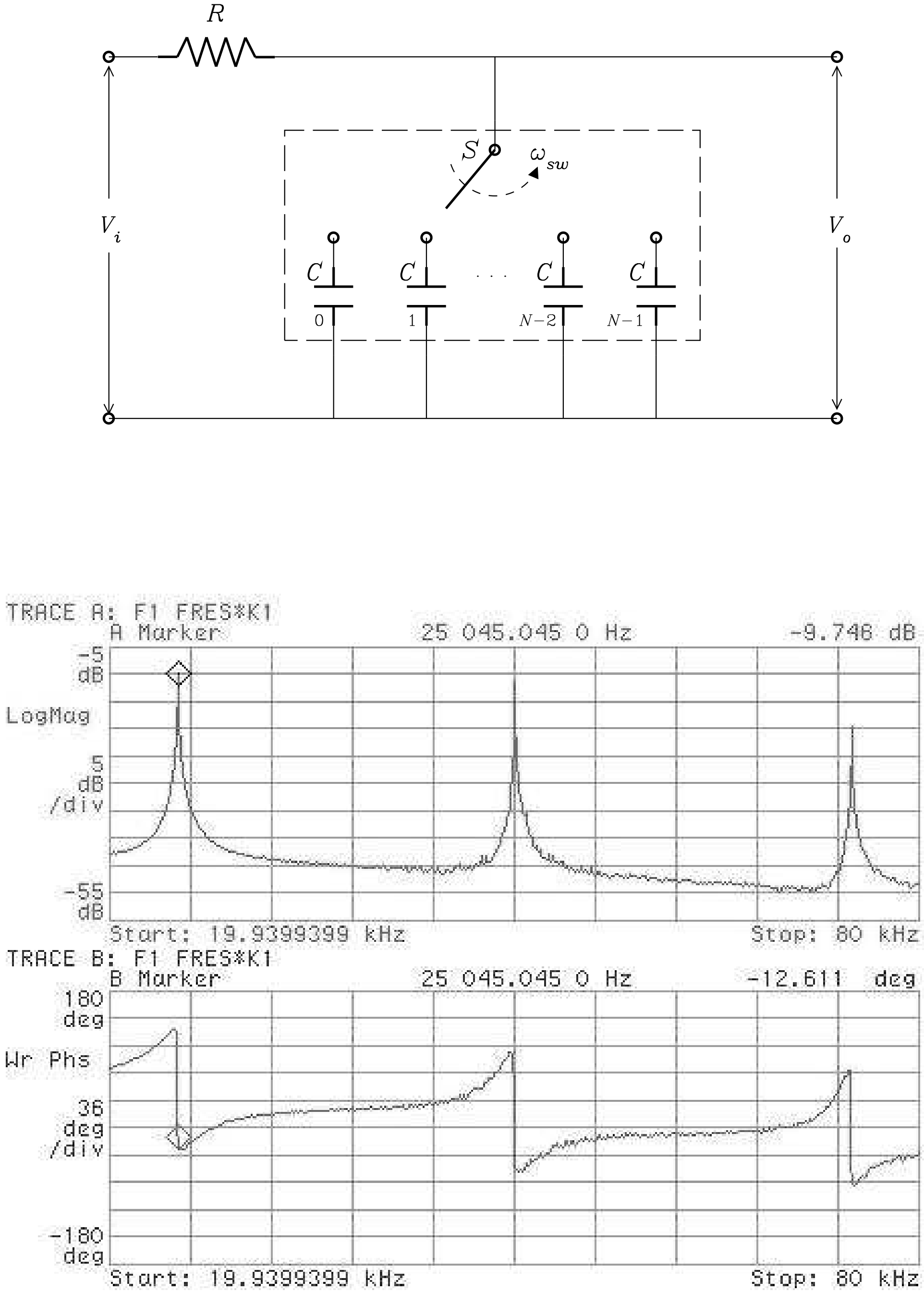

Fig. 5. The commutating filter consists of a resistor R and N capacitors C numbered from $0, 1, \ldots, N-1$. The switch S rotates at frequency ω_{sw} and connects each capacitor C for a time period of $2\pi/N\omega_{sw}$. The frequency response of this commutating filter is measured with a vector signal analyser (VSA) for $R = 10$ kΩ and $C = 0.1$ μF.

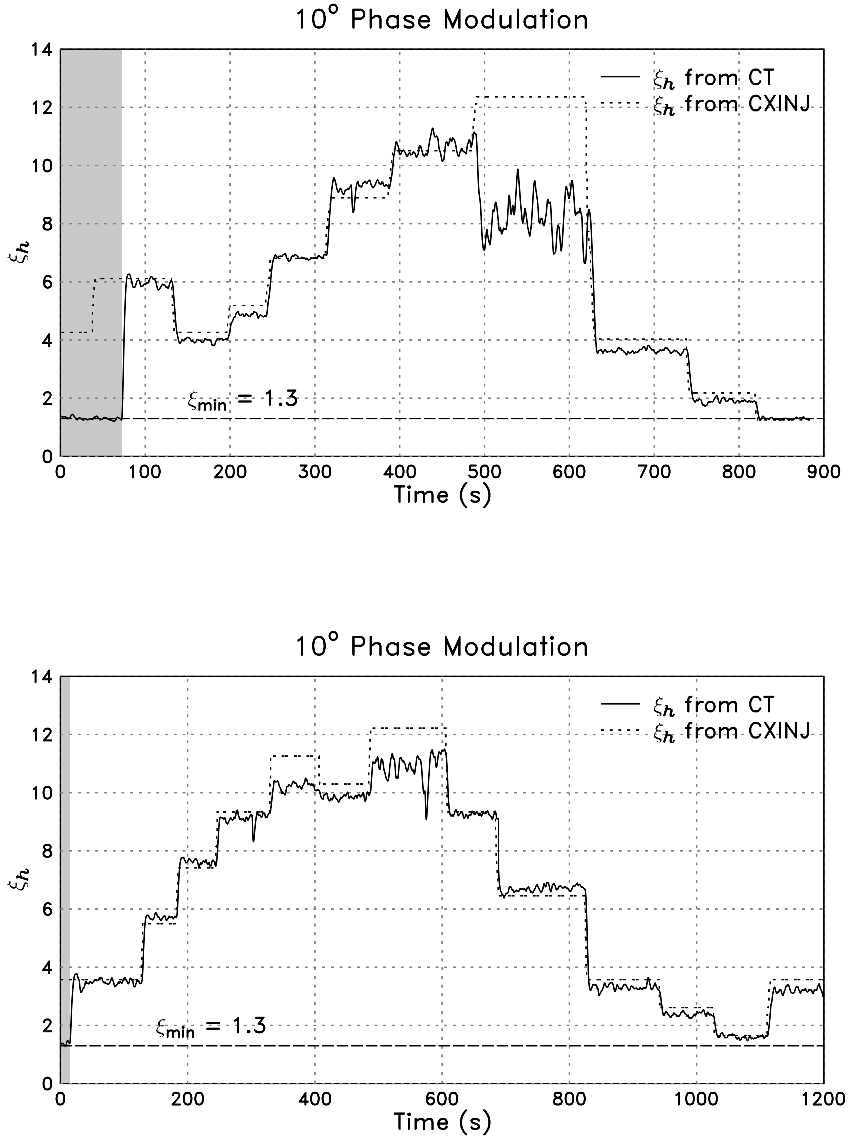

Fig. 6. The area shaded in grey is when the phase modulation is off. After it is turned on, the CT tracks the changes in chromaticity when the calibrated CXINJ is changed. Notice that when $\xi_h > 10$, the CT returns a smaller chromaticity than the actual ξ_h.

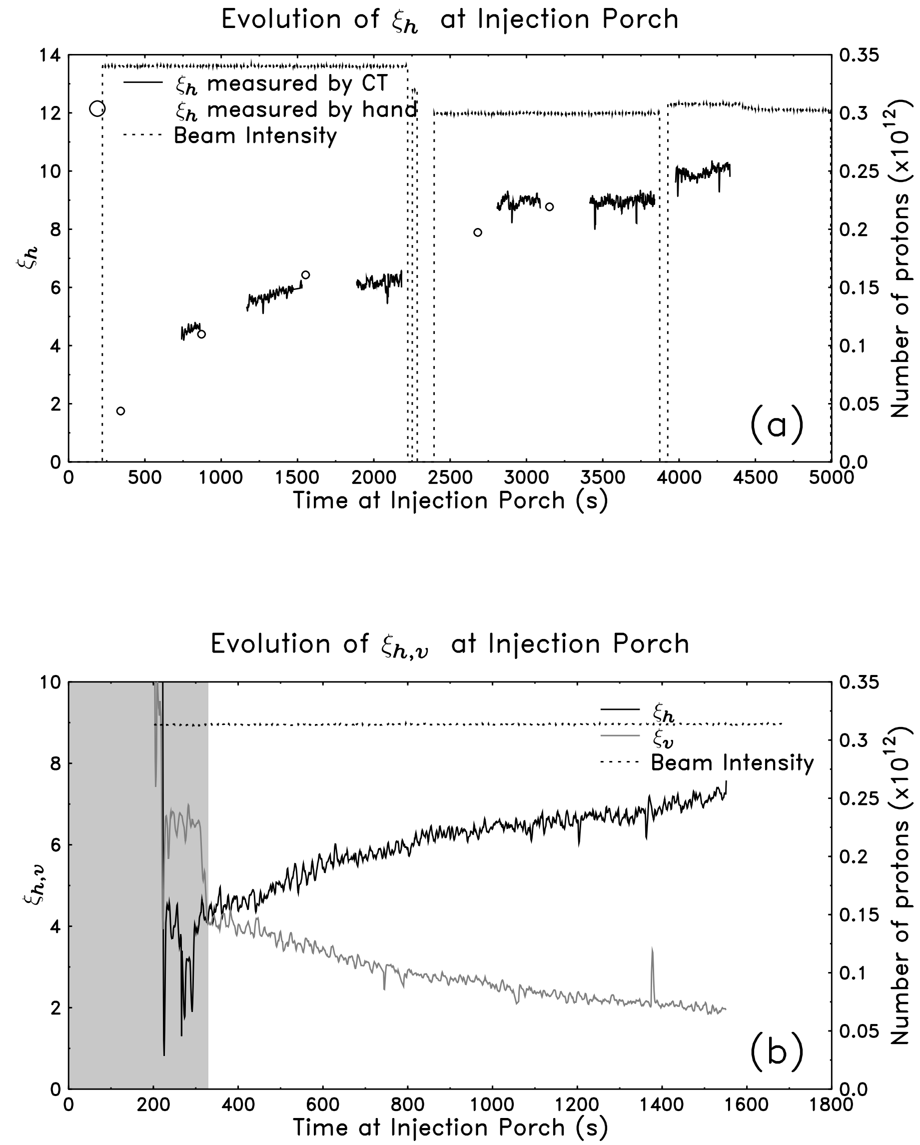

Fig. 7. The CT tracks the chromaticity drift at the injection porch of the Tevatron after 15 min at low beta, unsqueezed and ramped down from high energy. It is interesting to notice in (*a*) that after beam is re-injected at around 2500 s, ξ_h jumps from 6 units to 8 units with lower beam current. The area shaded in grey in (*b*) is when CT is turned on and chromaticities are adjusted to 4 units.

Table 1

Tevatron and CT Parameters

Parameter	**Value**	**Description**
η	0.0029	slip factor
k	448	mode number
q_0	0.55 – 0.6	fractional betatron tune
h	1113	harmonic number
$\Delta\phi_{\mathrm{mod}}$	2.8° – 11.2°	amplitude of phase modulation
Ω_{mod}	$2\pi \times 23\ \mathrm{s}^{-1}$	phase modulation frequency
Ω_s	$2\pi \times (35 - 84)\ \mathrm{s}^{-1}$	synchrotron frequency
ω_{RF}	$2\pi \times (53.1 \times 10^6)\ \mathrm{s}^{-1}$	RF frequency
ξ	4 – 15	Tevatron chromaticity range