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Achievable Magnetic Fields of Super-Ferric Helical Undulators for the ILC

S.H. Kim

Advanced Photon Source, Argonne National Laboratory

Abstract – The magnetic fields on the beam axis of helical undulators for the proposed International Linear Collider (ILC) gamma-ray production were calculated for undulator periods of 10 mm and 12 mm. The calculation assumed the use of low-carbon steel for the magnetic poles and a beam chamber outer diameter of 6.3 mm. Using *NbTi* superconducting coils at 4.2 K, the on-axis field for a 10-mm-period undulator was 0.62 T at the critical current density. The field for a 12-mm undulator period was 0.95 T, which gives a *K* value of 1.06. The *K* value for an 11-mm undulator with *Nb*₃*Sn* superconducting coils was estimated to about 1.1.

A transverse periodic helical magnetic field may be generated on the axis of a doublehelix coil with equal currents in opposite directions in each helix [1]. An ideal helical field may be expressed as

$$\mathbf{B}(z) = \left\{ \hat{x}B_0 \sin(kz) + \hat{y}B_0 \cos(kz) \right\},\tag{1}$$

where B_0 is the magnetic field modulus on the undulator axis, and $k = 2\pi / \lambda$ with λ as the undulator magnetic period length along the electron-beam direction on the *z*-axis. The radiated photon energy e_n for the *n*th harmonic is given by

$$e_n(keV) = n \frac{9.498E^2(GeV)}{\lambda(mm)\{1 + K^2 + (\gamma\theta)^2\}},$$
(2)

with *E* as the electron beam energy, γ is the relativistic factor of the electron energy, and θ is the angle between the z-axis and the radiated photon beam direction. The deflection parameter *K* is defined as

$$K = 0.0934\lambda(mm)B_0(T). \tag{3}$$

From Eq. (2), the photon energy may be calculated. Currently, the proposed International Linear Collider (ILC) lists $\lambda = 10$ mm and K = 1 for the helical undulator parameters [2]. For K = 1, the spectral power density of the photon beam is maximum at $\gamma \theta = 1$ for a single electron [1]. This note calculates the fields on the undulator axis with a beam chamber outer diameter of 6.3 mm for periods of 10 mm and 12 mm to evaluate whether the listed *K* value could be achieved.

Figure 1 shows a double-helix model coil for the low-carbon steel poles and the beam chamber. A double-helix superconducting (SC) coil with equal currents in opposite directions in each helix was inserted in between the steel coils for the field calculations. A

typical calculation for the on-axis fields corresponding to Eq. (1) is plotted in Fig. 2. The calculation used OPERA-3d [3]; the B(H) data of "low-carbon" steel included in the software were used for the steel poles shown in Fig. 1.

In Fig. 3, on-axis field modulus B_0 (*right axis*) calculated for periods $\lambda = 10$ mm and 12 mm, and the corresponding maximum fields in the coil (*left axis*) are plotted as a function of the average current density in the coil. The average critical current densities $J_c(NbTi)$ and $J_c(Nb_3Sn)$ at 4.2 K, for the *NbTi* and *Nb*₃Sn SC coils (*bottom axis*), respectively, are functions of the coil maximum field (*left axis*) and limit the coil current densities and the on-axis fields. The figure shows that, at $J_c(NbTi)$ around 1.15 kA/mm², the on-axis fields are approximately 0.55 T (K = 0.51) and 0.95 T (K = 1.06) for $\lambda = 10$ mm and 12 mm, respectively. The $J_c(Nb_3Sn)$ for $\lambda = 10$ mm is about 1.9 kA/mm², which gives an on-axis field of 0.85 T (K = 0.8). From the figure, one could estimate an achievable K value of about 1.1 for $\lambda = 11$ mm.

When the operating current density is close to the critical current density $J_c(NbTi)$ (see Fig. 3), the stability margins of the device may become an issue and may be simulated with some form of heat loads from inside the beam chamber [4]. For the development of planar-type SC undulators at the Advanced Photon Source (APS), an average $J_c(NbTi)$ up to 1.4 kA/mm² was achieved for a 14.5-mm-period short section. Considering the coil geometry for the helical undulator, $J_c(NbTi)$ and $J_c(Nb_3Sn)$ in Fig. 3 were reduced by 20% from that used in ref. [4]. The stability margin at the operating current density for $\lambda = 12$ mm may be enhanced by using an Nb_3Sn coil.

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References

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[2] http://www.linearcollider.org/cms/

[3] OPERA-3d, Vector Fields Ltd., Oxford, England. The author does not imply that similar software by other vendors cannot perform the work.

[4] S.H. Kim et al., IEEE Trans. Appl. Supercond. 15, 1240 (2005); S.H. Kim et al., R&D of Short-Period NbTi and Nb₃Sn Superconducting Undulators for the APS, Proc. 2005 PAC, 2410 (2005).



Fig. 1. A model of a double-helix coil for the low-carbon steel poles and beam chamber. A double-helix SC coil with equal currents in opposite directions in each helix is to be inserted between the steel coils.



Fig. 2. Plots of calculated B_x and B_y , two components in Eq. (1), for one 12-mm period along the undulator axis. The on-axis field B_0 was 0.88 T for an average coil current density of 1 kA/mm² and the beam chamber outer diameter as in Fig. 1.



Fig. 3. On-axis field modulus B_0 (*right axis*) calculated for periods $\lambda = 10$ mm and 12 mm, and the corresponding maximum fields in the coil (*left axis*) are plotted as a function of the average current density in the coil. The average critical current densities $J_c(NbTi)$ and $J_c(Nb_3Sn)$ at 4.2 K, for the *NbTi* and *Nb_3Sn* SC coils (*bottom axis*), respectively, are functions of the coil maximum field (*left axis*) and limit the coil current densities and the on-axis fields.