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J. R. Harris, P. G. O'Shea

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John R. Harris

Lawrence Livermore National Laboratory, Livermore, CA

Patrick G. O'Shea

Department of Electrical and Computer Engineering, University of Maryland, College Park, MD

Abstract

Passing an electron beam through an aperture can serve to reduce the beam current or change the transverse beam profile. For a sufficiently intense beam, space charge will drive a radial expansion of the beam, which may cause the current passing through the aperture to increase even though the current arriving at the aperture is decreasing. When a gridded electron gun is used, this may be expressed by stating that the transconductance of the apertured gun is negative. Here we explain this effect, and explore some of the key factors governing when it can occur and influencing its strength.

I. Introduction

In a gridded electron gun, the beam current I is adjusted by changing the voltage V_{GK} applied between the grid and cathode. The strength of this effect is given by the transconductance

$$g_m = \frac{\partial I}{\partial V_{GK}} . \tag{1}$$

Gridded guns generally behave as triodes, and so we expect their transconductance (when not in saturation or cutoff) to be positive, and to vary with the cube-root of beam current [1]. However, negative transconductance is sometimes observed in real electron guns, associated with the transition from space charge limited operation to emission limited operation [2]. As the grid-cathode voltage is adjusted to suppress emission from the cathode, the beam current is observed to increase to a level above the nominal space-charge limited current until it reaches some maximum value, and then begins to fall. Similar effects have been reported in ion sources [3]. In electron guns, other anomalous behavior is associated with this transition, including a decrease in beam current resulting from either an increase in cathode temperature, or from laser illumination of the cathode [4]. We previously proposed that these effects might be related to the use of apertures to reduce beam current extracted from electron guns [5]. A number of electron devices have previously been described which use apertured beams to produce negative transconductance. Examples include the combined use of apertures and secondary emission in dynatrons [6], deflection of beams across apertures [7], and quantum effects in nanotriodes [8]. In this paper we consider another approach, closely related to that of Ref. [5], which relies on space-charge-driven radial expansion of an electron beam incident on an aperture plate. Although we will discuss this effect in the context of a gridded electron gun, its key features depend solely on the distance traveled by the beam, the aperture size, and the beam parameters used. Modifying the beam current through any means, including changes in emission due to cathode temperature or illumination, can produce this effect.

II. Theory.

Consider a space-charge-dominated beam of current I_1, uniform current density J_1, and radius r_1 extracted from an electron gun and traveling through a drift section towards an aperture plate (Fig. 1). When it arrives, its radius will have changed to a new value r_2 under the influence of space charge and the beam's initial divergence, and the current density will have changed by the ratio r_1^2/r_2^2. If the beam radius is smaller than the aperture radius r_3, none of the beam will be intercepted and the current I_3 downstream of the aperture will be equal to the current I_1 produced in the gun. If the beam radius is larger than the aperture radius, the current passing through the aperture will be the product of the incident current density and the area of the aperture so that

$$I_3 = \left[J_1 \frac{r_1^2}{r_2^2} \right] \pi r_3^2 = I_1 \frac{r_3^2}{r_2^2} \qquad \text{(if } r_2 > r_3\text{)} \qquad (2)$$

Any increase in beam current I_1 will increase space charge forces in the beam, and therefore increase the radius of the beam incident on the aperture plate. If the cross-sectional area of the beam increases faster than the current I_1, the current I_3 extracted from the aperture will decrease (Fig. 2). This allows an increase in cathode temperature or illumination to have the paradoxical effect of reducing beam current measured downstream of the aperture. In a gridded gun, this will cause the transconductance of the current I_3 with respect to the grid-cathode voltage to become negative.

This transconductance can be calculated, beginning with the definition

$$g_{m,A} = \frac{\partial I_3}{\partial V_{GK}}, \tag{3}$$

where the subscript A denotes the apertured beam. The beam radius r_2 at the aperture plate depends on the beam current I_1, and both depend on the grid-cathode voltage V_{GK}. From eqs. (2) and (3),

$$g_{m,A} = \frac{\partial I_1}{\partial V_{GK}} \frac{r_3^2}{r_2^2} - 2 I_1 \frac{r_3^2}{r_2^3} \frac{\partial r_2}{\partial V_{GK}}. \tag{4}$$

The derivative in the second term is equivalent to $\frac{\partial r_2}{\partial I_1} \frac{\partial I_1}{\partial V_{GK}}$, and $\frac{\partial I_1}{\partial V_{GK}}$ is the transconductance g_m of the unapertured gun, so the transconductance of the apertured gun is

$$g_{m,A} = g_m \frac{r_3^2}{r_2^2} \left[1 - 2 \frac{I_1}{r_2} \frac{\partial r_2}{\partial I_1} \right]. \tag{5}$$

The values of r_2, I_1, and $\frac{\partial r_2}{\partial I_1}$ are functions of the grid-cathode voltage which can be determined numerically for particular cases, as will be done in the next section. Negative system transconductance will occur whenever

$$\frac{I_1}{r_2} \frac{\partial r_2}{\partial I_1} > \frac{1}{2}. \tag{6}$$

Eq. (5) also shows that the magnitude of $g_{m,A}$ increases with the aperture area; this holds unless $r_2 < r_3$, when the entire beam passes through the aperture and the system transconductance is simply that of the unapertured gun, g_m. Although smaller apertures result in smaller transconductances, they allow eq. (5) to be valid over a wider range of grid-cathode voltages.

Two other interesting quantities can be derived in a manner similar to eq. (5). The first is the transconductance for the current $\left(1-\frac{r_3^2}{r_2^2}\right)I_1$ which strikes the aperture plate; this value is $g_m\left[I_1\frac{2r_3^2}{r_2^3}\frac{\partial r_2}{\partial I_1}+1-\frac{r_3^2}{r_2^2}\right]$ when $r_3 < r_2$ and zero otherwise, and will never be negative. Notice that the transconductance of the intercepted current and the transconductance of the nonintercepted current sum to g_m.

The second quantity is the dynamic resistance $R_{p,A}=\frac{\partial V_A}{\partial I_3}$ of the apertured gun, where V_A is the anode voltage of the gun. This quantity is related to the dynamic resistance R_p of the unapertured gun by

$$R_{p,A}=R_p\frac{r_2^2}{r_3^2}\left[1-\frac{2I_1}{r_2}\frac{\partial r_2}{\partial I_1}\right]^{-1}. \qquad (7)$$

When the transconductance of the apertured gun is negative, the dynamic resistance of the apertured gun is also negative. Note also that $R_{p,A}g_{m,A}=R_pg_m$, which is equal to the amplification factor of the gun [1].

III. Converging and Nonconverging Beam Examples.

In the previous section, we considered the transconductance of apertured electron beams without considering the details of beam generation and expansion under space charge. In this section, we will directly calculate the current extracted from a gridded gun and the beam envelope resulting from various operating conditions. These will be used to show the presence of negative transconductance in the apertured current I_3.

To begin, consider a gridded electron gun with the parameters given in Table I. By analogy with triode operation, the current produced by this gun will depend on the grid-cathode voltage V_{GK} according to [1]

$$I(V_{GK}) = \frac{2.335\times10^{-6}}{d_{GK}{}^2\left[1+\frac{1}{\mu}\left(\frac{d_{AK}}{d_{GK}}\right)^{4/3}\right]^{3/2}}\left(V_{GK}+\frac{V_{AK}}{\mu}\right)^{3/2}\pi r_K^2 T_A T_G \tag{8}$$

in triode amplification mode and

$$I_{SCL} = \frac{4}{9}\varepsilon_0\left(\frac{2q}{m}\right)^{1/2}\frac{V_{AK}^{3/2}}{d_{AK}^2}\pi r_K^2 T_A T_G \tag{9}$$

in saturation (Fig. 3). In these equations, V_{AK} is the anode-cathode voltage, r_K is the cathode radius, T_A and T_G are the transparencies of the anode and grid meshes, μ is the amplification factor of the gun, d_{GK} is the grid-cathode distance, d_{AK} is the anode-cathode distance, ε_0 is the permittivity of free space, q is the fundamental charge, and m is the mass of the electron. As the beam extracted from this gun propagates through a drift section, it will expand under the influence of space charge. Its envelope is given by [9,10]

$$\frac{d^2r}{dz^2} = \frac{K}{r}, \tag{10}$$

where

$$K = \frac{Iq}{2\pi\varepsilon_0 mc^3\beta^3\gamma^3} \tag{11}$$

is the generalized perveance which depends on the beam current and the relativistic factors β and γ. Eq. (10) can be solved to find

$$z(r) = r_1\sqrt{\frac{2}{K}}\exp\left[-\left(\frac{r_1'}{\sqrt{2Kr_1}}\right)^2\right]\int_B^{\pm A}\exp(u^2)du, \tag{12}$$

where $z(r)$ gives the beam envelope, r_1 and r_1' are the initial beam radius and divergence, $A = \sqrt{\ln\frac{r}{r_1}+\left(\frac{r_1'}{\sqrt{2Kr_1}}\right)^2}$, and $B = \frac{r_1'}{\sqrt{2Kr_1}}$. In the integral, the positive root is taken if the beam is diverging, and the negative root is taken if the beam is converging.

Now, consider a beam extracted from the gun discussed above, which has an initial radius $r_1 = r_K$, and an initial divergence $r_1' = -0.0001$. Fig. 4 shows the envelope, calculated from Eq. (12), for several values of beam current. As the current from the gun is reduced, the beam radius at each location downstream will change. Fig. 5 and Fig. 6 show the beam radius and current density measured at four locations downstream from the gun. Provided that the beam radius is larger than the aperture radius, the current passing through the aperture is the product of the current density shown in Fig. 6 and the aperture area. In the case of an aperture plate at 15 cm or 20 cm from the gun, this current exhibits negative transconductance behavior. However, as the beam spot size changes, it may become smaller than the aperture radius, in which case the entire beam will pass through, and the observed negative transconductance will end. Fig. 7 shows the current emerging from the aperture as a function of grid-cathode voltage for several aperture sizes, with the aperture plate located 20 cm from the gun. Note that the negative transconductance effect shown in Fig. 7 is more pronounced for larger aperture sizes, as predicted by eq. (5).

We now repeat the calculation for a nonconverging beam ($r_0' = 0$). In this case, the variation of beam current with grid-cathode voltage is still given by Fig. 3, but the variation of beam radius with current is now given by Fig. 8. As the beam current approaches zero, space charge forces become negligible and the beam radius at all locations approaches the beam's initial radius. This is seen in Fig. 9, which shows the beam radius as a function of beam current I_1 at four distances downstream from the gun. Current density at the same four locations is shown in Fig. 10. Fig. 11 shows the apertured current I_3 for several values of aperture radius with the aperture located at 20 cm from the gun. The transconductance is only negative over a very small region of these curves, as shown in Fig. 12 for the 10 mm aperture.

Finally, we mention two assumptions implicit in these calculations. First, emittance was neglected in calculating the beam envelope. For high-perveance guns, this will be approximately correct at all but the lowest currents. Second, these calculations assumed a uniform current density in the beam. High-perveance electron guns frequently produce beams with nonuniform density profiles [11]; the shape of the $I_3(V_{GK})$ curves will be very sensitive to the transverse density profile of the beam as its radius approaches that of the aperture.

IV. Discussion.

We have shown how aperturing an intense electron beam may cause the current passing through the aperture to decrease, even though the current incident on the aperture has increased. When the beam is produced in a gridded electron gun, this will result in a negative transconductance. Our two examples showed that this can occur in both converging and nonconverging beams, but that the effect can be much more pronounced in the case of a converging beam. The most important mechanism for this is through a converging beam's ability to maximize the range of radii r_2 it can assume, and therefore to maximize $\frac{\partial r_2}{\partial I_1}$. Several other factors also play a key role in determining if this effect will occur and how strong it will be, including the aperture radius and the distance of the aperture from the gun. Although our discussion has primarily been framed in the context of gridded electron guns, the effect itself depends only on the beam dynamics and the system geometry, and should be applicable to intense electron beams generated from any source.

Acknowledgement

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References

[1] K.R. Spangenberg, *Vacuum Tubes*. New York: McGraw-Hill, 1948.

[2] J.R. Harris and P.G. O'Shea, IEEE Trans. Electron Devices **53** 2824 (2006).

[3] I.G. Brown, Review of Scientific Instruments **63** 2351-2356, figure 6 (1992).

[4] J.R. Harris, J.G. Neumann, and P.G. O'Shea, J. Appl. Phys. **99** 093306 (2006).

[5] J.R. Harris, Longitudinal Dynamics of an Intense Electron Beam, Ph.D. dissertation, University of Maryland, College Park, MD 2005. [Online] Available: http://hdl.handle.net/1903/2906

[6] A.W. Hull, Proceedings of the Institute of Radio Engineers **6** p. 5-35 (1918).

[7]E.W. Herold, U.S. Patent 2,294,659 (1942).

[8]V.A. Zhukov, Russian Microelectronics **34** 222 (2005).

[9] M. Reiser, *Theory and Design of Charged Particle Beams*, Section 4.2.1, New York: Wiley, 1994.

[10] J.R. Pierce, *Theory and Design of Electron Beams*, Section 9.2, Princeton: D. Van Nostrand, 1954.

[11] J.F. Gittins, *Power Travelling-Wave Tubes*, Section 5.2.7, New York: Elsevier, 1965.

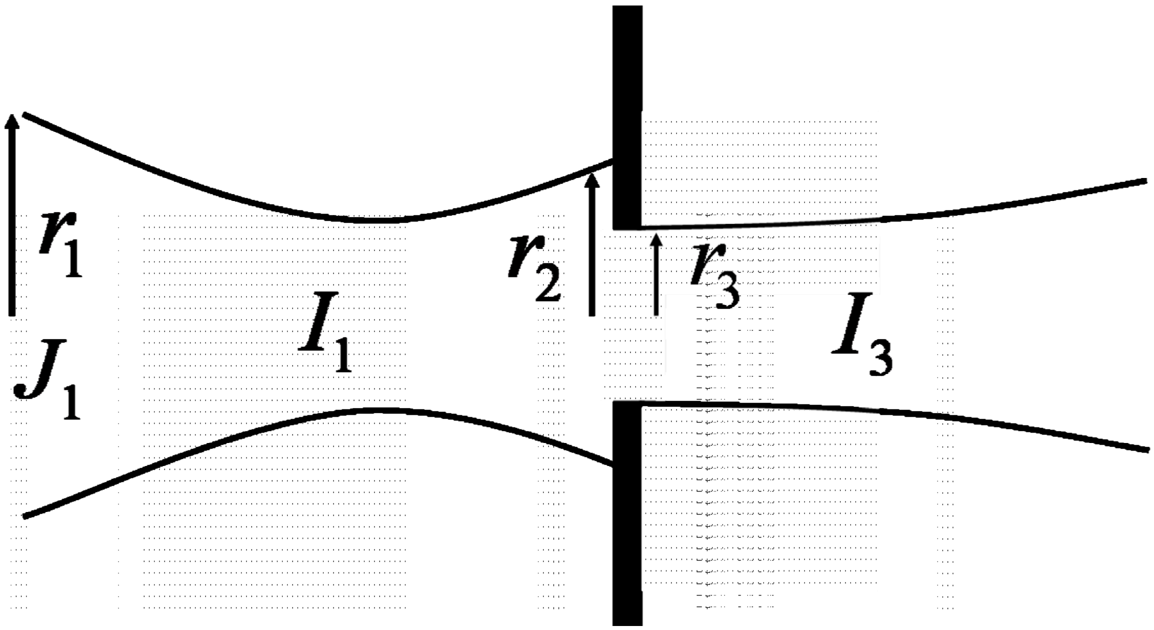


Fig. 1. Envelope of an intense, apertured electron beam.

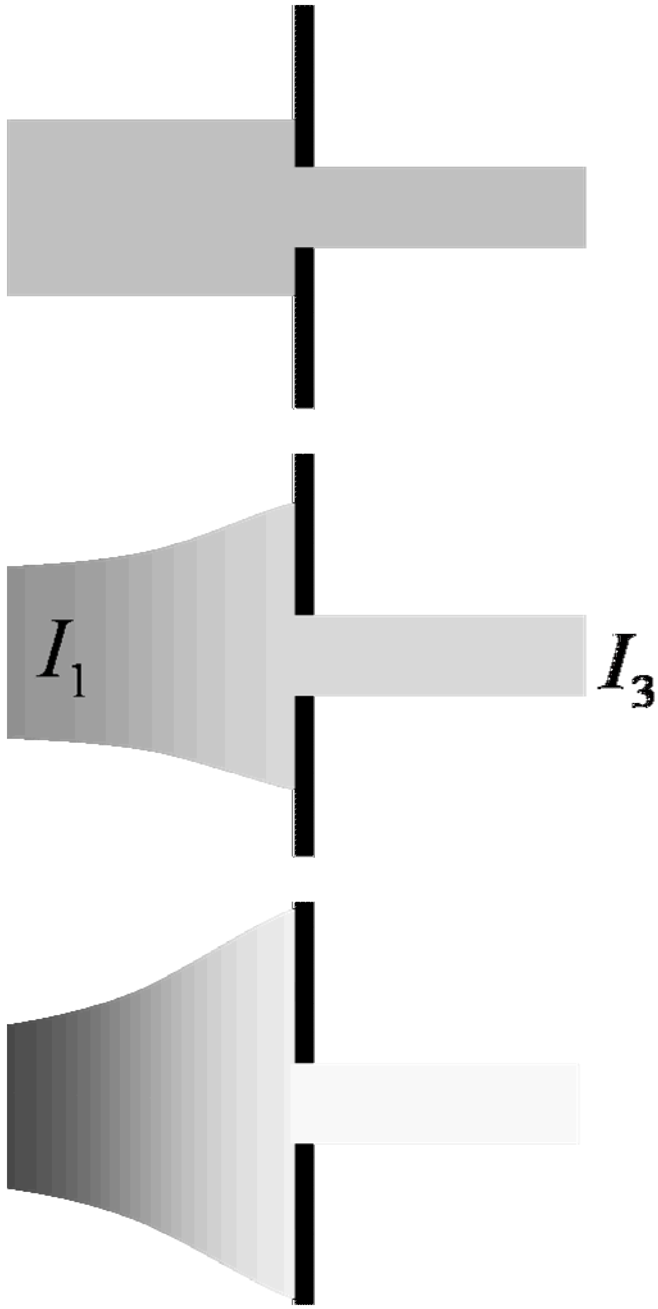
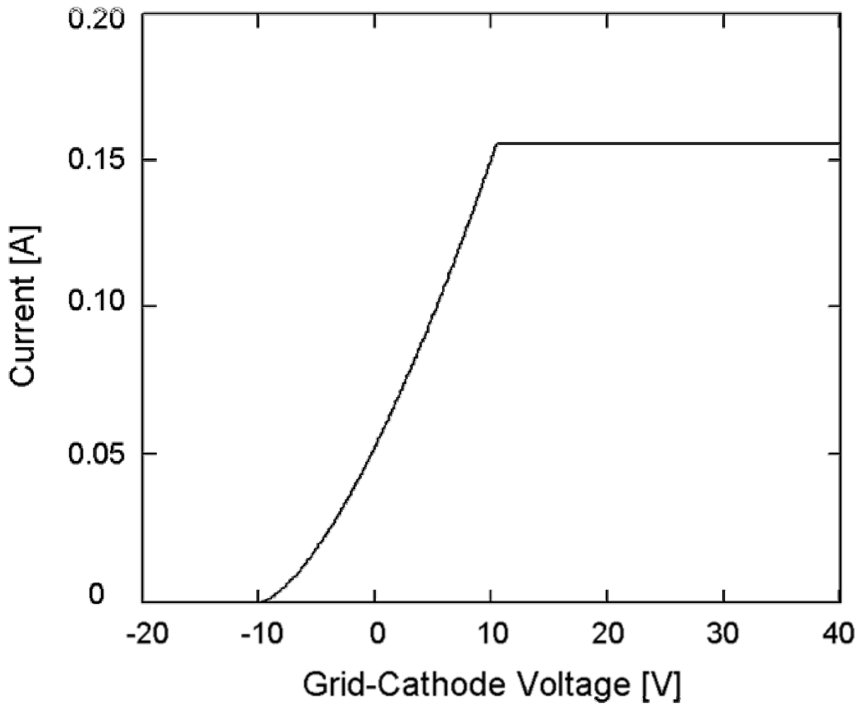


Fig. 2. Increasing the incident current I_1 can cause radial expansion of the beam, reducing the current density at the aperture, and therefore reducing the current passing through the aperture I_3.

Table I.

Cathode radius r_K	0.005 m
Anode-cathode voltage V_{AK}	10 kV
Amplification factor μ	1000
Anode-cathode separation d_{AK}	0.026 m
Cathode-grid separation d_{GK}	0.00015 m
Grid transparency T_G	0.66
Anode mesh transparency T_A	0.87



3. Current extracted from an electron gun with the parameters of Table I.

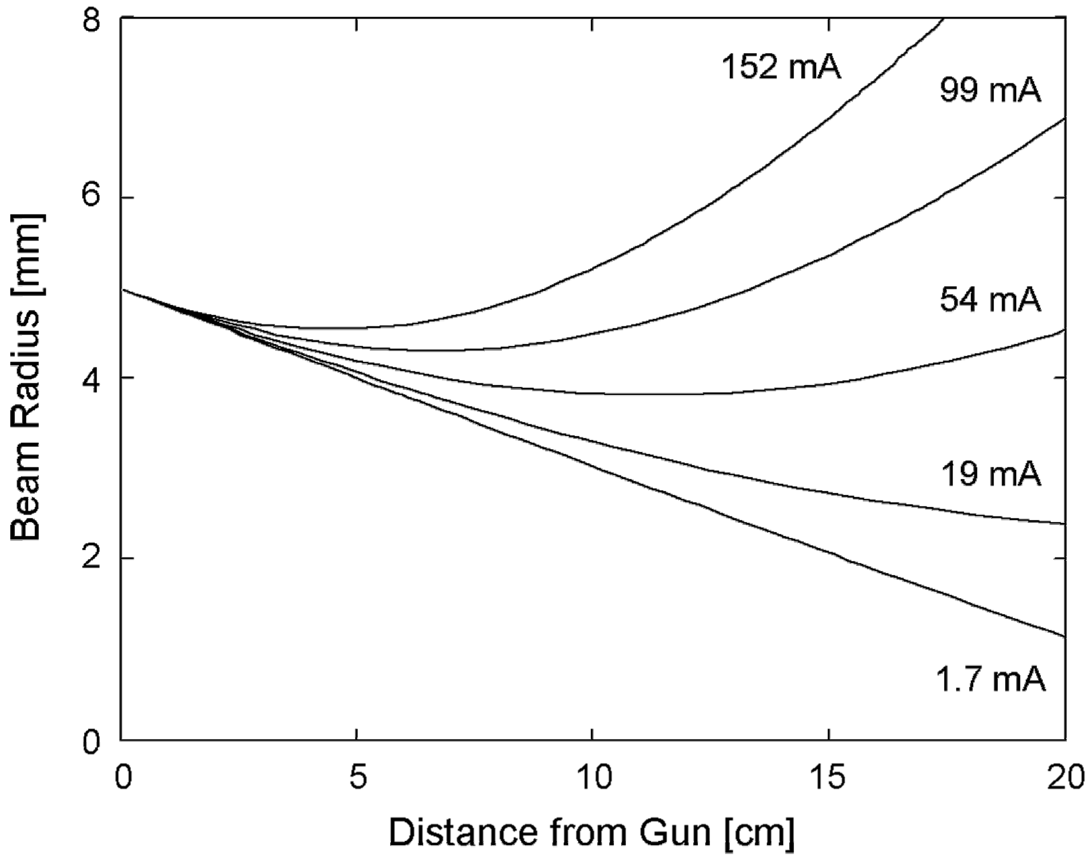


Fig. 4. Beam envelopes for converging beam ($r' = -0.0001$) currents of 152 mA (V_{GK} = 10 V), 99 mA (V_{GK} = 5 V), 54 mA (V_{GK} = 0 V), 19 mA (V_{GK} = -5 V), and 1.7 mA (V_{GK} = -9 V).

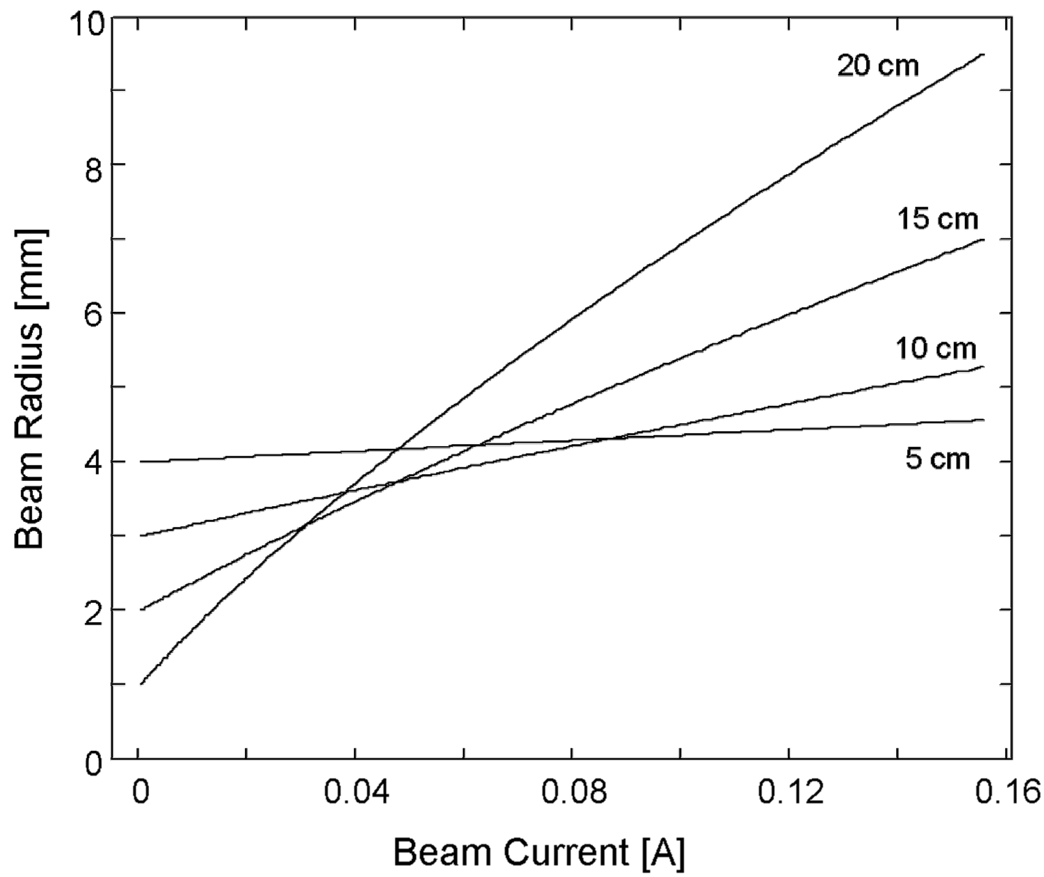
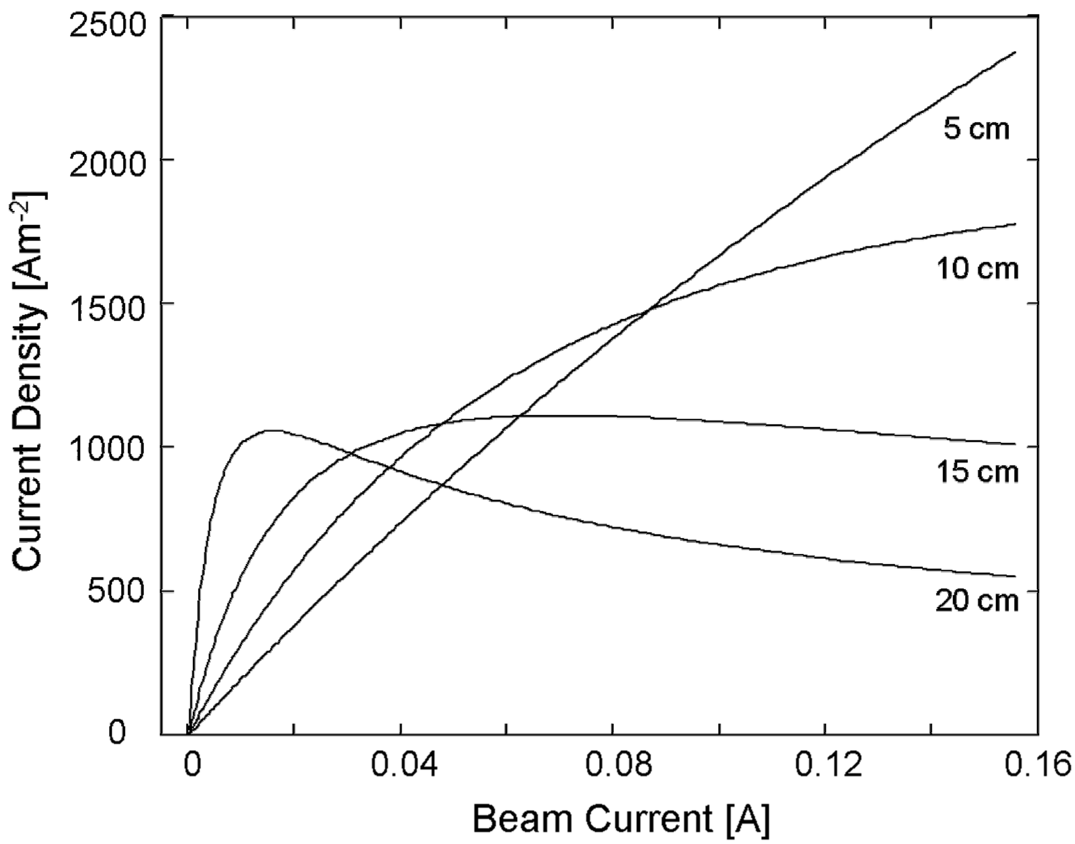


Fig. 5. Beam radius as a function of beam current at four distances from the gun.



6. Current density as a function of grid-cathode voltage for four distances downstream from the gun.

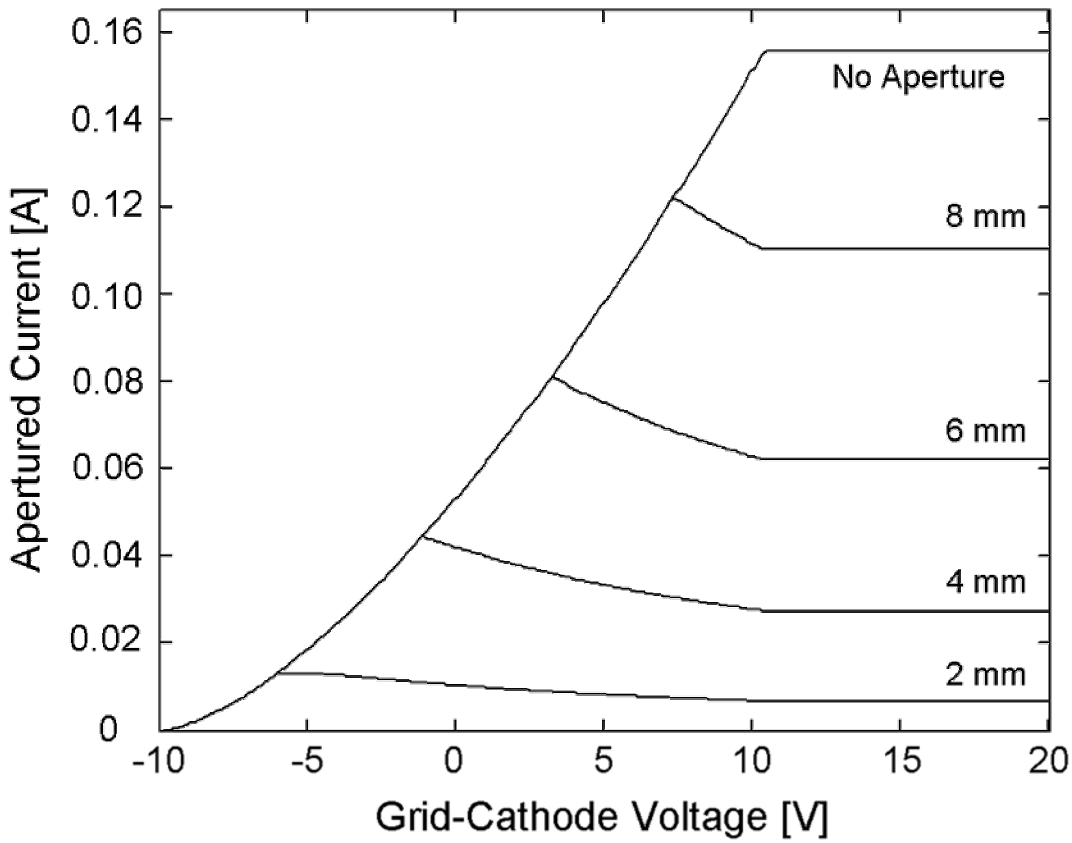


Fig. 7. Beam current downstream of an aperture at 20 cm from the gun, for four aperture radii. As the grid-cathode voltage is decreased from 20 V to -10 V, the beam current goes through four stages: 1) while the gun is in saturation the current remains unchanged; 2) when the gun enters triode amplification mode the current increases due to geometrical negative transconductance effects discussed above; 3) when the beam radius is smaller than the aperture radius the entire beam passes through, and the current is given by eq. (2); 4) as the grid-cathode voltage is decreased beyond -10 V the gun goes into triode cutoff and the current falls to zero.

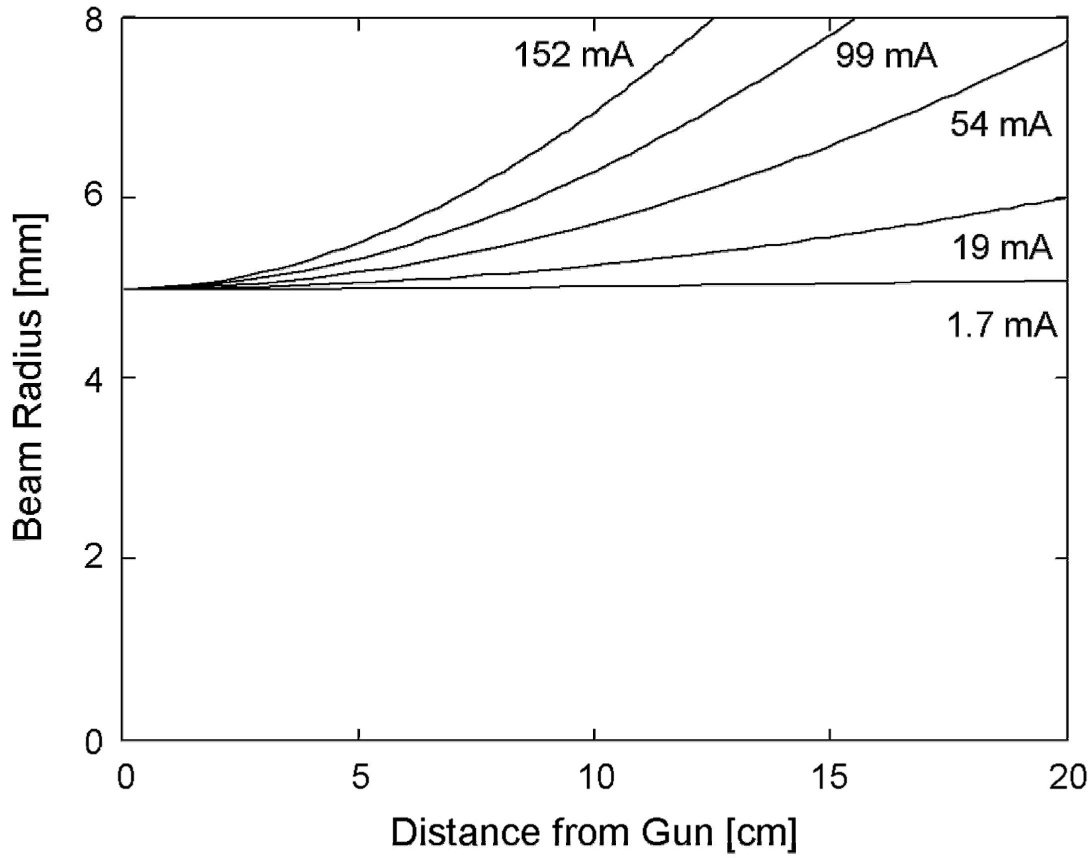


Fig. 8. Beam envelopes for nonconverging beam ($r' = 0$) currents of 152 mA (V_{GK} = 10 V), 99 mA (V_{GK} = 5 V), 54 mA (V_{GK} = 0 V), 19 mA (V_{GK} = -5 V), and 1.7 mA (V_{GK} = -9 V).

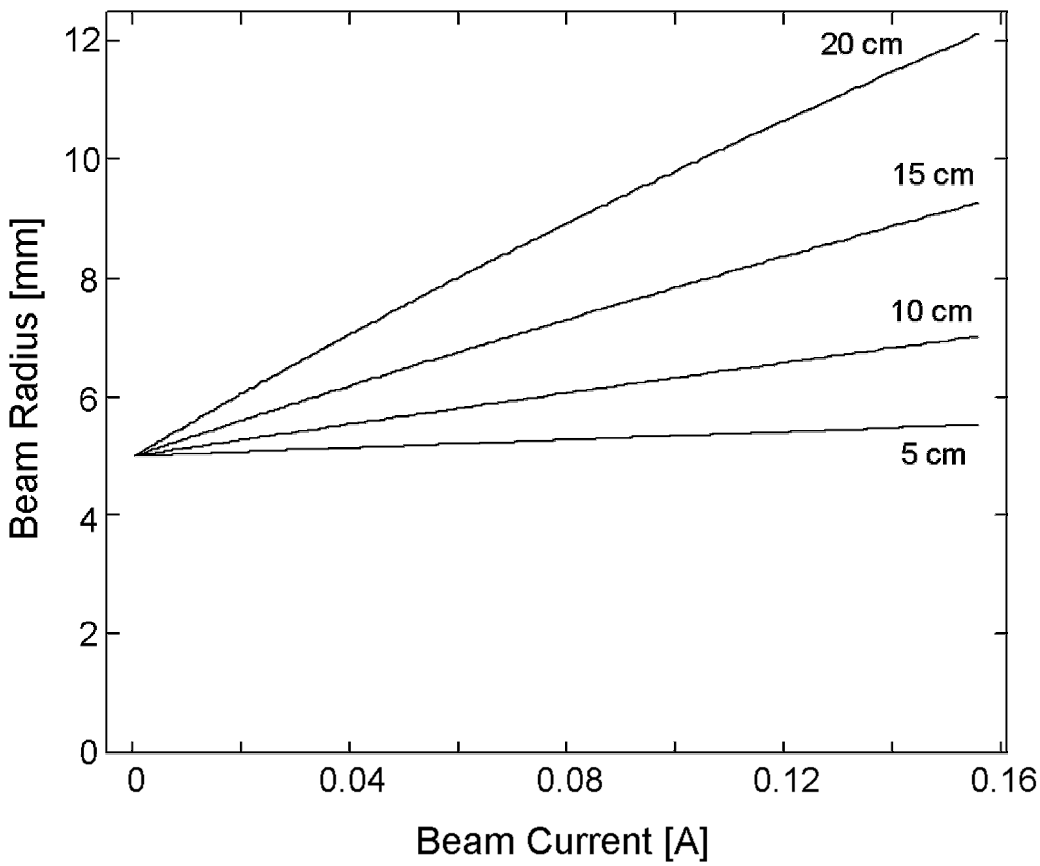
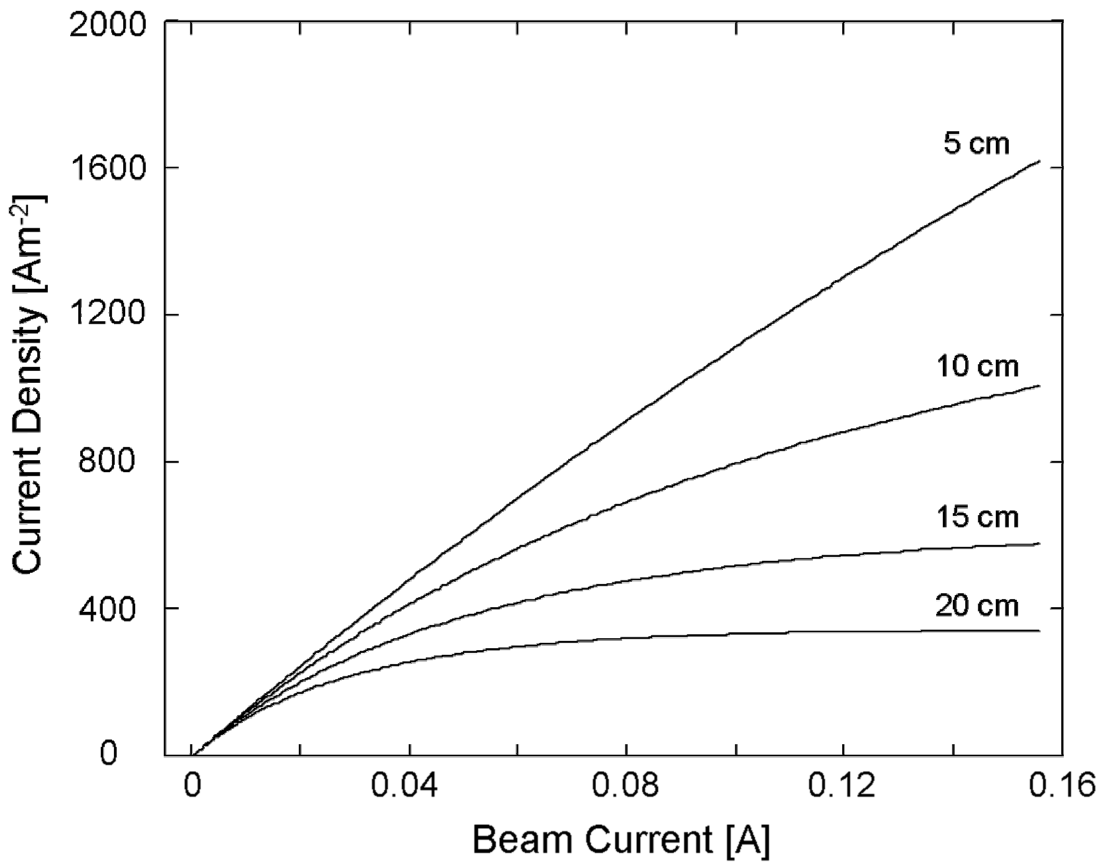


Fig. 9. Beam radius as a function of beam current in the unapertured, nonconverging electron beam, at four distances from the gun.



. Current density as a function of beam current in the unapertured, nonconverging beam at four distances from the gun.

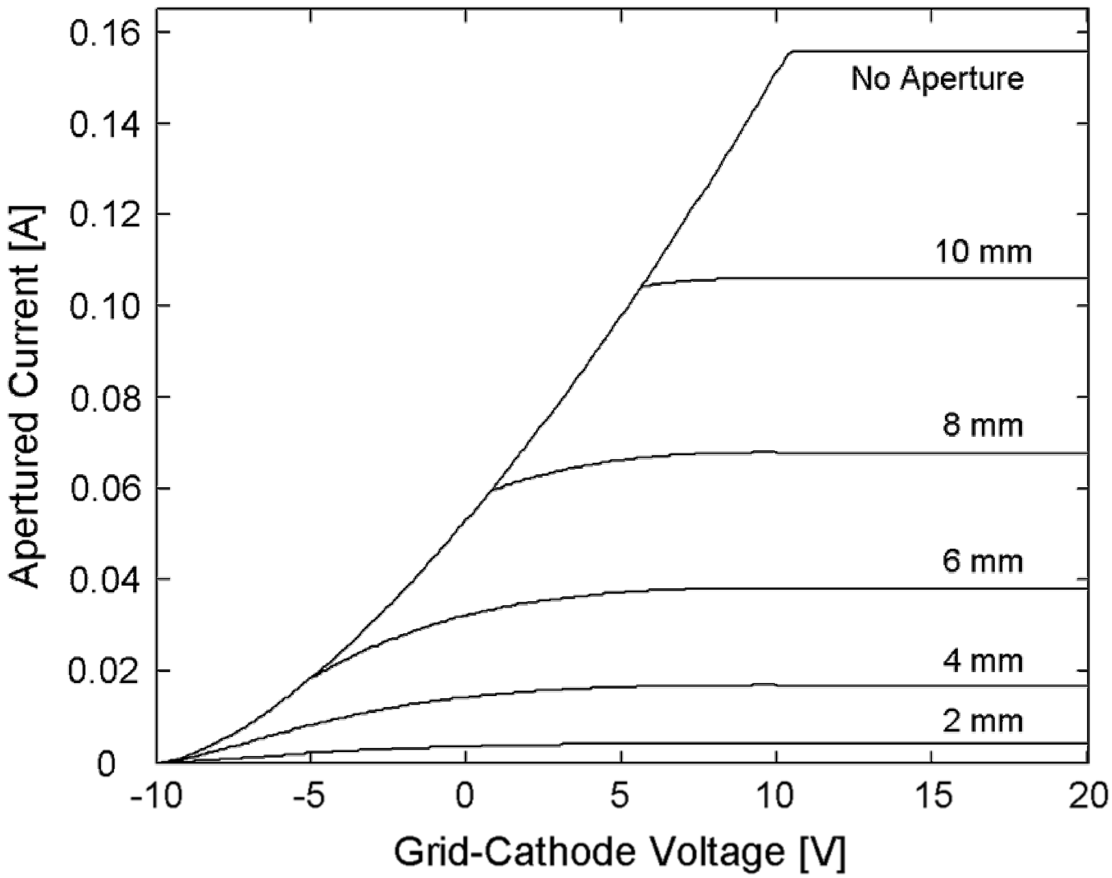


Fig. 11. Beam current I_3 extracted from apertures of various radii 20 cm downstream from the nonconverging gun.

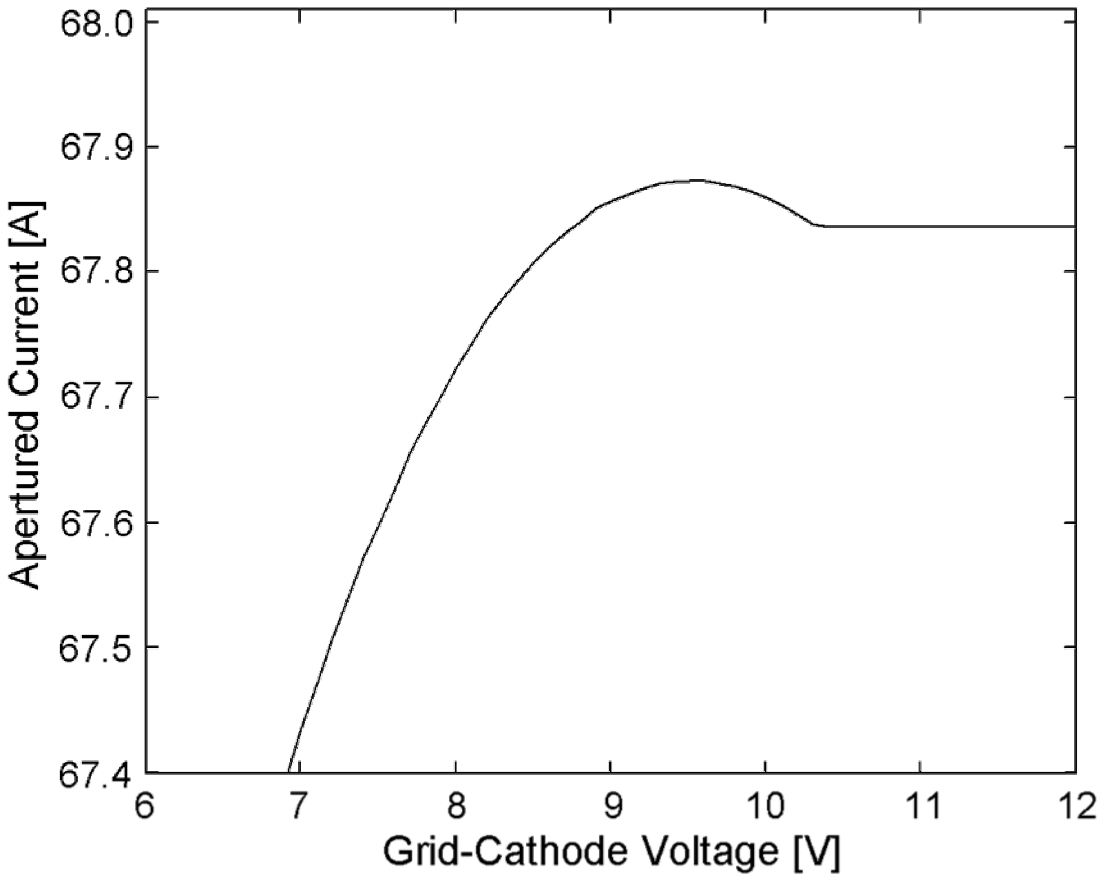


Fig. 12 Beam current I_3 extracted from 10 mm aperture 20 cm downstream from the nonconverging gun, showing the small region of negative transconductance.