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Collins Asymmetry at Hadron Colliders

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Abstract

We study the Collins effect in the azimuthal asymmetric distribution of hadrons inside a high energy jet in the single transverse polarized proton proton scattering. From the detailed analysis of one-gluon and two-gluon exchange diagrams contributions, the Collins function is found the same as that in the semi-inclusive deep inelastic scattering and e^+e^- annihilations. The eikonal propagators in these diagrams do not contribute to the phase needed for the Collins-type single spin asymmetry, and the universality is derived as a result of the Ward identity. We argue that this conclusion depends on the momentum flow of the exchanged gluon and the kinematic constraints in the fragmentation process, and is generic and model-independent.

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I. INTRODUCTION

Single-transverse spin asymmetries (SSA) in hadronic processes have a long history [1, 2]. Recent experimental measurements of SSAs in polarized semi-inclusive lepton-nucleon deep inelastic scattering (SIDIS) [3, 4], in hadronic collisions [5–7], and in the relevant e^+e^- annihilation process [8], have renewed the theoretical interest in SSAs and in understanding their roles in hadron structure and Quantum Chromodynamics (QCD). There are several approaches to understanding SSAs within the QCD framework [9–11]. Transverse-momentum-dependent (TMD) parton distributions and fragmentation functions, and their relevance for semi-inclusive DIS, the Drell-Yan process, di-hadron production in e^+e^- annihilations, and the single-inclusive hadron production at hadron colliders have been investigated [12–26].

Two important contributions from these TMD parton distributions and fragmentation functions have been mostly discussed in the last few years: the Sivers quark distribution and the Collins fragmentation function. The Sivers quark distribution [16] represents a distribution of unpolarized quarks in a transversely polarized nucleon, through a correlation between the quark's transverse momentum and the nucleon polarization vector. The existence of the Sivers function requires final/initial-state interactions [20], and an interference between different helicity Fock states of the nucleon [20, 27]. The Collins function represents a correlation between the transverse spin of the fragmenting quark and the transverse momentum of the hadron relative to the "jet axis" in the fragmentation process. Like the Sivers function, it vanishes when integrated over all transverse momentum.

One of the most nontrivial properties associated with the Sivers and Collins functions are their universality properties. Although they both belong to the so-called "naive-time-reversal-odd" functions, they do have different universality properties. For the quark Sivers function, because of the initial/final state interaction difference, they differ by signs for the SIDIS and Drell-Yan processes [20, 21, 28]. This non-universality has also been extended to other processes, such as the dijet-correlation in hadronic reactions, where it was found that both initial and final state interactions contribute to the SSA, and there exists non-trivial relation between this and those in the SIDIS and Drell-Yan processes [29–31], and a standard TMD factorization breaks down [31].

On the other hand, there have been several studies showing that the Collins function is universal between different processes, primarily in the SIDIS and e^+e^- annihilation [32–

34]. In these discussions, the gauge links in the fragmentation functions do not play a crucial role to leading to a nonzero Collins function though they are important to retain the gauge invariance, whereas it has been well understood that the gauge links in the parton distributions play very important roles to obtain non-zero quark Sivers function.

The Collins effect in the fragmentation process and its universality has been recently extended to the hadron production in pp collisions [35], where the azimuthal distribution of hadrons inside a high energy jet can probe the Collins fragmentation function and the quark transversity distribution [36] in the single transverse polarized nucleon-nucleon scattering. In this paper, we will give the detailed derivation of these results, and argue that the universality is in general and model-independent.

We are interested in the hadron production from the fragmentation of a transversely polarized quark which inherit transverse spin from the incident nucleon through transverse spin transfer in the hard partonic scattering processes [37–39]. As shown in Fig. 1, we study the process,

$$p(P_A, S_\perp) + p(P_B) \to jet(P_J) + X \to H(P_h) + X \ , \tag{1}$$

where a transversely polarized proton with momentum P_A scatters on another proton with momentum P_B, and produces a jet with momentum P_J (transverse momentum $P_\perp$ and rapidity y_1 in the Lab frame). The three momenta of P_A, P_B and P_J form the so-called reaction plane. Inside the produced jet, the hadrons are distributed around the jet axes. A particular hadron H will carry certain longitudinal momentum fraction z_h of the jet, and its transverse momentum P_{hT} relative to the jet axis will define an azimuthal angle with the reaction plane: ϕ_h, shown in Fig. 1. Thus, the hadron's momentum is defined as $P_h = z_h P_J + P_{hT}$. The relative transverse momentum P_{hT} is orthogonal to the jet's momentum P_J: $\vec{P}_{hT} \cdot \vec{P}_J = 0$. Similarly, we can define the azimuthal angle of the transverse polarization vector of the incident polarized proton: ϕ_s.

The leading order contribution to the jet production in pp collision comes from $2 \to 2$ sub-processes, where two jets are produced back-to-back in the transverse plane. For the reaction process of (1), one of the two jets shall fragment into the final observed hadron. In this paper, we study the physics in the kinematic region of $P_{hT} \ll P_\perp$. The unpolarized cross section contribution from the partonic $2 \to 2$ process $ab \to qc$ where the final state

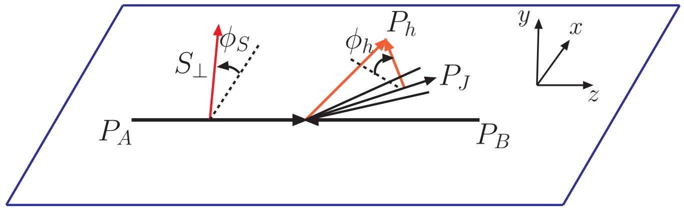

FIG. 1: *Illustration of the kinematics for the azimuthal distribution of hadrons inside a jet in pp scattering.*

quark q fragments into final observed hadron H, can be written as

$$\frac{d\sigma^{uu}}{dy_1 dy_2 dP_\perp^2 dz d^2P_{hT}} = \frac{d\sigma^{uu}}{d\mathcal{P}.\mathcal{S}.} = \sum_{b=q,g} x' f_b(x') x f_a(x) D_q(z_h, P_{hT}) \times H^{\rm uu}_{ab\to qc} \ , \tag{2}$$

where $d\mathcal{P}.\mathcal{S}. = dy_1 dy_2 dP_\perp^2 dz d^2P_{hT}$ represents the phase space for this process, y_1 and y_2 are rapidities for the jet P_J and the balancing jet, respectively, $P_\perp$ is the jet transverse momentum, and the final observed hadron's kinematic variables z_h and P_{hT} are defined above. Here, x and x' are the momentum fractions carried by the parton "a" and"b" from the incident hadrons, respectively. In the above equation, f_a and f_b are the associated parton distributions, and $D_q(z_h, P_{hT})$ is the TMD quark fragmentation function. The hard factors $H_{ab\to qc}$ are equal to the partonic differential cross section for the relevant subprocess: $H_{ab\to qc} = d\hat{\sigma}/d\hat{t}|_{ab\to qc}$. Similarly, the differential cross section for the transverse-spin dependent scattering process can be written as

$$\frac{d\sigma(S_\perp)}{d\mathcal{P}.\mathcal{S}.} = \sum_{b=q,g} x' f_b(x') x \delta q_T(x) \delta\hat{q}(z_h, P_{hT}) \frac{\epsilon^{\alpha\beta} S_\perp^\alpha}{M_h} \times \left[P_{hT}^\beta - \frac{P_B \cdot P_{hT}}{P_B \cdot P_J} P_J^\beta \right] \times H^{\rm Collins}_{qb\to qb} \ , \tag{3}$$

$\epsilon_\perp^{\alpha\beta} = \epsilon^{\mu\nu\alpha\beta} P_{A\mu} P_{B\nu} / P_A \cdot P_B$ with convention $\epsilon^{0123} = 1$, and $H^{\rm Collins}_{qb\to qb}$ is the hard factor for the partonic channel $qb \to qb$. Here, $\delta q_T(x)$ (also noted by δq, h_{1q} and $\Delta_T q$ in the literature) is the quark transversity distribution, and $\delta\hat{q}$ the Collins fragmentation function [17] (also noted as $\Delta\hat{D}$ or $H_1^\perp$ in the literature).

It was argued that the Collins function is universal between the above process and other processes such as e^+e^- annihilation and SIDIS [35]. As an example, we will demonstrate this universality for the particular partonic channel $qq' \to qq'$ contribution to our process, and all other channels will follow accordingly. For convenience, we list the hard factors for

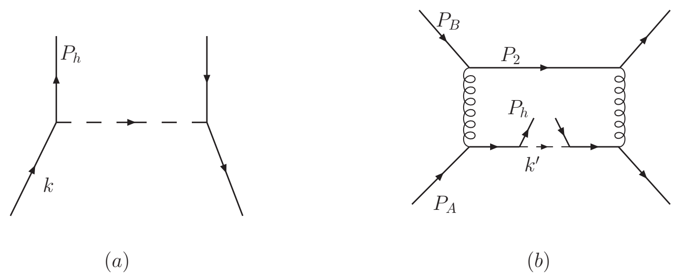

FIG. 2: *Quark fragmentation to pion production (a), and in pp scattering in a model described in [40] (b).*

this channel,

$$H^{\rm uu}_{qq'\to qq'} = \frac{\alpha_s^2\pi}{\hat{s}^2}\frac{N_c^2-1}{4N_c^2}\frac{2(\hat{s}^2+\hat{u}^2)}{-\hat{t}^2}, \quad H^{\rm Collins}_{qq'\to qq'} = \frac{\alpha_s^2\pi}{\hat{s}^2}\frac{N_c^2-1}{4N_c^2}\frac{4\hat{s}\hat{u}}{-\hat{t}^2} , \tag{4}$$

for the unpolarized and single-transverse-spin polarized cross sections, respectively. Here $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the usual partonic Mandelstam variables.

The rest of this paper is organized as follows. In Sec.II, we calculate the differential cross sections for the unpolarized and single-spin dependent scattering processes from $qq' \to qq'$ channel contributions, and demonstrate the universality of the Collins function. Especially, we will present a detailed calculation for one-gluon exchange diagrams which are essential for the universality argument. An extension to two-gluon exchange diagrams is presented in Sec.III. We summarize our paper in Sec.IV.

II. UNIVERSALITY OF THE COLLINS FUNCTION

We follow the model used in Ref. [17] to calculate the quark fragmentation into a pion. As shown in Fig. 2(a), a quark (with momentum k) fragments into a pion (with momentum P_h) by the vertex from a model described in [40]. A simple calculation will give the unpolarized quark fragmentation function [17],

$$D_q(z_h, P_{hT}) = \frac{g^2}{16\pi^3}\frac{z_h^2}{\vec{P}_{hT}^2 + z_h^2 M^2} , \tag{5}$$

where g is the coupling between the quark and pion, M is the quark mass.

We can also use this model to calculate pion production in hadronic process of (1). In Fig. 2(b), we show the Feynman diagram for the typical partonic channel $qq' \to qq'$ contribution, where the initial quarks have momenta P_A and P_B, respectively. In the final state, the produced pion has momentum P_h, and the associated final state quark has momentum k', whereas the balancing jet has momentum P_2. We further introduce a light-like momentum $\hat{k}$: $\hat{k}^2 = 0$, which represents the dominant component of the fragmenting quark's momentum. It can be parameterized as follows,

$$\hat{k} = -\frac{\hat{u}}{\hat{s}} P_A - \frac{\hat{t}}{\hat{s}} P_B + \vec{P}_\perp \ , \tag{6}$$

where $P_\perp$ is the transverse momentum for the fragmenting quark in the Lab frame, $\hat{s}$, $\hat{t}$ and $\hat{u}$ as mentioned above, are the usual partonic Madelstam variables for this partonic process: $\hat{s} = 2P_A \cdot P_B$, $\hat{t} = -2P_A \cdot \hat{k}$, and $\hat{u} = -2P_B \cdot \hat{k}$. In our discussions, the jet's transverse momentum $P_\perp$ (in the lab frame) is the large momentum scale at the same order as $\hat{s}$, $\hat{t}$ and $\hat{u}$. Of course, the full momentum of the fragmenting quark $P_1 = P_h + k'$ is off-shell in this diagram. However, its off-shellness is much smaller than $P_\perp$. In order to formulate the final state hadron's momentum, we introduce a conjugate light-like vector $\hat{n}$: $\hat{n}^0 = \hat{k}^0$ and $\vec{\hat{n}} = -\vec{\hat{k}}$. It is convenient to define this momentum in the center of mass frame of the two incident momenta P_A and P_B. In this frame, we have

$$\hat{n} = P_2 = -\frac{\hat{t}}{\hat{s}} P_A - \frac{\hat{u}}{\hat{s}} P_B - \vec{P}_\perp \ , \tag{7}$$

which happens to be the momentum of the balancing jet. From above, we have $\hat{k}^2 = \hat{n}^2 = 0$ and $\hat{k} \cdot \hat{n} = \hat{s}/2$. In the following calculations, we will work in this particular frame. We emphasize that our results do not depend on the frame.

With the above two momenta, we can formula the final state pion's momentum as

$$P_h = z_h \hat{k} + \frac{\vec{P}_{hT}^2}{2 z_h \hat{k} \cdot \hat{n}} \hat{n} + \vec{P}_{hT} \ , \tag{8}$$

where $z_h = P_h \cdot \hat{n} / \hat{k} \cdot \hat{n}$ is the momentum fraction of the fragmenting quark carried by the pion in the final state, P_{hT} is the transverse momentum relative to the fragmenting quark momentum $\hat{k}$: $P_{hT} \cdot \hat{k} = 0$ and $P_{hT} \cdot \hat{n} = 0$. In the above parameterization, we have neglect the pion mass, which is not relevant in our calculations. Similarly, we can formulate the associated final state quark momentum k' as,

$$k' = (1 - z_h) \hat{k} + \frac{\vec{P}_{hT}^2 + M^2}{2(1 - z_h) \hat{k} \cdot \hat{n}} \hat{n} - \vec{P}_{hT} \ , \tag{9}$$

where we have kept the quark mass, because it will be relevant for the nonzero single spin asymmetry discussed below.

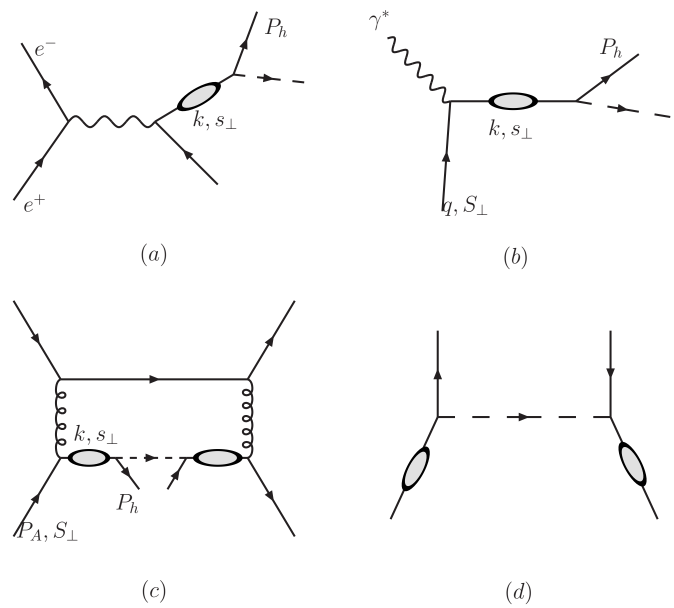

FIG. 3: *Universality of the Collins function in* e^+e^- *(a), deep inelastic scattering (b), and pp scattering (c), when we have dressed quark propagator associated with the fragmenting quark in these processes. The universal Collins function can be calculated from the diagram in (d). The blobs in the diagrams represent the dressed quark propagator in this model.*

With the above decompositions for the relevant momenta, it is straightforward to calculate the Feynman diagrams for this process in Fig. 2(b). In the calculations, we will utilize the power counting method to keep the leading order contributions, and neglect all higher order corrections of $P_{hT}/P_\perp$ or $M/P_\perp$. By doing that, we can separate the short distance physics (at the scale of $P_\perp$) from the long distance physics (at the scale of P_{hT} and M).

Finally, the cross section contribution from Fig. 2(b) will be,

$$\frac{d\sigma^{uu}}{d\mathcal{P}.\mathcal{S}.} = \frac{\alpha_s^2\pi}{\hat{s}^2}\frac{N_c^2-1}{4N_c^2}\frac{2(\hat{s}^2+\hat{u}^2)}{\hat{t}^2}\frac{g^2}{16\pi^2}\frac{z_h^2}{P_{hT}^2+z_h^2M^2} , \tag{10}$$

in the limit of $P_{hT} \ll P_\perp$. This result is indeed factorized into the hard factor $H^{\rm uu}_{qq'\to qq'}$ in Eq. (4) times the fragmentation function in Eq. (5) calculated from Fig. 2(a).

Now, we turn to discuss the SSA in this process. We need to generate a phase from the scattering amplitudes to have a non-vanishing SSA. As suggested in [17], the dressed quark propagator in this model may contribute to such a phase. Similarly, the vertex correction to the quark-pion vertex can also contribute a phase [41]. If the phase comes from the above sources, it is easy to argue the universality of the Collins function between our process and the SIDIS/e^+e^- process, because they are the same. For example, as we show in Fig. 3, the dressed quark propagator associated with the fragmenting quark can contribute to a nonzero phase [17], which will contribute the same to the Collins function in all these three processes: e^+e^- annihilation, SIDIS, and hadron production in pp scattering. This propagator can be parameterized as: $i(A \not{P}_1 + BM)/(P_1^2 - M^2)$ [17], where A and B are complex numbers. Following the above calculations for the unpolarized cross section, we will find the single-spin dependent cross section for process (1) from Fig. 3(c) can be written as

$$\frac{d\sigma(S_\perp)}{d\mathcal{P}.\mathcal{S}.} = \epsilon^{\alpha\beta} S_\perp^\alpha \left[P_{hT}^\beta - \frac{P_B \cdot P_{hT}}{P_B \cdot P_J} P_J^\beta \right] \frac{\alpha_s^2 \pi}{\hat{s}^2} \frac{N_c^2 - 1}{4N_c^2} \frac{4\hat{s}\hat{u}}{-\hat{t}^2} \frac{g^2}{16\pi^2} \frac{z_h^2(1-z_h) 2M \mathrm{Im}(A^* B)}{(P_{hT}^2 + z_h^2 M^2)^2} \ , \quad (11)$$

where again we only keep the leading order contribution in the limit of $P_{hT} \ll P_\perp$ and $M \ll P_\perp$. In the derivation of the above result, the following identity has been used to simplify the final expression,

$$\epsilon^{\alpha\beta} \left[\hat{s} P_\perp \cdot S_\perp P_\perp^\alpha P_{hT}^\beta + (\hat{u} - \hat{t}) P_B \cdot P_{hT} P_\perp^\alpha S_\perp^\beta - \hat{t}\hat{u} P_{hT}^\alpha S_\perp^\beta \right] = 0 \ , \quad (12)$$

which holds in our working frame. The above differential cross section can be factorized into the Collins function calculated from the dressed quark propagator from Fig. 3(d) [17] and the hard factor $H^{\rm Collins}_{qq' \to qq'}$ from Eq. (4) in this partonic channel $qq' \to qq'$, and this Collins function will be the same as that in e^+e^- and SIDIS processes in Fig. 3(a) and 3(b).

Similarly, the vertex corrections contributions to the Collins function can be analyzed accordingly, and the same factorization and universality of the Collins function will follow.

The main issue of the universality discussion concerns the extra gluon exchange contribution between the spectator and hard partonic part [32]. For example, in our case, because the hadron is colorless while the quark is colored, the remanet in the fragmentation process will be also colored. Thus the gluon exchanges between the remanet and the other parts of the scattering amplitudes become essential. In Fig. 4, we have shown all these interactions, including the gluon attachments to the incident quarks (a,c), and final state balancing quark (d) and the internal gluon propagator (b). These diagrams are much more complicated than those discussed in [32] for SIDIS and e^+e^- processes, where there is only one diagram contribution in both cases. Therefore, the universality argument for the Collins function is not straightforward. However, the dominant contribution to the fragmentation function comes from the kinematic region where the exchanged gluon is parallel to the final state hadron [42]. Otherwise, their contributions will be power suppressed in the limit of $P_{hT} \ll P_\perp$ or

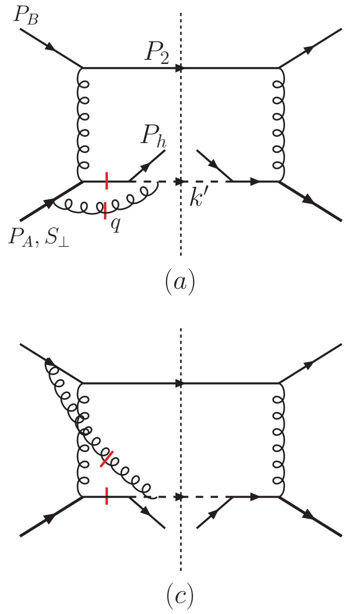

FIG. 4: *Gluon exchange diagrams contributions to the Collins asymmetry in pp collisions. The short bars indicate the pole contributions to the phase needed for a non-vanishing SSA. The additional two cuts in (d) cancel out each other.*

belong to a soft factor. For these collinear gluon interactions, we can use eikonal approximation and Ward identity to sum them together to form the gauge link in the definition of the fragmentation function [42].

Meanwhile, we notice that the contributing phases of the diagrams in Fig. 4 come from the cuts through the internal propagators in the partonic scattering amplitudes [20, 32]. In Fig. 4, we labeled these cut-poles by short bars in the diagrams. From our calculations, we find that all these poles come from a cut through the exchanged gluon and the fragmenting quark in each diagram, and all other contributions either vanish in the leading order contribution or cancel out each other. For example, in Fig. 2(d), we show two additional cuts, which contribute however opposite to each other and cancel out completely. To see this cancellation more clearly, we can write down the momentum integral of the exchange-gluon,

$$\int \frac{d^4q}{(4\pi)^4}\mathcal{M}(q)\frac{1}{(k'-q)^2-M^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{1}{(P_2+q)^2+i\epsilon}$$
$$\times\frac{1}{(P_2-P_B+q)^2+i\epsilon}\frac{1}{(P_h+k'-q)^2-M^2+i\epsilon}\ , \tag{13}$$

where $\mathcal{M}(q)$ represents the denominators coming from the scattering amplitude. By power counting analysis, the dominant contribution to the fragmentation comes from the kinematic region of q being parallel to the final state hadron's momentum $q \sim P_h$. From this fact, we can parameterize q in terms of $\hat{k}$ and $\hat{n}$, and define $q^+ = q \cdot \hat{n}/\hat{k}\cdot\hat{n}$ and $q^- = q \cdot \hat{k}/\hat{k}\cdot\hat{n}$. Thus the integral of momentum q becomes $d^4q = \hat{k}\cdot\hat{n} dq^+ dq^- d^2q_T$, where q_T is the transverse momentum relative to the jet momentum $\hat{k}$. Because q is parallel to $\hat{k}$, q^+ will be order 1, whereas q^- will be order of q_T^2/q^+. When we perform the integrals of dq^+dq^-, we need to take two poles from the above propagators to obtain a nonzero Collins asymmetry. These poles will form a cut through the Fenyman diagram. Physically, these cuts represent the kinematic allowed final state re-scattering in the diagram.

By examining the behaviors of the propagators in the above kinematic region, we further notice that the $t-$channel gluon propagator $1/(P_2 - P_B + q)^2$ does not contribute to a pole. This is because this propagator is far off-shell: $(P_2 - P_B)^2 = \hat{t} \sim -|\vec{P}_\perp|^2$. If we take a pole from this propagator, we have to constrain the momentum of q being proportional to P_2 and P_B, whose contribution will be power suppressed. Thus, we shall calculate the pole contributions from other propagators. In Fig. 4d, we show three possible cuts which are kinematic allowed for this diagram. Two of them are associated with the propagator $1/(P_2 + q)^2$. This propagator involves large momentum P_2, and can be simplified by using the eikonal approximation,

$$\frac{1}{(P_2+q)^2+i\epsilon} \approx \frac{1}{2P_2\cdot\hat{k}}\frac{1}{q^+ + i\epsilon} \ . \tag{14}$$

The pole contribution from this propagator is proportional to $\delta(q^+)$. With this delta function, the integral over q^- vanishes, because the rest poles are in the same half plane of q^-,

$$\begin{aligned} &\int \frac{dq^-}{2\pi}\frac{1}{(k'-q)^2 - M^2 + i\epsilon}\frac{1}{(P_h + k' - q)^2 - M^2 + i\epsilon}\cdots \\ &\sim \int \frac{dq^-}{2\pi}\frac{1}{2(k'^+ - q^+)q^- + \cdots + i\epsilon}\frac{1}{2(P_h^+ + k'^+ - q^+)q^- + \cdots + i\epsilon}\cdots = 0 \ , \end{aligned} \tag{15}$$

where we have used the fact that $k'^+ - q^+ > 0$ and $P_h^+ + k'^+ - q^+ > 0$. This means that the two cuts associated with the propagator $1/(P_2 + q)^2$ cancel out each other. The above result depends on the momentum flow of q in this diagram and the time-like process in the fragmentation region requiring that $k'^+ > 0$ and $P_h^+ > 0$.

Therefore, the only contribution to the nonzero SSA associated with the Collins effect comes from the cut going through the fragmenting quark and the exchange-gluon, as we labeled by short bars in this diagram. Summarizing the above analysis, we find that the contribution from this diagram can be written as

$$\frac{\hat{k}\cdot\hat{n}}{(P_2-k_1)^2(P_2-P_B)^2}\int\frac{dq^+dq^-d^2q_T}{(2\pi)^4}\frac{1}{q^+}\frac{1}{1-q^+}\frac{1}{(k'-q)^2-M^2}\mathcal{M}(q)$$
$$\times\delta(q^2)\delta\left((P_h+k'-q)^2-M^2\right)\ , \tag{16}$$

where we have made the eikonal approximation for the propagators $1/(P_2+q)^2$ and $1/(P_2-P_B+q)^2$.

Similar analysis can be done for all other diagrams, and we find that their contributions come from the same poles of the fragmenting quark and the exchange-gluon. Therefore, their contributions will have the similar expression as Eq. (16) with the same delta functions in the integral: $\delta(q^2)\delta\left((P_h+k'-q)^2-M^2\right)$ and the propagator $1/((k'-q)^2-M^2)$. Thus the contributions from all these diagrams can be summed together. In this sum, we notice that the different diagrams have different color-factors,

$$\begin{aligned}
4(\mathrm{a}):&\quad \frac{1}{N_c^2}\mathrm{Tr}[T^aT^cT^bT^c]\times\mathrm{Tr}[T^aT^b]=C_F\times\frac{N_c^2-1}{4N_c^2}+\frac{1}{N_c^2}\frac{if_{abc}}{2}\mathrm{Tr}[T^aT^bT^c]\ ,\\
4(\mathrm{b}):&\quad \frac{1}{N_c^2}\mathrm{Tr}[T^aT^cT^b]\times\mathrm{Tr}[T^aT^d]if_{dbc}=-\frac{1}{N_c^2}\frac{if_{abc}}{2}\mathrm{Tr}[T^aT^bT^c]\ ,\\
4(\mathrm{c}):&\quad \frac{1}{N_c^2}\mathrm{Tr}[T^aT^cT^b]\times\mathrm{Tr}[T^aT^bT^c]\ ,\\
4(\mathrm{d}):&\quad \frac{1}{N_c^2}\mathrm{Tr}[T^aT^cT^b]\times\mathrm{Tr}[T^aT^cT^b]\ .
\end{aligned}\tag{17}$$

We further find that the contributions (without the color-factors) from the diagrams (c) and (d) are opposite to each other. Thus, their total contribution will be the difference on the color-factor, which is $\frac{1}{N_c^2}\frac{if_{abc}}{2}\mathrm{Tr}[T^aT^bT^c]$. That means the contributions from all these four diagrams can be grouped into two terms with different color factors: one with $C_F\times\frac{N_c^2-1}{4N_c^2}$, and one with $\frac{1}{N_c^2}\frac{if_{abc}}{2}\mathrm{Tr}[T^aT^bT^c]$. The latter one vanishes in the leading order of $P_{hT}/P_\perp$ after we sum all diagrams contributions, and thus we are left with the first color-factor contribution.

After summing over all diagrams' contribution, the spin-dependent differential cross section coming from the Collins effect will be

$$\frac{d\sigma(S_\perp)}{d\mathcal{P}.\mathcal{S}.}=\frac{\alpha_s^2}{\pi}\frac{N_c^2-1}{4N_c^2}\frac{4\hat{s}\hat{u}}{-\hat{t}^2}\epsilon^{\alpha\beta}S_\perp^\alpha\left[g^{\beta\beta'}-\frac{P_B^{\beta'}}{P_B\cdot P_J}P_J^\beta\right]\frac{g^2}{(2\pi)^3}C_Fg_s^2\int\frac{dq^+dq^-d^2q_T}{(2\pi)^4}$$
$$\times\left(q^+P_{hT}^{\beta'}-z_hq_T^{\beta'}\right)\frac{1}{(k'-q)^2}\delta(q^2)\delta\left((P_h+k'-q)^2-M^2\right)\ . \tag{18}$$

where g_s is the strong coupling. From the above result, we find a clear separation of the short distance physics at the scale $P_\perp$ and long distance physics at the scale P_{hT}. The short distance part is just the hard factor $H^{\rm Collins}_{qq'\to qq'}$ for the spin-dependent cross section, which can be calculated from the partonic process with both initial and final state quarks transversely polarized [38], as we show in the left panel of Fig. 5. The long distance part of the above result can be factorized into the Collins fragmentation function calculated from the right panel of Fig. 5. In this part, because the $q_T^{\beta'}$ integral is proportional to $P_{hT}^{\beta'}$, we can combine the two terms in the integral into one expression contained in the Collins function. Therefore, the spin-dependent cross section Eq. (18) can be re-written as

$$\frac{d\sigma(S_\perp)}{d\mathcal{P.S.}} = \frac{\alpha_s^2}{\pi}\frac{N_c^2-1}{4N_c^2}\frac{4\hat{s}\hat{u}}{-\hat{t}^2}\epsilon^{\alpha\beta}S_\perp^\alpha\left[P_{hT}^\beta-\frac{P_B\cdot P_{hT}}{P_B\cdot P_J}P_J^\beta\right]\frac{\delta\hat{q}(z_h,P_{hT})}{M_h}\ , \tag{19}$$

where the Collins function $\delta\hat{q}$ is calculated from the Feynman diagram in the right panel of Fig. 5,

$$\begin{aligned}\delta\hat{q}(z_h,P_{hT}) &= \frac{M_h}{P_{hT}^\alpha}\frac{g^2}{(2\pi)^3}g_s^2C_F\int\frac{dq^+dq^-d^2q_T}{(2\pi)^4}\left(q^+P_{hT}^\alpha-z_hq_T^\alpha\right)\\ &\times\frac{1}{(k'-q)^2-M^2}\delta(q^2)\delta\left((P_h+k'-q)^2-M^2\right)\ ,\end{aligned} \tag{20}$$

where the index α is not understood as a sum. This final result demonstrates that we do have a factorization for the spin-dependent cross section into the hard factor $H^{\rm Collins}_{qq'\to qq'}$ times the Collins fragmentation function, and the Collins function is the same as that in e^+e^- and SIDIS processes [34].

Therefore, by using the Ward identity at this particular order, the final results for all the diagrams of Fig. 4 will sum up together into a factorized form as shown in Fig. 5, where the cross section is written as the hard partonic cross section for $q(S_\perp)q'\to q(s_\perp)q'$ subprocess multiplied by a Collins fragmentation function. The exchanged gluon in Fig. 4 is now attaching to a gauge link from the fragmentation function definition [14] as shown in the right panel of Fig. 5.

The key steps in the above derivation are the eikonal approximation and the Ward identity. The eikonal approximation is valid when we calculate the leading power contributions in the limit of $P_{hT}\ll k_\perp$. The Ward identity ensure that when we sum up the diagrams with all possible gluon attachments we shall get the eikonal propagator from the gauge link in the definition of the fragmentation function. The most important point to apply the Ward identity in the above analysis is that the eikonal propagator does not contribute to the phase

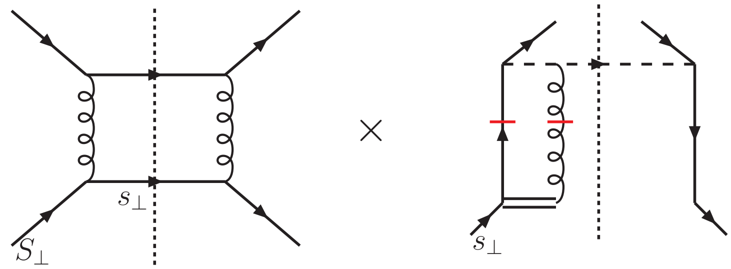

FIG. 5: *Factorize the contributions from Fig. 4 into the hard partonic cross section multiplied by the universal Collins fragmentation function. The short bars indicate the pole contribution to the Collins function.*

needed to generate a nonzero SSA. This is what we have shown for the Collins asymmetry in the above calculations, and the reason, as we mentioned above, is due to the momentum flow of the exchanged gluon and the kinematic constraints in the fragmentation process. We will show in the next section, that for the two-gluon exchange diagrams the eikonal propagators do not contribute to the phase for the nonzero Collins SSA in this process. Therefore, we conjecture that the above conclusions are valid to higher order contributions too.

This argument can not apply to the SSA associated with the parton distributions, where the eikonal propagator does contribute to the phase to generate a nonzero SSA. That is the reason we have sign differences for the Sivers functions in SIDIS and Drell-Yan processes.

III. TWO-GLUON EXCHANGE CONTRIBUTIONS

As we discussed in the last section, to demonstrate the universality of the Collins function, we have to apply the Ward identity to sum up all gluon exchange contributions into the gauge link from the definition of the fragmentation function. In order to use this argument, the eikonal propagator should not contribute to the phase needed to generate nonzero SSA associated with the Collins effects. This has been explicitly demonstrated in the last section for the one-gluon exchange contribution. In this section, we will extend the discussions to the two-gluon exchange contributions. Especially, we will show that these eikonal propagators do not contribute to the phase for the SSAs. The reason, again, is due to the time-like feature and the momentum flow in the fragmentation process.

We will focus our discussions on some representative diagrams from the two-gluon ex-

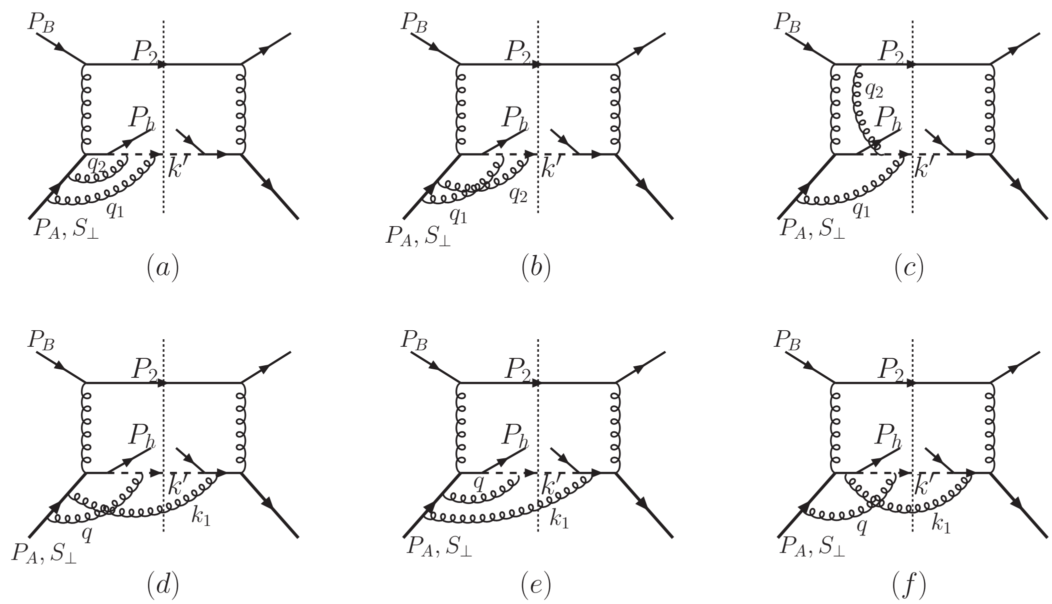

FIG. 6: *Example diagrams for two-gluon exchange contributions (a,b,c); and one real gluon radiation contributions (d,e,f).*

change contributions. All other diagrams will follow accordingly. We show these diagrams in Figs. 6(a,b,c). The contribution from Fig. 6(a) will depend on the following integral of the exchange gluons' momenta q_1 and q_2,

$$\int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\mathcal{M}(q_1,q_2)\frac{1}{(P_A-q_1)^2+i\epsilon}\frac{1}{(P_A-q_1-q_2)^2+i\epsilon}\frac{1}{(k'-q_1)^2+i\epsilon}$$
$$\frac{1}{(k'-q_1-q_2)^2+i\epsilon}\frac{1}{(k-q_1-q_2)^2+i\epsilon}\frac{1}{q_1^2+i\epsilon}\frac{1}{q_2^2+i\epsilon}\ , \tag{21}$$

where $k = P_1 = k' + P_h$ is the fragmenting quark's momentum and $\mathcal{M}(q_1, q_2)$ represents the numerators depending q_1 and q_2, especially their transverse momentum components. Following the arguments used in the last section, the first two propagators in the above expression can be further simplified by using the eikonal approximation, and then we will obtain the following expression

$$\int \frac{dq_1^-dq_1^+}{(2\pi)^2}\frac{dq_2^-dq_2^+}{(2\pi)^2}\frac{1}{-q_1^++i\epsilon}\frac{1}{-q_1^+-q_2^++i\epsilon}\frac{1}{(k'-q_1)^2+i\epsilon}\frac{1}{(k'-q_1-q_2)^2+i\epsilon}$$
$$\frac{1}{(k-q_1-q_2)^2+i\epsilon}\frac{1}{q_1^2+i\epsilon}\frac{1}{q_2^2+i\epsilon}\ , \tag{22}$$

where $q_i^\pm$ follow the definitions in the last section. The normalization of the above integral has been changed for convenience. This normalization is not relevant for our discussions, because we want to show that the eikonal propagators do not contribute to the phase needed for a nonzero SSA, not the actual contribution from this diagram. We will show if we take pole contributions from these two eikonal propagators, the final integral will vanish. Because of the existence of two eikonal propagators, the analysis will be more complicated than that in the last section. We discuss their contributions separately.

1. pole contribution from $\frac{1}{-q_1^+-q_2^++i\epsilon}$.

   If we take pole of this eikonal propagator, q_1^+ and q_2^+ will be constrained: $q_1^+ + q_2^+ = 0$, and the integral of (22) will become,

$$\int \frac{dq_1^+ dq_2^+}{2\pi}\frac{\delta(q_1^+ + q_2^+)}{q_1^+}\int \frac{dq_1^- dq_2^-}{(2\pi)^2}\frac{1}{-2(k'^+ - q_1^+)q_1^- + \Delta_1 + i\epsilon}\frac{1}{-2k'^+(q_1^- + q_2^-) + \Delta_2 + i\epsilon}$$
$$\frac{1}{-2k^+(q_1^- + q_2^-) + \Delta_3 + i\epsilon}\frac{1}{2q_1^+ q_1^- + \Delta_4 + i\epsilon}\frac{1}{2q_2^+ q_2^- + \Delta_5 + i\epsilon}\,, \tag{23}$$

   where Δ_i are some quantities depending on the transverse momenta of q_i, k' and P_h. The following analysis does not depend on the details of these numbers. In deriving the above equation, we have used the constraint of $q_1^+ + q_2^+ = 0$ to simplify the expression. We further notice that $k'^+ > 0$ and $k^+ > 0$. Thus, the poles of the second and third factor in the integral of q_1^- and q_2^- are both in the upper half plane. If $q_1^+ > 0$, which means that $q_2^+ < 0$, the pole of the fifth factor will be also in the upper half plane of q_2^-. Therefore, the poles of the three factors (the second, third and fifth) depending on q_2^- are all in the upper half plane of q_2^-, and the integral over q_2^- will vanish, and so will the above integral. Similarly, if $q_1^+ < 0$, the pole of the fourth factor will be in the upper half plane of q_1^-. Meanwhile, we will also have $k'^+ - q_1^+ > 0$, and the pole of the first factor will be in the upper half plane too. Therefore, the poles of the four factors (the first, second, third and fourth) depending on q_1^- are all in the upper half plane of q_1^-. The integral over q_1^- will vanish, and so will the above expression. In conclusion, in any case of $q_1^+ > 0$ or $q_1^+ < 0$, the above integral vanishes, and we do not have contribution from the pole of $\frac{1}{-q_1^+-q_2^++i\epsilon}$.

2. pole contribution from $\frac{1}{-q_1^++i\epsilon}$

   Because $q_1^+ = 0$, we can simplify the integral of (22) as follows,

$$\int \frac{dq_1^+ dq_2^+}{2\pi}\frac{\delta(q_1^+)}{q_2^+}\int \frac{dq_1^- dq_2^-}{(2\pi)^2}\frac{1}{-2k'^+ q_1^- + \Delta_1 + i\epsilon}\frac{1}{-2(k'^+ - q_2^+)(q_1^- + q_2^-) + \Delta_2 + i\epsilon}$$
$$\frac{1}{-2(k^+ - q_2^+)(q_1^- + q_2^-) + \Delta_3 + i\epsilon}\frac{1}{2q_2^+ q_2^- + \Delta_5 + i\epsilon}\,. \tag{24}$$

Again, the normalization has been changed for convenience. Because $k^+ > k'^+$, we will analyze the contributions of the above equation by classifying the different regions of q_2^+: (a) $q_2^+ > k^+$; (b) $q_2^+ < k'^+$; (c) $k'^+ < q_2^+ < k^+$. In the region of (a), we will have $k^+ - q_2^+ < 0$ and $k'^+ - q_2^+ < 0$. Therefore, the poles of the three factors (the second, third and fourth) are all in the lower half plane of q_2^-, and the integral over q_2^- vanishes. In the region of (b), we have $k'^+ > 0$, $k'^+ - q_2^+ > 0$ and $k^+ - q_2^+ > 0$. Thus, the poles of the three factors (the first, second and third) depending on q_1^- are all in the upper half plane, and the integral over q_1^- vanishes. In the region of (c), we have $q_2^+ > 0$, $k'^+ - q_2^+ < 0$ and $k^+ - q_2^+ > 0$. Therefore, the q_2^- integral will pick up the pole of the third factor, which actually determines the value of $q_1^- + q_2^-$. After substituting this back into the equation, we will find the second factor does not depend on q_1^- any more. The only dependence comes from the first factor. Obviously, this integral over q_1^- will vanish. In conclusion, in any case of (a,b,c), the above integral vanishes, and there is no contribution from the pole of $\frac{1}{-q_1^+ + i\epsilon}$.

In summary, there is no contribution to the SSA from the pole of the eikonal propagators in the diagram of Fig. 6(a). Similarly, the contribution from Fig. 6(b) depends on the following integral,

$$\int \frac{dq_1^- dq_1^+}{(2\pi)^2} \frac{dq_2^- dq_2^+}{(2\pi)^2} \frac{1}{-q_1^+ + i\epsilon} \frac{1}{-q_1^+ - q_2^+ + i\epsilon} \frac{1}{(k' - q_2)^2 + i\epsilon} \frac{1}{(k' - q_1 - q_2)^2 + i\epsilon}$$
$$\frac{1}{(k - q_1 - q_2)^2 + i\epsilon} \frac{1}{q_1^2 + i\epsilon} \frac{1}{q_2^2 + i\epsilon} , \tag{25}$$

where we have made the eikonal approximations for the two propagators along the incident quark line P_A. Comparing with Eq. (22), we find the only difference is the third factor $q_1 \to q_2$. Again, we can show that none of the two eikonal propagators will contribute to the phase needed for a nonzero SSA. We will discuss their contributions separately.

1. pole contribution from $\frac{1}{-q_1^+ - q_2^+ + i\epsilon}$.

   After taking this pole, the integral of (25) will become,

$$\int \frac{dq_1^+ dq_2^+}{2\pi} \frac{\delta(q_1^+ + q_2^+)}{q_1^+} \int \frac{dq_1^- dq_2^-}{(2\pi)^2} \frac{1}{-2(k'^+ - q_2^+)q_2^- + \Delta_1 + i\epsilon} \frac{1}{-2k'^+(q_1^- + q_2^-) + \Delta_2 + i\epsilon}$$
$$\frac{1}{-2k^+(q_1^- + q_2^-) + \Delta_3 + i\epsilon} \frac{1}{2q_1^+ q_1^- + \Delta_4 + i\epsilon} \frac{1}{2q_2^+ q_2^- + \Delta_5 + i\epsilon} , \tag{26}$$

   which is the same as Eq. (23) if we interchange $q_1^\pm$ and $q_2^\pm$. Thus, the above integral will vanish by the same arguments we have used for Eq. (23).

2. pole contribution from $\frac{1}{-q_1^+ + i\epsilon}$

   Because $q_1^+ = 0$, we can simplify the integral of (25) as follows,

$$\int \frac{dq_1^+ dq_2^+}{2\pi} \frac{\delta(q_1^+)}{q_2^+} \int \frac{dq_1^- dq_2^-}{(2\pi)^2} \frac{1}{-2(k'^+ - q_2^+)q_2^- + \Delta_1 + i\epsilon} \frac{1}{-2(k'^+ - q_2^+)(q_1^- + q_2^-) + \Delta_2 + i\epsilon}$$
$$\frac{1}{-2(k^+ - q_2^+)(q_1^- + q_2^-) + \Delta_3 + i\epsilon} \frac{1}{2q_2^+ q_2^- + \Delta_5 + i\epsilon} \ . \tag{27}$$

   Again, we classify three different regions of q_2^+ in the above equation: (a) $q_2^+ > k^+$; (b) $q_2^+ < k'^+$; (c) $k'^+ < q_2^+ < k^+$. The contributions from (a) and (b) regions vanish by the same reasons as we have shown for Eq. (24) in the above. In region (c), we have $k'^+ - q_2^+ < 0$ and $k^+ - q_2^+ > 0$. Therefore, the q_1^- integral will pick up the pole of the third factor, which again actually determines the value of $q_1^- + q_2^-$. After substituting this back into the equation, we will find the second factor does not depend on q_2^- any more. The only dependence comes from the first and last factors. Obviously, this integral over q_2^- vanishes because $k'^+ - q_2^+ < 0$ and $q_2^+ > 0$, and the poles of these two factors are both in the lower half plane. In conclusion, in any case of (a,b,c), the above integral vanishes, and there is no contribution from the pole of $\frac{1}{-q_1^+ + i\epsilon}$.

Similarly, the contribution from Fig. 6(c) will depend on the following integral,

$$\int \frac{dq_1^- dq_1^+}{(2\pi)^2} \frac{dq_2^- dq_2^+}{(2\pi)^2} \frac{1}{-q_1^+ + i\epsilon} \frac{1}{q_2^+ + i\epsilon} \frac{1}{(k' - q_1)^2 + i\epsilon} \frac{1}{(k' - q_1 - q_2)^2 + i\epsilon}$$
$$\frac{1}{(k - q_1 - q_2)^2 + i\epsilon} \frac{1}{q_1^2 + i\epsilon} \frac{1}{q_2^2 + i\epsilon} \ , \tag{28}$$

where again we have made the eikonal approximations. There are two eikonal propagators in the above equation, and as above we will discuss their contributions separately.

1. the pole contribution from $\frac{1}{-q_1^+ + i\epsilon}$

   After taking this pole, $q_1^+ = 0$, the above equation Eq. (28) will reduce to Eq. (24). According to the same arguments we used there, there will be no contributions from this pole.

2. the pole contribution from $\frac{1}{q_2^+ + i\epsilon}$

   This pole contribution means that $q_2^+ = 0$, and the integral of Eq. (28) become

$$\int \frac{dq_1^+ dq_2^+}{2\pi} \frac{\delta(q_2^+)}{q_1^+} \int \frac{dq_1^- dq_2^-}{(2\pi)^2} \frac{1}{-2(k'^+ - q_1^+)q_1^- + \Delta_1 + i\epsilon} \frac{1}{-2(k'^+ - q_1^+)(q_1^- + q_2^-) + \Delta_2 + i\epsilon}$$
$$\frac{1}{-2(k^+ - q_1^+)(q_1^- + q_2^-) + \Delta_3 + i\epsilon} \frac{1}{2q_1^+ q_1^- + \Delta_5 + i\epsilon} \ , \tag{29}$$

which will be identical to Eq. (27) if we interchange $q_1^\pm$ to $q_2^\pm$. Using the same arguments there, the above integral vanishes.

The above three examples are typical diagrams we encounter for the two-gluon exchange contributions for this channel. All these diagrams can be analyzed by a similar manner, and we will find that the eikonal propagators do not contribute to the phase needed to a nonzero SSA. Because of this fact, all these diagrams can be summed together to form the contributions from the gauge link in the fragmentation function, where the two gluons attach to the gauge link similar to the diagram we have shown in Fig. 5. Since there is no contributions from these eikonal propagators, the Collins function calculated from these diagrams will be the same as that in e^+e^- and SIDIS processes, and the universality preserved.

We have also drawn some other diagrams at this order in Fig. 6(d,e,f), which contribute to a real gluon radiation in addition to the gluon exchange. The analysis of these diagrams also show that we do not get contribution from the pole of the eikonal propagators. For example, the contribution from Fig. 6(d) depends on

$$\int \frac{d^4q}{(2\pi)^4}\frac{1}{-q^+ + i\epsilon}\frac{1}{-q^+ - k_1^+ + i\epsilon}\frac{1}{(k'-q)^2 + i\epsilon}\frac{1}{(k-q-k_1)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\ , \tag{30}$$

where k_1 is the momentum for the radiated gluon. We have two eikonal propagators in the above equation. However, none of them contributes to the phase needed to a nonzero SSA.

1. the pole contribution from $\frac{1}{-q^+ + i\epsilon}$

   This pole contribution means that $q^+ = 0$, and the integral of q^- will reduce to

   $$\int \frac{dq^-}{2\pi}\frac{1}{-2k'^+q^- + \Delta_1 + i\epsilon}\frac{1}{-2(k^+ - k_1^+)q^- + \Delta_2 + i\epsilon}\ . \tag{31}$$

   Because $k^+ = k_1^+ + k'^+ + P_h^+ > k_1^+$ and $k'^+ > 0$, the poles of the above two factors are both in the lower half plane of q^-, and the integral vanishes.

2. the pole contribution from $\frac{1}{-q^+ - k_1^+ + i\epsilon}$

   After taking this pole, we will have the following q^- integral

   $$\int \frac{dq^-}{2\pi}\frac{1}{-2(k'^+ - q^+)q^- + \Delta_1 + i\epsilon}\frac{1}{-2k^+q^- + \Delta_2 + i\epsilon}\frac{1}{2q^+q^- + \Delta_3 + i\epsilon}\ . \tag{32}$$

   Because the pole constrains that $q^+ = -k_1^+ < 0$ and $k'^+ - q^+ > 0$, the poles of the above three factors are all in the lower half plane. The integral over q^- vanishes.

In summary, there is no contribution from the pole of the eikonal propagators in the diagram of Fig. 6(d).

The contribution from Fig. 6(e) will depend on the following integral,

$$\int \frac{d^4q}{(2\pi)^4} \frac{1}{-k_1^+ + i\epsilon} \frac{1}{-q^+ - k_1^+ + i\epsilon} \frac{1}{(k'-q)^2 + i\epsilon} \frac{1}{(k-q-k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \ . \tag{33}$$

Because $k_1^+ > 0$, we only have one possible pole contribution from the eikonal propagator $1/(-q^+ - k_1^+ + i\epsilon)$, which vanishes by the same reason as above for diagram Fig. 6(d). Similarly, if the gluon with momentum q attaches to the radiated gluon instead of the incident quark line with momentum P_A (we did not show this diagram in Fig. 6), the contribution vanishes by the same reason.

The contribution from Fig. 6(f) depends on the following integral

$$\int \frac{d^4q}{(2\pi)^4} \frac{1}{-q^+ + i\epsilon} \frac{1}{(k'-q)^2 + i\epsilon} \frac{1}{(k-q-k_1)^2 + i\epsilon} \frac{1}{(k-q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \ , \tag{34}$$

after eikonal approximation. If we take the pole contribution from the eikonal propagator, the above integral will reduce to

$$\int \frac{dq^-}{2\pi} \frac{1}{-2k'^+q^- + \Delta_1 + i\epsilon} \frac{1}{-2(k^+ - k_1^+)q^- + \Delta_2 + i\epsilon} \frac{1}{-2k^+q^- + \Delta_3 + i\epsilon} \ . \tag{35}$$

Again, because $k^+ > k_1^+$, the poles of the above three factors are all in the lower half plane, and the integral over q^- vanishes. Thus, there is no contribution from the pole of the eikonal propagator for this diagram.

In summary, for the gluon radiation diagrams, there is no contributions from the pole of the eikonal propagators. Because of this fact, we can use Ward identity to sum all these diagrams together to form the gauge link contribution from the fragmentation function, similar to the diagram in Fig. 5 with an additional gluon radiation.

Concluding the analysis of the two-gluon exchange diagrams in Fig. 6, the eikonal propagators do not contribute to the phase needed for the nonzero SSA associated with the Collins effect. Therefore, we can apply the Ward identity at this order to sum all these diagrams plus other similar ones. This sum will lead to the gauge link contribution from the fragmentation function definition, and the fragmentation function will be the same as that in e^+e^- and SIDIS processes.

IV. SUMMARY AND DISCUSSIONS

In this paper, we have shown that the Collins function in hadron production in single-transverse-spin polarized pp scattering is the same as that in e^+e^- and SIDIS processes. This universality is a general and model-independent observation, and depends on the fact that the eikonal propagators do not contribute to the phase needed for a nonzero SSA. We have demonstrated this by explicit calculations for one-gluon exchange diagrams which corresponds to one eikonal propagator in the amplitudes, and two-gluon exchange diagrams which correspond to two eikonal propagators. Although our calculations were based on a model [17, 40], the analysis and arguments are quite general. The results, as we emphasized, depend on the momentum flow and kinematic constraints in the fragmentation process.

This observation is very different from the SSAs associated with the parton distributions, where the eikonal propagators from the gauge link in the parton distribution definition play very important role. It is the pole of these eikonal propagators contribute to the phase needed for a nonzero SSA associated with the naive-time-reversal-odd parton distributions, which also predicts a sign difference for the quark Sivers function between the SIDIS and Drell-Yan processes. More complicated results have been found for the SSAs in the hadronic dijet-correlation [29, 30], where a normal TMD factorization breaks down [31]. The reason is that the eikonal propagators from the initial and final state interactions in dijet-correlation process do contribute poles in the cross section [30, 31]. Because of this, the Ward identity is not applicable, and the standard TMD factorization breaks down, although a modified factorization may be valid if we modify the definition of the TMD parton distributions to take into account all the initial and final state interaction effects [29]. In the fragmentation process, as we discussed in our paper, the eikonal propagators do not contribute to an imaginary part, and the Ward identity is applicable. We have shown this in our explicit calculations including one-gluon and two-gluon exchange contributions.

There has been discussion about the twist-three quark-gluon correlation contribution in the fragmentation function, especially for the Collins effects [23, 43]. It will be interesting to further understand these contributions following the analysis in this paper, and discuss the universality issues in a more general ground [32, 33, 35].

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