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Application of chiral two- and three-nucleon interactions to the ⁴**He photo-disintegration**

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Abstract. We report on an *ab initio* calculation of the ⁴He total photo-absorption cross section using two- and three-nucleon interactions based upon chiral effective field theory. The microscopic treatment of the continuum problem is achieved using the Lorentz integral transform method, applied within the no-core shell model approach.

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INTRODUCTION

Chiral effective filed theory (χ EFT) [1, 2] represents our best opportunity to reach a consistent picture of the interaction among nucleons, that is based on the underlying and fundamental theory of quantum chromo-dynamics (QCD). Indeed, χ EFT may be thought as an effective theory for QCD in the low energy regime relevant for describing the properties of nuclei. In the framework of χ EFT the nucleon-nucleon (NN) interaction is predicted at the leading order, together with a three-nucleon (NNN) interaction at the next-to-next-to-leading order or N²LO [2, 3, 4], and even a four-nucleon (NNNN) interaction at the fourth order (N³LO) [5]. The details of QCD are contained in parameters, the so-called low-energy constants (LECs), that are not fixed by the symmetry, but can be constrained by experiment. Therefore, it is of the utmost importance to apply χ EFT to nuclei in an ab initio framework.

We performed an *ab initio* calculation [6] of the ⁴He total photo absorption cross section in unretarded dipole approximation, using the high quality NN potential at the fourth order (N³LO) in the χ EFT expansion of Ref. [7], and the NNN interaction at the highest order presently available (N²LO) [3, 5]. The two low-energy constants that parameterize the short-range NNN interaction were selected as discussed in Ref. [8]. The microscopic treatment of the continuum problem was achieved by means of the Lorentz integral transform (LIT) method [9], applied within the *ab initio* no-core shell model (NCSM) approach [10].

THE AB INITIO NO-CORE SHELL MODEL

The NCSM is a technique for the solution of the A-nucleon bound-state problem. Starting from an Hamiltonian containing realistic NN and NNN forces (both coordinateand momentum-space interactions can be equally handled), the Schrrödinger equation is solved by expanding the wave functions in terms of a complete set of A-nucleon harmonic oscillator (HO) basis states up to a maximum excitation $N_{max}\hbar\Omega$ above the minimum energy configuration, with Ω the HO frequency. Both Jacobi relative coordinates or Cartesian single-particle coordinates can be used. Indeed, in a complete $N_{max}\hbar\Omega$ space translational invariance is preserved even in the Slater-determinant basis. The convergence to the exact result with increasing N_{max} is accelerated by the use of an effective interaction tailored to the model-space truncation. The effective interaction is obtained using a unitary transformation in a n-body cluster approximation, where n is typically 2 or 3 [11].

TOTAL PHOTO-ABSORPTION CROSS SECTION VIA THE LORENTZ INTEGRAL TRANSFORM METHOD

In the long wave-length approximation, the total photo-absorption cross section

$$\sigma_{\gamma}(\omega) = 4\pi^2 \frac{e^2}{\hbar c} \omega R(\omega) , \qquad (1)$$

is proportional to the inclusive dipole response function:

$$R(\omega) = \int d\Psi_f \left| \left\langle \Psi_f \right| \hat{D} \left| \Psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega).$$
⁽²⁾

This is the sum of all the transitions from the ground state $|\Psi_0\rangle$ to the various allowed final states $|\Psi_f\rangle$ induced by the dipole operator:

$$\hat{D} = \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} \frac{\tau_i^z}{2} r_i Y_{10}(\hat{r}_i) .$$
(3)

In the above equations ground- and final-state energies are denoted by E_0 and E_f , respectively, whereas τ_i^z and $\vec{r}_i = r_i \hat{r}_i$ represent the isospin third component and center of mass frame coordinate of the *i*th nucleon. The direct calculation of such response function is extremely difficult, especially for energies beyond the three-body breakup threshold. However, it is possible to obtain the response function by following a small detour, the LIT method [12].

While in conventional approaches one usually starts from Eq. (2), in the LIT method one obtains $R(\omega)$ after the inversion of its integral transform with a Lorentzian kernel

$$L(\sigma_R, \sigma_I) = \int d\omega \frac{R(\omega)}{(\omega - \sigma_R)^2 + \sigma_I^2} = \langle \widetilde{\Psi} | \widetilde{\Psi} \rangle.$$
(4)

Indeed, in order to calculate such a transform it is sufficient to solve the in-homogeneous "Schrödinger-like" equation

$$(H - E_0 - \sigma_R + i\sigma_I)|\widetilde{\Psi}\rangle = \hat{D}|\Psi_0\rangle.$$
(5)

Because of the presence of an imaginary part σ_I in Eq. (5) and the fact that the righthand side of this same equation is localized ($\langle \Psi_0 | \hat{D}^{\dagger} \hat{D} | \Psi_0 \rangle < \infty$), one has an asymptotic



FIGURE 1. The ⁴He ground-state (*a*) energy, (*b*) point-proton root-mean-square radius $\langle r_p^2 \rangle^{1/2}$, and (*c*) total dipole strength $\langle \Psi_0 | \hat{D}^{\dagger} \hat{D} | \Psi_0 \rangle$ as functions of the model-space truncation N_{max} . Present results with χ EFT interactions for $\hbar \Omega = 22$ and 28 MeV.

boundary condition similar to a bound state. Thus, one can apply bound-state techniques for its solution, and, in particular, expansions over basis sets of localized functions such the NCSM basis. Moreover, the solution of Eq. (5) is unique. Once calculated $\langle \widetilde{\Psi} | \widetilde{\Psi} \rangle$ the response function can be obtained by numerical inversion of the integral equation (4) [13]. During this procedure all the final-state interaction of the problem is fully taken into account.

RESULTS

We start by discussing our results for the ground state of the α particle, which enters the driving term of the LIT equation (5). In Fig. 1 we show the convergence patterns for three different ground-state observables, calculated with and without inclusion of the NNN terms of the interaction: binding energy, point-proton radius and total dipole strength. For all of them we obtain very similar and smooth convergence patterns, using effective interactions at the three-body cluster level. In particular, already for $N_{max} = 18$ we find independence from both model space and harmonic oscillator parameter. At the ground-state level, the inclusion of the NNN force affects mostly the energy, providing 3.21 MeV additional binding, while the point-proton radius undergoes only a weak reduction. The total dipole strength follows the same pattern as the radius. Indeed, these two observables



FIGURE 2. The ⁴He total photo-absorption cross section as a function of the excitation energy ω . Present results with χ EFT interactions, and in particular: (upper panel) convergence pattern of the NN+NNN calculation with respect to the model-space truncation N_{max} for $\hbar\Omega = 28$ Me; (lower panel) frequency dependence of the best ($N_{max} = 18/19$) results with and without inclusion of the NNN force.

are correlated [14]:

$$\langle \Psi_0 | \hat{D}^{\dagger} \hat{D} | \Psi_0 \rangle \simeq \frac{ZN}{3(A-1)} \langle r_p^2 \rangle.$$
 (6)

This expression, which is exact for deuteron and triton, and for spatially symmetric systems, is violated of about 9% for the ⁴He calculated both with and without NNN interactions terms.

By applying the LIT method we have obtained the ⁴He total photo-absorption cross section shown in Fig. 2. Also for this observable we find a stable and accurate convergence thanks to the use of three-body effective interactions. From the bottom panel of the figure we also see that for the biggest model space used, $N_{max} = 18/19$, the dependence on the HO frequency is weak. The inclusion of the NNN interaction terms induces a suppression of the peak and an enhancement of the tail of the cross section. In particular, the reduction of the low-energy cross section is related, through the inverse energy-weighted sum rule to the the reduction found for the dipole strength

$$\int_{E_{th}}^{\infty} \frac{\sigma_{\gamma}(\omega)}{\omega} d\omega = 4\pi^2 \frac{e^2}{\hbar c} \langle \Psi_0 | \hat{D}^{\dagger} \hat{D} | \Psi_0 \rangle .$$
⁽⁷⁾



FIGURE 3. The ⁴He photo-absorption cross section as a function of the excitation energy ω . Present results with χ EFT interactions compared to the most recent experiments [15, 16, 17].

Our final results, presented in Fig. 3 together with the most recent experiments, show a peak around the excitation energy of $\omega = 27.8$ MeV, with a pick height mildly sensitive to the NNN force. The experimental situation in the near-threshold region is controversial: two direct measurements performed using quasi-mono-energetic photons [15, 16] show discrepancies up to a factor of two on the absolute height of the cross-section peak. We find an overall good agreement with the photo-disintegration data from bremsstrahlung photons of Nilsson *et al.* [16], while we reach only the last of the experimental points of Ref. [15]. In particular, the confused experimental situation drawn by these two data sets does not allow to asses the role of the NNN force effect. Recently Nakayama *et al.* performed an indirect measurements of the α -particle total photo-absorption cross section [17] by observing its analog via the ⁴He(⁷Li,⁷Be) reaction at an incident energy of 455 MeV and at forward scattering angles. Although the uncertainty on this extracted absolute cross section is 20% or more, the inclusion of the NNN terms of the interaction appear to improve the agreement of the calculated cross section with the latter indirect measurement.

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