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Current Advances in the Computational Simulation of the Formation of Low-Mass Stars

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Developing a theory of low–mass star formation ($\sim$ 0.1 to 3 $M_\odot$) remains one of the most elusive and important goals of theoretical astrophysics. The star–formation process is the outcome of the complex dynamics of interstellar gas involving non–linear interactions of turbulence, gravity, magnetic field and radiation. The evolution of protostellar condensations, from the moment they are assembled by turbulent flows to the time they reach stellar densities, spans an enormous range of scales, resulting in a major computational challenge for simulations. Since the previous Protostars and Planets conference, dramatic advances in the development of new numerical algorithmic techniques have been successfully implemented on large scale parallel supercomputers. Among such techniques, Adaptive Mesh Refinement and Smooth Particle Hydrodynamics have provided frameworks to simulate the process of low–mass star formation with a very large dynamic range. It is now feasible to explore the turbulent fragmentation of molecular clouds and the gravitational collapse of cores into stars self–consistently within the same calculation. The increased sophistication of these powerful methods comes with substantial caveats associated with the use of the techniques and the interpretation of the numerical results. In this review, we examine what has been accomplished in the field and present a critique of both numerical methods and scientific results. We stress that computational simulations should obey the available observational constraints and demonstrate numerical convergence. Failing this, results of large scale simulations do not advance our understanding of low–mass star formation.

1.   INTRODUCTION

Most of the stars in the galaxy exist in gravitationallly bound binary and low-order multiple systems. Although several mechanisms have been put forth to account for binary star formation, fragmentation has emerged as the leading mechanism for the past decade (*Bodenheimer et al.*, 2000). This point of view has been strengthened by observatons that have shown that the binary frequency among pre-main-sequence stars is comparable to or greater than that among nearby main-sequence stars (*Duchene et al.*, 1999). This suggests that most binary stars be formed during the protostellar collapse phase. Developing a theory for low mass star formation (0.2 to 3 solar masses) with the goal of understanding the nature and physical proper-

ties of the formation of binary and multiple stellar systems remains one of the most elusive and important goals of theoretical astrophysics. Until very recently, the extreme variations in length scale inherent in the star formation process have made it difficult to perform accurate calculations of fragmentation and collapse, which are intrinsicially three-dimensional in nature. Since the last review in *Protostars and Planets IV* by *Bodenheimer et al.*, 2000, dramatic advances in the development of new numerical algorithmic techniques including adaptive mesh refinement (AMR) coupled with radiation transport and MHD in three dimensions and Smooth Particle Hydrodynamics (SPH) have been successfully implemented on large scale parallel supercomputers to allow significant increases in the dynamic range pos-

sible for the problem of low mass star formation. It is now feasible to explore the collapse and fragmentation of turbulent molecular clouds down to the formation of turbulent cores and finally to the formation of stars. In this chapter, we examine what has been recently accomplished in the field of numerical simulation of low mass star formation and we critically review what we can believe of the results of such simulations. The review will not cover primordial star formation, nor will it address the problem of high mass star formation which will be addressed in a separate chapter by *Beauther et al.*. Since observations of binary and multiple star systems provide the crucial constraints of any theory of low mass star formation, we begin with a brief summary of the observations of low mass star forming regions.

1.a Key Questions Posed by the Observations

Major observational surveys that are now coming on line such as COMPLETE (Coordinated Molecular Probe Line Extinction Thermal Emission) (see *Goodman*, 2004), will provide us with a comprehensive statistical database with which to address key aspects of the star formation process in two phases, clouds and cores. A unified theory of star formation will of necessity have to explain how long a star remains in its birth core?; how long a star remains associated with filamentary structure in dark clouds?; what is the nature of the surrounding environment that a star-disk system must have to continue its accretion?; what process delineates the fragmentation of cores into binary or higher order systems? Existing observations of stellar birth sites within dark molecular clouds at infrared and millimeter wavelengths have provided us with detailed observational constraints that must be explained by any theory of low mass star formation (*Lada*, 2005). What dictates the basic properties of the dense cores in Giant Molecular Clouds (GMCs) with mean densities $\sim 10^4$ cm^{-3}, peak densities of 10^6 cm^{-3} and typical masses that range from 1–10^3 M$_\odot$? How does the frequency distribution of dense core masses select a power–law such that $dn/dm \sim m^{-1.5}$ (e.g., *Lada et al.*, 1991). Why is the overall efficiency of star formation in dense gas $\sim$ 10–20% so much higher than the overall star formation efficiency in GMCs $\sim$ 1–3% (*Myers et al.*, 1986)? Recent observation by *Lada and Lada,* (2003) have shown that embedded clusters span a range from groups of 10 or more stars to populations exceeding 1000 or more members with the large majority of stars forming in clusters with 100 members or more. At present, the physical process of star formation in clusters is not well understood observationally or theoretically. Observations have elucidated many properties of star forming cores that theory must explain. *Larson*(1981) first found scaling laws that established a relationship between the internal velocity dispersion σ_{1D} of star forming regions ranging from 0.1 to 10^2 pc. and the maximum projected lengthscale on the sky such that σ_{1D} (kms^{-1})=1.1L(pc)$^{0.38}$ interpreted as indicating turbulent motions on the scale of cores over a large dynamic range. *Goodman et al.* (1993) surveyed 43 molecular cloud cores in NH$_3$ and were able to estimate the ratio of the rotational energy to the gravitstional binding energy β and found that β varied from 10^{-3} to 10^{-1} with a mean value 0f .02. The density structure of molecular cores was determined by observations (*Motte and Andre*, 2001) concluding that core backgrounds were flatter than predicted by a singular isothermal sphere (*Shu*, 1977) but consistent with Bonnor-Ebert spheres (*Bonnor*, 1957; *Ebert*, 1955). An interesting and unexplained feature of these cores that can be seen from thier maps is the appearance of significant asymmetry suggestive of turbulent conditions (*Fisher*, 2002). Observations studying the multiplicity of stars in star forming regions (*Abt and Levy*, 1976) and more recent studies across different star forming regions (*Mathieu*, 1994; *Ghez et al.*, 1997; *Looney et al.*, 2000) have enabled comaprisons with the multiplicity in the field. These observations, when correlated against orbital separation and eccenticity can provide crucial constraints on the properities of stars resultant from different star formation theories. Correlation of multiplicity fraction with stellar age can provide clues to the formation mechanism of multiple systems. Recent observations of brown dwarfs binaries (*Basri*, 2000) have revealed the rarity of brown dwarf/stellar systems compared to the more typical stellar/stellar binaries. This so called "brown dwarf desert" provides key questions for theoretical models of multiple star formation. Observations of the orientation of the disks associated with star formation in multiple systems (*Monin et al.*, 1998) may provide a powerful constraint on theoretical models and indicate if multiple systems form within geoemtrcally thin flat disks (*Bodenheimer et al.*, 2000). Studies of the strength of magnetic fields in regions of star formaiton have shown that molecular clouds are marginally supercrtitical (*Crutcher*, 1999). Most simulations to date to date have neglected the effects of magnetic fields in the formation of binary and multiple systems. Some recent work that has attempted to include magnetic fields in 3D simulations (*Boss*, 1997, 1999, 2002), but this was included in a very approximate fashion. *Shu et al.*, 2000 has investigated thew effects of magnetic fields on the bifucation and fragmentation of singular isothermal magnetized disks. Observational studies of stellar feedback in the form of jets, winds and accretion shock luminosity (*snell et al.*, 1985) provide further constraints on simulations.

A major unsolved problem in the theory of star formation is that current theory cannot yet accurately predict the form of the IMF. A successful theory must be able to predict the initial distribution of stellar masses at birth (stellar IMF). Typical properties of the stellar IMF can be derived from detaield studies of the Trapezium cluster in Orion (*Lada*, 2005). *Meunch et al.*, (2002) found that the IMF has a broad peak extending fro 0.6 to 0.1 M$_\odot$ showing that there are relatively few high mass stars and few substellar objects. A key observational constraint for a modern theory of low mass stars is that no more than $\sim$ 22% of all objects formed are Brown Dwarfs.

1.b Initial Conditions Inferred from the Observations

Besides posing key questions, observations of interstellar clouds provide an insight into the initial conditions of the process of star formation. Observable physical properties that can be related to statistical models of star formation include i) scaling laws of velocity, density and magnetic fields; ii) mean relative values of turbulent, thermal, magnetic and gravitational energies (the normalization of the scaling laws); iii) general considerations about the morphology of density, velocity and magnetic fields.

Scaling Laws. Larson (1981) found that velocity and size of interstellar clouds are correlated over many orders of magnitude in size. This correlation has been confirmed by many more recent studies (e.g. Fuller and Myers 1992; Falgarone, Puget and Perault 1992). The most accepted interpretation is that the scaling law reflects the presence of supersonic turbulence in the ISM (Ossenkopf and Mac–Low 2002; Heyer and Brunt 2004). The density may also scale with size, implying constant surface density on the average. Starting with the work of Troland and Heiles (1986), a correlation between magnetic field strength and gas density, $B \propto n^{1/2}$, has been reported for mean densities larger than $n \sim 100$ cm^{-3}. Density and magnetic field scalings are very uncertain because both quantities are difficult to measure.

Mean Energies. A normalization of the velocity scaling representative of large molecular cloud samples is $\delta V \approx$ 1km/s$(L/1\mathrm{pc})^{0.4}$. Assuming an average temperature of $T = 10$ K, this normalization corresponds to an rms sonic Mach number $M_s \approx 5$ on the scale of 1 pc, and $M_s \approx 1$ at 0.02 pc. So, on the average, the turbulent kinetic energy is larger that the thermal energy. The gravitational energy is in approximate equipartition with the turbulent kinetic energy on the largest scales, while turbulence dominates the small scale dynamics. Indirect evidence of super–Alfvénic dynamics in giant molecular clouds has been presented by Padoan and Nordlund (1999) and Padoan et al. (2004), suggesting that on the average the magnetic energy has an intermediate value, between the thermal and the kinetic energies. The situation within dense prestellar cores is rather different from this average values of the turbulent cascade. Observations suggests that in dense cores gravitational, kinetic, thermal and magnetic energy are all comparable. However, the magnetic energy is very difficult to estimate. Accounting for both detections and upper limits, there is a large dispersion in the ratio of magnetic to gravitational energy of dense cores (Crutcher et al. 1993; Crutcher et al. 1999; Bourke et al. 2001; Nutter et al 2004). In the case of large scale super–Alfvénic turbulence, this dispersion and the B–n relation are predicted to be real (Padoan and Nordlund 1999).

Morphology. The morphology of molecular clouds has been often described as fractal (e.g. Elmegreen and Falgarone 1986; Youngung 2004) or multi–fractal (Chappell and Scalo 2001). Very small unbound clumps have been detected that are not the result of gravitational instabilities and are consistent with a turbulent origin (Falgarone,

Puget and Perault 1992; Langer et al. 1995). Large scale velocity–column density correlations from molecular line surveys of giant molecular clouds also suggest a turbulent origin of the observed density enhancements (Padoan et al. 2001). Dense prestellar cores have been studied with molecular line transitions and FIR and sub–mm continuum, to relate their morphology to star–formation theories. Cores are elongated and generally triaxial (Jones and Basu 2002), their density profiles are flat near the center, steeper at larger radii, and may show very sharp edges (e.g. Bacmann et al. 2000). The relative geometry of cores and magnetic field is generally not the one predicted by standard star–formation models (Ward–Thompson et al. 2000). A few cores with density profiles predicted by simple equilibrium considerations are known (Alves, Lada and Lada 2001; Keto and Field 2005), but radial density profiles may offer an over-simplified picture of the cores, being the result of two projections (line of sight and azimuthal). Indeed, Ballesteros-Paredes et al. (2003) have shown that even collapsing or rebounding cores may be exhibit Bonnor–Ebert like profiles, with the result that the temperature estimated from a Bonnor–Ebert analysis may be wrong by a factor of 10.

In summary, observations of molecular clouds and cores provide important constraints for initial conditions that numerical simulations of star formation should account for. Large scale simulations should be consistent with the turbulent nature of the ISM. On the scale of giant molecular clouds, the turbulence is on the average supersonic and super–Alfvénic and its kinetic energy is roughly equal to the cloud gravitational energy. Properties of dense cores are not considered as initial conditions for large scale simulations, because they should emerge self–consistently from the simulations. On smaller scale, simulations of the evolution of dense prestellar cores may assume a rough equipartition of kinetic, magnetic and gravitational energies, triaxial morphologies and shallow central density profiles. However, simulations of the collapse of individual cores should not be inconsistent with the large scale dynamics, which may be achieved for example by exploring a large range of initial and boundary conditions.

2. A BRIEF SURVEY OF LOW-MASS STAR FORMATION MECHANISMS

Although much progress in numerical simulation of collapse and fragmentation has been made in the intervening 6 years since PPIV, a self-consistent theory of binary and multiple star formation that addresses the key observations is still not at hand. In this section we will give a brief overview of some of the dominant mechanisms for low mass star formation. A critical review of detailed simulations of the leading theories will be presented in section IV.

As has been previously discussed by *Bodenheimer et al.*, 2000, binary formation and multiple formation can occur through the processes of (i) capture, (ii) fission, (iii) prompt initial fragmentation, (iv) disk fragmentation and (v) fragmentation during the protostellar collapse phase. To these

we may add more recently developed processes such as the bifurcation of strongly magnetized isopedic disks to multi–lobed equilibrium structures and competitve accretion. The reader is referred to *Bodenheimer et al.*, 2000 for a review of (i)–(iii). Here we will briefly review the dominant mechanisms.

A recent mechnaism for multiple star formation has been put forth by *Shu et al.*, (2000) and *Galli et al.*, (2001). They develop equilibrium models of strongly magnetized isopedic disks and explored their bifurcation to non-axi–symmetric, multi–lobed strcutures of increasing rotation rates. *Fisher*(2002) has pointed out possible problems with this mechanism. The rotational β that the model transitions to a binary structure is .16, considerably higher than observed median values of .02. The models predict that disks which form will all be exactly aligned in contradiction to observations that show nonalignment in some disks(*Monin et al.*,1998). The model requires the magnetic field to be lost on a dynamical timescale subsequent to multiple formation. This is believed to be possible on at high densities suggesting that the model is relevant only for short–period systems. Finally, the model has only been explored for 2D without turbulence. These conditions are too simplistic for star forming regions.

Disk fragmentation from gravitational instability can result in multiple systems in an equilibrium disk if the minimum Toomre Q parameter falls below ≈ 1. However, *Bodenheimer et al.*, 2000, have pointed out that the required initial conditions to drive Q < 1 may not be easily realized since the mass accretion timescale is significantly longer than the dynamical timescale throughout most of the evolution of the protostar. Disk fragmentation plays a key role in one of the theories of the formation of Brown Dwarfs (BDs). This scenario, known as the "failed embryo" scenario begins with a gravitationally unstable disk surrounding a protostar which subsequently fragments into a number of substellar objects. A variant of this mechanism relies upon the observation that if the crossing time of the cluster of embryos is much less than the free–fall time of the of the collapsing core, one or more of the members will be rapdily ejected forming a BD (*Reipurth and Clarke*, 2001). As the protostellar disk fragments interactions among the forming BDs rapidly eject them from the disk resulting in stripping the BDs of their surrounding disk material. Recent simulations of the evolution of turbulent cores, all performed with SPH (*Bate et al.*, 2002,2003;*Delgato-Donate et al.*, 2004; *Goodwin et al.*, 2004a,b) show multiple fragmentation ($N > 5 - 10$) of circumstellar disks and rapid ejection of BDs resulting in an exceess of BD and low mass companions. Increasingly, observational evidence of BD clustering (*Duchêne et al.*,2004) , Ly–α signatures (*Jayawardhana et al.*, 2002) and disk frequencies (*Mohanty et al.*, 2004) is showing major obstacles for the failed embryo scenario. *Goodwin and Kroupa*(2005) have considered the constraints on the number of stars that can form within a core posed by the decay or high–order multiple systems. They show that to be consistent with observations, cores must produce only 2 or 3 stars in contrast with the multiple fragmentation of disks resulting from virtually all SPH simulations. *Fisher et al.* (2005) have performed high resolution, converged AMR simulations of the collapse turbulent cores and in contrast to the SPH simulations find no evidence of disk fragmentation. These studies will be discussed and contrasted with SPH simulations in detail in section IV.

Currently, there are two dominant models of star formation. Gravitational collapse theory suggests that star–forming turbulent molecular clumps, typically hundreds to thousands of $M\odot$ in mass, fragment into cores that eventually collapse to make individual stars or small multiple systems (*Shu et al.*, 1987; *Padoan and Nordland*, 2002). In contrast, competitive accretion theory suggests that at birth all stars are much smaller than the typical stellar mass and that final stellar masses are determined by the subsequent accretion of unbound gas from the clump (*Bonnell et al.*,1998; *Bonnell et al.*, 2001; *Bonnell et al*,2001a; *Bate et al.*, 2005). Competitive accretion has suggested that BDs and free–floating planets are protostars ejected from clumps and predicts they should be stripped of their disks, have high velocity dispersions, form in dense star forming clouds and have a mean steller mass that should vary through the Galaxy(*Bate et al.*,2005). Recent observations however are increasingly ruling against competitive accretion theory. *Mohanty et al.*(2005) using Echelle spectra has discovered several BDs with associated disks contradicting competive accretion predictions and observations have failed to find any variation in typical stellar mass with environment, again in contrast to competive accretion. Recent theoretical work by *Krumholz et al.* (2005a) based on detailed simulations of Bondi–Hoyle accretion in a turbulent medium (*Krumholz et al.*, 2005b) have determined in what types of environments comperitive accretion can occur and show that no observed star–forming regiosn produce signinficant cpmpetive accretion, and that simulations that show comperitive acceretion come to this result because their properties differ substantially from those determined by observation. Their results explain why observations have failed to confirm the key predictions of the competitive accretion picture and appear to deal a serious blow to the competive accretion scenario pointing to gravitational collapse and fragmentation of turbulent clouds as the emerging dominant mechanism of low mass star formation.

3. THE SIMULATION OF LOW–MASS STAR FORMATION

3.a Physical Processes Necessary for Detailed Numerical Simulation

3.a.1 Turbulence

The Reynolds number estimates the relative importance of the nonlinear advection term and the viscosity term in the Navier–Stokes equation, $Re = V_0 L_0/\nu$, where V_0 is the flow rms velocity, L_0 is the integral scale of the turbulence

(say the energy injection scale) and ν is the kinematic viscosity that we can approximate as $\nu \approx v_{th}/(\sigma n)$, where v_{th} is the gas thermal velocity, n is the gas mean number density and $\sigma \sim 10^{-16}$ cm^2 is the typical gas collisional cross section. For typical molecular cloud values, $Re \sim 10^7$, which implies flows are highly unstable to turbulence. It is hard to exaggerate the importance of turbulent gas dynamics in astrophysical processes, as turbulence is a dominant transport mechanism. Turbulence is especially important in molecular clouds, where it is also a dominant fragmentation mechanism, because of its large Mach number and the short cooling time of the gas.

There has been significant progress in our understanding of supersonic turbulence in recent years. Phenomenological models of the intermittency of incompressible turbulence (e.g. She and Leveque 1994; Dubrulle 1994) have been extended to supersonic turbulence by Boldyrev (2002) and the predictions of the model have been confirmed by numerical simulations (Boldyrev, Nordlund and Padoan 2002, Padoan et al. 2004). The intermittency correction is small for the exponent of the velocity power spectrum (corresponding to the second order velocity structure function) and large only at high order. However, Boldyrev, Nordlund and Padoan (2002) have shown that low order density correlators are dependent on high order velocity statistics, so intermittency is likely to play a significant role in turbulent fragmentation, despite being only a small effect in the velocity power spectrum.

Because the turbulence is a dominant fragmentation and transport mechanism, its correct description is of paramount importance for simulations of molecular cloud fragmentation into low mass stars. At present, the least dissipative numerical codes and the most powerful supercomputers can only achieve a Reynolds number lower than $Re \sim 10^4$. This means that the scale of turbulence dissipation is much larger in numerical simulations (of order the computational mesh size) than in nature ($\sim 10^{14}$ cm). However, the ratio of the Kolmogorov dissipation scale and the Jeans length, for typical conditions in molecular clouds, is very small and remarkably independent of temperature and density, $\eta_{\rm K}/\lambda_{\rm J} \approx 10^{-4}(T/10{\rm K})^{-1/8}(n/10^3{\rm cm}^{-3})^{-1/4}$. This shows that it may be possible to successfully simulate the process of star formation without resolving numerically the Kolmogorov dissipation scale, unless the nature of turbulent flows varies dramatically between $Re \sim 10^3$ and $Re \sim 10^7$.

However, numerical simulations of large scale fragmentation are still required to i) generate a sizable inertial range (a power law power spectrum of the turbulent velocity over an extended range of scales); ii) provide a Jeans length larger than scales where dissipation is significant. Numerical dissipation becomes significant in turbulence simulations well above the mesh size. Velocity power spectra, even in the least dissipative codes, start to decay with increasing wavenumber faster than a power law already at approximately 15 to 30 times the Nyquist frequency. To satisfy the above requirements, the Jeans wavelength and outer scale of the turbulence should be larger than approximately 30 times and 300 times the Nyquist frequency respectively. This translates into a minimum mean Jeans length of approximately 60 computational zones and a minimum computational box size of at least $1,000^3$ computational zones, for a grid code. Assuming the standard SPH kernel of 50 particles this corresponds to at least $50 \times 1,000^3$ particles to describe the density field, and at least $few \times 1,000^3$ particles to describe the velocity field, if a Godunov SPH method is used (see below).

Grid code simulations are only recently starting to achieve this dynamical range, while particle codes appear unsuitable to the task. The calculation of Bate, Bonnell and Bromm (2003) has 3.5×10^6 particles, more than four orders of magnitude below the above estimate and therefore not adequate to describe the process of turbulent fragmentation. As turbulent fragmentation has become a fashionable research field, one should be very cautious with studies claiming to directly test the effect of turbulence on star formation using numerical simulations with resolution well below the above estimates. This warning applies to most of the numerical studies of large scale turbulent fragmentation cited in this book.

3.a.2 Gravity

Jeans Condition *Jeans* (1928) analyzed the linearized equations of 1-D isothermal self-gravitational hydrodynamic (GHD) for a medium of infinite extent and found that perturbations on scales larger than the Jeans length, $\lambda_J \equiv \left(\frac{\pi c_s^2}{G\rho}\right)^{1/2}$, are unstable. Thermal pressure cannot resist the self-gravity of a perturbation larger than λ_J, and runaway collapse results. *Truelove et al.*, 1997 showed that the errors generated by numerical GHD solvers can act as unstable perturbations to the flow. In a simulation with variable resolution, cell-scale errors introduced in regions of coarser resolution can be advected to regions of finer resolution, affording these errors the opportunity to grow. The unstable collapse of numerical perturbations can lead to substantial fragments, a process termed artificial fragmentation. The strategy for avoidance of artificial fragmentation is to maintain sufficient resolution of λ_J. Defining the Jeans number $J \equiv \frac{\Delta x}{\lambda_J}$, *Truelove et al.*, 1997 found keeping $J \leq 0.25$ avoided artificial fragmentation in isothermal evolution of a collapse spanning 7 decades of density, the approximate range separating typical molecular cloud cores from nonisothermal protostellar fragments. The constraint that λ_J be resolved is termed the Jeans condition by *Truelove et al.* and they were the first to introduce this crucial condition into finitie difference simulations. The Jeans condition arises because perturbations on scales above λ_J are physically unstable, and discretization of the GHD PDEs introduces perturbations on all scales above Δx. It is essential to keep the λ_J as resolved as possible in order to diminish the initial amplitude of perturbations that exceed this scale. Although it has been shown to hold only for isothermal evolution, it is reasonable to expect that it is necessary (although not necessarily sufficient) for nonisother-

mal collapse as well where the transition to non–isothermal evolution may produce structure on smaller scales than the local Jeans length. .

As a side effect of confining cell-sized perturbations to a length scale at which they are thermally damped, resolution of λ_J also ensures that gradients developed in isothermal flow by gravity are well resolved. Formation of structure on scales of λ_J and larger is a general feature of isothermal GHD flow since smaller fluctuations are damped but larger ones collapse. Lack of resolution of gradients within simulated flow triggers the injection of artificial viscosity. Introducing excess amounts of artificial viscosity renders the problem solved different from the inviscid problem posed. Continuous resolution of λ_J, however, keeps the flow inviscid and prevents artificial slowing of gravitational collapse.

Runaway Collapse The self-gravitational collapse in nearly spherical (i.e., three-dimensional) geometry tends to show a so-called "runaway collapse," where the denser central region collapse much faster than the less-dense surrounding region, decreasing the mass of that faster collapsing central region. The reason for this "runaway" can be found even in the simple formula of the free-fall time of a spherical object, $t_{\rm ff} \equiv \sqrt{\frac{3\pi}{32G\rho}}$, where ρ should be regarded as the mean density. The collapse timescale of the dense cental region is shorter than that of less-dense envelope, which makes the density profile more centrally peaked. A typical evolution of density profile can be seen in a well-known self-similar solution (*Larson* 1969, *Penston* 1970). The mass of the central fast collapsing region always stays on the order of Jeans mass $M_J = \rho\lambda_J^3 \sim G^{-3/2}C_s^{3/2}\rho^{-1/2}$, which decreases monotonically in this runaway stage. Thus, the description of this phenomena requires increasingly higher resolution, not only on spatial scale but also on the mass scale. Obviously, an accurate description is not gurranteed even in SPH-like methods that is based on Lagrangian particles.

The end of runaway stage corresponds to the deceleration of gravitational collapse. The pressure force and gravity force (per unit mass) scale with the radius of *homologously* collapsing sphere as $F_P = -\frac{1}{\rho}\frac{\partial P}{\partial r} \propto r^{-3\gamma+2}$ $F_G = \frac{GM}{r^2} \propto r^{-2}$, where γ is the effective specific heat ($P \propto \rho^\gamma$). If γ becomes larger than $\gamma_{\rm crit} = 4/3$, the increased pressure can decelerate the gravitational collapse.

3.a.3 Magnetic Fields

The magnetic field (B-field) increases the core critical mass for gravitational collapse, as it provides additional support besides thermal pressure. The maximum stable mass is the Bonnot–Ebert mass, $M_{\rm BE} = 1.18c_s^4/G^{3/2}p_{\rm ext}^{1/2}$, assuming thermal pressure support (*Bonnor* 1956; *Ebert* 1957), while it is proportional to the magnetic flux, $M_{mag} \sim \Phi_B/2\pi G^{1/2}$, assuming magnetic support. Detailed self–consistent calculations accounting for both thermal and magnetic support (*Mouschovias & Spitzer* 1976; *Tomisaka et al.* 1988) show that the maximum stable mass can be expressed as $M_{\rm mag,max} \sim$ $M_{\rm BE}\left\{1 - \left[0.17/(G^{1/2}M/\Phi)_c\right]^2\right\}^{-3/2}$, where $(M/\Phi)_c$ is the central mass–to–flux ratio. A similar formula was proposed by McKee (1989), $M_{\rm mag,max} \sim M_{\rm BE} + \Phi_B/2\pi G^{1/2}$.

Further support is provided by rotation. For a core with specific angular momentum j, the maximum stable mass is given by $M_{\rm max} \sim \left[M_{\rm mag,max}^2 + (4.8c_s j/G)^2\right]^{1/2}$ (*Tomisaka et al.* 1989). The dynamical runaway collapse begins when the core mass exceeds this maximum stable mass (magnetically supercritical cloud). Quasi–static equilibrium configurations exist for cores less massive than the maximum stable mass. The evolution of these subcritical cores is controlled by the processes of ambipolar diffusion and magnetic braking, both of which have longer timescales than the gravitational free fall.

The ionization fraction in molecular clouds is very low, because of their high opacity to ionizing photons, and controlled primarily by cosmic rays. The predominantly neutral gas is coupled to the B-field only indirectly, through friction with ions and charged dust grains (see *Nakano* 1984). This indirect coupling is the reason for the process of ambipolar diffusion, that is the relative motion between the neutral gas and the charged ions and grains attached to the B-field. Due to the interplay of pressure and gravitational forces, magnetic flux and charged components can gradually escape the central region of a dense core, thus increasing the mass–to–flux ratio and decreasing the maximum stable mass. Ambipolar diffusion can therefore turn a magnetically subcritical core into an unstable supercritical one.

The frictional force is proportional to the ion–neutral drift velocity, $v_{\rm D}$, $F = \rho v_{\rm D}\tau_{ni}^{-1}$, where τ_{ni} is the mean collision time of a neutral particle with ions. While in a static configuration the gravitational force is balanced by magnetic forces (besides thermal pressure), in a configuration contracting under the effect of ambipolar drift one may assume that the gravitational force is roughly balanced by the frictional force. With this assumption, the timescale for magnetic flux loss from a quasi–static high–density core with a scale–height of R is given by $\tau_{\rm P} \sim R/v_{\rm D} \sim R/[F\tau_{ni}/\rho] \sim \tau_{\rm ff}^2/\tau_{ni}$, where $\tau_{\rm ff}$ is the free–fall time (*Mouschovias* 1989; *Tomisaka et al.* 1990). Thus, the ambipolar diffusion time scale is much longer than the free–fall time scale: $\tau_{\rm P} \sim 15.7(G\rho_c)^{-1/2} \sim 43\,{\rm Myr}(\rho_c/2 \times 10^{-21}{\rm g\,cm^{-3}})^{-1/2}$, where the density of ions is assumed to be $\propto \rho^{1/2}$. As the core contracts, the density grows and, when $n \gtrsim 10^{12}{\rm cm^{-3}}$, the magnetic field is effectively decoupled from the gas. At these densities, Joule dissipation becomes important and particle drifts are qualitatively different from ambipolar diffusion (*Nakano et al.* 2002).

The B-field is also responsible for transfer of angular momentum in magnetized rotating cores, by a process called magnetic braking. Magnetic breaking is caused by the azimuthal component of the Lorentz force $(\vec{j} \times \vec{B})_\phi$. In the case of a core rotating in a static

medium with uniform density, $\rho_{\rm amb}$, and uniform B-field, $B_{\rm amb}$, a torsional Alfvén wave transfers angular momentum from the core to the ambient medium. The magnetic braking timescale of a core with column density $\Sigma_{\rm core}$ is given by $\tau \sim (4\pi/\rho_{\rm amb})^{1/2}\Sigma_{\rm core}/B_{\rm amb} \sim 1{\rm MGyr}(M/5M_\odot)(R/0.3{\rm pc})^{-2}(\rho_{\rm amb}/10^{-22}{\rm g\,cm}^{-3})^{-1/2}(B_{\rm amb}/30\mu{\rm G})^{-1}$ (*Mouschovias & Paleologou* 1980). Since the toroidal B-field is amplified from the poloidal one in the rotating core, a strong rotational motion induces an effective magnetic braking and angular momentum transfer. In the evolution of subcritical cores, the magnetic braking is important during the quasi–static contraction phase controlled by the ambipolar diffusion (*Basu and Mouschovias* 1994). In the dynamical runaway collapse, the rotational speed is smaller than the inflow speed (*Tomisaka* 2000)

3.a.4 Outflows

The B-field plays crucial roles in the star formation process, one of which is generating an outflow. Magnetohydrodynamical simulations of the contraction of the molecular cores (*Tomisaka* 1998, 2000, 2002; *Allen et al.* 2003; *Banerjee & Pudritz* 2005) have shown that after the formation of the first core the gas rotates around the core, the toroidal B-field is induced and thus the magnetic torque transfers the angular momentum from the disk-midplane to the surface. The gas which has received enough angular momentum compared with the gravity is ejected by the excess centrifugal force, which is called "magnetocentrifugal wind mechanism" (*Blandford & Payne* 1982), along the magnetic field line squeezed by the laterally contracting disk. In the case of weak B-field, magnetic pressure gradient of the toroidal B-field accelerates the gas and the outflow forms in the perpendicular direction to the disk. The axisymmetric 2D simulations show that (1) at least 10% of the accreted mass is ejected; (2) angular momentum has reduced a factor 10^{-4} from that of the parent cloud at the age of $\simeq 7000{\rm yr}$ from the core formation. This resolves the problem that the excess angular momentum should be removed in the course of star formation (*Tomisaka* 2000). In 7000 yr from the first core formation, mass of the core reaches $\sim 0.1M_\odot$ and the outflow extends to a distance from the core of $\simeq 2000{\rm AU}$ with a speed of $\sim 2{\rm km\,s}^{-1}$ (*Tomisaka* 2002). If the accretion continues and the core mass grows to one solar mass, the outflow expands and its speed is accelerated further. After the first core is fragmented into binary or multiple cores, each fragment spins and multiple outflows are ejected (*Machida et al.* 2004; *Ziegler* 2005).

3.a.5 Thermal Budget

Low Density Regime In the low density regime the gas temperature can be determined by equating gas cooling and heating rates (e.g. *Juvela, Padoan & Nordlund*, 2001). The heating is usually assumed to be from cosmic rays, but ambipolar drift heating in turbulent clouds can also contribute significantly (*Padoan, Zweibel & Nordlund*, 2000). Above a gas density of 10^4–10^5 cm^{-3}, gas and dust grains are thermally coupled. The thermal balance of dust grains can be described by the equation $4\pi\kappa\rho I + \Gamma_{\rm g} = 4\pi\kappa\rho\sigma_{\rm SB}T^4 \equiv \Lambda_{\rm thin}$, where κ, I, $\Gamma_{\rm g}$ denote the opacity, the intensity of interstellar radiation field, and the heating rate due to collision with gas respectively. With typical ISM parameters, the above equation gives $T \sim 10$ K. The strong temperature sensitivity of the cooling rate makes the dust grains (hence the gas) nearly isothermal. During the gravitational collapse, gas and dust are isothermal until a density of 10^{10}–10^{11} cm^{-3}, when the compressional heating rate becomes larger than the cooling rate.

High Density Regime The further evolution of a collapsing core and the formation of a protostar are radiation–hydrodynamical (RHD) processes that should be modeled by solving radiation transfer and hydrodynamics simultaneously in multi dimensions. The most sophisticated multi–dimensional models are based on the (flux–limited) diffusion approximation (*Bodenheimer et al.* 1990, *Krumholtz et al.* 2005).

Based on results of detailed radiative transfer computations in *spherical* symmetry (*Larson*, 1969; *Winkler & Newman* 1980; *Masunaga et al.*, 1998; *Masunaga & Inutsuka* 2000a,b) we can outline the following stages. Once the compressional heating dominates the radiative cooling, the central temperature increases gradually from the initial value of ~ 10 K. The initial slope of the temperature as a function of gas density corresponds to an effective ratio of specific heats $\gamma = 5/3$: $T(\rho) \propto \rho^{2/3}$ for $10{\rm K} < T < 100{\rm K}$. This monatomic gas property is due to the fact that the rotational degree of freedom of molecular hydrogen is not excited in this low temperaure regime: e.g., $E(J = 2 - 0)/k_{\rm B} = 512K$. When the temperature becomes larger than $\sim 10^2 K$, the slope corresponds to $\gamma = 7/5$, as for diatomic molecules. This value of γ is larger than the minimum required for thermal pressure support against gravitational collapse: $\gamma > \gamma_{\rm crit} \equiv 4/3$. The collapse is therefore decelerated and a shock is formed at the surface of a quasi–adiabatic core, called "the first core". Its radius is about 1 AU in spherical symmetric models, but can be significantly larger in more realistic multi–dimensional models. It consists mainly of H_2. The increase of density and temperature inside the first core is slow but monotonic. When the temperature becomes $> 10^3 K$, the dessociation of H_2 starts. The binding energy of H_2 is about 4.5 eV, much larger than the thermal energy per hydrogen molecule in this temperature regime. Therefore the dessociation of H_2 acts as an efficient cooling of the gas, which makes $\gamma < 4/3$, triggering the second dynamical collapse. In this second collapse phase, the collapsing velocity becomes very large and engulfs the first core. As a result, the first core lasts only for $\sim 10^3$ years. In the course of the second collapse, the central density attains the stellar value, $\rho \sim 1{\rm g/cc}$, and the second adiabatic core, or "protostar", is formed. The time evolution of the SED obtained from the self–consistent RHD calculation can be found in *Masunaga & Inutsuka* (2000a).

Core Fragmentation Stability analysis of rotating poly-

tropic gas shows that gas with the rotation-to-gravitational energy ratio $T/|W| > 0.27$ is unstable for non-axisymmetric perturbations (for example, *Imamura et al.* 2000). Assuming a barotropic equation of state representing the thermal energy budget, nested grid hydro simulations (*Matsumoto & Hanawa* 2003) show that the first-core disk increases in $T/|W|$ by mass accretion. If the first-core disk rotates fast enough as the angular speed $\times$ free-fall time exceeds $\Omega_c(4\pi G\rho_c)^{1/2} \gtrsim (0.2 - 0.3)$, fragments appear and grow into binaries and multiples in the first core phase. The non-axisymmetric nonlinear spiral pattern can transfer the angular momentum of the accreting gas.

B-field affects the rotation motion (magnetic braking) and thus fragmentation. Whether B-field stabilizes the first core against the fragmentation or not (*Machida et al.* 2005b; *Ziegler* 2005) is attracting attention in relation to the binary formation. The runaway collapse conserves Ω-to-B ratio. (As long as the core contracts in a spherical fashion, conservations of angular momentum and magnetic flux require Ω_c and $\vec{B}_c$ to increase $\propto \rho_c^{2/3}$. Both Ω and $\vec{B}$ increase proportional to the column density, after a disk is formed perpendicular to Ω and $\vec{B}$ is contracting laterally. Thus, the runaway collapse conserves Ω-to-B ratio.) In order to achieve enough rotation to fragment, the initial Ω-to-B ratio must satisfy the condition $(\Omega/B)_{init} > 0.39G^{1/2}c_s \sim 1.7 \times 10^{-7}(c_s/0.19\mathrm{km\,s^{-1}})^{-1}\mu\mathrm{G}^{-1}$, (*Machida et al.* 2005b). Simulations show that increasing the B-field strength, fragmentation is stabilized by the suppression of rotation motion by the magnetic braking. *Boss* (2002) predicted contrarily. However, it should be noted that this comes from the fact that his model equation is not fully consistent with MHD and magnetic braking is not taken into account.

3.a.6 Radiative Transfer in Multi–Dimensions

Radiation transport has been shown to play a significant role in the outcome of fragmentation to binary and multiple systems for low mass star formation (*Boss et al.*, 2000) as well as determining the limiting mass in high mass star formation in both 2D (*Yorke & Sonnhalter*, 2002); and more recently in the first 3D simulations (*Krumholz et al.*,2005). The strong dependence of the evolution of the isothermal and nonisothermal cloud models on the handling of the cloud's thermodynamics implies that collapse calculations will need to treat the thermodynamics accurately in order to obtain the correct solution (*Boss et al.*, 2000). Because of the great computational burden imposed by solving the mean intensity equation in the Eddington approximation, which increases the computational time by a factor of 10 (or more) during the latter phases of collapse, it is tempting to sidestep the Eddington approximation solution altogether and employ a simple barotropic prescription designed to mimic the behavior of the Eddington approximation (e.g., *Boss*, 1981; *Bonnell*, 1994; *Bonnell and Bate*, 1994a,; *Burkert et al.*, 1997; *Klein et al.*, 1999). *Boss et al.* (2000) have demonstrated this is problematical. By making detailed comparisons of cloud collapse and fragmenta-

tion of Gaussian peaked clouds with both the barotropic approximation and an implementation of radiation transport in the Eddington approximation, they show that the outcome of such a collapse can be a single star in the former approximation and a binary or triple system in the latter. The differences occur because the barotropic approximation (a stiffened equation of state) assumes that the coupled self-gravitating, radiation–hydrodynamical equations can be closed by specifying the thermal properties of the soley as a function of density; that is the specific entropy of the gas depends strictly on the density. In reality, the specific entropy of the a gas particle depends on the thermal history of the parcel, so that while the temperature field tends to follow the density field, it is not an exact correspsondance and the temperature in reality is determined not only by adiabatic compression, but by compressional heating in a 3D volume with a strongly varying optical depth. Thus in reality, the temperature dependence cannot be represented with a simple barotropic approximation with any great accuracy and radiative transfer must be used. We discuss the various methods of radiation transport in section 3.b6.

3.b Methodology of Numerical Simulations

3.b.1 Complexity of the Problem of Low Mass Star Formation

The overriding grand computational challenge for simulations to develop a predictive theory of low mass star formation is that star formation occurs in clouds as a result of the force of gravity over a huge dynamic range of spatial scales, with different physical mechanisms playing different degrees of importance on varying scales. The gas densities in these clouds themselves vary over many orders of magnitude. Gravity, turbulence, radiation and magnetic fields all contribute to the delicate balance of forces that result in gravitational collapse and fragmentation of clouds, clumps and cores to produce stars. Thus the problem becomes one of treating multi–scale, multi–physics where the physics is highly coupled over the large range of scale. To develop a feel for the range of scale a simulation must trarverse, we can consider the internal structure of GMCs as hierarchical, consisting of smaller subunits within larger ones (*Elmegreen et al.*, 2000). GMCs vary in size from 20–100 pc., in density from 50–100 H_2 cm^{-3} and mass 10^4–10^6 M$_\odot$. Self–gravity and turbulence are equally important in controlling the structure and evolution of these clouds. Magnetic fields are likely to play and important role as well (*Heiles et al.*, 1993; *McKee et al.*, 1993). *Cruthcher* (1999) has suggested that there may be an equipartition between the magnetic energy and the turbulent energy in molecular clouds implying that a combination of static magnetic fields and MHD turbulence might together support the clouds against gravity. Embedded within the GMCs are dense clumps which may form clusters of stars. These clumps are few pc. in size, masses of a few thousand M$_\odot$ and mean densities $\sim 10^3$ H_2 cm^{-3}. The clumps

have dense core substructures with radii ~ 0.1pc., densities 10^4–10^6 H_2 cm^{-3} and a masses that range from 1 to several $M_\odot$. These cores likely form individual stars or low order multiple systems. Turbulence and magnetic fields are likely to play important roles alongside gravity in fragmenting molecular clouds into the observed star-forming cores. The role of MHD turbulence has been investigated by numerical simulation in 2D and 3D (*Ostriker et al.*, 1999; *Vasquez–Semadeni et al.*, 2000; *Ballesteros–Paredes*, 2003; *Mac Low and Klessen*, 2003; *Nordland and Padoan*, 2003). Using a computational domain that starts from dense turbulent clumps within GMCs (R $\sim$ 2 pc.) and evolves the simulation through the isothermal core formation and collapse phase into the regime where density peaks become opaque to the thermal radiation from the dust grains at densities 2×10^{10} H_2 cm^{-3} and through to the formation of the first hydrostatic core at densities of 10^{13} H_2 cm^{-3} requires an accurate calculation across 10 orders of magnitude of density and 4–5 orders of magnitude on spatial scale. This would require a resolution of about 10 AU, enough to resolve 100 AU separation binaries. To follow the collapse all the way to an actual star would require a further 10 order of magnitude increase in density and 2–3 orders of magnitude further spatial collapse. Clearly such extraordinary computational demands rule out fixed grid simulations entirely and require highly accurate dynamic regriddung strategies or SPH approaches that can maintain the resoltuion required by the numerical Jeans criterion (*Truelove et a.*, 1997) throughout space for all time.

3.b.2 Smooth Particle Hydrodynamics

The description of the gravitational collapse requires a large dynamic range of spatial resolution, and thus, one of the efficient ways for this is to use Lagrangian methods. Smoothed particle hydrodynamics (SPH) is a fully Lagrangian particle method designed for describing compressible flud dynamics. This method is economical in handling hydrodynamic problems that have large (almost) empty regions. A variety of astrophysical problems including star formation (see, e.g., *Goodwin et al.* and *Whitworth et al.* in this volume) have been studied by SPH because of its simplicity in programming two- and three-dimensional codes and its versatility of incorporating various physical effects, such as self-gravity, radiative cooling, and chemical reactions. A broad discussion of the method can be found in a review by *Monaghan* (1994). In order to further increase the dynamic range of spatial resolution, *Kitsionas and Whitworth* (2002) introduced particle splitting, which is an adaptive approach in SPH.

The "standard" SPH formalism adopted artificial viscosity that mimics the classical von-Neumann Richtmeyer viscosity, so that it tends to give poor performance in describing strong shocks. In the two- or three-dimensional calculation of colliding gases, standard-SPH particles often penetrate to the opposite side. This unphysical effect can be partially eliminated by the so-called XSPH prescription *Monaghan* (1989) which does not introduce the (required)

additional *dissipation* but results in additional *dispersion* of the waves. As a more efficient method for handling strong shocks in the SPH framework, so-called "Godunov SPH" is proposed by *Inutsuka* (2002) who implemented the exact Riemann solver in the strictly conservative particle method, and used in the collapse and fragmentation of self-graviatating objects (*Tsuribe and Inutsuka* 1999; *Cha and Whitworth* 2003a,b).

The implementation of self-gravity in SPH is relatively easy, and one can use various acceleration methods, such as *Tree-Codes*, and special purpose processors (e.g., GRAPE board). The flux-limited diffusion radiative transfer is incorporated in SPH by *Whitehouse and Bate* (2004) and *Bastien, Cha, and Viau* (2004).

Some of the authors are now using "sink particles" to follow the subseqent evolution even after protostars are formed (see *Krumholtz et al.*, 2004 for the corresponding technique in Eulerian grid-based method). This is a prescription to continue the calculations without resolving the phenomera around the stellar objects that have extremely short timescales. The validity of various treatment of the sink particles are discussed in literature, but seems to remain controversial.

3.b.3 Fixed grid Hydrodynamics

Since the time of its introduction, the numerical code of choice for supersonic hydrodynamic turbulence has been the Piecewise Parabolic Method[1] (PPM) of Colella and Woodward (1984). PPM is based on a Rieman solver (the discretized approximation to the solution is locally advanced analytically) with a third order accurate reconstruction scheme, which allows an accurate and stable treatment of strong shocks, while maintaining numerical viscosity to a minimum away from discontinuities. Because the physical viscosity is neglected (PPM solves the Euler equation), large scale PPM flows are characterized by a very large effective Reynolds number (Porter and Woodward 1994). Direct numerical simulations (DNS) of the Navier–Stokes equation, where the physical viscosity is explicitly computed, require a linear numerical resolution four times larger than PPM to achieve the same wave–number extension of the inertial range of turbulence as PPM (Sytine et al. 2000). From this point of view, therefore, PPM has a significant advantage over DNS codes, which are generally designed for incompressible turbulence anyway, and hence of limited use for simulations of the ISM. Versions of the PPM scheme have been incorporated in community codes such as ENZO[2] and FLASH[3].

Codes based on straightforward finite difference methods, rather than Rieman solvers, have also been used in applications to star formation and interstellar turbulence, such as the Zeus code[4] (Stone and Norman 1992a,b) and

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[1] http://www.lcse.umn.edu/

[2] http://cosmos.ucsd.edu/enzo/

[3] http://flash.uchicago.edu/website/home/

[4] http://cosmos.ucsd.edu/

the Stagger Code[5] (Nordlund and Galsgaard 1995). Finite difference codes address the problem of supersonic turbulence with the introduction of localized numerical viscosity to stabilize the shocks while keeping viscosity as low as possible away from shocks. The main advantages of this type of codes, compared with Rieman solvers, are their flexibility in incorporating new physics and their computational efficiency.

Fixed–grid codes cannot achieve the dynamical range required for problems involving the gravitational collapse of protostellar cores. To address these problems numerical methods used for fixed–grid codes must be generalized into AMR schemes.

3.b.4 Adaptive Mesh Refinement Hydrodynamics and Nested Grids

Self-gravitational hydrodynamics involves the collapse and fragmentation of an unstable cloud into smaller condensations. Fragmentation is essential to understanding the formation of stars and galaxies, yet gaseous flows undergoing frgamentation naturally involve a substantial three-dimensional variation in length and density scale. This enormous dynamic range presents a formidable obstacle to obtaining an accurate numerical solution, as the flow must remain well-resolved throughout the evolution. The resolution required after considerable collapse is well in excess of that needed initially. Dynamic range in scale of 10^4 and density of 10^9 or more is not unusual. Fixed-resolution methods cannot be used to simulate such a 3-D collapse in a practical amount of time using current computers. The adaptive mesh refinement (AMR) scheme utilizes underlying rectangular grids at different levels of resolution. Linear resolution varies by integral refinement factors—usually 4—between levels, and a given grid is always fully contained within one at the next coarser level (excluding the coarsest grid). The origin of the method stems from the seminal work of *Berger and Oliger* (1984) and *Berger and Collela* (1989). The AMR method can employ multiple spatially unconnected grids at a given level of refinement. Most importantly, the AMR method dynamically resizes and repositions these grids and inserts new, finer ones within them according to adjustable refinement criteria such as the numerical Jean condition (*Truelove et al.*, 1997). Fine grids are automatically removed as flow conditions require less resolution. During the course of the calculation, some pointwise measure of the error is computed at frequent intervals – typically every other time step. At those times, the cells that are identified are covered by a relatively small number of rectangular patches, which are refined by some even integer factor. Refinement is in both time and space, so that the calculation on the refined grids is computed at the same CFL number as that on the coarse grid. This procedure is applied recursively. The overall algorithm is fully conservative: the finite difference approximations on each level are in conservation form, as is the

coupling at the interface between grids at different levels of refinement. AMR has three substantial advanatges over standard SPH. Combined with high order Godunov methods, AMR acheives a much higher resolution of shocks. This is important in obtaining accuracy in supersonic turbulent flows in star forming clumps and cores and in accretion shocks onto forming protostars. AMR allows high resolution at *all* points in the flow as dictated by the physics. Unlike SPH, whereby particles are taken from low mass regions to follow high mass regions thereby deteriorating the accuracy elsewere else in the domain, AMR maintains high order accuracy everwhere. AMR has as its basis, fixed Eulerian grids and thus can take advantage of sophisticated algorithms to incoporate magnetic fields and radiative transfer. This is far more difficult in a particle based scheme. AMR was first introduced into astrophysics by *Klein et al.* (1990, 1994) and has been used extensively both in low mass and high mass star formation simulations (*Truelove et al.*, 1998; *Klein*, 1999; *Klein et al.*, 2000; *Klein et al.*, 2003; *Klein*, 2004; *Krumholz et al.*, 2005.

There are several ways to implement AMR. They can broadly be divided into two categories: Meshes with fixed number of cells, such as in lagrangian or rezoning approaches, and meshes with variable number of cells, such as unstructured finite elements, structured cell–by–cell and structured sub–grid blocks. For various reasons the most widely adopted approaches in astrophysics are structured sub–grid blocks and cell–by–cell. The first was developed by *Berger & Oliger* (1984) and *Berger & Collela* (1989) and uses variable sized patches of refined regions. It is used in the AMR code developed by Klein and collaborators (*Klein*, 1999; *Crockett et al.*, 2005) and in the community code ENZO *Bryan & Norman* 2000). The cell–by–cell approach is based on the refinement of individual mesh cells into 8 adjacent child cells of half the size of the parent cell (a so called oct–tree in 3D), such as in PARAMESH (Mac-Neice et al., 2000), which is used in the community code Flash (*Banerjee et al.* 2004). A hybrid approach used in the code NIRVANA (*Ziegler* 2005) uses refinement blocks of fixed size (4^3 in 3D) and is again based on an oct–tree data structure as in PARAMESH. The cell–by–cell method has the advantages of flexible and efficient refinement patterns and low memory overhead and the disadvantages of expensive interpolation and derivation formulae and large tree data structures. The sub–grid block method is more efficiency and more suitable for shock capturing schemes than the cell–by–cell method, at the price of some memory overhead.

Finally, nested grids consisting of concentric hierarchical rectangular subgrids can also be very effective for problems of well defined geometry (*Yorke et al.* 1993). These methods are suitable for tracing the non–homologous runaway collapse of an initially symmetrical cloud in which the coordinates of a future dense region are known in advance (*Tomisaka* 1998). The finest subgrid is added dynamically when spatial resolution is needed as in AMR methods.

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[5]www.astro.ku.dk/StaggerCode/

3.b.5 Approaches for Magneto–Hydrodynamics

Since strong shocks often appear in the astrophysical phenomena, a shock–capturing scheme is needed also in MHD. Upwind schemes based on the Riemann solver are used as the MHD engine. Schemes well known in hydrosimulation, such as Roe's approximate Riemann solver (*Brio & Wu* 1988; *Ryu & Jones* 1995; *Nakajima & Hanawa* 1996), piecewise parabolic method (PPM; *Dai & Woodward* 1994), are also applicable to MHD.

Divergence-free B-field: Special attention should be paid to guarantee $\mathrm{div}\vec{B} = 0$ in MHD simulations. To ensure that the divergence of Maxwell stress tensor $T_{ij} = -(1/4\pi)B_i B_j + (1/8\pi)B^2\delta_{ij}$ gives the Lorentz force, the first term of right-hand side $\partial_j(B_i B_j)$ must equal to $B_j\partial_j B_i$. This requires $B_i\partial_j B_j = 0$ and means that a fictitious force will appear along the B-field if the condition of divergence-free is broken. Divergence of B-field amplifies the instability of the solution even for a linear wave. Thus, it is necessary for the MHD scheme to keep the divergence of B-field zero within a round-off error or at least small enough. This divergence-free nature should be satisfied for the boundaries of subgrids in AMR and nested grid schemes.

One realization has been done by "constrained transport (CT)" (*Evans & Hawley* 1988), in which the staggered collocation of the components of B-field on the cell faces makes the numerical divergence vanish exactly. In the staggered collocation, the electric field $-v \times \vec{B}$ of the induction equation $\partial_t\vec{B} = \nabla \times (\vec{v} \times B)$ is evaluated on the edge of the cell-face and the line integral along the edge gives the time difference of a component of the B-field. Note that the electric field on one edge appears twice to complete the induction equation. To vanish the divergence of B-field, CT requires the two evaluations to coincide each other. To utilize the Godunov-type Riemann solver along the context of CT, *Balsara & Spicer* (1999) proposed a scheme as follows: (1) face-centered B-field is interpolated to the cell center; (2) from the cell-centered variables, numerical flux at the cell face is obtained using a Riemann solver; (3) the flux is interpolated to the edge of the cell-face and the electric field in the induction equation is obtained; (4) new face-centered B-field is obtained from the induction equation. Variants of this method are widely used [see also *Ryu et al.* (1998) and *Ziegler* (2004)].

Avoiding staggered collocation of B-field requires divergence cleaning. In this case, divergence cleaning is realized by replacing the B-field every step as $\vec{B}^{new} = \vec{B} - \vec{\nabla}\Phi$, where $\vec{\nabla}^2\Phi = \mathrm{div}\vec{B}$ (Hodge projection), or by solving a diffusion equation for div $\vec{B}$ as $\partial_t\vec{B} = \eta\nabla(\nabla \cdot \vec{B})$. The former is combined with pure Godunov-type Riemann solvers using only cell-centered variables (*Ryu et al.* 1995). *Crockett et al.* (2004) reported that the divergence cleaning of the face-centered B-field appearing in the numerical flux based on an unsplit, cell-centered Godunov scheme improves its accuracy and stability.

Powell et al. (1999) proposed a different formalism, in which div $\vec{B}$ term is kept in the MHD equations as a source (e.g. the Lorents force $(\nabla \times \vec{B}) \times \vec{B}/4\pi$ gives an extra term related to div $\vec{B}$ as $-\vec{B}\nabla \cdot \vec{B}$ beside the Maxwell stress tensor term $-T_{ij}$, in the equation for momentum density.) In this formalism, div $\vec{B}$ is not amplified but advected along the flow.

Comparison between various methods is found in *Tóth* (2000), *Dedner et al.* (2002), *Balsara & Kim* (2004), and *Crocket et al.* (2004).

There have been attempts to solve the induction equation with SPH methods (e.g. *Stellingwerf & Peterkin*, 1994; *Byleveld & Pongracic*, 1996; *Price & Monaghan*, 2003, 2005). A major obstacle is an instability that develops when the momentum and energy equations are written in conservation form. As a result, the equations must be written in a way that does not conserve momentum (*Phillips & Monaghan*, 1985; *Morris*, 1996), which is a major concern for the accurate treatment of shocks. Results of recent tests of the state–of–the–art SPH MHD code (*Price & Monaghan*, 2005) appear to be rather poor even for very mild shocks.

3.b.6 Approaches for Radiation Transport

Several levels of improvement can be made over replacing a barotropic EOS with a formulation of the radiation transport but each level comes with increased cost and complexity. Excellent discussions of the details of these methods may be found in *Mihalas and Mihalas* (1984) and *Castor* (2004). Here we will briefly describe the methods.

The simplest improvement of radiation transfer beyond a barotropic stiffened EOS is the diffusion approximation which pertains to the limit in which radiation can be treated as an ideal fluid with small corrections. The approximation holds when the photon mean free path is small compared with other length scales. The combined energy equation for the gas and radiation results in an implicit non-linear diffusion equation for the temperature. The weakness of the diffusion approximation is that it is strictly applicable to optically thick regimes and performs poorly in optically thin regions. This can be severe in optically thin regions of an inhomogeneous turbulent core or in the optically thin atmosphere surrounding a developing protostar. The next level of approximation is the Eddington approximation (*Boss and Myhill*, 1992;*Boss et al.*, 2000). It can be shown that the diffusion approximation leads directly to Eddington's approximation $P_\nu = \frac{1}{3}E_\nu I$ where P_ν is the pressure tensor moment of the specific intensity of radiation, E_ν is the scalar energy density of radiation an I is the isotropic identity tensor. This approximation, coupled with dropping the time dependent term in the 2nd moment equation of transfer results in a combined parabolic 2nd order time dependent diffusion equation for the energy density of the radiation field. This formulation of the Eddington appoximation is used in *Boss et al.* (2000). The approximation results in a loss of the finite propagation speed of light c and a loss of the radiation momentum density, thus there is an error in the total momentum budget. In optically thin regions, the radiation flux can increase without limit. An alternative approach that

modifies the Eddington approximation, compensates for the errors made in dropping the time dependent flux term by including a correction factor in the diffusion coefficient for the radiation flux. This correction factor, called a flux limiter, is in general a tensor and has the property that the flux goes to the diffusion limit at large optical depth and it correctly limits the flux to be no larger than cE in the optically thin regime. This improvement over the Eddington approximation has been used by *Klein et al.* (2004) for the simulation of both low mass and high mass star formation. The resulting sparse matrices introduced by the diffusion like terms are solved by multi-grid iterative methods in and AMR framework. Both the Eddington approximation and the flux-limiting correction make errors on the order of 20%. The next level of approximation, the variable Eddington tensor method, removes all of the inaccuracies of the Eddington approximation and the flux limiter modification. It was first formulated in multi-dimensions for astrophysical problems by Dykema et al. (1996). In essence, if the precise ratio of the pressure tensor to the energy density were included as an ad hoc multiplier in the Eddington approximation equations they would represent an exact closure of the system. The tensor ratio is obtained iteratively from either an auxialliary solution of the exact transport equation for the specific intensity or using an approximate analytic representation of the tensor. This method holds promise for future simulations but has yet to be used in star formation. The final two approaches which are highly accurate and deal with the angle dependent transport equation directly are S_N methods and Monte Carlo methods. They have not yet been developed for simulations in star formation because the cost in 3D is prohibitive. The S_N method is a short characteristic method in which a bundle of rays is created at every mesh point and are extended in the upwind direction only as far as the next spatial cell. The main problem is in finding the efficient angle set to represent the radiation field in 2 or 3 dimensions *Castor* (2004). Finally, one might consider MonteCarlo methods to solve the transport equation. Although simple to implement (its great advantage), this method suffers from needing a vast number of operations per timestep to get accurate statistics in following the particles used to track the radiation field.

Radiative transfer implementations have recently been developed also for SPH methods, based on the diffusion approximation (*Whitehouse, Bate and Monaghan*, 2005) or the Monte Carlo method (*Stamatello % Whitworth*, 2003,2005).

3.b.7 In–Depth Contrast of Various Methods

Based on the physical processes and numerical methodologies discussed in the previous sections we can compare numerical schemes according to their ability to handle the following problems both accurately and efficiently: (a) turbulence, (b) strong socks, (c) self–gravity, (d) magnetic fields, (e) radiative transfer.

The standard SPH method has been successful with (c) and implementations of (e) have been recently developed in the diffusion approximation (*Whitehouse, Bate and Monaghan*, 2005) and with a Monte Carlo method (*Stamatello % Whitworth*, 2003,2005). It does not include (d), it is well known to be inadequate for (b) and has had virtually no applications to (a) to date. As any Lagrangian particle methods, SPH provides good resolution in high density regions, but very poor in low density ones, where a correct description of the velocity field is therefore compromised. The Godunov SPH method improves the standard SPH codes because of its ability to address (b), but does not provide a solution to (d) and is untested for (a) as well. Altough MHD is currently under development in SPH, results of standard MHD tests with a state–of–the–art code show the need for significant improvements even in the case of very mild shocks (*Price & Monaghan*, 2005). Current applications of SPH should therefore be limited to non–MHD problems and the accuracy and performance of SPH with turbulent flows must be thourughly tested.

In hydrodynamical problems, the state–of–the–art grid–based methods such as MUSCL (*van Leer* 1979) and PPM (*Colella and Woodward* 1984) use exact Riemann Solvers to construct the numerical fluxes and provide very accurate description in astrophysical flows with strong shocks (b). They have also been thourughly tested with compressible turbulent flows, particularly PPM, for which they are the undisputed methods of choice, and MHD versions have been developed that can address both (d) and (e). Traditional finite–difference grid–based schemes are still viable alternative though, because the best of them can also accurately address (a), (b), (d) and (e), at a lower cost of code development and computer resources. Point (c) can also be efficiently dealt with by grid–based codes thanks to AMR methods. However, the development of AMR schemes that satisfy (c) and at the same time (d) has begun only recently. These schemes exist and have been succesfully tested, but it is unclear at present which approach will provide the best trade off between accuracy and performance.

The constrained transport method appears to be the ideal one to guarantee the $\nabla \cdot B = 0$ condition. Various schemes have been proposed even in the category of Godunov–type methods with a linearized Riemann Solver. An exact MHD Riemann Solver would be more adequate to handle strong shocks, but it is not available yet. In MHD we have to solve seven characteristics even in one–dimensional problems, which hinders an efficient construction of numerical fluxes based on the non–linear waves. Furthermore, the discovery of the existence of the MHD intermediate shocks (*Brio and Wu* 1988) brings an additional difficulty in the categorization and prediction of the emerging non–linear waves. Among possible solutions, a linearized Riemann Solver with artificial viscosity may still be a useful option.

The Godunov MHD code of *Crockett et al* (2004) has been merged with the AMR RHD code of *Klein et al.* (2004) into the first fully devloped AMR magneto–radiation–hydrodynamic code to be used in simulations of star formation.

4. RESULTS OF NUMERICAL SIMULATIONS AND CONFRONTATION WITH THE OBSERVATIONS

4.a Fragmentation of molecular clouds into filaments and cores from the interaction of turbulence, gravity and magnetic fields

Simulations of supersonic turbulence generate complex density fields reminiscent of observed star forming clouds. Because random supersonic flows are directly observed in molecular clouds, there is little doubt that the observed density structure is related to the density enhancements found in numerical simulations of supersonic turbulence. A careful comparison between models and observations may even shed light on the relative importance of turbulence, gravity and magnetic fields.

Such a comparison was pioneered by Falgarone et al. (1994) and later continued by many others (e.g. Padoan et al. 1998,1999; Padoan et al. 2001; Ostriker, Stone and Gammie 2001; Ballestero-Paredes and Mac Low 2002; Ossenkopf 2002; Padoan et al. 2004; Gammie et al. 2003; Klessen et al. 2005). Important results have been obtained by these comparisons. However, with the exception of the various works by Padoan et al. and the work of Ossenkopf (2002), where post–processed three dimensional non–LTE radiative transfer calculations were carried out, all other studies are based on a superficial comparison between densities and velocities in the simulations and the observed quantities. Only detailed radiative transfer calculations on the results of numerical simulations can turn the observational data into a useful probe of the dynamical conditions in molecular clouds. For example, Padoan et al (1998) predicted the ratios of line width and intensities of different molecular transitions. By computing synthetic spectra of the J=1–0 transition of 13CO and comparing them with 13CO surveys of molecular clouds, Padoan et al (2001) found strong evidence that density enhancements in molecular clouds originate from turbulent shock compressions, like in the simulations, and not from gravitational instabilities. With similar radiative transfer calculations, but also involving the self–consistent calculation of the three–dimensional equilibrium temperature distribution, Padoan et al (2004) were able to accurately compare the projected density power spectra in numerical simulations and in observations of molecular cloud complexes. That work ruled out a large value of the mean magnetic field strength in the observed regions.

Important works that neglected radiative transfer effects include i) the study of velocity scaling, showing that molecular cloud turbulence is driven on large scale (e.g. Ossenkopf and Mac Low 2002; Heyer and Brunt 2004); ii) the study of core properties, showing that turbulent flows naturally generate dense cores with shapes, internal turbulence, rotation velocity and magnetic field strength consistent with the observations (e.g. Padoan et al. 1999; Gammie et al. 2003; Tilley and Pudritz 2004,2005; Li et al. 2004).

In principle the stellar IMF is the final outcome of the same fragmentation process responsible for the cloud structure and morphology. However, the comparison between the mass distribution of sink particles in SPH simulations of cloud fragmentation and the stellar IMF are not considered here as a tool to constrain the dynamical models of star–forming cloud. As explained in section III.a1, a description of the process of turbulent fragmentation accurate enough to correctly describe the initial conditions for the gravitational collapse is beyond the capability of present day SPH simulations that largely underestimate the effect of small scale turbulence.

4.b Collapse and Fragmentation of Molecular Cloud Cores into Low Mass Stars

As we have stated, over the past several years two dominant models of how stars form have emerged and these two camps can be described as gravitational collapse and competitive accretion. In both theories, a star initially forms when a gravitational bound gas collapses, but there is a crucial distinction between them as to what they predict will happen subsequently (*Krumholz et al.*, 2005a). In the gravitational collapse scenario, after a protostar has consumed or expelled all the gas in its initial core, it may continue accreting from the parent clump, however, it will not accrete enough to significantly alter its mass (*McKee and Tan*, 2003; *Padoan et al.*, 2005). Competitive accretion, in contrast, requires that the amount accreted after consuming the initial core be substantially larger than the protostar mass. Simply put, a clean way to look at this following *Krumholz et al.*, 2005a is to define $f_m \equiv \dot{m}_* t_{dyn}/\dot{m}_*$ as the fractional change in mass that a protostar of mass m_* undergoes each dynamical time t_{dyn} of its parent clump, starting after the initial core has been consumed by the accreting protostar. Gravitational theory suggests that $f_m \ll 1$, while competitive accretion *requires* $f_m \gg 1$. In recent work examining the plausibility of competitive accretion, *Krumholz et al.* 2005a considered two possible geometries; both spherical clumps and filaments. It is reasonable that these extremes bracket realistic star-forming clumps. In the first scenario they examined, they suppose that the gas the protostar is accreting is not accumulated into bound structures on scales smaller than the entire clump. For unbound gas, self–gravity may be neglected and the entire problem can be treated as Bondi-Hoyle accretion in a turbulent medium of non–self–gravitating gas onto a point particle. In a companion paper (*Krumholz et al.*, 2005b) they develop the theory for Bondi-Hoyle accretion in a turbulent medium and show that such accretion is bi–modal, at some points resembling classical Bondi-Hoyle flow, and in other cases being closer to the vorticity–dominated flows recently consdiered by *Krumholz et al.*, 2004. Using this newly developed theory they derive the accretion rate for such a turbulent medium and they confirm their theory with detailed, high resolution, converged AMR simulations. By using this accretion rate and the definitions of the virial parameter $\alpha_{\rm vir} \equiv M_{\rm vir}/M$

and the dynamical time $t_{\mathrm{dyn}} \equiv R/\sigma$ where σ is the velocity dispersion in the gas, they show that the accretion of unbound gas gives $f_{\mathrm{m-BH}} = (14.4, 3.08\frac{L}{R})\phi_{\mathrm{BH}}\alpha_{\mathrm{vir}}^{-2}(\frac{m_*}{M})$ for a (spherical, filamentary) star–forming region where ϕ_{BH} represents the effects of turbulence (*Krumholz et al.*, 2005a). From this it is clear that competitive accretion is most effective in low mass clumps with virial parameters $\alpha_{\mathrm{vir}} \ll 1$. They then examined the observed properties of a large range of star forming regions spanning both low mass and high mass stars and computed the properties for each region yielding α_{vir}, ϕ_{BH} and $f_{\mathrm{m-BH}}$. In virtually every region examined, the virial parameter $\alpha_{\mathrm{vir}} \geq 1$ and $f_{\mathrm{m-BH}} \ll 1$. Thus *none* of the star–forming regions are consistent with competitive accretion, but all are consistent with gravitational collapse. The Bondi-Hoyle rate is an upper limit on the accretion so that estimates for $f_{\mathrm{m-BH}}$ are the most favorable for competitive accretion and its true value is most likely even lower. As has been shown by *Bonnell et al.* (2001) in a study of competitive accretion in embedded stellar clusters, if the stars are sufiently close-packed, their tidal radii will be smaller than their Bondi-Hoyle radii and the accretioon will be lower. Furthermore, if one considers stars more massive than $\sim 10M_\odot$ radiation pressure will halt Bondi-Hoyle accretion onto the stars (*Edgar and Clarke*, 2004). In a second possible competitive accretion scenario *Krumholz et al.* (2005a) examined another way that a star could increase its mass by capturing and accreting other gravitationally bound cores. By analyzing the critical velocity below which any collision leads to a capture and above which it will not, and observing that cores within a molecular clump have roughly the same surface density as the parent clump (*Larson*, 191) they compute the escape velocity from the surface of a core in terms of the properties of the clump. This enables them to compute the amount of mass that a protostar can gain by capture of ther cores. This results in the calculation of $f_{\mathrm{m-cap}}$, the fractional change in mass that a protostar undergoes by capturing bound cores. The evaluation of $f_{\mathrm{m-cap}}$ depends on the fraction of the parent clump mass that is in bound cores. This is generally observed through surveys to be ~ 0.1 (*Motte et al.*, 1998; *Johnstone et al.*, 2001). As found with $f_{\mathrm{m-BH}}$, all the values are estimated to be three more orders of magnitude below unity. One can ask the question is there any region in parameter space for a cloud clump to have $f_{\mathrm{m}} = f_{\mathrm{m-BH}} + f_{\mathrm{m-cap}} \geq 1$. *Krumholz et al.* (2005a) find that $f_{\mathrm{m}} \geq 1$ only for $\alpha_{\mathrm{vir}}^2 M < 8.4M_\odot$. Observed star forming regions typically have $\alpha_{\mathrm{vir}} \approx 1$ and $M \approx 10^2 - 10^4 M_\odot$. Therefore no known star–forming region has $\alpha_{\mathrm{vir}}^2 M < 8.4M_\odot$ for competitive accretion to work.

If competitive acceretion is clearly not supported by observations in any known star forming region, why do the simulations (*Bonnell et al.*,1998; *Bonnell et al.*, 2001; *Bonnell et al*,2001a; *Bate et al.*, 2005) almost invariably find competitive accretion to work? Is there a fundamental flaw in the methodology used in competitive accretion scenarios (SPH) or is the problem one of physics and initial conditions? As *Krumholz et al.* (2005a) point out, all competitive accretion have virial parameters $\alpha_{\mathrm{vir}} \ll 1$. Some of the simulations start with $\alpha_{\mathrm{vir}} \approx 0.01$ as a typical choice (Bonnell et al., 2001a; *Bonnell et al.*, 2001b; *Klessen and Burkert*, 2000; *Klessen and Burkert*, 2001). For other simulations the virial parameter is initially of order unity but decreases to $\ll 1$ in a crossing time as turbulence decays (*Bonnell et al.*, 2004; *Bate et al.*, 2002a; *Bate et al.*, 2003; *Bate et al.*, 2002b). It is also noteworthy that many of the simulations begin with clumps of mass considerably smaller ($M \leq 100M_\odot$) than that typically found in star forming regions $\sim 5000M\odot$ (*Plume et al.*, 1997). As a result these simulations have $\alpha_{\mathrm{vir}}^2 M < 10M_\odot$ which is why they erroneously find competitive accretion to be important (*Krumholz et al. 2005a*). *Krumholz et al.*, 2005a also point to three other features of the simulations that would further increase their estimate of f_{m}. One such example is that the small virial parameters found in competitive accretion simulations causes a too rapid global collapse of clumps resulting in a large fraction of their mass to becme stars in a crossing time. As a result, this deprives the clump the time needed for large cores to assemble and only small cores form. Without large cores, large stars can only form via competitive accretion. As an example a recent simulation of a $\sim 1000M_\odot$ clump produced cores no larger than 1 $M\odot$ (*Bonnell et al.*, 2004), substantially inconsistent with observations that find many cores more massive than this in similar star forming regions (*Johnstone et al.*, 2001; *Plume et al.*, 1997.

The results of this work indicate that there are three serious problems with simulations of competitive accretion leading to their inconsistency with observations of star–forming regions. Paramount among them, the chief reason they evolve to $\alpha_{\mathrm{vir}} \ll 1$ is that they omit feedback from star formation. Observations by *Quillen et al.*, (2005) show that outflows inject enough energy to sustain turbulence thereby keeping the virial parameter from declining to values much less than unity. Another problem is that the simulations consider isolated clumps with too little material. Real clumps embedded in larger molecular clouds have large scale turbulent motions that can cascade down to the clump scale preventing turbulent decay.

In conclusion, it appears that seed protostars cannot gain mass efficiently by competitive accretion processes in any observed star–forming clump that is approximately in virial balance. There is no observational evidence for the existence of any regions that are far from virial balance that competitive accretion models require to be effective. This suggests that current simulations showing the importance of competitive accretion are missing crucial physics and this is the key reason that they are to date, unable to model the observations of star–forming regions. It follows that competitive accretion is not a viable mechanism for producing the IMF. This recent theoretical work combined with observational evidence gives strong support to gravitational collapse over competitive accretion as the leading model for low mass star formation.

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