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***Muon  $g-2$  and Electric Dipole Moments in Storage  
Rings: Powerful Probes of Physics Beyond the  
Standard Model***

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# Muon $g-2$ and Electric Dipole Moments in Storage Rings: Powerful Probes of Physics Beyond the Standard Model

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## Dipole Moments in Storage Rings

### Muon $g-2$ : Theoretical Significance

The  $g$ -factor of a particle is defined as

$$g \equiv \frac{\frac{\text{magnetic moment}}{e\hbar/2mc}}{\frac{\text{angular momentum}}{\hbar}} \quad (1)$$

The Dirac equation predicts  $g = 2$  for point-like, spin 1/2 particles. The quantity  $g - 2$  probes the difference between the mass and charge distributions of a particle and  $g - 2 = 0$  when they are the same at all times. The proton ( $g_p = 5.586$ ) and the neutron ( $g_n = -3.826$ ) differ significantly from the value 2, indicating they are composite particles. Their ratio ( $g_p/g_n = -1.46$ ) being close to the predicted value of  $-3/2$  was the first success of the constituent quark model. The anomalous magnetic moment ( $a \equiv \frac{g-2}{2}$ ) value of the electron is known 350 times more precisely than the muon ( $a$ ) in agreement with the theoretical prediction involving only QED. The  $(g-2)/2$  value of the muon is known with 0.5ppm accuracy and because its sensitivity to a certain class of particles scales as  $(m_\mu/m_e)^2 \approx 40,000$  it is necessary to add the contribution of strong and weak interactions of the standard model (SM). Speculative extensions of the SM, like super-symmetry (SUSY), could also have a significant contribution.

The estimated contributions to the anomalous magnetic moment of the muon are the sum of the SM contributions and those coming from new physics:  $a_\mu(\text{theo}) = a_\mu(\text{QED}) + a_\mu(\text{had}) + a_\mu(\text{weak}) + a_\mu(\text{new physics})$ , with

- $a_\mu(\text{QED}) = 11658470.6(0.3) \times 10^{-10}$
- $a_\mu(\text{had}) = 694.9(8.) \times 10^{-10}$  (based on  $e^+e^-$  data)
- $a_\mu(\text{had}) = 709.6(7.) \times 10^{-10}$  (based on  $\tau$  data)
- $a_\mu(\text{weak}) = 15.4(0.3) \times 10^{-10}$

with the sum of the SM contributions being

- $a_\mu(\text{SM}) = 11\,659\,181(8)(8.) \times 10^{-10}$  (based on  $e^+e^-$  data)
- $a_\mu(\text{SM}) = 11\,659\,196(8) \times 10^{-10}$  (based on  $\tau$  data)

Speculative extensions like SUSY also have a potentially large contribution given by [1]

$$a_\mu(\text{SUSY}) = \text{sgn}(\mu) \times 13 \times 10^{-10} \left[ \frac{100\text{GeV}}{m_{\text{SUSY}}} \right]^2 \tan \beta \quad (2)$$

with  $\text{sgn}(\mu)$  the sign of the SUSY ( $\mu$ ) parameter and  $\tan \beta$  the ratio of the vacuum expectation values of the two Higgs doublets.

## Muon g-2: Experimental Method

There are three major components of the muon g-2 experimental method [2]:

- Polarize: Using the parity violating decay  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
- Interact: Precess in a uniform magnetic field
- Analyze: Using the parity violating decay  $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

Pions are produced by colliding energetic protons onto a fixed target. Pions of a certain momentum range are collected and directed into a pion decay channel. The muons resulting from the pion decay can be highly polarized (of the order of 95%) when a small muon momentum bite of 1% is used. These longitudinally polarized muons are directed into a large super-conducting magnet of 7.11m radius. The magnetic field is vertical and has a strength of 1.5T. The radius and strength of this magnet are very specific and driven by the requirement to use muons of a specific momentum 3.1GeV/c, a.k.a. magic momentum, with  $\gamma \approx 29.3$  [3].

For the non-relativistic case the g-2 principle is just the difference between the momentum precession and the spin precession of the muon. The cyclotron (angular) frequency is

$$\omega_c = \frac{eB}{m} \quad (3)$$

while the spin precession is

$$\omega_s = \frac{g eB}{2 m} \quad (4)$$

and their difference

$$\omega_a = \omega_s - \omega_c = \left(\frac{g}{2} - 1\right) \frac{eB}{m} = \frac{g - 2}{2} \frac{eB}{m} \Rightarrow \omega_a = a \frac{eB}{m} \quad (5)$$

It turns out that this equation is also valid in the relativistic case when taking into account Thomas' precession of the accelerated system. For a positive a, like it is the case with the muon, it means that the spin vector gets ahead of the momentum vector in every turn. In order to be able to determine the anomalous magnetic moment a with high accuracy we need to determine with at least the same accuracy  $\omega_a$ ,  $e/m$  and  $B$ . The last requirement places severe restrictions on the possible magnetic field configurations, with the simplest being that of a high uniformity. A highly uniform magnetic field is not very efficient in storing a large number of muons because of the absence of vertical focusing. This vertical focusing is provided for by an electrical

focusing system (quadrupoles) without adding a significant systematic error when the muon momentum used is the magic one. At this momentum the influence of the horizontal electric field on both the muon momentum vector and the muon spin vector is the same. A small correction arising from the finite momentum width of the stored beam is applied at the end of the data analysis. At low momentum the radial electric field influences the momentum vector more than it influences the spin vector. The opposite happens at very high momentum values (the E-field looks like a magnetic field and it precesses the spin vector more than the momentum vector by the anomalous magnetic moment factor  $a$ ). In between, at  $\gamma = 29.3$ , there is a happy coincidence where the E-field influences both the momentum and spin vectors the same!

The (positive) muons decay to one positron and two neutrinos, with the preferred direction of the positron (in the muon rest frame) being along the muon spin. This fact has very interesting consequences on the energy spectrum of the positrons depending on the *relative* angle between the muon spin and momentum vectors. When the muon spin and momentum are parallel, the energy of the positron in the lab frame is maximum (on average) and when they are anti-parallel it is minimum (see Figure 1). The time spectrum of the detected positrons is given in (Figure 2).

## Energy Spectrum of Detected Positrons

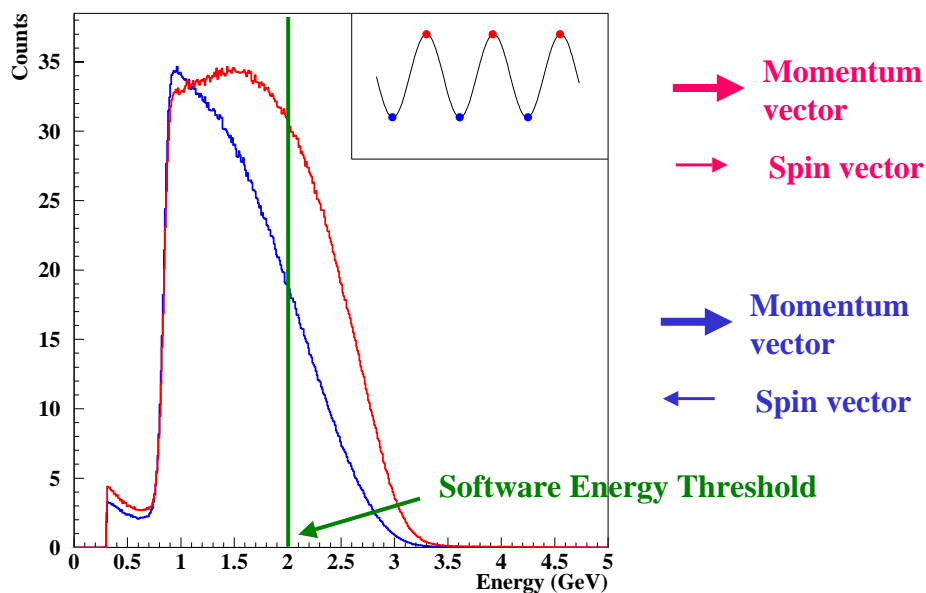


Figure 1: When the spin and momentum vectors are aligned the energy of the positrons in the lab frame is (on average) larger than when the two vectors are anti-parallel. Setting an energy threshold of, e.g., 2GeV modulates the number of particles above that energy with the g-2 frequency.

The main experimental accomplishments that made the muon g-2 experiment at Brookhaven National Laboratory particularly sensitive are several:

1. High muon intensity made possible by the large proton intensity available from the AGS with the proper timing structure.

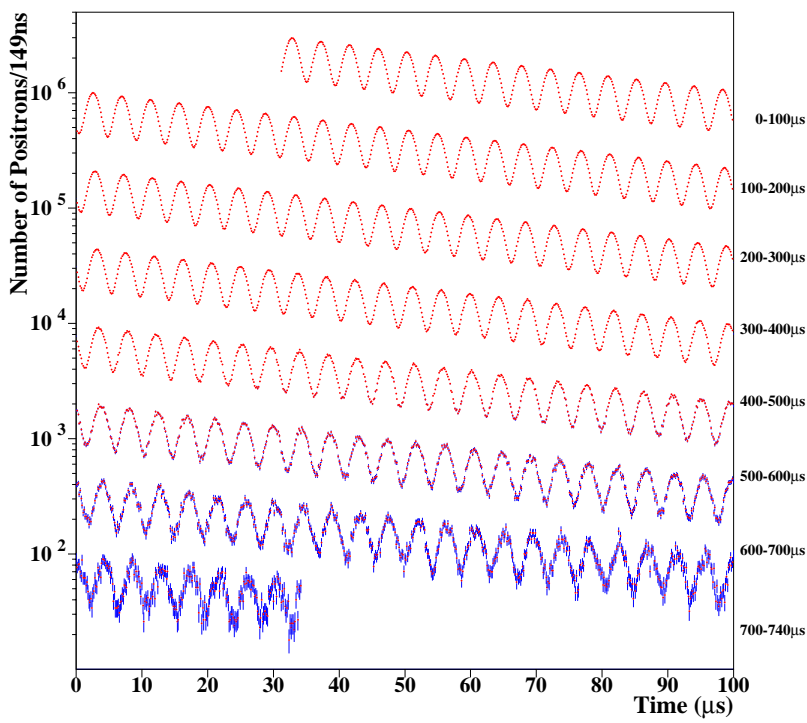


Figure 2: The number of detected positrons as a function of time. The spectrum is a traditional radioactive decay modulated with the  $g-2$  frequency.

2. The construction of the largest diameter super-conducting magnet in the world providing a stable magnetic field and the superb NMR system as its monitoring device [4].
3. Direct muon injection, made possible with the fast magnetic kicker with minimal residual magnetic field [5].
4. A super-conducting inflector with a superconducting shield that provided 1.5T DC magnetic field to counter-part the main magnet field and yet it allowed no measurable fringe field in the storage region, just a couple of cm away [6]!
5. Pulsed electric quadrupoles providing vertical focusing with an innovative lead design to quench the trapping of low energy electrons [7].
6. Electromagnetic calorimeters and their detection electronics that could operate at high and low counting rates with adequate gain and timing stability [8].

The final experimental results as well as the theoretical predictions based on both the  $e^+e^-$  and  $\tau$  data are shown in Figure 3; the total experimental error is 0.5ppm [9]. The experimental value is found to be  $11\,659\,203(8) \times 10^{-10}$ . Clearly it would be beneficial to reduce both the theoretical and experimental uncertainties. It is expected that the theoretical uncertainty will be reduced by a factor of 2 in the next years. On the experimental side we have already put in a proposal for an upgraded experiment at BNL with a goal of 0.2ppm, which received scientific approval and is currently awaiting funding approval from the DOE.

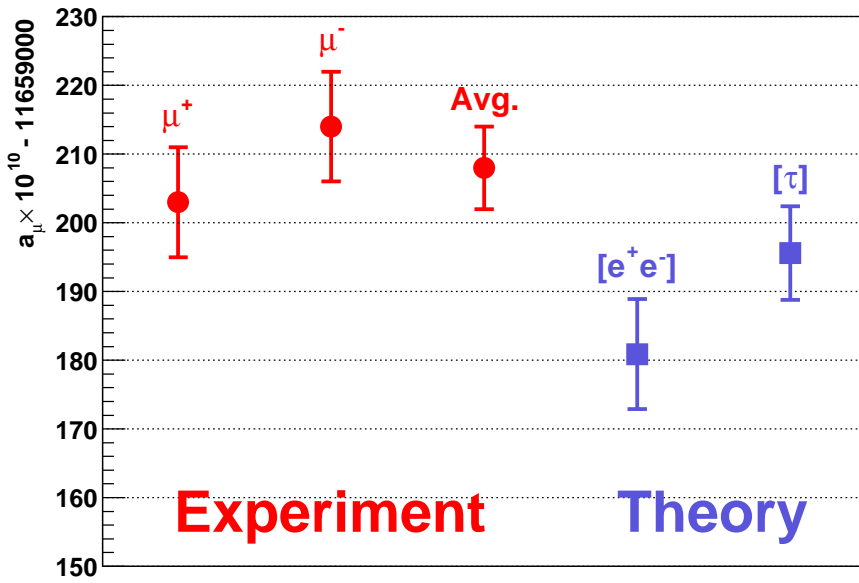


Figure 3: The experimental value as well as the theoretical predictions of the anomalous magnetic moment of the muon are shown here. The experimental error is dominated by its statistical error.

## Electric Dipole Moments in Storage Rings

The search for an electric dipole moment (EDM) of fundamental particles has been going on since the 1950s. If an EDM was to exist it would have to violate separately parity (P) and time reversal (T) invariance (see Figure 4). T-violation, assuming CPT conservation, means CP-violation which is one of the required conditions, according to Sakharov, for an initially matter-antimatter symmetric universe to evolve to the present day matter dominated universe. EDMs are particularly sensitive to

**A Permanent EDM Violates both  
T & P Symmetries:**

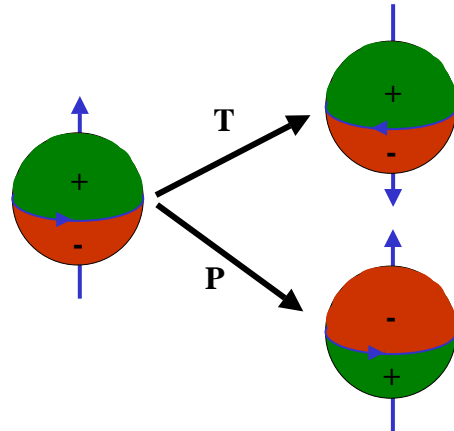
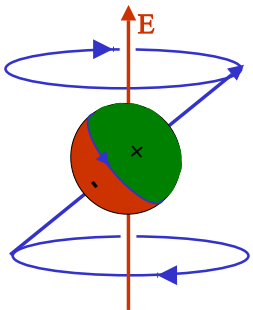


Figure 4: The existence of an EDM in a particle with spin would violate separately parity (P) and time (T) reversal invariance.

physics beyond the SM. Due to the structure of the SM (a single CP-violating phase

plus the fact that the  $W^-$  boson comes only in one handedness) the EDMs in the SM have negligible values (from a  $10^{-31}e \cdot \text{cm}$  to  $10^{-32}e \cdot \text{cm}$  for the neutron to  $< 10^{-38}e \cdot \text{cm}$  for the electron) as they are a high order effect. In most extensions of the SM the above mentioned restrictions do not exist (e.g. in SUSY there are 42 CP-violating phases, with both right and left handed bosons present) and their EDM contributions are first order effects. The expected numerical values are in the neighborhood of the current experimental limits. Obviously the discovery of an EDM will constitute a significant progress in understanding the origin of our cosmos. The usual method for searching for an EDM is depicted in Figure 5.

### Usual Experimental Method

$$\frac{d\vec{s}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}$$


Compare the Zeeman Frequencies  
When E-field is Flipped:

$$\hbar(\omega_1 - \omega_2) = 4dE$$

$$\sigma_d \propto \frac{1}{E} \frac{1}{\sqrt{N \tau T}}$$

Figure 5: Strong electric fields are used to probe EDMs. Flipping the electric field direction should change the particle spin precession rate in the presence of an EDM.

The current EDM limits are dominated by the electron [10] ( $< 1.6 \times 10^{-27}e \cdot \text{cm}$ ), the neutron [11] ( $< 6.3 \times 10^{-26}e \cdot \text{cm}$ ) and  $^{199}\text{Hg}$  [12] ( $< 2.1 \times 10^{-28}e \cdot \text{cm}$ ) with approximately the same physics sensitivity. Even though the  $^{199}\text{Hg}$  is numerically the best limit it only translates to a lesser limit for the neutron due to screening (Schiff theorem). As a matter of fact screening is the reason that all the EDM experiments have been performed in neutral systems and then the EDM of the individual charged particles has been deduced using theory.

We have now introduced a new sensitive EDM method using directly charged particles in magnetic storage rings. This method uses the fact that magnetic fields Lorentz transform into electric field in the rest frame of the moving particles and therefore they couple to the EDM ( $d$ ) causing the spin ( $S$ ) vector to precess according to:

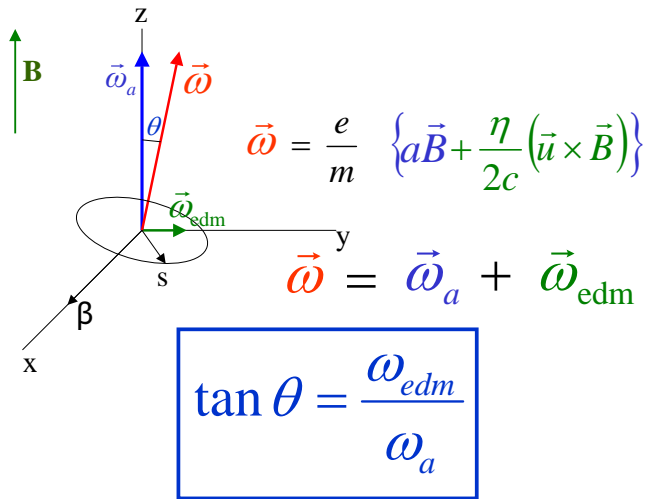
$$\frac{d\vec{S}}{dt} = \vec{d} \times (\vec{V} \times \vec{B}) \quad (6)$$

where, e.g., 1T is equivalent to 300MV/m for relativistic particles, much more than it is possible to achieve for electric fields in the laboratory.

In our muon g-2 experiment we used this effect to place an indirect limit on the muon EDM (see Figure 6). Due to the tilt in the angular velocity vector with respect to the magnetic field direction there is a modulated vertical spin component. The



## Indirect Muon EDM limit from the g-2 Experiment



Ron McNabb's Thesis 2003:  $< 2.7 \times 10^{-19} e \cdot \text{cm}$  95% C.L.

Figure 6: The direction of the angular velocity vector is slightly tilted with respect to the magnetic field direction in the presence of a muon EDM.

amplitude of the vertical component depends on the g-2 period, the larger the period the stronger the signal. This is the reason we have proposed to use particles with a momentum lower than the magic momentum (where a radial electric field influences the momentum direction more than the spin direction) to “freeze” the spin in the forward direction (in effect making the g-2 period as long as possible) [13]. We have applied this method to produce a letter of intent with a sensitivity of  $10^{-24} e \cdot \text{cm}$  for the muon [14] (statistics limited) and a proposal with a sensitivity of  $10^{-27} e \cdot \text{cm}$  for the deuteron nucleus [15] (systematics limited) as it is most applicable to particles with small anomalous magnetic moments. The deuteron EDM is the sum of the proton, and neutron EDM plus the CP-violating nuclear forces:

$$d_D = (d_p + d_n) + d_D^{\pi NN} \quad (7)$$

A limit of  $3 \times 10^{-27} e \cdot \text{cm}$  was used to compare it with the the present limits of the neutron, electron (Tl) and  $^{199}\text{Hg}$  [16] limits. The deuteron EDM is most favorable by a factor of 10 to 1000 depending on the value of the SUSY parameters and the models. The deuteron polarimetry (the analyzer that probes the spin polarization state as a function of time) is described by Onderwater in the same proceedings.

The cost of the proposal was estimated to be \$34M mainly due to the large size of the storage ring and the Brookhaven PAC felt it was too expensive for the physics reach.

Since then Yuri Orlov invented [17] another variation (resonance method, see the contribution by Orlov in the current proceedings) of the Storage Ring EDM method which reduces the size of the ring by an order of magnitude and eliminates the major systematic error limitation of the previous method. The potential reach of the method is  $10^{-29} e \cdot \text{cm}$  an un-precidented sensitivity level. The buildup of the vertical spin component is achieved by using RF to modulate the particle velocity in resonance with the g-2 frequency. The method is also applicable to particles with large anomalous magnetic moments. It eliminates the need for the radial electric field, the major source of systematic error of the previous method.

## Conclusion

We have shown that the study of dipole moments, both magnetic and electric, in storage rings offer unique opportunities in probing physics beyond the SM. Both methods use similar techniques (particle and spin precession in magnetic storage rings). We are currently investigating the systematic errors associated with the resonance EDM method. So far it looks very promising.

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