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A PRELIMINARY STUDY OF THE EFFECT OF SHIFTS IN PACKING FRACTION ON K-EFFECTIVE IN PEBBLE-BED REACTORS

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ABSTRACT

A preliminary examination of the effect of pebble packing changes on the reactivity of a pebble-bed reactor (PBR) is performed. As a first step, using the MCNP code, the modeling of a PBR core as a continuous and homogenous region is compared to the modeling as a collection of discrete pebbles of equal average fuel density. It is shown that the two modeling approaches give the same trends inasmuch as changes in k_{eff} are concerned. It is thus shown that for the purpose of identifying trends in k_{eff} changes, the use of a homogeneous model is sufficient. A homogenous model is then used to assess the effect of pebble packing arrangement changes on the reactivity of a PBR core. It is shown that the changes can be large enough to result in prompt criticality. It is shown that for uranium fueled PBRs, thermal feedback could have the potential to offset the increase in reactivity, whereas for plutonium fueled systems, thermal feedback may not be sufficient for totally offsetting the packing-increase reactivity insertion and could even exacerbate the initial response. It is thus shown that a full study, including reactor kinetics, thermal feedback, and the dynamics of energy deposition and removal is warranted to fully characterize the potential consequences of packing shifts.

1. INTRODUCTION

The packing variations in a randomly filled bed of spheres in a cylindrical region were summarized by El-Wakil (1982). In pebble-bed reactor applications, it is commonly assumed that the packing realized in practice is one that corresponds to the most probable random packing fraction of 60% to 62%. Although experimental proof exists that such packing is the most probable, there exists neither experimental evidence nor theoretical proof to support the assertion that other packing arrangements are impossible. Furthermore, the likelihood (or probability) of occurrence of any particular packing arrangement is not quantified. A shift from the most probable packing arrangement to any packing arrangement with a higher density of fuel could lead to an increase in reactivity and, possibly, to the occurrence of an undesirable transient. Such a change in packing could arise, for example, as a consequence of an earthquake. The absence of quantification for the probability of occurrence of such higher density states requires that shifts to these states be considered in safety analyses, since they cannot be ignored as scenarios beyond the design basis for the reactor under consideration. Since the scenarios that would arise from the shift in packing fraction cannot be discarded as beyond the design basis, their consequences must be ascertained. In this paper, a preliminary assessment of the reactivity effect of changes in packing fractions is carried out.

In the next section, the various possible packing arrangements that can exist in a bed of pebbles are discussed. The following section presents a survey of the literature found thus far on the subject of changes in the packing fraction of fuel pebbles in a pebble-bed nuclear reactor. In Section 4, the methodology followed in conducting this preliminary assessment is presented. In particular, it is shown that the use of a homogenous MCNP model is sufficient for reproducing the trends that would be obtained using a heterogeneous model that explicitly represents discrete pebbles. In the following section, the results of this study are presented. The final section presents concluding remarks and recommendations for further study.

2. PACKING ARRANGEMENT VARIATIONS IN A BED OF SPHERES

2.1 Average Packing Fraction of Identical Spheres

In a PBR, the core occupies a cylindrical or annular vat filled with billiard-ball-sized fuel pebbles that are dropped onto the top of the cylinder or annulus and removed from the bottom. In his summary of experimental findings on pebbles packing, El-Wakil (1982) shows that the average distribution of void fractions (and conversely of packing fractions) depends on the ratio D/d of the diameter D of the cylindrical container (vessel diameter) to the diameter d of the pebble. The distribution seems to approach an asymptotic packing fraction value of 0.60 as the ratio increases to 15 and greater. Thus, in the case of a pebble-bed reactor with a core diameter of 3 m and a pebble diameter of 6 cm, the ratio D/d would be 50 and the expected average packing fraction would be 0.60 or 60%. This value is most often used in models of pebble-bed reactors such as the AVR or the current design by ESKOM. In contrast to this accepted practice, recent computational results (Wu and Lee, 2000) show that the void fraction depends on the postulated average coordination number (i.e., the number of spheres that are expected to be in contact with a given sphere) and on the mean contact area between the spheres (though this latter parameter is not fully applicable to highly rigid spheres). The principal conclusion relevant to the present study is that the void fraction is expected to be variable. It is also important to acknowledge that many arrangements are possible, both ordered and random. Of these arrangements, several are remarkable and are discussed in the next subsection.

2.2 Some Remarkable Higher Packing Fractions of Identical Spheres

In a recent paper, Torquato et al. (2000) introduced the concept of the “maximally random jammed” state of a bed of identical spheres. A jammed state is a configuration of the spheres in which none can move if all the others are fixed. This state is of practical interest to pebble beds, as explained next. Assume a random bed of pebble undergoes densification because of the motion of its constituent pebbles. If the bed reaches a random jammed state, no further densification can occur, unless a pebble is removed. This assumes that upward motion of pebbles is not possible (i.e., pebbles move only because of gravity, so the top layer of pebbles can be viewed as fixed). Of course, all jammed states would have this property and could be considered as "stops" on the densification path from 60% (the most likely random packing fraction) up to the maximum density (i.e., packing) that can be achieved. Torquato and co-workers showed that the maximally random jammed state corresponds to a packing fraction of about 64%. They further remarked on other jammed states (not necessarily random). Among the remarkable jammed states are (i) the cubic crystalline lattice (about 52%, corresponding to $\pi/6$) and (ii) the fcc crystalline lattice (about 74.05%, exactly $\pi/\sqrt{18}$) recently proven to be the densest possible packing of identical spheres (Hales et al., 1998). If it is assumed that a random bed with a 60% packing fraction undergoes a shift in the positions of its pebbles that results in densification, all states (up to 74.01%) are theoretically possible, and the densification may be interrupted any time the pebble arrangement reaches a jammed state. If it is assumed that the pebbles arrangement remains maximally random, then the first possible "stop" is at 64%. Reactivity changes corresponding to many states denser than 60% are considered in this paper, including that of the maximally random jammed state. The principal conclusions to be retained from the paper of Torquato and co-workers are summarized next. First, it must be recognized that jammed states are possible. Second, though not reviewed here, many random density states can be achieved, depending on the pebble pouring rate, the inter-pebble forces (friction, hard-sphere repulsion), and amplitude and frequency of vibration during and after pouring. Third, the probability of any one state is given by a density probability function $P(\mathbf{r}^N)$ associated with finding the system in a state $\mathbf{r}^N$. Finally, such complete information is never available. This conclusion is of particular importance to the safety analysis of a pebble-bed reactor. Since the probability of the various packing states cannot be known, such packing patterns cannot be eliminated from consideration on the basis of having a likelihood that makes them beyond the design basis.

3. BRIEF SURVEY OF PREVIOUS WORK

Recently, Karriem et al. (2000) investigated the effects of the packing fraction. Their investigation postulated either an infinite array or a spherical region of a given size. They computed the values of k-effective in that array or region with various packing arrangements. They considered body-centered tetragonal and close-packed hexagonal lattice arrangements, and varied the packing fraction from 52% to 74% in each case. (In any given lattice arrangement, only one packing fraction is possible when all pebbles are in the closest possible contact in that arrangement, but if the spacing is artificially increased, lower values of packing fraction can be specified.). They showed that, for a given packing fraction, the type of lattice arrangement made little difference, but that for either lattice arrangement, the difference in k-effective between the least-packed and most-packed cases was about 60% in an infinite medium where the voids between pebbles were filled with water. Their study shows that the effects of packing fraction can be important, but they did not address all the phenomena of practical relevance related to pebble packing fraction. It is of greater practical interest to find what would happen if a finite, cylindrically shaped PBR core filled with a given number of pebbles were to undergo a change in its packing arrangement to a more tightly packed state as a result, for example, of shaking produced by an earthquake. An increase in k-effective so induced could conceivably result in a reactivity-insertion accident.

Brogli et al. (1991) investigated this "slumping" issue briefly, using the S_n code TWODANT, in planning an experimental program for the PROTEUS facility. Their investigation considered a small reflected core (0.125 m diameter X 0.128 m initial height) loaded with fresh fuel. They found that changes in packing fraction from 62% to 67% and 74% produced increases in k-effective from 1.0001 to 1.0162 and 1.0368, respectively. In the absence of feedback, such reactivity increases would be very serious events in an operating reactor, as the reactor would become prompt supercritical. It is important to examine the consequences of such packing shifts in reactors of power plant scale, and also to estimate the ability of feedback mechanisms to mitigate these consequences. This note reports the results of an inquiry into these questions.

The purpose of this inquiry was not to produce a definitive study of the issue. Instead, the inquiry was intended to determine if a potential safety problem exists because of the possibility of shifts in the packing arrangement. We have found sufficient indications of such a problem to justify recommending that the consequences of packing arrangement shifts be studied as a part of any specific design program for pebble-bed reactors.

4. METHOD OF APPROACH

The MCNP code (Briesmeister, 1997) was used to model five PBR configurations, of which four are realistic, and one is an interesting bounding case. Up to five values of packing fraction were considered in each case. The lowest, 52%, corresponds to a cubic lattice in which the centers of the pebbles are located at the corners of the cubes, and the diameter of the pebbles is equal to the length of the edge of the cubes. The next value, 60%, represents a loosely packed random arrangement. The third value, 64%, corresponds to the "maximally random jammed" state, which has recently been shown (Torquato et al., 2000) to be the most disordered arrangement possible in which all the pebbles are immobilized (as explained above). The fourth value, 68%, is a simple body-centered cubic arrangement, where the centers of the pebbles are at the center and at the corners of the lattice cube, and the pebbles are in contact on the diagonals of the cube. The last value, 74%, is the face-centered cubic arrangement, exemplified by the organization of oranges in a crate. This is the densest possible way to pack identical spheres. In several cases, the core was represented as initially critical with the 52% packing fraction, and then k-effective was found after the pebbles had settled into more densely packed states. In all cases, the core diameter is 3 m. In the fourth case, initial critical packing fractions of 52% and 61% were both considered. In the fifth case, the initial critical packing fraction was 61%.

The pebbles are 6.0 cm in diameter, and in the UO_2-fueled cases they are 8.0% enriched in U-235. In the fresh-fuel cases, they each contain 7.065 g of uranium. The graphite matrix contains a small amount of silicon; the composition was taken from an unpublished PBR design study being performed at the Idaho National Engineering and Environmental Laboratory (Weaver, 2000).

The first case is an approximate representation of a steady-state core in a PBR reactor with recirculating fuel. Based on typical parameters for such a reactor, the core height is assumed to be 9 m. The core is surrounded by a reflector 1 m thick. The partially depleted condition of this steady-state core is represented approximately by setting the fuel composition in the pebbles as that which would produce a critical height of 9 m in the 52-percent-packed configuration. Even though the actual core would have a nonuniform composition, with the average burnup increasing in the direction of pebble flow, the conclusions obtained from this case are qualitatively useful. In this case, the core is modeled twice, once as a homogeneous cylindrical region and once as an array of discrete pebbles, with the pebbles in the appropriate lattice positions for each packing fraction. Since it is impractical to model the random arrangements with MCNP, only the 52%, 68%, and 74% packing fractions were modeled in the discrete version. Where results for the discrete version appear in graphs below, the missing values are interpolated to force the plotting software to produce lines between the points. In the discrete version, the total pebble mass is conserved by adjustment of the core height only, since the pebbles remain the same. It is assumed that there is initially no cavity above the top of the core, but when the pebbles settle to a more tightly packed configuration, a cavity appears because the bottom of the upper reflector does not move. In the homogeneous version, the total mass of pebbles is conserved by adjustment of the core density as the core upper surface shifts downwards.

The second case is the configuration of initial criticality for a “peu-à-peu” core (Teuchert et al., 1991) – i.e., a core in which all the pebbles are fresh and fully loaded with fuel, but are added to the core “little-by-little.” In this core, no fuel is withdrawn from the bottom, and as the fuel is depleted, fresh fuel is added to the top to maintain criticality. This core is surrounded by a graphite reflector 1 m thick, with a cavity 10 m high above the top of the core to provide space for the addition of fresh fuel. In this case, the core is modeled as a uniform homogeneous cylinder. The total pebble mass is conserved for all five values of packing fraction by adjustment of the material density and core height.

The third case is a bare core with all fresh pebbles. The initial critical height of this core, with a packing fraction of 52%, is 8.5 m. This is not a fully realistic core, because there is no reflection at all, but it is interesting as the largest possible core with all fresh fuel and no absorber or poison pebbles or shim rods. If the only solid material around the core were a metal pressure vessel, it would not provide very much reflection, so this case is physically meaningful, although it would not have good neutron economy. As in the first case, this core is assumed uniform, and the individual pebbles are not modeled. All five values of packing fraction are included, and the total pebble mass is conserved once again by adjustments of the core height and material density.

In the first three cases, no account is taken of temperature effects, except that the materials are specified by the selection of cross sections evaluated at 294 K. The fourth case is formulated to assess the effects of an increase in temperature with increased packing fraction. At a packing fraction of 52%, the cross sections in this case are once again evaluated at 294 K. The temperature at which the cross sections are evaluated accounts for the resonance widths. Also at the 52% packing fraction, the temperatures are specified at 294 K in the cell cards; this specification defines the temperature for the free-gas treatment of low-energy-neutron scattering. Finally, at the 52% packing fraction, the grph.01t library (300 K) is used to select the S(α,β) treatment. At greater values of packing fraction, it is assumed that the fuel temperature rises because of increased reactor power to about 600 K, a value chosen arbitrarily. Cross sections at 587 K are used for the uranium nuclides and oxygen (because cross-section libraries were readily available for these temperatures), a temperature of 600 K was specified in the cell cards, and the grph.04t library (600 K) was used for the S(α,β) treatment. Nuclide densities are adjusted to give a 10-m critical height at a packing fraction of 52% with a 1-m graphite reflector (the reflector temperature is not increased). This case was repeated with an initial critical packing fraction of 61%. As in the first and third cases, the core is represented as uniform. The temperatures considered here apply to cold startup conditions or periods when the reactor is shut down; however, a full analysis of temperature feedback effects must address operating conditions as well.

The fifth case is like the fourth (with a core initially critical at a packing fraction of 61%), except that all the uranium in the fuel is replaced by Pu-239. As in the fourth case, the atom density of the plutonium is adjusted to produce a critical height of 10 m. Because the temperature reactivity coefficient in this case is positive, the additional temperature effect of graphite thermal expansion is included in the modeling. This effect is small and was, therefore, neglected in the fourth case. The effects of temperature on reactivity in PBRs fueled only by plutonium in graphite pebbles have been studied recently by Bende (2000). His extensive calculations showed that the temperature coefficient of reactivity is positive in a broad range of operating regimes, particularly at low temperature and low fuel concentration (e.g., high burnup). Bende found that there is a critical temperature T_l above which the reactivity coefficient becomes negative, and he predicted that once the temperature exceeds this value, the reactor would adopt a stable operating condition, probably after damped oscillations. However, he did not study the kinetics of this phenomenon exhaustively.

5. RESULTS

In all the cases considered, the critical core height was first determined for the initial packing fraction. As shifts in packing were postulated, the new core heights were computed, assuming the same amount of fuel is present, but at an increased density. In Table 1 and in Figure 1, the core height as a function of the packing fraction is shown for each of the five cases considered in this study.

Table 1. Core Height versus Packing Fraction

Packing Fraction	0.52	0.6	0.64	0.68	0.74
Core Model	**Core height (cm)**				
"Peu-a-peu"	126	109.2	102.4	96.4	88.5
Typical PBR	900	780	731	688	632
Bare Core	850	737	695	654	601
Temperature effects (1)	1000	867	813	765	703
All remaining cases	N/A	1000	953	897	824

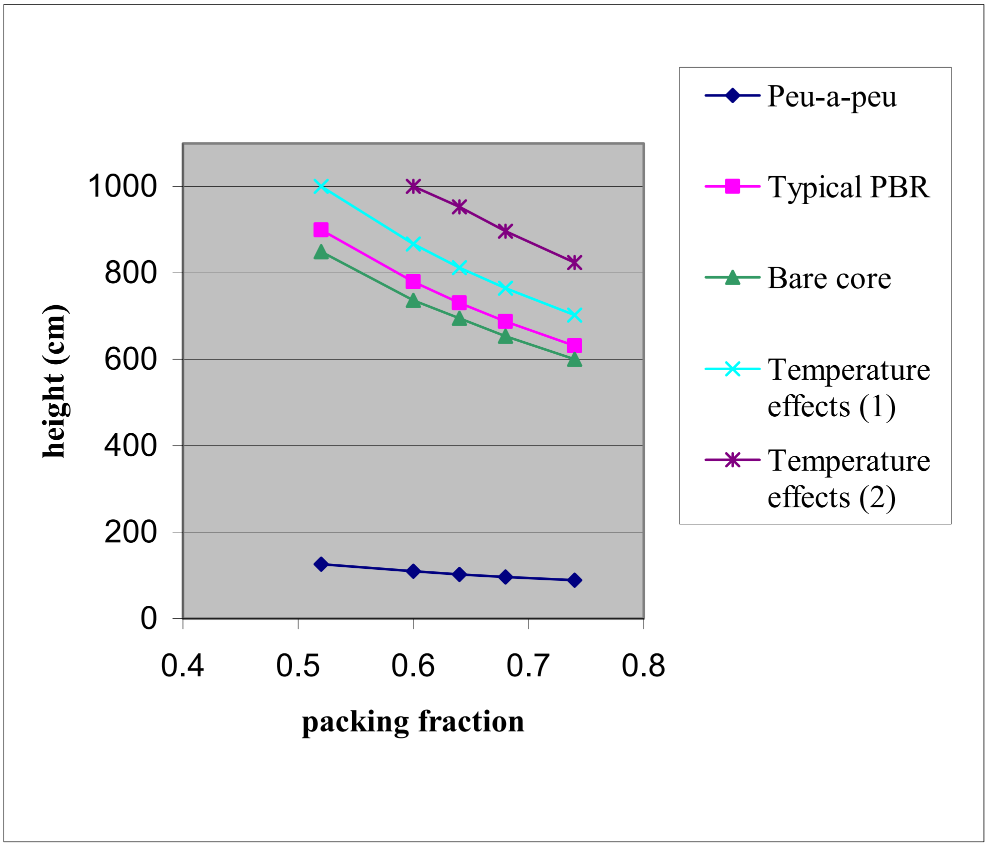

Figure 1: Reactor height versus packing fraction

The results for the two models of the "Typical PBR" are shown in Figure 2. The discrete model captures detail that is lost in the homogeneous model, but the figure shows that the trends are roughly similar. Since the goal in this paper is a qualitative estimate of the potential for pebble slumping to pose criticality safety problems, the loss of detail in the homogeneous model was accepted in return for the ability it confers to perform

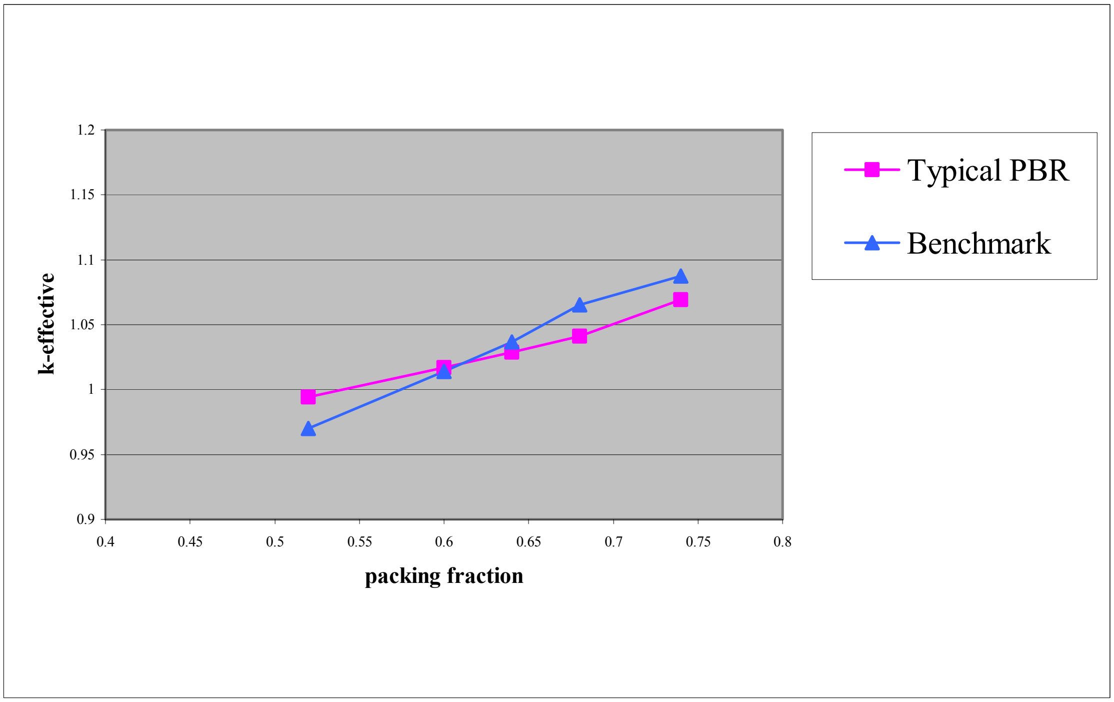

Figure 2: k-effective versus packing fraction in Typical PBR (discrete pebbles and homogeneous benchmark)

numerous calculations quickly and at arbitrary densities. However, any design calculations for an actual PBR would need to be performed with a model that not only preserves the discrete pebble lattice structure, but also accounts for spatial variation of such quantities as fuel burnup. The remainder of the study is predicated on the recognition that the homogenous model produces correct trends, even if not a full fidelity representation of a pebble-bed core.

The "Peu-à-peu" core, the "Typical PBR" core, and the bare unreflected core are compared in Figure 3. In all of these cases, the fractional change in volume is the same for a shift in between the same two values of packing fraction, so the explanation for the differences in the slopes of the curves must be sought elsewhere than in the volume change. The "Typical PBR" and the bare core are similar in height, yet their responses to slumping are quite different. The most substantial differences among the cases are in the nature of the reflectors. The bare core has none, the "Typical PBR" has no cavity above the core at first, and the "Peu-à-peu" core has a very large cavity to begin with. The reflective and absorptive properties of the cavity probably account more than anything else for the differences among these cases.

The effects of temperature feedback in a uranium-fueled PBR are shown in Figure 4. The reactor configuration is similar to that in the "Typical PBR" case, although not identical.

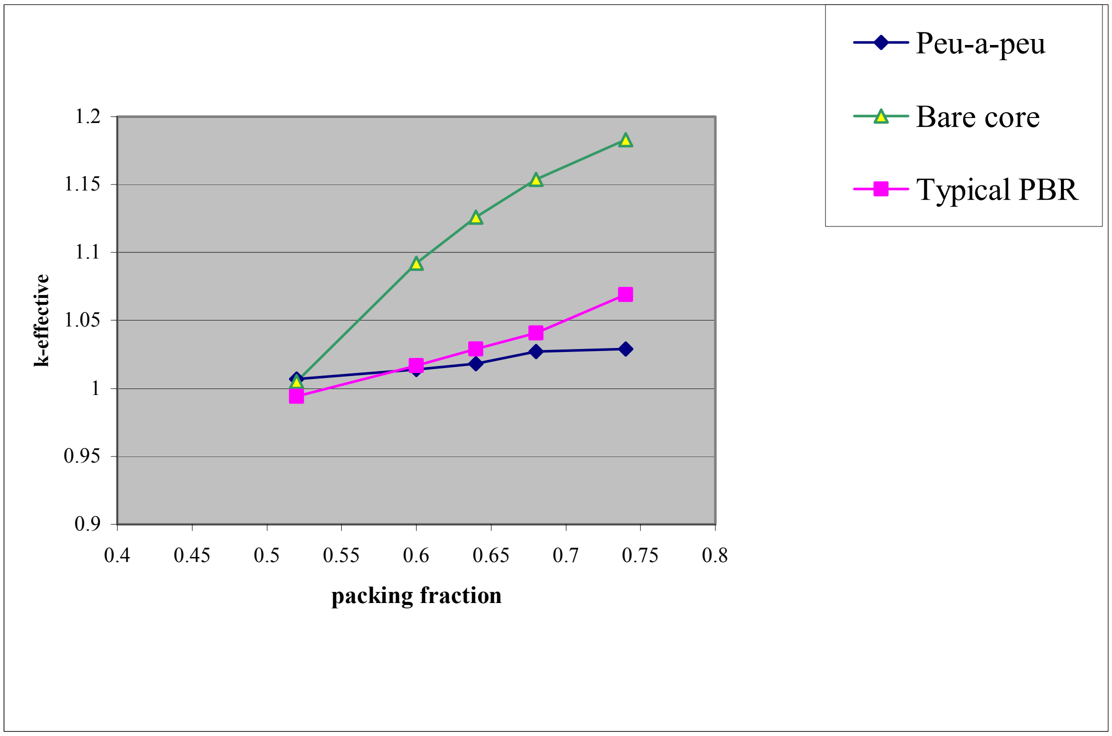

Figure 3: k-effective versus packing fraction in three example PBRs

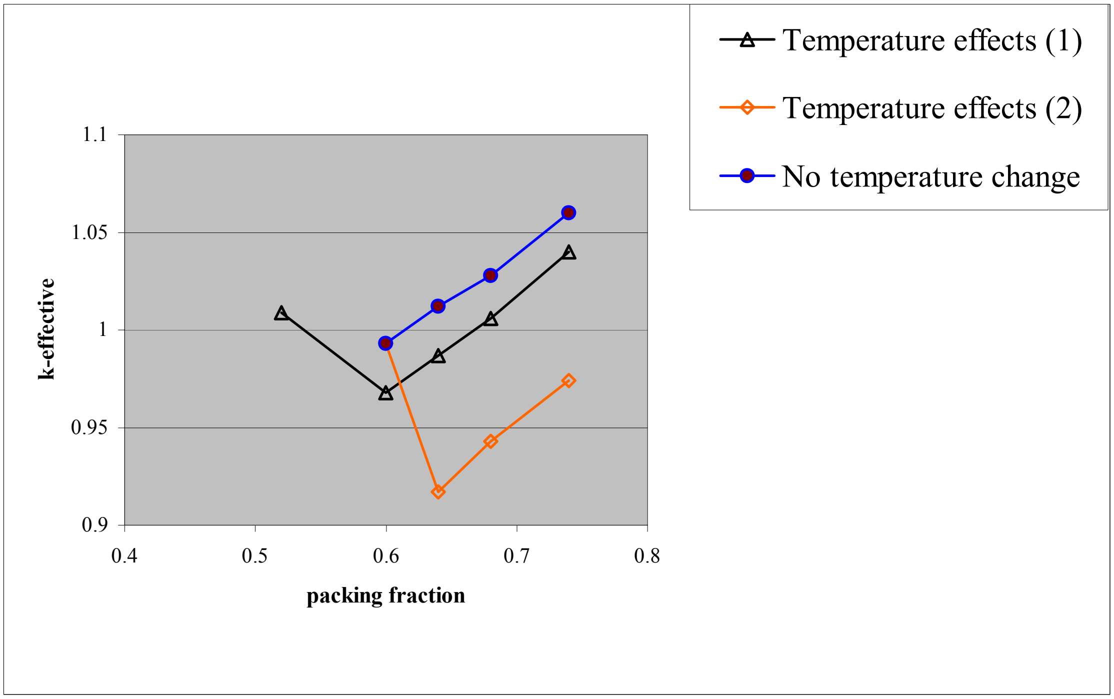

Figure 4: k-effective vs packing fraction showing temperature feedback effects in uranium-fueled PBRs

In each case, the temperature increase of 300 K is assumed to apply, regardless of the packing shift postulated from the initial packing fraction (i.e., the packing fraction is assumed to shift in one step from the initial value to the value in question, and the power increase caused by this shift produces a temperature increase of 300 K). When the initial packing fraction is 52%, the maximum possible change in packing fraction is almost 50% of the initial value, so that the corresponding change in k-effective is relatively large, and this maximum packing shift overcomes temperature feedback and produces a prompt supercritical condition. However, it is very unlikely that the pebbles would ever be in the simple cubic arrangement of a 52% packing fraction. It is much more likely that the initial packing fraction would be in the range of 60-62%, which is usually seen in practice. Therefore, this case was repeated with an initial packing fraction of 61%. The calculation with the initial value of 61% was also performed without temperature feedback effects. In this case, temperature feedback is sufficient to maintain a subcritical reactor even in a packing shift to 74%. However, it should be remembered that the 300 K temperature increase is chosen arbitrarily, and not from a thermal calculation that would show the actual measure of the temperature increase.

Results for plutonium-fueled cases are shown in Figure 5. In the plutonium-fueled PBR, the increase in k-effective in the absence of temperature increases is comparable to that in uranium-fueled PBRs, but in this case the temperature

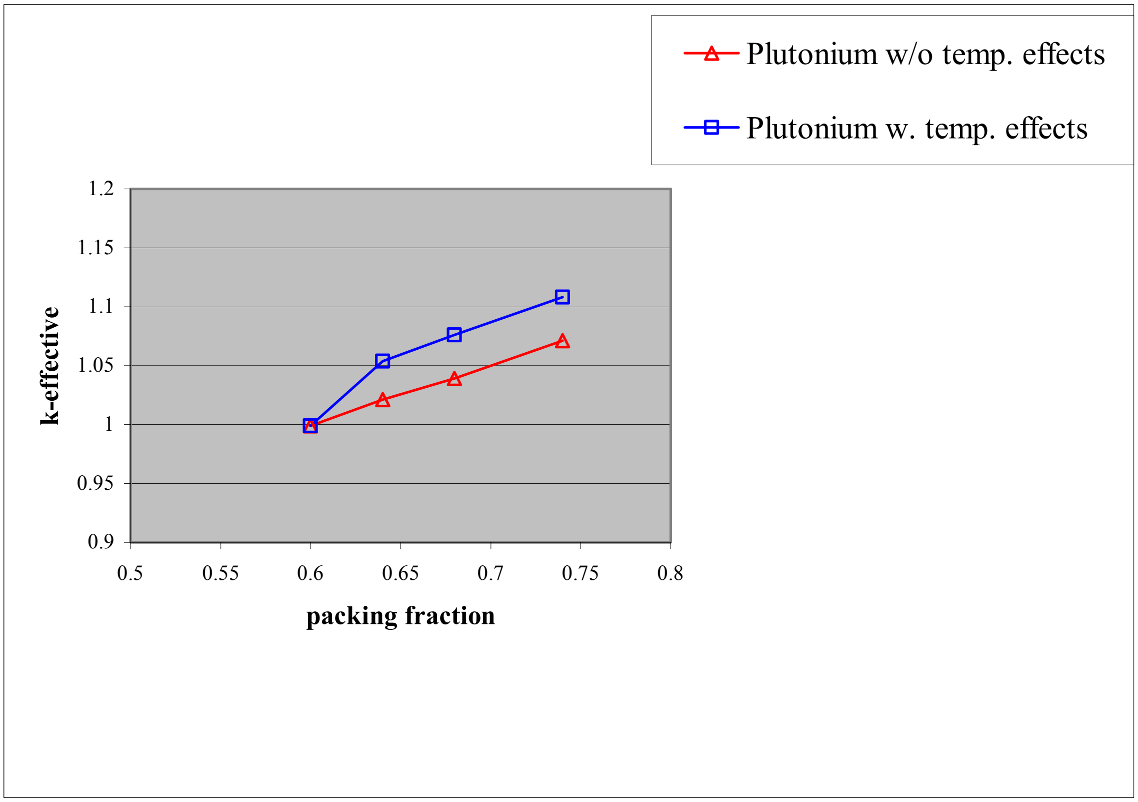

Figure 5: k-effective versus packing fraction showing temperature feedback effects in plutonium-fueled PBRs

feedback makes the slumping-induced reactivity change even worse. Despite Bende's finding that a stable operating condition will eventually be reached, the time-dependence of the temperature and reactor power must be analyzed to ensure that no fuel damage occurs before the reactor power is stabilized.

In thermal fission of U-235 the total delayed neutron fraction β is about 0.0065. Table 2 shows that the changes in k-effective exceed this value in almost every case, the exceptions being in the peu-à-peu case for the packing fraction shifts from 0.60 to 0.64 and from 0.68 to 0.74, and in the case where temperature effects are considered for uranium fuel. The other shifts in the peu-à-peu case are only slightly greater than β, but in the other two cases without temperature effects, the increases in k-effective vary from about 80% of β to more than a factor of 10β. Prompt criticality occurs when the increase in k-effective is equal to β.

Table 2. Changes in k-effective with packing fraction

Shift in Packing Fraction	0.52 - 0.6	0.6 - 0.64	0.64 - 0.68	0.68 - 0.74
Core Model	**Changes in k-effective**			
"Peu-a-peu"	0.007	0.004	0.009	0.002
Typical PBR*	0.023*	0.012*	0.012*	0.028
Bare Core	0.087	0.034	0.028	0.029
Temperature effects (1)	-0.041	0.019	0.019	0.034
Temperature effects (2)		-0.076	0.026	0.031
594 K, no temp. change		0.019	0.016	0.032
Pu Fuel, no temp. change		0.022	0.018	0.032
Pu Fuel with temp. change		0.055	0.022	0.032
* the first three values are interpolated for the Typical PBR case, but their sum is correct				

It is extremely unlikely that the pebbles could ever be packed initially with such a low packing fraction as 0.52, and it is also extremely unlikely that they would ever be so neatly organized as the face-centered cubic arrangement with its packing fraction of 0.74. It seems much more likely that a shift would occur between the relatively loose random packing of about 0.6 and the maximally random jammed state of 0.64. However, in most cases, even this small shift could cause the reactor to go from just critical to prompt supercritical, according to these results, unless temperature feedback effects are sufficient to overcome the reactivity insertion from the packing shift.

The minimum ("lower bounding") time required for each shift in packing fraction is shown for the five cases in Table 3. These time intervals are the times required for free-fall from rest between the two elevations. Except for the peu-à-peu case, these time intervals are generally in the range from about 0.3-0.5 seconds; the time intervals for the peu-à-peu case are shorter because the core is shorter and the height increments are shorter accordingly. Time intervals of the order of 0.3 s are long enough for emergency scram rods to be driven into the core in hollow guide tubes by compressed gas or electromagnetic action, but the introduction of an active emergency shutdown system violates the requirement of passive safety for Generation IV reactors. To preserve passive safety in the PBR, some passive means must be found to counteract the reactivity insertion imposed by shifts in packing fraction. Temperature feedback may be sufficient for this requirement (even with plutonium fuel, according to Bende), but a self-consistent coupled solution of the thermal and kinetics behavior would be required to show this for any given design.

Table 3: Minimum Times to Effect Packing Shift

Shift in Packing Fraction	0.52 - 0.6	0.6 - 0.64	0.64 - 0.68	0.68 - 0.74
Core Model	**Times to Shift**			
Peu-a-peu	0.1852	0.1178	0.1107	0.1270
Typical PBR	0.4949	0.3162	0.2962	0.3381
Bare Core	0.4802	0.2928	0.2893	0.3289
Temperature effects (1)	0.5210	0.3320	0.3130	0.3557
Temperature effects (2)		0.3097	0.3381	0.3860
594 K, no temp. change		0.3097	0.3381	0.3860
Pu Fuel, no temp. change		0.3097	0.3381	0.3860
Pu Fuel with temp. change		0.3097	0.3381	0.3860

6. CONCLUDING REMARKS AND RECOMMENDATIONS

It has been shown that shifts in packing fraction caused by such phenomena as earthquakes could produce reactivity insertions large enough, in the absence of feedback, to cause a PBR to become prompt-supercritical. It is very likely that temperature feedback effects can counteract such reactivity insertions. However, the adequacy of temperature feedback to protect a PBR in events of this kind should be demonstrated by a coupled reactor kinetics and thermal analysis for each PBR design. Such an analysis should address not only cold startup conditions, which were investigated in the work reported here, but also operating conditions.

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