*INEEL/CON-02-01208*
*PREPRINT*

# An Inverse Model For TETRAD: Preliminary Results

**G. Michael Shook**
**J. L. Renner**

**September 23, 2002**

**Geothermal Resource Council Annual Meeting**

# An Inverse Model for TETRAD: Preliminary Results

G. Michael Shook and J.L. Renner
Idaho Engineering and Environmental Laboratory
PO Box 1625 Idaho Falls, ID 83415-2107
ook@inel.gov; rennerjl@inel.gov

**Abstract**
A model-independent parameter estimation model known as PEST has been linked to the reservoir simulator TETRAD. The method of inverse modeling is briefly reviewed, and the link between PEST and TETRAD is discussed. A single example is presented that illustrates the power of parameter estimation from well observations.

**Keywords:** PEST, TETRAD, Parameter Estimation, Inverse Modeling

## Introduction

Proper geothermal reservoir management requires substantial knowledge of subsurface properties (permeability, porosity, saturation distribution, etc.), accurate estimations of fluid recharge locations and amounts, constitutive relations, and many other parameters. These properties are typically neither well known nor easy to estimate, yet are required for accurate numerical modeling of the reservoir. The number of unknowns far outweighs the number of parameters that can be estimated, and so a vast number of approximate solutions may exist that yield agreement between field observations (e.g., temperature histories in wells, production rates, etc.) and responses predicted by numerical modeling. Such solutions may or may not be sufficiently accurate estimators of reservoir behavior, depending in large part on the goals of the modeling exercise at hand. In any case, a reservoir model used for reservoir management decisions requires some semblance of history matching to estimate reservoir properties. That history match effort may be manual (i.e., whereby reservoir properties are modified by the reservoir engineer to match observed field behavior), or automatic (where reservoir properties are estimated via mathematical methods). Either method can be a time-consuming process.

Automatic history matching, also known as inverse modeling or parameter estimation, was first introduced to the geothermal community by Unocal through the AIM simulator (L. Murray, personal communication, 1992). AIM coupled automatic history matching methods to the TETRAD simulator, using input and output files containing well observations and comparing these against predictions obtained through forward simulations of TETRAD. AIM remains proprietary software, and is thus not available to the TETRAD community at large. More recently, Finsterle (1993) developed the inverse model ITOUGH2 for the numerical model TOUGH2 (Pruess, 1991). The overriding goals of these inverse models are to: 1) automate the time-consuming process of estimating reservoir properties for use in reservoir management, 2) remove the modeler's possible bias in parameter estimation, and 3) provide property correlation and uncertainty statistics of the property estimations themselves.

A recent project initiated at INEEL is developing a public domain parameter estimation model (named $TET^{-1}$) that is coupled to TETRAD (Vinsome and Shook, 1993). The inverse model used in this effort is known as PEST, short for Parameter ESTimation (Doherty, 2000). PEST is a model-independent inverse model, that runs in a script mode. The advantage of using such a model is that any number of models (instructions) can be run within the script; this property is discussed more below. Below we give a brief description of inverse modeling, discuss why PEST is most amenable to the project goals, and present an example of the power of this new code. The coupled reservoir inverse model is expected to be completed and available to interested users in late 2002; further project goals are discussed in the Summary section.

**Inverse Modeling**
The following discussion is a brief overview of inverse modeling as implemented in PEST (Doherty, 2000). More detail can be found in the references provided.

The inverse problem, i.e., determining reservoir properties from field observations, is solved by minimizing the differences between field observations and predictions

$$\mathbf{r} = \mathbf{c} - \mathbf{b}(\mathbf{x})$$

Where **c** is a vector containing actual field observations, and **b** is a vector of simulated (predicted) results, which are a function of the parameter field **x** we wish to estimate. The sensitivity of updated predictions is determined by running the forward model a number of times in order to calculate the Jacobian matrix, **J**:

$$J_{ij} = \frac{\partial b_i}{\partial x_j}$$

If the error structure of the residuals is assumed to be Gaussian, the objective function to be minimized is simply the sum of the squared differences between observations and predictions; we therefore must minimize the function Φ:

$$\Phi = (\mathbf{c} - \mathbf{J}\mathbf{x})^t (\mathbf{c} - \mathbf{J}\mathbf{x})$$

We obtain an updated parameter set dx by solving the system of equations below

$$\delta\mathbf{x} = (\mathbf{J}^t \mathbf{J})^{-1} \mathbf{J}^t (\mathbf{c}\text{-}\mathbf{b})$$

The above discussion is intentionally simplified, ignoring observation weighting (e.g., Doherty, 2000; Finsterle and Pruess, 1995) and various methods of accelerating parameter updates via the Marquardt parameter, λ (Marquardt, 1963; Levenberg (1944)). These features exist in PEST, and are discussed in detail by Doherty (2000). Another, vastly important, by-product of the optimization process is an extensive sensitivity and error analysis which can identify the key parameters governing system responses and identify parameter correlations. These particular features will be illustrated in a follow-up paper.

**The PEST/TETRAD Interface**

Several features of PEST made it especially useful in this project. First, PEST is in the public domain, and therefore available to a wide range of users at a nominal price. PEST is also model independent, working only on input and output files. Because most TETRAD users do not have access to source code, this feature preserves the code developer's interests. Also, PEST operates in a script mode; any number of commands (models) can be executed in the script. This is the single most important feature of PEST for later stages of this project, since we plan to combine both reservoir modeling and geophysical modeling in future parameter estimation analyses.

An instruction file is written that calls TETRAD and manipulates the well output file (creates a standard output format that allows fast comparison to the observed datafile). TETRAD is called many times (at least as many times as the number of parameters being estimated), and the resulting Jacobian is calculated. This is used in the parameter update estimates, $\delta \mathbf{x}$, and a residual is calculated. Depending on user-selected termination criteria, the inverse model either stops (parameter estimations sufficiently correct), or the process continues by calling TETRAD, calculating the Jacobian, updating parameter estimates, etc. Additional details on the process are given by Doherty (2002).

**Example: Cool Water Injection into a Superheated Steam Reservoir**

The example given below is a simplified problem of geothermal reservoir management. It is well recognized that fluid injection is required for prudent reservoir management. Injection is needed both to maintain reservoir pressure as well as enhance energy extraction from the rock matrix. Therefore, while injection results in enhanced energy recovery, care must be taken to place injection wells in such a way that the cooler injectate does not travel through preferential flowpaths (i.e., poor sweep efficiency) and break through to extraction wells prematurely. Knowledge of the subsurface flow paths (the permeability and porosity fields) is crucial to mitigating premature thermal breakthrough.

Tracer tests are frequently used to identify flow paths in geothermal reservoirs, with more than 40 such tests having been conducted on domestic fields in the last decade. However, attempts at correlating tracer recovery data to thermal velocities have met with limited success. Shook (2001a) shows that fluid and temperature velocities are related in permeable media, with the ratio of velocities equal to the ratio of volumetric heat capacities. It has also been shown that similar calculations can be made in fractured media, as long as the fracture geometry is relatively simple (Shook, 2001b). These methods, however, have not been field tested as yet.

We are, therefore, left with the following issues. We recognize that well production rates are related to the (local) permeability distribution, while tracer histories are related to the interwell-averaged permeability and porosity fields. Thus, by measuring (observing) well rates and tracer effluent curves, we should be able to estimate permeability and porosity distributions. The example described below shows a preliminary attempt at doing so.

A description of the problem is as follows. The domain is square and two-dimensional, with dimensions of 400m by 400m by 5m. Extraction wells were placed in each corner of the domain, and an injection well in the center. Initial conditions of pressure and temperature are such that the reservoir is filled with highly superheated steam (P = 1500 kPa, T = 170°C). As this was a preliminary attempt at coupling TETRAD with PEST, fairly simple heterogeneity was invoked. The reservoir was divided into four quadrants corresponding to each of the four production wells' drainage pattern. Permeability and porosity were constant within any given quadrant, but varied between the quadrants. The domain and petrophysical properties are shown schematically in Figure 1.

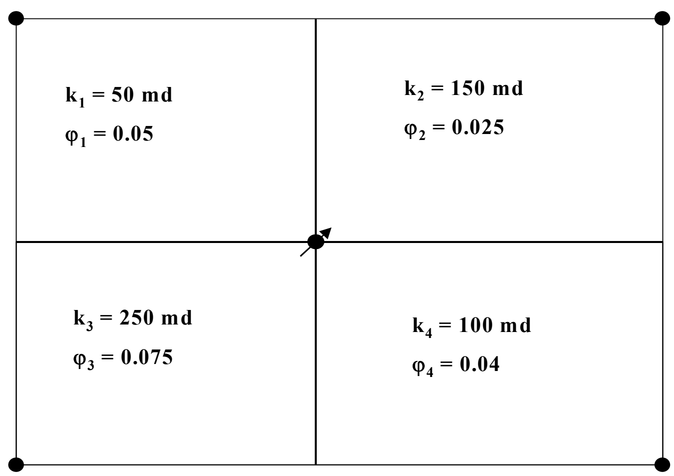



**Figure 1. Schematic of reservoir description for example problem. Production wells are at each corner; injection well is in the middle.**

At t=0, production and injection commence. The production wells were placed on a pressure constraint of 900 kPa. The injection well was rate-constrained to inject 1000 kg/hr of fluid at 35°C. During the first day of injection an ideal tracer was also injected; after 1 day, injection continued but the fluid was only fresh water. Because of the high degree of superheat, a substantial amount of the injectate boils, resulting in phase disappearances, transport of tracer in the vapor phase, isotherms that lag substantially behind the injection front, etc. This problem was specifically selected for its highly nonlinear nature and numerical problems.

This example is purely numerical in nature. That is, we assume the description above is that of the "real" reservoir, and $TET^{-1}$ must obtain an approximation of that description. The data to be used in the inverse modeling includes well production rates, tracer histories, and temperature histories for each production well. These results are shown in Figure 2a-c.

Figure 2a.  Well Flow Rates

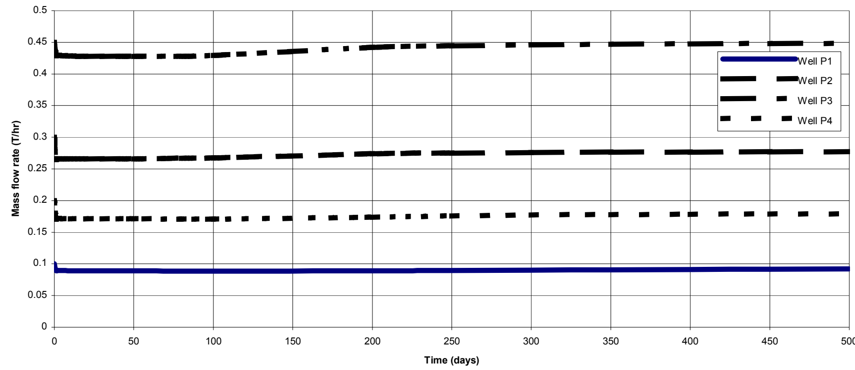



Figure 2b.  Tracer Histories

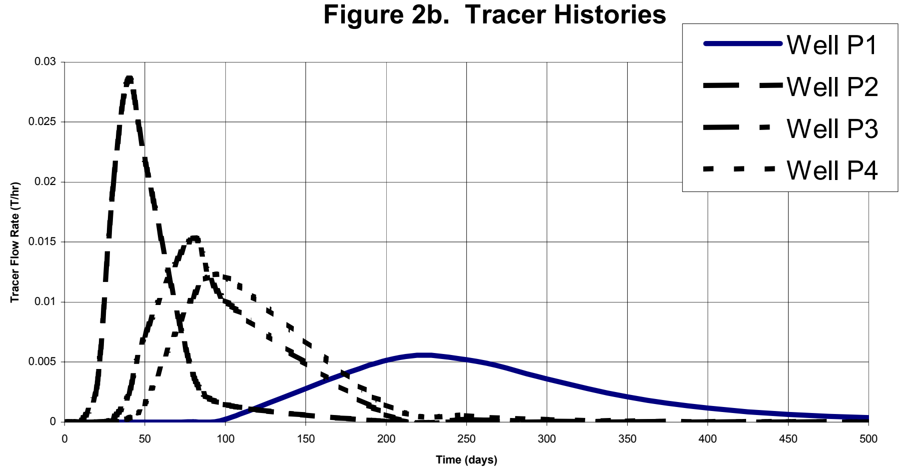
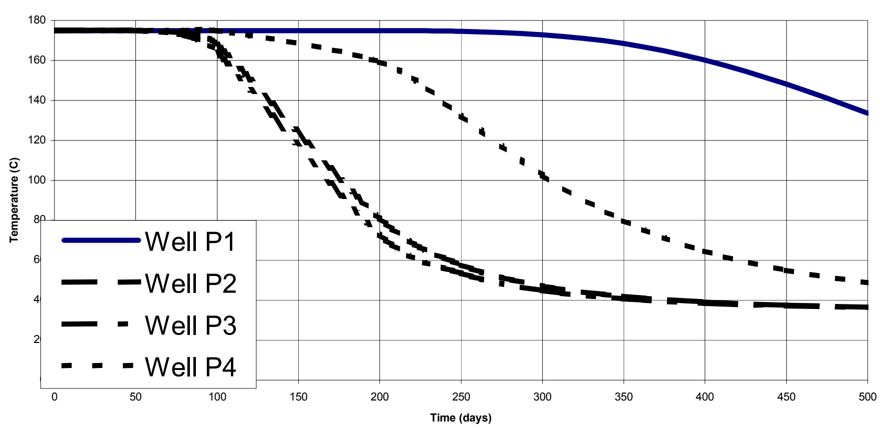



Figure 2c.  Temperature Histories



Inverse modeling of course requires an initial estimate of reservoir properties in order to begin. For this example, the reservoir was assumed to be homogeneous and isotropic, with a permeability of 750 md and porosity of 0.1. These properties resulted in well histories as shown in Figure 3 (note that all four wells show identical behavior because the domain is symmetric. One can see the significant differences between Figures 2 and 3. In this case, the residual (sum of the squared differences between predicted an observed) is 50.

The parameter estimation scheme was then initiated. TETRAD was called by the script file eight times for each iteration. A plot of the residual vs. iteration number is shown in Figure 4. After the ninth iteration, convergence criteria were satisfied, and the program stopped. A comparison between the "real" and estimated parameters is given in Table 2. Differences in the well histories (i.e., between real and final estimated cases) are not discernable, so individual histories are not shown. Note from Table 2 that the largest relative error is 0.1%.

**Figure 3a. Well Flow Rates (all wells equal)**

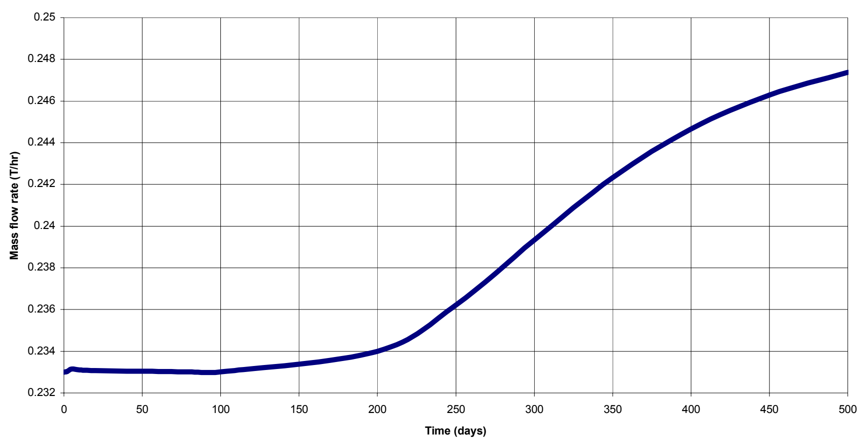



**Figure 3b. Tracer Histories (all wells equal)**

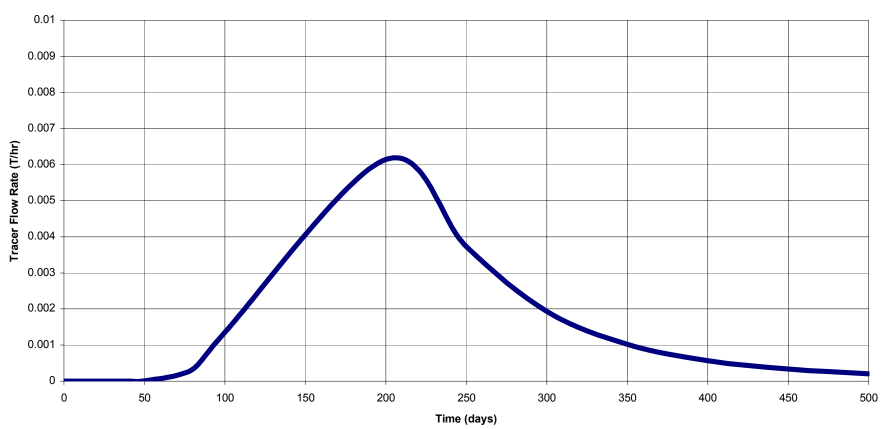


Figure 3c.  Temperature Histories (all wells equal)

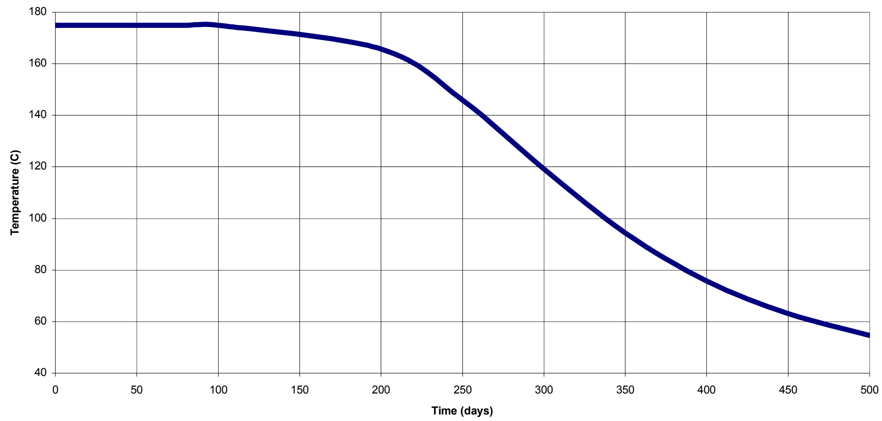



Figure 4.  Residual as a function of iteration number.

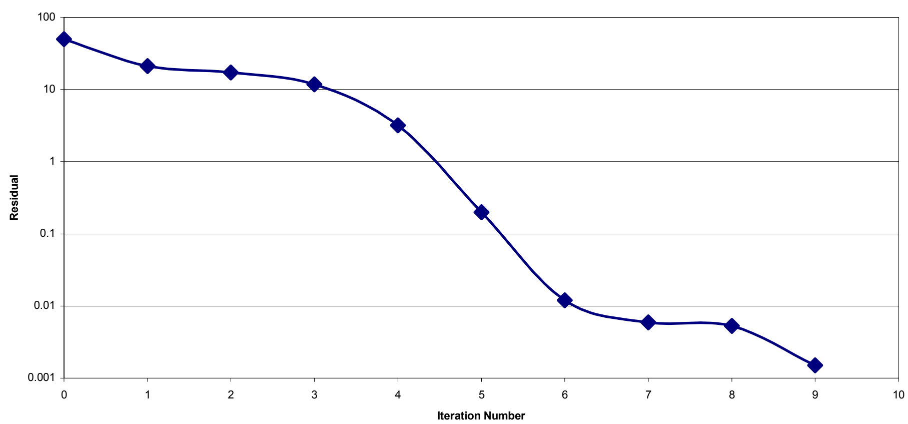



**Table 2.  Summary results for example**

| In 9 Iterations: | $k_i$ | $\varphi_i$ | Max Rel. Error |
|---|---|---|---|
| Region 1 | 49.95 | 0.05 | 0.1 % |
| Region 2 | 150.1 | 0.025 | 0.07% |
| Region 3 | 249.9 | 0.075 | 0.04% |
| Region 4 | 100.03 | 0.04 | 0.03% |

**Summary and Future Work**

Preliminary efforts at coupling the reservoir simulator TETRAD with the inverse model PEST have shown great promise.  This joint model, $Tet^{-1}$ gave excellent results on a numerically challenging problem. While the reservoir description was relatively simple (only 8 parameters to be estimated), the non-linearity of the problem added significant difficulty. An immediate goal in this project is to explore the statistics generated by PEST during the optimization scheme for use in sensitivity analysis and uncertainty propagation.

We are also in the midst of coupling TETRAD with a suite of geophysical codes (Shook, 2002). Validation and verification studies are being completed now. Beginning this fall we plan to begin coupling this combined (reservoir + geophysics) model with PEST. By including additional predictions and observations, we expect this approach to obtain better and less uncertain parameter estimates. The new code, TetGeo$^{-1}$, is expected to be available in 2003, and should find application is a variety of fields, including design and interpretation of lab-scale experiments, tracer test interpretation, and reservoir management schemes.

**Acknowledgements**
The authors gratefully thank John Doherty of Watermark Numerical Computing for his support in linking PEST to TETRAD, and for illuminating the nuances of PEST. Discussions with Stefan Finsterle regarding inverse modeling were also most helpful. Funding for this work was provided by the U.S. Department of Energy, Office of Geothermal and Wind Technologies, under contract DE-AC07-99ID13727.

**References**

Doherty, J.L, 2000, "PEST: Model Independent Parameter Estimation, Preface to the 4$^{th}$ Edition," Watermark Numerical Computing.

Doherty, J.L, 2002, "Assessing the Use of PEST with TETRAD," Internal INEEL Status Report, 12 pp.

Finsterle, S., 1993, "ITOUGH Users Guide," Report LBL-34581, Lawrence Berkeley Laboratory.

Finsterle, S., and K. Pruess, 1995, "Automatic History Matching of Geothermal Field Performance," Proc. 17$^{th}$ New Zealand Geothermal Workshop, Auckland, New Zealand.

Levenberg, K., 1944, "A Method for the Solution of Certain Nonlinear Problems in Least Squares," *Q. Appl. Math*, V. 2, pp 164-168.

Marquardt, D.W., 1963, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," *J. Soc. Ind. Appl. Math*., V. 11(2), pp 431-441.

Pruess, K., 1991, "TOUGH2-A general-purpose numerical simulator for multiphase fluid and heat flow," Report LBL-29400, Lawrence Berkeley Laboratory.

Shook, G.M., 2001a, "Prediction of Thermal Breakthrough in Heterogeneous Media from Tracer Tests," *Geothermics*, Vol. 30, pp 573-589.

Shook, G.M., 2001b, "Prediction of Thermal Velocities from Tracer Tests in Fractured Media," **Trans**., Geothermal Resources Council, Vol. 25, pp 465-468.

Shook, G.M., 2002, "Preliminary Efforts to Couple TETRAD with Geophysics Models," **Trans**., 27th Stanford Workshop on Geothermal Reservoir Engineering, in press.

Vinsome, P.K.W. and G.M. Shook, 1993, "Multi-purpose simulation," *J. Petroleum Science and Engineering*, 9, pp 29-38.