INL/CON-06-01321
PREPRINT

Uncertainty Evaluation of Available Energy and Power

52nd International Instrumentation Symposium

Jon P. Christophersen
John L. Morrison

May 2006

The INL is a
U.S. Department of Energy
National Laboratory
operated by
Battelle Energy Alliance

UNCERTAINTY EVALUATION OF AVAILABLE ENERGY AND POWER

Jon P. Christophersen[1] and John L. Morrison[2]

[1]Idaho National Laboratory, Idaho Falls, ID
[2]Montana Tech, Butte, MT

Keywords: uncertainty, available energy, available power, linearity, offset

Abstract

The Idaho National Laboratory does extensive testing and evaluation of advanced technology batteries and ultracapacitors for applications in electric and hybrid vehicles. The testing is essentially acquiring time records of voltage, current and temperature from a variety of charge and discharge time profiles. From these three basic measured parameters, a complex assortment of derived parameters (resistance, power, etc.) is computed. Derived parameters are in many cases functions of multiple layers of other derived parameters that eventually work back to the three basic measured parameters. The purpose of this paper is to document the methodology used for the uncertainty analysis of the most complicated derived parameters broadly grouped as available energy and available power. This work is an analytical derivation. Future work will report the implementation of algorithms based upon this effort.

Introduction

Previous work [1-4] reported the method and results of the uncertainty study performed at the Idaho National Laboratory (INL) for the Energy Storage Technology Laboratory. That work developed all the relationships for the measured parameters (voltage, current, and temperature) and an array of simple multivariable derived parameters (resistance, power, etc.). This work describes the methodology for the uncertainty of the two most complicated multilayered nested parameter expressions, available energy (AE) and available power (AP). Neither AE nor AP is a simple derived parameter function of measured variables such as Equations 45 through 52 from Section 3.2 in Reference 1. They are rather analytical procedures involving some of those derived parameters. The analytical procedures are described in detail in Reference 2. This development will describe the analysis techniques used to obtain a closed form analytical expression for the uncertainty of AE and AP.

We start by describing the expressions for available energy and available power. This is followed by a description of the basic assumed error model for the uncertainty. The generalized Taylor Series approach will be modified to accommodate situations whereby many of the multi-layered variables are not necessarily statistically independent of each other. The application of the Taylor Series uncertainty technique will be outlined for calibration errors of either offset or linearity. We conclude with an overview description of the analytical results for AE and AP, and a recommendation of how it can be implemented.

Available Energy and Available Power

The U.S. Council for Automotive Research is participating with the Department of Energy to develop advanced battery technologies for electric and hybrid-electric vehicle applications. This work is directed by the Office of FreedomCAR (Freedom Cooperative Automotive Research) and Vehicle Technologies, which have established performance goals and requirements as defined in their Battery Test Manual [5]. The manual was written to characterize the performance of energy storage devices relative to the hybrid-electric vehicle goals and requirements. For the minimum power assist goals, a battery must be able to simultaneously deliver 25 kW pulse discharge power (or 20 kW peak regenerative, or "regen," pulse power) over an energy range of 300 Wh.

For devices under test, these parameters are determined from a hybrid pulse power characterization test, which consists of a 10-s discharge and regen pulse separated by a 40-s rest interval [5]. This profile is repeated at each 10% depth-of-discharge increment starting from a fully charged state and ending at the minimum allowable voltage. From this test, the discharge and regen resistances are determined from Equation (1),

$$R = \left| \frac{\left(V(t_1) - V(t_2)\right)}{\left(I(t_2) - I(t_1)\right)} \right| \tag{1}$$

where t_1 is the start of the discharge or regen pulse, and t_2 is the voltage or current at 10-s. From these data, the pulse power capabilities are calculated as defined in Equations (2) and (3),

$$P_{DIS} = V_{MIN} \frac{(OCV_{DIS} - V_{MIN})}{R_{DIS}} \tag{2}$$

$$P_{REG} = V_{MAX} \frac{(V_{MAX} - OCV_{REG})}{R_{REG}} \tag{3}$$

where V_{MAX} and V_{MIN} are the maximum and minimum voltage the cell can safely operate (usually specified by the manufacturer). For single cells, pulse power is usually scaled by a battery size factor for direct comparisons with the performance goals.

These power data are then related to the cumulative energy removed during the test [5]. Figure 1 shows an example of a scaled power versus energy curve for a cell at beginning of life. The regen power capability (right y-axis) is scaled by a ratio of the regen to discharge power goals (i.e., 20 kW / 25 kW) to normalize all calculations to the discharge power (this is denoted as P^*_{REG}). The point at which the discharge and regen curves cross each other is known as the pulse power limit. The available energy is the difference between the discharge and regen curves at a given discharge power (PP_{DIS}), as defined in Equation (4), where PP_{DIS} is less than the pulse power limit. The available power is the discharge power at a given range of energy (ΔEE_{AE}), as defined by Equation (5), where ΔEE_{AE} is zero at the pulse power limit.

$$\Delta EE_{AE} =$$

$$ER_{DIS_i} - ER_{REG_j} + \left\{ \frac{ER_{DIS_{i+1}} - ER_{DIS_i}}{P_{DIS_{i+1}} - P_{DIS_i}} (PP_{DIS} - P_{DIS_i}) - \frac{ER_{REG_{j+1}} - ER_{REG_j}}{P^*_{REG_{j+1}} - P^*_{REG_j}} (PP_{DIS} - P_{REG_j}) \right\} \quad (4)$$

$$PP_{DIS} =$$

$$\left\{ \frac{\Delta EE_{AE} - ER_{DIS_i} + ER_{REG_j} + \left(\frac{ER_{DIS_{i+1}} - ER_{DIS_i}}{P_{DIS_{i+1}} - P_{DIS_i}} \right) P_{DIS_i} - \left(\frac{ER_{REG_{j+1}} - ER_{REG_j}}{P^*_{REG_{j+1}} - P^*_{REG_j}} \right) P_{REG_j}}{\left(\frac{ER_{DIS_{i+1}} - ER_{DIS_i}}{P_{DIS_{i+1}} - P_{DIS_i}} \right) - \left(\frac{ER_{REG_{j+1}} - ER_{REG_j}}{P^*_{REG_{j+1}} - P^*_{REG_j}} \right)} \right\} \quad (5)$$

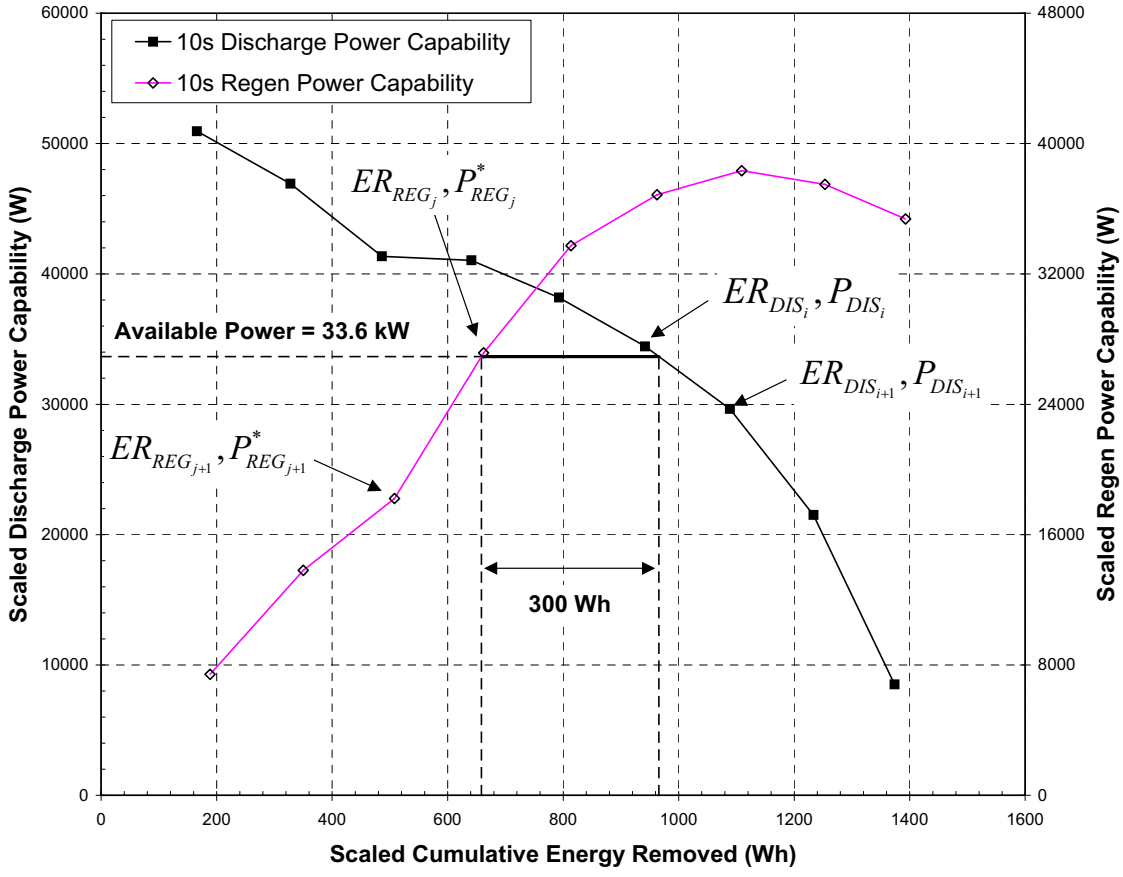


Figure 1. Scaled power versus energy curve for a representative cell

Using the minimum power assist goals as an example, the available power is the point at which the available energy is 300 Wh (i.e., ΔEE_{AE} = 300 Wh). Thus, the available power is determined by linearly interpolating between the two closest points on the discharge and regen curves as shown in Figure 1. To meet the goals, the available power needs to be greater than or equal to 25 kW, and in this example, it is 33.6 kW.

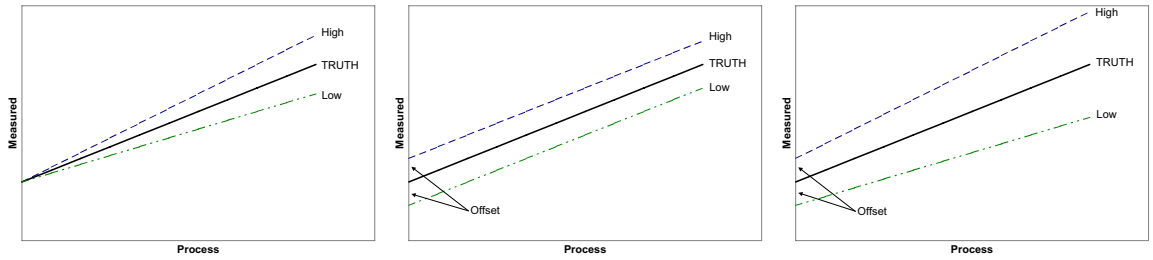
Basic Calibration Error Model

We use a simple straight line model to relate the measurement to the process. In this section we consider uncertainty associated with S_P and M_{OS}. Equation 6 gives the generalized relationship,

$$M = S_P P + M_{OS} \tag{6}$$

where M is the measurement of P process parameter, P is the physical process parameter, S_P is the calibration sensitivity constant, and M_{OS} is the calibration offset constant.

The calibration process establishes the constants S_P and M_{OS}. Uncertainty of these constants is the calibration error. The calibration error is random at calibration and then assumed fixed for each measurement until the next calibration. The uncertainty of S_P is the linearity error and the uncertainty of M_{OS} is the offset error, where the specified uncertainty of a measurement channel is usually given as "full scale" error. Figure 2 illustrates error for a measurement that is (a) all linearity, (b) all offset, and (c) a mix of linearity and offset.



(a) (b) (c)

Figure 2. Uncertainty from (a) linearity, (b) offset, and (c) both linearity and offset

The proper mix of linearity and offset error may not be available. In the derivation of the uncertainty of some of the less complex derived parameters [1-2], the mix between linearity and offset for the calibration error was addressed with two derivations: calibration error all offset; calibration error all linearity. The analyst was expected to process the error both ways and then pick the worst case. This same approach is applied to the uncertainty analysis for AE and AP.

In the Taylor Series partial derivative expressions that are not multi-layered nested and do not involve integrals (always implemented as summations for these battery data reduction algorithms), the offset error term is,

$$\Delta P_i(x_{meas}) = \frac{\partial P_i}{\partial x_{meas}} \Delta x_{os} \tag{7}$$

where P_i is the derived parameter, ΔP_i is the uncertainty of P_i, x_{meas} is the independent measured variable for P_i, and Δx_{os} is the calibration offset error for x_{meas}.

That same expression as all linearity is,

$$\Delta P_i(x_{meas}) = \frac{\partial P_i}{\partial x_{meas}} \alpha_x x_{meas} \tag{8}$$

where α_x is the linearity error.

In general α_x will simply be Δx_{os} divided by the full scale of x_{meas}. Observe that with linearity error, the net uncertainty in x_{meas} is proportional to x_{meas}. This is important because with common mode errors in expressions with terms that cancel each other out (i.e., in the case of only offset), they might sum to zero. However, with a linearity error those terms will not necessarily sum to zero.

The general implementation described in Equations 7 and 8 is easily applied to all of the various functional relationships used for the battery parameters given in Section 3 of Reference 1.

Assumptions

All of the voltage and current measurements are generally made by the same measurement channel at different times. Consequently, we assume that the voltage errors are statistically independent from current errors. Additionally, we assume that both the voltage and current error is a common mode calibration offset or linearity component along with a repeatability component. Repeatability accounts for calibration drift, bit granularity, and noise, and is included in the Taylor Series error expression, but assumed to be statistically independent from everything. The linearity and offset error make up the "full scale" error, while the repeatability error is assumed random at each measurement. Thus, all the various terms that have the presence of a common mode voltage or current error are not independent of each other. However, any term with a random voltage or current error is independent from anything else. Lastly, as described in Reference 1, we consider perturbation errors small and thus the application of the Taylor Series will only have the first derivative terms.

Taylor Series Error Terms for Multi- Layered Nested Parameters

We will develop the concept by example with a simple case of nested parameters two layers deep. Consider the function F of variables X, Y, and Z which in turn are functions of nested variables x_1, x_2, x_3; y_1, y_2, y_3; z_1, z_2, z_3 respectively as illustrated by Equation 9.

$$\begin{aligned} F &= F(X,Y,Z) \\ X &= X(x_1,x_2,x_3) \\ Y &= Y(y_1,y_2,y_3) \\ Z &= Z(z_1,z_2,z_3) \end{aligned} \tag{9}$$

We apply the Taylor Series to obtain the perturbation of the function F,

$$\begin{aligned} &\Delta F = \\ &\frac{\partial F}{\partial X}\left(\frac{\partial X}{\partial x_1}\Delta x_1 + \frac{\partial X}{\partial x_2}\Delta x_2 + \frac{\partial X}{\partial x_3}\Delta x_3\right) + \\ &\frac{\partial F}{\partial Y}\left(\frac{\partial Y}{\partial y_1}\Delta y_1 + \frac{\partial Y}{\partial y_2}\Delta y_2 + \frac{\partial Y}{\partial y_3}\Delta y_3\right) + \\ &\frac{\partial F}{\partial Z}\left(\frac{\partial Z}{\partial z_1}\Delta z_1 + \frac{\partial Z}{\partial z_2}\Delta z_2 + \frac{\partial Z}{\partial z_3}\Delta z_3\right) \end{aligned} \tag{10}$$

Suppose in the above example x_1, y_1, z_1 is temperature T, all measured by the same temperature channel at different times; x_2, y_2, z_2 is voltage V, all measured by the same voltage channel at different times; and x_3, y_3, z_3 is current I, all measured by the same current channel also at different times. Let us consider for Δx_i, Δy_i, Δz_i that the calibration error is all from offset. Thus the typical measurement error becomes,

$$\Delta p_i = P_{OS} + P_{\sigma^2} \tag{11}$$

where P_{OS} is the calibration offset error, and P_{σ^2} is the repeatability error.

When Equation 11 is plugged into Equation 10 (for temperature, voltage and current), all the terms that have an offset variable (V_{OS}, I_{OS}, T_{OS}) are not statistically independent and must add. The terms that have a repeatability variable ($V_{\sigma^2}, I_{\sigma^2}, T_{\sigma^2}$) are statistically independent and combine as the Root Sum of the Squares (RSS). This yields a measurement error of,

$$\Delta F_{Tos} = \left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_1} + \frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_1} + \frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_1} \right) T_{OS}$$

$$\Delta F_{Vos} = \left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_2} + \frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_2} + \frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_2} \right) V_{OS}$$

$$\Delta F_{Ios} = \left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_3} + \frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_3} + \frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_3} \right) I_{OS}$$

$$\left(\Delta F_{T\sigma^2}\right)^2 = \left(\left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_1} \right)^2 + \left(\frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_1} \right)^2 + \left(\frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_1} \right)^2 \right) \left(T_{\sigma^2}\right)^2 \tag{12}$$

$$\left(\Delta F_{V\sigma^2}\right)^2 = \left(\left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_2} \right)^2 + \left(\frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_2} \right)^2 + \left(\frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_2} \right)^2 \right) \left(V_{\sigma^2}\right)^2$$

$$\left(\Delta F_{I\sigma^2}\right)^2 = \left(\left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_3} \right)^2 + \left(\frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_3} \right)^2 + \left(\frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_3} \right)^2 \right) \left(I_{\sigma^2}\right)^2$$

Equation 12 can then be compiled into a total error of,

$$\Delta F = \left(\left(\Delta F_{Tos}\right)^2 + \left(\Delta F_{Vos}\right)^2 + \left(\Delta F_{Ios}\right)^2 + \left(\Delta F_{T\sigma^2}\right)^2 + \left(\Delta F_{V\sigma^2}\right)^2 + \left(\Delta F_{I\sigma^2}\right)^2 \right)^{\frac{1}{2}} \tag{13}$$

where the main terms of Equation 13 are all statistically independent, thus the resulting RSS.

Now consider that the calibration error is assumed all linearity error and thus Equation 11 becomes Equation 14,

$$\Delta p_i = \alpha_p p_i + P_{\sigma^2} \tag{14}$$

where α_p is the calibration linearity error, P_{σ^2} is the repeatability error, and p_i is the measured parameter.

Observe in Equation 14 the linearity part of the error is proportional to the measured parameter p_i. In Equation 12, within the three offset terms, the sub-terms are additive and could cancel each other out. However, for linearity, each of those sub-terms have a factor of the measured parameter all at different times and all likely different magnitudes and thus are unlikely to cancel each other out. This is why it is not clear if the most conservative uncertainty is based upon an assumption of calibration error all offset or all linearity and thus the computation must be done both ways. Combining Equation 10 and Equation 14 results in the following:

$$\Delta F_{Tlin} = \left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_1} x_1 + \frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_1} y_1 + \frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_1} z_1 \right) \alpha_T$$

$$\Delta F_{Vlin} = \left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_2} x_2 + \frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_2} y_2 + \frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_2} z_2 \right) \alpha_V$$

$$\Delta F_{Ilin} = \left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_3} x_3 + \frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_3} y_3 + \frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_3} z_3 \right) \alpha_I$$

$$\left(\Delta F_{T\sigma^2}\right)^2 = \left(\left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_1} \right)^2 + \left(\frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_1} \right)^2 + \left(\frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_1} \right)^2 \right) \left(T_{\sigma^2}\right)^2 \tag{15}$$

$$\left(\Delta F_{V\sigma^2}\right)^2 = \left(\left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_2} \right)^2 + \left(\frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_2} \right)^2 + \left(\frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_2} \right)^2 \right) \left(V_{\sigma^2}\right)^2$$

$$\left(\Delta F_{I\sigma^2}\right)^2 = \left(\left(\frac{\partial F}{\partial X}\frac{\partial X}{\partial x_3} \right)^2 + \left(\frac{\partial F}{\partial Y}\frac{\partial Y}{\partial y_3} \right)^2 + \left(\frac{\partial F}{\partial Z}\frac{\partial Z}{\partial z_3} \right)^2 \right) \left(I_{\sigma^2}\right)^2$$

Equation 13 gets modified for linearity as follows:

$$\Delta F = \left(\left(\Delta F_{Tlin}\right)^2 + \left(\Delta F_{Vlin}\right)^2 + \left(\Delta F_{Ilin}\right)^2 + \left(\Delta F_{T\sigma^2}\right)^2 + \left(\Delta F_{V\sigma^2}\right)^2 + \left(\Delta F_{I\sigma^2}\right)^2 \right)^{\frac{1}{2}} \tag{16}$$

Summary of Taylor Series Applied to AE and AP Uncertainty

The data reduction algorithms for AE and AP are realized as a single functional relationship of eight primary derived parameters. Each derived parameter has functions of many more sub-variables nested through up to seven layers until finally arriving at measured voltage and/or current. As a result, AE and AP have the functional relationship of both dependant and independent variables. The general functional relationship is given by Equation 17 (where P_1 through P_8 are the derived parameters). The exact relationships are given by Equations 4 and 5.

$$AE = F\left(AP, P_1, ...P_8\right) \tag{17}$$

In the actual relationship represented by Equation 4, deriving the uncertainty ΔAE was significantly less complicated than deriving the same for AP. Given that ΔAE is obtained, Equation 17 becomes,

$$AE + \Delta AE = F\left(AP + \Delta AP, P_1, ...P_8\right) \tag{18}$$

Equation 18 is easily solved for ΔAP and is of the form of Equation 19.

$$\Delta AP = F\left(\Delta AE, P_1, ...P_8\right) \tag{19}$$

Thus the Taylor series process need only be performed once for AE and Equation 19 used to obtain the uncertainty of AP. These derivations are described in detail in References 6 and 7 (Reference 6 shows the linearity assumption and Reference 7 shows the offset assumption) and summarized in this paper. The eight primary derived parameters for available energy are shown in Equation 20.

$$AE\left\{ER_{DIS_i}, ER_{DIS_{i+1}}, ER_{REG_j}, ER_{REG_{j+1}}, P_{DIS_i}, P_{DIS_{i+1}}, P^*_{REG_j}, P^*_{REG_{j+1}}\right\} \tag{20}$$

Therefore, the error terms are shown in Equation 21.

$$\begin{aligned}
&\delta\left(\Delta EE_{AE}\right): \\
&\frac{\partial \Delta EE_{AE}}{\partial ER_{DIS_i}}\Delta ER_{DIS_i}, \frac{\partial \Delta EE_{AE}}{\partial ER_{DIS_{i+1}}}\Delta ER_{DIS_{i+1}}, \frac{\partial \Delta EE_{AE}}{\partial ER_{REG_j}}\Delta ER_{REG_j}, \frac{\partial \Delta EE_{AE}}{\partial ER_{REG_{j+1}}}\Delta ER_{REG_{j+1}}, \\
&\frac{\partial \Delta EE_{AE}}{\partial P_{DIS_i}}\Delta P_{DIS_i}, \frac{\partial \Delta EE_{AE}}{\partial P_{DIS_{i+1}}}\Delta P_{DIS_{i+1}}, \frac{\partial \Delta EE_{AE}}{\partial P^*_{REG_j}}\Delta P^*_{REG_j}, \frac{\partial \Delta EE_{AE}}{\partial P^*_{REG_{j+1}}}\Delta P^*_{REG_{j+1}}
\end{aligned} \tag{21}$$

Each primary variable disturbance (Equation 20) can be expressed as partial derivative sequences that lead to measured voltage or current with the general form given by Equation 22 (offset) or Equation 23 (linearity).

$$\begin{aligned}
&\Delta P_i: \\
&\frac{\partial P_i}{\partial P_{q1}}\frac{\partial P_{q1}}{\partial P_{q2}}....\frac{\partial P_{qR}}{\partial I}\left(I_{OS}+I_{\sigma^2}\right), \frac{\partial P_i}{\partial P_{r1}}\frac{\partial P_{r1}}{\partial P_{r2}}....\frac{\partial P_{rS}}{\partial V}\left(V_{OS}+V_{\sigma^2}\right)\rightarrow \\
&\Delta P_i(I_{OS}), \Delta P_i(I_{\sigma^2}), \Delta P_i(V_{OS}), \Delta P_i(V_{\sigma^2})
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\Delta P_i: \\
&\frac{\partial P_i}{\partial P_{q1}}\frac{\partial P_{q1}}{\partial P_{q2}}....\frac{\partial P_{qR}}{\partial I}\left(\Delta I_{LIN}+I_{\sigma^2}\right), \frac{\partial P_i}{\partial P_{r1}}\frac{\partial P_{r1}}{\partial P_{r2}}....\frac{\partial P_{rS}}{\partial V}\left(\Delta V_{LIN}+V_{\sigma^2}\right)\rightarrow \\
&\Delta P_i(I_{LIN}), \Delta P_i(I_{\sigma^2}), \Delta P_i(V_{LIN}), \Delta P_i(V_{\sigma^2})
\end{aligned} \tag{23}$$

In Equations 22 and 23, the terms to the right of the arrow are shown as functions and not products. Those resultant functions can be combined to obtain the overall square error for offset or linearity. The general forms of the error solution are shown in Equations 24 (offset assumption) and 25 (linearity assumption). Each of these terms contain multiple partial derivatives, and the final forms of the uncertainty expressions are too big to include in this paper, but they are shown in References 6 and 7. Observe that both the offset and linearity voltage and current terms sum together and then are squared. This is because those terms relative to each other are not statistically independent. Contrast this with the terms for the variance of current and voltage. These errors are independent of everything else, and are squared prior to summing.

$$\left(\delta\left(\Delta EE_{AE}\right)\right)^2 =$$

$$\underbrace{\left(\sum_{i=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_i}\left(\Delta P_i(V_{OS})\right)\right)^2}_{1:V_{OS}\ Terms} + \underbrace{\left(\sum_{j=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_j}\left(\Delta P_j(I_{OS})\right)\right)^2}_{2:I_{OS}\ Terms} + \tag{24}$$

$$\underbrace{\sum_{n=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_n}\Delta P_n(V_{\sigma^2})\right)^2}_{3:V_{\sigma^2}\ Terms} + \underbrace{\sum_{m=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_m}\Delta P_m(I_{\sigma^2})\right)^2}_{4:I_{\sigma^2}\ Terms}$$

$$\left(\delta\left(\Delta EE_{AE}\right)\right)^2 =$$

$$\underbrace{\left(\sum_{i=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_i}\left(\Delta P_i(V_{LIN})\right)\right)^2}_{1:V_{LIN}\ Terms} + \underbrace{\left(\sum_{j=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_j}\left(\Delta P_j(I_{LIN})\right)\right)^2}_{2:I_{LIN}\ Terms} + \tag{25}$$

$$\underbrace{\sum_{n=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_n}\Delta P_n(V_{\sigma^2})\right)^2}_{3:V_{\sigma^2}\ Terms} + \underbrace{\sum_{m=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_m}\Delta P_m(I_{\sigma^2})\right)^2}_{4:I_{\sigma^2}\ Terms}$$

Conclusions and Recommendations

Extensive testing of battery and ultracapacitor technologies is underway at the Idaho National Laboratory. The established performance goals and requirements for available energy and available power necessitate complex calculations of derived parameters. This effort demonstrates that the Taylor Series approach for uncertainty can be applied to these very complicated data processing procedures to obtain closed form expressions for uncertainty. Since it is difficult to find one uncertainty expression that accounts for both linearity and offset errors, the analysis needs to be done twice (once with only offset error, and once with only linearity error), and the worst of the two results is picked. These uncertainty expressions must still undergo a thorough peer review, and be implemented as part of the standard data processing algorithms.

Acknowledgement

This work was prepared as an account of work sponsored by an agency of the United States Government under US DOE Contract DE-AC07-05ID14517. Funding for this work was provided by the U.S. DOE Office of FreedomCAR and Vehicle Technologies.

References

[1] "Uncertainty Study of INEEL EST Laboratory Battery Testing Systems Vol 1 Background and Derivation of Uncertainty Relationships", INL document INEEL/EXT-01-00505 December 2001

[2] "Uncertainty Study of INEEL EST Laboratory Battery Testing Systems Vol 2 Application of Results to INEEL Testers", INL document INEEL/EXT-01-00505 March 2003

[3] "Evaluation of the Uncertainty of Measured and Derived Parameters Used for Modeling & Analysis of Advanced High Power Batteries", by Gary Hunt and John Morrison, ISA 48th IIS 2002

[4] "A Measurement Uncertainty Study for a Battery Testing Laboratory", by Gary Hunt and J. Morrison, 47th IIS 2001

[5] "Freedom Car Battery Test Manual For Power-Assist Hybrid Electric Vehicles" INL document DOE/ID-11069 October 2003

[6] "Uncertainty Evaluation of Available Energy and Power (Assuming Measurement Errors are all Linearity)" by J. Morrison, letter report to INL, December 05.

[7] "Uncertainty Evaluation of Available Energy and Power (Assuming Measurement Errors are all Offset)" by J. Morrison, letter report to INL, December 05.