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Abstract

The Idaho National Laboratory does extensive testing and evaluation of state-of-the-art batteries and ultracapacitors for hybrid-electric vehicle applications as part of the FreedomCAR and Vehicle Technologies Program. Significant parameters of interest include Available Energy and Available Power. Documenting the uncertainty analysis of these derived parameters is a very complex problem. The uncertainty is an unknown combination of both linearity and offset error; the analysis presented in this paper computes the uncertainty both ways and then the most conservative method is assumed (which is the worst case scenario). Each method requires the use of over 134 equations, some of which are derived and some are measured values. This includes the measurement device error (calibration error) and bit resolution and analog noise error (standard deviation error). The implementation of these equations to acquire a closed form answer was done using Matlab (an array based programming language) and validated using Monte Carlo simulations.

Introduction

Previous work [1-4] reported the method and results of the uncertainty study performed at the Idaho National Laboratory (INL). That work developed all the relationships for the measured parameters (voltage, current, and temperature) and an array of simple multivariable derived parameters (resistance, power, etc.). Additionally, References [5, 6] developed in detail the Taylor Series (TS) uncertainty analysis used to obtain a closed form analytical expression for the uncertainty of Available Energy (AE) and Available Power (AP). These parameters consist of multivariable and multi-level derived parameters and the TS analysis was extremely complicated. In addition to the TS derivation, References [5, 6] give a complete description of the resulting algorithm for AE and AP uncertainty with a system of equations that can be implemented in computer code. Reference [7] gives a summary overview of that analysis.

This work describes an overview of the implementation and validation of the results of References [5, 6] realized in Matlab code. It will form the basis of what will finally be integrated into the battery testing and reporting regimen at the INL. To facilitate the validation, an alternative uncertainty methodology based on Monte Carlo (MC) was developed and implemented. Overall validation was achieved by processing typical example sets of data and comparing the results of both methods.

We start by describing an overview for AE and AP. This is followed by a summary description of the basic TS uncertainty expression for AE and AP. A description of the MC analysis follows using a simple voltage divider example that will be compared to a TS analysis. Then a very high level overview of the Matlab code is given along with a brief description of the inputs and outputs. Sample battery test data analysis results follow using both TS and MC. Finally, conclusions and recommendations for future efforts are presented.

AE and AP Overview

The FreedomCAR Battery Test Manual [8] provides various test and analysis procedures that are designed to verify a battery's performance relative to the established goals and requirements. Available energy is defined as the discharge energy available over the depth-of-discharge range where both the discharge and regenerative pulse power goals are precisely met. Available power is the discharge pulse power capability at which the usable energy is equal to the available energy goal [8]. A sample usable energy curve at beginning of life is shown in Figure 1 using the minimum Power Assist goals of 25 kW discharge pulse power and 300 Wh available energy. As the battery ages, the usable energy curve shifts to the left and down. Once the curve crosses the intersection between the available energy and pulse power goals, the battery has reached end of life. Since the calendar life goal for hybrid-vehicle applications is 15 years, battery testing is usually accelerated using higher temperatures and the AP and AE data are used to estimate calendar life under normal usage. Obviously, the uncertainty associated with AE and AP will impact these life estimations.

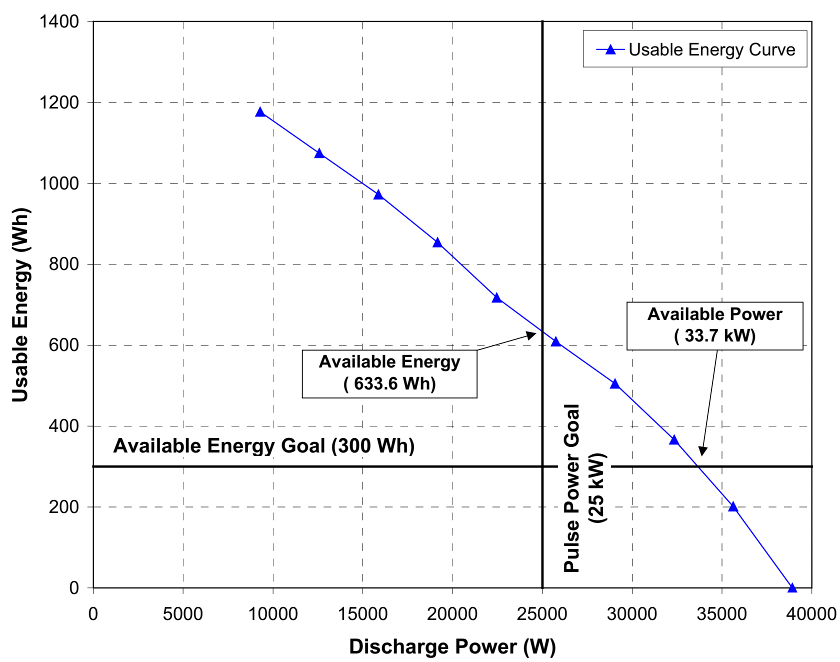

Figure 1. Sample usable energy curve at beginning of life

Taylor Series Overview for AE and AP

As seen in the previous section, a battery's performance envelope described by AE and AP is critical to fitting an application. Thus credibility of the battery's fit to the application in regards to the power vs. energy envelope depends upon the uncertainty of AE and AP.

The TS analysis approach is essentially the determination of all the parameter input sensitivities to the data processing algorithm for AE and AP via chain rule partial derivatives. As described in [7] the uncertainty of a measurement is due to an unknown mix of offset and linearity errors. Thus the TS uncertainty analysis was done for each assuming all or nothing. Reference [5] shows the TS analysis assuming all linearity error and Reference [6] gives the same analysis assuming all offset error. Equations 1 and 2 summarize the high level TS uncertainty for linearity and offset, respectively [7]. Both of these equations are very high level simplifications of those given in [5,6]. For example, Group 1 of Equation 1 (i.e., the V_{LIN} terms) shows a sum of only 8 terms, but it expands to 43 terms when all of the parameter sub-variables are included to determine the voltage effects, and each term is a string of factors of partial derivatives of about 6 or more in length. The complete system of equations in [5, 6] describe a TS uncertainty algorithm that can be implemented in computer code.

$$\left(\delta\left(\Delta EE_{AE}\right)\right)^2 = \underbrace{\left(\sum_{i=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_i}\left(\Delta P_i(V_{LIN})\right)\right)^2}_{1:\,V_{LIN}\ Terms} + \underbrace{\left(\sum_{j=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_j}\left(\Delta P_j(I_{LIN})\right)\right)^2}_{2:\,I_{LIN}\ Terms} + \underbrace{\sum_{n=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_n}\Delta P_n(V_{\sigma^2})\right)^2}_{3:\,V_{\sigma^2}\ Terms} + \underbrace{\sum_{m=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_m}\Delta P_m(I_{\sigma^2})\right)^2}_{4:\,I_{\sigma^2}\ Terms} \quad (1)$$

$$\left(\delta\left(\Delta EE_{AE}\right)\right)^2 = \underbrace{\left(\sum_{i=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_i}\left(\Delta P_i(V_{OS})\right)\right)^2}_{1:\,V_{OS}\ Terms} + \underbrace{\left(\sum_{j=1}^{8}\frac{\partial \Delta EE_{AE}}{\partial P_j}\left(\Delta P_j(I_{OS})\right)\right)^2}_{2:\,I_{OS}\ Terms} + \underbrace{\sum_{n=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_n}\Delta P_n(V_{\sigma^2})\right)^2}_{3:\,V_{\sigma^2}\ Terms} + \underbrace{\sum_{m=1}^{8}\left(\frac{\partial \Delta EE_{AE}}{\partial P_m}\Delta P_m(I_{\sigma^2})\right)^2}_{4:\,I_{\sigma^2}\ Terms} \quad (2)$$

Monte Carlo (MC) Analysis for Validation

Validation is a process that is undertaken at the completion of a test or analysis that attempts to answer the question: "Do these results make sense?" The method chosen for validation was to assess uncertainty for AE and AP for typical example sets of data by both MC and TS. The TS process is considered to be validated if the results agree. MC analysis is a process whereby an initial set of input data are statistically perturbed so as to generate a large number of input data sets. The statistics of the perturbation are set by measurement and calibration errors of the input data. These data are processed using the AE and AP algorithm for both offset and linearity, and the resultant statistical variation is the MC error. This is accomplished as follows:

We assume that the measurement system error is specified as only a full-scale error, and this leads to a simple relationship that allows switching between all linearity or all offset errors. Let N be the number of MC analysis runs to be made. Let the typical parameter be given by P. Let the offset error in units of P be given by ΔP_{OS}. Let the full scale of P be given by P_{FS}. Then the linearity error as a unit-less fraction is given as:

$$\alpha_{LIN} = \frac{\Delta P_{OS}}{P_{FS}} \tag{3}$$

The Matlab function "randn(N,1)" will make an array of N normally distributed numbers with zero mean and unity standard deviation. Let $r_P(i)$ be the i^{th} number from the array obtained from "randn(N,1)". Then the i^{th} parameter, $P_{LIN}(i)$, that has been statistically perturbed by linearity error is given by:

$$P_{LIN}(i) = P + P\alpha_{LIN} r_P(i) \tag{4}$$

Additionally, the i^{th} parameter, $P_{OS}(i)$, that has been statistically perturbed by offset error is given by:

$$P_{OS}(i) = P + \Delta P_{OS} r_P(i) \tag{5}$$

Each time the Matlab function "randn(N,1)" is executed (within the same code) a different array of N normally distributed numbers with zero mean and unity standard deviation is obtained. Using a similar technique, the bits of resolution error was obtained using the standard form as shown in Equation 6.

$$Err = \frac{1}{2^n - 1} \tag{6}$$

Where: n is the number of bits
Err is the bits of resolution error as a unit-less fraction

A simple voltage divider example, as shown in Figure 2, is given whereby the uncertainty is computed via TS and MC for both linearity and offset. Both uncertainties will be plotted and compared for agreement. The gain is given by Equation 7.

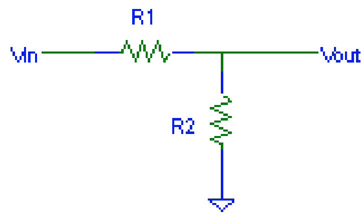

Figure 2. Simple voltage divider

$$A = \frac{V_{OUT}}{V_{IN}} = \frac{R_2}{R_1 + R_2} \tag{7}$$

It can be shown that the TS uncertainty expression for Equation 7 is given by Equation 8 when resistors R_1, R_2 are assumed to be statistically independent.

$$errA = \frac{1}{(R_1 + R_2)^2}\sqrt{(R_1 \Delta R_2)^2 + (R_2 \Delta R_1)^2} \tag{8}$$

Where: ΔR_1 is the error of R_1 in ohms

ΔR_2 is the error of R_2 in ohms

To keep the analysis simple we let the resistors have the same tolerance and be statistically independent. The tolerance error of the resistors is given by Equation 9. Observe that this resistor uncertainty is in the form of a linearity error.

$$\Delta R_{1,2} = \alpha R_{1,2} \tag{9}$$

Where: α is the tolerance as a unit less fraction

R is the resistance, ohms

Using the stated conditions and assumptions, Equations 8 and 9 combine to Equation 10, which is the result for the TS linearity error.

$$errA_{LIN} = \alpha\sqrt{2}\frac{R_1 R_2}{(R_1 + R_2)^2} \tag{10}$$

Resistor uncertainties are almost always considered as a linearity error, but for this example we will assume otherwise, and also model it as a resistor offset error as a proof of principle, as shown in Equation 11.

$$\Delta R_{1,2} = \alpha R_{FS} \tag{11}$$

Where: α is the tolerance as a unit less fraction

R_{FS} is the full scale resistance, ohms

The TS uncertainty expression for offset is given by Equation 12.

$$errA_{OS} = \frac{\alpha R_{FS}}{\left(R_1 + R_2\right)^2}\sqrt{R_1^2 + R_2^2} \tag{12}$$

For MC the gain A must be computed many times using statistically perturbed resistor values. Using the stated assumptions, the perturbed resistor values for linearity are given by Equation 13.

$$R_1(i) = R_1\left(1+\alpha r_A(i)\right), \quad R_2(i) = R_2\left(1+\alpha r_B(i)\right) \tag{13}$$

where $r_A(i), r_b(i)$ are random numbers obtained from 2 different zero mean and unity variance normally distributed arrays of numbers. Plugging Equation 13 into Equation 7 we obtain the expression for members of the set MC linearity perturbed gain $A_{LIN}(i)$.

$$A_{LIN}(i) = \frac{R_2\left(1+\alpha r_B(i)\right)}{\left\{R_1\left(1+\alpha r_A(i)\right) + R_2\left(1+\alpha r_B(i)\right)\right\}} \tag{14}$$

Using the stated assumptions, the perturbed resistor values for offset are given by Equation 15.

$$R_1 = R_1 + \alpha R_{FS} r_A(i),\ R_2 = R_2 + \alpha R_{FS} r_B(i) \tag{15}$$

Plugging Equation 15 into Equation 7 we obtain the expression for members of the set MC offset perturbed gain $A_{OS}(i)$.

$$A_{OS}(i) = \frac{\left(R_2 + \alpha R_{FS} r_B(i)\right)}{\left\{R_1 + R_2 + \alpha R_{FS}\left(r_A(i) + r_B(i)\right)\right\}} \tag{16}$$

Using Matlab to perform the analysis, Equation 10 and Equation 14 are analyzed for the linearity case, the results compared, then Equations 12 and 16 are analyzed for the offset case and the results compared. We let the resistor tolerance be 1%, which is the assumed standard deviation in a Gaussian distribution, and we let the sum of the resistors (treated as R_{FS}) be 10K, we pick 1K steps starting with $R_1 = 1K,\ R_2 = 9K$ and ending with $R_1 = 9K,\ R_2 = 1K$. Finally, we pick 1000 samples of the MC gain $A(i)$ to be statistically processed for mean and standard deviation. The standard deviation of the set $A(i)$ is the MC error. The Matlab results are plotted in Figure 3 for linearity and Figure 4 for offset. Clearly the assumptions of linearity or offset cause the figures to be very different. For the linearity error, the MC estimates are consistently larger than the corresponding TS error, whereas the offset MC estimate is initially larger, but crosses over the TS error mid-way through. However, although the MC and TS results are not identical, they are very close and the trends track. Consequently, these data show that the MC and TS techniques agree.

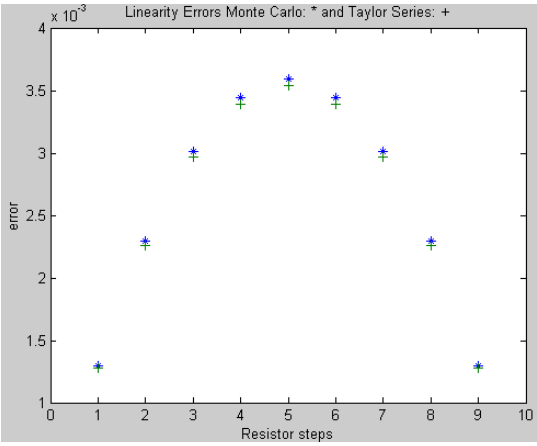

Figure 3. TS and MC linearity gain errors

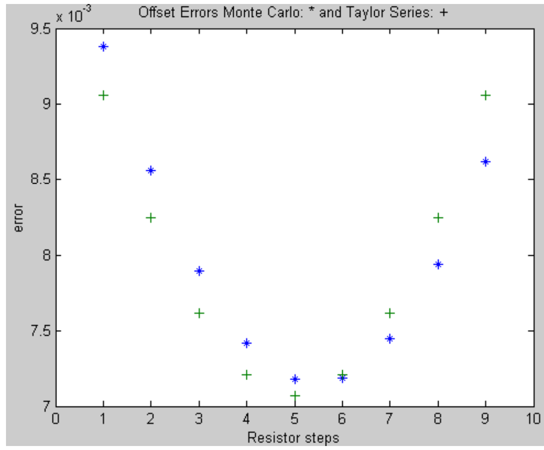

Figure 4. TS and MC offset gain errors

Matlab Code Overview

A simplified flow diagram of the Matlab implementation is shown in Figure 5. The user interface prompts the user to enter the raw data file, min and max battery voltage, full scale voltage and current, voltage and current calibration tolerance, the number of bits of resolution and the number of MC runs. The output of the code provides the usable energy curves with the upper and lower uncertainty range as determined from both the MC and TS analyses.

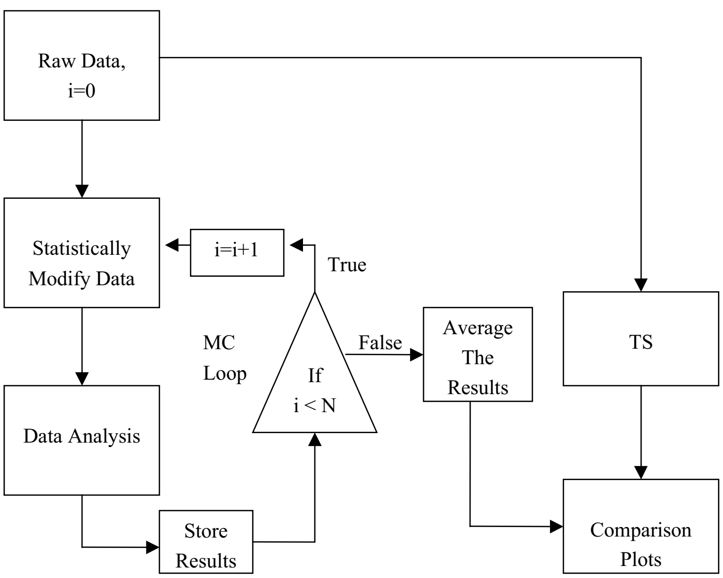

Figure 5. Block diagram

Matlab Analysis Results

The analysis results were obtained from sample data files using the parameter values shown in Table 1. Five different cases were analyzed to determine the effects of the various parameters on the overall uncertainty results. Case 1 is based on the actual values used for the sample data files. Case 2 doubled the full-scale voltage and current, and Case 3 doubled the voltage and current tolerance. Cases 4 and 5 looked at the effects of uncertainty by switching from a 16-bit tester to a 12-bit or 20-bit tester, respectively. Figures 6 and 8 show the resulting usable energy curves with high and low uncertainty bounds from both MC and TS for Case 1. Figures 7 and 9 show the usable energy curves for Case 2. Because of space limitations only Case 1 and 2 plots are shown.

Table 1. User input parameter settings for five cases of Matlab analysis

Case	Parameter	Value
1	Full Scale Volts	10V
	Full Scale Current	12.5A
	Voltage Tolerance	.0002
	Current Tolerance	.0002
	Digitizer Bits	16
2	Full Scale Volts	**20V**
	Full Scale Current	**25A**
	Voltage Tolerance	.0002
	Current Tolerance	.0002
	Digitizer Bits	16
3	Full Scale Volts	10V
	Full Scale Current	12.5A
	Voltage Tolerance	**.0004**
	Current Tolerance	**.0004**
	Digitizer Bits	16
4	Full Scale Volts	10V
	Full Scale Current	12.5A
	Voltage Tolerance	.0002
	Current Tolerance	.0002
	Digitizer Bits	**12**
5	Full Scale Volts	10V
	Full Scale Current	12.5A
	Voltage Tolerance	.0002
	Current Tolerance	.0002
	Digitizer Bits	**20**

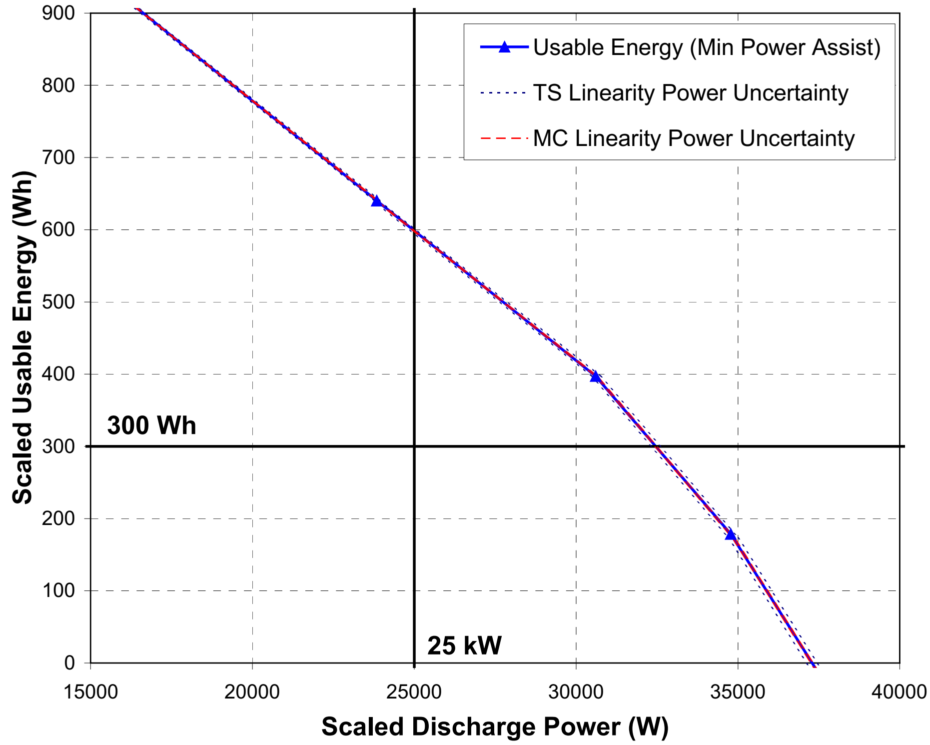

Figure 6. Case 1 linearity error

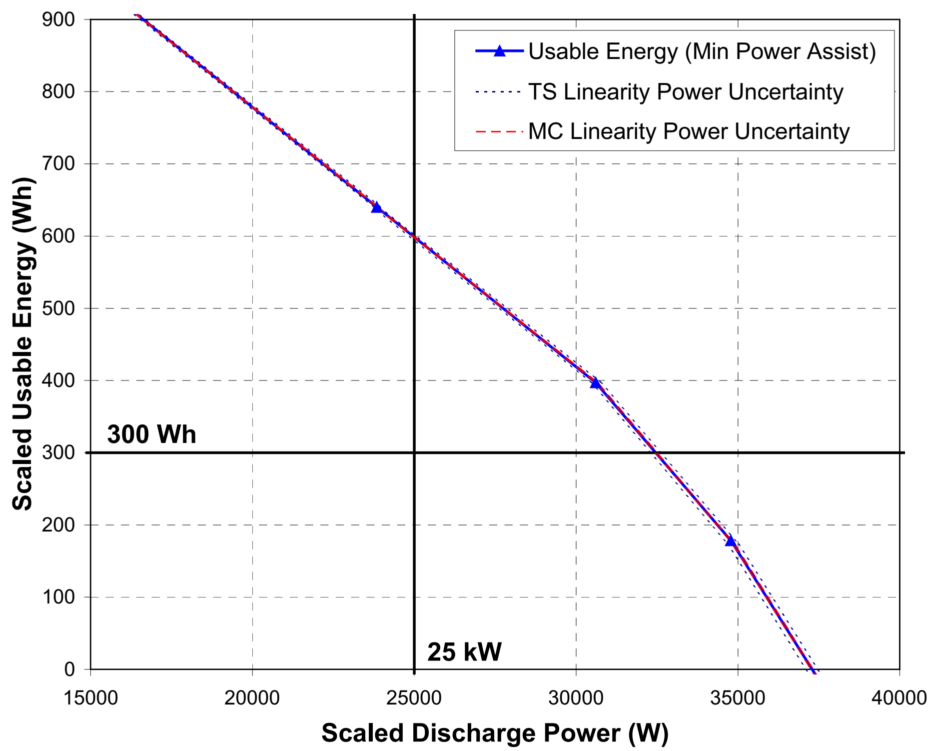

Figure 7. Case 2 linearity error

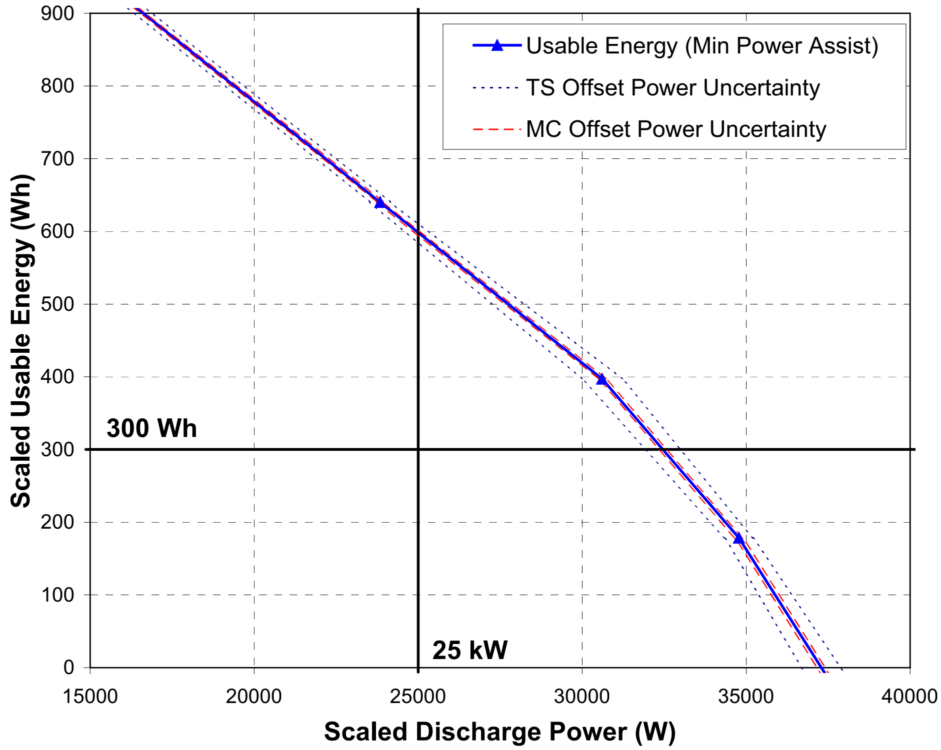

Figure 8. Case 1 offset error

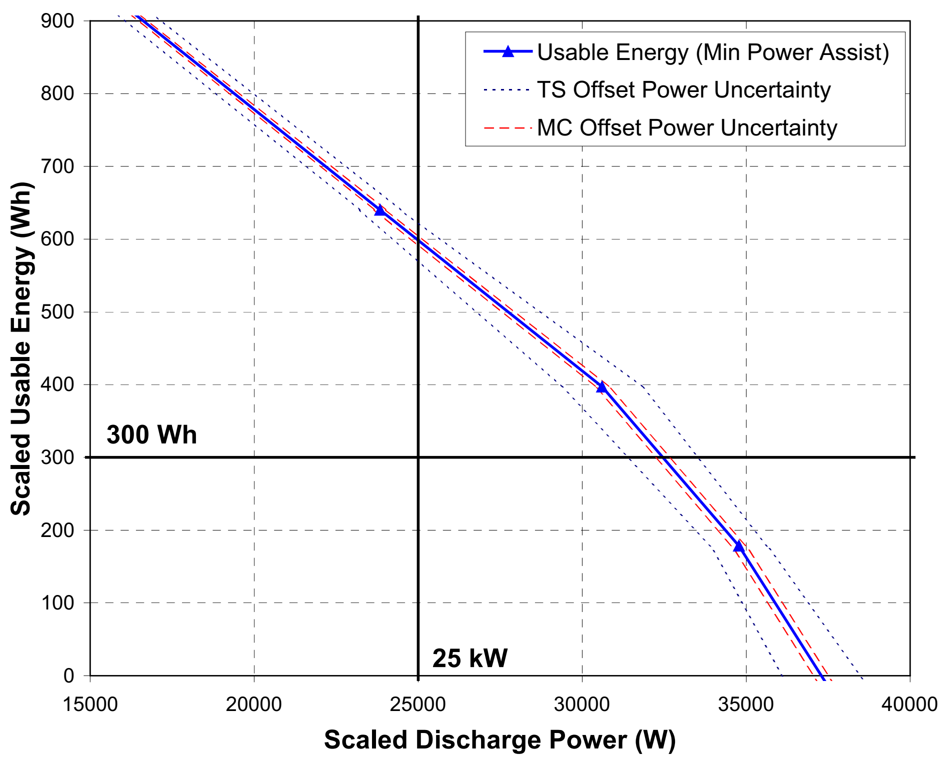

Figure 9. Case 2 offset error

Conclusions and Recommendations

Upon examining the results of the five cases, the following observations are made.

1. In all five cases, the TS and MC trends were generally similar, though not identical.
2. The offset assumption for both TS and MC tended to produce greater uncertainties.
3. TS tended to produce greater uncertainties than MC.
4. The total MC and TS uncertainty generally doubled for both Case 2 (doubling the full-scale range) and Case 3 (doubling tolerances).
5. The effect of decreasing the bits of resolution in Case 4 appeared to mask the difference between linearity and offset for both TS and MC.
6. Using a high bit resolution (Case 5) yielded virtually identical results as Case 1.

Based principally on the first observation, we conclude that the analysis is validated. Upon examining the results of all the cases we also conclude that the analysis provides a very good way to quantify the impact on overall uncertainty of measurement full scale, calibration accuracy and bits of resolution. The apparent conservatism of the TS values compared to MC indicate that using a coverage factor of one (i.e., only one standard deviation in the uncertainty values) is sufficient for our application. An open issue is how to combine the different errors. All linearity using MC is probably too optimistic and all offset using TS is probably too pessimistic. A reasonable compromise would be the average of linearity and offset errors.

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