

# Illuminating the $1/x$ moment of parton distribution functions

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## Abstract

The Weisberger relation, an exact statement of the parton model, elegantly relates a high-energy physics observable, the  $1/x$  moment of parton distribution functions, to a nonperturbative low-energy observable: the dependence of the nucleon mass on the value of the quark mass or its corresponding quark condensate. We show that contemporary fits to nucleon structure functions fail to determine this  $1/x$  moment; however, deeply virtual Compton scattering can be described in terms of a novel  $F_{1/x}(t)$  form factor which illuminates this physics. An analysis of exclusive photon-induced processes in terms of the parton-nucleon scattering amplitude with Regge behavior reveals a failure of the high  $Q^2$  factorization of exclusive processes at low  $t$  in terms of the Generalized Parton-Distribution Functions which has been widely believed to hold in the past. We emphasize the need for more data for the DVCS process at large  $t$  in future or upgraded facilities.

## 1 The Weisberger relation

The importance of the  $1/x$  moment of parton distribution functions (pdf's) was stressed in a 1972 paper by W. Weisberger [1]. There he derived a relation between the  $1/x$  moment and the derivative of the squared proton mass with

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respect to the squared parton mass defined at the same renormalization scale  $\mu$ . In modern notation and normalization [2], Weisberger's result reads

$$\frac{\delta M_N^2}{\delta m_i^2(\mu)} = \int_0^1 \frac{dx}{x} (f_i(x)_\mu + \bar{f}_i(x)_\mu) . \quad (1)$$

Here  $f_i$  is the  $i$ th-quark distribution function, and since CPT invariance implies that the mass of quark and antiquark are equal and must be varied together, we have also included the antiquark pdf  $\bar{f}$ .

With the advent of QCD and the Hellmann-Feynman theorem, one can see that Weisberger's result holds simply by noting that in light front quantization the Hamiltonian contains a kinetic energy term

$$M_{\text{kin}}^2 = \sum_i \frac{k_\perp^2 + m_i^2}{x_i} . \quad (2)$$

where  $x = k^+/P^+ = (k^0 + k^z)/(P^0 + P^z)$  is the light-front momentum fraction. After regularization and renormalization [3], a  $c_8$  mass counterterm appears, but no mass dependence in any of the other counterterms, so that the formal manipulation of the Hellmann-Feynman theorem remains valid in the regulated Hamiltonian.

Upon taking the expectation value  $\langle \delta M^2 / \delta m_i^2 \rangle_\psi$ , the trivial integration over the  $k_\perp$  transverse variables leads to the  $\int f/x$  result. Thus the Weisberger relation Eq. 1 relates the variation of the proton mass to the quark mass terms which appear specifically in the LF kinetic energy. (The Weisberger relation is an exact statement of the parton model, but in full QCD there could be an additional term due to an implicit mass dependence of the fields, which is under investigation).

In chiral perturbation theory the quark mass dependence of the nucleon mass is parameterized in terms of a contact term with an unknown constant  $c_1$  [4]

$$M_N(m_q) = M_N(0) - 4c_1 m_\pi^2 - \frac{3g_A^2 m_\pi^3}{32\pi f_\pi^2} + O(m_\pi^4) . \quad (3)$$

The constant counterterm  $4c_1 m_\pi^2$  is related to the  $\sigma$  term of the nucleon, the expectation value of a scalar current. The accurate determination of the  $1/x$  moment of pdf's, including its scale dependence, would thus allow a determination of the  $\sigma$  term independently of chiral perturbation theory, or alternatively, provide a constraint between high and low-energy physics which tests the way mass terms enter the QCD Hamiltonian. One can also combine independent evaluations of the  $\sigma = \hat{m} \partial m_N / \partial \hat{m}$  term and the  $1/x$

moment to provide an evaluation of the quark mass via (for isospin averaged light quarks)

$$\hat{m}^2 = \frac{M_N \sigma}{\langle 1/x \rangle}. \quad (4)$$

The spontaneous chiral symmetry breaking pattern of QCD also allows us to write down a new sum rule for the pion distribution function by combining the Weisberger relation with the Gell-Mann-Oakes-Renner relation, which links the quark mass to the quark condensate in the chiral limit:

$$\int_0^1 \frac{dx}{x} f_u^\pi(x) = \frac{\langle \bar{\Psi} \Psi \rangle_{m=0}^2}{m_\pi^2 f_\pi^4}. \quad (5)$$

The left and right-hand sides vary with the scale in the same way (since  $m \langle \bar{\psi} \psi \rangle$  is renormalization-group invariant). This result is independent of the light-front formalism since  $\delta M_N^2 / \delta m_q^2$  can be studied in any framework. The light-front formalism, however, provides the tools needed to demonstrate the relation. This new sum rule can be of use to constrain models of the pion as well as Deep Inelastic Scattering data.

## 2 Regularization of the Weisberger relation

The parton distribution functions measured in deep inelastic lepton scattering are observed to diverge at small  $x_{bj}$  due to the Regge behavior of the forward virtual Compton amplitude and simple analytic arguments. In fact, modern fits to deep inelastic scattering data at small  $Q^2$  routinely employ a parameterization of pdf's which is a simple variation of the Kuti-Weisskopf model [5], namely

$$x f_i(x) = A_i x^{\eta_i} (1-x)^{\lambda_i} (1 + \gamma_i \sqrt{x} + \epsilon_i x) \quad (6)$$

where all parameters are left free for the fit. The phenomenology of deep inelastic scattering generally requires  $\eta$  to be smaller than 1 for several pdf's. In fact, for the valence flavors,  $\eta_i = 1 - \alpha(0)$ , a typical Regge intercept  $\alpha(0) = 1/2$  makes the integral in eq. 1 to be manifestly divergent. This is the case for the GRV98 pdf set [6] which has exponents  $-0.85$  and  $-0.52$  for the light sea and valence pdf's respectively. Notice that the  $\sqrt{x}$  in eq. 6 gives rise to subleading Regge power laws. For the MRST98 [2] pdf sets, an also widely used alternative, the power-law exponents have higher variation around classical Regge theory and the  $u$  proton's valence component has a somewhat high intercept  $\alpha_u(0) \in (0.53, 0.59)$ , the  $d$  valence component being definitely at odds with other phenomenology with  $\alpha_d(0) \simeq 0.73$  as large as

the sea component. The subleading Regge behavior is also given by the  $\sqrt{x}$  factor in eq. 6, and having an intercept larger than zero, it also causes a divergence. In both GRV98, MRST98 sets the gluon pdf behaves as a valence-like parton with a very small intercept at this low scale, indication of the gluon degrees of freedom being truncated at low energy [7].

The Weisberger relation is thus formally divergent and needs to be properly regulated. This can be done either by analytical continuation from large  $t$  where the amplitudes are convergent [8] or by studying the spectral representation of the parton-nucleon scattering amplitude which underlie the parton-distribution functions. Both topics will be discussed briefly below, but meanwhile let us give the correctly regulated relation as found by Brodsky, Close, and Gunion(BCG) [9],

$$\begin{aligned} \frac{\delta M_N^2}{\delta m_i^2} &= \int_0^1 \frac{dx}{x} \left( f_i - f_i^{\text{Regge}} + \bar{f}_i - \bar{f}_i^{\text{Regge}} \right) (x) - \sum_{\alpha} \frac{\gamma_{\alpha}}{\alpha(0)} - \sum_{\bar{\alpha}} \frac{\bar{\gamma}_{\bar{\alpha}}}{\bar{\alpha}(0)} \quad (7) \\ f_i^{\text{Regge}}(x) &= \sum_{\alpha} \gamma_{\alpha} x^{-\alpha(0)} \quad \alpha(0) > 0 \\ \bar{f}_i^{\text{Regge}}(x) &= \sum_{\bar{\alpha}} \bar{\gamma}_{\bar{\alpha}} x^{-\bar{\alpha}(0)} \quad \bar{\alpha}(0) > 0 . \end{aligned}$$

Notice that the particular form of this subtraction entails that there can be no Regge pole with exactly  $\alpha = 0$  in the input fit (this is set by the BCG choice of the subtraction point in the parton-nucleon scattering matrix formalism). As a consequence we cannot currently examine the pion sum rule with standard pion distribution functions [10] since  $\alpha = 0$  constant terms do appear in those parametrizations. It may be possible to develop an equivalent formula with a different subtraction point to avoid this inconvenience.

The result of computing the properly regularized  $1/x$  moment from Eq. 7 for a few standard pdf sets is given in table 1. As can be seen, there is considerable spread in the results, and much room for improvement in the determination of the moments.

### 3 The $1/x$ Form Factor of the Nucleon

An important empirical way to access the  $1/x$  moment of parton distribution functions is by utilizing the forward limit of the generalized parton distribution (GPD) functions measured in deeply virtual Compton scattering (DVCS); specifically,

$$H(x, \zeta = 0, t = 0) = f(x) \quad (8)$$

Table 1: Weisberger integral  $\int_0^1 dx \frac{f(x)}{x}$  for MRST98 [2] and GRV parton distribution functions. Following BCG, we have analytically continued in  $t$  as in eq. (7) by adding and subtracting the divergent Regge terms. The sets are taken at low-energy input scales  $1\text{GeV}^2$  (MRST) and  $0.26(0.4)\text{GeV}^2$  for the LO(NLO) GRV set. The latter has no strange sea component at this low scale.

quark flavor	MRST Low gluon	MRST Central gluon	MRST Upper gluon	LO GRV	NLO GRV
$u$	34	8.6	12	133	26
$\bar{u}$	-1.3	-5.2	-7.1	62	5.8
$\frac{\delta M_N^2}{\delta m_u^2}$	33	3.4	4.9	195	32
$d$	0.98	-0.4	0.33	-20	-5.7
$\bar{d}$	-0.46	-0.75	-1.8	-62	-17
$\frac{\delta M_N^2}{\delta m_d^2}$	0.52	-1.1	-1.5	-82	-22
$s$	-0.43	-1.5	-2.2	0	0
$\frac{\delta M_N^2}{\delta m_s^2}$	-0.86	-3.0	-4.4	0	0
$g$	$\simeq 600$	$\simeq 400$	$\simeq 2900$	10	12

so that the  $F_{1/x}(t)$  form factor defined by

$$F_{1/x}(t) = \sum e_q^2 \int_0^1 dx \frac{H(x, 0, t)}{x} \quad (9)$$

should take in the  $t \rightarrow 0$  limit a value given by a sum of the  $1/x$  moments for various flavors. Unfortunately this equation is known to be rigorously valid for sizeable  $t$  only. In that case, the  $F_{1/x}(t)$  form factor is accessible via DVCS; the required DVCS amplitude in the GPD-collinear factorization formalism is given by

$$\mathcal{M}^{++}(s, t, Q^2) = \frac{-e_q^2 \sqrt{1-\zeta}}{2(1-\zeta/2)} \int_{\zeta-1}^1 dx \left[ \frac{1}{x-i\epsilon} + \frac{1}{x-\zeta+i\epsilon} \right] H(x, \zeta, t) . \quad (10)$$

(Here we have ignored the contribution of the  $E(x, \zeta, t)$  GPD, and work in the asymmetric frame).

The experimental determination of the  $F_{1/x}(t)$  form factor would in principle allow an analytic continuation in  $t$  to  $t = 0$ , thus providing  $1/x$  moment. However, as noted in the next section, it is not trivial to carry out such an extrapolation through the low  $t$  region due to Regge divergences.

A prediction for the  $1/x$  form factor of the nucleon has been given for a

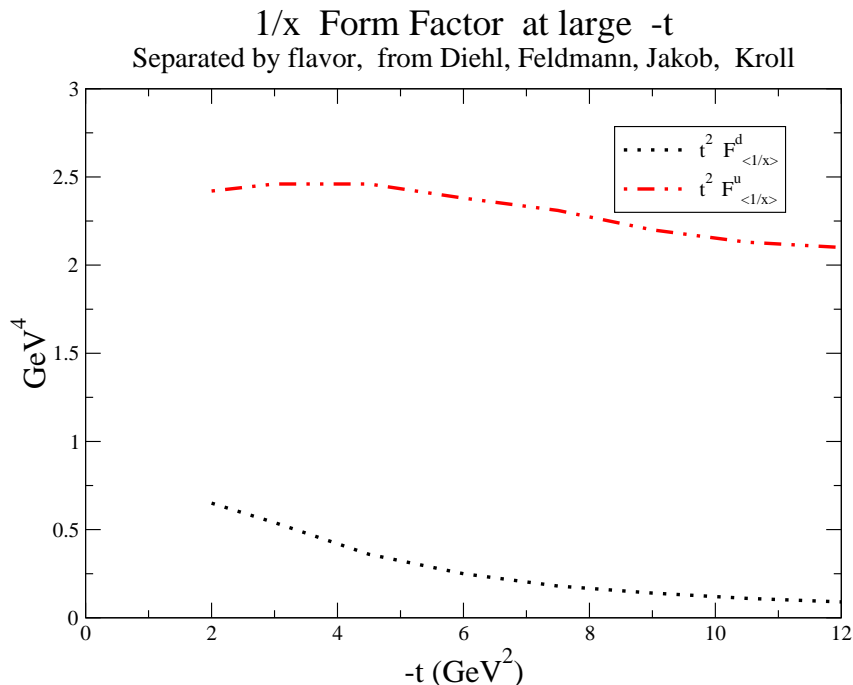


Figure 1: An evaluation of the  $1/x$  form factor of the proton assuming the Gaussian light-front constituent quark model utilizing flavor-separated form factors obtained from a set of GPD's fit to a number of conventional Dirac and Pauli form factors. Data from [11] is given only at large  $-t$  by direct calculation.

particular model, the Gaussian light-front constituent quark model; this is illustrated in figure 1.

## 4 Loss of collinear factorization in deeply virtual Compton scattering

It has been recently shown [12, 13] that the DVCS amplitudes can be most efficiently described in terms of  $t$ -channel Regge exchanges. The analysis proceeds along the lines of ref. [9] by employing a representation of the leading twist amplitude as an integral over the underlying parton-nucleon scattering amplitude [14]. The DVCS amplitude can then be written in terms of a

subtracted spectral representation

$$A^\pm = \sum_n c_n (2\pi)^4 \int dm^2 \left[ \frac{\rho_n}{s_{pp} - m^2 + i\epsilon} - \frac{\rho_R^n}{-m^2 + i\epsilon} \pm (s_{pp} \rightarrow u_{pp}) \right] f(k^2, k'^2) \quad (11)$$

The convergence needed for the handbag diagram is provided by the regularization procedure  $I_n = (m^2)^n \frac{d^n}{(dm^2)^n}$ ,  $n > 1$  and the functional form of  $f$ . The Regge behavior follows from the form  $\rho^R \propto (m^2)^{\alpha(t)}$  (under the assumption that the quark-nucleon matrix element has standard hadronic physics properties [14]).

At large  $t$ , one can eschew Regge behavior and think of such representation as a dispersive integral over diquark exchanges of varying mass. The form of the resulting nucleon GPDs (see figure 2) are similar to those found in ref. [15] and ref. [16] for the GPD of the pion.

However, if one now proceeds to study the low- $t$  region, one finds Regge poles at the break-points of the GPD, for example, approaching  $x = \zeta$  from higher values of  $x$  one finds

$$H(x \rightarrow \zeta^+, \zeta, t \simeq 0) \rightarrow \left[ \pi^2 m_q^2 I_{n-1} \beta \int_0^\infty \frac{d\phi \phi^\alpha}{(\phi^2 + m_q^2)^2} \right] \left[ \frac{(1-\zeta)^\alpha}{1+\alpha} (x-\zeta)^{-\alpha} - \frac{\zeta^{-\alpha}}{1+\alpha} \right] \quad (12)$$

where the function on the left bracket will be discussed in detail in our upcoming publication. The function on the right however shows clearly the Regge pole  $(x-\zeta)^{-\alpha}$ . As a result, whereas the DVCS amplitude correctly exhibits Regge scaling in  $s$ , its  $Q^2$  dependence does not track with the same power; the amplitude at nonzero- $t$  thus cannot scale with Bjorken  $\zeta$  alone. Such Regge contributions were not contemplated in the original proof of the collinear factorization theorem [18] and thus apparently make it fail. Current models, such as the one presented in in figure 2, which have soft behavior at the break-points also must be improved.

## 5 The $J = 0$ fixed-pole in Compton Scattering.

In Regge theory, hadron scattering amplitudes scale as  $s^\alpha(t)$ , where the exponent of the Regge pole evolves with  $t$ . A fixed pole at  $J = \alpha = 0$  corresponds to a constant real amplitude. Such behavior was proposed [19] and found [8] in Compton scattering in the late sixties. In their analysis Damashek and Gilman [8] used the forward dispersion relation for the Compton amplitude and measurements of total photoabsorption cross section  $\sigma(\gamma p \rightarrow X)$  to show

$H(x, \zeta, t = -1 \text{ GeV}^2)$  in perturbative diquark model  
various  $\zeta$

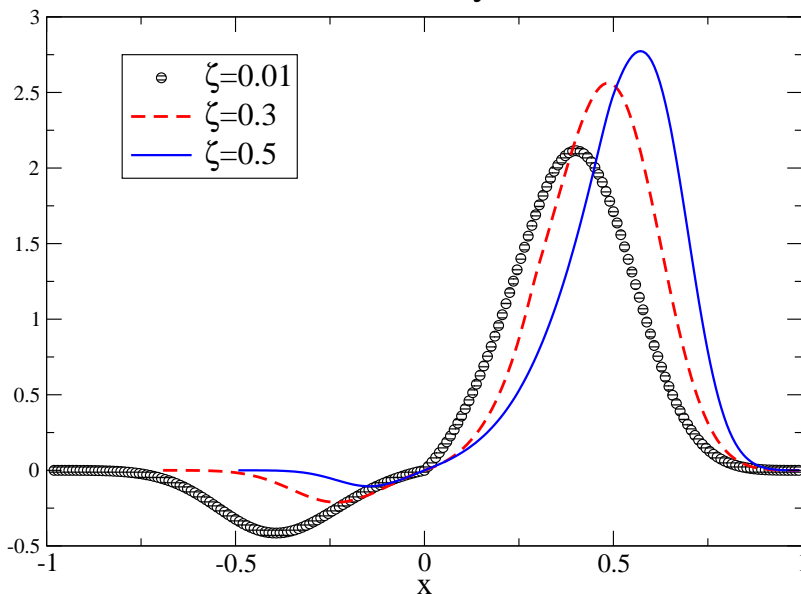


Figure 2: The GPD  $H(x, \zeta, t)$  at fixed  $-t = 1 \text{ GeV}^2$  in a perturbative quark-diquark model [17] with masses  $400 \text{ MeV}$  and  $800 \text{ MeV}$  for the quark and diquark, a vertex coupling  $g = 25$  and both  $u$  and  $s$  channel exchanges (the latter, covering the antiquark region  $x < 0$ , have been suppressed by an ad-hoc 0.2 factor since this is a valence-like model)



that the forward Compton scattering on the proton has a  $J = 0$  contribution. A formal proof that the Compton amplitude must present fixed pole behavior was given in ref. [20]. Physically it arises from the local four-point seagull interaction in scalar QCD or from the instantaneous fermion exchange interaction in the light-front QCD Hamiltonian [21, 22]. The  $J = 0$  contribution to the DVCS amplitude is thus independent of  $s$  for any photon virtuality and any momentum transfer  $t$ .

In general, the contribution to Compton scattering (real or virtual) which is directly sensitive to the  $1/x$  moment can be identified with the “handbag” diagram in QCD where the incoming and outgoing photons interact on the same valence quark line. Note that in the case of fixed  $\theta_{CM}$  angle Compton scattering, where  $t, u$ , and  $s$  are all large, the outgoing photon can be equally well emitted by another valence quark (see figure 3). Therefore, Compton

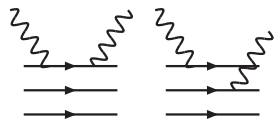


Figure 3: In the handbag mechanism the left diagram dominates the cross-section over all others such as, say, the right one. This is testable through the ratio of the Compton scattering cross sections of the neutron over the proton.

scattering at fixed angle does not isolate the handbag diagram. The optimum experimental approach is thus to work in the Regge regime for DVCS. As shown in ref. [9], the  $J = 0$  fixed pole has  $t$  dependence given precisely by the  $1/x$  form factor.

Thus, a good experimental strategy to extract the  $J = 0$   $F_{1/x}(t)$  form factor is to fix  $t$  and let  $s$  increase until only the constant fixed-pole amplitude remains. Since the contribution to the DVCS amplitude is real, it can be extracted from interference with the Bethe-Heitler amplitude [21]. In addition, if one wants to interpret this form factor in terms of a moment of GPD’s, one needs to demand  $Q^2 \gg -t$ . An upgraded Jefferson Laboratory with a 12 GeV beam should be able to reach perhaps  $s \simeq 40 \text{ GeV}^2$ ,  $Q^2 \simeq 6 \text{ GeV}^2$ ,  $t \simeq -3 \text{ GeV}^2$  and thus should be able to report a first measurement in a regime where the virtual Compton amplitude should become  $Q^2$  and  $s$  independent. The extracted  $t$  dependence would provide the first

measurement of the  $F_{1/x}(t)$  nucleon form factor. It is also possible that current measurements by the Hall A collaboration at JLab [23] of  $R_V(t)$  in real Compton scattering also yield the same physics, but there is no kinematic limit where one can perform the needed checks. The kinematically stringent Regge limit of DVCS at sizeable  $t$  provides further motivation for a future electron-proton collider.

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