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Flux Compactification

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We review recent work in which compactifications of string and M theory are constructed in which all scalar fields (moduli) are massive, and supersymmetry is broken with a small positive cosmological constant, features needed to reproduce real world physics. We explain how this work implies that there is a "landscape" of string/M theory vacua, perhaps containing many candidates for describing real world physics, and present the arguments for and against this idea. We discuss statistical surveys of the landscape, and the prospects for testable consequences of this picture, such as observable effects of moduli, constraints on early cosmology, and predictions for the scale of supersymmetry breaking.
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Contents

I. INTRODUCTION

It is an old idea that unification of the fundamental forces may be related to the existence of extra dimensions of space-time. Its first successful realization appears in the works of Kaluza (1921) and Klein (1926), which postulated a fifth dimension of space-time, invisible to everyday experience.

In this picture, all physics is described at a fundamental level by a straightforward generalization of general relativity to five dimensions, obtained by taking the metric tensor $g_{\mu\nu}$ to depend on five-dimensional indices $\mu = 0, 1, 2, 3, 4$, and imposing general covariance in five dimensions. Such a theory allows five-dimensional Minkowski space-time as a solution, a possibility in evident contradiction with experience. However, it also

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allows many other solutions with different or less symmetry. As a solution which could describe our universe, consider a direct product of four-dimensional Minkowski space-time, with a circle of constant periodicity, which we denote $2\pi R$. It is easy to check that at distances $r >> R$, the gravitational force law reduces to the familar inverse square law. Furthermore, at energies $E << \hbar/Rc$, all quantum mechanical wave functions will be independent of position on the circle, and thus if R is sufficiently small (in 1926, subatomic), the circle will be invisible.

The point of saying this is that the five dimensional metric $g_{\mu\nu}$, regarded as a field in four dimensions, contains additional, non-metric degrees of freedom. In particular, the components $g_{\mu 5}$ transform as a vector field, which turns out to obey the Maxwell equations in a curved background. Thus, one has a unified theory of gravitation and electromagnetism.

The theory contains one more degree of freedom, the metric component g_{55}, which parameterizes the radius R of the extra-dimensional circle. Since the classical Einstein equations are scale invariant, in the construction as described, there is no preferred value for this radius R. Thus, Kaluza and Klein simply postulated a value for it consistent with experimental bounds.

Just like the other metric components, the g_{55} component is a field, which can vary in four-dimensional space-time in any way consistent with the equations of motion. We will discuss these equations of motion in detail later, but their main salient feature is that they describe a (non-minimally coupled) massless scalar field. We might expect such a field to lead to physical effects just as important as those of the Maxwell field we were trying to explain. Further analysis bears this out, and quickly leads to effects such as new long range forces, or time dependence of parameters, in direct conflict with observation.

All this would be a historical footnote were it not for the discovery, which emerged over the period 1975–1985, that superstring theory provides a consistent quantum theory of gravity coupled to matter in ten space-time dimensions (Green *et al.*, 1987a,b). At energies low compared to its fundamental scale (the string tension), this theory is well described by ten-dimensional supergravity, a supersymmetric extension of general relativity coupled to Yang-Mills theory. But the nonrenormalizability of that theory is cured by the extended nature of the string.

Clearly such a theory is a strong candidate for a higher dimensional unified theory of the type postulated by Kaluza and Klein. Around 1985, detailed arguments were made, most notably by Candelas *et al.* (1985), that starting from the heterotic superstring theory, one could derive supersymmetric grand unified theories (GUTs) of the general class which, for completely independent reasons, had already been postulated as plausible extensions of the Standard Model up to very high energies. This construction, the first quasi-realistic string compactification, took ten-dimensional space-time to be a direct product of four-dimensional Minkowski space-time, with a six dimensional Ricci flat manifold, one of the so-called Calabi-Yau manifolds. Performing a Kaluza-Klein type analysis, one obtains a four-dimensional theory unifying gravity with a natural extension of the Standard Model, from a single unified theory with no free parameters.

However, at this point, the problem we encountered above rears its ugly head. Just like the classical Einstein-Maxwell equations, the classical supergravity equations are scale invariant. Thus, if we can find any solution of the type we just described, by rescaling the size R of the compactification manifold, we can obtain a one-parameter family of solutions, differing only in the value of R. Similarly, by making a rescaling of R with a weak dependence on four-dimensional position, one obtains approximate solutions. Thus, again R corresponds to a massless field in four dimensions, which is again in fatal conflict with observation.

In fact, the situation is even worse. Considerations we will discuss show that typical solutions of this type have not just one but hundreds of parameters, called moduli. Each will lead to a massless scalar field, and its own potential conflict with observation. In addition, the interaction strength between strings is controlled by another massless scalar field, which by a long-standing quirk of terminology is called the dilaton. Since this field is present in all string theories and enters directly into the formulas for observable couplings, many proposals for dealing with the other moduli problems, such as looking for special solutions without parameters, founder here.

On further consideration, the moduli are tied up with many other interesting physical questions. The simplest of these is just the following: given the claim that all of known physics can arise from a fundamental theory with no free parameters, how do the particular values we observe for the fundamental parameters of physics, such as the electron mass or the fine structure constant, actually emerge from within the theory? This question has always seemed to lie near the heart of the matter and has inspired all sorts of speculations and numerological observations, some verging on the bizarre.

This question has a clear answer within superstring theory, and the moduli are central to this answer. The answer may not be to every reader's liking, but let us come back to this in due course.

To recap, we now have a problem, a proliferation of massless scalar fields; and a question, the origin of fundamental parameters. Suppose we ignore the dynamics of the massless scalar fields for a moment, and simply freeze the moduli to particular values, in other words restrict attention to one of the multi-parameter family of possible solutions in an *ad hoc* way. Now, if we carry out the Kaluza-Klein procedure on this definite solution, we will be able to compute physical predictions, including the fundamental parameters. Of course, the results depend on the details of the assumed solution for the extra dimensions of space, and the particular values of the moduli.

Now returning to the problem of the massless scalar fields, a possible solution begins with the observation that the equations of motion of general relativity and supergravity are scale invariant only at the classical level. Defining a quantum theory of gravity (in more than two space-time dimensions) requires introducing a preferred scale, the Planck scale, and thus there is no reason that the quantum theory cannot prefer a particular value of R, or of the other moduli. Indeed, this can be demonstrated by simple considerations in quantum field theory. For example, given a conducting cavity, even one containing vacuum, one can measure an associated Casimir energy, which depends its size and shape. This agrees with the theoretically predicted vacuum energy of the zero-point fluctuations of the quantum electromagnetic field. Very similar computations show that a quantum field in a compactified extra dimensional theory will have a Casimir energy which depends on the size and other moduli parameters of the extra dimensions, and which contributes to the four-dimensional stress-energy tensor.

In a more complete treatment, this Casimir energy would be the first term in a systematic expansion of the quantum vacuum energy, to be supplemented by higher order perturbative and nonperturbative contributions. In higher dimensional theories, it is also possible to turn on background field strengths in the extra dimensions without breaking Lorentz invariance, and these contribute to the vacuum energy as well. All of these effects can be summarized in an effective potential, defined as the total vacuum energy, considered as a function of assumed constant values for the moduli fields.

We now work on the assumption that this effective potential, defined in precise analogy to the effective potentials of conventional quantum field theory and many-body physics, can be used in a very similar way: to determine the possible (metastable) vacuum states of the theory, as the local minima of the effective potential. Any configuration not at such a minimum will roll down to one, converting its excess potential energy into other entropically favored forms, such as radiation. This argument is very general and applies to all known physical systems with many degrees of freedom; it is widely accepted in cosmology as well, so there is no evident reason not to accept it in the present context.

Almost all effective potentials for systems in the real world have more than one local minimum. The consequences of this fact depend on the time scales of transitions between minima (quantum or thermally induced) compared to the time scales under study. If transitions proceed rapidly, the system will find the global minimum of the potential, and if this changes upon varying parameters the system undergoes a phase transition. On the other hand, if transitions between vacua proceed slowly, local minima are effectively stable, and one speaks of a system with multiple configurations. Both phenomena are ubiquitous; examples of extremely long-lived metastable configurations include most organic molecules (which "decay" to hydrocarbons and carbon dioxide), and all nuclei except ^{62}Ni, the nucleus which minimizes the binding energy per nucleon.

The structure of effective potentials responsible for multiple minima, metastability and transitions is central to a good deal of real world physics and chemistry. Although details are always essential, there are also principles which apply with some generality, which make up the theory of energy landscapes (Wales, 2003). The picturesque term "landscape" actually originated in evolutionary biology (Wright, 1932).

For reasons we will discuss in Sec. II, string vacua with small positive cosmological constant, as would fit present astronomical observations, are believed to be metastable and extremely long-lived even compared to cosmological time scales. Thus, if we find multiple local minima of the effective potentials derived from string/M theory compactification, the appropriate interpretation is that string/M theory has multiple configurations, the vacua.

Now, ever since the first studies of string compactification, it has appeared that choices were involved, at the very least the choice of compactification manifold, and other discrete choices, leading to multiple vacua. However, it was long thought that this might be an artifact of perturbation theory, or else not very interesting, as the constraints of fitting the data would pick out a unique candidate solution. While occasional suggestions to the contrary were made, as in Banks (1995b); Linde (2005); Schellekens (1998); Smolin (1997), these were not supported by enough evidence to attract serious attention.

This has changed in recent years, as increasingly detailed arguments have been developed for the existence of a large number of candidate vacua within string/M theory. (The bulk of our review will be devoted to these arguments, so we defer the references to there.) These vacua realize different values of the cosmological term, enabling an "anthropic" solution of the cosmological constant problem, along the lines set out by Banks (1984); Bousso and Polchinski (2000); Linde (1984b); Weinberg (1987), which can naturally accomodate the growing evidence for dark energy (see Copeland *et al.* (2006) for a recent overview).

Does anything pick out one or a subset of these vacua as the preferred candidates to describe our universe? At this point, we do not know. But, within the considerations we discuss in this review, there is no sign that any of the vacua are preferred. So far as we know, any sufficiently long-lived vacuum which fits all the data, including cosmological observations, is an equally good candidate to describe our universe. This is certainly how we proceed in analogous situations in other areas of physics. The analogy leads to the term "landscape of string vacua" and a point of view in which we are willing to consider a wide range of possibilities for what selected "our vacuum." Indeed, an extreme point of view might hold that, despite the evident centrality of this choice to all that we will ever observe, nevertheless it might turn out to be an undetermined, even "random" choice among many equally consistent alternatives.

Of course, such a claim would be highly controversial. And, while in our opinion the idea must be taken very seriously, it is far outrunning the present evidence. String/M theory is a theory of quantum gravity, and given our present limited understanding both of general principles of quantum gravity and of its microscopic definition, it is too early to take any definite position about such claims. Rather, in this review, we will try to state the evidence from various sides. To start with, since there is as yet no precise definition of the effective potential in string theory, we need to state our working definition, and justify it within our present understanding of the theory. Then, there are important differences between other physical theories and quantum gravity, which suggest various speculations about why some of the vacua which appear consistent at the level of our discussion, actually should not be considered. Another point in which quantum gravity plays an essential role is the idea that early cosmology leads to a "measure factor," an *a priori* probability distribution on the vacua which must be taken into account in making predictions.

We discuss all of these points in Sec. III. While pointing out many incomplete aspects of the theory, whose development might significantly change our thinking, we conclude that at present there is no clear evidence against, or well-formulated alternative to, the "null hypothesis" which states that each of these vacua is *a priori* a valid candidate to describe our universe. In fact, many of the suggested alternatives, at least within the general framework of string theory, would themselves require a significant revision of current thinking about effective field theory, quantum mechanics, or inflationary cosmology. Compared to these, the landscape hypothesis appears to us to be a fairly conservative option. We will argue as well that it can lead to testable predictions, perhaps by finding better selection principles, or perhaps by thinking carefully about the situation as it now appears.

To summarize the situation, while we have a criterion that determines preferred values for the size and other moduli, namely that our vacuum is a long-lived local minimum of the effective potential, this criterion does not determine the moduli uniquely, but instead gives us a set of possibilities, the vacua. Let us make an *ad hoc* choice of one of these vacua, and ask what physics it would predict.

To first address the question of massless scalar fields, while the moduli would still be fluctuating scalar fields, as in other physical problems, their effects could be estimated by a linearized analysis. As always, this allows for small fluctuations with frequency ω proportional to the second derivatives of the effective potential. Then, by quantum mechanics, the minimum energy of such a fluctuation is $E = \hbar\omega$, and thus we might expect to need to do experiments at energies at least E or at distances less than c/ω in order to see their effects. These are standard ideas in particle physics, summarized by the phrase that the effective potential could "lift" (give mass to) the moduli fields and make them unobservable at energies less than E. Their only remaining effect is that this physics, referred to as "moduli stabilization," sets the parameters in the actual solution, which enter into computing physical predictions.

What do we expect for the energy scale E? Although detailed computations may not be easy, the energy scales which enter into such a computation include the Planck scale, the string tension, and the inverse size of the extra dimensions $\hbar c/R$ (often referred to as the "Kaluza-Klein scale" or M_{KK}). There is no obvious need for the lower energy scales of present-day physics to enter, and thus it seems plausible that a detailed analysis would lead to all moduli gaining masses comparable to the new scales of string theory. In this case, the prospects for direct observation of physical effects of the moduli would be similar to those for direct observation of excited string modes or of the extra dimensions, in other words a real possibility but not a particularly favored one.

It is possible that some moduli might gain lower masses and thus have more direct experimental consequences. One class of observational bounds on the masses of moduli arise from from fifth-force experiments; these are important for masses less than about 10^{-3}eV. A stronger bound comes from cosmology; masses up to 10TeV or more are constrained by the requirement that energy trapped in oscillations of the moduli fields should relax before primordial nucleosynthesis (Banks *et al.*, 1994; de Carlos *et al.*, 1993). Both bounds admit loopholes, and thus this theoretical possibility is of particular interest for phenomenology.

How does one compute the effective potential in string theory? For a long time, progress in this direction was very slow, due to the belief that in the supersymmetric compactifications of most interest, the effective potential would arise entirely from nonperturbative effects. This brought in the attractive possibility of using asymptotic freedom and dimensional transmutation to solve the hierarchy problem (Witten, 1981a), but also the difficulty that such effects could only be computed in the simplest of theories.

Other possibilities were occasionally explored. A particularly simple one is to turn on background magnetic fields (or generalized p-form magnetic fields) in the extra dimensions. These contribute the usual B^2 term to the energy, but since they transform as scalars in the observable four dimensions this preserves Poincaré symmetry, and thus such configurations still count as "vacua." Furthermore, writing out the B^2 term in a curved background, one sees that it depends non-trivially on the metric and thus on the moduli, and thus it is an interesting contribution to the effective potential for moduli stabilization. However, while this particular construction, usually called the flux potential, is simple, the lack of understanding of other terms in the effective potential and of any overall picture inhibited work in this direction.

Over the last few years, this problem has been solved, by combining this simple idea with many others: the concepts of superstring duality, other techniques for computing nonperturbative effects such as brane instantons, and mathematical techniques developed in the study of mirror symmetry, to compute a controlled approximation to the effective potential in a variety of string and M theory vacua. The basic result is that these effective potentials can stabilize moduli and lead to supersymmetry breaking with positive cosmological constant, just as is required to get a vacuum which could describe our universe. One can go on to get more detailed results, with applications in particle physics and cosmology which we will discuss.

We have now finished the non-technical summary of the basic material we will cover in this review, and turn to an outline. In Sec. II, we assume a general familiarity with particle physics concepts, but not necessarily with string theory. Thus, we begin with an overview of the basic ingredients present in the different 10d string theories, and the known types of compactification. We then discuss some of the data needed to specify a vacuum, such as a choice of Calabi-Yau manifold, and a choice of moduli. We then explain in general terms how the fluxes can be expected to induce potentials for moduli of the metric of the extra-dimensional manifold, M. Finally, we describe some of the current applications of flux vacua: to models of the cosmological constant, to particle physics, and to early universe cosmology.

In Sec. III, we begin to assume more familiarity with string theory, and critically examine the general framework we will use in the rest of the paper: that of 10d and 4d effective field theory. While our present day understanding of physics fits squarely into this framework, there are conceptual reasons to worry about its validity in a theory of quantum gravity.

In Sec. IV, we turn to detailed examples of concrete constructions of flux vacua. These include the simplest constructions which seem to fix all moduli, in both the IIb and the IIa theories. We also comment on recent progress, which suggests that there are many extensions of these stories to unearth.

It will become clear, from both the general arguments in Sec. II, and the concrete examples in Sec. IV, that the number of apparently consistent quasi-realistic flux vacua is extremely large, perhaps greater than 10^{500}. Therefore, we will need to use statistical reasoning to survey broad classes of vacua. In Sec. V, we describe a general framework for doing this, and give an overview of the results.

We conclude with a discussion of promising directions for further research in Sec. VI.

II. A QUALITATIVE PICTURE

We begin by briefly outlining the various known classes of quasi-realistic compactifications, to introduce terminology, give the reader a basic picture of their physics, and explain how observed physics (the Standard Model) is supposed to sit in each. A more detailed discussion of each class will be given in Sec. IV, while far more complete discussions can be found in (Green *et al.*, 1987a,b; Johnson, 2003; Polchinski, 1998a,b; Zwiebach, 2004).

We then introduce some of the mathematics of the compactification manifolds, particularly the Calabi-Yau manifolds, to explain why moduli are more or less inevitable in these constructions. Even more strikingly, this mathematics suggests that the number of types of matter in a typical string/M theory compactification is of order hundreds or thousands, far more than the 15 or so (counting the quarks, leptons and forces) which we have observed to date. Thus, a central problem in string compactification is to explain why most of this matter is either very massive or hidden (so far), and give us a good reason to believe in this seemingly drastic exception to Occam's razor.

In the next subsection, we explain flux compactification, and how it solves the problem of moduli stabilization. In particular, it becomes natural that almost all moduli fields should be very massive, explaining why they are not seen.

We then explain, following Bousso and Polchinski (2000), why flux compactifications in string theory lead to large numbers of similar vacua with different values of the cosmological constant, leading to an "anthropic" solution of the cosmological constant problem. This solution depends crucially on having the many extra types of unobserved matter we just mentioned and might be regarded as the "justification" of this generic feature of string compactification.

Finally, we outline some of the testable consequences this picture might lead to. These include not just observable effects of the moduli, but also calculable models of inflation, and new mechanisms for solving the hierarchy problem of particle physics.

A. Overview of string and M theory compactification

String/M theory is a theory of quantum gravity, which can at present be precisely formulated in several weakly coupled limits. There are six such limits; five of these are the superstring theories in ten space-time dimensions (Polchinski, 1998a,b), called type IIa, type IIb, heterotic $E_8 \times E_8$, heterotic $SO(32)$ and type I. In addition, there is an eleven dimensional limit, usually called M theory (Duff, 1996). All of these limits are described at low energies by effective higher dimensional supergravity theories. Arguments involving duality (Bachas *et al.*, 2002; Polchinski, 1996) as well as various partial nonperturbative definitions (Aharony *et al.*, 2000; Banks, 1999) strongly suggest that this list of these weakly coupled limits is complete, and that all are limits of a single unified theory.

A quasi-realistic compactification of string/M theory is a solution of the theory, which looks to a low energy observer like a four dimensional approximately Minkowski space-time, with physics roughly similar to that of the Standard Model coupled to general relativity. The meaning of "roughly similar" will become apparent as we proceed, but certainly requires obtaining the correct gauge group, charged matter content and symmetries, as well as arguments that the observed coupling constants and masses can arise.

Now, of the six weakly coupled limits, the type II theories and M theory have 32 supercharges and (at least at first sight) do not include Yang-Mills sectors, a problem which must be solved to get quasi-realistic compactifications. The other three theories have 16 supercharges and include Yang-Mills sectors, $SO(32)$ from the open strings in type I, and either $E_8 \times E_8$ or $SO(32)$ in the heterotic strings. On the other hand, by ten-dimensional supersymmetry, the only fermions with Yang-Mills quantum numbers are the gauginos, transforming in the adjoint of the gauge group. Thus, we must explain how such matter can give rise to the observed quarks and leptons, to claim we have a quasi-realistic compactification.

Although there is a rich theory of string compactification to diverse dimensions, here we restrict attention to quasi-realistic compactifications. The original models of this type, the $E_8 \times E_8$ heterotic string compactified on a Calabi-Yau manifold (Candelas *et al.*, 1985), have $\mathcal{N} = 1$ supersymmetry, and unify $SU(3) \times SU(2) \times U(1)$ into a simple grand unified gauge group such as $SU(5), SO(10)$ or E_6.

Let us briefly discuss the important physical scales in a compactification. Of course, one of the main goals is to explain the observed four dimensional Planck scale, which we denote $M_{P,4}$ or simply M_P. By elementary Kaluza-Klein reduction of D-dimensional supergravity,

$$M_{P,D}^{D-2} \int_{M \times \mathbb{R}^{3,1}} d^D x \, \sqrt{g} R^{(D)} \rightarrow$$
$$(\mathrm{Vol} M) M_{P,D}^{D-2} \int_{\mathbb{R}^{3,1}} d^4 x \, \sqrt{g} R^{(4)} + \cdots ,$$

this will be related to the D-dimensional Planck scale $M_{P,D}$, and the volume of the compactification manifold Vol(M). Instead of the volume, let us define the Kaluza-Klein scale

$$M_{KK} = 1/Vol(M)^{1/(D-4)},$$

at which we expect to see Kaluza-Klein excitations; the relation then becomes

$$M_{P,4}^2 = \frac{M_{P,D}^{D-2}}{M_{KK}^{D-4}}. \quad (1)$$

In the simplest (or "small extra dimension") picture, used in the original work on string compactification, all of these scales are assumed to be roughly equal. If the Yang-Mills sector is also D-dimensional, this is forced upon us, to obtain an order one four-dimensional gauge coupling; there are other possibilities as well.

1. Supersymmetry

There are many reasons to focus on compactifications with low energy $\mathcal{N} = 1$ supersymmetry.[1] The best reasons to focus on supersymmetric candidates for weak scale physics, come from the bottom-up perspective. SUSY suggests natural extensions of the Standard Model such as the minimal supersymmetric Standard Model (MSSM) (Dimopoulos and Georgi, 1981), or non-minimal SSM's with additional fields. These models can solve the hierarchy problem, can explain coupling unification, can contain a dark matter candidate, and have other attractive features. But so far, all this is only suggestive, and these models tend to have other problems, such as reproducing precision electroweak measurements and a (presumed) Higgs mass $M_H \geq 113$GeV. Thus, many alternative models which can explain the hierarchy, and even the original "desert" scenario which postulates no new matter below the GUT scale, are at this writing still in play.

Since collider experiments with a good chance of detecting TeV scale supersymmetry are in progress at Fermilab and scheduled to begin soon at Cern, the question of what one can expect from theory has become very timely. We have just given the standard bottom-up arguments for low energy SUSY, and these were the original motivation for the large effort devoted to studying such compactifications of string/M theory over the past twenty years. From this study, other top-down reasons to focus on SUSY have emerged, having more to do with the calculational power it provides. Let us summarize some of these motivations.

First, there are fairly simple scenarios in which an assumed high scale $N = 1$ supersymmetry, is broken by dynamical effects at low energy. In such compactifications, supersymmetry greatly simplifies the computation of the four dimensional effective Lagrangian, as powerful physical and mathematical tools can be brought to bear. Now this may be more a question of theoretical convenience than principle, as in many models (such as the original GUT's) perturbation theory works quite well at high energy. But, within our present understanding of string/M theory, it is quite important.

Second, as we will discuss in Sec. II.F.2, supersymmetry makes it far easier to prove metastability, in other words that a given vacuum is a local minimum of the effective potential. In particle physics terms, metastability is the condition that the scalar field mass terms satisfy $M_i^2 \geq 0$. Now in supersymmetric theories, there is a bose-fermi mass relation, $M_{Bose}^2 = M_{Fermi}(M_{Fermi} - X)$, where X is a mass scale related to the scale of supersymmetry breaking. Thus, all one needs is $|M_{Fermi}| >> X$, to ensure metastability.

[1] As is well known, $N > 1$ supersymmetry in $d = 4$ does not allow for chiral fermions.

At first, this argument may not seem very useful, as in many realistic models the observed fermions all have $M_{Fermi} \leq X$. But, of course, this is why these fermions have already been observed. As it will turn out, typical string compactifications have many more particles, and this type of genericity argument will become very powerful.

Note that neither of these arguments refers directly to the electroweak scale and the solution of the hierarchy problem. As we formulate them more carefully, we will find that their requirements can be met even if supersymmetry is broken so far above the electroweak scale that it is irrelevant to the hierarchy problem.

This will lead to one of the main conclusions of the line of work we are reviewing, which is that *TeV scale supersymmetry is not inevitable* in string/M theory compactification. Rather, it is an assumption with a variety of good theoretical motivations, which we can expect to hold in some string/M theory compactifications. However, there are a priori three other qualitatively distinct classes of models. There are models where supersymmetry is broken at scales which are well described by four-dimensional effective field theory, allowing us to use 4d tools to control the analysis, but in which it is not directly relevant to solving the hierarchy problem; there are models where SUSY is broken at the KK scale, and there are even arguments in self-consistent perturbation theory, such as (Silverstein, 2001), for other classes of models, in which supersymmetry is broken at the string scale.

In any case, for the reasons we just discussed, we will proceed with the assumption that our compactification preserves $d = 4$, $N = 1$ supersymmetry at the KK scale, and ask what this implies for the compactification manifold M. In the compactifications we discuss, this is related to the existence of covariantly constant spinors on M, which is determined by its holonomy group, denote this $\mathrm{Hol}(M)$. We omit the details of this standard argument for lack of space (see Green *et al.* (1987b)), but cite the main result. This is that the number of supersymmetries in $d = 4$, is equal to the number of supercharges in the higher dimensional theory, divided by 16, times the number of singlets in the decomposition of a spinor $\mathbf{4}$ of $SO(6)$ under $\mathrm{Hol}(M)$. In the generic case of $\mathrm{Hol}(M) \cong SO(6)$ this is zero, so to get low energy supersymmetry we require $\mathrm{Hol}(M) \subset SO(6)$, a condition on the manifold and metric referred to as *special holonomy.*

All possible special holonomy groups were classified by Berger (1955), and the results relevant for supersymmetry in $d = 4$ are the following. For dim $M = 6$, as would be needed to compactify the string theories, the special holonomy groups are $U(3)$ and $SU(3)$, and subgroups thereof. The only choice of $\mathrm{Hol}(M)$ for which the spinor of $SO(6)$ contains a unique singlet is $SU(3)$. Spaces which admit a metric with this special holonomy are known as Calabi-Yau manifolds; their existence was proven in Yau (1977), and we will discuss some of their properties later. One can show that the special holonomy metric is Ricci flat, and thus this choice takes us a good part of the way towards solving the ten-dimensional supergravity equations of motion.

For dim $M = 7$, as would be used in compactifying M theory, the only choice leading to a unique singlet is $\mathrm{Hol}(M) \cong G_2$. Again, manifolds of G_2 holonomy carry Ricci flat metrics, so this leads to another new class of compactifications.

These are two of three classes of manifold which are of particular interest for quasi-realistic string/M theory compactification, the G_2 holonomy manifolds and the Calabi-Yau threefolds (these are complex manifolds, and the standard nomenclature refers to their complex dimension, which is three). The third class are the "elliptically fibered Calabi-Yau fourfolds," used in F theory compactification. We will defer discussion of these to Sec. IV; physically they are closely related to certain type IIb compactifications.

The outcome of the discussion so far, reflecting the state of the field in the late 1980's, is that the three theories with 16 supercharges and Yang-Mills sectors, all admit compactification on Calabi-Yau manifolds to $d = 4$ vacua with $N = 1$ supersymmetry, and gauge groups of roughly the right size to produce GUT's. On the other hand, the theories with 32 supercharges have various problems; the type II theories seem to lead to $N = 2$ supersymmetry and too small gauge groups, while M theory on a smooth seven dimensional manifold cannot lead to chiral fermions (Witten, 1981b). In fact all of these problems were later solved, but let us here follow the historical development.

2. Heterotic string

The starting point for Candelas *et al.* (1985) (CHSW) was the observation that the grand unified groups are too large to obtain from the Kaluza-Klein construction in ten dimensions, forcing one to start with a theory containing ten-dimensional Yang-Mills theory; furthermore the matter representations $5 + \bar{10}$, 16 and 27 can be easily obtained by decomposing the E_8 adjoint (and not from $SO(32)$), forcing the choice of the $E_8 \times E_8$ heterotic string.

General considerations of effective field theory make it natural for the two E_8's to decouple at low energy, so in the simplest models, the Standard Model is embedded in a single E_8, leaving the other as a "hidden sector." But what leads to spontaneous symmetry breaking from E_8 to E_6 or another low energy gauge group? This comes because we can choose a non-trivial background Yang-Mills connection on M, let us denote this V. Such a connection is not invariant under E_8 gauge transformations and thus will spontaneously break some gauge symmetry, at the natural scale of the compactification M_{KK}. The remaining unbroken group at low energies is the commmutant in E_8 of the holonomy group of V. Simple group theory, which we will see in an example below, implies that to realize the GUT groups E_6, $SO(10)$ and $SU(5)$, the holonomy of V must be $SU(3)$, $SU(4)$ or $SU(5)$ respectively.

Not only is E_8 gauge symmetry breaking possible, it is actually required for consistency. As part of the Green-Schwarz anomaly cancellation mechanism, the heterotic string has a three-form field strength $\tilde{H}_3$ with a modified Bianchi identity,

$$d\tilde{H}_3 = \frac{\alpha'}{4}\left(Tr(R \wedge R) - Tr(F_2 \wedge F_2)\right) \ . \tag{2}$$

In the simplest solutions, $\tilde{H}_3 = 0$, and then consistency requires the right hand side of (2) to vanish identically. The solutions of Candelas *et al.* (1985) accomplish this by taking the "standard embedding," in which one *equates* the E_8 gauge connection on M (in one of the two E_8's) with the spin connection ω, i.e. considers an E_8 vector bundle $V \to M$ which is $V = TM$. In this case, since $F = R$ for one of the E_8's, and vanishes for the other, (2) is trivially satisfied, and (by considerations we give in Sec. IV) so are the Yang-Mills equations.

Thus, any Calabi-Yau threefold M, gives rise to at least one class of heterotic string compactifications, the CHSW compactifications. The holonomy of V is the same as that of M, namely $SU(3)$, and thus this construction leads to an E_6 GUT. Below the scale M_{KK}, there is a 4d $\mathcal{N} = 1$ supersymmetric effective field theory governing the light fields. In the CHSW models, these include

- A pure E_8 $\mathcal{N} = 1$ SYM theory, the hidden sector.

- An "observable" E_6 gauge group. One can also make simple modifications to V (tensoring with Wilson lines) to accomplish the further breaking to $SU(3) \times SU(2) \times U(1)$ at M_{KK}, so typically in these models $M_{GUT} \sim M_{KK}$.

- Charged matter fields. The reduction of the E_8 gauginos will give rise to chiral fermions in various 4d matter (chiral) multiplets. The adjoint of E_8 decomposes under $E_6 \times SU(3)$ as

  $$\mathbf{248} = (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \bar{\mathbf{3}}) \oplus (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \ . \tag{3}$$

  Thus we need the spectrum of massless modes arising from charged matter on M in various $SU(3)$ representations. As explained in Green *et al.* (1987b), this is determined by the Dolbeault cohomology groups of M; thus

  $$n_{27} = h^{2,1}(M), \quad n_{\overline{27}} = h^{1,1}(M) \tag{4}$$

  are the numbers of chiral multiplets in the 27 and $\overline{27}$ representations of E_6. Since for a Calabi-Yau manifold the Euler character $\chi = 2(h^{1,1} - h^{2,1})$, we see that the search for three-generation E_6 GUTs in this framework will be transformed into a question in topology: the existence of Calabi-Yau threefolds with $|\chi| = 6$. This problem was quickly addressed, and quasi-realistic models were constructed, beginning with Greene *et al.* (1986); Tian and Yau (1986).

- Numerous gauge neutral **moduli** fields. The Ricci-flat metric on the Calabi-Yau space M is far from unique. By Yau's theorem (Yau, 1977), it comes in a family of dimension $2h^{2,1}(M) + h^{1,1}(M)$. As we will describe at much greater length below, the parameters for this family, along with $h^{1,1}(M)$ axionic partners, are moduli corresponding to infinitesimal deformations of the complex structure and the Kähler class of M. In addition, there is also the dilaton chiral multiplet, containing the field which controls the string coupling, and an axion partner.

- More model dependent modes arising from the $(\mathbf{1}, \mathbf{8})$ in (3). These correspond to infinitesimal deformations of the solutions to the Yang-Mills field equations, and thus are also moduli, in this case moduli for the gauge connection V. By giving vacuum expectation values to these scalars, one moves out into a larger space of compactifications with $V \neq TM$.

It should be emphasized that the CHSW models, based on the standard embedding $V \cong TM$, are a tiny fraction of the heterotic Calabi-Yau compactifications. More generally, a theorem of Donaldson, Uhlenbeck and Yau relates supersymmetric solutions of the Yang-Mills equations (equivalently, solutions of the hermitian Yang-Mills equations) to stable holomorphic vector bundles V over M. Many such bundles exist which are not in any way related to TM.

Solutions of the Bianchi identity (2) then become more involved. Instead of solving it exactly, one can argue that if one solves (2) in cohomology, then there is no obstruction to extending the solution order-by-order in an expansion in the inverse radius of M (Witten and Witten, 1987).

These more general models are of great interest because they allow for more general phenomenology. Instead of GUTs based on E_6, which contain many unobserved particles per generation, one can construct $SO(10)$ and $SU(5)$ models. The technology involved in constructing such bundles is quite sophisticated; some state of the art constructions appear in Donagi *et al.* (2005) and references therein.

One can then go on to compute couplings in the effective field theory at the compactification scale. Perhaps the most characteristic feature of weakly coupled heterotic models is a universal relation between the four dimensional Planck scale, the string scale, and the gauge coupling, which follows because all interactions are derived from the same closed string diagram. At tree level, this relation is

$$M^2_{Planck,4} \sim \frac{M^2_{KK}}{g^{8/3}_{YM}}. \tag{5}$$

Since the observed gauge couplings are order one, this clearly requires the extra dimensions to be very small. Actually, if we put in the constants, this relation leads to a well known problem, as discussed in Witten (1996c) and references there: if we take a plausible grand unified coupling $g_{YM}^2 \sim 1/25$, one finds $M_{KK} \sim 10^{18}$GeV which is far too large for the GUT scale. Various solutions to this problem have been suggested, such as large one-loop corrections to Eq. (5) (Kaplunovsky, 1988).

Perhaps the most interesting of these proposed solutions is in the so-called "heterotic M theory" (Horava and Witten, 1996; Witten, 1996c). Arguments from superstring duality suggest that the strong coupling limit of the ten-dimensional $E_8 \times E_8$ heterotic string is eleven-dimensional M theory compactified on an interval; the two ends of the interval provide ten-dimensional "end of the world branes" each carrying an E_8 super Yang-Mills theory. In this theory, while much of the previous discussion still applies, the relation Eq. (5) is drastically modified.

Finally, there are "non-geometric" heterotic string constructions, based on world-sheet conformal field theory, such as (Antoniadis *et al.*, 1987; Kawai *et al.*, 1986, 1987; Narain *et al.*, 1987). In some cases these can be argued to be equivalent to geometric constructions (Gepner, 1987). The full picture is not at all clear at present.

3. Brane models

Following the same logic for type II theories leads to $\mathcal{N} = 2$ supersymmetric theories. Until the mid 1990s, the only known way to obtain $\mathcal{N} = 1$ supersymmetry from type II models was through "stringy" compactifications on asymmetric orbifolds (Narain *et al.*, 1987). A powerful theorem of Dixon *et al.* (1987) demonstrated that this would never yield the Standard Model, and effectively ended the subject of type II phenomenology for 8 years.

After the discovery of Dirichlet branes (Polchinski, 1995) this lore was significantly revised, and quasi-realistic compactifications can also arise in both type IIa and type IIb theories. A recent review with many references appears in Blumenhagen *et al.* (2005a). Since it is in the type II case that flux compactifications are presently most developed, we need to discuss this in some detail.

Dirichlet branes provide a new origin for non-abelian gauge symmetries (Witten, 1996a). Furthermore, intersecting branes (or branes with worldvolume fluxes) can localize chiral matter representations at their intersection locus (Berkooz *et al.*, 1996). And finally, an appropriate choice of D-branes can preserve some but not all of the supersymmetry present in a type II compactification. Thus, type II strings on Calabi-Yau manifolds, with appropriate intersecting brane configurations, can give rise to chiral $\mathcal{N} = 1$ supersymmetric low energy effective field theories.

There are three general classes of type II $\mathcal{N} = 1$ brane compactifications on Calabi-Yau manifolds:

- IIa orientifold compactifications with D6 branes.

- IIb orientifold compactifications with D7 and D3 branes.

- Generalized type I compactifications; in other words IIb orientifold compactifications with D9 and D5 branes.

After the choice of Calabi-Yau, a particular compactification is specified by a choice of orientifold projection (Gimon and Polchinski, 1996), and a choice of how the various Dirichlet branes are embedded in space-time. Each of the branes involved is "space-filling," meaning that they fill all four Minkowski dimensions; the remaining spatial dimensions ($p - 3$ for a Dp-brane) must embed in a supersymmetric cycle of the compactification manifold (to be further discussed in Sec. IV). Finally, since Dirichlet branes carry Yang-Mills connections, just as in the heterotic construction one must postulate the background values for these fields. The nature of this last choice depends on the class of model; it is almost trivial for IIa, and for the generalized type I model with D9 branes one uses essentially the same vector bundles as in the heterotic construction, while in the IIb model with D7 and D3 branes, the number of choices here are intermediate between these extremes.

In a full analysis, a central role is played by the so-called tadpole conditions. We will go into more detail about one of these (the D3 tadpole condition) later. These conditions have more than one physical interpretation. In a closed string language, they express the condition that the total charge on the compactification manifold, including Dirichlet branes, orientifold planes and all other sources, must vanish, generalizing the Gauss' law constraint that the total electric charge in a closed universe must vanish. In an open string language, they are related to anomalies, and generalize the condition (2) related to anomaly cancellation in heterotic strings. In any case, a large part of the general problem of finding and classifying brane models, is to list the possible supersymmetric branes, and then to find all combinations of such branes which solve the tadpole conditions.

The collection of all of these choices (orientifold, Dirichlet branes and vector bundles on branes) is directly analogous to and generalizes the choice of vector bundle in heterotic string compactification. In some cases, such as the relation between heterotic $SO(32)$ and type I compactification, there is a precise relation between the two sets of constructions, using superstring duality. There are also clear relations between the IIa and both IIb brane constructions, based on T-duality and mirror symmetry between Calabi-Yau manifolds.

The predictions of the generic brane model are rather different from the heterotic models. Much of this is because the relation between the fundamental scale and the gauge coupling, analogous to Eq. (5), takes the form

$$g_{YM}^2 = \frac{g_s l_s^{p-3}}{\mathrm{Vol}X},$$

where Vol$X$ is the volume of the cycle wrapped by the particular branes under consideration. Since *a priori,* volumes of cycles have no direct relation to the total volume of the compactification manifold, one can have many more possible scenarios for the fundamental scales in these theories, including large extra dimension models. A related idea is that the branes responsible for the observed (standard model) degrees of freedom can be localized to a small subregion of the compactification manifold, allowing its energy scales to be influenced by "warping" (Randall and Sundrum, 1999a).

Even if one has small extra dimensions, coupling unification is generally not expected in brane models. This is because the different gauge groups typically arise from stacks of branes wrapping different cycles, with different volumes, so the couplings have no reason to be equal.

To conclude this overview, let us mention two more classes of compactification which can be thought of as strong coupling limits of the brane constructions, and share many of their general properties. First, there are compactifications of M theory on manifolds of G_2 holonomy; these are related to IIa compactification with D6-branes by following the general rules of IIa–M theory duality. Second, there are F theory compactifications on elliptically fibered fourfolds; these can be obtained as small volume limits of M theory on Calabi-Yau fourfolds, or by varying parameters in IIb compactification with D7 and D3 branes. Both of these more general classes have duality relations with the heterotic string constructions, so that (in a still only partially understood sense) all of the $\mathcal{N} = 1$ compactifications are connected via superstring dualities, supporting the general idea that all are describing vacuum states of a single all-encompassing theory.

B. Moduli fields

In making any of the string compactifications we just described, in order to solve the Einstein equations, we must choose a Ricci-flat metric g_{ij} on the compactification manifold M. Now, given such a metric, it will always be the case that the metric λg_{ij} obtained by an overall constant rescaling is also Ricci-flat, because the Ricci tensor transforms homogeneously under a scale transformation. Thus, Ricci-flat metrics are never unique, but always come in families with at least one parameter.

Mathematically, the parameter space of distinct (diffeomorphism inequivalent) Ricci-flat metrics is by definition the moduli space of Ricci-flat metrics. This is a manifold, possibly with singularities, and thus we to fully describe it we would need to introduce charts and local coordinates in each chart. Let us work locally, and call these coordinates t^α with $1 \leq \alpha \leq n$.

What is the physics of this? In general treatments of Kaluza-Klein reduction, one decomposes the D-dimensional equations of motion as a sum of terms, say for a massless scalar field as ϕ

$$0 = (\eta^{\mu\nu}\partial_\mu\partial_\nu + \Delta_M)\phi,$$

where Δ_M is the scalar Laplacian on M. One then writes the higher dimensional field ϕ as a sum over eigenfunctions f_k of Δ_M,

$$\phi(x, y) = \sum_k \phi_k(x) f_k(y).$$

Substituting, one finds that the eigenvalues λ_k become the masses squared of an infinite set of fields, the "Kaluza-Klein modes."

Doing the same in the presence of moduli, we might consider the parameters t^i as undetermined, and write

$$\int_{M\times\mathbb{R}^{3,1}} d^D x\, \sqrt{G} R^{(D)} \left[\, G(\vec{t}) + \delta G \,\right],$$

where $G(\vec{t})$ is an explicit parameterized family of Ricci flat metrics on M, and δG are the small fluctuations around it. We then expand δG in eigenfunctions, to find the spectrum of the resulting low energy effective field theory.

However in doing this, we should be careful not to double count degrees of freedom. Variations δG which correspond to varying moduli,

$$\delta G_\alpha \equiv \frac{\partial G(\vec{t})}{\partial t^\alpha}$$

are perfectly good variations – which must therefore correspond to fields in the four-dimensional theory. Working out the linearized equations, one finds that each coordinate t^i becomes a massless field. Rather than do this computation explicitly, one can simply note that compatibility between the equations of motion in D dimensions and four dimensions, requires that a small constant variation of t^i must lead to a solution in four dimensions. This will only be the case if it is massless.

Intuitively, since we get a valid compactification for any particular choice of Ricci-flat metric, locality demands that we should be able to vary this choice at different points in four dimensional space-time. By general principles, such a local variation must be described by a field. The situation is analogous to that of a spontaneously broken symmetry. By locality, we can choose the symmetry breaking parameter to vary in space-time, and if the parameter was continuous it will lead to a massless field, a Goldstone mode.

However, this is only an analogy; there is a crucial difference between the two situations. The origin of the Goldstone mode in symmetry breaking implies of course that the physics of any constant configuration of this field must be the same (since all are related by a symmetry). On the other hand, moduli can exist *without a symmetry.*

In this case, physics can and usually will depend on its value. Thus, one finds a parameterized family of *physically distinct vacua*, the moduli space $\mathcal{M}$, connected by simply varying massless fields.

While this situation is almost never encountered in real world physics, this is not because it is logically inconsistent. Rather, it is because in the absence of symmetry, there is no reason the effective potential should not depend on all of the fields. Thus, even if we were to find a family of vacua at some early stage of our analysis, in practice the vacuum degeneracy would always be broken by corrections at some later stage.

A well known loophole to this statement is provided by supersymmetric quantum field theories. Due to non-renormalization theorems, such moduli spaces often persist to all orders in perturbation theory or even beyond. These theories manifest different low energy physics at distinct points in $\mathcal{M}$, and thus provide a theoretical example of the phenomenon we are discussing here. [2]

Conversely, one might argue that, given that supersymmetry is broken in the real world, any moduli we find at this early stage will be lifted after supersymmetry breaking. We will come back to this idea later, once we have more of the picture. We will eventually argue that while this is true, in models with low energy supersymmetry breaking, it is more promising to consider stabilization of many of the moduli *above* the scale of supersymmetry breaking. However, this is a good illustration of the general idea that it is acceptable to have moduli at an early stage in the analysis, which can be lifted by corrections to the potential at some lower energy scale.

Finally, we should note that, whether or not the moduli play an important role in observable physics, they are very important in understanding the configuration space of string theory. In particular, in many of the explicit constructions we discussed above, as well as in the explicit non-geometric constructions we briefly mentioned, one finds that apparently different constructions in fact lead to vacua which differ only in the values of moduli, and thus one can be turned into another by varying moduli. In this situation, there need be no direct relation between the number of constructions, and the final number of vacua after moduli stabilization.

In early exploratory work, this point was not fully appreciated. As a relevant example, in Lerche *et al.* (1987), the number of lattice compactifications was estimated to be 10^{1500}. Thus already this work raised the possibility that the number of string vacua might be very large. However, these were very simple vacua with unbroken supersymmetry,[3] and at the time it was generally thought that the number of quasi-realistic vacua would be much smaller. An argument to this effect was that since moduli were not stabilized in these models, it might be (as is now thought to be the case) that this large number of compactifications were simply special points contained in a far smaller number of connected moduli spaces of vacua. Then, in similar quasi-realistic models with broken supersymmetry and an effective potential, the number of actual vacua would be expected to be comparable to the (perhaps small) number of these connected moduli spaces.

Such debates could not be resolved at that time. To make convincing statements about the number and distribution of vacua, one needs to understand the effective potential and moduli stabilization.

C. Calabi-Yau manifolds and moduli spaces

While our main concern is moduli spaces of Ricci flat metrics, we should first give the reader some examples of Calabi-Yau threefolds, so the discussion is not completely abstract. Let us describe the simplest known constructions, as discussed in Green *et al.* (1987b).

1. Examples

The simplest example to picture mentally is the "blown-up $T^6/\mathbb{Z}_3$ orbifold." We start with a six-torus with a flat metric and volume V, chosen to respect a discrete $\mathbb{Z}_3$ symmetry. To be more precise, we take three complex coordinates z^1, z^2, and z^3, and define the torus by the identifications

$$z^i \cong z^i + 1 \cong z^i + e^{2\pi i/3}.$$

We then identify all sets of points related by the symmetry

$$z^i \to e^{2\pi i/3} z^i \quad \text{for all } i.$$

A generic orbit of this action contains 3 points (the order of the group) and thus the resulting manifold has volume $V/3$. One can easily show that if all orbits had this property (the group is freely acting), the quotient space would be a manifold. However, it is not so; for example the point $(0, 0, 0)$ is a fixed point. In fact there are 27 such fixed points.

While one can use such an "orbifold" directly for string compactification, one can also modify it to get a smooth Calabi-Yau, the "blown-up orbifold." To do this, one removes a neighborhood of each of the fixed points, and

[2] Some have argued that this is in fact a good reason to consider models with high-energy supersymmetry breaking, where non-renormalization theorems do not interfere with the perturbative generation of sufficiently generic potentials to stabilize all moduli at a high scale (Silverstein, 2004b).

[3] Or with supersymmetry broken at the string scale with vanishing dilaton potential, so that the perturbative construction is unstable against loop corrections.

replaces it with a Ricci flat space whose metric asymptotes to that of the original quotient space. One can then "smooth out" this metric to achieve Ricci flatness everywhere. In a sense, the string does this automatically, so the blow-up is a correct guide to the resulting physics.

The blow-up process introduces topology at each of the fixed points; as it turns out, a two-cycle and a four-cycle. Thus, the final result is a smooth Calabi-Yau with second Betti number $b^2 = \dim H^2(M, \mathbb{R}) = 27$. One can also show that the third Betti number $b^3 = 2$.

A second simple example is the "quintic hypersurface" in $\mathbb{P}^4$. This is the space of solutions of a complex equation of degree five in five variables z_i, such as

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0, \qquad (6)$$

where the variables are interpreted as coordinates on complex projective space, *i.e.* we count the vectors $(z_1, z_2, z_3, z_4, z_5)$ and $(\lambda z_1, \lambda z_2, \lambda z_3, \lambda z_4, \lambda z_5)$ as representing the same point, for any complex $\lambda \neq 0$. One can show that the Euler character $\chi = -200$ for this manifold by elementary topological arguments ((Green *et al.*, 1987b), vol. II, 15.8). With a bit more work, one finds all the Betti numbers, $b^0 = b^2 = b^4 = b^6 = 1$, and $b^3 = 204$. We omit this here, instead computing b^3 by other means in the next subsection.

The main point we take from these examples is that it is easy to find Calabi-Yau threefolds with Betti numbers in the range 20–300; indeed, as we will see later in Sec. V.D.3, this is true of most known Calabi-Yau threefolds.

2. Moduli space - general properties

The geometry of a moduli space of Calabi-Yau manifolds as they appear in string theory has been nicely described in Candelas and de la Ossa (1990) (see also Seiberg (1988); Strominger (1990)). Locally, it takes a product form

$$\mathcal{M} = \mathcal{M}_C \times \mathcal{M}_K \qquad (7)$$

where the first factor is associated with the complex structure deformations of M and the second is associated with the Kähler deformations of M, complexified by the B-field moduli.

These two factors enter into physical string compactifications in rather different ways. At the final level of the effective $N = 1$ theory, the most direct sign of this is that the gauge couplings are primarily controlled by a subset of the moduli:

- $\mathcal{M}_K$ for heterotic and IIb compactifications;
- $\mathcal{M}_C$ for IIa compactifications.

The main results we need for the discussion in this section, are the relations between the Betti numbers of the Calabi-Yau manifold M, and the dimensions of these moduli spaces:

$$b_2 = \dim \mathcal{M}_K; \qquad b_3 = 2\dim \mathcal{M}_C + 2. \qquad (8)$$

The first relation follows from Yau's theorem, and is not hard to explain intuitively. Since the Ricci flatness condition is a second order PDE, at a linearized level, it reduces to the condition that a deformation of a Ricci flat metric must be a harmonic form. The Kähler moduli space parameterizes deformations which come from deforming the Kähler form, and thus its dimension is the same as that of the space of harmonic two-forms, which by Hodge's theorem is b_2. The second relation can be understood similarly by relating the remaining metric deformations to harmonic three-forms, given a bit more complex geometry.

Mathematically, one can understand these moduli spaces in great detail, and in principle exactly compute many of the quantities which enter into the flux potential we will discuss shortly. Without going into the details of this, let us at least look at an example, the complex structure moduli space of the quintic hypersurfaces we just discussed.

The starting point is the observation that we do not need to take the precise equation Eq. (6), to get a Calabi-Yau manifold. In fact, a generic equation of degree five in the variables,

$$f(z) \equiv \sum_{1 \le i,j,k,l,m \le 5} c^{ijklm} z_i z_j z_k z_l z_m = 0, \qquad (9)$$

can be used. This equation contains $5 \cdot 6 \cdot 7 \cdot 8 \cdot 9/5! = 126$ adjustable coefficients, denoted c^{ijklm}, and varying these produces Calabi-Yau manifolds with different complex structures. To be precise, there is some redundancy at this point. One can make linear redefinitions $z_i \to g_i^j z_j$ using an arbitrary 5×5 matrix g_i^j, to absorb 25 of the coefficients. This leaves 101 undetermined coefficients, so $\dim \mathcal{M}_C = 101$. By Eq. (8), this implies that the Betti number $b^3 = 2 \cdot 101 + 2 = 204$.

One can continue along these lines, defining the meaning of a "generic equation," and taking into account the redundancy we just mentioned, to get a precise definition of the 101-dimensional moduli space $\mathcal{M}_C$ for the quintic, and results for the moduli space metric, periods and other data we will call on in Sec. IV and Sec. V. Similar results can be obtained for more or less any Calabi-Yau moduli space, and many examples can be found in the literature on mirror symmetry. While the techniques are rather intricate, it seems fair to say that at present this is one of the better understood elements of the theory.

D. Flux compactification: Qualitative overview

Each of the weakly coupled limits of string/M theory has certain preferred "generalized gauge fields," which are sourced by the elementary branes. For example, all closed string theories (type II and heterotic) contain the "universal Neveu-Schwarz (NS) 2-form potential" B_{ij} or $B^{(2)}$ (henceforth, the superscript notation will always indicate the degree of a form). Just as a one-form Maxwell potential can minimally couple to a point particle, this two-form potential minimally couples to the fundamental string world sheet. At least in a quadratic (free field) approximation, the space-time action for the B field is a direct generalization of the Maxwell action; we define a field strength

$$H^{(3)} = dB^{(2)},$$

in terms of which the action is

$$S = \int d^{10}x\sqrt{g}\left(R - (H^{(3)})_{ijk}(H^{(3)})^{ijk} + \dots\right),$$

leading to an equation of motion

$$\partial^i H^{(3)}_{ijk} = \delta_{jk} + \dots$$

where δ_{jk} is a source term localized on the world-sheets of the fundamental strings.

The analogy with Maxwell theory goes very far. For example, recall that some microscopic definitions of Maxwell theory contain magnetic monopoles, particles which are surrounded by a two-sphere on which the total magnetic flux $g = \int F^{(2)}$ is non-vanishing. This magnetic monopole charge must satisfy the Dirac quantization condition, $e \cdot g = 2\pi$ (in units $\hbar = 1$). So too, closed string theories contain five-branes, which are magnetically charged. A five-brane in ten space-time dimensions can be surrounded by a three-sphere, on which the total generalized magnetic flux $\int H^{(3)}$ is non-vanishing. Again, it must be quantized, in units of the inverse electric charge (Nepomechie, 1985; Teitelboim, 1986).

Besides the NS two-form, the type II string theories also contain generalized gauge fields which are sourced by the Dirichlet branes, denoted $C^{(p+1)}$ with $p = 0, 2, 4, 6$ for the IIa theory, and $C^{(p+1)}$ with $p = 1, 3, 5$ for the IIb. We denote their respective field strengths $F^{(p+2)}$; these are not all independent but satisfy the general "self-duality" condition

$$*F^{(p+2)} = F^{(10-p-2)} + \mathrm{non-linear\ terms}.$$

To complete the catalog, the type I theory has $C^{(2)}$ (as it has a D-string), while M theory has a three-form potential $C^{(4)}$, which minimally couples to the supermembrane.

We are now ready to discuss flux compactification. The general idea makes sense for any higher dimensional theory containing a $p+1$ form gauge field for any p. Let us denote its field strength as $F^{(p+2)}$.

Now, suppose we compactify on a manifold with a nontrivial $p+2$ cycle Σ; more precisely the homology group $H_{p+2}(M)$ should be non-trivial, and Σ should be a nontrivial element of homology. In this case, we can consider a configuration with a non-zero *flux* of the field strength, defined by the condition

$$\int_\Sigma F^{(p+2)} = n \neq 0 \qquad (10)$$

As a simple illustration – not directly realized in string theory – we might imagine starting with Maxwell's theory in six dimensions, and compactifying on $M = S^2$. In this case, $H_2(S^2, \mathbb{Z}) \cong \mathbb{Z}$, and we can take a generator σ to be the S^2 itself. Thus, we are claiming that there exists a field configuration whose magnetic flux integrated over S^2 is non-zero. Indeed there is; we can see this by considering the field of an ordinary magnetic monopole at the origin of $\mathbb{R}^3$, and restricting attention to an S^2 at constant radius R, to obtain the field strength

$$B_{\theta\phi} = g \sin\theta \; d\theta \; d\phi.$$

While this solves Maxwell's equations in three dimensions by construction, one can easily check that such a magnetic field actually solves Maxwell's equations restricted to the S^2.[4] Thus, this is a candidate background field configuration for compactification on S^2.

Note that we have defined a flux which threads a nontrivial cycle in the extra dimensions, with no charged source on the S^2. The monopole is just a pictorial device with which to construct it. Appealing to the monopole also allows us to call on Dirac's argument, to see that quantum mechanical consistency requires the flux n to be integrally quantized (in suitable units).

The same construction applies for any p. Furthermore, if we have a larger homology group, we can turn on an independent flux for each basis element σ_i of $H_{p+2}(M, \mathbb{R})$,

$$\int_{\sigma_i} F^{(p+2)} = n_i, \qquad (11)$$

where $1 \leq i \leq \dim\, H_{p+2}(M, \mathbb{R}) \equiv b_{p+2}$, the $p{+}2$'th Betti number of the manifold M. In the case $p = 0$ of Maxwell theory, one can see that any vector of integers n_i is a possible field configuration, by appealing to the mathematics of vector bundles (these numbers define the first Chern class of the $U(1)$ bundle). Equally precise statements for $p > 0$, or for the case in which the homology includes torsion, are in the process of being formulated (Moore, 2003).

Now, in Maxwell's theory and its generalizations, turning on a field strength results in a potential energy proportional to $\mathbf{B}^2$, the square of the magnetic field. Of course, the presence of nontrivial $\mathbf{E}$ or $\mathbf{B}$ in our observed four dimensions would imply spontaneous breaking of Lorentz symmetry. By contrast, in our case, we can turn

[4] One can avoid this (short) computation by noting that while the magnetic field is a rotationally symmetric two-form on S^2, there is no rotationally symmetric one-form, so $\partial^i F_{ij}$ must vanish.

on magnetic fluxes in the extra dimensions *without* directly breaking 4d Lorentz invariance. However, there will still be an energetic cost, now proportional to F^2, the square of the flux.

Now, the key point is that because the fluxes are threading cycles in the compact geometry, this energetic cost will depend on the precise choice of metric on M. In other words, it will generate a potential on the moduli space $\mathcal{M}$. If this potential is sufficiently generic, then minimizing it will fix the metric moduli.

In principle, this potential can be computed by starting from the standard Maxwell lagrangian coupled to a curved metric. One finds for the potential energy

$$V = \int_M d^D y \, \sqrt{G} G^{ij} G^{kl} F^{(2)}_{ik} F^{(2)}_{jl} \tag{12}$$

$$= \int_M F^{(2)} \wedge (*F)^{(D-2)} \tag{13}$$

where G is the metric on M. The second version, in differential form notation and where $*$ denotes the D-dimensional Hodge star, applies for any $F^{(p+2)}$ with the replacement $2 \to p+2$; here the metric enters in the definition of $*$.

Now, if we substitute for G the family of Ricci flat metrics $G(\vec{t})$ introduced in Sec. II.B, and do the integrals, we will get an explicit expression for $V(t)$, which we can minimize. This is the definition of the flux potential; we now have the technical problem of computing it.

At first, it is not clear that this can be done at all; indeed we cannot even get started as no closed form expression is known for any Ricci flat metric on a compact Calabi-Yau manifold. In principle the computations could be done numerically, but working with solutions of six dimensional nonlinear PDE's is not very easy either, and this approach is in its infancy (Douglas *et al.*, 2006a; Headrick and Wiseman, 2005). Fortunately, by building on many mathematical and physical works, we now have an approach which leads to a complete analytical solution of this problem, as we will discuss in Sec. IV.

1. Freund-Rubin compactification

There are other Kaluza-Klein theories in which the technical problem of computing Eq. (12) is far simpler, and was solved well before string theory became a popular candidate for a unified theory. While these theories are too simple to be quasi-realistic, they serve as good illustrations. Let us consider one here, leaving more detailed discussion to Sec. IV.

After it was realized that Nature employs non-abelian gauge fields, the earliest idea of 5d unification was augmented. Instead, theorists considered $4+D$ dimensional theories, with D of the dimensions compactified on a space with a non-abelian isometry group. In this case, dimensional reduction leads to a gauge group which contains the isometry group. One can even find seven dimensional manifolds for which this is the Standard Model gauge group, although chiral fermions remain a problem for this idea.

In any case, the problem of explaining how and why the extra D dimensions were stabilized in whatever configuration was required to obtain 4d physics was first studied in this context. A collection of historically significant articles on Kaluza-Klein theory, with modern commentary, can be found in Appelquist *et al.* (1987).

The first serious attempt we know of to explain the "spontaneous compactification" of the extra D-dimensions appeared in the work of Cremmer and Scherk, who realized that by including extra fields beyond gravity in $4+D$ dimensions, stabilization of the compact dimensions could be achieved in a reliable classical approximation (Cremmer and Scherk, 1976). This work was extended by Luciani (1978) and reached more or less modern form with the seminal paper of Freund and Rubin (1980).

Let us see how the Freund-Rubin mechanism works by again considering six dimensions, now in Einstein-Maxwell theory. Compactifying to 4d on an S^2, they found that inclusion of a magnetic flux piercing the S^2 allows one to stabilize the sphere. One can understand this result by a scaling argument; such arguments are discussed in modern contexts in Giddings (2003); Kachru *et al.* (2006); Silverstein (2004b). We start with a 6d Einstein/Yang-Mills Lagrangian

$$S = \int d^6 x \sqrt{-G_6} \left(\mathcal{R}_6 - |F^{(2)}|^2 \right), \tag{14}$$

where all dimensions are made up with powers of the fundamental scale M_6. We then consider reduction to 4d on a sphere of radius R:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 g_{mn}(y) dy^m dy^n \tag{15}$$

where m, n run over the two extra dimensions, and g is the metric on a two-sphere of unit radius. Let us then thread the S^2 with N units of $F^{(2)}$ flux

$$\int_{S^2} F = N \,. \tag{16}$$

In the 4d description, $R(x)$ should be viewed as a field. Naively reducing, we will find a Lagrangian where $R^2(x)$ multiplies the curvature tensor $\mathcal{R}_4$. To disentangle the graviton kinetic term from the kinetic term for the modulus $R(x)$, we should perform a Weyl rescaling. After this rescaling, we find an effective potential with two sources:

- Before the rescaling, the 6d Einstein term would contribute to the action a term proportional to the integrated curvature of the S^2, which gives the Euler character. In particular, the positive curvature makes a negative contribution to the potential. After the rescaling, this term is no longer a constant; instead it scales like $-R^{-4}$.

- The N units of magnetic flux through the S^2, of course, contribute a positive energy. It arises by reducing

$$\int d^6x - F_{mn} \frac{g^{mp}}{R^2} \frac{g^{nq}}{R^2} F_{pq} \ . \tag{17}$$

  By flux quantization, $F \sim \frac{N}{R^2}$, while the integral over the internal space contributes a factor of R^2. Therefore, the flux potential scales like N^2/R^6. The dimensions are made up by powers of the fundamental scale, in terms of which the flux quantum is defined.

Therefore, the potential as a function of $R(x)$ takes the schematic form

$$V(R) \sim \frac{N^2}{R^6} - \frac{1}{R^4} \ . \tag{18}$$

It is not hard to see that this function has minima where $R \sim N$. So with moderately large flux, one can achieve radii which are large in fundamental units, and curvatures which are small. This means that the vacua found in this way are reliable.

Strictly speaking, the original Freund-Rubin vacua are *not* compactifications which yield lower-dimensional effective field theories. The vacuum energy following from (18) is negative, and gives rise to a 4d curvature scale comparable to the curvature of the S^2. Therefore, 4d effective field theory is not obviously a valid approximation scheme in these vacua. In Sec. IV of the review, we will focus on models where over some range of energies, the effective description of physics really is four-dimensional. It is plausible, however, that by using more complicated manifolds and tuning parameters to decrease the 4d vacuum energy, one could use the Freund-Rubin idea to obtain quasi-realistic vacua (Acharya *et al.*, 2003).

E. A solution of the cosmological constant problem

Einstein's equations, relating the curvature of space-time to the stress-energy of matter, admit an additional term on the right hand side,

$$g_{ij} = 8\pi G_N \left(T_{ij} + \Lambda g_{ij}\right).$$

The additional "cosmological constant" term Λ is a Lorentz-invariant vacuum energy and is believed to be generically present in any theory of quantum gravity; it receives corrections from known quantum effects (somewhat analogous to the Casimir effect) at least of order $(100\text{GeV})^4$. On the other hand, elementary considerations in cosmology show that any value $|\Lambda| > 1(\text{eV})^4$ or so is in violent contradiction with observation. More recently, there is observational evidence of various types (the acceleration of the expansion of the universe; and detailed properties of the cosmic microwave background spectrum) which can be well fit by assuming $\Lambda \sim 10^{-10}(\text{eV})^4 > 0$.

This is by now a very long-standing question with which most readers will have some familiarity; we refer to (Carroll, 2001; Nobbenhuis, 2004; Padmanabhan, 2003; Weinberg, 1989) for introductory overviews, and the history of the problem. A very recent discussion from the same point of view we take here is in Polchinski (2006), along with general arguments against many of the other approaches which have been taken towards the problem.

One approach which cannot be ruled out on general grounds is to simply assert that the fundamental theory contains the small observed parameter Λ. More precisely, the large quantum contributions Λ_q from all types of virtual particles (known and unknown), are almost precisely compensated by an adjustable "bare cosmological constant" $\Lambda_{bare} \sim -\Lambda_q$. However, besides being unesthetic, this idea *cannot* be directly realized in string/M theory, which is formulated without free parameters. Rather, to address this problem, we must find out how to compute the vacuum energy, and argue that the energy of the vacuum we observe takes this small value.

Of course, taken purely as a problem in microscopic physics, the prospects for accurately computing such a small vacuum energy seem very distant; furthermore it seems very unlikely that any vacuum would exhibit the remarkable cancellations between the large known contributions to the vacuum energy, and unknown contributions, required to make such an argument. But here is precisely the loophole; what is indeed very unlikely for a single vacuum, can be a likely property for *one* out of a large set of vacua.

Simple toy models in which this is the case were proposed in Abbott (1985); Banks *et al.* (1991). The general idea is to postulate a potential with a large number of roughly equally spaced minima, for example

$$V(\phi) = a\phi - b\sin\phi + \Lambda_q,$$

whose minima $\phi = 2\pi n$ have energies $\Lambda_n = \Lambda_q + 2\pi a n - b$. Thus, if a is very small, then no matter what value Λ_q takes, at least one minimum will realize the small observed Λ. By postulating more fields, one can even avoid having to postulate a small number a (Banks *et al.*, 1991). For example, consider

$$V(\phi) = a_1\phi_1 + a_2\phi_2 - \sin\phi_1 - \sin\phi_2 + \Lambda_q.$$

The reader may enjoy checking that if the ratio a_1/a_2 is irrational, any Λ can be approximated to any desired accuracy.

While in effective field theory terms these models might be reasonable, the actual potentials arising from string/M theory compactification appear not to take this form. Besides verifying this in explicit expressions (for example coming from Eq. (12)), there is a conceptual problem. This is that these models assume that the field ϕ can take extremely large values, of order $1/\Lambda$. However, taking a modulus ϕ to be so large, implies that the Calabi-Yau manifold is decompactifying, or undergoing some similar limit. In such a limit, the potential can be computed more directly and (at least in known examples) does not take the required form.

However, there is another mechanism for producing potentials with large numbers of minima, introduced by Bousso and Polchinski (2000), which relies on having a very large number of degrees of freedom.[5] Let us consider a toy model of flux compactification, where there are N different p-cycles in the compact geometry that may be threaded by the flux of some p-form field F

$$\int_{\Sigma^i} F = n^i, \; i = 1, \cdots, N \tag{19}$$

Let us also assume a simple *ad hoc* cutoff on the allowed values of the fluxes, of the form

$$\sum_i n_i^2 \leq L \tag{20}$$

where L is some maximal amount of flux. One can view Eq. (20) as a toy model of the more complicated tadpole conditions that arise in real string models. Finally, let us assume that for each value of the fluxes F, the resulting potential function for moduli admits a minimum with energy

$$V \sim -V_0 + c_i n_i^2 \tag{21}$$

Here we take the c_i to be distinct order one constants, while $-V_0$ is assumed to be a large fixed negative energy density, for example representing the quantum contribution to the cosmological constant Λ_q we discussed earlier.

A striking fact follows from these simple assumptions and known facts about compactification topologies – the number of vacua will be huge. As we discussed, typical Calabi-Yau threefolds have betti numbers of order 100. For a space with $N = 100$ and $L = 100$, Eq. (20) indicates that the number of vacua can be approximated by the volume of a sphere of radius $\sqrt{L} \sim 10$ in a 100-dimensional space. This is roughly $\frac{\pi^{50}}{50!} \times 10^{100} \sim 10^{60}$. Here it was important that $\sqrt{L}$ is much larger than the unit of flux quantization, so that one can approximate the number of possible flux choices by computing the volume in flux space of the region defined by Eq. (20).

We will justify this toy model in detail in Sec. V, by showing that the real counting of flux vacua – while differing in details – is similar, that concrete examples with significantly larger N and L exist, and that the fractions of flux choices for which vacua exist in some approximation scheme are large enough not to significantly alter the estimate above.

Now, let us consider the cosmological constant in this model. In a vacuum with flux vector n_i, this will be given by Eq. (21). Thinking of the quadratic term in Eq. (21) as defining a squared distance from the origin in N-dimensional space, we see that to have a small vacuum energy of order ϵ, a flux vacuum must sit within a shell bounded by two ellipsoids, of radius $\sqrt{V_0}$ and $\sqrt{V_0 + \epsilon}$. (These are ellipsoids because the c_i are not all equal, though we assume them all to be $\mathcal{O}(1)$).)

As argued in Bousso and Polchinski (2000), if the number of vacua exceeds $\sim 10^{120}$, quite plausibly this shell is populated by some choices of flux. The simplest argument for this is that, given that the fluxes n_i and the postulated coefficients c_i are independent, we can expect the values of the vacuum energy attained by Eq. (21) to be roughly uniformly distributed over scales much smaller than the coefficients c_i. Thus, in a set of N_{vac} vacua, we might expect the typical "level spacing" to be $1/N_{vac}$, and that a vacuum energy of order $1/N_{vac}$ will be realized by at least one vacuum. We will make more precise arguments of this type in Sec. V.

Thus, this toy model can explain why at least some vacua exist with the very small cosmological constant consistent with observation. Furthermore, the essential features of the toy model, namely a very large number of vacua with widely distributed vacuum energies, distinguished by the values of hundreds of microscopic parameters, seem to be shared by more realistic stringy flux compactifications. This energy landscape of potential string vacua has been called the "string landscape" (Susskind, 2003); a detailed and very clear qualitative discussion can be found in Susskind (2005).

1. Anthropic selection

Suppose we grant that a few, rare vacua will have small Λ. How do we go on to explain why we find ourselves in such a vacuum?

There have been many attempts to find dynamical mechanisms which strongly prefer the small Λ solutions (including relaxation mechanisms (Brown and Teitelboim, 1987, 1988; Feng *et al.*, 2001; Steinhardt and Turok, 2006), peaking of the wave-function of the Universe (Coleman, 1988; Hawking, 1984), and many others (Itzhaki, 2006; Rubakov, 2000)). Each seems in some sense problematic: for instance the relaxation proposals typically suffer from an "empty universe problem," whereby they favor completely empty vacuum solutions with small Λ, incompatible with our cosmological history which presumably includes inflationary, radiation dominated, and matter dominated epochs. For a much more detailed discussion of the problems that bedevil various dynamical selection mechanisms, and possible loopholes, see Polchinski (2006).

Absent a dynamical selection mechanism, one can try to use so-called "anthropic" criteria to explain why we inhabit a vacuum with small Λ. Perhaps a better term for the generally accepted criteria of this type is *selection effect*; in other words we take the evident fact that the circumstances of a particular experiment or observation might skew the distribution of observed outcomes, and apply it to the problem at hand of why we observe "our

[5] Amazingly, this idea was anticipated in Sakharov (1984).

universe" instead of another.

In practice, what is meant by this, is an argument which focuses on some macroscopic property of our universe, and derives constraints on microphysics by requiring the microphysics to be consistent with the macroscopic phenomenon. The most famous example, and the one we will cite here, is Weinberg's argument (Weinberg, 1987) that the existence of structure (i.e. galaxies) puts stringent bounds on the magnitude of the cosmological term.[6] For positive cosmological constant, the bound arises due to two competing effects. On the one hand, primordial density perturbations gravitate and attract each other; in a universe with vanishing Λ, the Jeans instability will then eventually lead to the formation of large scale structure. On the other hand, a large Λ and the consequent accelerated expansion, could lead to such rapid dilution of matter that structure can never form. The requirement that structure have time to form before the accelerated expansion takes over, leads to a bound on Λ within an order of magnitude or two of the observed value.

Weinberg's logic suggests that if structure is required for observers, and if there are many possible vacua with different values of Λ, then selection effects will explain why any given observer sees an atypically small value of Λ. It is also important that since the scales of microphysics differ so drastically from the scale of the required Λ, one can expect the distribution of vacua in Λ-space to be reasonably flat over the anthropically acceptable range. Hence, all else being equal, one should expect to find a value of Λ close to the upper bound compatible with structure formation.

Indeed, this seems to be true in our universe. It is notable that Weinberg's bound was published well before the detection of dark energy, and the amount of dark energy is very close to his estimate of the maximal value compatible with the formation of structure. There is some controversy about how close our universe is to the bound (which of course depends on the precise formulation of the macroscopic requirement); see e.g. Loeb (2006) and references therein.

An important question which must be asked before accepting this logic is: are these vacua all part of a single theory, and a single cosmology, or does superselection operate to make the choice of vacuum a unique initial condition? In this context, inflation and particularly eternal inflation (see Guth (2000) for a nice discussion with further references) suggests strongly that the correct picture involves a "multiverse" with many different inflating regions, corresponding to the different de Sitter critical points in the set of vacua.

From this point of view, it is important to ask whether different flux vacua can be connected by physical processes in string theory. The answer is yes; the fluxes are dynamical variables in the full string theory, so vacua with different discrete choices can tunnel to one another. For instance, in any set of de Sitter flux vacua obtained from a single compactification manifold in the IIb theory, one can connect vacua with different values of F_3 flux through a 3-cycle, by considering the domain bubble formed by wrapping a D5 brane on the dual 3-cycle (times a 2+1 dimensional slice of dS_4).

Anthropic arguments are typically met with suspicion for the simple reason that it seems hard to convincingly and quantitatively verify a physical theory based on such arguments. There are many reasons (discussed in e.g. Arkani-Hamed *et al.* (2005b); Banks *et al.* (2004); Wilczek (2005)) to believe that more traditional, dynamical explanations will be required to resolve some of the outstanding mysteries of physics. But unless another convincing solution to the cosmological constant problem is found, this one is likely to stay with us.

F. Other physical consequences

While explaining the cosmological constant would be an important achievement, the resolution provided by the landscape of flux vacua does not suggest immediate tests. Furthermore, anthropic arguments are (understandably) not considered clean ways to test or verify a theory.

Happily, the same techniques which are used in the study of flux vacua, and which make it natural to consider the anthropic explanation of the CC problem, also allow one to derive new string models of particle physics and cosmology. Much of the interest in studying flux vacua has indeed been driven by the goal of finding new classes of explicit models which make testable predictions. Over the past few years, these studies have motivated new testable models of TeV scale particle physics (Arkani-Hamed and Dimopoulos, 2005; Arkani-Hamed *et al.*, 2005b; Giudice and Rattazzi, 2006; Giudice and Romanino, 2004), new models of inflation (Kachru *et al.*, 2003; Silverstein and Tong, 2004) which can have testable signatures via cosmic strings (Copeland *et al.*, 2004; Dvali and Vilenkin, 2004b; Jones *et al.*, 2003; Sarangi and Tye, 2002) or non-gaussianities of the spectrum of density perturbations (Alishahiha *et al.*, 2004; Babich *et al.*, 2004; Chen *et al.*, 2006), and new testable proposals for the mediation of supersymmetry breaking (Choi *et al.*, 2005). Of course, one should not view these models as inevitable top-down consequences of string theory, which they certainly aren't. Instead, they are special choices made out of a wide range of possibilities in the fundamental theory, proposed in part because they have clearly identifiable or at least unusual characteristic signatures. The hope is that the influx of new data on TeV scale particle physics and inflationary cosmology expected in the next decade, will help select between these ideas or (more likely) suggest new, testable proposals.

[6] While we cannot fully review the history here, important earlier works along these lines include (Banks, 1984, 1985; Barrow and Tipler, 1988; Linde, 1984a).

Here, we briefly describe, at a very qualitative level, three areas where studies of flux vacua may be directly relevant to phenomenological questions in string theory. We will need to call upon some basic results from the theory of supersymmetry breaking, so we review this first.

1. Overview of spontaneous supersymmetry breaking

By spontaneous supersymmetry breaking, we mean that although the vacuum breaks supersymmetry, at some high energy scale dynamics is described by an effective $\mathcal{N} = 1$ supergravity theory. As discussed in Wess and Bagger (1992), the effective potential in such a theory is determined by the superpotential W, a holomorphic "function" of the chiral fields,[7] and the Kähler potential K, a real-valued function of these fields. Let us denote the chiral fields as ϕ^i, then the effective potential takes the form

$$V = e^K \left(\sum_i |F_i|^2 - 3\frac{|W|^2}{M_{Pl,4}^2} \right) + \frac{1}{2} \sum_\alpha D_\alpha^2 \qquad (22)$$

where $F_i = DW/D\phi^i \equiv \partial W/\partial\phi^i + (\partial K/\partial\phi^i)W$ are the so-called F terms, associated to chiral fields, while the D terms $D_\alpha \sim \sum \phi^\dagger t_\alpha \phi$ are associated to generators of the gauge group.

While any solution of $\partial V/\partial\phi^i = 0$ with $\partial^2 V/\partial\phi^i\partial\phi^j$ positive definite is a metastable vacuum, spontaneous supersymmetry breaking is characterized by non-zero values for some of F_i and D_α. The most basic consequence of this is that the gravitino gains a mass $m_{3/2} = e^K|W|/M_{Pl,4}^2$ by a super-Higgs mechanism. If we assume that the cosmological constant $V \sim 0$, $|W|$ and thus $m_{3/2}$ are determined by Eq. (22) in terms of $|F|^2$ and $|D|^2$.

Another common although model-dependent consequence of supersymmetry breaking is the generation of soft supersymmetry breaking terms, such as masses for the gauginos and scalars. One fairly generic source for scalar masses is coupling through irrelevant terms in the Kähler potential, with the general structure

$$\int d^2\theta d^2\bar{\theta} \frac{c^2}{M_P^2} X^\dagger X (\phi^i)^\dagger \phi^i \qquad (23)$$

Such terms are not forbidden by any symmetry (unless ϕ_i is a Goldstone boson, but compactification moduli in general are not, with the notable exception of axions). If they are present and $F_X \neq 0$, the field ϕ^i obtains a mass

$$m_i \sim \frac{c\, F_X}{M_P}. \qquad (24)$$

Similarly, if X appears in the gauge-coupling function f for some gauge group G, i.e. in the term

$$\int d^4x\ d^2\theta f(X) Tr(W_\alpha W^\alpha), \qquad (25)$$

then $F_X \neq 0$ gives rise to a gaugino mass as well. Another generic source of masses for charged particles is anomaly mediation (Giudice *et al.*, 1998; Randall and Sundrum, 1999b); in particular this produces gaugino masses

$$m_{1/2} \sim b\ g_{YM}^2\ m_{3/2}$$

where b is a beta function coefficient. Quite generally, these effects lead to masses proportional to F/M_P.

Finally, given a soft mass term for charged fields X, their one loop diagrams produce soft mass terms for charged gauginos, and at higher loop order soft masses for all charged particles. This is known as gauge mediation; for references and a review see Giudice and Rattazzi (1999). Unlike the previous mechanisms, this effect is not suppressed by M_P, but by M_X, the mass of the X fields.

Let us now consider a quasi-realistic model which solves the hierarchy problem by spontaneous supersymmetry breaking. In general, one expects the EFT to be a sum of several parts; a supersymmetric Standard Model (SSM); a sector responsible for supersymmetry breaking; possibly a messenger sector which couples supersymmetry breaking to the SSM; and finally sectors which are irrelevant for this discussion. After integrating out all non-SSM fields, one obtains an SSM with soft supersymmetry breaking terms, such as masses for the gauginos and scalars. The first test of the model is that the resulting potential leads to electroweak symmetry breaking. This depends on two general features of the supersymmetric extension. Recall that an SSM must have at least two Higgs doublets; let us suppose there are two, H_u and H_d. First, the Higgs doublets can get a supersymmetric mass term

$$W = \ldots + \mu H_u H_d,$$

the so-called μ term. This must be small, $\mu \sim M_{EW}$. In addition, one must get soft supersymmetry breaking masses coupling the two Higgs doublets (the b term), also of order M_{EW}. Of course, there are many, many more constraints to be satisfied by a realistic model, most notably on flavor changing processes.

Now, one can distinguish two broad classes of supersymmetry breaking models. In the first class, generally known as "gravity mediated" models,[8] supersymmetry breaking is mediated only by effects which are suppressed by powers of M_P. In this case, to obtain soft masses at M_{EW}, the natural expectation is $F \sim (10^{11}\text{GeV})^2$, the so-called intermediate scale, and $m_{3/2} \sim M_{EW}$.

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[7] To be more precise, the superpotential in supergravity is a section of a holomorphic line bundle.

[8] We are oversimplifying here.

On the other hand, if the SSM soft masses come from gauge mediation, the sparticle masses are suppressed by powers of M_X, not M_P. Therefore, depending on M_X, one can get by with a much smaller F breaking, perhaps as low as $F \sim (100 \text{ TeV})^2$. Such a gauge mediated model will have $m_{3/2} << M_{EW}$ as well as many other differences from the first class.

This more or less covers the basic facts we will need for this review; further discussion can be found in many good reviews such as Giudice and Rattazzi (1998); Luty (2005); Martin (1997)

2. The moduli problem

As we discussed, string compactifications preserving 4d $\mathcal{N}=1$ supersymmetry typically come with dozens or hundreds of moduli fields. These are chiral multiplets ϕ_i which have gravitational strength couplings and a flat potential to all orders in perturbation theory.

In general, all scalar fields, including the moduli, will receive mass after supersymmetry breaking. In a few cases, namely the moduli which control the Standard Model (or grand unified) gauge couplings, we can put a lower bound on this mass, around 100GeV, just by considering quantum effects in the Standard Model. As pointed out in Banks *et al.* (2002), this precludes any observable variation of the fine structure constant (and the other SM gauge couplings), even on cosmological time scales. Thus, while the underlying theory allowed for such time variation in principle, it is inconsistent with known properties of our vacuum combined with the effective potential hypothesis. This is perhaps the simplest testable prediction of string/M theory for which contrary evidence has ever been reported (Murphy *et al.*, 2003); the present status is discussed in Uzan (2003, 2005).[9]

More generally, one can estimate moduli masses in particular models of supersymmetry breaking. Using Eq. (24), and assuming a gravity mediated model with $F \sim (10^{11}\text{GeV})^2$, we find a rough upper bound

$$m_{moduli} \sim 1 \text{ TeV}.$$

As for gauge mediated models, since moduli which do not couple directly to the Standard Model also get their leading masses from Eq. (24), their masses will be far lower, even down to the eV range.

In general, such particles would not be subject to direct detection, because of their very weak (nonrenormalizable) coupling to the Standard Model. One can construct optimistic scenarios (including the large-extra dimensions scenario (Arkani-Hamed *et al.*, 1998) and models of gauge mediation with very low SUSY breaking scale) in which the moduli masses come down to 10^{-3} eV, so that one could hope to detect such fields in fifth-force experiments studying the strength of gravity at short distances (Dimopoulos and Giudice, 1996). Of course the moduli must be coupled gravitationally to the SUSY breaking sector to obtain such a small mass $\sim (\text{TeV})^2/M_P$.

However, granting the usual discussion of inflationary cosmology, scalar fields masses less than about 100TeV will cause significant phenomenological problems. In particular, they cause a Polonyi problem – the oscillations of such scalars about the minima of their potential, in a cosmological setting, will overclose the universe (Banks *et al.*, 1994; de Carlos *et al.*, 1993). One way of understanding this is as follows. The equation of motion for a modulus ϕ in the early universe is

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \ . \tag{26}$$

Taylor expanding $V(\phi) \sim m^2\phi^2 + \cdots$, we see that the "Hubble friction" (the second term on the LHS) dominates over the restoring force from the potential energy, if $H >> m$. Via the relaton $H^2 M_P^2 \sim V_{tot}$ (where V_{tot} is the total energy density of the Universe), we see that in the early Universe, Hubble friction will dominate for light fields. This means that until H decreases to $H < m$, such fields will *not* reach the minima of their potential; they will be trapped by Hubble friction at some random point.[10] After the Hubble constant drops below m, the energy density in these fields can dominate the Universe, leading to a variety of possible problems (overclosure, modifications of the successful predictions of BBN, etc.).

There are scenarios with moduli in this mass range where the cosmological problems are avoided, say by a stage of low-scale inflation (Dvali, 1995; Randall and Thomas, 1995)). In general, however, this suggests that the idea that string moduli get their mass through radiative corrections after SUSY breaking is disfavored. Rather, we should look for the physics of moduli stabilization at higher energy scales.

As we discussed, we can expect the flux potential to produce moduli masses. A first naive estimate for the energy scale of this potential would be M_{KK}, since this is an effect of compactification. However, this neglects the fact that the unit of quantization of the fluxes is set by the fundamental scales, in string theory the string scale. This discussion is somewhat model dependent (Kachru *et al.*, 2006); in Sec. IV we will discuss the case of IIb flux vacua. In general in such vacua, the complex structure

[9] String/M theory also leads to many testable predictions for which we have no reason at present to expect contrary evidence, for example CPT conservation, unitarity bounds in high energy scattering, and so forth.

[10] This discussion is oversimplified, since V itself may receive significant thermal corrections. The point then is that for a modulus field, the true minimum only appears, typically very far away ($\sim M_P$ in field space) from the finite-temperature minimum, after H drops below the typical scales of the zero-temperature potential.

moduli which get a mass from fluxes end up with a typical mass M_F

$$M_F \sim \frac{\alpha'}{R^3} \tag{27}$$

which satisfies $M_F << M_{KK}$ at moderately large radius, but is still well above the supersymmetry breaking scale M_{SUSY} (and far above the even smaller gravitino mass $M_{3/2} \sim M_{SUSY}^2/M_P$) for low energy supersymmetric models with moderate R.

In a top-down discussion, one must check that these masses squared are positive, *i.e.* metastability. Actually, one can argue that this is generic in supersymmetric theories, in the following sense. The mass matrix V'' following from Eq. (22) takes the form

$$M_{boson}^2 = M_{fermion}(M_{fermion} - \alpha M_{3/2}) \tag{28}$$

for some order one α.[11] Thus, any bosonic partner to a fermion with $|M_{fermion}| >> M_{3/2}$ will automatically have positive mass squared. Since for moduli, $M_{fermion} \sim M_F >> M_{3/2}$, this entire subsector will be stable.

a. Quintessence There is one cosmological situation in which the existence of an extremely light, weakly coupled scalar field has been proposed as a feature instead of a bug. One of the standard alternatives to a cosmological constant in explaining the observed dark energy is “quintessence” (Peebles and Ratra, 1988). In this picture, a slowly rolling scalar field dominates the potential energy of the Universe, in a sort of late-time analogue of early Universe inflation (though perhaps lasting only for $\mathcal{O}(1)$ e-foldings). In light of our discussion, it is natural to ask, can string theory give rise to natural candidates for quintessence?

The observational constraints on time variation of coupling constants make it necessary to keep the relevant scalar very weakly coupled to observable physics. The necessary mass scale of the scalar, comparable to the Hubble constant today, means also that this scalar must *not* receive the standard $\sim M_{SUSY}^2/M_P$ mass from SUSY breaking. The most natural candidate is therefore a pseudo Nambu-Goldstone boson, and in string theory, these arise plentifully as axions. An axion with weak enough couplings and whose shift symmetry is broken by dynamics at very low energies, could conceivably serve as quintessence; it has been Hubble damped on the side of its potential until the present epoch, and may just be beginning its descent.

The prospects for this scenario are described in the recent paper Svrcek (2006). While it is plausible, the scenario suffers from all of the tuning problems of the cosmological constant scenario, and an additional “why now” problem – there is no good reason for the field to become undamped only in the recent past. Still, the special case of axion quintessence does seem possible in string theory.

3. The scale of supersymmetry breaking

Perhaps the most fundamental question in string phenomenology is the scale of supersymmetry breaking. As we discussed, there are many hints in the present data which point towards TeV scale supersymmetry. It has long been thought that low energy supersymmetry would also follow from a top-down point of view. One of the simplest arguments to this effect uses the concept of “naturalness,” according to which an effective field theory can contain a small dimensionful parameter, only if it gains additional symmetry upon taking the parameter to zero. This is not true of the Higgs mass in the Standard Model, but can be true for supersymmetric theories.

On the other hand, the solution we just described for the cosmological constant problem seems to have little to do with this sense of naturalness; indeed it may seem in violent conflict with it.[12] Should this not give us pause? How do we know that the small ratio $M_{EW}^2/M_{Pl,4}^2 \sim 10^{-33}$ might not have a similar explanation? Following this line of thought, one might seek an anthropic explanation for the hierarchy, as has been done in several works (Arkani-Hamed and Dimopoulos, 2005; Arkani-Hamed *et al.*, 2005a,b; Giudice and Rattazzi, 2006; Giudice and Romanino, 2004). While interesting, the possibility of such an explanation would not bear directly on whether the underlying theory has low energy supersymmetry, unless we could argue that our existence required this property (or was incompatible with it), which seems implausible.

However, there is a different set of arguments, which we will now describe, that low energy supersymmetry, and the naturalness principle which suggested it, may not be the prediction of string/M theory. Rather, one should define a concept of *stringy naturalness,* based on the actual distribution of vacua of string/M theory, which leads to a rather different intuition about fine-tuning problems.

The starting point is the growing evidence that there are many classes of string vacua with SUSY breaking at such high scales that it does not solve the hierarchy problem, starting with early works such as (Alvarez-Gaume

[11] The reader should not confuse this with formulae governing the sparticle partners of standard model excitations, for which the soft-breaking terms give the dominant effects, and can lead to splittings much larger than this estimate in various scenarios.

[12] Actually, in Sec. V, we will show that the c.c. is uniformly distributed in some classes of vacua, in a way consistent with traditional naturalness. In our opinion, anthropic arguments are *not* in contradiction with naturalness, rather they presuppose some idea of naturalness.

et al., 1986; Dixon and Harvey, 1986; Scherk and Schwarz, 1979; Seiberg and Witten, 1986), and more recently models with stabilized moduli such as (Saltman and Silverstein, 2006; Silverstein, 2001). Despite their disadvantage in not solving the hierarchy problem, might such vacua "entropically" overwhelm the vacua with low-scale breaking? Let us illustrate how one can study this question with the following top-down approach to deriving the expected scale of supersymmetry breaking, along the lines advocated in Douglas (2004b); Susskind (2004).

Suppose for a moment that one has classified the full set of superstring vacua, obtaining some set with elements labelled by i. Suppose, for sake of argument, that we had a complete model of how early cosmology produces these vacua, which leads to the claim that "the probability to observe vacuum i is $P(i)$." Finally, suppose that the SUSY breaking scale in the i'th vacuum is F_i. Then, we could use this data to define a probability distribution over SUSY breaking scales. Similarly, if we have more observables for each vacuum we could define a joint distribution over all of them.

To make a simple discussion, let us focus on two parameters, the supersymmetry breaking scale F, and the scale of electroweak symmetry breaking $M_{EW} \sim 100\text{GeV}$. Now, imagine that we are about to do an experiment which will detect superpartners if $F < F_{exp} = 1\text{TeV}$. Then, the probability with which we expect to discover supersymmetry would be

$$P_{susy} = \sum_{F_i \leq F_{exp}, M_{EW,i}=100\text{GeV}} P(i). \tag{29}$$

If this probability were high, we would have derived a top-down prediction of TeV scale supersymmetry.

But, from what we know about string theory, do we know it will be high? Might it instead be low, so that the discovery of TeV scale supersymmetry would in some sense be evidence against string theory?

Before continuing, we hasten to say that any top-down "prediction" of this sort would only be as good as the assumptions which went in, and furthermore would probably rely on drastic simplifications of the full problem. We fully expect that the problem of testing string theory, like any other theory, will involve the same sort of interaction between theory and experiment which characterizes all successful science. Our goal here is to make an idealization of this complex problem, in order to gain understanding. We will discuss the assumptions and simplifications which would go into any such prediction in Sec. V, here let us continue in order to make the point that *given what we know now, TeV scale supersymmetry is not an inevitable prediction of string theory.*

First, given our ignorance of the correct probabilities $P(i)$, a simple hypothesis to get a feel for the problem is to set the probability $P(i) = 1/N$ for each of the N vacua in the landscape. In other words, we assume that the more string/M theory vacua realize a certain property, the more likely we are to observe it. In Sec. V, we will critically examine this hypothesis, and see how far one can go without making any appeal to probabilities at this point, but let us grant it for the moment.

Now, let us rephrase the usual argument from naturalness in this language. We focus attention on the subset of string/M theory vacua which, while realizing all the other properties of the Standard model, may have a different value for the electroweak scale M_{EW}. Since this is quadratically renormalized, in the absence of any other mechanism, we expect that the fraction of theories with $M_{EW} < M_{EW,max}$ should be roughly

$$\frac{M^2_{EW,max}}{M^2_{cutoff}} \sim 10^{-30}$$

taking $M_{cutoff} \sim M_{GUT}$ for definiteness. While small, of course given enough vacua, we will find vacua in which the hierarchy is a result of fine tuning.[13]

Let us now grant that we have some subset of the string theory vacua in which the Higgs mass is determined by supersymmetry breaking in the general way we discussed in Sec. II.F.1. More specifically, let us grant that the Higgs mass satisfies a relation like Eq. (24), with $F_X \sim 10^{11}\text{GeV}$, the intermediate scale, so that we can expect to see supersymmetry at the TeV scale. Then, while there are further conditions to check, one might expect an order one fraction of these models to work.

Now, the naturalness argument is the claim that, since most of the TeV scale supersymmetry vacua work (fit the data), while only 10^{-30} of the fine tuned vacua work, we should expect to live in a universe with TeV scale supersymmetry, or at least prefer this alternative to the fine-tuned models.

Of course, we arranged our discussion in order to make the essential gap in this argument completely evident. It is that, even though the fraction of fine-tuned vacua which work is relatively small, if their number is large, we might find in the end that far more of these vacua work than the supersymmetric vacua. Given our hypothesis, string theory would then predict that we should *not* see supersymmetry at the TeV scale.

Is this what we expect or not? Before taking a position, one should realize that the additional structures being postulated in the supersymmetric models – the scale of susy breaking, a solution to the μ problem, a mediation mechanism in which FCNC and the other problems of generic supersymmetric models are solved, and so on – each come with a definite cost, not in terms of some subjective measure of the complexity or beauty of the theory, but in terms of what fraction of the actual string/M theory vacua contain these features. Is this cost greater than 10^{-30} or not?

We will describe some results bearing on this question in Sec. V, but at present we are still far from having sufficient knowledge of the set of string vacua to make

[13] See Silverstein (2004a) for a toy model of how fluxes can do this.

convincing statements. But given toy models which incorporate some of the detailed structure of flux vacua in computable limits, there are already interesting suggestions about how the computation might turn out (Dine *et al.*, 2004; Douglas, 2004c; Silverstein, 2004a; Susskind, 2004)

What is already clear, is that claims that string theory naturally 'prefers' low energy supersymmetry are, as yet, far from being justified. Indeed, the simplest toy models suggest the opposite. It would be very important to improve our understanding of this point.

4. Early universe cosmology

There is substantial and growing evidence for a period of early universe inflation to explain the homogeneity, isotropy, and large-scale structure of our Hubble volume (Linde, 2005; Spergel *et al.*, 2006). However, obtaining a reasonable model of inflation in string theory requires a detailed understanding of moduli stabilization (Kachru *et al.*, 2003). The reason is as follows.

The dynamics of scalar fields ϕ_i evolving in a scalar potential in an FRW cosmology with Hubble constant $H = (\frac{\dot{a}}{a})$, is governed by the equations

$$\ddot{\phi}_i + 3H\dot{\phi}_i = -\frac{\partial V}{\partial \phi_i} \tag{30}$$

where V is the potential for the scalar fields. To obtain slow-roll inflation, one needs to require that the steepest gradient in the potential is not very steep, since (30) basically describes gradient flow in V. However, in string models at moderately weak coupling $g_s \to 0$ and/or large volume $R >> l_s$ for the internal dimensions, one *knows* that this is not true. *All* known sources of potential energy fall rapidly to zero as R^{-n} with $n \geq 6$ in models that are well described by 10d supergravity (see e.g. (Giddings, 2003; Kachru *et al.*, 2006; Silverstein, 2004b)). Similarly all known sources vanish as a positive power g_s. These power laws are far too fast to allow slow-roll inflation (or late-time acceleration for that matter (Fischler *et al.*, 2001; Hellerman *et al.*, 2001)).

The lesson is that in order to achieve inflation, one must either work in a regime of strong coupling/small radius where it is difficult at present to compute (Brustein *et al.*, 2003), or one must find models where the radii/dilaton and other rapidly rolling moduli, have been stabilized by a computable potential. Even then, achieving inflation in a controlled manner is quite challenging (Kachru *et al.*, 2003). But given our earlier comments about flux vacua and moduli stabilization, this makes flux vacua a logical place to try to construct models of slow-roll inflation in string theory.

Let us illustrate these issues by discussing a concrete proposal for inflation. Dvali and Tye proposed that natural models of inflation may be obtained by considering branes and anti-branes (or more generally, branes which do not preserve the same supersymmetry), separated on a compact space M (Dvali and Tye, 1999). The branes and anti-branes attract one another via the higher-dimensional analogue of Coulomb exchange of RR and gravitational forces. The candidate inflaton is the brane separation mode on M, while the exit from inflation can occur when the brane and anti-brane reach a distance $\sim l_s$ from one another, where the lightest stretched string becomes tachyonic. This picture is thoroughly reminiscent of hybrid inflation (Linde, 1994), with the tachyon playing the role of the "waterfall field" that causes the exit from inflation. Such brane inflation models were generalized and explored in Alexander (2002); Burgess *et al.* (2002, 2001); Dasgupta *et al.* (2002); Dvali *et al.* (2001); Garcia-Bellido *et al.* (2002); Gomez-Reino and Zavala (2002); Herdeiro *et al.* (2001); Jones *et al.* (2002); Shiu and Tye (2001) without addressing the issue of moduli stabilization; a review of these models appears in Quevedo (2002).

It was argued in Kachru *et al.* (2003) that considering brane inflation in the absence of moduli stabilization does not make sense; that is, that the predictions derived from considerations of the open string potential ignoring the closed string modes, would be corrected very significantly by inclusion of the closed strings. The interbrane potential, for D3 and anti-D3 branes separated by a distance d in M, is given by

$$V(d) = 2T_3 \left(1 - \frac{1}{(2\pi)^3}\frac{T_3}{M_{10}^8 d^4}\right) \tag{31}$$

where T_3 is the brane tension and d is the interbrane distance. Note that d is related to a canonically normalized scalar field via the relation $\phi = \sqrt{T_3}d$.

It is well known that to obtain standard slow-roll inflation, the inflaton potential must satisfy the slow-roll conditions, that

$$\epsilon = \frac{M_P^2}{2}(\frac{V'}{V})^2 << 1, \quad \eta = M_P^2\frac{V''}{V} << 1 \ . \tag{32}$$

Primes denote derivatives with respect to the inflaton ϕ; the first condition roughly guarantees that the Universe will undergo accelerated expansion, while the second guarantees that this period of accelerated expansion will last sufficiently long to explain the horizon and flatness problems (for reviews of basic facts about inflationary cosmology, see Linde (2005); Lyth and Riotto (1999)).

Can (31) plausibly satisfy these conditions? There is a well known problem. On a space of radius R, using the relation between M_{10} and the 4d Planck scale M_P, one can quickly see

$$\eta \sim (R/d)^6 \times \mathcal{O}(1) \ . \tag{33}$$

Since one expects $d \leq R$, such models will have trouble giving rise to slow-roll inflation. Many clever model building tricks were postulated to surmount this kind of difficulty in the papers cited previously; arguments presented in Kachru *et al.* (2003) show that *generically*, the problem persists.

However, it masks an even more basic problem. Even supposing one did engineer a flat inter-brane potential, the correct 4d Einstein frame potential is not quite given by (31). Instead, it is rescaled by the Weyl-rescaling to reach 4d Einstein frame, which multiplies (31) by an overall factor of $\frac{1}{R^{12}}$. Now regardless of the interbrane potential, when the manifold M is in the $R >> 1$ regime where the potential is expected to take this form, the system of equations (30) will lead to rapid decompactification!

A similar argument would show that one must find a way to avoid relaxing to $g_s \to 0$. It is believed that both of these problems can be solved, by stabilizing the radion and the dilaton, in some classes of flux compactifications.

This will leave the problems involved in engineering a flat enough interbrane potential to satisfy (32), and indeed generic mechanisms of moduli stabilization do not yield sufficiently flat interbrane potentials. It has become a problem of great interest in recent years to find natural models of flat inflationary potentials in string theory. While some tuning is involved, construction of pseudo-realistic models seems well within reach in many scenarios. The state of the art in engineering concrete examples of brane inflation where one can hope to find such flat potentials is described in Baumann *et al.* (2006).

We have focused on brane inflation here as an illustration of the issues which tie inflation to moduli stabilization, but similar issues arise in other inflationary models using moduli fields (Banks, 1995a; Binetruy and Gaillard, 1986; Blanco-Pillado *et al.*, 2004, 2006) or axions (Adams *et al.*, 1993; Arkani-Hamed *et al.*, 2003; Banks *et al.*, 2003; Dimopoulos *et al.*, 2005; Easther and McAllister, 2006; Freese *et al.*, 1990) to inflate in string compactifications.

III. QUANTUM GRAVITY, THE EFFECTIVE POTENTIAL AND STABILITY

As the subsequent discussion will be quite technical, before going more deeply into details we should ask more basic questions, such as

- What are our implicit assumptions? Can we trust them, and the formalism which they lead to?
- Might there be *a priori* arguments that the type of vacuum we seek (with stabilized moduli and positive cosmological constant) does not exist, or is extremely rare?
- Related to this, might there be unknown additional consistency conditions, which are satisfied by only a few of the vacua?

Since as yet we have no fully satisfactory nonperturbative definition of any string theory or M theory, clearly our discussion cannot start from first principles; we need to make assumptions about how the theory works and what constitutes a "solution" to proceed. Thus, our arguments will not be conclusive, but rather are meant to summarize existing work and suggest new approaches to addressing these questions.

A. The effective potential

Our point of view, as we explained in the introduction and implicitly assumed throughout Sec. II, is that the vacuum structure of string/M theory is determined by an effective potential V_{eff}. This is a function of the many scalar fields which parameterize the local choices (moduli) determining a particular solution, and whose value is the exact vacuum energy of that solution. Granting this, our problem is to define V_{eff} incorporating all classical and quantum contributions to the energy, compute it in a controlled way, and find its local minima.

While this is how all known physical theories work, there are good reasons not to accept this uncritically in a quantum theory of gravity, as has been particularly emphasized in Banks (2004); Banks *et al.* (2004); Dine (2004a). Let us cite a few of these reasons, and then consider the various candidates we have for complete definitions of the theory, to try to evaluate them.

We begin by asking whether the concepts which enter into the effective potential are really well defined. First, as is well known, there is no universal way to define energy in a generally covariant theory. The standard formal definition of energy is the dynamical variable conjugate to time translation, in the sense of Hamiltonian mechanics, or in quantum commutation relations. However, in a generally covariant theory, time translation invariance is simply an arbitrariness of the choice of global time coordinate, on which no observable can depend. The logical conclusion is therefore that the energy, and thus the effective potential, in any such theory must be identically zero.

Of course, this conclusion is not really acceptable in a theory which can describe conventional non-gravitational physics, as clearly the concept of energy is sensible and useful in that context. Formally, the simplest way around it is to consider only asymptotically flat solutions, which at large distances (in any space-like direction) approach Minkowski space-time (or its product with the internal dimensions). In such a solution, one can define the generators of the Poincaré group purely in terms of the asymptotic fields; in particular the energy E is related to the term in the metric $g_{00} \sim -2E/r$ which expresses the Newtonian potential of a source with mass E. Since the vacuum solutions we are interested in as models of our universe in the present epoch are extremely close to asymptotically flat, this definition would seem entirely adequate.

Actually, there are major loopholes in the argument we just made, coming from caveats such as "in the present epoch," and "extremely close to asymptotically flat." We will discuss these below in Sec. III.C, with the conclusion that they all rely on some sort of non-locality in the theory. While this does not make them unthinkable, let us postpone this discussion and proceed to discuss the definition of the effective potential in Minkowski space-time.

Let us recall the standard definition of the effective potential in a quantum field theory, for definiteness a theory of a single scalar field ϕ. We first couple ϕ to a source J, and compute (say using the functional integral) the partition function $Z(J)$, to define the generating function of connected Green's functions $F(J) = \log Z(J)$. We then set the expectation value ϕ_0 of ϕ by solving the equation

$$\frac{\partial F}{\partial J} = \phi_0,$$

which formally amounts to a Legendre transform. The resulting functional $\Gamma(\phi_0)$, specialized to constant ϕ_0, is the effective potential.

In trying to repeat this definition in string theory, we face the problem that it is not possible to couple a string theory to a local source, nor to a local current; this was one of the main problems with the early proposals for using strings to describe hadronic physics. This led to the general observation that the theory tends not to provide natural definitions of "off-shell" quantities, meaning quantities defined in terms of space-time histories which are not solutions. For example, computations of scattering amplitudes using the string world-sheet formalism are unambiguous only if all of the external states are on mass shell. While off-shell amplitudes can be defined, these depend on additional arbitrary choices (the world-sheet conformal factor), which leads to ambiguous results. This is not considered a flaw in the theory, as the S-matrix is defined purely in terms of scattering of on-shell external states. However, the effective potential is an off-shell quantity.

Two general ways around this problem are known. The first approach is to do without the coupling to a local source, instead manipulating the value of ϕ_0 by adjusting the boundary conditions. This is not completely general, but can be satisfactory in some situations. For example, if the effective potential is zero, any constant ϕ_0 will be a solution, and we can pick a particular solution by choice of boundary conditions. More generally, if we know the effective potential in advance, we can find the solutions of the effective field theory, and pick one by choice of boundary conditions.

This is implicitly what is done in most work on string compactifications with extended supersymmetry. For example, in a family of compactifications to Minkowski space, supersymmetry guarantees that the effective potential is zero, so there is no difficulty in adjusting moduli by varying boundary conditions. This is relevant for us as our type II flux compactifications have extended supersymmetry at a high scale, broken by the fluxes, and we can appeal to this argument to justify our computations of other quantities in the effective Lagrangian, such as the kinetic terms.

Another class of examples is flux compactifications with $N \geq 4$ supersymmetry in anti-de Sitter space. Again, supersymmetry determines the effective potential uniquely, so that one can study solutions with prescribed boundary conditions without detailed string theoretic computation. This is used implicitly in many works on the AdS/CFT correspondence.

While at first this definition seems inadequate for the problem at hand, in which we want to compute a non-trivial effective potential which we do not know in advance, one can still try to follow this route. One would start with a known extended supersymmetry background, and then postulate boundary conditions (probably time-dependent) which, if it were the case that the effective potential described a second non-trivial metastable vacuum, would lead to a solution matching on to this solution in the interior. However, this approach generally fails, for an interesting reason which we will discuss in Sec. III.C.

This brings us to the second approach, which is simply to couple the string theory to a non-local source. For example, one can do this in string field theory, the framework which is most directly analogous to quantum field theory (Zwiebach, 1993). Just as QFT can defined in terms of an operator $\phi(x)$ which creates or destroys a particle at a point in space-time x, here one introduces a string field operator, call it $\Phi[L]$, which creates or destroys a string on a one-dimensional loop L in space-time. One can then introduce a source $J[L]$ for the string field into the action in the standard way, say as

$$S = S_0 + \int dL\ \Phi[L]J[L],$$

where the definition of the integral over loops is taken from the string field theory framework. One then follows the same reasoning which led to the field theory definition, to get a string field theoretic effective potential $\Gamma[\Phi_0]$.

While such a definition would be rather difficult to use in practical computations, the point is to have a precise definition which underlies and could be used to justify our approximate considerations. To do this, the next step would be to identify the light modes in $\Gamma[\Phi_0]$, and solve for all of the others, to obtain an effective potential which is a function of a finite number of fields. To the extent that we could do this, we would have made precise the intuition that string theory reduces to field theory at long distances, where the effective potential is a valid concept.

However, there are formidable obstacles to making such a definition precise. At present there is very little understanding of string field theory beyond its perturbative expansion, and just as for quantum field theory, this expansion is only asymptotic (Shenker, 1990). It is also not obvious that all of the nonperturbative effects we will call upon below are contained in string field theory, see Schnabl (2005) for relevant progress on this. In any case, verifying or refuting the approximate discussion we will make below would be an important application of a nonperturbative definition.

For four-dimensional quantum field theory, such a definition is made by appealing to the renormalization group. One must find some asymptotically free UV completion of the theory of interest, and then find some approximate finite description of the weakly coupled short distance theory, such as lattice field theory.

While we have no comparable theoretical understanding of string theory, there is a widely shared intuition that, at least in considering low energy processes and vacuum structure, string theory is weakly coupled at short distances (the string scale and below). This intuition has several sources: first, the extended nature of the string cuts off interactions at these distances. Second, asymptotic supersymmetry makes the leading contributions of massive states to the effective action cancel. Finally, other effects of massive states are suppressed by inverse powers of the fundamental scales. Presumably, this intuition justifies matching on to a field theoretic description at distances around the string scale, and then following the standard RG paradigm.

B. Approximate effective potential

Let us now grant that the problems we just discussed are only technical, and consider how we would make a precise definition of the effective potential we use in this review, namely in weakly coupled string field theory, taking into account nonperturbative effects in a semiclassical expansion.

We start with ten-dimensional effective field theory, *i.e. supergravity* with α' and g_s corrections. We then compactify to get a four-dimensional effective action with massive KK modes, string modes and the like. At this level, the discussion is precise. Even in the presence of fluxes, in many models, the leading results can be inferred from supersymmetry and considerations of 4d $N = 2$ supergravity.

We then need to add in semiclassical nonperturbative effects, such as instantons and wrapped branes. Our control and understanding of such computations improved dramatically in the mid 1990s. The first computations of instanton effects in 4d supersymmetric field theories were made in the 1980s (Affleck *et al.*, 1984), when the vacuum structure of supersymmetric QCD was elucidated. By the mid 1990s, it became possible to find exact superpotentials for a wide variety of $N = 1$ field theories (Intriligator *et al.*, 1994). In $N = 2$ field theories, infinite series of instanton contributions to the prepotential were also summed using holomorphy arguments and symmetries, starting with the seminal work of Seiberg and Witten (1994).

In string theory, mirror symmetry relates exact prepotentials in type IIa Calabi-Yau models and type IIb Calabi-Yau models, where an infinite (worldsheet) instanton sum in the prepotential on one side maps to a completely classical geometric computation on the other (Candelas *et al.*, 1991). In the duality revolution, it was found that in suitable circumstances, string duality maps these worldsheet instanton sums to spacetime instanton sums, allowing one to recover directly string theory and (in the decoupling limit) field theory non-perturbative effects from string duality (Kachru *et al.*, 1996; Kachru and Vafa, 1995). This grew into the realization that one could design stringy configurations of branes or singularities to give rise to a given low energy field theory, and compute the instanton sums via string techniques, in both $N = 2$ and $N = 1$ field theories (Katz *et al.*, 1997; Katz and Vafa, 1997).

More generally, holomorphy arguments allow one to classify which kinds of Euclidean branes, wrapping which kinds of topologies, can contribute to a holomorphic superpotential. With the introduction and understanding of D-branes in the mid 1990s (Polchinski, 1995), the full list of possible BPS instantons relevant for a variety of $N \geq 1$ vacua was known. A prototypical example of a macroscopic argument classifying (under some clearly stated assumptions) which kinds of branes and topologies are relevant for instanton effects in F-theory, appears in Witten (1996b). While exact computation of the superpotential in a general compactification is highly nontrivial and still beyond our reach, this does allow for principled estimates of the leading instanton contributions in many backgrounds. In the particular cases where the instanton effect can be re-interpreted in the low energy effective theory as a dynamical effect in quantum field theory, even the coefficient can be estimated with some confidence, by matching to exact field theory results. In many examples, even this crude level of understanding suffices to exhibit vacua in the reliable regime of weak coupling and large volume, under moderate assumptions about the precise coefficients appearing in the nonperturbative superpotential.

The main issue we now have to address, is that we want to take the sum of various terms, some inferred directly from supergravity or world-sheet physics, and others computed (or even inferred) from nonperturbative effects. Typically, a solution of $\partial V_{eff}/\partial\phi^i = 0$ for the full effective potential will not be a critical point of the various terms which enter into V_{eff}, so these terms will be ambiguous. But if there are ambiguities, how can we be sure that we have fixed them for every term in a consistent way?

Our eventual answer to this question in this review will be to exhibit examples of solutions in which contributions to the effective potential with different origins have parametrically different scales. Thus, although individual terms may have some ambiguity, a very weak control over this ambiguity will suffice to prove that the full effective potential has minima.

We see no reason that such a separation of scales should be needed for consistency, so this type of argument is not completely satisfactory; it does not apply to large numbers (perhaps the vast majority) of solutions. A fully satisfactory argument has to be based on a complete formulation of string theory in which the effective potential has a precise definition, as we discussed earlier. However, already within the limits of this argument, we will find sufficiently many stabilized vacua to justify the basic claims of Sec. II.

Even restricting attention to these solutions, we are still not done. Another pitfall to guard against in extrapolating results for the effective potential is the possibility of phase transitions. This is especially worrisome for first order transitions, which unlike second order transitions have no clear signal such as a field or order parameter becoming massless. Such transitions are not possible in global supersymmetry, in which the energy of a supersymmetric vacuum is always zero; however this is not true after supersymmetry breaking and in supergravity. Should we worry about this possibility?

Actually, the rules here are somewhat different from equilibrium statistical mechanics and field theory, in that sufficiently long-lived metastable configurations will count as vacua. However, a possibility which needs to be considered is that additional fields, perhaps arising from the Kaluza-Klein modes of dimensional reduction, or composite fields expressing quantum correlations, might destabilize our candidate vacua.

The first possibility, that Kaluza-Klein modes destabilize vacua, will be considered in Sec. IV. The basic argument that this generically does not happen was given in Sec. II.F.2.

The second possibility is handled by a combination of arguments. In most of the effective field theory, quantum fluctuations are controlled by the string coupling, which we have assumed to be small. Thus, mass shifts for composite fields will be small, so given that the moduli are all massive, we do not expect phase transitions. This argument has the flaw that some subsectors of the theory must be strongly coupled at low energy (after all we know this is the case for QCD). For these sectors, we appeal to existing field theory analyses, and the assumption that the supersymmetry breaking scale is smaller than the fundamental scale, so that supergravity effects are a small correction.

C. Subtleties in semiclassical gravity

In our discussion so far, we assumed that our local region of the universe can be well modelled as Minkowski space-time. Of course, no matter how slow the time evolution of the universe, or how small the cosmological constant, if these are non-zero, at sufficiently large scales the nature of the solution will be radically different from Minkowski space-time. Thus, we might wonder whether even if a solution looks consistent on cosmological scales, it could be inconsistent as a full solution of the theory.

At first this might sound like it could only happen if the underlying framework were non-local. However, while string/M theory is believed to be in some sense non-local at the fundamental (Planck and string) length scales, in all known formalisms and computations these effects are either exponentially small at longer distances, or appear to be gauge artifacts, analogous to the apparent instantaneous force at a distance one finds in Coulomb gauge. Thus it is hard to see how they could be relevant. Still, some feel that paradoxes involving black hole evaporation and entropy point to non-locality (Giddings, 2006).

Even in a local theory, a solution which is consistent on short time scales, can in principle be inconsistent on longer time scales, by developing a singularity with no consistent physical interpretation or "resolution." Although one often hears the slogan that "string theory resolves space-time singularities," there are examples of space-like singularities with no known resolution, nor any proof that this cannot be done, making this an active field of research.

Now in an ordinary physical theory, one would say that the possibility of developing a singularity with no consistent interpretation shows that the theory is not fundamental; rather it should be derived from a more fundamental theory in which the corresponding solution is not singular. Familiar examples include Navier-Stokes and other phenomenological many-body theories, and of course classical general relativity.

In the present context, one might attempt a different interpretation. If it turned out here that some subset of vacua generically led to singularities, while another subset did not, it might be reasonable to exclude the first set of vacua as inconsistent. Now it seems strange to us, indeed acausal, to throw out a solution because of an inconsistency which appears (say) $10^{10^{10}}$ years in the future. Still, if such an approach led to interesting claims, it might be worth pursuing.

Another idea along these lines is that there might be approximately Minkowski solutions which, while themselves consistent, cannot be embedded in a solution with a sensible cosmological origin. This test seems better as it is consistent with causality. It could be further refined by asking not just that the cosmology be theoretically consistent, but that it agree with observation. Of course, we will eventually need to address this issue in the course of testing any given solution, but we might ask if there are simple arguments that some solutions cannot be realized cosmologically, or cannot satisfy the constraints discussed in Sec. II.F.4, before going into details. We know of no results in this direction however.

Let us now come back to a point raised in Sec. III.A, and explain the obstacles to performing thought experiments which prove the existence of multiple (isolated) vacua of an effective potential (Banks, 2000; Farhi *et al.*, 1990). For instance, suppose the effective potential for a single scalar ϕ has two vacua at $\phi_\pm$. One can make a vacuum bubble interpolating between the two vacua, whose surface tension we can call σ. Starting from the ϕ_+ vacuum, suppose one nucleates a bubble of radius R in the ϕ_- phase. The Schwarzschild radius of the bubble is $\sigma(R/M_P)^2$. So the bubble will be smaller than its Schwarzschild radius unless $R > \sigma(R/M_P)^2$, i.e. unless

$$R < \frac{M_P^2}{\sigma} \ . \tag{34}$$

This is interesting for the following reason. A ϕ_+ experimentalist can only use the bubble to infer the existence of the ϕ_- vacuum and study its properties, if (34) is satisfied. We would expect that the potential barrier between two typical vacua in a quantum gravity theory should be $\sim M_P$, as there is no small parameter to change the scaling in typical solutions. Then, one would also find $\sigma \sim M_P^3$, and only bubbles smaller than the Planck length would be outside their Schwarzschild radius! Of course such bubbles are not a priori meaningful solutions, and could not be used by an experimentalist to verify the existence of other vacua.

This argument is a bit quick, for example because the vacua we will discuss in Sec. IV do have small parameters, but the conclusion is largely correct, as explained further in (Banks, 2000; Farhi *et al.*, 1990).

D. Tunneling instabilities

We have argued that in string theory, the effective potentials one infers from direct computation typically have many minima. It then makes sense to discuss physics in any of the metastable vacua, only if the lifetime τ of such a vacuum is parametrically long compared to the string time. While for generic vacua arising at radii or couplings of $\mathcal{O}(1)$ this may rarely be the case, it is believed that there are very large numbers of vacua that in fact easily satisfy this criterion, and even the more stringent criterion $\tau > \tau_{today}$, which is roughly 15 billion years.

The quantitative theory of the decay of false vacua in field theories with many vacuum states was worked out by Coleman and collaborators in a series of classic papers (Callan and Coleman, 1977; Coleman, 1977; Coleman and De Luccia, 1980). Let us consider a toy model consisting of a single scalar field ϕ, with a metastable de Sitter vacuum of height V_0 at ϕ_0, and a second Minkowski vacuum at infinity in field space. This can be thought of as a rough toy model of the potential for a volume modulus in a string compactification, where the second vacuum represents the decompactification limit (Kachru *et al.*, 2003a). Suppose the barrier height separating the dS vacuum from infinity is V_1.

The tension of the bubble wall for the bubble of false vacuum decay is easily computed to be

$$T = \int_{\phi_0}^{\infty} d\phi \sqrt{2V(\phi)} \tag{35}$$

The dominant tunneling process differs depending on whether $V_0 M_P^2 >> T^2$ or $V_0 M_P^2 << T^2$.

Since we see that $T \sim \sqrt{V_1}\Delta\phi$, this translates into the question of whether $\Delta\phi << \sqrt{\frac{V_0}{V_1}}$ or $\Delta\phi >> \sqrt{\frac{V_0}{V_1}}$. The former regime is called the "thin wall limit" for obvious reasons. In this limit, the analysis of Coleman et al applies. The tunneling probability, is given by

$$P = \exp\left(-\frac{27\pi^2 T^4}{2V_0^3}\right) \tag{36}$$

For dS vacua with small $V_0 << V_1$, the rate is clearly highly suppressed, easily yielding a lifetime in excess of 10^{10} years.

In the opposite regime of a low but thick potential barrier, $V_0 M_P^2 << T^2$, the dominant instanton governing vacuum decay would instead be the more enigmatic Hawking-Moss instanton (Hawking and Moss, 1982). The physical interpretation of this instanton is unclear; a description in terms of thermal fluctuations of the ϕ field which yields the same estimate for the rate can be found in Linde (2005) and references therein. The action of this instanton is the difference between the dS entropies of dS vacua with vacuum energies V_0 and V_1, resulting in a tunneling rate

$$P \simeq \exp\left(-S(\phi_0)\right) = \exp\left(-\frac{24\pi^2}{V_0}\right) \ . \tag{37}$$

For small V_0, this again is completely negligible. The formula Eq. (37) neglects a small multiplicative correction factor of $\exp(\frac{24\pi^2}{V_1})$ which accounts for the entropy at the "top of the hill."

For $V_0 << V_1$, this factor is not numerically important, but its presence serves to prove a conceptual point. Because of the existence of the Hawking-Moss instanton, *any* dS vacuum which is accessible in the effective field theory approximation to string theory, will have a lifetime which is parametrically short compared to the Poincare recurrence time of de Sitter space (considered as a thermal system with a number of degrees of freedom measured by the de Sitter entropy) (Kachru *et al.*, 2003a).

This discussion illustrates how, within the regime of effective field theory, one can find long-lived vacua. However, a point which already appeared in Bousso and Polchinski (2000), and has not been settled in more realistic models, is that besides the approximate Minkowski vacua at infinity we just discussed, there are many other possible endpoints for the decay of a vacuum, both dS and AdS vacua. Some of these tunneling rates have been computed in Ceresole *et al.* (2006); Frey *et al.* (2003); Kachru *et al.* (2003a, 2002), and generically they are also very small. However, one might wonder whether the large degeneracy of possible targets could lead to enough "accidentally" low barriers to substantially increase the overall decay rates. This might be addressed using the statistical techniques of Sec. V.

E. Early cosmology and measure factors

In any theory with many vacua, one could ask: are some vacua preferred over others? A natural answer in the present context is that if so, it will be for cosmological reasons: perhaps the "big bang" provides a preferred initial condition, or perhaps the subsequent dynamics favors the production of certain vacua.

This type of question has been studied by cosmologists for many years; some recent reviews include Guth (2000); Linde (2005); Tegmark (2005); Vilenkin (2006). At present the subject is highly controversial and thus we are only going to sketch a few of the basic ideas here.

One general idea is that a theory of quantum gravity will have a preferred initial condition. The most famous example is the wave function of Hartle and Hawking (1983), which is defined in terms of the Euclidean functional integral. Presumably, time evolving this wave function and squaring it would lead to a probability distribution on vacua. In the present context, this suggests looking for a natural wave function on moduli space, or on some larger configuration space of string/M theory. An idea in this direction appears in Ooguri *et al.* (2005).

Another idea, more popular in recent times, is that the distribution of vacua is largely determined by the dynamics of inflation. Inflation involves an exponential expansion of spatial volume, which tends to wash away any dependence on initial conditions. In particular, many of the standard arguments for inflation in our universe, such as the explanations of homogeneity, flatness, and the non-observation of topological defects, rely on this property. While these standard arguments do not in themselves bear on the selection of a particular vacuum, it is widely believed that inflation also washes away all dependence on the initial conditions relevant for vacuum selection (say the choice of compactification manifold, moduli and fluxes), because of the phenomenon of *eternal inflation* (Linde, 1986a,b; Vilenkin, 1983).

Without going into details, eternal inflation leads to a picture in which any initial vacuum, will eventually nucleate bubbles containing all the other possible vacua, sometimes called "pocket universes." Because of the exponential volume growth, the number distribution of these pocket universes will "very quickly" lose memory of the initial conditions and, one hopes, converge on some universal distribution.

For this to happen, the microscopic theory must satisfy certain conditions. First, the effective potential must either contain multiple de Sitter vacua, or contain regions in which inflation leads to large quantum fluctuations (essentially, one needs $\delta\rho/\rho \sim 1$). Then, to populate all vacua from any starting point, and thus have any hope to get a universal distribution, all vacua must be connected by transitions. These conditions are fairly weak and very likely to be true in string/M theory. The first can already be satisfied by models of the type we discussed in Sec. II.F.4. One can find much evidence for the second condition, that all vacua are connected through transitions, from the theory of string/M theory duality. For example, it is true for a wide variety of models with extended supersymmetry (for example, $N = 2$ type II compactifications on Calabi-Yau (Avram *et al.*, 1996; Greene *et al.*, 1995)), and thus will be true for flux compactifications built from these, since the potential goes to zero in the large volume limit.

Either way, the result of such considerations would be a probability distribution on vacua, usually referred to as a "measure factor." This probability distribution would then be used to make probabilistic predictions, along the lines we suggested in an example in Sec. II.F.3.

At this point, many difficult conceptual questions arise. After all, our (the observable) universe is a unique event, and most statisticians and philosophers would agree that the standard "frequentist" concept of probability, which assumes that an experiment can be repeated an indefinite number of times, is meaningless when applied to unique events. While this may at first seem to be only a philosophical difficulty, it will become practical at the moment that our theoretical framework produces a claim such as "the probability with which our universe appears within our theory is .01", or perhaps 10^{-10}, or perhaps 10^{-1000}. How should we interpret such results?

A serious discussion of this question would be lengthy, and would require calling upon many topics out of the main line of our review. In particular, many cosmologists have argued that the interpretation of a measure factor requires taking into account the selection effects we discussed in Sec. II.E.1 in a *quantitative* way, estimating the "expected number of observers" contained in each pocket universe, to judge whether a "typical observer" should expect to make a certain observation. Doing this would involve a good deal of astrophysics, and perhaps even input from other disciplines such as chemistry, biology and so on. Indeed, the complexity of actually implementing such a program has led many to doubt that any generally accepted conclusions could be reached this way.

It may be better to restrict one's ambitions, to avoid this complexity. One step in this direction would be to restrict attention to vacua which can reproduce all observations to date (*e.g.* the Standard Model and relevant cosmological observations), as we know this is anthropically allowed, and not ask why we do not see something else, instead only deriving predictions for yet to be measured observables. Of course, if we go on to see something new with "small probability," we will be left wondering what this means.

Another would be to consider a probability as significant only if it is extremely small, and consider such unlikely vacua as "impossible." In other words, we choose some ϵ, and if our observations can only be reproduced by vacua with probability less than ϵ, we consider the theory with this choice of measure factor as falsified. While one might debate the appropriate choice of ϵ, since some ideas for measure factors lead to extremely small probabilities for some vacua (say proportional to tunneling rates, which as we saw in Sec. III.D are extremely small), this might be interesting even with an ϵ so small as to meet general acceptance.

Another answer, which is probably the most sound philosophically, is not to try to interpret absolute probabilities defined by individual theories, but only compare probabilities between different theories, considering the theory which gives the largest probability as preferred. Even without a competitor to string/M theory, this might be useful in judging among proposed measure factors, or dealing with other theoretical uncertainties. Following up this line of thought would lead us into Bayesian statistics; see MacKay (2003) for an entertaining and down-to-earth introduction to this topic.

Anyhow, these questions are somewhat academic at this point, as general agreement has not yet been reached about how to define a measure factor, or what structure the result might have. In particular, doing this within eternal inflation is notoriously controversial, though recent progress is reported in Bousso (2006); Vilenkin (2006) and references there, and perhaps generally accepted candidate definitions will soon appear.

Thus let us conclude this subsection by simply listing a few of the claims for measure factors which appear in existing literature. One such is the entropy $\exp 24\pi^2 M_P^4/E$ of de Sitter space with vacuum energy E. This is the leading approximation to the Hartle-Hawking wave function, where E is usually interpreted as the vacuum energy in some initial stage of inflation. Since such a factor is extremely sharply peaked at small E, its presence is more or less incompatible with observed inflation, ruling this wave function out. There are some ideas for how corrections in higher powers in E could fix this, see Firouzjahi *et al.* (2004); Sarangi and Tye (2006).

Another common result is $\exp 24\pi^2 M_P^4/\Lambda$, formally the same entropy factor, but now as a function of the cosmological constant at the present epoch. This arose in the early attempts to derive a measure from eternal inflation, and has a simple interpretation there: the probability that a randomly chosen point sits in some vacuum, includes a factor of the average lifetime of that vacuum, as predicted by Eq. (37). This interpretation suggests that this measure factor is also incorrect, as during almost all of this lifetime the universe is cold and empty, so this factor has no direct bearing on the expected number of observers. More technical arguments have also been made against it.

If we ignore this problem, since this measure is heavily peaked on small Λ, we might claim to have a dynamical solution to the c.c. problem (one also needs to argue that $\Lambda < 0$ is not possible). From the point of view we are taking, this proposal has the amusing feature that it predicts that the total number of $\Lambda > 0$ vacua is roughly 10^{120}, which presumably could be checked independently. If so, this would seem superficially attractive, as in principle it predicts a unique overwhelmingly preferred vacuum, the one with minimum positive Λ. On the other hand, the prospects for computing Λ accurately enough to find this vacuum seem very dim. Even if we could get exact results for Λ, there are arguments from computational complexity theory that the problem of finding its minimum is inherently intractable (Denef and Douglas, 2006), making this measure factor nearly useless in practical terms.

Perhaps a better response to the problem is to define away the entropy factor. There are various closely related ways to do this (Vilenkin, 2006); for example in Vanchurin and Vilenkin (2006) it is argued that it can be done by restricting attention to the world-line of a single "eternal observer," and counting the number of bubbles it enters. This leads to a prescription in terms of the stationary distribution of a Markov process constructed from intervacuum tunneling rates; its detailed properties are being explored, but at first sight this appears to lead to a wildly varying probability factor $P(i)$ which, since it is determined by the structure of high energy potential barriers, would have little correlation to most observable properties of the vacua themselves. As we discuss in Sec. V.F, this might still allow making probabilistic predictions.

Other factors which have appeared in such proposals, and while probably subleading to the ones we covered might be important, include a volume expansion factor (the overall growth in volume during slow-roll inflation), the volume in configuration space of the basin of attraction leading to the local minimum (Horne and Moore, 1994), a canonical measure on phase space (Gibbons and Turok, 2006), dynamical symmetry enhancement factors (Kofman *et al.*, 2004), and the volume of the extra dimensions (Firouzjahi *et al.*, 2004).

F. Holographic and dual formulations

The advent of string/M theory duality in the mid-90's led to an entirely novel perspective on many questions, and several new candidate nonperturbative frameworks, such as matrix models, Matrix theory, and the AdS/CFT correspondence. While at present it is not known how to use any of them to directly address the problem at hand, perhaps the most general of these (and certainly the best-studied one) is the AdS/CFT correspondence, which bears on the definition of solutions with negative cosmological constant.

Specializing to the case of present interest, consider a maximally symmetric four-dimensional solution of string theory with negative cosmological constant, in other words a product of $3+1$ dimensional anti-de Sitter spacetime with a 6 dimensional internal space. According to AdS/CFT, there will exist a dual $2+1$-dimensional conformal field theory (without gravity), which is precisely equivalent to the quantum string theory in this spacetime. This can be made more concrete for questions which only involve observables on the boundary of AdS; for example a scattering amplitude in AdS maps into a correlation function in the CFT, and boundary conditions of the fields in AdS map into the values of couplings in the CFT.

This dictionary has been much studied. The most important entries for present purposes are the relation between the $3+1$ dimensional AdS c.c. and the number of degrees of freedom of the CFT, and the relation between masses in AdS and operator dimensions in the CFT. For example, in Freund-Rubin compactification of IIb string theory on $AdS_5 \times S^5$, the curvature radius $R^4/(\alpha')^2 \sim g_s N$, so the number of degrees of freedom N^2 scales to very large values for weakly curved, weakly coupled vacua. Similarly, the map between operators and gravity modes shows that operators with dimension $\Delta \sim \mathcal{O}(1)$ map to KK modes with masses $\sim 1/R$.

For the Freund-Rubin examples, the AdS curvature radius and the radius of the internal sphere are equal. For the AdS_4 vacua which arise in discussions of the landscape, one is usually interested instead in theories with compact dimensions having $R_{KK} << R_{AdS}$, so there is an effective 4d description. Such theories will have dual CFTs that differ *qualitatively* from those appearing in standard examples of AdS/CFT. By the mapping from gravity modes to field theory operators, we see for instance that the number of operators with $\Delta \sim \mathcal{O}(1)$ will be much smaller in these theories. Instead of an infinite tower of operators with regularly spaced conformal dimension (dual to the KK tower in Freund-Rubin models), these dual CFTs will have a sporadic set of low dimension operators (dual to the compactification moduli), and then a much larger spacing between the operators dual to KK modes.

So, given a class of AdS vacua in the landscape, it seems reasonable to search for candidate dual CFTs that could provide their exact definition. Further thought leads to difficulties with this idea. First, the AdS vacua whose existence is established using effective potential techniques, by definition lie in the regime in which the gravity description is weakly coupled. Since they have no moduli, they do not extrapolate (along lines of fixed points) to a dual regime where the field theory would be weakly coupled. So trying to find the dual field theory, involves working on the wrong (strongly coupled) side of the duality, a difficult procedure at best.

Second, we are not primarily interested in typical landscape vacua. Rather, we are most interested in those highly atypical vacua in which fortuitous cancellations gives rise to small Λ, as in the Bousso-Polchinski argument. Such vacua rely on complicated cancellations between many terms, and there are reasons to think they are exceedingly hard to find explicitly even in the more computable gravity description. This is the familiar problem, that one would need to include Standard Model and other loop corrections to very high orders in perturbation theory, to claim that one had found a specific vacuum with small Λ. Even worse, a small variation to one of these complicated solutions (such as changing a flux by one unit) will spoil the cancellation and give a large cosmological term. This suggests that the CFT's we would be most interested in finding (which are dual to the AdS vacua with atypically small Λ) are also complicated, and furthermore that we might need to compute very precisely to see the cancellations which single out the few solutions with small cosmological term.

Nevertheless, in principle we should be able to get the general features of the problem to agree on both sides. The basic picture would seem to be that we start with QFT's with many, many degrees of freedom, perhaps the dual theory of Silverstein (2003), and then flow down to CFT. To recover agreement with our effective potential analyses, we would need to find that a generic RG flow either loses almost all the degrees of freedom, and thus is dual to large Λ, or else has no weakly coupled space-time interpretation at all. On the other hand, given appropriate tunings in the bare theory, more degrees of freedom would survive the flow, leading to a theory whose dual had a tuned small Λ. It would be very interesting to have a quantitative version of this argument.

IV. EXPLICIT CONSTRUCTIONS

We now add some flesh to our previous qualitative considerations by describing how flux vacua can be constructed in type II string theories. The most studied case involves IIb/F-theory vacua, so we will begin there. We then present more recent results about IIa flux vacua, discuss mirror symmetry in this setting, and provide some definite indications that many new classes of vacua are waiting to be explored. We make no pretense to completeness in reviewing all approaches to the subject; rather, we hope that this review, together with the excellent reviews (Frey, 2003; Grana, 2006; Silverstein, 2004b), will provide a good overview of various approaches and classes of models. In particular, for discussions of models without low energy supersymmetry, the reader should consult Silverstein (2004b).

A. Type IIb D3/D7 vacua

In this subsection, we consider type IIb / F-theory vacua whose 4d $\mathcal{N} = 1$ supersymmetry is of the type preserved by D3/D7 branes in a Calabi-Yau orientifold. The $\mathcal{N} = 1$ vacua which preserve D5/D9 type supersymmetry are less explored, though some examples will appear in a later subsection.

1. 10d solutions

Here, we describe the 10d picture of flux compactifications in the supergravity limit. We follow the treatment in Giddings *et al.* (2002). Closely related solutions (related to the IIb solutions via the F-theory lift of M-theory) were first found in M-theory compactifications on Calabi-Yau fourfolds in Becker and Becker (1996), and some aspects of their F-theory lift were described in Dasgupta *et al.* (1999); Gukov *et al.* (2000).

The type IIb string in 10 dimensions has a string frame action

$$L = \frac{1}{2\kappa_{10}^2} \int d^{10}x \, (-G)^{1/2} e^{-2S} \, (R + 4\partial_\mu S \partial^\mu S$$

$$-\frac{1}{2}|F_1|^2 - \frac{1}{12}G_3 \cdot \overline{G_3} - \frac{1}{4 \cdot 5!}\tilde{F}_5^2\Big)$$
$$+\frac{1}{8i\kappa_{10}^2}\int e^S C_4 \wedge G_3 \wedge \overline{G_3} + S_{loc}\ . \tag{38}$$

The theory has an NS field strength H_3 (with potential B_2) and RR field strengths $F_{1,3,5}$ (with corresponding potentials $C_{0,2,4}$).

$$G_3 = F_3 - \phi H_3 \tag{39}$$

is a combination of the RR and NS three-form fields,

$$\phi = C_0 + ie^{-S} \tag{40}$$

is the axio-dilaton, and

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3\ . \tag{41}$$

The 5-form field is actually self-dual; one must impose the constraint

$$\tilde{F}_5 = *\tilde{F}_5 \tag{42}$$

by hand when solving the equations of motion. Finally, S_{loc} in (38) allows for the possibility that we include the action of any localized thin sources in our background; possible objects which could appear there in string theory include D-branes and orientifold planes.

We will start by looking for solutions with 4d Poincare symmetry. The Einstein frame metric should take the form

$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n \tag{43}$$

μ, ν run over $0, \cdots, 3$ while m, n take values $4, \cdots, 9$ and $\tilde{g}_{mn}$ is a metric on the compactification manifold M. We have allowed for the possibility of a warp factor $A(y)$. In addition one should impose

$$\phi = \phi(y), \quad \tilde{F}_5 = (1 + *)[d\alpha(y) \wedge dx^0 \cdots \wedge dx^3] \tag{44}$$

and allow only *compact* components of the G_3 flux

$$F_3, H_3 \in H^3(M, \mathbb{Z})\ . \tag{45}$$

The G_3 equation of motion then tells one to choose a harmonic representative in the given cohomology class.

One can show by using the trace-reversed Einstein equations for the $\mathbb{R}^4$ components of the metric, that

$$\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\overline{G}^{mnp})}{12Im(\tau)} + e^{-6A}[\partial_m\alpha\partial^m\alpha + \partial_m e^{4A}\partial^m e^{4A}]$$
$$+\kappa_{10}^2 e^{2A}(T_m^m - T_\mu^\mu)_{loc}\ . \tag{46}$$

We have denoted the stress-energy tensor of any localized objects (whose action appears in S_{loc}) by T_{loc}.

This equation already tells us something quite interesting. The first two terms on the right hand side are ≥ 0, but on a compact manifold, the left hand side integrates to zero (being a total derivative). Therefore, in compact models, and in the *absence* of localized sources, there is a no-go theorem: the only solutions have $G_3 = 0$ and e^A = constant, and IIb supergravity does not allow nontrivial warped compactifications. This is basically the no-go theorem proved in various ways in Gibbons (1984); Maldacena and Nunez (2001); de Wit *et al.* (1987).

This does not mean that one cannot find warped solutions in the full string theory. String theory does allow localized sources. It was emphasized already in Verlinde (2000) that one can make warped models by considering compactifications with N D3 branes, and stacking the D3 branes at a point on the compact space; then as is familiar from the derivation of the AdS/CFT correspondence (Maldacena, 1998), the geometry near the branes can become highly warped.

For this loophole to be operative, one needs

$$(T_m^m - T_\mu^\mu)_{loc} < 0 \tag{47}$$

to evade the global obstruction to solving Eq. (46). Before finding nontrivial warped solutions with flux, we will also need one more fact. The Bianchi identity for $\tilde{F}_5$ gives rise to a constraint

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa_{10}^2 T_3 \rho_3^{loc} \tag{48}$$

where T_3 is the D3-brane tension, and ρ_3^{loc} is the local D3 charge density on the compact space. The integrated Bianchi identity then requires, for tadpole cancellation,

$$\frac{1}{2\kappa_{10}^2 T_3}\int_M H_3 \wedge F_3 + Q_3^{loc} = 0 \tag{49}$$

where Q_3^{loc} is the sum of all D3 charges arising from localized objects.

Now, one can re-write the equation (48) more explicitly in terms of the function $\alpha(y)$ as

$$\tilde{\nabla}^2\alpha = ie^{2A}\frac{G_{mnp} *_6 \overline{G}^{mnp}}{12Im(\tau)} + 2e^{-6A}\partial_m\alpha\partial^m\alpha + 2\kappa_{10}^2 e^{2A}T_3\rho_3^{loc}. \tag{50}$$

Subtracting this from the Einstein equation (46), one finds

$$\tilde{\nabla}^2(e^{4A} - \alpha) = \frac{e^{2A}}{24Im(\tau)}|iG_3 - *_6G_3|^2 + e^{-6A}|\partial(e^{4A} - \alpha)|^2$$
$$+2\kappa_{10}^2 e^{2A}[\frac{1}{4}(T_m^m - T_\mu^\mu)^{loc} - T_3\rho_3^{loc}]. \tag{51}$$

Let us make the *assumption*

$$\frac{1}{4}(T_m^m - T_\mu^\mu)^{loc} \geq T_3\rho_3^{loc}\ . \tag{52}$$

This serves as a constraint on the kind of localized sources we will want to consider in finding solutions. The inequality is saturated by D3 branes and O3 planes, as well as by D7 branes wrapping holomorphic cycles; it is satisfied by $\overline{D3}$ branes; and it is violated by O5 and $\overline{O3}$ planes.

Assuming we restrict our sources as above, it follows from (51) that G_3 must be imaginary self-dual

$$*_6 G_3 = iG_3, \tag{53}$$

that the warp factor and C_4 are related

$$e^{4A} = \alpha, \tag{54}$$

and that the inequality (52) is actually *saturated.* So solutions to the tree-level equations should include only D3, O3 and D7 sources. In the quantum theory, one can obtain solutions on compact M with $\overline{D3}$ sources as well; we will describe this when we discuss supersymmetry breaking.

We did not write out the extra-dimensional Einstein equation and the axio-dilaton equation of motion yet; their detailed form will not be important for us. Imposing them, we find that this class of solutions describes F-theory models (Vafa, 1996) in the supergravity approximation, including the possibility of background flux. As noted earlier, these solutions are closely related to those of Becker and Becker (1996), whose F-theory interpretation has also been described in Dasgupta *et al.* (1999); Gukov *et al.* (2000).

The simplest examples of such solutions are perturbative IIb orientifolds. An argument of Sen (1997) shows that every compactification of F-theory on a Calabi-Yau fourfold has, in an appropriate limit, an interpretation as a IIb orientifold of a Calabi-Yau threefold. We will therefore develop the story in the language of IIb orientifolds, but the formulae generalize in a straightforward way to the more general case. In this special case of perturbative orientifolds, at leading order, the metric on the internal space is *conformally* Calabi-Yau; it differs by the warp factor e^{2A}. For this reason, these flux vacua are often described as Calabi-Yau compactifications with flux, although strictly speaking the metric on the internal space is not Calabi-Yau (and in the case of general F-theory models, is related indirectly only to the base of an elliptic Calabi-Yau fourfold, not a Calabi-Yau threefold).

2. 4d effective description

In this section we describe the construction of the 4d effective action for IIb orientifolds with RR and NS flux, following Giddings *et al.* (2002). The main result will be an explicit and computable result for the 4d effective potential, which can be analyzed using analytical, numerical or statistical techniques. Earlier work in this direction appeared in Dasgupta *et al.* (1999); Gukov *et al.* (2000); Mayr (2001); Taylor and Vafa (2000)), while related results in gauged supergravity were presented in Andrianopoli *et al.* (2002a,b,c, 2003a,b); Angelantonj *et al.* (2004, 2003); Dall'Agata (2001, 2004a,b); D'Auria *et al.* (2003a,b, 2002); Ferrara (2002); Ferrara and Porrati (2002); Michelson (1997); Polchinski and Strominger (1996). Generalizations of this formalism to include effects of the warp factor appear in DeWolfe and Giddings (2003); Frey and Maharana (2006); Giddings and Maharana (2006).

We consider a Calabi-Yau threefold M with $h^{2,1}$ complex structure deformations, and choose a symplectic basis $\{A^a, B_b\}$ for the $b_3 = 2h^{2,1} + 2$ three-cycles $a, b = 1, \cdots, h_{2,1}+1$, with dual cohomology elements α_a, β^b such that:

$$\int_{A^a} \alpha_b = \delta^a_b, \quad \int_{B_b} \beta^a = -\delta^a_b, \quad \int_M \alpha_a \wedge \beta^b = \delta^b_a\ . \tag{55}$$

Fixing a normalization for the holomorphic three-form Ω, we then define the periods

$$z^a = \int_A^a \Omega, \quad \mathcal{G}_b = \int_{B_b} \Omega \tag{56}$$

and the period vector $\Pi(z) = (\mathcal{G}_b, z^a)$. The z^a are projective coordinates on the complex structure moduli space of the Calabi-Yau threefold, with $\mathcal{G}_b = \partial_b G(z)$. The Kähler potential $\mathcal{K}$ for the z^a as well as the IIb axio-dilaton $\phi = C_0 + \frac{i}{g_s}$ is given by

$$\mathcal{K} = -\log\left(i\int_M \Omega\wedge\bar{\Omega}\right) - \log(-i(\phi-\bar{\phi})) \tag{57}$$

Note that given the period vector, one can re-write

$$\int_M \Omega\wedge\bar{\Omega} = -\Pi^\dagger \Sigma \Pi \tag{58}$$

where Σ is the symplectic matrix. This structure on the complex structure moduli space follows from so-called special geometry, as derived in Candelas and de la Ossa (1991); Dixon *et al.* (1990); Strominger (1990). The special geometry governs the moduli space of vector multiplets in $\mathcal{N} = 2$ supersymmetric compactifications. However, it also governs the complex structure moduli space of $\mathcal{N} = 1$ orientifolds of these models, which is the application of interest here. (In general, some of the complex structure moduli could be projected out in any given orientifold construction; in this circumstance, one should appropriately restrict the various quantities to the surviving submanifold of the moduli space).

Now, we consider turning on fluxes of the RR and NS-NS 3-form field strengths F_3 and H_3. In a self-explanatory notation, we define these via integer-valued b_3-vectors f, h:

$$F_3 = -(2\pi)^2\alpha'(f_a\alpha^a + f_{a+h_{2,1}+1}\beta_a), \tag{59}$$

$$H_3 = -(2\pi)^2\alpha'(h_a\alpha^a + h_{a+h_{2,1}+1}\beta_a)\ . \tag{60}$$

These fluxes generate a superpotential for the complex structure moduli as well as the axio-dilaton (Gukov *et al.*, 2000)

$$W = \int_M G_3\wedge\Omega(z) = (2\pi)^2\alpha'(f-\phi h)\cdot\Pi(z) \tag{61}$$

where $G_3 = F_3 - \phi H_3$.

To write down a general expression for the potential, we need to introduce one more ingredient. Thus far, we have described only a Kähler potential on the complex structure moduli space. In general models, there are also Kähler moduli (up to $h^{1,1}(M)$ of them, depending on how many survive the orientifold projection). However, they will cancel out of the tree-level effective potential in the IIb supergravity, in the following way. The Kähler potential for these moduli is

$$\mathcal{K}_k = -2\log(V) \tag{62}$$

Given a basis of divisors $\{S_\alpha\}$, $\alpha = 1, \cdots, h_{1,1}$, the volume V is determined in terms of the Kähler form $J = t^\alpha S_\alpha$ by

$$V = \frac{1}{6} S_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma \tag{63}$$

Note however that for this class of vacua, the flux superpotential (61) does *not* depend explicitly on the Kähler moduli.

Now, on general grounds, the expression for the potential in $\mathcal{N} = 1$ supergravity takes the form (Freedman *et al.*, 1976)

$$V = e^{\mathcal{K}+\mathcal{K}_k} \left(\sum_{i,j} g^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right) \tag{64}$$

where i, j runs over a, ϕ, α. $D_i W$ is the Kähler covariantized derivative $D_i W = \partial_i W + K_{,i} W$. At this point, because of the special structure where W is independent of the ts at tree level, in the expression (64) the $-3|W|^2$ term precisely cancels the terms where i, j run over α, β. Therefore, one can express the full tree-level flux potential as (Giddings *et al.*, 2002)

$$V = e^{\mathcal{K}_{tot}} \left(\sum_{a,b} g^{a\bar{b}} D_a W \overline{D_b W} \right) \tag{65}$$

where here the sum over a also includes ϕ.

So surprisingly, despite the fact that we are working in an $\mathcal{N} = 1$ supergravity, the potential is positive semi-definite with vacua precisely when $V = 0$! Furthermore, one sees immediately that generic vacua are not supersymmetric; supersymmetric vacua have $D_a W = D_\phi W = D_\alpha W = 0$, while non-supersymmetric vacua have $D_\alpha W \neq 0$ for some α. This is precisely a realization of the cancellation that occurs in a general class of supergravities known as no-scale supergravities (Cremmer *et al.*, 1983; Ellis *et al.*, 1984). Unfortunately, the miracle of vanishing cosmological constant for the non-supersymmetric vacua depended on the tree-level structure of the Kähler potential (62) which is not radiatively stable. Therefore this miracle, while suggestive, does not lead to any mechanism of attacking the cosmological constant problem. The potential (65) receives important corrections both in perturbation theory and nonperturbatively.

A simple characterization of the points in moduli space which give solutions to $V = 0$ for a given flux arises as follows. We need to solve the equations

$$D_\phi W = D_a W = 0 \tag{66}$$

which, more explicitly, means

$$(f - \bar{\phi} h) \cdot \Pi(z) = (f - \phi h) \cdot (\partial_a \Pi + \Pi \partial_a \mathcal{K}) = 0 \ . \tag{67}$$

In fact, these equations have a simple geometric interpretation: for a given choice of the integral fluxes f, h, they require the metric to adjust itself (by motion in complex structure moduli space) so that the (3,0) and (1,2) parts of G_3 vanish, leaving a solution where G_3 is "imaginary self-dual" (ISD), as in (53).

At this stage, since we are solving $h_{2,1} + 1$ equations in $h_{2,1} + 1$ variables for each choice of integral flux, it seems clear that generic fluxes will fix all of the complex structure moduli as well as the axio-dilaton. Furthermore, one might suspect that the number of vacua will diverge, since we have not yet constrained the fluxes in any way.

However, the fluxes also induce a contribution to the total D3-brane charge, arising from the term in the 10d IIb supergravity Lagrangian

$$\mathcal{L} = \cdots + \frac{1}{8i\kappa_{10}^2} \int \frac{C_{(4)} \wedge G_3 \wedge G_3}{Im\phi} + \cdots \tag{68}$$

where C_4 is the RR four-form potential which couples to D3 branes. This results in a tadpole for D3-brane charge, in the presence of the fluxes:

$$N_{\rm flux} = \frac{1}{(2\pi)^4 (\alpha')^2} \int_M F_3 \wedge H_3 = f \cdot \Sigma \cdot h \tag{69}$$

This is important because: i) one can easily check that for ISD fluxes $N_{\rm flux} \geq 0$, and ii) in a given orientifold of M, there is a tadpole cancellation condition (49), which we can write in the form

$$N_{\rm flux} + N_{D3} \ = \ L \tag{70}$$

where L is some total negative D3 charge which needs to be cancelled, arising by induced charge on D7 and O7 planes (Giddings *et al.*, 2002), and/or explicit O3 planes. In practice, for an orientifold which arises in the Sen limit (Sen, 1997) of an F-theory compactification on elliptic fourfold Y, one finds (Sethi *et al.*, 1996)

$$L = \frac{\chi(Y)}{24} \ . \tag{71}$$

What this means is that the allowed flux choices in an orientifold compactification on M, and hence the numbers of flux vacua, are stringently constrained by the requirement $N_{\rm flux} \leq L$. This will be important later in the review, when we discuss vacuum statistics for this class of models.

We note here that in describing this classical story, we have simplified matters by turning on only the background closed string fluxes. In general orientifold or F-theory models, D7 branes with various gauge groups are also present, and one can turn on background field strengths of the D7 gauge fields, generating additional contributions to the tadpole condition (70) and the spacetime potential energy. Because our story is rich enough without considering these additional ingredients, we proceed with the development without activating them, but discussions which incorporate them in this class of vacua can be found in *e.g.* Burgess *et al.* (2003); Haack *et al.* (2006); Jockers and Louis (2005a,b); Garcia del Moral (2006).

a. Example: The conifold We now exemplify our previous considerations by finding flux vacua in one of the simplest non-compact Calabi-Yau spaces, the deformed conifold. The metric of this space is known explicitly (Candelas and de la Ossa, 1990). The vacua we discuss below have played an important role in gauge/gravity duality (Klebanov and Strassler, 2000), the study of geometric transitions (Vafa, 2001), and warped compactifications of string theory (Giddings *et al.*, 2002), including models of supersymmetry breaking (Kachru *et al.*, 2002). We will encounter some of these applications as we proceed.

The deformed conifold is a noncompact Calabi-Yau space, defined by the equation

$$P(x,y,v,w) = x^2 + y^2 + v^2 + w^2 = \epsilon^2 \tag{72}$$

in $\mathbb{C}^4$. As $\epsilon \to 0$, the geometry becomes singular: the origin is non-transverse, since one can solve $P = dP = 0$ there. It is not difficult to see that an S^3 collapses to zero size at this point in moduli space; e.g. for real ϵ^2, the real slice of (72) defines such an S^3. In this limit, the geometry can be viewed as a cone over $S^3 \times S^2$. There are two topologically nontrivial three-cycles; the A-cycle S^3 we have already discussed, which vanishes when $\epsilon \to 0$, and a dual B-cycle swept out by the S^2 times the radial direction of the cone.

The singularity (72) arises locally in many compact Calabi-Yau spaces (at codimension one in the complex structure moduli space). In such manifolds, the B-cycle is also compact; the behavior of the periods of Ω is partially universal, being given by

$$\int_A \Omega = z, \quad \int_B \Omega = \frac{z}{2\pi i}\log(z) + \text{regular} = \mathcal{G}(z)\ . \tag{73}$$

Here $z \to 0$ is the singular point in moduli space where A collapses, and the regular part of the B-period is non-universal.

We can now study flux vacua using the periods (73) and the explicit formulae (61), (65) for the superpotential and potential energy function. Choosing

$$\int_A F_3 \sim M, \quad \int_B H_3 \sim -K \tag{74}$$

we find that the superpotential takes the form

$$W(z) = -K\phi z + M\mathcal{G}(z) \tag{75}$$

Given the logarithmic singularity in $\mathcal{G}$, this superpotential bears a striking resemblance to the Veneziano-Yankielowicz superpotential of pure $\mathcal{N} = 1$ supersymmetric $SU(M)$ gauge theory, conjectured many years ago (Veneziano and Yankielowicz, 1982). We'll see that this is no accident.

The Kähler potential can be determined using the equation (57). We will be interested in vacua which arise close to the conifold point where z is exponentially small; to obtain such vacua we will consider K/g_s to be large. In this limit, the dominant terms in the equation for classical vacua are

$$D_z W = \frac{M}{2\pi i}\log(z) - i\frac{K}{g_s} + \cdots \tag{76}$$

where $\cdots$ are $\mathcal{O}(1)$ terms that will be negliible in a self-consistent manner. For K/g_s large, one finds that

$$z \sim \exp(-2\pi K/g_s M) \tag{77}$$

So there are flux vacua exponentially close to the conifold point in moduli space. In fact, due to the ambiguity arising from the logarithm when one exponentiates to solve for z, there are M vacua, distributed in phase but with z of the magnitude given above.

For the noncompact Calabi-Yau, these are good flux vacua. In fact, the conifold with fluxes (74) is dual, via gauge/gravity duality, to a certain $\mathcal{N} = 1$ supersymmetric $SU(N+M) \times SU(N)$ gauge theory, with $N = KM$ (Klebanov and Strassler, 2000). While it is beyond the scope of our review to discuss this duality in detail, the IR physics of the gauge theory involves gluino condensation in pure $SU(M)$ $\mathcal{N} = 1$ SYM. This fact, together with the duality, explains the appearance of the Veneziano-Yankielowicz superpotential in (75). The M vacua we found in the z-plane, are the M vacua which saturate the Witten index of pure $SU(M)$ SYM.

In a compact Calabi-Yau, the dilaton ϕ is also dynamical and we would need to solve the equation $D_\phi W = 0$ as well. Naively, one would find an obstruction to doing this in the limit described above (large K/g_s and exponentially small z). In fact, one *can* do this even in compact situations, as described in Giddings *et al.* (2002). The details do not matter for us here, however.

While this example is quite simple, we will use it to illustrate many points in our review. In later subsections, we will re-encounter these solutions in constructing examples of warped compactification, novel models of supersymmetry breaking, and attractors in the space of flux vacua. In the literature, one can find many other examples of explicit vacua, both in toroidal orientifolds (Blumenhagen *et al.*, 2003; Cascales and Uranga, 2003a,b; Dasgupta *et al.*, 1999; Frey and Polchinski, 2002; Greene *et al.*, 2000; Kachru *et al.*, 2003c) and in more nontrivial Calabi-Yau threefolds (Aspinwall and Kallosh, 2005; Conlon and Quevedo, 2004; Curio *et al.*, 2002, 2001; DeWolfe, 2005; DeWolfe *et al.*, 2005a; Giryavets *et al.*, 2004a,b; Tripathy and Trivedi, 2003).

3. Quantum IIb flux vacua

At the classical level, the Kähler moduli of IIb orientifolds with flux remain as exactly flat directions of the no-scale potential. However, quantum corrections will generally generate a potential for these moduli. This potential will have at least two different sources:

1. In every model, there will be corrections to the Kähler potential which depend on Kähler moduli. The leading such corrections have been computed in *e.g.* Becker *et al.* (2002); Berg *et al.* (2005, 2006). As soon as $\mathcal{K}_k$ takes a more general form than (62), the no-scale cancellation disappears and the scalar potential will develop dependence on the Kähler moduli.

2. The superpotential in these models enjoys a nonrenormalization theorem to all orders in perturbation theory (Burgess *et al.*, 2006). Nonperturbatively, it can be violated by Euclidean D3-brane instantons. The conditions for such instantons to contribute in the absence of G_3 flux, and assuming they have smooth worldvolumes, with vanishing intersection with other branes in the background, are described in Witten (1996b). The basic condition is familiar also from supersymmetric gauge theory: there should be precisely two fermion zero modes in the instanton background. Witten argues that these zero modes can be counted as follows. One can lift the Euclidean D3 brane to an M5 brane wrapping a divisor D in the M-theory dual compactification on a Calabi-Yau fourfold. Then, the number of fermion zero modes can be related to the holomorphic Euler character χ of the divisor:

$$\text{number of zero modes} = 2\chi(D) = 2\sum_{p=0}^{3} h^{0,p}(D)\ . \quad (78)$$

   In the simplest case of an isolated divisor with $h^{0,0} = 1$ and other $h^{0,p}$ vanishing, the contribution is definitely nonzero. For more elaborate cases where $\chi = 1$ but the divisor has a moduli space, it is conceivable that the integral over the instanton moduli could vanish.

The conditions under which such instantons contribute in the presence of various fluxes and/or space-filling D-branes (whose worldvolumes they may intersect) remain a subject of active investigation (Blumenhagen *et al.*, 2006; Florea *et al.*, 2006; Gorlich *et al.*, 2004; Haack *et al.*, 2006; Ibanez and Uranga, 2006; Kallosh *et al.*, 2005; Lust *et al.*, 2006c; Saulina, 2005). The condition (78) is certainly modified. More generally, there can be contributions from nonperturbative dynamics in field theories arising on D7-brane worldvolumes, whose gauge coupling is Kähler-modulus dependent (Gorlich *et al.*, 2004; Kachru *et al.*, 2003a).

It was argued in Kachru *et al.* (2003a) (KKLT) that such corrections will allow one to find flux compactifications of the IIb theory that manifest landscapes of vacua with all moduli stabilized. As a simple toy model for how such corrections may be important let us consider a model with a single Kähler modulus ρ, with

$$\mathcal{K}_k = -3\,\log(-i(\rho-\bar\rho)) \quad (79)$$

Here one should think of $\mathrm{Im}(\rho) \sim \frac{R^4}{(\alpha')^2}$ where R is the radius of M, while $Re(\rho)$ is related to the period of an axion arising from C_4 (Giddings *et al.*, 2002). If there is a D7 stack which gives rise to a pure SYM sector, whose gauge coupling depends on ρ, one finds a superpotential of the general form

$$W = W_0 + Ae^{ia\rho}\ . \quad (80)$$

One should view W_0 as being the constant arising from evaluating the flux superpotential (61) at its minimum in complex structure moduli space. A is a determinant which a priori depends on complex structure moduli, and a is a constant depending on the rank of the D7 gauge group. We noted above that A would generally depend on complex structure moduli. However, the scales in the flux superpotential make it clear that complex structure moduli receive a mass at order $\frac{\alpha'}{R^3}$, while any Kähler modulus potential arising from the correction in (80) will be significantly smaller. Therefore, one can view the supergravity functions above as summarizing the effective field theory of the light mode ρ, having integrated out the heavy complex structure modes and dilaton. For a detailed discussion of possible issues with such a procedure, see *e.g.* de Alwis (2005); Choi *et al.* (2004).

It is then straightforward to show that one can solve the equation $D_\rho W = 0$, yielding a vacuum with all moduli stabilized and with unbroken supersymmetry (Kachru *et al.*, 2003a). For small W_0, this vacuum moves into the regime of control (large $\mathrm{Im}(\rho)$) with logarithmic speed. (Small a arising from large rank gauge groups also helps). Given the expectations for which kinds of gauge theories we can realize in string compactifications, this provides a loose proof-of-principle that one can find models with all moduli stabilized. This picture has been substantially fleshed out and extended in further work; the most explicit examples to date appear in Denef *et al.* (2004, 2005); Lust *et al.* (2006b, 2005a).

Before moving on to summarize further detailed considerations, we discuss here two important questions which may concern the reader. Firstly, under the assumptions above, one requires an exponentially small value of W_0 to obtain a vacuum which is in the regime of computational control, where further corrections are expected to be small. Is it reasonable to expect such a small value? We would like to point out that in all string models of SUSY GUTs, such a tune of the "constant" in the superpotential is inevitable, for other reasons. The vacuum energy in supergravity is of the schematic form

$$V \sim \left(|F|^2 - 3\frac{|W|^2}{M_P^2} \right) \tag{81}$$

The largest F term which is allowed in a model where SUSY explains the gauge hierarchy is roughly $(10^{11}\mathrm{GeV})^2$. Therefore, to get moderately small vacuum energy (not the full cancellation required for the absurdly small CC), one clearly requires

$$|W|^2 \leq M_P^2 (10^{11}\mathrm{GeV})^4 \rightarrow (\frac{W}{M_P^3}) \leq 10^{-14} \tag{82}$$

For models of gauge mediation with low-scale breaking, the tune becomes even larger. This tune is absolutely necessary in the standard supergravity picture of unification, and enters directly into cosmology via the gravitino mass. It is therefore an *inevitable* problem in standard SUSY scenarios with high string scale, that one will be required to tune W to be small at any minimum.

This does not answer the question of whether such small values of W_0 are in fact attainable in actual flux vacua. For this, the statistical theory to be developed in the next section is a useful tool; it indicates that given the impressively large number of flux vacua, and the distribution of W_0 within that set, very small values should be attainable. We will be more quantitative about this in later sections, since it is an important point for making contact with the phenomenology of SUSY GUTs regardless of our technique of moduli stabilization.

As an aside, we note here that $\langle W \rangle$ serves as the order parameter for R-symmetry breaking in supersymmetric models. Therefore, it is suggested by 't Hooft naturalness that there will exist classes of models which naturally manifest very small values of $\langle W \rangle$. Such flux vacua have only, thus far, been constructed in non-compact Calabi-Yau models.

A second interesting question is: when is one justified in using the tree-level Kähler potential while including the nonperturbative correction to W? Clearly, at very large volume, corrections to $\mathcal{K}_k$ (which are power-law suppressed) are more important than instanton effects. However, in the spirit of self-consistent perturbation theory, this is not the relevant question. The relevant question is, given the estimates above, if one then includes a first correction to $\mathcal{K}_k$ and then re-expands around the solution one has obtained with the tree level $\mathcal{K}_k$, how much does the solution shift? It is easy to verify that for large $\mathrm{Im}(\rho)$, the perturbative corrections to $\mathcal{K}_k$ (expanded around the minimum to the potential) shift the solution by a small amount, which can be tuned by tuning W_0.

Naturally, however, this suggests that the corrections to $\mathcal{K}$ themselves may cause interesting new features at large volume, giving rise to further critical points in the potential distinct from the KKLT minima. Such critical points have indeed been observed (Balasubramanian and Berglund, 2004; Balasubramanian *et al.*, 2005; Berg *et al.*, 2006; von Gersdorff and Hebecker, 2005), using estimates for the first few quantum corrections to $\mathcal{K}$. These can yield vacua with very large volume, even realizing the large-extra dimensions scenario of Arkani-Hamed *et al.* (1998). The phenomenology of such models has been described in Conlon and Quevedo (2006); Conlon *et al.* (2005).

4. Supersymmetry Breaking

The vacua we have discussed so far are supersymmetric. One would hope to learn also about vacua which have supersymmetry breaking at or above the TeV scale, and have positive cosmological constant. Here we discuss three ideas in this direction: one in some detail (largely because it is novel and uses stringy ingredients), and two more standard ideas quite briefly. We will be parochial in our interests, focusing on theories with low energy breaking (i.e. breaking far below the KK scale, possibly relevant to explaining the electroweak hierarchy). There are also a host of interesting theories manifesting supersymmetry breaking at the KK scale (Saltman and Silverstein, 2006) or even higher scales (Silverstein, 2001). Examples of this type are discussed in a pedagogical way in the excellent review (Silverstein, 2004b).

We will also only discuss the mechanisms of SUSY-breaking that have been explored in the IIb landscape. One of the most important consequences of SUSY-breaking is of course the generation of soft terms. For flux-induced breaking, these terms have been investigated in the important works (Allanach *et al.*, 2005; Camara *et al.*, 2004, 2005b; Font and Ibanez, 2005; Ibanez, 2005; Lawrence and McGreevy, 2004a,b; Lust *et al.*, 2006a, 2005b,c; Marchesano *et al.*, 2005). More generally, constructions of models incorporating a standard-like model together with flux stabilization have appeared in Cvetic *et al.* (2005); Cvetic and Liu (2005); Marchesano and Shiu (2004, 2005).

a. Warped Supersymmetry Breaking The idea originally presented in Kachru *et al.* (2003a), making use of Kachru *et al.* (2002) and Giddings *et al.* (2002), is as follows. Calabi-Yau compactification, at leading order in α', gives rise to a compactification metric of the form

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n \tag{83}$$

with μ, ν running over coordinates in our $\mathbb{R}^4$, and $m, n = 1, \cdots, 6$ parametrizing the coordinates on the "extra" six dimensions.

However, in the presence of fluxes, one finds a more general metric of the form

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n \ . \quad (84)$$

$A(y)$ is a warp factor, which allows the "scale" in the 4d Minkowski space to vary as one moves along the compact dimensions y^m. [14] The equation determining $A(y)$ in terms of the flux compactification data (the CY metric, the choice of fluxes, and the value of the axio-dilaton) can be found in Giddings *et al.* (2002). Compactifications where $A(y)$ varies significantly as one moves over the compact six-manifold M, are often called "warped compactifications."

An important toy model of warped compactification is the Randall-Sundrum model (Randall and Sundrum, 1999a). This is a 5d model where a warp factor which varies by an exponential amount over the 5th dimension (which is compactified on an interval), can be used to explain exponential hierarchies in physics. The basic idea is that scales at the end of the 5th dimension where e^A has a minimum, are exponentially smaller than those at the UV end where e^A is maximized.

The simplest realization of this idea in string theory uses precisely the same kinds of (deformed) conical throats that arise in describing string duals of confining gauge theories (Klebanov and Strassler, 2000). We found for instance that in the conifold geometry, one can stabilize moduli exponentially close to a conifold point in moduli space without tuning

$$\int_A F_3 = M, \quad \int_B H_3 = -K, \quad z = \exp(-2\pi K/g_s M) \ . \quad (85)$$

But the fluxes here are precisely those of the warped deformed conifold solution which appears in gauge/gravity duality; hence the warp factor $e^{2A(y)}$ at the "tip" of the deformed conifold, will take the same value it does there. This gives rise to an exponential warping

$$e^A \sim e^{-2\pi K/3g_s M} \ . \quad (86)$$

As a result, compactifications of the conifold with flux, can give rise to string theory models which accomodate the exponential warping of scales used in Randall-Sundrum scenarios (Giddings *et al.*, 2002). The possibilities for making realistic R-S models in this general context have been investigated in much more detail in the interesting works (Cascales *et al.*, 2004, 2005; Franco *et al.*, 2005; Gherghetta and Giedt, 2006) and references therein.

Instead of using the redshifting of scales to explain the Higgs mass directly, this warping can also be used in another way. Imagine now that instead of engineering the Standard Model in the region of minimal warp factor, one arranges for SUSY breaking to occur there. The Standard Model, or a supersymmetric GUT extension thereof, can be localized in the bulk of the Calabi-Yau space, where $e^A \sim \mathcal{O}(1)$. In this situation, the exponentially small scale of supersymmetry breaking can be explained by warping, instead of by instanton effects. It can be transmitted via gravity mediation or other mechanisms to the observable sector. Precisely this scenario, combined with other assumptions, has been explored in a phenomenological context in *e.g.* Brummer *et al.* (2006); Choi *et al.* (2005); Kitano and Nomura (2005).

To justify and flesh out such scenarios, finding explicit microscopic models of such SUSY breaking is an interesting question. In fact, such a model was proposed already in Kachru *et al.* (2002). The idea is to consider the conifold with flux, in the presence of a small number $p << M$ of anti-D3 branes. While the throat carries $\int H \wedge F = KM$ units of D3 brane charge, this is not obviously available to perturbatively annihilate with the anti-branes. It then becomes interesting to work out the dynamics of this non-supersymmetric but controlled system.

For $p << M$, we can consider the p anti-branes as probes of the exact solution given in Klebanov and Strassler (2000). Then their dynamics will be governed by their worldvolume action in the fixed supergravity background. This action is a function of the six matrix-valued fields Φ^i, which are adjoints of $SU(p)$ and parametrize the brane positions on M. In an appropriate duality frame, it is given by the sum of two terms: a Born-Infeld term

$$S_{BI} = -\frac{\mu_3}{g_s}\int d^4x Tr\sqrt{det(G_{||})det(Q)} \quad (87)$$

and a Chern-Simons term

$$S_{CS} = -\mu_3 \int d^4x Tr(2\pi i_\Phi i_\Phi B_6 + C_4) \ . \quad (88)$$

Here $\frac{\mu_3}{g_s} = T_3$, $G_{||}$ is the pullback of the induced metric along the anti-branes, i_Φ is the interior derivative so

$$i_\Phi i_\Phi B_6 = \Phi^n \Phi^m B_{mnpqrs}\frac{1}{4!}dy^p \wedge \cdots \wedge dy^s, \quad (89)$$

Q is the matrix

$$Q^i_j = \delta^i_j + \frac{2\pi i}{g_s}[\Phi^i, \Phi^k](G_{kj} + g_s C_{kj}) \quad (90)$$

and B_6 is given in an ISD flux background by

$$dB_6 = \frac{1}{g_s^2} *_{10} H_3 = -\frac{1}{g_s}dV_4 \wedge F_3 \quad (91)$$

where dV_4 is the volume form on $\mathbb{R}^4$ at the brane location in the compact dimensions.

It is best to summarize the dynamics in three steps (DeWolfe *et al.*, 2004; Kachru *et al.*, 2002).

[14] The fact that fluxes generate warping was described in Becker and Becker (1996); Strominger (1986), this was discussed in the IIb context in Dasgupta *et al.* (1999); Greene *et al.* (2000), and concrete ideas about Randall-Sundrum scenarios in string theory were first developed in Chan *et al.* (2000); Verlinde (2000).

1. Weight loss The non-commutator terms in the ISD flux background yield the action

$$-\frac{\mu_3}{g_s}\int d^4x\sqrt{g_4}e^{4A}Tr\left(2+\frac{1}{2}e^{-2A}\partial_\mu\Phi^i\partial^\mu\Phi^j g_{ij}\right) \ . \tag{92}$$

Therefore, the leading potential is

$$V(y) = 2e^{4A(y)} \ . \tag{93}$$

It arises by adding the BI and CS terms; for a D3-brane these would instead cancel, as D3-branes in the ISD flux backgrounds feel no force.

It is a feature of the Klebanov-Strassler solution (Klebanov and Strassler, 2000) (and a wide class of other conical and deformed conical geometries) that the warp factor depends only the "radial" direction in the cone, $A(y) = A(r)$ for some radial direction r. Then the potential (93) simply yields a force in the radial direction

$$F_r(r) = -2\frac{\mu_3}{g_s}\partial_r e^{4A}(r) \ . \tag{94}$$

The warp factor monotonically decreases as one goes towards the smooth (deformed) tip of the cone, so in the first step of evolution, the p anti-branes are drawn quickly to the region of minimal warp factor, the tip of the deformed conifold. This result is intuitively clear; the branes wish to minimize their energy, and the minimal energy can be obtained by going to the region where $e^A << 1$, where the branes enjoy maximal weight loss.

2. Embiggening Now, let us analyze the dynamics of the p anti-D3 branes at the tip. The metric at the tip of the warped deformed conifold is given by

$$ds^2 \simeq (e^{-2\pi K/3Mg_s})^2 dx_\mu dx^\mu + R^2 d\Omega_3^2 + (dr^2 + r^2 d\tilde{\Omega}_2^2)\times b_0^2 \ . \tag{95}$$

Here, b_0 is a number of order 1, and $R^2 \sim g_s M$. In particular at the tip $r=0$, the geometry is well approximated by an S^3 of radius $\sqrt{g_s M}$.

The flux is also easy to determine; the H_3 flux is spread over the radial direction, while the F_3 flux threads the S^3 at the tip. In the supergravity regime where $g_s M >> 1$, we can solve $\int_A F_3 = M$ by just setting F proportional to the warped volume form ϵ on the S^3:

$$F_{mnp} = f\epsilon_{mnp}, \quad f \sim \frac{1}{\sqrt{g_s^3 M}} \ . \tag{96}$$

So the system we are studying consists of p anti-D3 branes transverse to a diffuse magnetic 3-form flux, or equivalently, p anti-D3 branes in an electric 7-form flux.

This system is T-dual to D0-branes in an electric 4-form flux. But these D0-branes undergo the famous Myers effect (Myers, 1999); p D0-branes in a background flux expand into a fuzzy D2-brane carrying p units of worldvolume gauge flux (to encode the D0 charge). Similarly here, the anti-D3 branes should be expected to expand into 5-branes, carrying p units of worldvolume flux. Because we are working in a duality frame where S_{CS} contains a coupling to B_6, in fact the anti-D3s will expand into an NS 5 brane.

We can see this in equations as follows. On the large S^3, one can approximate

$$C_{kj} \sim \frac{2\pi}{3}F_{kjl}\Phi^l, \quad G_{kj} \sim \Delta_{kj} \ . \tag{97}$$

Therefore

$$Q^i_j = \delta^i_j + \frac{2\pi i}{g_s}[\Phi^i,\Phi_j] + i\frac{4\pi^2}{3}F_{kjl}[\Phi^i,\Phi^k]\Phi^l \ . \tag{98}$$

Then

$$Tr(\sqrt{detQ}) \simeq p - i\frac{2\pi^2}{3}F_{kjl}Tr\left([\Phi^k,\Phi^j]\Phi^l\right) - \frac{\pi^2}{g_s^2}Tr[\Phi^i,\Phi^j]^2 \ . \tag{99}$$

Now the B_6 term in S_{CS} would *cancel* the cubic term in the potential if we were considering D3 branes; they do not undergo a Myers effect in this background. On the other hand, for anti-D3 branes, the B_6 term *adds* and we find an effective potential

$$\begin{aligned} V_{eff}(\Phi) \ &= \ e^{-8\pi K/3Mg_s}\frac{\mu_3}{g_s}\left(p - i\frac{4\pi^2 f}{3}\epsilon_{kjl}Tr[\Phi^k,\Phi^j]\Phi^l \right. \\ &\left. \qquad - \frac{\pi^2}{g_s^2}Tr[\Phi^i,\Phi^j]^2 + \cdots\right) \ . \end{aligned} \tag{100}$$

It is important to emphasize that this potential is exponentially small, due to the warp factor at the tip of the cone.

Now, demanding $\frac{\partial V_{eff}}{\partial \Phi} = 0$, we find the equation

$$[[\Phi^i,\Phi^j],\Phi^j] - ig_s^2 f\epsilon_{ijk}[\Phi^j,\Phi^k] = 0 \ . \tag{101}$$

We can solve this equation by choosing constant matrices Φ^i that satisfy

$$[\Phi^i,\Phi^j] = -ig_s^2 f\epsilon_{ijk}\Phi^k \ . \tag{102}$$

This is a very familiar equation. Up to a rescaling of fields, (102) is just the commutation relation satisfied by $p\times p$ matrix representations of the $SU(2)$ generators! Therefore, we can find extrema of the anti-brane potential, by simply choosing (generally reducible) $p\times p$ matrix representations of $SU(2)$, i.e. there is an extremum for each partition of p. The full "landscape" of these extrema is somewhat complicated (see *e.g.* DeWolfe *et al.* (2004); Jatkar *et al.* (2002) for some remarks about its structure, and Gomis *et al.* (2005) for a more general discussion of open string landscapes). What is clear is that the energetically preferred solution is the p dimensional irreducible representation, for which

$$V_{eff} \simeq e^{-8\pi K/3Mg_s}\times p\frac{\mu_3}{g_s}\left(1 - \frac{8\pi^2}{3}\frac{(p^2-1)}{M}\frac{1}{b_0^{12}}\right). \tag{103}$$

The *radius* of the fuzzy S^2 the branes unfurl into, is given by

$$\tilde{R}^2 = \frac{4\pi^2}{b_0^8}\frac{(p^2-1)}{M^2} \times R^2 \tag{104}$$

where $R^2 \sim g_s M$ controls the size of the S^3 at the tip of the geometry.

It is clear from (104) that we can only trust this solution for $p << M$; for larger p, the radius $\tilde{R}$ approaches the radius of the S^3, and global features of the geometry may become important.

3. Deflation We now comment on the ultimate fate of these non-supersymmetric anti-D3 states in the Klebanov-Strassler throat. The throat is characterized by $\int_A F_3 = M$, $\int_B H_3 = -K$. At very large values of the radial coordinate r (the UV of the dual quantum field theory), the charge Q_{tot} characterizing the throat with the p probe antibranes is then:

$$\int_A F_3 = M, \ \int_B H_3 = -K, N_{\overline{D3}} = p$$
$$\rightarrow Q_{tot} = KM - p \ . \tag{105}$$

But there are also supersymmetric states carrying this same total charge; for instance, one could consider

$$\int_A F_3 = M, \ \int_B H_3 = -(K-1), \ N_{D3} = M - p$$
$$\rightarrow Q_{tot} = KM - p \ . \tag{106}$$

Since the two charge configurations (105) and (106) have the same behavior at infinity in the radial coordinate, they should be considered as two distinct states in the same theory. In fact, one can explicitly write down a vacuum bubble interpolating between them; it consists of an NS 5-brane wrapping the A-cycle, and was studied in detail in DeWolfe *et al.* (2004); Kachru *et al.* (2002). This bubble can be interpreted as a bubble of false vacuum decay, carrying the metastable non-supersymmetric vacuum (105) to a stable supersymmetric vacuum. Because the scale of supersymmetry breaking in the initial vacuum is exponentially small, one can control these states quite well for $1 << p << M$. A detailed study shows that as p approaches M, the metastable vacuum disappears; the critical value of p/M is $\mathcal{O}(1/10)$.

This situation is reminiscent of some recent examples where direct study of 4d supersymmetric field theories has uncovered metastable non-supersymmetric vacua (Intriligator *et al.*, 2006). There, Seiberg duality plays a crucial role in unveiling the non-supersymmetric vacua, while here it is gauge/gravity duality. Inasmuch as the large gM gravity solution is dual to a field theory, however, this system can probably be thought of as an example of the same phenomenon, now in strongly coupled field theory. Extending our knowledge of such states (using either gauge/gravity duality or 4d field theory techniques), and the interrelations between them, remains a very active area of research.

In addition to their interest as an example of the intricate dynamics that can occur with branes in flux backgrounds, these states have also been used in the KKLT proposal to obtain de Sitter vacua in string theory (Kachru *et al.*, 2003a), and play an important role in some models of string inflation (Kachru *et al.*, 2003). [15] Of course, for the former role, other mechanisms of supersymmetry breaking could serve as well. We now discuss two less stringy, but very well motivated, ideas.

b. Dynamical Supersymmety Breaking An alternative to using warped compactification to obtain an exponentially small scale of supersymmetry breaking, is to use dimensional transmutation and instanton effects (Witten, 1981a). Many examples of field theories which dynamically break supersymmetry have been discovered over the years, starting with the work of Affleck, Dine and Seiberg summarized in Affleck *et al.* (1985). More recent reviews include (Poppitz and Trivedi, 1998; Shadmi and Shirman, 2000).

It is clear that one can incorporate these dynamical breaking sectors as part of the low energy physics of a string compactification. The extra-dimensional picture then does not a priori add much to the 4d discussion, although to some extent it can be useful in "geometrizing" criteria for different mediation mechanisms to dominate (Diaconescu *et al.*, 2006). Discussions of DSB with gauge or gravity mediation of SUSY breaking to the Standard Model, in fairly concrete pseudo-realistic string compactifications, appear in Braun *et al.* (2006); Diaconescu *et al.* (2006); Franco and Uranga (2006); Garcia-Etxebarria *et al.* (2006).

c. Breaking by fluxes Perhaps the most direct analog of the original Bousso-Polchinski proposal, in the IIb flux landscape, is the following. We saw in the previous subsection that one can supersymmetrically stabilize all moduli after including nonperturbative corrections to the superpotential which depend on Kähler moduli. Previous to stabilizing Kähler moduli, it would have seemed that one must solve the no-scale equation $V = 0$ to find a IIb flux vacuum. However, given that one will stabilize Kähler moduli anyway, it is no longer necessary to do this. Instead, consider the potential $V_{\rm flux}(z_a, \phi)$ arising from the three-form fluxes. If one finds a critical point of this potential in the a, ϕ directions, with

$$\partial_a V = \partial_\phi V = 0, \ \partial^2 V \geq 0 \tag{107}$$

[15] The argument of section IV.A.1 that one cannot solve the IIb equations of motion in warped Calabi-Yau flux compactifications if one includes anti-D3 sources, is true only at tree level. The same effects which allow one to stabilize the Kähler moduli, also allow the incorporation of anti-branes with sufficiently small (warped) tension, as shown in Kachru *et al.* (2003a).

then the vacuum would be stable in the a, ϕ directions (despite the tree-level instability in the Kähler modulus directions). Now, including the instanton contributions to V, it becomes clear that one may stabilize the Kähler moduli and complex/dilaton moduli while using a "flux vacuum" for the complex/dilaton moduli which is *not* of the ISD type, as long as the departure from the ISD condition is not too severe. However, any violation of the ISD equations yields, via (61), a nonzero F-term for some complex/dilaton modulus. This means that the resulting vacuum will yield spontaneous supersymmetry breaking. A toy model vacuum of this type has been exhibited in Saltman and Silverstein (2004).

Because the effects being used to stabilize Kähler moduli are exponentially small, this mechanism is only viable if one "tunes" in flux space to find proto-vacua with a very small violation of the ISD condition. This was shown to be generic in IIb vacua in Denef and Douglas (2005), as we will discuss in Sec. V.C. It is also possible that some classes of fluxes would naturally yield exponentially small F-terms for complex/dilaton moduli; while such flux vacua have yet to be exhibited, they would be expected to arise via geometric transitions from simple wrapped D-brane gauge theories with dynamical supersymmetry breaking. It would be very interesting to exhibit such flux vacua.

B. Type IIa flux vacua

In this section, we briefly discuss the construction of Calabi-Yau flux vacua in type IIa string theory. Our exposition follows the notation and strategy of DeWolfe *et al.* (2005b), using the $\mathcal{N} = 1$ supersymmetric formalism developed in Grimm and Louis (2005). Closely related work developing the basic formalism for IIa flux compactification and presenting explicit examples also appears in Acharya *et al.* (2006b); Aldazabal *et al.* (2006); Benmachiche and Grimm (2006); Bovy *et al.* (2005); Camara *et al.* (2005a); Derendinger *et al.* (2005a,b); House and Palti (2005); Ihl and Wrase (2006); Kachru and Kashani-Poor (2005); Saueressig *et al.* (2006); Villadoro and Zwirner (2005). Candidate M-theory vacua which are in many ways similar to these IIa models were first described in Acharya (2002).

1. Qualitative considerations

Before we launch into a detailed study, it is worth contrasting the present case with the class of IIb vacua we just described. In the IIa string compactified on a Calabi-Yau space M, one can imagine turning on background fluxes of both the NS three-form field H_3 and the RR $2p$ form fields $F_{0,2,4,6}$. The basic intuition that 3-form fluxes should yield complex structure dependent potentials, while even-form fluxes should yield Kähler structure dependent potentials, then suggests that the IIa flux superpotential will depend on all geometric moduli already at tree level. This is correct, and is in stark contrast to the no-scale structure which governs IIb flux vacua.

This suggests that it may be possible to stabilize all moduli in IIa compactification just by turning on fluxes. If we focus for a moment on just the dilaton and the volume modulus, which are normally two of the more vexing moduli in string constructions, we can see by a simple scaling argument that the flux potential will suffice to stabilize them in a regime of control.

To find the potentials due to fluxes, one should reduce the flux kinetic and potential terms from 10d to 4d, remembering to perform the necessary Weyl rescalings to move to 4d Einstein frame. These are discussed in a pedagogical way in Silverstein (2004a). The results are as follows. If the compactification manifold has radius R and string coupling $g_s = e^{\phi}$, then:

• N units of RR p-form flux contributes to the scalar potential with the scaling

$$V_{RR} = N^2 \frac{e^{4\phi}}{R^{6+2p}} \tag{108}$$

• N units of NS 3-form flux contribute

$$V_{NS} = N^2 \frac{e^{2\phi}}{R^{12}} \tag{109}$$

• N orientifold p+3 planes wrapping a p-cycle in the compact manifold and filling spacetime, contribute

$$V_{O(p+3)} = -N \frac{e^{3\phi}}{R^{12-p}} \tag{110}$$

(while of course N D-branes would, up to an overall coefficient, make the same contribution with a positive sign).

The simplest class of $\mathcal{N} = 1$ supersymmetric IIa orientifolds arise by acting with an anti-holomorphic involution $\mathcal{I}$ on a Calabi-Yau space M. The fixed locus of $\mathcal{I}$ is some collection of special Lagrangian cycles, which are wrapped by O6 planes. Let us assume for a moment that there are $\mathcal{O}(1)$ O6 planes in our construction. The tadpole condition for D6-brane charge takes the schematic form

$$N_{D6} + \int_{\Sigma} F_0 \wedge H_3 = 2N_{06} \ . \tag{111}$$

where Σ is the three-cycle pierced by H_3 flux. We can therefore cancel the tadpole by introducing $\mathcal{O}(1)$ units of F_0 and H_3 flux, without adding D6 branes. The other fluxes are unconstrained by tadpole conditions; so we can, for instance, also turn on N units of F_4 flux. The overall result is a potential that takes the schematic form

$$V = \frac{e^{4\phi}}{R^6} - \frac{e^{3\phi}}{R^9} + \frac{e^{2\phi}}{R^{12}} + N^2 \frac{e^{4\phi}}{R^{14}} \ . \tag{112}$$

This potential has minima with $R \sim N^{1/4}$ and $g_s \sim N^{-3/4}$. Hence, as emphasized in DeWolfe *et al.* (2005b), the IIa theory can be expected to admit flux vacua with parametrically large values of the compactification volume and parametrically weak string coupling, in a $1/N$ expansion. Unlike standard Freund-Rubin vacua (Freund and Rubin, 1980), these theories are effectively four-dimensional; the 4d curvature scale is parametrically less than the compactification radius.

It is still important to verify that the qualitative considerations here are born out in detail in real Calabi-Yau models. We now describe the relevant formalism.

2. 4d multiplets and Kähler potential

To find the chiral multiplets in a 4d $\mathcal{N} = 1$ supersymmetric orientifold of M, we proceed as follows. The $\mathcal{N} = 2$ compactification on M gives rise to $h^{1,1}$ $\mathcal{N} = 2$ vector multiplets and $h^{2,1} + 1$ hypermultiplets (including the universal hyper). The projection will choose an $\mathcal{N} = 1$ vector or chiral multiplet from each $\mathcal{N} = 2$ vector, and an $\mathcal{N} = 1$ chiral multiplet from each hyper.

Let us first analyze the projected vector multiplet moduli space. If in a basis of (1,1) forms on M there are $h^{1,1}_-$ that are odd under the involution, then the surviving moduli space of Kähler forms is $h^{1,1}_-$ dimensional. (The even basis elements give rise to $\mathcal{N} = 1$ vector multiplets, which contain no moduli and will not enter in our discussion). We can write the complexified Kähler form on the quotient as

$$J_c = B_2 + iJ = \sum_{a=1}^{h^{1,1}_-} t_a \omega_a \tag{113}$$

where $t_a = b_a + iv_a$ are complex numbers, and ω_a form a basis for $H^{1,1}_-$.

The Kähler potential for the reduced moduli space is inherited from the $\mathcal{N} = 2$ parent Calabi-Yau theory, and is given by

$$K^K(t_a) = -\log\left(\frac{4}{3}\int_M J \wedge J \wedge J\right) = -\log\left(\frac{4}{3}\kappa_{abc} v^a v^b v^c\right) \tag{114}$$

where κ_{abc} is the triple intersection form

$$\kappa_{abc} = \int_M \omega_a \wedge \omega_b \wedge \omega_c \ . \tag{115}$$

Now, we turn to the projected hypermultiplet moduli space. Here, the formalism is more intricate (Grimm and Louis, 2005). Choose a basis for the harmonic three-forms $\{\alpha_A, \beta_B\}$ where $A, B = 0, \cdots, h^{2,1}$ and

$$\int_M \alpha_A \wedge \beta_B = \delta_{AB} \ . \tag{116}$$

Without loss of generality, one can expand Ω as

$$\Omega = \sum_A Z_A \alpha_A - G_B \beta_B \ . \tag{117}$$

The Z_A are homogeneous coordinates on complex structure moduli space; we will denote by z_C $(C = 1, \cdots, h^{2,1})$ the *inhomogeneous* coordinates on this same space.

The complex structure moduli are promoted to quaternionic multiplets in the $\mathcal{N} = 2$ parent theory by adjoining RR axions. If we expand the C_3 gauge potential whose field strength is F_4

$$C_3 = \xi_A \alpha_A - \tilde{\xi}_B \beta_B \tag{118}$$

then we get $h^{2,1}+1$ axions. The axions from $\xi_0, \tilde{\xi}_0$ join the axio-dilaton to yield the universal hypermultiplet, while the other $h^{2,1}$ axions quaternionize the z_C.

The orientifold involution splits $H^3 = H^3_+ \oplus H^3_-$. Each of these eigenspaces is of (real) dimension $h^{2,1}+1$. Let us split the basis for H^3 so $\{\alpha_k, \beta_\lambda\}$ span the even subspace, while $\{\alpha_\lambda, \beta_k\}$ span the odd subspace. Here $k = 0, \cdots, \tilde{h}$ while $\lambda = \tilde{h} + 1, \cdots, h^{2,1}$. Then the orientifold restricts one to the subspace of moduli space (Grimm and Louis, 2005)

$$ImZ_k = Reg_k = ReZ_\lambda = Img_\lambda = 0 \ . \tag{119}$$

C_3 is also even under the orientifold action; this projects in the axions ξ_k and $\tilde{\xi}_\lambda$ while projecting out the others. In addition, the orientifold projects in the dilaton ϕ and one of $\xi_0, \tilde{\xi}_0$. So as expected, from each hypermultiplet, we get a single chiral multiplet, whose scalar components are the real or imaginary part of the complex structure modulus, and an RR axion.

We can summarize the surviving hypermultiplet moduli in terms of the object

$$\Omega_c = C_3 + 2i\mathrm{Re}\left(C\Omega\right) \ . \tag{120}$$

Here, C is a "compensator" which incorporates the dilaton dependence via

$$C = e^{-D + K^{cs}/2}, \quad e^D = \sqrt{8}e^{\phi + K^K/2} \ . \tag{121}$$

One should think of e^D as the four-dimensional dilaton; K^{cs} is the Kähler potential for complex structure moduli

$$\begin{aligned} K^{cs} &= -\log\left(i\int_M \Omega \wedge \bar{\Omega}\right) \\ &= -\log 2(ImZ_\lambda Reg_\lambda - ReZ_k Img_k) \ . \end{aligned} \tag{122}$$

The surviving chiral multiplet moduli are then the expansion of Ω_c in a basis for H^3_+:

$$N_k = \frac{1}{2}\int_M \Omega_c \wedge \beta_k = \frac{1}{2}\xi_k + i\mathrm{Re}(CZ_k) \tag{123}$$

and

$$T_\lambda = i\int_M \Omega_c \wedge \alpha_\lambda = i\tilde{\xi}_\lambda - 2\mathrm{Re}(Cg_\lambda) \ . \tag{124}$$

The Kähler potential which governs the metric on this moduli space is

$$K^Q = -2\log\left(2\int_M \mathrm{Re}(C\Omega) \wedge *\mathrm{Re}(C\Omega)\right) \ . \tag{125}$$

3. Fluxes and superpotential

Now, we can contemplate turning on the fluxes which are projected in by the anti-holomorphic involution. It turns out that H_3 and F_2 must be odd, while F_4 should be even. So we can write

$$H_3 = q_\lambda \alpha_\lambda - p_k \beta_k, \quad F_2 = -m_a \omega_a, \quad F_4 = e_a \tilde{\omega}^a \tag{126}$$

where $\tilde{\omega}^a$ are the 4-form duals of the $H^{1,1}_-$ basis ω_a. There are in addition two parameters m_0 and e_0, parametrizing the F_0 and F_6 flux on M.

In the presence of these fluxes, one can write the 4d potential after dimensional reduction as (DeWolfe *et al.*, 2005b; Grimm and Louis, 2005)

$$V = e^K \left(\sum_{t_a, N_k, T_\lambda} g^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right) \tag{127}$$

Here the total Kähler potential is

$$K = K^K + K^Q. \tag{128}$$

and $D_i W = \partial_i W + W \partial_i K$ is the Kähler covariantized derivative.

The superpotential W is defined as follows. Let

$$W^Q(N_k, T_\lambda) = \int_M \Omega_c \wedge H_3 = -2p_k N_k - i q_\lambda T_\lambda \tag{129}$$

and

$$W^K(t_a) = e_0 + \int_M J_c \wedge F_4 - \frac{1}{2} \int_M J_c \wedge J_c \wedge F_4 - \frac{m_0}{6} \int_M J_c \wedge J_c \wedge J_c \tag{130}$$

The full superpotential is then

$$W(t_a, N_k, T_\lambda) = W^Q(N_k, T_\lambda) + W^K(t_a)\ . \tag{131}$$

Our first qualitative point is now clear: the potential depends, in general, on all geometric moduli at tree level. Detailed examination of the system of equations governing supersymmetric vacua

$$D_{t_a} W = D_{N_k} W = D_{T_\lambda} W = 0 \tag{132}$$

shows that under reasonable assumptions of genericity, one can stabilize all geometric moduli in these constructions (DeWolfe *et al.*, 2005b). These same considerations show that in the leading approximation, $h^{2,1}_+$ *axions* will remain unfixed. An orientifold of a rigid Calabi-Yau model (i.e., one with $h^{2,1} = 0$) was studied in detail in DeWolfe *et al.* (2005b), where it was shown that this flux potential gives rise to an infinite number of 4d vacua with all moduli (including all axions) stabilized. Furthermore, as suggested by the scaling argument in (IV.B.1), these solutions can be brought into a regime where g_s is arbitrarily weak and the volume is arbitrarily large.

4. Comments on 10d description

The 10d description of the IIa solutions is less well understood than the description of their IIb counterparts. That is because in the IIb case, one special class of solutions is conformally Calabi-Yau, at leading order (Giddings *et al.*, 2002). In the IIa case, on the other hand, the metrics of the supersymmetric compactifications are those of $half-flat$ manifolds with $SU(3)$ structure. The definition of such spaces can be found in Chiossi and Salamon (2002), and their relation to supersymmetric IIa compactification is described in Behrndt and Cvetic (2005a,b); House and Palti (2005); Lust and Tsimpis (2005).

It is natural to wonder what relation these half-flat solutions bear to the Calabi-Yau flux vacua we have been discussing, where the fluxes are viewed as a perturbation of a IIa Calabi-Yau compactification. This issue has been considerably clarified in the beautiful work (Acharya *et al.*, 2006b). The description in terms of a Calabi-Yau metric perturbed by backreaction from the flux (and inclusion of thin-wall brane sources) is valid at asymptotically large volume. Finite (but large) volume analysis of the supergravity solution with localized O6-planes, indicates that the backreaction deforms the metric to a half-flat, non Calabi-Yau metric with $SU(3)$ structure, outside a small neighborhood of the O-planes. The detailed formulae for the stabilization of moduli derived from the considerations of the previous subsection, can be recovered precisely from the supergravity solution in the approximation that the O6 charge is smeared.

C. Mirror symmetry and new classes of vacua

The constructions we have reviewed in detail here, are based at the start on type II Calabi-Yau models. Such models famously enjoy mirror symmetry, a duality exchanging the IIb string on M with a IIa string on a "mirror manifold" W. Dual theories must give rise to the same 4d physics (though in different regimes of parameter space one or the other may be a better description). Therefore, we see immediately that simply to match dimensions of hyper and vector multiplet moduli spaces, one must have $h^{1,1}(M) = h^{2,1}(W)$ and $h^{2,1}(M) = h^{1,1}(W)$. This can be viewed as a mirror reflection on the hodge diamond of a Calabi-Yau space, which explains the name of the duality.

It is natural to wonder whether, since the parent Calabi-Yau theories enjoy mirror symmetry, the classes of flux vacua we have constructed above also come in mirror pairs. Are they dual to one another in some way? This seems unlikely, given the qualitative differences between the two classes of 4d effective field theories. We will see that it isn't the case, but that nevertheless exploring analogues of mirror symmetry for these vacua may lead us to interesting conclusions. In fact, it provides a strong indication that the known portion of the landscape (even when restricting consideration to compactifications with a useful description in the language of traditional differential geometry) is a small piece of a much larger structure.

While mirror symmetry was used to great effect already in the early 1990s (Candelas *et al.*, 1991), methods of constructing W given M were known only for very special classes of models (Greene and Plesser, 1990). An important advance came during the duality revolution, when it was realized that in fact, mirror symmetry is a simple generalization of T-duality (Strominger *et al.*, 1996). The known examples of Calabi-Yau spaces admit, in some limit, a fibration structure where T^3 fibers vary over an S^3 base. By matching of BPS states (in particular, the $D3$ wrapping the T^3 fiber of M with the $D0$ brane on W), it was shown that mirror symmetry can be considered as a generalization of T-duality, where one T-dualizes the T^3 fibers of M to obtain W and vice-versa.

Since the study of these T^3 fibrations is in its infancy, it may seem surprising that this construction will be of use to us. However, it is a conceptually simple relation of a space and its mirror, and it will allow us to check whether the mirrors of IIb Calabi-Yau flux vacua are IIa Calabi-Yau flux vacua. This general subject has been explored in Bouwknegt *et al.* (2004); Chiantese *et al.* (2006); Fidanza *et al.* (2004, 2005); Grana *et al.* (2006a); Gurrieri *et al.* (2003); Kachru *et al.* (2003b); Tomasiello (2005).

1. A warm-up: The twisted torus

Before moving on to real vacua and their duals, we provide an illustrative example that should make our conclusions intuitively clear. We follow the discussion in Kachru *et al.* (2003b). Imagine string compactification on a square T^3, M, with metric

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{133}$$

and a nonzero NS three-form flux

$$\int_M H_3 = N \ . \tag{134}$$

Since $H = dB$, we are free to choose a gauge in which

$$B_{yz} = Nx \tag{135}$$

with other components vanishing. It will not escape the reader's attention that this configuration is not a static solution of the equations of motion; the T^3 is flat so there is no curvature contribution to the lower-dimensional effective potential, while the H_3 flux energy can be diluted by expanding the volume of the T^3. We ignore this for now; we will use this setup as a module in a more complicated configuration that provides a static solution of the full equations of motion momentarily.

With the data at hand, we can proceed to T-dualize in the z direction. Applying Buscher's T-duality rules (Buscher, 1987, 1988) (their generalizations to include RR fields (Bergshoeff *et al.*, 1995; Hassan, 2000) will also play a role momentarily), we find a new background with:

$$B = 0, \quad ds^2 = dx^2 + dy^2 + (dz + Nxdy)^2 \tag{136}$$

The coordinate identifications to be made in interpreting the metric are

$$(x, y, z) \simeq (x, y+1, z) \simeq (x, y, z+1) \simeq (x+1, y, z-Ny) \tag{137}$$

This space is an example of a Nilmanifold – it has $h^1 = 2$, and in particular is topologically distinct from T^3, which would have been the expected T-dual target space in the absence of H_3 flux. So, we see that T-dualizing along a leg of an H_3 flux, one can exchange the NS flux for other NS data – namely, topology, as encoded by the metric. Here, loosely speaking, the nontrivial topology arises because as one winds around the x circle, one performs an $SL(2, \mathbb{Z})$ transformation mixing the y, z directions.

If we are foolish enough to T-dualize again, now along the y direction, straightforward application of the rules leads us to the metric

$$ds^2 = \frac{1}{1 + N^2x^2}(dz^2 + dy^2) + dx^2 \tag{138}$$

and the B-field

$$B_{yz} = \frac{Nx}{1 + N^2x^2} \ . \tag{139}$$

Making sense of this data is not as simple as interpreting the Nilmanifold metric above. In particular, as you wind around the circle coordinatized by x, the metric g and B are not periodic in any obvious sense. There is a "stringy" sense in which they *are* periodic; there is an $O(2, 2; \mathbb{Z})$ transformation that relates the values at $x = 1$ to the values at $x = 0$. However, this $O(2, 2; \mathbb{Z})$ transformation is not an element of $SL(2, \mathbb{Z})$, and so this data can at best make sense as the target space of a "stringy" sigma model. Discussions of such non-geometric backgrounds (including and generalizing asymmetric orbifolds) are a subject of current interest; see for instance Dabholkar and Hull (2003, 2006); Flournoy *et al.* (2005); Flournoy and Williams (2006); Hellerman *et al.* (2004); Hull (2005, 2006a,b); Hull and Reid-Edwards (2005); Lawrence *et al.* (2006); Shelton *et al.* (2005); Silverstein (2001).

In the following, we will focus our discussion on open questions about the geometric vacua. Clearly, however, these considerations suggest that once one considers general vacua, novel "stringy" geometric structures will play an important role in obtaining a thorough understanding. This was also clear already in the discussion of IIb flux vacua, where presumably generic vacua arise at radii of $\mathcal{O}(1)$ where stringy geometry will be important, and only tuning in flux space (e.g. to obtain small W_0) allows one to obtain vacua in the geometric regime.

2. A full example

We will now provide full string solutions which incorporate the previous phenomena. We follow Kachru *et al.* (2003b); see also (Grana *et al.*, 2006b; Schulz, 2004, 2006) for further discussion of these models.

Consider IIb string theory on the $T^6/\mathbb{Z}_2$ orientifold, where the $\mathbb{Z}_2$ inverts all six circles (and is composed with the operation of worldsheet parity reversal). For simplicity, focus attention on a $(T^2)^3$, with complex moduli $\tau_{1,2,3}$:

$$dz^i = dx^i + \tau_i dy^i, \quad \Omega = \Pi_i dz^i \tag{140}$$

Flux vacua in this model were studied in *e.g.* Dasgupta *et al.* (1999); Frey and Polchinski (2002); Kachru *et al.* (2003c). One example from Kachru *et al.* (2003b) will suffice for us. Let

$$F_3 = 2\left(dx^1 \wedge dx^2 \wedge dy^3 + dy^1 \wedge dy^2 \wedge dy^3\right) \tag{141}$$

and

$$H_3 = 2\left(dx^1 \wedge dx^2 \wedge dx^3 + dy^1 \wedge dy^2 \wedge dx^3\right). \tag{142}$$

The factor of 2 is inserted in order to avoid subtleties with flux quantization of the sort described in Frey and Polchinski (2002). We can easily read off the flux superpotential

$$W = 2(\tau_1\tau_2\tau_3 + 1) + 2\phi(\tau_1\tau_2\tau_3 + \tau_3)\ . \tag{143}$$

It is easy to see that along the locus

$$\phi\tau_3 = -1, \quad \tau_1\tau_2 = -1 \tag{144}$$

the equations $\frac{\partial W}{\partial \tau_i} = \frac{\partial W}{\partial \phi} = 0$ are satisfied, as is $W = 0$. Therefore, there is a moduli space $\mathcal{M}$ of supersymmetric vacua. In fact, these vacua preserved $\mathcal{N} = 2$ supersymmetry – this is a special feature which arises because the torus is a non-generic Calabi-Yau space. The non-genericity of the torus also implies that one should impose primitivity conditions $J \wedge G_3 = 0$ on the Kähler form (these are trivially satisfied in generic Calabi-Yau orientifolds). This leaves a moduli space of Kähler structures parametrizing $\mathcal{M}$ as well.

We have chosen our fluxes so that in appropriate regions of $\mathcal{M}$, the best description (which involves the weakest couplings and largest KK masses) is either the model above, or its T-dual on one, two or three circles. In the gauge

$$B_{x^1x^3} = 2x^2, \quad B_{y^1x^3} = 2y^2 \tag{145}$$

the relevant T-dual descriptions are the following.

One T-duality along x^1:

This gives rise to a IIa model with metric

$$ds^2 = \frac{1}{R^2_{x^1}}(dx^1 + 2x^2dx^3)^2 + R^2_{x^2}(dx^2)^2 + R^2_{x^3}(dx^3)^2 + \cdots \tag{146}$$

Here, $x^{1,2,3}$ sweep out a Nilmanifold over the T^3 spanned by the y^i. There are also nonzero fluxes remaining:

$$B_{y^1x^3} = 2y^2 \tag{147}$$

in the NS sector and

$$F_2 = 2dx^2 \wedge dy^3, \quad F_4 = 2(dx^1 + 2x^2dx^3) \wedge dy^1 \wedge dy^2 \wedge dy^3 \tag{148}$$

in the RR sector.

This manifold has $h^{1,1} = 5$ and is Non-Kähler. In particular, it isn't just that one does not use a Calabi-Yau metric in describing the physical theory (that is true even for Calabi-Yau compactification, where α' corrections deform the metric even in the absence of flux). There is a topological obstruction to putting such a metric on this space.

Second T-duality along y^1:

Now we find the IIb theory with metric

$$\begin{aligned} ds^2 \;=\; & \tilde{R}^2_{x^1}(dx^1 + 2x^2dx^3)^2 + R^2_{x^2}(dx^2)^2 \\ & + R^2_{x^3}(dx^3)^2 + \frac{1}{R^2_{y^1}}(dy^1 + 2y^2dx^3)^2 \\ & + R^2_{y^2}(dy^2)^2 + R^2_{y^3}(dy^3)^2 \end{aligned} \tag{149}$$

and with fluxes

$$\begin{aligned} B = 0, \quad F_3 \;=\; & 2(dx^1 + 2x^2dx^3) \wedge dy^2 \wedge dy^3 \\ & + 2(dy^1 + 2y^2dx^3) \wedge dx^2 \wedge dy^3\,. \end{aligned} \tag{150}$$

This space is also Non-Kähler.

Third T-duality along y^3:

This just flips the radius of the y^3 circle in (149) and changes the flux to

$$F_2 = 2(dx^1 + 2x^2dx^3) \wedge dy^2 + 2(dy^1 + 2y^2dx^3) \wedge dx^2\ . \tag{151}$$

At this point, we have T-dualized on some T^3 in the original starting model, and so we can consider this a (somewhat trivial) analog of mirror symmetry in the spirit of Strominger *et al.* (1996). We see that this example suggests that the IIb Calabi-Yau flux vacua of the general class studied in (IV.A) are *not* mirror to the IIa Calabi-Yau flux vacua described in (IV.B). In less simple examples, we expect that dualizing flux vacua with one leg of the H_3 flux along the T^3 fiber could lead to a geometric but non Calabi-Yau dual, while duals of theories with more legs of H_3 on the T^3 fiber will in general be "non-geometric" vacua. It is important to keep in mind that such duals should only be considered (and will only in general be meaningful) when they are, in some regime of moduli, the best description of the low energy physics.

One might wonder whether every geometric flux vacuum admits *some* dual description that brings it into one of the two large classes we've explored in the IIb and IIa theories in the previous subsections. A study of examples strongly suggests that this is false; we have seen only the tip of the iceberg so far. For instance, the class of vacua described in Chuang *et al.* (2005) does not admit a dual description involving IIa, IIb or heterotic Calabi-Yau compactification, to the best of our knowledge. Furthermore, Grana *et al.* (2006b) exhibit an explicit vacuum based on Nilmanifold compactification that is not dual (via any known duality) to a Calabi-Yau with flux.

V. STATISTICS OF VACUA

Given a systematic construction of a set of string vacua, besides working out individual examples, one can try to get some understanding of the possibilities from statistical studies. As we mentioned earlier, such studies date back to the late 1980's, and while at that time moduli stabilization was not understood, still interesting results were obtained. Perhaps the most influential of these came out of the related study of the set of Calabi-Yau threefolds, which provided the first evidence that mirror symmetry was a general phenomenon (Candelas *et al.*, 1995; Kreuzer *et al.*, 1992). We will briefly review some of these results in V.D.3.

The systematic constructions we have discussed of flux superpotentials and other effective potentials enable us for the first time to find statistics of large, natural classes of stabilized vacua. In this section, we describe a general framework for doing this (Douglas, 2003), and some of these results. See Douglas (2004a); Kumar (2006) for other recent reviews.

The large number of flux vacua suggests looking for commonalities with other areas of physics involving large numbers, such as statistical mechanics. As it turns out, there are very close analogies with the theory of disordered systems, in which one constructs idealized models of crystals with impurities, spin glasses and other disordered systems, by taking a "random potential." In other words, one chooses the potential randomly from an ensemble of potentials chosen to reflect general features of the microscopic physics, and does statistical studies. Given a simple choice of ensemble, one can even get analytic results, which besides adding understanding, are particularly important in studying rare phenomena. As we will explain, by treating the ensemble of flux superpotentials as a random potential, one can get good analytical results for the distribution of flux vacua, which bear on questions of phenomenological interest.

We begin our treatment with a careful explanation of the definitions, as while they are simple, they are different from those commonly used in statistical mechanics and quantum cosmology. This is so that we can avoid ever having to postulate that a given vacuum "exists" or "is created" with a definite probability, an aspect of the theory which, as we discussed in Sec. III.E, is not well understood at present. Rather than a probability distribution, we will discuss vacuum counting distributions, which can be unambiguously defined.

One reason to be careful about these definitions, is that the need for making theoretical approximations will lead us to introduce approximate vacuum counting distributions, which are also interpreted in probabilistic terms. However, the underlying definition of probability in this case is clear; it expresses our confidence in the particular theoretical arguments being used, and in this sense is subjective. The payoff for this methodological interlude will be a clear understanding of how statistics of string theory vacua can lead to a precise definition of *stringy naturalness*, as introduced in Sec. II.F.3.

We proceed to describe counting of flux vacua, and some of the exact results. This will enable us to continue the discussion of Sec. II.F.3 on the scale of supersymmetry breaking. We continue with a brief survey of what is known about other distributions, such as of Calabi-Yau manifolds, and distributions governing the matter content. We then survey various simpler distributions which have been suggested as models for the actual distributions coming from string theory. Finally, we discuss the general interpretation of statistical results, and the prospects for making arguments such as those in Sec. II.F.3 precise.

A. Methodology and basic definitions

Suppose we have a large class of vacua, constructed along the lines of Sec. IV or otherwise. As we discussed, we have no *a priori* reason to prefer one over the other. While we have many *a posteriori* ways to rule out vacua, by fitting data, computing measure factors, or otherwise, this requires detailed analysis to do. In this situation, we may need to know the distributions of vacua, or of their observable properties, to make theoretical progress.

As in Sec. II.F.3, it is useful to motivate the subject as an idealization of the problem of testing string theory: if we had a list of the observables for every single vacuum, call these V_i, then all we would need to do this, is to check whether the actual observables appear on this list.

To be a bit more concrete, let us grant that each of our vacua can be described in terms of some four-dimensional effective field theory (EFT) T defined at an energy scale E. This is presently true, and until fairly recently we would have said T is the Standard Model; the more recent observations of neutrino mixings, etc. need not concern us at the moment.[16] In this situation, our list of predictions could be replaced by a list of the EFT's T_i which are low energy limits of the string vacua V_i, and we want to know whether T appears on this list.

To be even more concrete, let us consider the data which goes into explicitly specifying a particle physics EFT. This will include both discrete and continuous

[16] Of course, a string vacuum might predict phenomena which are not best described using four-dimensional EFT, such as extra dimensions. We leave the necessary generalizations of our discussion as an exercise for the interested reader.

choices. Discrete choices include the gauge group G and matter representation R of fermions and bosons. Choices involving parameters include the effective potential, Yukawa couplings, kinetic terms and so on; let us denote the vector containing these parameters as $\vec{g}$. While we will not do it here, in a complete discussion, we would need to specify the cutoff prescription used in defining the EFT as well. In any case, we can regard the sum total of these choices as defining a point $T_i = (G, R, \vec{g})$ in a "theory space" $\mathcal{T}$.

Now, given a set of vacua $\{T_i\}$, the corresponding vacuum counting distribution is a density on $\mathcal{T}$,

$$dN_{vac}(G, R, \vec{g}) = \sum_i \delta(G, G_i)\delta(R, R_i)\delta^{(n)}(\vec{g}-\vec{g}_i), \quad (152)$$

or, for conciseness,

$$dN_{vac}(T) = \sum_i \delta(T - T_i). \quad (153)$$

Its integral over a subset of theory space $\mathcal{R} \subset \mathcal{T}$ is the number of vacua contained in this subset,

$$N(R) = \int_R dN_{vac} \quad (154)$$

It should be clear that Eq. (152) contains the same information as the set of vacua $\{T_i\}$. What may be less obvious, but will emerge from the discussion below, is that one can find useful approximations to such distributions, which are far easier to compute than the actual vacuum counting distribution. This is because these distributions show a great deal of structure, which is *not* apparent if one restricts attention to quasi-realistic models from the start. This observation is the primary formal motivation for introducing the definition.

At this point, if the definition Eq. (152) is clear, one can proceed to the next subsection. However, since many similar but different definitions can be made, and the issue of interpretation may confuse some readers, let us briefly expand on these points.

To eliminate one possible source of confusion at the start, the list we are constructing is of "possible universes" within string theory. Our own universe at the present epoch is supposed to correspond to one of these universes, not some sort of superposition or dynamical system which explores multiple vacua. The point of the list, or the equivalent distribution Eq. (152), is simply to have a precise way to think about the totality of possibilities.

Another possible confusion is between Eq. (152), and the definition of a measure factor used in quantum cosmology. As we discussed in Sec. III.E, to define a measure factor, we need to assign a "probability factor" to each vacuum, call this $P(i)$. The measure factor corresponding to a given list $\{T_i\}$ and probability factor $P(i)$ is then

$$d\mu_P(T) = \sum_i P(i)\ \delta(T - T_i). \quad (155)$$

Its integral over a region R, gives the probability with which we expect a vacuum in R to be produced by the cosmological model which gave rise to this measure factor.

We already discussed some aspects of the interpretation of such distributions in Sec. III.E, and we will continue this in Sec. V.F. The main point we want to make here is simply that, unlike a measure factor, a vacuum counting distribution is **not** a probability distribution, and does not require any concept of a "probability that a universe of type T exists" for its definition. Rather, it summarizes information about the set of consistent vacua of the theory.

1. Approximate distributions and tuning factors

A reason to be careful about the difference between a vacuum counting distribution and a measure factor, is so that we can properly introduce the idea of an approximate vacuum counting distribution. To motivate this, suppose that we know how to construct a set of vacua V_i, but that our theoretical technique is not adequate to compute the exact value of a coupling g in each vacuum, only some approximation to it. In practice this will always be true, but it gains particular significance for parameters which we must fit to an accuracy far better than our computational abilities, with the prime example being the cosmological constant as we discussed earlier.

Suppose for sake of discussion that we are interested in the cosmological constant Λ, but can compute it only to an accuracy roughly $\Delta\Lambda$. We might model our relative ignorance by modifying our definition Eq. (152) to

$$dN_{vac}(\Lambda) = \sum_i \frac{1}{\sqrt{\pi}\Delta\Lambda} \exp - \frac{(\Lambda - \Lambda_i)^2}{(\Delta\Lambda)^2}, \quad (156)$$

a sum of Gaussian distributions of unit weight. The choice of the Gaussian, while not inevitable, would follow if the total error was the sum of many independent terms, which is reasonable as the cosmological constant receives corrections from many sectors in the theory.

If we use the resulting approximate vacuum counting distribution as before to compute integrals like Eq. (154), we will get results like "we expect region R to contain half a vacuum," or perhaps 10^{-10} vacua. What could this mean?

Of course, given that string theory and the effective potential have a precise definition, any particular vacuum has some definite cosmological constant $\Lambda_{i,true}$. The problem is just that we don't know it. In modelling our ignorance with a Gaussian (or any other distribution), we have again introduced probabilities into the discussion – but note that this is a different and less problematic sense of probability than the $P(i)$ we introduced in discussing the measure factor. It is not intrinsic to string theory or cosmology, but rather it expresses our judgement of how accurate we believe our theoretical computations to be, and nothing else. As such, it is a technical device, but a useful one as we shall see.

Having understood this, the meaning of results like "we expect region R to contain 10^{-10} vacua" in this context becomes clear. In actual fact, the region must contain zero, one or some other definite number of vacua. While given the theoretical information to hand, we do not know the actual number, we now have good reasons to think R probably does not contain any vacua. However, this conclusion is not ironclad; perhaps numerical coincidences in the computations will put one or more vacua into R. If our model for the errors is correct, the probability of this happening is 10^{-10}, in the usual "frequentist" sense: if we have 10^{10} similar regions to consider, we expect one of them to actually contain a vacuum.

The reader will probably have already realized that what we have just discussed, gives a precise sense within string theory to the usual discussion of fine tuning made in effective field theory. Although in principle every coupling constant in every string vacuum has some definite value, and in this sense is "tuned" to arbitrary precision, in practice we cannot compute to this precision, and need to work with approximations. The preceding discussion gives us a way to do this and to combine the results of various approximations. This could be used to justify the style of discussion we made in II.F.3, where we compared hypothetical numbers of vacua with and without low energy supersymmetry. In combining the various ingredients of an approximate vacuum counting distribution, small tuning factors can be compensated by multiplicity factors, to produce apparently counterintuitive results. We will come back to this idea after discussing some concrete results.

Of course, the specific ansatz Eq. (156) was a way to feed in explicit knowledge about computational accuracy and tuning. As we will see, there are many other approximations one might make in computing a vacuum counting distribution, sometimes with explicit control parameters and sometimes not, but with the same general interpretation. We will discuss the "continuous flux approximation" in some detail below.

Finally, let us cite the standard statistical concept of a *representative* sample. This is a sample from a larger population, in which the distribution of properties of interest well approximates the distribution in the larger population. Given a representative sample of N_{rep} vacua, their distribution dN_{rep}, and the total number of vacua N_{vac}, we could infer an approximation to the total vacuum counting distribution,

$$dN_{vac}(T) \sim \frac{N_{vac}}{N_{rep}} dN_{rep}(T).$$

While elementary, this idea is probably our main hope of ever characterizing the true dN_{vac} of string/M theory in practice, so making careful use of it is likely to become an increasingly important element of the discussion.

B. Counting flux vacua

The simplest example of the general framework we are about to describe is the counting of supersymmetric IIb flux vacua for a Calabi-Yau with no complex structure moduli. This leads to flux vacua with stabilized dilaton-axion, and a one-parameter distribution which can be worked out using elementary arguments. One can explicitly see the nature of the continuous flux approximation.

To go further, we need to introduce some formalism. This is modelled after similar problems in random potential theory, as well as the mathematical theory of random sections of holomorphic line bundles. We then discuss general results for supersymmetric vacua, and some explicit two parameter distributions. We then give similar results for nonsupersymmetric vacua. Finally, we summarize some of the most important general conclusions from this type of analysis.

1. IIb vacua on a rigid CY

This problem was studied in Ashok and Douglas (2004); Denef and Douglas (2004). We write τ for the dilaton-axion; by definition it must satisfy $\text{Im}\,\tau > 0$, in other words it takes values in the upper half plane. A rigid CY, for example the resolved $T^6/\mathbb{Z}_3$ orbifold, has $b^{2,1} = 0$ and thus $b^3 = 2$; thus there are two NS fluxes a_i and two RR fluxes b_i, which we take to be integrally quantized.

The flux superpotential Eq. (61) is

$$W = (a_1 + \Pi a_2)\tau + b_1 + \Pi b_2 \equiv A\tau + B,$$

where we group the NS and RR fluxes into two complex combinations A and B. Here

$$\Pi \equiv \frac{\int_B \Omega}{\int_A \Omega}$$

is a complex number which is determined by the geometry of the CY; let us take $\Pi = i$ for simplicity.

The Kähler potential on this moduli space is $K = \log \text{Im}\,\tau$, and it is very easy to solve $DW = 0$ for the location of the supersymmetric vacuum as a function of the fluxes; it is

$$DW = 0 \leftrightarrow \bar{\tau} = -\frac{B}{A}, \tag{157}$$

so there will be a unique vacuum if $\text{Im}\,B/A > 0$, and otherwise none.

The tadpole condition Eq. (70) becomes

$$\text{Im}\,A^* B \leq L. \tag{158}$$

Finally, the $SL(2,\mathbb{Z})$ duality symmetry of IIb superstring theory acts on the dilaton and fluxes as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}: \quad \tau \to \tau' = \frac{a\tau + b}{c\tau + d}; \quad \begin{pmatrix} A \\ B \end{pmatrix} \to \begin{pmatrix} aA + bB \\ cA + dB \end{pmatrix}. \tag{159}$$

Two flux vacua which are related by an $SL(2,\mathbb{Z})$ transformation are physically equivalent, and should only be counted once. Since the duality group is infinite, gauge fixing this symmetry is essential to getting a finite result.

A direct way to classify these flux vacua, is to first enumerate all choices of A and B satisfying the bound Eq. (158), taking one representative of each orbit of Eq. (159), and then to list the flux vacua for each. Now it is not hard to see (Ashok and Douglas, 2004) that this can be done by taking $a_2 = 0$, $0 \le b_1 < a_1$ and $a_1 b_2 \le L$. By Eq. (157), each choice of flux stabilizes a unique vacuum, and thus the total number of vacua is finite,

$$N_{vac}(L) = 2\sigma(L) = 2\sum_{k|L} k, \tag{160}$$

where $\sigma(L)$ is a standard function discussed in textbooks on number theory, with the asymptotics

$$\sum_{L\le N} \sigma(L) = \frac{\pi^2}{12} N^2 + \mathcal{O}(N log N)$$

Finally, we can use Eq. (157) to get the distribution of vacua in configuration space. Let us suppose that in the resulting low energy theory, τ controls a gauge coupling, but there is no direct dependence on the values A, B of the fluxes. In this case, it is useful to use $SL(2,\mathbb{Z})$ transformations to bring all of the vacua into the fundamental region $|\tau| \ge 1$ and $|\text{Re } \tau| \le 1/2$, as this is the moduli space of physically distinct theories, ignoring the flux.

We plot the results for $L = 150$ in Figure V.B.1. Each point on this graph is a possible value of τ in some flux vacuum; many of the points correspond to multiple vacua.

While the figure clearly displays a great deal of structure, one might worry about its intricacy and ask: if this is what comes out of the simplest class of models, what hope is there for understanding the general distribution of vacua in string theory? Fortunately, there is a very simple approximate description, which captures much of the structure of this distribution. It is a uniform distribution, modified by a sort of "symmetry enhancement" phenomenon.

We first discuss the uniform distribution. A very naive first guess might be $d^2\tau$, but of course this is not invariant under field redefinitions; rather we must look at the geometry of the configuration space to decide what is a natural "uniform" distribution. Now the configuration space of an effective field theory always carries a metric, the "sigma model metric," defined by the kinetic terms in the Lagrangian,

$$\mathcal{L} = G_{ij}\partial\phi^i\partial\phi^j + \dots. \tag{161}$$

Thus, the natural definition of a "uniform measure" on configuration space, is just the volume form associated to the sigma model metric,

$$d\mu = d^n\phi\sqrt{\det G(\phi)}. \tag{162}$$

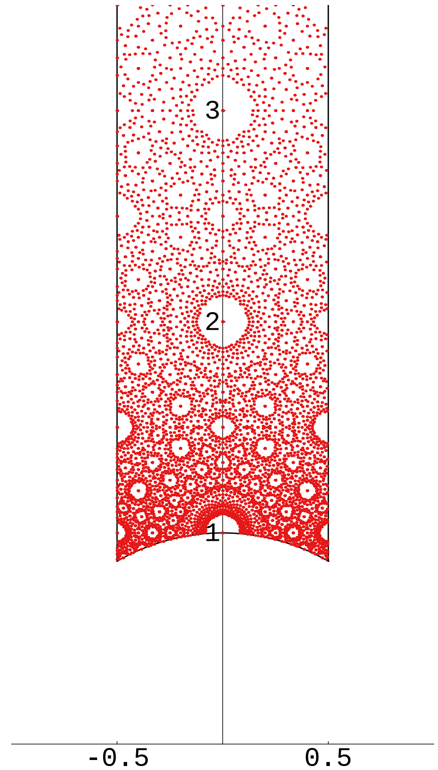

FIG. 1 Values of τ for rigid CY flux vacua with $L_{max} = 150$. From Denef and Douglas (2004).

In the problem at hand, this is

$$d\mu = \frac{d^2\tau}{4(\text{Im } \tau)^2}. \tag{163}$$

Of course, this is a continuous distribution, unlike the actual vacuum counting distributions which are sums of delta functions. However, if we take a limit in which the number of vacua becomes arbitrarily large, it might be that the limiting distribution of vacua could be approximated by a continuous distribution. Since the discreteness of the allowed moduli values was due to flux quantization, and it is intuitively clear that the effects of this should become less important as L increases, a reasonable conjecture would be that in the limit $L \to \infty$, the distribution of flux vacua in moduli space approaches Eq. (163).

If we are a bit more precise and keep track of the total number of vacua, we can make a similar conjecture for the vacuum counting distribution itself. Normalizing Eq. (163) so that its integral over a fundamental region is Eq. (160), we find

$$\lim_{L\to\infty} dN_{vac} = \pi\ L\ \frac{d^2\tau}{(\text{Im } \tau)^2}. \tag{164}$$

For example, a disc of area A should contain $4\pi AL$ vacua in the large L limit.

While this is true, as can be deduced from the formalism we will describe shortly, at first glance the finite

L distribution may not look very uniform. Comparing with the $L = 150$ figure, we see that around points such as $\tau = ni$ with $n \in \mathbb{Z}$, there are holes of various sizes containing no vacua. Where do these holes come from, and how can they be consistent with the claim?

In fact, at the center of each of these holes, there is a large degeneracy of vacua, which after averaging over a sufficiently large region recovers the uniform distribution. For example, there are 240 vacua at $\tau = 2i$, which compensate for the lack of vacua in the hole. As discussed in Denef and Douglas (2004), while this leads to a local enhancement, just beyond the radius of the hole the uniform approximation becomes good.

This behavior can be understood as coming from alignments between the lattice of quantized fluxes and the constraints following from the equations $DW = 0$. Using this, one can argue that the continuous flux approximation will well approximate the total number of vacua in a region of radius r satisfying

$$L > \frac{K}{r^2}. \qquad (165)$$

Another rough model for the approximation might be a Gaussian error model as in Eq. (156), with variance $\sigma \sim K/L$. Finally, one can also understand the corrections to the large L approximation as a series in inverse fractional powers of L, using mathematics discussed in Douglas *et al.* (2006b).

2. General theory

The result we just discussed is a particular case of a general formula for the large L limit of the index density of supersymmetric flux vacua in IIb theory on an arbitrary Calabi-Yau manifold M (Ashok and Douglas, 2004),

$$\sum_{L \le L_{max}} dI_{vac}(L) = \frac{(2\pi L_{max})^{b_3}}{\pi^{b_3/2} b_3!} \det(-R - \omega). \qquad (166)$$

We will explain what we mean by "index density" shortly; like the vacuum counting distribution, it is a density on moduli space, here a $b_3/2$ complex dimensional space which is the product of axion-dilaton and complex structure moduli spaces. The prefactor depends on the tadpole number L defined in Eq. (70), and on b_3, the third Betti number M. Instead of the density for a single L, we have added in all $L \le L_{max}$; in the large L limit the relation between these is the obvious one, but such a sum converges to the limiting density far more quickly than results at fixed L.

The density $\det(-R - \omega \cdot 1)$ is entirely determined by the metric on moduli space Eq. (161); all the dependence on the other data entering the flux superpotential Eq. (61) cancels out of the result. It is a determinant of a $(b_3/2) \times (b_3/2)$ dimensional matrix of two-forms, constructed from the Kähler form ω on $\mathcal{M}$, with the matrix valued curvature two-form R constructed from the metric on $\mathcal{M}$.

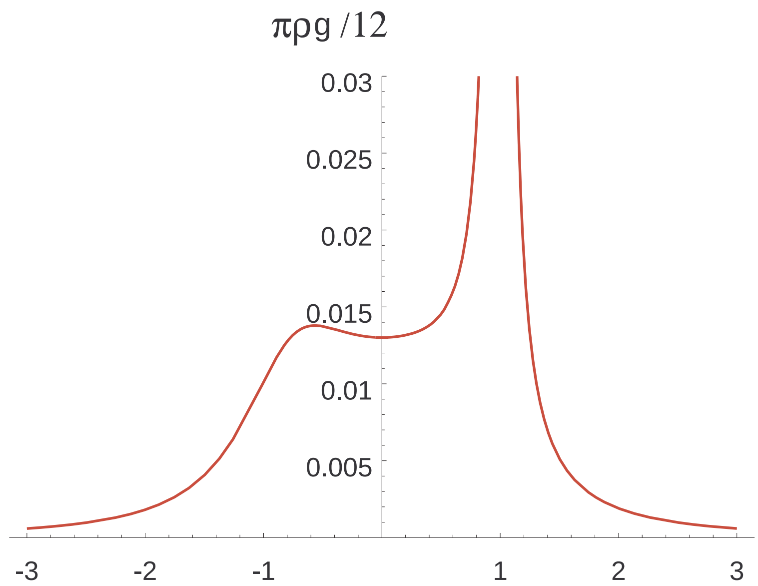

FIG. 2 The susy vacuum number density per unit ψ coordinate volume, on the real ψ-axis, for the mirror quintic. From Denef and Douglas (2004).

Thus, while the volume form Eq. (162) was a natural first guess for the distribution of flux vacua, as we will see the actual distribution can be rather different. The agreement between Eq. (164) and Eq. (163) in the example of Sec. V.B.1 was particular to this case, and follows from $R \propto \omega$ for that moduli space. Similar, though more complicated, explicit results can be obtained for the actual vacuum counting distribution (Denef and Douglas, 2004; Douglas *et al.*, 2006b), and distributions of nonsupersymmetric flux vacua (Denef and Douglas, 2005).

Let us plot the number density in another example, compactification on the mirror of the quintic CY (Candelas *et al.*, 1991; Greene and Plesser, 1990). Here $\mathcal{M}_C(M)$ is one complex dimensional and thus the distribution depends on two parameters; however it is a product distribution whose dependence on the dilaton-axion is again Eq. (163) for symmetry reasons. The dependence on the complex structure modulus is non-trivial; if we plot it along a real slice, we get Figure V.B.2.

The striking enhancement as $\psi \to 1$ is because this limit produces a conifold singularity as discussed in Sec. IV.A.2.a. As discussed in Denef and Douglas (2004), near the conifold point Eq. (166) becomes[17]

$$dN_{vac} \sim \frac{d^2\psi}{|\psi-1|^2(\log|\psi-1|)^3} \tag{167}$$

As discussed in Sec. IV.A.2.a, under flux-gauge duality, the parameter $\psi - 1$ is dual to the dynamically induced scale μ in the gauge theory, and thus dimensional transmutation explains the leading $d\mu/\mu$ dependence here. However the log factors have to do with details of the sum over fluxes.

This distribution is (just barely) integrable; doing so over a disc, the number of susy vacua with $L \le L_*$ and $|\psi - 1| \le R$ is

$$\mathcal{N}_{vac} = \frac{\pi^4 L_*^4}{18 \ln \frac{\mu^2}{R^2}}. \tag{168}$$

The logarithmic dependence on R implies that a substantial fraction of vacua are extremely close to the conifold point. For example when $L_* = 100$ and $\mu = 1$, there are still about one million susy vacua with $|v| < 10^{-100}$.

Despite this enhancement, from the figure one sees that the majority of vacua are not near the conifold point. On the other hand, in many parameter models, a sizable fraction of vacua can be expected to contain conifold limits, by a simple probabilistic argument we give in Sec. V.E.3.

Many of the other general results for flux vacuum distributions which we called upon in Sec. IV also follow from Eq. (166), by inserting known behaviors of moduli space metrics, introducing further constraints and so on. For example,

- The fraction of flux vacua with string coupling $g_s \le \epsilon << 1$ goes as ϵ. This follows from the expression Eq. (163) for the tree level metric on dilaton-axion moduli space.
- The fraction of weakly coupled vacua with $e^K|W|^2 \le \epsilon$ goes as ϵ. This is particular to IIb flux vacua, for reasons we discuss at the end of this subsection.

a. Definition of the index density This is a sum over vacua, weighed by ± 1 factors,

$$dN_{vac}(T) = \sum_i \delta(T - T_i)(-1)^F_i. \tag{169}$$

[17] While in general this formula is the index density Eq. (169), it is not hard to show that all vacua near the conifold point have $(-1)^F = +1$, so that in this case it is also the number density. More generally, globally supersymmetric vacua (which do not depend on the $(\partial K)W$ term in DW) always have $(-1)^F = +1$. Conversely, the $(-1)^F = -1$ vacua are in a sense "Kähler stabilized."

The factor $(-1)^F_i$ will be defined shortly; it is essentially the sign of the determinant of the fermionic mass matrix.

The primary reason to consider this quantity is that it leads to much simpler explicit results than Eq. (152). To explain why, we recall the general formula for the distribution of critical points of a random potential V. As is well-known in the theory of disordered systems, this is

$$dN_{vac}(z) = \langle \delta(V'(z)) |\det V''(z)| \rangle, \tag{170}$$

where the expectation value is taken in the ensemble of random potentials; here the ensemble of flux potentials. Formally, such a density is proportional to the delta function $\delta(V'(z))$, however the integral of such a delta function over field space is not 1. To get a normalized density in which each vacuum has unit weight, we multiply by the Jacobian factor.

Now, upon incorporating the sign factor in Eq. (169), this becomes

$$dI_{vac}(z) = \langle \delta(V'(z)) \det V''(z) \rangle, \tag{171}$$

and the somewhat troublesome absolute value sign from the Jacobian is removed. The virtue of this is that the index turns out to be much simpler to compute than dN_{vac}, yet provides a lower bound for the actual number of vacua. There is some evidence that the ratio of the index to the actual number of vacua is of order $1/c^K$ for some order one c (Douglas *et al.*, 2004).

We can use essentially the same formulae Eq. (170) and Eq. (171) to count supersymmetric vacua, by replacing V with a flux superpotential $W(z)$, taking into account that it and the chiral fields are complex. Combining these ideas, and taking the continuous flux limit as in the previous subsection, leads to the integral formula

$$\lim_{L\to\infty} dI_{vac}(z;L) = \tag{172}$$

$$\int d^{2K} N_{N\eta N=L} \quad \delta^{(2n)}(DW(z)) \det \begin{pmatrix} D_i\bar{D}_jW & D_iD_jW \\ \bar{D}_i\bar{D}_j\bar{W} & \bar{D}_iD_j\bar{W} \end{pmatrix} \tag{173}$$

where the tadpole constraint was schematically written $N\eta N = L$ in terms of a known quadratic form η.

b. Computational techniques Without going into the details of the subsequent computations leading to Eq. (166), two general approaches have been used. One is to formally represent the integral over fluxes satisfying the tadpole constraint as a Laplace transform of a Gaussian integral with weight $\exp -N\eta N$. In this way, one can think of the random superpotential as defined by its two-point function,

$$\langle W(z_1)\, \bar{W}(\bar{z}_2) \rangle = \exp -K(z_1, \bar{z}_2),$$

where $K(z_1, \bar{z}_2)$ is the formal continuation of the Kähler potential $K(z, \bar{z})$ to independent holomorphic and antiholomorphic variables. In this sense, the flux superpotential is a Gaussian random field, however a rather peculiar one as its correlations can grow with distance. Still, one can proceed formally despite this, and then justify the final results.

The other approach (Denef and Douglas, 2004) is to make a direct change of variables from the original fluxes F, H to the relevant derivatives of the superpotential. Since this provides more physical intuition for the results, let us discuss it a bit.

One of the main simplifications which allows obtaining explicit results for a density such as Eq. (166), is that its definition restricts attention to the neighborhood of a point in configuration space, the point z. Because of this, we only need a finite amount of information about the effective potential, namely the superpotential $W(z)$ and some finite number of its derivatives at z, to compute it.

For example, to evaluate Eq. (172), we only need $D_iW(z)$, $D_iD_jW(z)$, $D_i\bar{D}_jW(z)$, and their complex conjugates. Standard results in supergravity (or, the fact that W is a holomorphic section), imply that $D_i\bar{D}_jW = g_{i\bar{j}}W$, so this is known in terms of W. Thus, we only need the joint distribution of $1 + n + n(n+1)/2 = (n+1)(n+2)/2$ independent parameters derived from the potential to compute the vacuum counting index. Let us give these names; in addition to $W \equiv W(z)$ we have[18]

$$F_A = D_AW(z); \qquad Z_{AB} = D_AD_BW(z). \tag{174}$$

By substituting Eq. (61) into these expressions and fixing z, we get F and Z as functions of the fluxes N; in fact they are *linear* in the N.

Now, we can rewrite Eq. (172) as

$$\lim_{L\to\infty} dI_{vac}(z;L) = \tag{175}$$

$$\int [d^2W\ d^2F\ d^2Z]_L\ \delta^{(2n)}(F_i)\ \det\begin{pmatrix} g_{i\bar{j}}W & Z_{ij} \\ \bar{Z}_{\bar{i}\bar{j}} & g_{\bar{i}j}\bar{W}\end{pmatrix} \tag{176}$$

where the notation $[d^2W\ d^2F\ d^2Z]_L$ symbolizes the integral over whatever subset of these variables corresponds to the original integral over fluxes satisfying the tadpole condition.

What makes this rewriting very useful, is that the change of variables $N \to (W, F, Z)$ turns out to be very simple (Denef and Douglas, 2004); it is just

$$\int_{N\eta N=L} d^{2K}N \to \int_{L=|W|^2-|F|^2+|Z|^2} d^2W\ d^{2n}F\ d^{2n-2}Z_{0i} \tag{177}$$

where the index $i = 0$ denotes the dilaton-axion. In particular, the Jacobian $\det \partial N/\partial(W, F, Z)$ is a constant (in appropriate conventions, unity), and the only constraint is the tadpole constraint, which is also simple. Of course, since we have only $4n$ fluxes, only a subset of $n-1$ of the Z variables can appear; however the others are also simple:

$$Z_{ij} = \mathcal{F}_{ijk}g^{k\bar{l}}\bar{Z}_{\bar{l}},$$

where $\mathcal{F}_{ijk}$ are the standard “Yukawa couplings” of special geometry (Candelas *et al.*, 1991). It is a fairly short step from these formulae to Eq. (166) and its generalizations.

The rewriting Eq. (177) is the simplest way to describe the ensemble of IIb flux vacua, if one only needs to find distributions of single vacua and their properties (formally, one-point functions). On the other hand, the approach in which W is a generalized Gaussian random field, could also be used to compute distributions depending on the properties of more than one vacuum, or on the effective potential away from its critical points, for example average barrier heights between vacua, or the average number of e-foldings of slow-roll inflationary trajectories. In fact, modelling the inflationary potential as a Gaussian random field has been tried in cosmology (Tegmark, 2005); it would be interesting to do the same with this more accurate description of the effective potentials for flux vacua.

All of these precise results are in the continuous flux approximation. As before, the general theory suggests that this should be good for $L >> K$. The results have been checked to some extent by numerical study (Conlon and Quevedo, 2004; Giryavets *et al.*, 2004a), finding agreement with the distribution in z, and usually (though not always) the predicted scaling with L. It should be said that numerous subtleties had to first be addressed in the works which eventually found agreement; such as the need to avoid double-counting flux configurations related by duality, and the need to consider fairly large values of the flux.

c. Other ensembles of flux vacua These can be treated by similar methods, say by working out the analog to Eq. (177). This was done for G_2 compactifications in Acharya *et al.* (2005). A useful first picture can be formed by considering the ratio (DeWolfe *et al.*, 2005b)

$$\eta \equiv \frac{\text{number of independent fluxes}}{\text{number of (real) moduli}},$$

as this determines the number of parameters $(W, F_i, Z_{ij}, \ldots)$ which can be considered as roughly independent. While for IIb flux vacua $\eta = 2$, for all of the other well understood flux ensembles (M theory, IIa,heterotic) $\eta = 1$ as there is only one type of flux.

For $\eta = 1$, one generally finds the uniform distribution Eq. (162), and $|W|$ is of order the cutoff scale. This is because the conditions $D_iW = F_i = 0$ already set almost all of the fluxes, so there are too few fluxes to tune W

[18] Strictly speaking, one needs to include the Kähler potential in these definitions, to get quantities which are invariant under Kähler-Weyl transformations. An alternate convention, which saves a good deal of notation and which we follow here, is to do a Kähler-Weyl transformation to set $K(z,\bar{z}) = 0$ at the point z under consideration, and use an orthonormal frame for the tangent space to z; see Denef and Douglas (2004) for more details.

to a small value. This is perhaps the main reason why controlled small volume compactifications are easier to discuss in the IIb theory. Of course, it may yet turn out that additional choices in the other theories, less well understood at present, allow similar constructions there.

C. Scale of supersymmetry breaking

Let us now resume the discussion of Sec. II.F.3, combining results from counting flux vacua with various general observations, to try to at least identify the important questions here. We would like some estimate of the number distribution of vacua described by spontaneously broken supergravity,[19]

$$dN_{vac}[M_{susy}, M_{EW}, \Lambda] \qquad (178)$$

at the observed values of Λ and M_{EW}. If this were approximately a power law,

$$dN_{vac}[M_{susy}, 100\mathrm{GeV}, 0] \sim dM_{susy} M_{susy}^{\alpha},$$

then for $\alpha < -1$ we would predict low scale susy, while for $\alpha \geq 1$ we would not. Here we will define

$$M_{susy}^4 = \sum_i |F_i|^2 + \frac{1}{2}\sum_\alpha D_\alpha^2,$$

the energy scale associated to supersymmetry breaking in the microscopic theory. Note that many authors use a different definition in which $M_{susy} \sim m_{3/2}$.

For purposes of comparison, let us begin with the prediction of field theoretic naturalness. This is

$$dN_{vac}^{FT} \sim \left(\frac{M_{EW}^2\ M_{Pl}^2}{M_{susy}^4}\right)\left(\frac{\Lambda}{M_{susy}^4}\right) f(M_{susy}), \qquad (179)$$

where the first factor follows from Eq. (24). As for $f(M_{susy})$, if we grant that this is set by strong gauge dyanmics, a reasonable ansatz might be dM_{susy}/M_{susy}, analogous to Eq. (167). This would lead to $\alpha = -9$ and a clear prediction.

Now, while we cannot say we have a rigorous disproof of Eq. (179), the approach we are discussing gives us many reasons to disbelieve it, based both on computation in toy models, and on simple intuitive arguments. Let us explain these in turn.

The simplest problem with Eq. (179) is the factor Λ/M_{susy}^4. Instead, distributions of flux vacua generally predict Λ/M_{KK}^4, Λ/M_{Pl}^4 or some other fundamental scale. In other words, *tuning the cosmological constant is not helped by supersymmetry.*

[19] There are also vacua with no such description, because supersymmetry is broken at the fundamental scale. While these might further disfavor TeV scale supersymmetry, at present it is hard to be quantitative about this.

To see this, we start from Eq. (22), and the claim that Λ is the value of the potential at the minimum, so that $\Lambda = M_{susy}^4 - 3|W|^2/M_{Pl,4}^2$. Intuitively, this formula expresses the cancellation between positive energies due to supersymmetry breaking (the F and D terms), and a negative "compensating" energy from the $-3|W|^2$ term. However, one should not fall into the trap of thinking that any of these terms are going to "adjust themselves" to cancel the others. Rather, there is simply some complete set of vacua with some distribution of Λ values, out of which a $\Lambda \sim 0$ vacuum will be selected by some *other* consideration (anthropic, cosmological, or just fitting the data). For the purpose of understanding this distribution, it is best to forget about this later selection effect, only bringing it in at the end.

On general grounds, since the cosmological constant is a sum of many quasi-independent contributions, it is very plausible that it is roughly uniformly distributed out to some cutoff scale M, so that the basic structure we are looking for in Eq. (178) is this scale. Clearly by Eq. (22) this is set by the cutoffs in the F, D and W distributions; more specifically by the largest of these.

Let us now focus on the W distribution, coming back to the F and D distributions shortly. According to the definition Eq. (61), the effective superpotential W receives contributions from all the fluxes, including those which preserve supersymmetry. Because of this, the distribution of W values has little to do with supersymmetry breaking; rather it is roughly uniform (as a complex variable) out to a cutoff scale set by flux physics, namely M_F as defined in Sec. II.F.2. Since

$$d(|W|^2) = 2|W|d|W| = \frac{1}{\pi}d^2W,$$

this implies that $|W|^2$ is uniformly distributed out to this scale, and thus that Λ will be uniformly distributed at least out to this scale, leading to a tuning factor Λ/M_F^4.

To summarize, the distribution of the cosmological constant is not directly tied to supersymmetry breaking, because it receives contributions from supersymmetric sectors as well. This correction to Eq. (179) would result in $\alpha = -5$, still favoring low scale supersymmetry, but rather less so.

Now, there is a clear loophole in this argument, namely that there might be some reason for the supersymmetric contributions to W to be small. In fact, one can get this by postulating an R symmetry, which is only broken along with supersymmetry breaking. However, within the framework we are discussing, it is not enough just to say this to resolve the problem. Rather, one now has to count the vacua with the proposed mechanism (here, R symmetry), and compare this to the total number of vacua, to find the cost of assuming the mechanism. Only if this cost is outweighed by the gain (here a factor M_F^4/M_{susy}^4) will the mechanism be relevant for the final prediction. We will come back and decide this shortly.

Before doing this, since the correction we just discussed would by itself not change the prediction of low scale supersymmetry, we should discuss the justification of the other factors in Eq. (179). First, we will grant the factor M_{EW}^2/M_{susy}^4, not because it is beyond question – after all this assumes some generic mechanism to solve the μ problem – but because the information we would need about vacuum distributions has not yet been worked out.

On the other hand, the claim that the distribution of supersymmetry breaking scales among string/M theory vacua is dM_{susy}/M_{susy}, can also be questioned. While this sounds like a reasonable expectation for theories which break supersymmetry dynamically, one has to ask whether there are other ways to break supersymmetry, what distributions these lead to, and how many vacua realize these other possibilities.

Given the definition of supersymmetry breaking vacuum we used in Sec. II.F.1, namely a metastable minimum of the effective potential with F or $D \neq 0$, one might well expect a generic effective potential to contain many supersymmetry breaking vacua, not because of any "mechanism," but simply because generic functions have many minima. We discussed this idea in IV.A.4.c, and it was shown to be generic for IIb flux vacua in Denef and Douglas (2005), leading to the distribution

$$dN_{vac}[M_{susy}] \sim \left(\frac{M_{susy}}{M_F}\right)^{12}. \qquad (180)$$

Although the high power 12 may be surprising at first, it has a simple explanation (Dine *et al.*, 2005; Giudice and Rattazzi, 2006). Let us consider a generic flux vacuum with $M_{susy} << M_F$. Since one needs a goldstino for spontaneous susy breaking, at least one chiral superfield must have a low mass; call it ϕ. Generically, the flux potential gives order M_F masses to all the other chiral superfields, so they can be ignored, and we can analyze the constraints in terms of an effective superpotential reduced to depend on the single field ϕ,

$$W = W_0 + a\phi + b\phi^2 + c\phi^3 + \ldots.$$

The form of the Kähler potential $K(\phi, \bar{\phi})$ is also important for this argument; however one can simplify this by replacing (a, b, c) by invariant variables generalizing Eq. (174),

$$F \equiv D_\phi W;\ Z \equiv D_\phi D_\phi W;\ U \equiv D_\phi\ D_\phi D_\phi W.$$

In terms of these, the conditions for a metastable supersymmetric vacuum are $|F| = M_{susy}^2$ (by definition), $|Z| = 2|F|$ (this follows from the equation $V' = 0$), and finally $|U| \sim |F|$ (as explained in Denef and Douglas (2005) and many previous discussions, this is necessary so that $V'' > 0$. This also requires a lower bound on the curvature of the moduli space metric).

Now, the distribution of the (F, Z, U) parameters in flux superpotentials can be worked out; we gave the result for F and Z in Eq. (175), and one can also find U in terms of (F, Z) and moduli space geometry. A good zeroth order picture of the result is that (F, Z, U) are independent and uniformly distributed complex parameters, up to the flux potential cutoff scale M_F. All three complex parameters must be tuned to be small in magnitude, leading directly to Eq. (180).

The upshot is that "generic" supersymmetry breaking flux vacua exist, but with a distribution heavily favoring the high scale, enough to completely dominate the $1/M_{susy}^4$ benefit from solving the hierarchy problem. Indeed, this would be true for any set of vacua arising from generic superpotentials constructed according to the rules of traditional naturalness with a cutoff scale M_F.

The flaw in the naturalness argument in this case is very simple; one needs to tune several parameters in the microscopic theory to accomplish a single tuning at the low scale. Of course, if the underlying dynamics correlated these parameters, one could recover natural low scale breaking. This would be a reasonable expectation if W was entirely produced by dynamical effects, or perhaps in some models in which it is a combination of dynamical and high scale contributions. Besides models based on gauge theory, it is entirely possible that a more careful analysis of the distribution of flux vacua on Calabi-Yau, going beyond the "zeroth order picture" we just described by taking into account more of the structure of the actual moduli spaces, would predict such vacua as well.

Of course, even if such vacua exist, we must go on to decide how numerous they are. Following Dine (2004b); Dine *et al.* (2004, 2005), we can summarize the picture so far by dividing the set of supersymmetry breaking vacua into "three branches,"

1. Generic vacua; *i.e.* with all of the F, D and W distributions as predicted by the flux vacuum counting argument we just discussed.

2. Vacua with dynamical supersymmetry breaking (DSB). Here we assume the distribution dM_{susy}/M_{susy} for the breaking parameters; however W is uniformly distributed out to high scales.

3. Vacua with DSB and tree level R symmetry. Besides the dM_{susy}/M_{susy} distribution, we also assume W is produced by the supersymmetry breaking physics.

In option (1), TeV scale supersymmetry would seem very unlikely. While both (2) and (3) lead to TeV scale supersymmetry, they can differ in their expectations for $|W|$ and thus the gravitino mass: in (3) this should be low, while in (2) the prior distribution is neutral, so the prediction depends on the details of mediation as discussed in Sec. II.F.1.

What can we say about which type of vacuum is more numerous in string/M theory? There is a simple argument against (3), and indeed against most discrete symmetries in flux vacua (Dine and Sun, 2006). First, a discrete symmetry which acts on Calabi-Yau moduli space, will have fixed points corresponding to particularly symmetric Calabi-Yau manifolds; at one of these, it acts as a discrete symmetry of the Calabi-Yau. Such a symmetry of the Calabi-Yau will also act on the fluxes, trivially on some and non-trivially others. To get a flux vacuum respecting the symmetry, one must turn on only invariant fluxes. Now, looking at examples, one finds that typically an order one fraction of the fluxes transform non-trivially; for definiteness let us say half of them. Thus, applying Eq. (166) and putting in some typical numbers for definiteness, we might estimate

$$\frac{N_{vac\ symmetric}}{N_{vac\ all}} \sim \frac{L^{K/2}}{L^K} \sim \frac{10^{100}}{10^{200}}.$$

Thus, discrete symmetries of this type come with a huge penalty. While one can imagine discrete symmetries with other origins for which this argument might not apply, since W receives flux contributions, it clearly applies to the R symmetry desired in branch (3), and probably leads to suppressions far outweighing the $(M_F/M_{susy})^4$ gain.

Thus, R symmetry appears to be heavily disfavored, with the exception of R parity: since the superpotential has R charge 2, it is invariant under a $\mathbb{Z}_2$ R symmetry. While crucial for other phenomenology, R parity does not force small W.

What about branches (1) versus (2) ? Among the many issues, we must estimate what fraction of vacua realize dynamical supersymmetry breaking. Looking at the literature on this, much of it adopts a very strong definition of supersymmetry breaking, in which one requires that no supersymmetric vacua exist. And, although the situation is hardly clear, it appears that very few models work according to this criterion. This might be regarded as evidence against (2).

However, this is a far stronger definition of supersymmetry breaking than we used elsewhere in our review. Rather, the question we want to answer is the difficulty of realizing metastable dynamical supersymmetry breaking vacua. Recent work such as (Dine *et al.*, 2006; Intriligator *et al.*, 2006) suggests that this is not so difficult, but it is still a bit early to evaluate this point.

Again, according to the point of view taken here, the goal is to show that metastable dynamical supersymmetry breaking vacua are generic in a quantitative sense. Doing this requires having some knowledge about the distributions of gauge theories among string/M theory vacua, to which we turn.

D. Other distributions

Understanding the total number and distribution of vacua requires combining information from all sectors of the theory. Here we discuss some of the other sectors, while the problem of combining information from different sectors is discussed in Douglas (2003).

1. Gauge groups and matter content

By now, the problem of trying to realize the Standard Model has been studied in many classes of constructions. Let us consider type IIa orientifolds of a Calabi-Yau M, (see Blumenhagen *et al.* (2005a) for a recent review). In the vast majority of such vacua which contain the SM, one finds that the tadpole and other constraints force the inclusion of "exotic matter," charged matter with unusual Standard Model quantum numbers or with additional charges under other gauge groups. One also finds hidden sectors, analogous to the second E_8 of the original CHSW models. While less well studied, other constructions such as more general heterotic vacua, F and M theory vacua, often contain exotic matter as well.

All this might lead to striking predictions for new physics, if we could form a clear picture of the possibilities, and which of them were favored within string/M theory. One is naturally led to questions like: Should we expect to see such exotic matter at low energies? Could the extra matter be responsible for supersymmetry breaking? Could the hidden sectors be responsible for some or all of the dark matter, or have other observable consequences?

A systematic base for addressing these questions would be to have a list of all vacua, with their gauge groups and matter content, as well as the other EFT data. While this is a tall order, finding statistics of large sets of vacua, such as the number of vacua with a given low energy gauge group G and matter representation R, is within current abilities (Blumenhagen *et al.*, 2005b; Dienes, 2006; Dijkstra *et al.*, 2005; Douglas and Taylor, 2006; Gmeiner, 2006a,b; Gmeiner *et al.*, 2006; Kumar and Wells, 2005a,b). Besides providing a rough picture of the possibilities, such statistics can guide a search for interesting vacua, or used to check that samples are representative.

Thus, let us consider a vacuum counting distribution

$$N_{vac}(G, R). \tag{181}$$

To be precise, G and R should refer to all matter with mass below some specified energy scale μ. The existing results count $N = 1$ supersymmetric vacua and ignore quantum effects, considering gauge groups which remain unbroken at all scales, and massless matter.

Most systematic surveys treat intersecting brane models (IBM's), in which the possible gauge groups G are products of the classical groups $U(N)$, $SO(N)$ and $Sp(N)$, while all charged matter transforms as two-index tensors: adjoint, symmetric and anti-symmetric tensors, and bifundamentals. In a theory with r factors in the gauge group, the charged matter content can be largely summarized in an $r \times r$ matrix I_{ij}, whose (i, j) entry denotes the number of bifundamentals in the $(N_i, \bar{N}_j)$, called the "generalized intersection matrix." Thus, we can rewrite Eq. (181) as

$$N_{vac}(\{N_i\}, I_{ij}). \tag{182}$$

Following the procedure outlined in Sec. II.A.3, one can make lists of models, and compute Eq. (182). The results from the studies so far are rather intricate, so a basic question is to find some simple approximate description of the result. In particular, one would like to know to what extent the data (N_i, I_{ij}) shows structure, such as preferred patterns of matter content, or other correlations, which might lead to predictions.

Alternatively, one might propose a very simple model, such as that the (N_i, I_{ij}) are (to some approximation) independent random variables.[20] While one might be tempted to call this a "negative" claim, of course we should not be prejudiced about the outcome, and methodologically it is useful to try to refute "null hypotheses" of this sort. Actually, since even in this case there would be preferred distributions of the individual ranks and multiplicities, such a result would carry important information.

The studies so far (Blumenhagen *et al.*, 2005b; Dijkstra *et al.*, 2005; Douglas and Taylor, 2006) in general are consistent with the null hypothesis, but suggest some places to look for structure. As yet they are rather exploratory and show only partial agreement, even about distributions within the same model classes.

In Blumenhagen *et al.* (2005b), the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold (and simpler warm-up models) were studied, and all gauge sectors enumerated. Simple analytical models were proposed in which Eq. (182) is governed by the statistics of the number of ways of partitioning the total tadpole among supersymmetric branes. For example, the total number of vacua with tadpole L goes roughly[21] as $\exp \sqrt{L}$, and the fraction containing an $SU(M)$ gauge group goes as $\exp -M/\sqrt{L}$. Computer surveys supported these claims, and found evidence for a anticorrelation between total gauge group rank and the signed number of chiral matter fields, and for a relative suppression of three generation models. However, it is not clear whether these surveys used representative samples, for reasons discussed in Douglas and Taylor (2006).

In Douglas and Taylor (2006), algorithms were developed to perform complete enumerations of "k-stack models," in other words the distribution of k of the gauge groups and associated matter. These obey power law distributions such as $N_{vac} \sim L^n/N_i^\alpha$ with α depending on the types of branes involved.

The work (Dijkstra *et al.*, 2005) enumerated orientifolds of Gepner models, and restricted attention to the SM sector, again finding that the majority of models had exotic matter, and multiple Higgs doublets.

[20] While this cannot literally be true of the entire spectrum as this must cancel anomalies, these constraints are relatively simple for brane models, so the simplest model of the actual distribution is to take a distribution of matter contents generated by taking these parameters independent, and then keeping only anomaly free spectra.

[21] There are $\log L$ corrections in the exponent.

An element not fully discussed in any of these works is that to compute Eq. (181) as defined in Sec. V.A, one needs to stabilize all other moduli, and incorporate multiplicities from these sectors. One can try to estimate these multiplicities in terms of the number of degrees of freedom in the "hidden" (non-enumerated) sectors by using generic results such as Eq. (166), for example as is done in Kumar and Wells (2005a).

To state one conclusion on which all of these works agree, the fraction of brane models containing the Standard Model gauge group and matter representations is somewhere around 10^{-10}, as first suggested by heuristic arguments in Douglas (2003). In this sense, reproducing the SM is not the hard part of model construction, and indeed has been done in all model classes with "sufficient complexity" (for example, enough distinct homology classes) which have been considered.

Among the many open questions, it would be very interesting to know if the heterotic constructions, which one might expect to favor GUTs and thus work more generically, are in fact favored over the brane models. The one existing survey (Dienes, 2006), of non-supersymmetric models, indeed finds GUT and SM gauge groups with far higher frequency. However a mere 10^{10} advantage here might well be swamped by multiplicities from fluxes and other sectors.

2. Yukawa couplings and other potential terms

These terms have a variety of sources in explicit constructions: world-sheet instantons in IIa models; overlap between gauge theoretic wave functions in IIb and heterotic models; all with additional space-time instanton corrections. While clearly very interesting for phenomenology, at this point none of this is understood in the generality required to do statistical surveys.

However one can suggest interesting pictures. As an example, one might ask the following question. Suppose, as might fit with the type of results we are discussing, that in some large class of vacua, quark and lepton masses were independent "random" variables, each with distribution $d\mu(m)$. Is there any $d\mu(m)$ with both plausible top-down and bottom-up motivations? In Donoghue (2004); Donoghue *et al.* (2006), the distribution $d\mu(m) \sim dm/m$ was proposed, both as a best fit among power law distributions to the observed masses, and as naturally arising from the combination of (1) uniform distributions of moduli z, and (2) the general dependence of Yukawa couplings

$$m \sim \lambda \sim \exp -z$$

expected if they arise from world-sheet instantons.

3. Calabi-Yau manifolds

All the explicit results we discussed assumed a choice of Calabi-Yau manifold. Now we do not know this choice *a priori*, so to count all vacua we need to sum over it, and thus we need the distribution of Calabi-Yau manifolds. Of course, we might also use statistics to try to decide *a priori* what is the most likely type of Calabi-Yau to contain realistic models, or use this data in other ways. In any case it is very fundamental to this whole topic.

Unfortunately, we do not at present know this distribution. The only large class of Calabi-Yau manifolds which is understood in any detail at present is the subset which can be realized as hypersurfaces in toric varieties. In more physical terms, these are the Calabi-Yau manifolds which can be realized as linear sigma models with a superpotential of the form $W = Pf(Z)$, leading to a single defining equation. Mathematically, the toric varieties which can be used are in one-to-one correspondence with reflexive polytopes in four dimensions. Such a polytope encodes the geometry and determines the Betti numbers, intersection forms, prepotential and flux superpotential, and supersymmetric cycles; for examples of how this information is used in explicitly constructing vacua see Denef *et al.* (2004, 2005).

We leave the definitions for the references, but the main point for our present discussion is that this is a combinatorial construction, so that the set of such polytopes can be shown to be finite, and in principle listed. In practice, the number of possibilities makes this rather challenging. Nevertheless, this was done by Kreuzer and Skarke (2002a,b), who maintain databases and software packages to work with this information (Kreuzer and Skarke, 2004).

This data, as illustrated by Figure V.D.3, is the evidence for our earlier assertion that "most Calabi-Yau manifolds have $b \sim 20 - 300$," in the range needed to solve the cosmological constant problem along the lines of Bousso and Polchinski (2000), but not leading to drastically higher vacuum multiplicities.

At present, the number of topologically distinct toric hypersurface Calabi-Yau manifolds is not known. While the 15122 points on this plot are clearly distinct, one point can correspond to several polytopes; furthermore the correspondence betweena polytopes and Calabi-Yau manifolds is not one-to-one; thus one has only lower and upper bounds. Furthermore, this set is known not to include all Calabi-Yau manifolds. One can at least hope that it is a representative subset; most but not all mathematicians would agree that this is reasonable.

4. Absolute numbers

Combining the various sectors and multiplicities we discussed, leads to rough estimates for numbers of vacua arising in different classes of constructions. The exploratory nature of much of the discussion, combined with the theoretical uncertainties outlined in Sec. III, make these estimates rather heuristic at present. Let us quote a few numbers anyways.

To the extent that we can estimate numbers of other

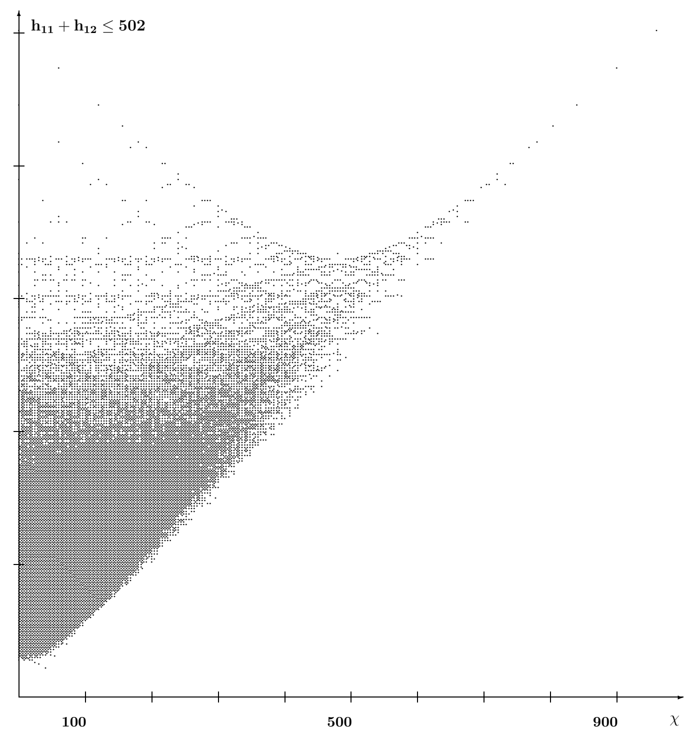

FIG. 3 The toric hypersurfaces with $\chi \geq 0$, from Kreuzer and Skarke (2002a). The vertical axis is $h^{1,1} + h^{2,1}$, while the horizontal axis is $\chi = 2(h^{1,1} - h^{2,1})$. The full set also contains the mirror manifolds obtained from these by taking $\chi \to -\chi$.

choices in heterotic and IIa, they are subleading to numbers of IIb flux vacua. One can get a lower bound on this from Eq. (166), if one can compute the integral over moduli space. This has only been done in one and two parameter examples, and for T^6 moduli space in Ashok and Douglas (2004), and in these cases gave $\pi^{\dim \mathcal{M}_C}$ times order one factors (one over the order of a discrete symmetry group), and thus were subleading to the prefactor. We will assume this is generally true, but it would be worth checking, as it is not inconceivable that CY moduli spaces have very large symmetry groups, and this would drastically reduce the numbers.

The number L can be computed either by choosing a IIb orientifolding, or using the relation to F theory on an elliptically fibered fourfold N, for which $L = \chi(N)/24$. While it would be interesting to survey the expected number of flux vacua over all the manifolds we discussed in Sec. V.D.3, at present it is not entirely clear that all of these allow stabilizing Kähler moduli. For the three examples which were shown to do so in Denef *et al.* (2004), one finds 10^{307}, 10^{393}, and 10^{506}. While this does not take into account the need for small $|W_0|$ and g_s, to check metastability under varying moduli, and so on, these are comparatively small factors. Thus, one can take 10^{500} as a reasonable estimate at present, unless and until we can argue that further conditions of the sort discussed in Sec. III are required.

One might worry that this is an underestimate, as we have left out many other known (and unknown) constructions. The only handle we have on this is the set of F theory compactifications, which are so similar to IIb that the same formulas might be applied. Since typical fourfolds have $K \sim 1000$, this could drastically increase the numbers, to say 10^{1000}. On the other hand, the additional moduli (compared to IIb orientifolds) correspond to charged matter, and one should take their superpotential and gauge dynamics into account in counting these vacua, so it is not clear that this is correct. More generally, while one might expect that as more constructions come under control the estimate will increase, this need not be, as new dualities between these constructions will also come into play.

E. Model distributions and other arguments

As we have seen, the computation of any distribution from microscopic string theory considerations is a lot of work. Since it is plausible that many results will have simple explanations, having to do with statistics and general features of the problem, it is tempting to try to guess them in advance.

The simplest examples are the uniform distributions, such as Eq. (162). At first these may not look very interesting; for example Eq. (163) for the dilaton-axion prefers order one couplings. Another well-known example is a mass parameter in an EFT, such as a boson mass $m^2\phi^2$. The standard definition of naturalness includes the idea that in a natural theory, this parameter will be uniformly distributed up to the cutoff scale. To some extent, this is a good model of one parameter flux vacuum distributions away from singular points.

Even so, on combining many such simply distributed parameters, one finds structure, which can lead to peaking and predictions.

1. Central limit theorem

As is very familiar, random variables which arise by combining many different independent sources of randomness, tend to be Gaussian (or normally) distributed. This observation is made mathematically precise by central limit theorems. Thus, if we find that some observable in string theory is the sum (or combination) of many moduli, or many independent choices in our definition of vacuum, it becomes plausible that this observable will be normally distributed as well.

One can design model field theory landscapes in which this postulate holds (Arkani-Hamed *et al.*, 2005b; Dienes *et al.*, 2005; Distler and Varadarajan, 2005). A simple example is to take a large number N of scalar fields ϕ_i, with scalar potential

$$V = \sum_i V_i(\phi_i) \tag{183}$$

and where each V_i is a quartic potential with two vacua, at $\phi_i^\pm$. This kind of model would arise if the N fields are localized at distinct points in extra dimensions, for instance, so their small wavefunction overlaps highly suppress cross terms in the potential. For simplicity, we will further take the quartics to be identical, though our considerations would hold more generally.

It follows immediately from the central limit theorem that, despite the fact that there are 2^N vacua, it is very hard to find vacua of this system with small cosmological constant! More concretely, let V_{av} be the average of the energies of the $\phi_\pm$ vacua, and V_{diff} be the difference. Then the distribution governing the vacuum energies of the vacua is

$$\rho(\Lambda) = \frac{2^N}{\sqrt{2\pi N V_{diff}}} \exp\left(-\frac{(\Lambda - N V_{av})^2}{2 N V_{diff}^2}\right) \tag{184}$$

In a non-supersymmetric system with UV cutoff M_*, we would a priori expect $V_{av} \sim M_*^4$, and therefore the distribution of vacua peaks at cosmological constant NM_*^4, with a width of order $\sqrt{N}M_*^4$. Vacua around zero cosmological constant are *not* scanned. In some fraction of such ensembles of order $1/\sqrt{N}$, where for some reason one fortuitously found $V_{av} \le \frac{1}{\sqrt{N}}M_*^4$, one *would* be able to scan around zero cosmological constant. In a trivial supersymmetric generalization of this landscape, with an unbroken R-symmetry (which guarantees that $V = 0$ is special), again one *would* be able to scan around zero cosmological constant, while supersymmetric theories without R-symmetry would not in general be expected to allow such scanning.

Suppose now the vevs $\phi_i^\pm$ characterizing a given vacuum also enter in the physical coupling constants c_a governing low energy physics (as *e.g.* the moduli do in the couplings of standard-like models in string theory). The same logic would teach one that despite the vast landscape of 2^N vacua, the coupling constants *don't* scan very much – they fluctuate by $\frac{\delta c_a}{c_a} \sim \frac{1}{\sqrt{N}}$ around their mean value.

In a landscape with this behavior, despite the large number of vacua, many physical quantities could be predicted with $1/\sqrt{N}$ precision. Since in nature only a few quantities seem plausibly to be environmentally determined, while many others beg for explanations based on dynamics and symmetries, one could hope that the cosmological term is one of a few variables that is scanned, while other quantities of interest do not scan (Arkani-Hamed *et al.*, 2005b).

To decide whether this is a good model for a particular parameter, one must look at microscopic details. As we mentioned, it is very plausible that the cosmological constant works this way. On the other hand, there is no obvious sense in which a modulus is a sum of independent random variables, and indeed the AD distribution Eq. (166) does not look like this. This would also be true for an observable which was a simple function of one or a few moduli, for example a gauge coupling in a brane model, proportional to the volume of a cycle. On the other hand, a hypothetical observable which was a sum or combination of many moduli, might be well modelled in this way. These observables would be the ones that are most clearly amenable to prediction (or post-diction) using statistical techniques.

2. Random matrix theory

Other universal distributions which appear very often in physics are the random matrix ensembles such as the GUE, GOE and so on. In the large N limit, these "peak" and exhibit universal properties such as the semicircle law, level repulsion and so on.

On general grounds, one might expect moduli masses to be modelled by a random matrix distribution. This was made more precise in Denef and Douglas (2005), who observed that since the matrix of fermion masses D_iD_jW in supersymmetric field theories is a complex symmetric matrix, it can be modelled by the CI distribution of Altland and Zirnbauer (1997). This leads to level repulsion between eigenvalues, characterized by the distribution

$$d\mu[\lambda] = \prod_a d(\lambda_a^2) \prod_{a<b} |\lambda_a^2 - \lambda_b^2|. \tag{185}$$

In particular, degenerate eigenvalues are non-generic. This was important in the arguments for Eq. (180) as degenerate eigenvalues would have led to an even larger exponent.

Another model for moduli masses was proposed in Easther and McAllister (2006). They considered the large volume limit, in which the superpotential is a sum of a flux term with nonperturbative corrections, as in Eq. (80). In this limit, while most fields (complex structure moduli, dilaton and others) obtain large masses, the axionic parts of the Kähler moduli obtain small masses, depending on the expectation values of the first set of fields. Taking the number of Kähler moduli as K and the number of the others as N, a reasonable model for the resulting mass matrix is

$$(M^2)_{ij} = \sum_{\alpha \le K+N} H_{i\alpha} H^\dagger_{\alpha j},$$

where H is a $K \times (K+N)$ matrix with randomly distributed entries. For large K, N, the limiting distribution for M^2 is very generally the Marcenko-Pastur distribution, a simple distribution depending on the ratio K/N.

While these are interesting universal predictions, they apply to moduli masses at scales M_F, and it is not completely obvious how they would relate to observable physics. In Easther and McAllister (2006) it was proposed that they favor "N-flation," a mechanism for slow-roll inflation (Dimopoulos *et al.*, 2005).

Similar ansatzes assuming less structure appear in Aazami and Easther (2006); Holman and Mersini-Houghton (2005); Kobakhidze and Mersini-Houghton (2004); Mersini-Houghton (2005).

3. Other concentrations of measure

This is the general term in mathematics for the "large N limits" and other universal phenomena exhibited by integrals over high dimensional spaces.

As a simple example, recall from Figure V.B.2 that in a one parameter model, most flux vacua are not near a conifold point. Suppose the probability of a given modulus being *away* from a conifold point is $1-\epsilon$, then the probability of n moduli being away from conifold points should be $(1-\epsilon)^n$, which for $n\epsilon >> 1$ will be small. In this sense, most vacua with many moduli will be near some conifold point; some numbers are given in Hebecker and March-Russell (2006).

Another example is that the vast bulk of an n-parameter CY moduli space is at order one volume (of the CY itself); the fraction which sits at volume greater than Vol falls off as $(\mathrm{Vol})^{-n/3}$ (Denef *et al.*, 2004). This applies for example to the large volume regime we discussed in Sec. V.E.2; it is also relevant for IIb flux vacua in its mirror interpretation.

4. Non-existence arguments

Instead of doing statistics on explicit constructions, another approach to constrain the set of vacua is to find consistency conditions or other *a priori* arguments that vacua with certain properties cannot exist.

Perhaps the best known example is the statement that vacua of string theory cannot have continuous global symmetries (Banks *et al.*, 1988). One argument for this is based on general properties of theories of quantum gravity, specifically the fact that absorption and radiation of particles by black holes will violate these symmetries. A very different argument, from string world-sheet perturbation theory, is that such a symmetry must correspond to a world-sheet conserved current, and such a current can be used to construct a vertex operator for a vector boson gauging the symmetry.

Recently, Arkani-Hamed *et al.* (2006) have proposed that this result can be made quantitative: in any theory of quantum gravity containing a $U(1)$ gauge theory sector, there should be a lower bound on the gauge coupling,

$$g > \frac{m}{M_P},$$

where m is the mass of the lightest charged particle. Besides verifying this in examples, they argue for this by considering entropy bounds on the end states of charged black holes; see also (Banks *et al.*, 2006). It may be that such arguments, using only general features of quantum gravity, can lead to further interesting constraints (Ooguri and Vafa, 2006; Vafa, 2005).

5. Finiteness arguments

In counting vacua, one is implicitly assuming that the number of quasi-realistic vacua of string/M theory is finite. As it is easy to write down effective potentials with an infinite number of local minima, clearly this is a nontrivial hypothesis, which must be checked. Actually, if interpreted too literally, it is probably not true: there are many well established infinite series of compactifications, such as the original Freund-Rubin example of Sec. II.D.1. While the well-established examples are not quasi-realistic, at first one sees no obvious reason that such series cannot exist.

A basic reason to want a finite number of quasi-realistic vacua, under some definition, is that if this is not true, one runs a real risk that the theory can match any set of observations, and in this sense will not be falsifiable. Again, this may not be obvious at first, and one can postulate hypothetical series which would not lead to a problem, or even lead to more definite predictions. Suppose for example that the infinite series had an accumulation point, so that almost all vacua made the same predictions; one might argue that this accumulation point was the preferred prediction (Dvali and Vilenkin, 2004a).

However, the problem which one will face at this point, is that any general mechanism leading to infinite series of vacua in the observable sector, would also be expected to lead to infinite sets of choices in every other sector of the theory, including hidden sectors. Now, while a hidden sector is not directly observable, still all sectors are coupled (at least through gravity; in our considerations through the structure of the moduli space as well), so choices made there do have a small influence on observed physics. For example, the precise values of stabilized moduli in flux vacua, will depend on flux values in the hidden sector. Thus, an infinite-valued choice in this sector, would be expected to lead to a set of vacua which densely populates even the observable sector of theory space, and eliminates any chance for statistical predictions.

This argument comes with loopholes of course; one of the most important is that the measure factor can suppress infinite series. Still, finiteness is one of the most important questions about the distribution of vacua.

Let us consider the example of Sec. II.D.1, in which the flux N can be an arbitrary positive integer. Analogous infinite series exist in its generalizations to the G_2 holonomy and IIa examples of Sec. IV.B, and so on. In all of these series, the compactification volume goes as a positive power of N. Thus, if our definition of "quasi-realistic" includes an upper bound on this volume, these infinite series will not pose a problem. Such a bound follows from Eq. (1) and a phenomenological lower bound on the fundamental scale, say $M_{P,D} > 1\text{TeV}$.

Various arguments have been given that the number of choices arising from a particular part of the problem are finite in this sense: the number of generations (Douglas and Zhou, 2004); the number of IIb flux vacua (Douglas and Lu, 2006; Eguchi and Tachikawa, 2006); the choice of compactification manifold (Acharya and Douglas, 2006), and the choice of brane configuration (Douglas and Taylor, 2006). This rules out postulated infinite series such as that of Dvali and Vilenkin (2004a), as well as various others which have been suggested. However at present there is no completely general argument for finiteness, so this is an important point to check in each new class of models.

F. Interpretation

We come finally to the question of how to use distributions such as Eq. (152) or Eq. (155). One straightforward answer is that that they are useful in guiding the search for explicit vacua. For example, if it appears unlikely that a vacuum of some type exists, one should probably not put a major effort into constructing it.

Going beyond this, distributions give us a useful shortcut to finding explicit vacua with desired properties. As one example, in the explicit construction of Sec. IV.A.3, we needed to assert that IIb flux vacua exist with a specified small upper bound on $|W_0|$. For many purposes, one does not need to know an explicit set of fluxes with this property; a statistical argument that one exists would be enough. The cosmological constant itself is a very important example because, for the reasons we discussed earlier, there is little hope in this picture to find the actual vacua with small Λ.

Going further, it would be nice to know to what extent arguments such as those in Sec. II.F.3 and Sec. V.C could be made precise, and what assumptions we would need to rely on. At first, one may think that such arguments require knowing the measure factor, plunging us into the difficulties of Sec. III.E. However, if the absolute number of vacua is not too large, this is not so; one could get strong predictions which are independent of this. After all, if we make an observation X, and one has a convincing argument that no vacuum reproducing X exists, one has falsified the theory, no matter what the probabilities of the other vacua might be.

These comments may seem a bit general, but when combined with the formalism we just discussed, and under the hypothesis that there are not too many vacua, could have force. Let us return to the problem of the scale of supersymmetry breaking. According to the arguments of Sec. II.E and the distribution results of Sec. V.B, tuning the cosmological constant requires having 10^{120} vacua which, while realizing a discretuum of cosmological constants, are otherwise identical. Now suppose we found only 10^{100} vacua with high scale supersymmetry breaking; since finding the observed c.c. would require an additional 10^{-20} tuning, we would have a good reason to believe that high scale supersymmetry breaking is not just disfavored, but *inconsistent* with string/M theory. While there would be a probabilistic aspect to this claim, it would not be based on unknowns of cosmology or anthropic considerations, but the theoretical approximations which were needed to get a definite result. If this were really the primary source of uncertainty in the claim, one would have a clear path to improving it.

This argument is a simple justification for defining the "stringy naturalness" of a property in terms of the number of string/M theory vacua which realize it, or theoretical approximations to this number. Of course, to the extent that one believes in a particular measure factor or can bring in other considerations, one would prefer a definition which takes these into account; however at present one should probably stick to the simplest version of the idea.

The downside of this type of argument is, not having made additional probabilistic assumptions, if there are "too many" vacua, so that each alternative is represented by at least one vacuum, one gets no predictions at all. How many is "not too many" for this to have any chance of succeeding? A rough first estimate is, fewer than 10^{230}. This comes from multiplying together the observed accuracies of dimensionless couplings, the tuning factors of the dimensionful parameters, and the estimated 10^{-10} difficulty of realizing the Standard Model spectrum. This produces roughly[22] $10^{-70-120-30-10} \sim 10^{-230}$. Neglecting all the further structure in the problem, one might say that, if string/M theory has more than 10^{230} vacua, there is no obvious barrier to reproducing the SM purely statistically, so one should not be able to falsify the theory, on the basis of present data, using statistical reasoning. Conversely, if there are fewer vacua, in principle this might be possible.

The number 10^{230} is a lower bound; if the actual distribution of vacua were highly peaked, or if we were interested in a rare property, we could argue similarly with more vacua. Let us illustrate this by supposing that we find good evidence for a varying fine structure constant. As we discussed in Sec. II.F.2, fitting this would require an effective potential which is almost independent of α_{EM}, and this is highly non-generic; in Banks *et al.* (2002), it was argued that the first 8 coefficients in the series expansion of $V(\alpha_{EM})$ would have to be tuned away. However, in a large enough landscape, even this might happen statistically. Taking the cutoff at a hypothetical $M_{susy} \sim 10\text{TeV}$, this is a tuning factor of order 10^{-600}, so if string/M theory had fewer than 10^{800} or so vacua, such an observation would rule it out with some confidence, while if it had more, we would be less sure.

This is an instructive example, both because the point is clear, and because the stated conclusions taken literally sound absurd. If we really thought the observations required a varying fine structure constant, we would quickly proceed to the hypothesis that the framework we are discussing based on the effective potential is wrong, that there is some other mechanism for adjusting the c.c., or perhaps some mechanism other than varying moduli for varying the apparent fine structure constant. Any such prediction depends on all of the assumptions, including the basic ones, which should be suspected first. However, we can start to see how statistical and/or probabilistic claims of this sort, might unavoidably enter the discussion.

But what if there are 10^{1000} vacua? And what hope is there for estimating the actual number of vacua? All one can say about the second question is that, while there are too many uncertainties to make a convincing estimate at present, we have a fairly good record of eventually answering well-posed formal questions about string/M theory.

Regarding the first question, in this case one probably needs to introduce the measure factor, which will increase the predictivity. This might be quantified by the standard concept of the entropy of a probability distribution,

$$S = \sum_i P_i \ \log(1/P_i).$$

The smaller the entropy, the more concentrated the measure, and the more predictive one expects the theory to be. To some extent, one can repeat the preceding discussion in this context, by everywhere replacing the number of vacua with the total statistical weight e^S. However, justifying this would require addressing the issues raised in Sec. III.E.

There is another reason to call on the measure factor, namely the infinite series of M theory and IIa vacua discussed in Sec. IV.B. Since these run off to large volume, all but a finite number are already ruled out, as discussed in Sec. V.E.5. However, since their number appears to grow with volume, any sort of probabilistic reasoning is likely to lead to the prediction that extra dimensions are just about to be discovered, an optimistic but rather suspicious conclusion.

An alternate hypothesis (Douglas, 2005) is that the correct measure factor suppresses large extra dimensions, which would be true for example if it had a factor $\exp -\text{vol}(M)$. Possible origins for such a factor might be whatever dynamics selects $3+1$ dimensions (some of the many suggestions include Brandenberger and Vafa

[22] The exponent 70 includes α_1 (10), α_2 (6), α_3 (2), m_{proton} (10), m_n (10), and 14 less well measured SM parameters, contributing say 32.

(1989); Easther *et al.* (2005)), or decoherence effects as suggested in Firouzjahi *et al.* (2004).

One cannot go much further in the absence of more definite information about the measure factor. But an important hypothesis to confirm or refute, is that its only important dependence is on the aspects of a vacuum which are important in early cosmology, while for all other aspects one can well approximate it by a uniform measure, in which the probability that one of a set of N similar vacua appears, is taken to be $1/N$.

The former clearly include the scale of inflation and the size of the extra dimensions, and may include other couplings which enter into the physics of inflation and reheating. However, since the physics of inflation must take place at energy scales far above the scales of the Standard Model, most features of the Standard Model, such as the specific gauge group and matter content, the Yukawa couplings, and perhaps the gauge couplings, are probably decorrelated from the measure factor. For such parameters, the uniform measure $P(i) \sim 1/N$ should be a good approximation. Regarding selection effects, we can try to bypass this discussion with the observation that we know that the Standard Model allows for our existence, and we will not consider the question of whether in some other vacuum the probability or number of observers might have been larger.

It is not *a priori* obvious whether a measure factor will depend on two particularly important parameters, the cosmological constant and the supersymmetry breaking scale. As we discussed in Sec. III.E, the cosmological constant does enter into some existing claims, but this leads to its own problems. As for supersymmetry breaking, one might argue that this should fall into the category of physics below the scale of inflation and thus not enter the measure factor, but clearly the importance of this point makes such pat arguments unsatisfying. See Kallosh and Linde (2004) for arguments suggesting a link between these two scales.

Let us conclude by suggesting that, while an understanding of the measure factor is clearly essential to put these arguments on any firm footing, it might turn out that the actual probabilities of vacua are essentially decorrelated from almost all low energy observables, perhaps because they are determined by the high scale physics of eternal inflation, perhaps because they are controlled by the value of the c.c. which is itself decorrelated from other observables, or perhaps for other genericity reasons. In any of these cases, decorrelation and the large number of vacua would justify using the uniform measure, and the style of probabilistic reasoning we used in Sec. II.F.3 would turn out to be appropriate.

VI. CONCLUSIONS

The primary goal of superstring compactification is to find realistic or quasi-realistic models. Real world physics, both the Standard Model and its various well motivated extensions, is rather complicated, and thus it should not be surprising that this goal is taking time to achieve.

Already when the subject was introduced in the mid-1980's, good plausibility arguments were given that the general framework of grand unified theories and low energy supersymmetry could come out of string theory. While there were many gaps in the picture, and some of the most interesting possibilities from a modern point of view were not yet imagined, it seems fair to say that the framework we have discussed in this review is the result of the accumulation of many developments built on that original picture.

In this framework, we discussed how recent developments in flux compactification and superstring duality, along with certain additional assumptions such as the validity of the standard interpretation of the effective potential, allow one to construct models which solve more of the known problems of fundamental physics. Most notably, this includes models with a small positive cosmological constant, but also models of inflation and new models which solve the hierarchy problem.

We emphasize that our discussion rests on assumptions which are by no means beyond question. We have done our best in this review to point out many of these assumptions, so that they can be critically examined. But we would also say that they are not very radical or daring assumptions, but rather simply follow general practice in the study of string compactification, and more generally in particle physics and other areas. Any of them might be false, but in our opinion that would in itself be a significant discovery.

Even assuming that the general framework we have discussed is correct, there are significant gaps in our knowledge of even the most basic facts about the set of string vacua. Our examples were largely based on Calabi-Yau compactification of type II theories, where there are tools inherited from $\mathcal{N} = 2$ supersymmetry that make the calculations particularly tractable. General $\mathcal{N} = 1$ flux vacua in these theories, which involve "geometric flux" (discretely varying away from the Calabi-Yau metric) or even non-geometric compactifications (as briefly discussed in IV.C), are poorly understood. In the heterotic string, Calabi-Yau models do not admit a sufficiently rich spectrum of fluxes to stabilize moduli in a regime of control. The more general Non-Kähler compactifications, which are dual to our type II constructions and should eventually lead to similar moduli potentials, are being intensely investigated as of this writing (Becker *et al.*, 2003a,b, 2006, 2004; Fu and Yau, 2005, 2006; Goldstein and Prokushkin, 2004; Li and Yau, 2004; Lopes Cardoso *et al.*, 2004, 2003). There has been less work on moduli potentials in G_2 compactifications of M-theory, though these also provide a promising home for SUSY GUTs; see *e.g.* Acharya *et al.* (2006a); Acharya (2002); Beasley and Witten (2002).

In fact, these investigations may still be of too limited a scope: in a full survey of models which stabilize moduli, not requiring a strict definition in terms of worldsheet conformal field theory, other limits of string theory such as non-critical strings (Myers, 1987) and their compactifications, should also be explored. There have been some interesting investigations in this direction (Maloney *et al.*, 2002), but as yet little is yet known about the possible phenomenology of these models.

We think many readers will agree that what has emerged has at least answered Pauli's famous criticism of a previous attempt at unification. The picture is strange, perhaps strange enough to be be true. But is it true? That is the question we now face.

Let us briefly recap a few areas in which we might find testable predictions of this framework, as outlined in Sec. II.F. Perhaps the most straightforward application, at least conceptually, is to inflation, as the physics we are discussing determines the structure of the inflationary potential. There are by now many promising inflationary scenarios in string theory, involving brane motion, moduli, or axions as inflatons. In each scenario, however, there are analogues of the infamous eta problem (Copeland *et al.*, 1994), where Planck-suppressed corrections to the inflaton potential spoil flatness and require mild (1 part in 100) tuning to achieve 60 e-foldings. While this may be a small concern relative to other hierarchies we have discussed, it has nevertheless made it difficult to exhibit very explicit inflationary models in string theory. In addition to surmounting these obstacles through explicit calculation in specific examples, it will be important to develop some intuition for which classes of models are most generic; this will involve sorting out the vexing issues of measure that were discussed in III.E. Even lacking this top-down input, clear signatures for some classes of models have been found, via cosmic string production (Copeland *et al.*, 2004; Sarangi and Tye, 2002) or non-Gaussianities of the perturbation spectrum (Alishahiha *et al.*, 2004); perhaps our first clue will come from experiment.

Moduli could in principle lead to observable physics at later times, such as a varying fine structure constant, or quintessence. The first is essentially ruled out, while the second appears even less natural than a small cosmological term, with no comparable "anthropic" motivation.

Implicit in the word "natural," is the fact that many predictions in this framework are inherently statistical, referring to properties of large sets but not all vacua. The statistics of vacua provides precise definitions of "stringy naturalness," which take into account not just values of couplings and the renormalization group, but all of the choices involved in string compactification. This shares some features of "traditional naturalness," but may differ dramatically in others.

In particular, TeV scale supersymmetry is not an inevitable prediction of string/M theory in this framework. While we discussed many of the ingredients which would go into making a well motivated string/M theory prediction, we are not at present taking a position as to what the eventual prediction might be. Conceivably, after much further theoretical development, we might find that TeV scale supersymmetry is disfavored. Of course, a successful prediction that Cern and Fermilab will precisely confirm the Standard Model would be something of a Pyrrhic victory. As physicists, we would clearly be better off with new data and new physics.

For the near term, the main goal here is not really prediction, but rather to broaden the range of theories under discussion, as we will need to keep an open mind in confronting the data. The string phenomenology literature contains many models with TeV scale signatures; as examples inspired by this line of work, we can cite Arkani-Hamed and Dimopoulos (2005); Arkani-Hamed *et al.* (2005b); Giudice and Rattazzi (2006); Giudice and Romanino (2004); Kane *et al.* (2006). In the longer term, a statistical approach may become an important element in bridging the large gap between low energy data and fundamental theory.

We may stand at a crossroads; perhaps much more direct evidence for or against string/M theory will be found before long, making statistical predictions of secondary interest. Or perhaps not; nature has hidden her cards pretty well for the last twenty years, and perhaps we will have to play the odds for some time to come.

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