Geometric Transitions and Dynamical SUSY Breaking

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We show that the physics of D-brane theories that exhibit dynamical SUSY breaking due to stringy instanton effects is well captured by geometric transitions, which recast the non-perturbative superpotential as a classical flux superpotential. This allows for simple engineering of Fayet, Polonyi, O'Raifeartaigh, and other canonical models of supersymmetry breaking in which an exponentially small scale of breaking can be understood either as coming from stringy instantons or as arising from the classical dynamics of fluxes.

1. Introduction

It is of significant interest to find simple examples of dynamical supersymmetry breaking in string theory. One class of examples, where stringy D-instanton effects play a starring role, was described in [1]. These models exhibit "retrofitting" of the classic SUSY breaking theories (Fayet, Polonyi and O'Raifeartaigh) [2], without incorporating any nontrivial gauge dynamics. Instead, stringy instantons [3] automatically implement the exponentially small scale of SUSY breaking in theories with only Abelian gauge fields. A related idea using disc instantons instead of D-instantons appears in [4]. These models are simpler in many ways than their existing field theory analogues [5].

In this paper, we show that these results (and many generalizations) admit a clear and computationally powerful understanding using geometric transition techniques [6](see also [7,8]). Such techniques are well known to translate quantum computations of superpotential interactions in non-trivial gauge theories to classical geometric computations of flux-induced superpotentials [9]. They are most powerful when the theories in question exhibit a mass gap. While the classic models we study *do* manifest light degrees of freedom (and hence do not admit a complete description in terms of geometry and fluxes), we find that a mixed description involving small numbers of D-branes in a flux background – which arises after a geometric transition from a system of branes at a singularity – nicely captures the relevant physics of supersymmetry breaking[1]. In the original theory without flux, the SUSY breaking effects are generated by D-instantons either in $U(1)$ gauge factors or on unoccupied, but orientifolded, nodes of the quiver gauge theory (analogous to those studied in [1,15,16]). Both are in some sense "stringy" effects. Simple generalizations involve more familiar transitions on nodes with large N gauge groups.

The geometric transition techniques we apply have two advantages over the description using stringy instantons in a background without fluxes. First, they allow for a classical computation of the relevant superpotential instead of requiring a nontrivial instanton calculation. Second, they incorporate higher order corrections (due to multi-instanton effects in the original description) which had not been previously calculated.

The organization of this paper is as follows. In section 2, we remind the reader of the relevant background about geometric transitions. In section 3, we discuss the geometries we will use to formulate our DSB theories. In sections 4-6, we give elementary examples

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[1] For an application of geometric transitions to the study of supersymmetry breaking in the context of brane/anti-brane systems see [10-14].

that yield Fayet, Polonyi, and O'Raifeartaigh models that break SUSY at exponentially low scales. In section 7, we present a single geometry that unifies the three models, reducing to them in various limits. In section 8, we provide a more general, exact analysis of the existence of these kinds of susy-breaking effects. In section 9, we give a few other examples of simple DSB theories (related to recent or well known literature in the area). Finally, in section 10, we extend the technology to orientifold models, in particular recovering models which are closely related to the specific examples of [1].

2. Background: Geometric Transitions

Computing non-perturbative corrections in string theory, even to holomorphic quantities such as a superpotential, is in general very difficult. A surprising recent development [6,17] is that in some cases – namely for massive theories – these non-perturbative effects can be determined by perturbative means in a dual language[2].

Consider, for example, N D5 branes in type IIB string theory wrapping an isolated, rigid $\mathbb{P}^1$ in a local Calabi-Yau manifold. In the presence of D5 branes, D1 brane instantons wrapping the $\mathbb{P}^1$ generate a superpotential for the Kähler moduli[3]. The instanton effects are proportional to

$$\exp\left(-\frac{t}{Ng_s}\right)$$

where $t = \int_{S^2}(B^{NS} + ig_s B^{RR})$. For general N, these D1 brane instantons are gauge theory instantons. More precisely, they are the fractional $U(N)$ instantons of the low energy $\mathcal{N} = 1$ $U(N)$ gauge theory on the D5 brane. However, on the basis of zero mode counting, one expects that stringy instanton effects are present even for a single D5 brane.

In the absence of D5 branes, the theory has $\mathcal{N} = 2$ supersymmetry, and the Kähler moduli space is unlifted. In that case, the local Calabi-Yau with a rigid $\mathbb{P}^1$ is known to have another phase where the S^2 has shrunk to zero size and has been replaced by a finite S^3. The two branches meet at $t = 0$, where there is a singularity at which the D3 branes wrapping the S^3 become massless.

What happens to this phase transition in the presence of D5 branes? Classically, we can still connect the S^2 to the S^3 side by a geometric transition. The only difference is

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[2] For a two-dimensional example see [18].

[3] This is a slight misnomer, since t is a parameter, and not a dynamical field for a non-compact Calabi-Yau.

that to account for the D5 brane charge, we need there to be N units of RR flux through the S^3,

$$\int_{S^3} H^{RR} = N.$$

Quantum mechanically the effect is more dramatic. In the presence of D5 branes there is no sharp phase transition at all between the S^2 and the S^3 sides; the interpolation between them is completely *smooth*. As a consequence, the two sides of the transition provide *dual* descriptions of the same physics. Since the theory is massive now, the interpolation occurs by varying the coupling constants of the theory. The fact that the singularity where the S^3 shrinks to zero size is eliminated is consistent with the fact that D3 branes wrapping an S^3 with RR flux through it are infinitely massive. The most direct proof of the absence of a phase transition is in the context of M-theory on a G_2 holonomy manifold [19,20,21]. This is related to the present transition by mirror symmetry and an M-theory lift. In M-theory, the transition is analogous to a perturbative flop transition of type IIA string theory at the conifold, except that in M-theory the classical geometry gets corrected by M2 brane instantons instead of worldsheet instantons [19]. The argument that the two sides are connected smoothly is analogous to Witten's argument for the absence of a sharp phase transition in IIA [22]. In both cases, the presence of instantons is crucial for the singularities in the interior of the classical moduli space to be eliminated.

The fact that the two sides of the transition are connected smoothly implies that the superpotentials have to be the same. The instanton-generated superpotential has a dual description on the S^3 side as a *perturbative* superpotential generated by fluxes. The flux superpotential

$$\mathcal{W} = \int H \wedge \Omega$$

is perturbative, given by

$$\mathcal{W} = \frac{t}{g_s} S + N \, \partial_S \mathcal{F}_0 \tag{2.1}$$

where $\mathcal{F}_0(S)$ is the prepotential of the Calabi-Yau, and

$$S = \int_{S^3} \Omega.$$

The first term in (2.1) comes from the running of the gauge coupling t/g_s which implies that there is an H^{NS} flux turned on on the Calabi-Yau through a 3-chain on the S^2 side.

This three-chain becomes the non-compact 3-cycle dual to the S^3 after the transition. Near the conifold

$$\partial_S \mathcal{F}_0 = S \left(\log \left(\frac{S}{\Delta^3} \right) - 1 \right) + \dots$$

where the omitted terms are a model dependent power series in S, and Δ is a high scale at which t is defined. Integrating out S in favor of t, the superpotential $\mathcal{W}$ becomes

$$\mathcal{W}_{inst} = -\Delta^3 exp(-\frac{t}{Ng_s}) + \dots$$

up to two and higher order instanton terms that depend on the power series in $\mathcal{F}_0(S)$. The duality should persist even in the presence of other branes and fluxes, as long as the S^2 that the branes wrap remains isolated, and the geometry near the branes is unaffected. As we'll discuss in section 10, this can also be extended to D5 branes wrapping $\mathbb{P}^1$'s in Calabi-Yau orientifolds.

3. The Theories

To construct the models in question, we will consider type IIB on non-compact Calabi-Yau 3-folds which are A_r ADE type ALE spaces fibered over the complex plane $\mathbb{C}[x]$. These are described as hypersurfaces in $\mathbb{C}^4$ as follows

$$uv = \prod_{i=1}^{r+1} (z - z_i(x)). \tag{3.1}$$

This geometry is singular at points where $u, v = 0$ and $z_i(x) = z_j(x) = z$. At these points, there are vanishing size $\mathbb{P}^1$'s which can be blown up by deforming the Kähler parameters of the Calabi-Yau. There are r 2-cycle classes, which we will denote

$$S_i^2.$$

These correspond to the blow-ups of the singularities at $z_i = z_{i+1}$, $i = 1, \dots r$. It is upon these $\mathbb{P}^1$'s that we wrap D5 branes to engineer our gauge theories.

The theory on the branes can be thought of as an $\mathcal{N} = 2$ theory, corresponding to D5 branes wrapping 2-cycles of the ALE space, which is then deformed to an $\mathcal{N} = 1$ theory by superpotentials for the adjoints. For the branes on S_i^2 this superpotential is denoted $\mathcal{W}_i(\Phi_i)$. The adjoint Φ_i describes the positions of the branes in the x-direction, and the

superpotential arises because the ALE space is fibered nontrivially over the x plane. The superpotential can be computed by integrating [23,24]

$$\mathcal{W} = \int_{\mathcal{C}} \Omega$$

over a 3-chain with one boundary as the wrapped S^2. In this particular geometry, it takes an extra simple form (the details of the computation appear in appendix A)

$$\mathcal{W}_i(x) = \int (z_i(x) - z_{i+1}(x)) dx. \tag{3.2}$$

In addition to the adjoints, for each intersecting pair of two-cycles S_i^2, S_{i+1}^2 there is a bifundamental hypermultiplet at the intersection, consisting of chiral multiplets $Q_{i,i+1}$ and $Q_{i+1,1}$, with a superpotential interaction inherited from the $\mathcal{N} = 2$ theory

$$\mathrm{Tr}(Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i,i+1}Q_{i+1,i}\Phi_i).$$

Classically, the vacua of the theory correspond to the different ways of distributing branes on the minimal $\mathbb{P}^1$'s in the geometry [25]. When one of the nodes is massive, the instantons corresponding to D1 branes wrapping the S^2 can be summed up in the dual geometry after a geometric transition. As explained in [1], and as we'll see in the next section, this can trigger supersymmetry breaking in the rest of the system.

As an aside, we note that the systems we are studying are a slight generalization of those described in [15,1]. Those geometries are related to the family of geometries studied here, but correspond to particular points in the parameter space where the adjoint masses have been taken to be large and the branes and/or O-planes have been taken to coincide in the x-plane. In addition, we allow the possibility of $U(1)$ (or in some cases higher rank) gauge groups on the transitioning node, whereas in [15,1] the instanton effects were associated with nodes that were only occupied by O-planes. Nevertheless, we will find the same qualitative physics as in [1] in this broader class of theories.

4. The Fayet Geometry

We now turn to the specific geometry which will engineer the Fayet model at low energies. This will be an A_3 geometry, and (3.1) can be written explicitly as

$$uv = (z - mx)(z + mx)(z - mx)(z + m(x - 2a)). \tag{4.1}$$

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After blowing up, we wrap M branes each on S_1^2 at $z_1(x) = z_2(x)$, on S_2^2 at $z_2(x) = z_3(x)$ and one brane on S_3^2 at $z_3(x) = z_4(x)$. The tree-level superpotential (3.2) is now given by

$$W = \sum_{i=1}^{3} \mathcal{W}_i(\Phi_i) + \mathrm{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathrm{Tr}(Q_{23}\Phi_3 Q_{32} - Q_{32}\Phi_2 Q_{23}) \qquad (4.2)$$

where

$$\mathcal{W}_1(\Phi_1) = m\Phi_1^2, \qquad \mathcal{W}_2(\Phi_2) = -m\Phi_2^2, \qquad \mathcal{W}_3(\Phi_3) = m(\Phi_3 - a)^2.$$
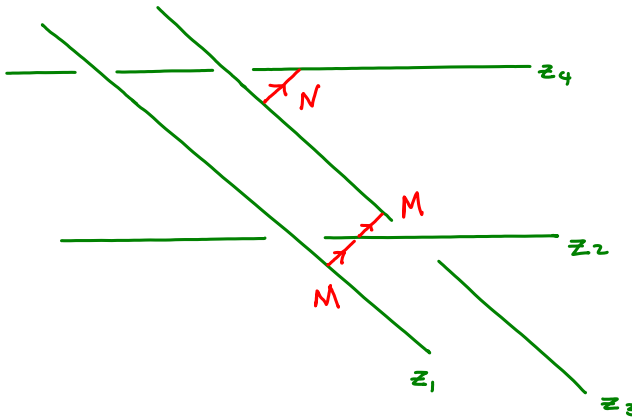

Fig. 1. The A_3 geometry used for retrofitting the Fayet model, *before* the geometric transition. The red lines represent the $\mathbb{P}^1$'s, wrapped by D5 branes. The third node does not intersect the other two and is massive. The geometry after the transition sums up the corresponding instantons. For $N = 1$ branes on S_3^2, the instantons are stringy. For $N > 1$, these are fractional instantons associated with gaugino condensation in the pure $U(N)$ $\mathcal{N} = 1$ gauge theory on that node.

The branes on nodes one and two intersect, since both of the corresponding $\mathbb{P}^1$'s are at $x = 0$. However, the third node, and the single brane on it, is isolated at $x = a$, and the theory living on it is massive. Correspondingly, the the instantons effects due to D-instantons wrapping the third node can be summed up in a dual geometry where we trade S_3^2 for a three-cycle S^3 with one unit of flux through it

$$\int_{S^3} H^{RR} = 1 \ .$$

The geometry after the transition is described by the deformed equation

$$uv = (z - mx)(z + mx)((z - mx)(z + m(x - 2a)) - s) \qquad (4.3)$$

where the size of the S^3

$$\int_{S^3} \Omega = S$$

is given by $S = s/m$. It is fixed to be exponentially small by the flux superpotential, as we shall see shortly. The third brane is gone now, and so are the fields Q_{23}, Q_{32} and Φ_3. The effective superpotential can now be written to leading order in S as

$$\mathcal{W}_{eff} = \mathcal{W}_1(\Phi_1) + \tilde{W}_2(\Phi_2, S) + \text{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathcal{W}_{flux}(S).$$

In this geometry, the exact flux superpotential is

$$\mathcal{W}_{flux} = \frac{t}{g_s} S + S\left(\log \frac{S}{\Delta^3} - 1\right)$$

without any polynomial corrections in S. It is crucial here that the superpotential for Φ_2 has changed due to the change in the geometry to $\tilde{\mathcal{W}}_2(\Phi_2)$, where

$$\tilde{\mathcal{W}}_2(x) = \int (z_2(x) - \tilde{z}_3(x))dx,$$

while the superpotential for Φ_1 is unaffected. We have defined

$$(z - \tilde{z}_3(x))(z - \tilde{z}_4(x)) = (z - z_3(x))(z - z_4(x)) - s$$

with $\tilde{z}_3(x)$ being the branch which asymptotically looks like $z_3(x)$ at large values of x. In other words,

$$\tilde{\mathcal{W}}_2(x) = \int_\Delta^x (-m(x' + a) - \sqrt{m^2(x' - a)^2 + s})dx'.$$

This superpotential sums up the instanton effects due to Euclidean branes wrapping node three.

Before the transition, the vacuum was at $\Phi_2 = 0$. At the end of the day, we expect it to be perturbed by exponentially small terms $\sim S$, so the relevant part of the superpotential is

$$\tilde{W}_2(\Phi_2) = -m\text{Tr}\Phi_2^2 - \frac{1}{2}S\,\text{Tr}\log\frac{a - \Phi_2}{\Delta} + \dots. \tag{4.4}$$

where we've omitted terms of order S^2 and higher and dropped an irrelevant constant. We comment on the form of these corrections in appendix B.

The theories on nodes one and two are asymptotically free. If the fields S and $\Phi_{1,2}$ have very large masses, we can integrate them out and keep only the light degrees of freedom. Keeping only the leading instanton corrections, the relevant F-terms are

$$F_{\Phi_1} = 2m\Phi_1 - Q_{12}Q_{21}$$

$$F_{\Phi_2} = -2m\Phi_2 + Q_{21}Q_{12} + \frac{S}{2(a - \Phi_2)} \tag{4.5}$$

$$F_S = t/g_s + \log S/\Delta^3 - \frac{1}{2}\text{Tr}\log(a - \Phi_2)/\Delta$$

Setting these to zero, we obtain

$$S_* = \Delta^3 \exp(-\frac{\tilde{t}}{g_s}) + \dots$$

where

$$\tilde{t} = t - \frac{1}{2}Mg_s\log(a/\Delta)$$

and

$$\Phi_{1,*} = -\frac{1}{2m}Q_{12}Q_{21}$$

$$\Phi_{2,*} = \frac{1}{2m}Q_{21}Q_{12} + \frac{1}{4ma}S_* + \dots. \tag{4.6}$$

The omitted terms are higher order in $Q_{21}Q_{12}/ma$ and $\exp(-\frac{\tilde{t}}{g_s})$. The low energy, effective superpotential is

$$\mathcal{W}_{eff} = \frac{1}{m}\text{Tr}(Q_{12}Q_{21}Q_{12}Q_{21}) - \frac{S_*}{4ma}\text{Tr}Q_{12}Q_{21} + \dots$$

where we have neglected corrections to the quartic coupling, and the higher order couplings of $Q's$, all of which are exponentially suppressed. As shown in [1], in the presence of a generic FI term for the off-diagonal $U(1)$ under which Q_{12} and Q_{21} are charged,

$$D = Q_{12}Q_{12}^\dagger - Q_{21}^\dagger Q_{21} - r,$$

the exponentially small mass for Q will trigger F-term supersymmetry breaking with an exponentially low scale; we can put $Q_{12,*} = \sqrt{r}$, and then

$$F_{Q_{21}} \sim \frac{\sqrt{r}}{4ma}S_* \ .$$

Geometrically, turning the FI term corresponds to choosing the central charges of the branes on the two nodes to be miss-aligned. Combined with the fact that the nodes one

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and two have become massive with an exponentially low mass, this provides an extremely simple mechanism of breaking supersymmetry at a low scale. The non-supersymmetric vacuum we found classically is reliable, as long as the scale of supersymmetry breaking is far above the strong coupling scales of the $U(M) \times U(M)$ gauge theory. Had we taken N branes on the massive node instead of one, the story would have been the same, apart from the fact that the flux increases, and correspondingly the vacuum value of S changes to $S_* \sim \Delta^3 \exp(-\frac{\tilde{t}}{Ng_s})$. In this case however, the instantons that trigger supersymmetry breaking are the fractional $U(N)$ instantons.

5. The Polonyi Model

In this section we construct the Polonyi model with an exponentially small linear superpotential term for a chiral superfield Φ. This will turn out to be somewhat more subtle, and the existence of the (meta)stable vacuum will depend sensitively on the Kähler potential. We describe specific cases where we know the relevant Kähler potential does yield a stable vacuum in section 7.

Consider an A_2 geometry given by

$$uv = (z - mx)(z - mx)(z + m(x - 2a)) \tag{5.1}$$

which has one D5-brane wrapped on the S_1^2 blowing up $z_1(x) = z_2(x)$, and one D5-brane wrapped on the S_2^2 blowing up $z_2(x) = z_3(x)$. This system has a tree-level superpotential

$$\mathcal{W} = \mathcal{W}_1(\Phi_1) + \mathcal{W}_2(\Phi_2) + Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}. \tag{5.2}$$

where

$$\mathcal{W}_1(\Phi_1) = 0, \qquad \mathcal{W}_2(\Phi_2) = m(\Phi_2 - a)^2$$

This theory has a classical moduli space of vacua parameterized by the expectation value of Φ_1 and where $Q_{12,*} = 0 = Q_{21,*}$, and $\Phi_{2,*} = a$.

At a generic point in the moduli space, away from $\Phi_1 = a$, the theory on the branes wrapping S_2^2 is massive. Then, the instanton effects associated with D1 branes wrapping this node can be summed up by a geometric transition, that replaces S_2^2 by an S^3 with one unit of flux through it. This deforms the Calabi-Yau geometry to

$$uv = (z - mx)((z - mx)(z + m(x - 2a)) - s).$$

which has an S^3 of size

$$\int_{S^3} \Omega = S$$

where $S = s/m$. With this deformation, the superpotential for node 1 is altered as well:

$$\tilde{\mathcal{W}}_1(x) = \int (-m(a-x) + \sqrt{m^2(a-x)^2 + s})dx.$$

The effective superpotential after the transition is simply

$$\mathcal{W}_{eff} = \tilde{W}_1(\Phi_1, S) + \mathcal{W}_{flux}(S)$$

where the flux superpotential has the simple form:

$$\mathcal{W}_{flux}(S) = \frac{t}{g_s} S + S(\log S/\Delta^3 - 1)$$

Note that there is no supersymmetric vacuum, since $F_{\Phi_1} \neq 0$ always.

Suppose at a point in the moduli space, centered say at $\Phi_1 = 0$, the Kähler potential takes the form

$$K = |\Phi_1|^2 + c|\Phi_1|^4 + \ldots$$

where the higher order terms are suppressed by a characteristic mass scale (which we set to one). Then, provided:

$$|ca^2| \gg 1, \qquad c < 0,$$

the theory has a non-supersymmetric vacuum at

$$\Phi_{1,*} = \frac{1}{ca^*}, \tag{5.3}$$

which breaks SUSY at an exponentially low scale.

This can be seen as follows. Expanded about small Φ_1, the superpotential $\tilde{W}_1$ takes the form

$$\tilde{\mathcal{W}}_1(\Phi_1) = -\frac{S}{2} \log(a - \Phi_1)/\Delta + \ldots$$

where the subleading terms are suppressed by additional powers of S, but are otherwise regular at the origin of Φ_1 space. Integrating out S first, by solving its F term constraint, we find

$$S_* = \Delta^3 exp(-\tilde{t}/g_s) + \ldots$$

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where

$$\tilde{t} = t - \tfrac{1}{2}g_s \log(a/\Delta)$$

and the subleading terms are of order Φ_1/a which will turn out to be small in the vacuum. For large $\tilde{t}$, S is generically very massive, so integrating it out is justified.

The potential for Φ_1 now becomes

$$V_{eff}(\Phi_1) = \frac{1}{1 + c|\Phi_1|^2} \frac{|S_*|^2}{|a - \Phi_1|^2} + \dots$$

It is easy to see that, up to corrections of order $1/|a^2 c|$ and $S_*/(ma^2)$, this has a non-supersymmetric vacuum at (5.3) where Φ_1 has a mass squared of order

$$-c|\frac{S_*}{a}|^2.$$

This is positive, and the vacuum is (meta)stable, as long as $c < 0$. Note that we could have obtained the Polonyi model as a limit of the Fayet model where we turn on a very large FI term for the off-diagonal gauge group of nodes one and two. In this case, the stability of the Fayet model for a generic (effectively canonical) Kähler potential guarantees that the Polonyi model obtained from it is stable. In fact [1], as we'll review in section 7 , one can show this directly by computing the relevant correction to the Kähler potential, arising from loops of massive gauge bosons.

6. An O'Raifeartaigh model

To represent the third simple classic class of SUSY breaking models, we engineer an O'Raifeartaigh model. Consider the A_3 fibration with

$$z_1(x) = mx, \ z_2(x) = mx, \ z_3(x) = mx, \ z_4(x) = -m(x - 2a) \ . \tag{6.1}$$

The defining equation of the non-compact Calabi-Yau is then

$$uv = (z - mx)(z - mx)(z - mx)(z + m(x - 2a)) \ . \tag{6.2}$$

We wrap 1 D5 brane on each of $S^2_{1,2,3}$. The adjoints Φ_1 and Φ_2 are massless, while Φ_3 obtains a mass from its superpotential

$$\mathcal{W}_3(x) = \int (z_3(x) - z_4(x)) \ dx \tag{6.3}$$

which gives

$$\mathcal{W}_3(\Phi_3) = m(\Phi_3 - a)^2 \ .$$

Of course, there are also quarks Q_{12}, Q_{21} and Q_{23}, Q_{32}. They couple via the superpotential couplings

$$Q_{12}\Phi_1 Q_{21} - Q_{12}\Phi_2 Q_{21} + Q_{23}\Phi_2 Q_{32} - Q_{23}\Phi_3 Q_{32} \ . \tag{6.4}$$

Because Φ_3 is locked at a, for generic values of Φ_2, Q_{23} and Q_{32} are massive. Then node 3 is entirely massive, and we can perform a geometric transition.

The resulting theory has a new "glueball superfield" S, and effective superpotential

$$\mathcal{W}_{eff} = Q_{12}\Phi_1 Q_{21} - Q_{12}\Phi_2 Q_{21} - \frac{1}{2}S\log(a - \Phi_2)/\Delta + S(\log(S/\Delta^3) - 1) + \frac{t}{g_s}S + \ldots \tag{6.5}$$

Integrating out the S field yields (at leading order)

$$S_* = \Delta^3 e^{-\tilde{t}/g_s} \ . \tag{6.6}$$

where

$$\tilde{t} = t + \frac{1}{2}g_s \log(a/\Delta).$$

Plugging this into the superpotential yields:

$$\mathcal{W}_{eff} = Q_{12}\Phi_1 Q_{21} - Q_{12}\Phi_2 Q_{21} - \frac{1}{2}S_*\Phi_2/a \ + \ldots \tag{6.7}$$

The omitted terms are suppressed by more powers of Φ_2/a. We recognize (6.7) as the superpotential for an O'Raifeartaigh model, very similar to the one considered in [1]. We see that setting $F_{\Phi_1} = F_{\Phi_2} = 0$ is impossible, so one obtains F-term supersymmetry breaking, with a small scale set by $\Delta e^{-t/3g_s}$.

The stability of the non-supersymmetric vacuum again depends on the form of (technically) irrelevant corrections to the Kähler potential. As in the case of Polonyi model, corrections which yield a stable vacuum can be arranged by embedding the model in a slightly larger theory. We'll turn to this in the next section.

7. A Master Geometry

It is possible to construct one configuration of branes on an A_4 geometry which in appropriate limits can be made to reduce to any of the three simple models discussed in the previous sections. The geometry is described by the defining equation

$$uv = (z - mx)(z - mx)(z + mx)(z - mx)(z + m(x - 2a)) \tag{7.1}$$

which has superpotential given by

$$\mathcal{W}_{master} = \sum_{i=1}^{4} \mathcal{W}_i(\Phi_i) + \sum_{i=1}^{3} \text{Tr}(Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i+1,i}\Phi_i Q_{i,i+1}). \tag{7.2}$$

where we wrap N branes on nodes one, two and three, and a single brane on node four. The superpotentials for the adjoints are given by

$$\mathcal{W}_1(\Phi_1) = 0, \quad \mathcal{W}_2(\Phi_2) = -m\text{Tr}(\Phi_2^2), \quad \mathcal{W}_3(\Phi_3) = m\text{Tr}(\Phi_3^2), \quad \mathcal{W}_4(\Phi_4) = -m(\Phi_4 - a)^2 \ .$$

For simplicity of the discussion, we'll set $N = 1$ in this section. The non-abelian generalization is immediate, since all the nodes are asymptotically free (for large adjoint masses). As long as the scale of supersymmetry breaking driven by the geometric transition is high enough, we can ignore the non-abelian gauge dynamics on the other nodes.
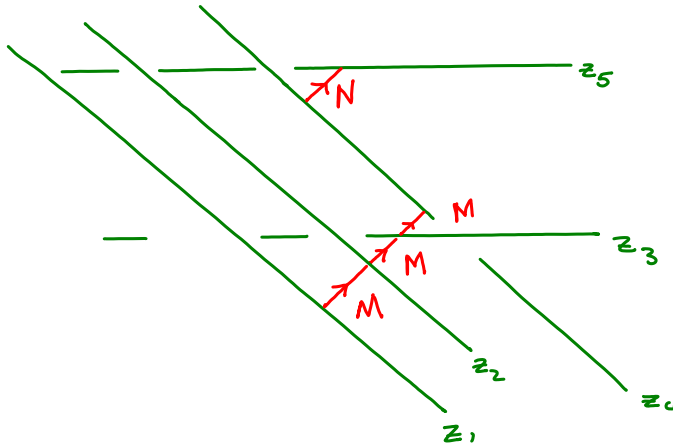

Fig. 2. The master A_4 geometry that gives rise to Fayet, Polonyi and O'Raifeartaigh models by turning on suitable FI terms. The stringy instantons associated with the massive fourth node generate the non-perturbative superpotential that triggers dynamical supersymmetry breaking in the rest of the theory.

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The master theory has a metastable non-supersymmetric vacuum for generic, non-zero FI terms. We can recover all three of the models discussed above by introducing large Fayet-Iliopoulos terms for certain pairs of quarks, so we expect that these will have non-supersymmetric vacua as well. This approach to obtaining the canonical models is particularly useful in the case of Polonyi and O'Raifeartaigh models, for which we needed to assume a particular sign for the subleading correction to the Kähler potential. By obtaining the theories from the master theory, we can compute the corrections to the Kähler potential directly and show that they are of the type required to stabilize the susy-breaking vacua.

To see that the master theory has a metastable non-supersymmetric vacuum, we can proceed as in the Fayet model. Node four is massive, and the corresponding non-perturbative superpotential can be computed in the geometry after transition. The effective superpotential after the transition and integrating out the massive adjoints $\Phi_{2,3}$ is then easily seen to be

$$\mathcal{W}_{eff} = Q_{12}Q_{21}\Phi_1 + \frac{S_*}{4ma}(Q_{23}Q_{32} + \ldots)$$

where we have omitted quartic and higher order terms in the Q's which do not affect the status of the vacuum. With generic FI terms setting

$$|Q_{12}|^2 - |Q_{21}|^2 = r_2, \qquad |Q_{23}|^2 - |Q_{32}|^2 = r_3 \ ,$$

this is easily seen to have an isolated vacuum which breaks supersymmetry.

We'll now show that we can recover all of the the three models studied so far in particular regimes of large FI terms.

7.1. O'Raifeartaigh

We can recover the O'Raifeartaigh construction by turning on a large FI term for Q_{23} and Q_{32} – that is, for the U(1) under which these are the only charged quarks. This generates a D-term

$$D_{O'R} = |Q_{23}|^2 - |Q_{32}|^2 - r_3. \tag{7.3}$$

Taking $r_3 >> 0$, this requires that Q_{23} acquire a large expectation value. Additionally there is an F-term for Q_{32}

$$F_{Q_{32}} = Q_{23}(\Phi_3 - \Phi_2) \tag{7.4}$$

which, in light of the D-term constraint, will set Φ_2 equal to Φ_3. The superpotential then becomes just the O'Raifeartaigh superpotential of the previous section (with certain indices renamed),

$$\mathcal{W}_{O'R} = m(\Phi_4 - a)^2 + Q_{12}Q_{21}(\Phi_2 - \Phi_1) + Q_{24}Q_{42}(\Phi_4 - \Phi_2) \ . \tag{7.5}$$

By performing a geometric transition on the massive node, we recover the superpotential (6.5).

7.2. Fayet

Alternatively, we could have turned on a large FI term for Q_{12} and Q_{21}, generating a D-term

$$D_{Fayet} = |Q_{12}|^2 - |Q_{21}|^2 - r_2. \tag{7.6}$$

In conjunction with the F-term for Q_{21}, by the same process as in the O'Raifeartaigh model, Φ_1 is set equal to Φ_2. This time, the remaining superpotential is given by

$$\mathcal{W}_{Fayet} = m\Phi_2^2 - m\Phi_3^2 + m(\Phi_4 - a)^2 + Q_{23}Q_{32}(\Phi_3 - \Phi_2) + \ldots \tag{7.7}$$

which is precisely the superpotential associated with the Fayet geometry (4.1). Performing a geometric transition on S_4^2, we recover the Fayet model as discussed in section 4.

7.3. Polonyi

From the Fayet model above, before the geometric transition, we can turn turn on another D-term for the quarks Q_{23} and Q_{32}, which along with the F-term for Q_{32} sets $\Phi_2 = \Phi_3$. The superpotential becomes

$$\mathcal{W} = -m\Phi_3^2 + m(\Phi_4 - a)^2 + Q_{34}Q_{43}(\Phi_4 - \Phi_3)$$

which reproduces the Polonyi model of section 5. Again performing the geometric transition on S_4^2 results in the actual Polonyi model.

7.4. The Kähler potential

The O'Raifeartaigh and Polonyi models have flat directions at tree level. As we discussed for e.g. the Polonyi model, the existence of a stable SUSY-breaking vacuum depends on the sign of the leading, quartic correction to the Kähler potential. When we obtain the model as a suitable limit of our master model as above, we can compute this correction and verify explicitly that the vacuum is stable. Let us go through this in some detail. In fact, for simplicity, let's focus on obtaining a stable Polonyi model as a limit of a Fayet model [1].

After the geometric transition in the Fayet model, the effective theory is characterized by a superpotential

$$\mathcal{W} = \frac{S_*}{ma} Q_{23} Q_{32} + \dots \tag{7.8}$$

and D-term

$$D = |Q_{32}|^2 - |Q_{23}|^2 - r_3 \ . \tag{7.9}$$

Here r_3 is the FI term for the $U(1)$ under which only Q_{23} and Q_{32} carry a charge. We can expand this theory about the vev $Q_{23} = \sqrt{r_3}$. Renaming

$$X = Q_{32} \ ,$$

the effective theory then has

$$\mathcal{W} = \frac{S_*}{ma} \sqrt{r_3} X \ . \tag{7.10}$$

To find the Kähler potential for X, we should integrate out the massive $U(1)$ gauge multiplet. What happens to the potential contribution from the D-term, (7.9)? As explained in [26], in the theory with the $U(1)$ gauge field, gauge invariance relates D-term and F-term vevs at any critical point of the scalar potential. When one integrates out the $U(1)$ gauge field, there is a universal quartic correction to the Kähler potential which (using the relation) precisely reproduces the potential contribution from the D-term. For the theory in question, the quartic correction to the Kähler potential for X is just

$$\Delta K = -\frac{g_{U(1)}^2}{M_{U(1)}^2} (X^\dagger X)^2 \ . \tag{7.11}$$

Here $M_{U(1)}$ is the mass of the $U(1)$ gauge boson, $M_{U(1)} \sim g_{U(1)} \sqrt{r_3}$. The result is a quartic correction to K

$$\Delta K = -\frac{1}{r_3} (X^\dagger X)^2 \ . \tag{7.12}$$

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So in the notation of section 5,

$$c = -\frac{1}{r_3}$$

and the sign $c < 0$ results in a stable vacuum, as expected. Plugging in the F-term $F_X \sim \frac{S_*}{ma}\sqrt{r_3}$, (7.12) gives X a mass

$$m_X \sim \frac{S_*}{ma},$$

in agreement with what it was in the full, Fayet model. Note that while one would obtain other quartic couplings in K after integrating out the $U(1)$ gauge boson, they don't play any role. They involve powers of the heavy field Q_{34}, and since $F_{Q_{34}} \ll F_X$, cross-couplings of the form $Q_{34}^\dagger Q_{34} X^\dagger X$ in K do not correct the estimate for the X mass above appreciably.

8. Generalization

We now present a very general argument for the existence of supersymmetry-breaking effects in a class of stringy quiver gauge theories which includes those just discussed. Suppose we have such an A_r quiver theory in which the last node is isolated and undergoes a transition. Note that this is the case in the master geometry considered in the previous section.

In this case, the transition deforms the geometry to the following:

$$uv = \left(\prod_{i=1}^{r-1}(z - z_i(x))\right)((z - z_r(x))(z - z_{r+1}(x)) - s)$$

where in which case the superpotential for the branes on the second-to-last node becomes

$$\tilde{\mathcal{W}}_{r-1}(\Phi_{r-1}) = \int dx(\tilde{z}_r(x) - z_{r-1}(x))$$

where $\tilde{z}_r(x)$ is the solution to the equation

$$(z - z_r(x))(z - z_{r+1}(x)) = s \tag{8.1}$$

which asymptotically approaches $z_r(x)$. We can re-write the superpotential as a correction to the pre-transition superpotential as

$$\tilde{\mathcal{W}}_{r-1}(\Phi_{r-1}) = \int dx(\tilde{z}_r(x) - z_r(x)) + \mathcal{W}_{r-1}(\Phi_{r-1})$$

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and the F-term for Φ_{r-1} and the remaining adjoints are then given by

$$F_{\Phi_{r-1}} = \mathcal{W}'_{r-1}(\Phi_{r-1}) + (\tilde{z}_r(\Phi) - z_r(\Phi)) + Q_{r-1,r}Q_{r,r-1}$$
$$F_{\Phi_i} = \mathcal{W}'_i(\Phi_i) + Q_{i-1,i}Q_{i,i-1} - Q_{i,i+1}Q_{i+1,i}$$
(8.2)

which we can combine to obtain the constraint

$$\sum_i^{r-1} \mathcal{W}'_i(\Phi_i) = z_r(\Phi_{r-1}) - \tilde{z}_r(\Phi_{r-1})$$
(8.3)

Note that the right hand side here cannot vanish for any value of Φ_{r-1} since $z_r(x)$ can never solve (8.1), the solution to which defines $\tilde{z}_r(x)$

If we now consider turning on generic FI terms for the $U(1)$ gauge groups, the D-term constraints will require that, say, the $Q_{i,i+1}$'s acquire vevs while the $Q_{i+1,i}$'s get fixed at zero. The F-terms for the $Q_{i+1,i}$'s will then in turn require

$$\Phi_i = \Phi_j$$

for all i, j. When the brane superpotentials for the first $r - 1$ nodes are of the form

$$\mathcal{W}_i(\Phi_i) = \epsilon_i m \, \Phi_i^2, \qquad i = 1, \ldots r - 1.$$

where $\epsilon_i = 0 \pm 1$, the left hand side of (8.3) vanishes, while the right hand side is strictly non-zero. It is exponentially small, as long as the last node was isolated

$$\mathcal{W}_r(\Phi_r) = m(\Phi_r - a)^2$$

before the transition. This generically triggers low-scale susy breaking.

In terms of the classic models discussed in this paper, one can immediately see that the susy breaking in the Fayet model and in the master geometry can be explained by the above analysis. In the case of the Polonyi and O'Raifeartaigh models, it is even simpler, since the left hand side of (8.3) vanishes *identically* for those models. One could conduct a similar analysis for configurations with more complicated superpotentials and non-generic F-terms on a case-by-case basis. What we see is that often the susy-breaking effects caused by the geometric transition can be understood at an exact level.

9. SUSY breaking by the rank condition

Here, we exhibit models which break supersymmetry due to the "rank condition." This class of models is very similar to those arising in studies of metastable vacua in SUSY QCD [27]. However, we work directly with the analogue of the magnetic dual variables, and the small scale of SUSY breaking is guaranteed by retrofitting [2].

Consider the A_3 fibration with

$$z_1(x) = mx, \qquad z_2(x) = -mx, \qquad z_3(x) = -mx, \qquad z_4(x) = -m(x - 2a) . \qquad (9.1)$$

Then the defining equation is

$$uv = (z - mx)(z + mx)(z + mx)(z + m(x - 2a)) . \qquad (9.2)$$

We choose to wrap $N_f - N_c$ D5 branes on S_1^2, N_f D5 branes on S_2^2, and a single D5 on S_3^2. The tree level superpotential is

$$\mathcal{W} = \sum_{i=1}^{3} \mathcal{W}_i(\Phi_i) + \sum_{i=1}^{2}(Q_{i,i+1}\Phi_{i+1}Q_{i+1,i} - Q_{i+1,i}\Phi_i Q_{i,i+1}). \qquad (9.3)$$

where

$$\mathcal{W}_1(\Phi_1) = m\mathrm{Tr}(\Phi_1)^2, \qquad \mathcal{W}_2(\Phi_2) = 0, \qquad \mathcal{W}_3(\Phi_3) = -m(\Phi_3 - a)^2$$
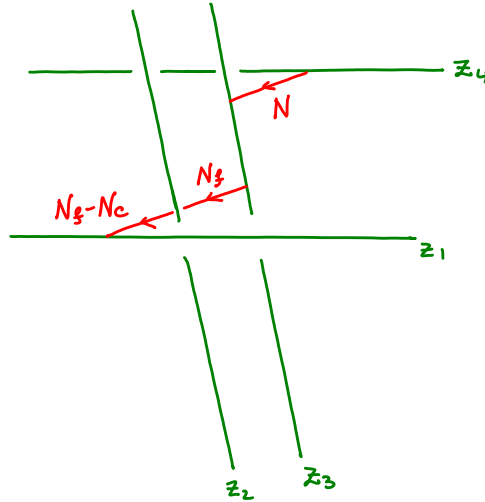

Fig. 3. The (magnetic) A_3 geometry that retrofits the ISS model.

Now, we replace the third ($U(1)$) node with an S^3 with flux, and integrate out Φ_1 trivially (we can take the mass to be very large). The result is:

$$\mathcal{W} = S(\log{(S/Delta^3)} - 1) + \frac{t}{g_s}S - \frac{1}{2}S \, \mathrm{Trlog}(a - \Phi_2)/\Delta - Q_{12}\Phi_2 Q_{21} \ + \dots \qquad (9.4)$$

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where the omitted terms are suppressed by additional powers of S. Integrating out S in a Taylor expansion about $\Phi_2 = 0$, produces a theory with superpotential

$$\mathcal{W} = S_* \mathrm{Tr}\Phi_2/a - \mathrm{Tr}Q_{12}\Phi_2 Q_{21} + \ldots \tag{9.5}$$

where

$$S_* = \Delta^3 exp(-\tilde{t}/g_s) \, , \tag{9.6}$$

and $\tilde{t} = t - N_f \frac{1}{2} g_s \log(a/\Delta)$. Computing F_{Φ_2}, we see that the contribution from the first term in (9.5) has rank N_f, while the contribution from the second term has maximal rank $N_f - N_c < N_f$. The two cannot cancel and SUSY is broken. However, due to the small coefficient of the $\mathrm{Tr}\Phi_2$ term, the breaking occurs at an exponentially small scale.

This model is very similar to the theories analyzed in [27] (for $N_c + 1 \leq N_f < \frac{3}{2}N_c$) and in section 4 of [28]. One difference is that the origin of the small parameter is dynamically explained. The discussion of corrections due to gauging of the $U(N_f)$ factor (which is a global group in [27]) is identical to that in [28] up to a change of notation, and we will not repeat it here. For large a, the higher order corrections to (9.5) (which are suppressed by powers of Φ_2/a) should not destabilize the vacuum at the origin, described in [27,28].

We could also replace the $U(1)$ at node 3 with a $U(N)$ gauge group, still in the same geometry. Then, in (9.4), the coefficient of the $S\log S$ term is changed to N. The only effect, after a geometric transition at node 3, is the replacement of replacement $e^{-t/g_s} \to e^{-t/g_s N}$ in (9.6). This model, where the node upon which we perform the geometric transition has non-Abelian gauge dynamics, is a literal example of the retrofitting constructions of [2]. The Φ_2 field appears in the gauge coupling function of the $U(N)$ gauge group at node 3, because it controls the masses of the quarks Q_{23} and Q_{32} which are charged under $U(N)$. At energies below the quark mass, the $U(N)$ is a pure $\mathcal{N} = 1$ gauge theory and produces a gaugino condensation contribution Λ_N^3 in the superpotential. The standard result for matching the dynamical scale of the low-energy pure $U(N)$ theory to the scale Λ_{N,N_f} of the higher energy theory with N_f quark flavors with mass matrix $\tilde{m}$ is[4]

$$\Lambda_N^{3N} = \Lambda_{N,N_f}^{3N - N_f} \det\tilde{m} \, . \tag{9.7}$$

---

[4] Here, we are assuming the adjoints are very massive $m \to \infty$ and are just matching the QCD theories with quark flavors.

With the identification of S with the gaugino condensate [6]

$$S \sim tr(W_\alpha^2) = \Lambda_N^3,$$

and identifying the mass matrix $\tilde{m} = a - \Phi_2$, we see that we predict

$$S^N = \Lambda_{N,N_f}^{3N-N_f} \det(a - \Phi_2) . \tag{9.8}$$

This is precisely what carefully integrating S out of (9.4) produces, with $\Lambda_{N,N_f}^{3N-N_f} = \Delta^{3N-N_f} e^{-t/g_s}$. So in our model with $N > 1$, the small $\text{Tr}(\Phi_2)$ term in (9.5) can really be thought of as arising from the presence of Φ_2 in the gauge coupling function for the $U(N)$ factor.

10. Orientifold models

In the presence of orientifold 5-planes, we expect D1 brane instantons wrapping 2-cycles that map to themselves to contribute to the superpotential. The D1 brane instanton contributions should again be computable using a geometric transition that shrinks the S^2, and replaces it with an S^3. Geometric transitions with orientifolds have been studied for e.g. in [29,30].

After the transition we generally get 2 different contributions to the superpotential. First, the charge conservation of the D5/O5 brane that disappear after the transition, requires a flux through the S^3 equal to the the amount of brane charge:

$$\mathcal{W}_{flux} = \frac{t}{g_s} S + N_{D5/O5} \partial_S \mathcal{F}_0$$

Second, there can be additional O5 planes that survive as the fixed points of the holomorphic involution after the transition. The O5 planes, just like D5 branes generate a superpotential [31]

$$\mathcal{W}_{O5} = \int_\Sigma \Omega,$$

where the integral is over a three-chain with a boundary on the orientifold plane. The contributions to the superpotential due to O5 planes and RR flux of the orientifold planes are both computed by topological string RP^2 diagrams. The contributions of physical brane charge come from the sphere diagrams.

In this way, geometric transitions can be used to sum up the instanton generated superpotentials in orientifold models. Analogously to our discussion of the previous sections, this can be used for dynamical supersymmetry breaking. We'll discuss in detail the Fayet model below; others can be seen to follow in similar ways.

10.1. The Fayet model

Consider orientifolding the theory from section 3, by combining the worldsheet orientation reversal with an involution I of the Calabi-Yau manifold. For this to preserve the same supersymmetry as the D5 branes, the holomorphic involution I of the Calabi-Yau has to preserve the holomorphic three-form $\Omega = du/udzdx = -dv/vdzdx$.

An example of such an involution is one that takes

$$x \to -x$$

and

$$u \to v, \qquad v \to u$$

A simplest Fayet-type model built on this orientifold is an A_5 geometry that is roughly a doubling of that in section 4:

$$uv = (z - mx)^2(z + mx)^2(z - m(x - 2a))(z + m(x - 2a)).$$

We'll blow this up in a sequence:

$$z_1(x) = mx, \qquad z_2(x) = -m(x - 2a), \qquad z_3(x) = mx,$$
$$z_4(x) = -mx, \qquad z_5(x) = +m(x + 2a), \qquad z_6(x) = -mx,$$

It can be shown that the orientifold projection ends up mapping

$$S_i^2 \to S_{6-i}^2,$$

fixing S_3^2. Consider wrapping M branes on S_i^2 for $i = 1, 2$, and their mirror images, and $2N$ branes on S_3^2. With a particular choice of orientifold projection, the gauge group on the branes is going to be

$$U(M) \times U(M) \times Sp(N)$$

Since the orientifold flips the sign of x, on the fixed node S_3^2 it converts Φ_3 to an adjoint of $Sp(N)$. (Having chosen that the orientifold sends x to minus itself, the action on the rest of the variables is fixed by asking that it preserve the same susy as the D5 branes, *and* that it be a symmetry after blowing up.) In the model at hand, the tree-level superpotential is

$$W = \sum_{i=1}^{3} \mathcal{W}_i(\Phi_i) + \text{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \text{Tr}(Q_{23}\Phi_3 Q_{32} - Q_{32}\Phi_2 Q_{23}).$$

where

$$\mathcal{W}_1(\Phi_1) = m\mathrm{Tr}(\Phi_1 - a)^2, \qquad \mathcal{W}_2(\Phi_2) = -m\mathrm{Tr}(\Phi_2 - a)^2, \qquad \mathcal{W}_3(\Phi_3) = m\mathrm{Tr}\Phi_3^2.$$

Note that, even though the $\mathbb{P}^1$ is fixed by the orientifold action, it is not fixed point-wise. This means there is no $O5^+$ plane charge on it. Instead, there are two $non-compact$ orientifold 5-planes. This model is T-dual [32] to the O6-plane models of [15].

After the geometric transition that shrinks node three and replaces it with an S^3

$$S_3^2 \to S^3$$

the geometry becomes:

$$uv = (z - mx)(z + mx)(z - m(x - 2a))^2(z + m(x - 2a))^2((z - mx)(z + mx) - s).$$

where

$$\int_{S^3} \Omega = S$$

with $S = s/m$. Since the orientation reversal acted freely on the S_3^2, there are only N units of D5 flux through the S^3

$$\int_{S^3} H^{RR} = N$$

which gives a superpotential

$$\mathcal{W}_{flux} = \frac{t}{2g_s}\, S + NS(\log \frac{S}{\Delta^3} - 1)$$

the overall factor of $1/2$ comes from the fact that both the charge on the S^2 and its size has been cut in half by the orientifolding. Above, $t = \int_{S_3^2} k + iB^{RR}$ is the combination of Kahler moduli that survives the orientifold projection. In addition, the two non-compact $O5^+$ planes get pushed through the transition. Because the space still needs 2 blowups to be smooth, to give a precise description of the O5 planes would require using a geometry covered with 4 patches. At the end of the day, effectively, the O5 planes correspond to non-compact curves over the two points on the Riemann surface

$$(z - \tilde{z}_3(x))(z - \tilde{z}_4(x)) = ((z - mx)(z + mx) - s) = 0.$$

located at $x = 0$, and the corresponding values of z, $z_\pm(0)$. They generate a superpotential

$$\mathcal{W}_{O5^+} = \int^{z_-(0)} (\tilde{z}_3 - \tilde{z}_4)dx + \int^{z_+(0)} (\tilde{z}_3 - \tilde{z}_4)dx.$$

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One can show that the contribution of the O5 planes is

$$\mathcal{W}_{O5+} = +S(\log \frac{S}{\Delta^3} - 1)$$

The fact that the RP^2 contribution is proportional to that of the sphere is not an accident. It has been shown generally that the contribution of the O5 planes in these classes of models is $\pm\partial_S \mathcal{F}_{S^2}$ [33,30]. This means that the O5 planes and the fluxes add up to

$$N + 1$$

units of an "effective" flux on the S^3.

After the transition, the branes on node three have disappeared and with them Φ_3 and Q_{23}, Q_{32}. In addition, the deformation of the geometry induces a deformation of the superpotential for node 2:

$$\tilde{\mathcal{W}}_2(x) = \int (z_2(x) - \tilde{z}_3(x)) dx$$

where one picks for $\tilde{z}_3$ the root that asymptotes to $+mx$. This is

$$\tilde{\mathcal{W}}_2(x) = \int (-m(x - 2a) - \sqrt{(mx)^2 + s}) dx,$$

which, when expanded near the vacuum at $x = a$, gives

$$\tilde{\mathcal{W}}_2(\Phi_2) = -\text{Tr}\, m(\Phi_2 - a)^2 - \tfrac{1}{2}S\,\text{Tr}\log(\Phi_2/\Delta) + \ldots$$

The effective superpotential that sums up the instantons is thus

$$\mathcal{W}_{eff} = \mathcal{W}_1(\Phi_1) + \tilde{\mathcal{W}}_2(\Phi_2, S) + \text{Tr}(Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12}) + \mathcal{W}_{flux} + \mathcal{W}_{O5}$$

Up to an overall shift of both $\Phi_{1,2}$ by a, this is the same model as in section 3.

We expect a transition here even when $N = 0$, and there are no D5 branes on the S^2. The transition for $Sp(0)$ is analogous to the transition that occurs for a single D-brane on the S^2, and a $U(1)$ gauge theory. In both cases, the smooth joining of the S^2 and the S^3 phases is due to instantons that correct the geometry. In the orientifold case at hand, it is important to note that, while there is no flux through the S^3, the D3 brane wrapping it is absent: the orientifold projection projects out [34] the $\mathcal{N} = 1$ $U(1)$ vector multiplet associated with the S^3, and with it the D3 brane charged under it.

Picking the other orientifold projection, the $Sp(N)$ gauge group gets replaced with an $SO(2N)$ with Φ_3 becoming the corresponding adjoint. In this case, much of the story remains the same, except that the RP^2 contribution becomes

$$\mathcal{W}_{O5^-} = -S(\log \frac{S}{\Delta^3} - 1).$$

This means that the O5$^-$ planes and the fluxes add up to

$$N - 1$$

units of an "effective" flux on the S^3. This is negative or zero for $N \leq 1$. Naively, the negative effective flux breaks supersymmetry after the transition. This is clearly impossible. It has been argued in [30] that the correct interpretation of this is that in fact $SO(2)$, $SO(1)$ and $SO(0)$ cases do not undergo the geometric transition. This has to correspond to the statement that in these cases there are no D1 brane instantons on node three, and that the classical picture is *exact* in these cases. This translates in the statement that in these cases, in

$$\mathcal{W}_{eff} = \mathcal{W}_{eff}|_{S=0}$$

S should not be extremised, but rather set to zero identically in the effective superpotential.

Note that with the SO projection on the space-filling branes, a D-instanton wrapping the same node enjoys an Sp projection. As discussed in [35,15], in this situation direct zero-mode counting also suggests that the instanton should *not* correct the superpotential. There are more than two fermion zero modes coming from the Ramond sector of strings stretching from the instanton to itself. This is in accord with the results of [30]. In contrast, when one has an Sp projection on the space-filling branes, the instanton receives an SO projection, and the instanton with $SO(1)$ worldvolume gauge group has the correct zero mode count to contribute. The presence of the instanton effects when one has this projection (and their absence when one does not), was also confirmed by direct studies of the renormalization group cascade ending in the appropriate geometry in [15].

Acknowledgements

Appendix A. Brane superpotentials

We can compute the superpotential $\mathcal{W}(\Phi)$ as function of the wrapped 2-cycles Σ by using the superpotential [23,24]

$$\mathcal{W} = \int_{\mathcal{C}} \Omega$$

where C is a three-chain with one boundary being Σ and the other being a reference two-cycle Σ_0 in the same homology class. It is easy to show [23] that the critical points of the superpotential are holomorphic curves. We will evaluate it for the geometries at hand. We can write the holomorphic three-form of the non-compact Calabi-Yau in the usual way,

$$\Omega = \frac{dv \wedge dz \wedge dx}{\frac{dF}{du}} = \frac{dv}{v} \wedge dz \wedge dx. \tag{A.1}$$

Now for fixed values of x and z, the equation for the CY threefold becomes $uv = const$, which is the equation for a cylinder. By shifting the definition of u or v by a phase, we can insist that the constant is purely real, and then by writing $u = x + iy$, $v = x - iy$, the equation can be reformulated as two real equations in terms of the real (x_R, y_R) and imaginary (x_I, y_I) parts of x and y.

$$x_R^2 + y_R^2 = C + x_I^2 + y_I^2, \qquad x_R x_I = y_R y_I. \tag{A.2}$$

The first of these can be solved for any given values of x_I and y_I to give an S^1. The second equation restricts the possible values which we choose for x_I and y_I to a one-dimensional curve in the (x_I, y_I) plane, and so we have the topology of $S^1 \times \mathrm{R}$, where the size of the S^1 degenerates at the points where $z = z_i(x)$ for any i. By simultaneously shifting the phases of u and v according to

$$u \rightarrow e^{i\theta} u$$
$$v \rightarrow e^{-i\theta} v$$

the equation for the cylinder remains unchanged, and we simply rotate about the S^1 factor. We can thus integrate Ω around the circle and obtain

$$\int_{S^1} \Omega = dz \wedge dx$$

up to an overall constant. Now the $\mathbb{P}^1$'s on which we are wrapping the D5 branes are the product of the S^1 just discussed and an interval in the z direction between values where

the S^1 fiber degenerates. Thus, for a given $\mathbb{P}^1$ class in which the vanishing S^1 occurs for $z_i(x)$ and $z_j(x)$, we can integrate $dz \wedge dx$ over the interval in the z-plane and obtain

$$\int_{S^1 \times I_{ij}} \Omega = (z_i(x) - z_j(x))dx.$$

The superpotential for the D-branes then becomes a superpotential for the location of the branes on the t-plane. Defining an arbitrary reference point t_*, we then have

$$\mathcal{W}(x) = \int_{t_*}^{t} (z_i(x) - z_j(x))dx \tag{A.3}$$

Of course, the contribution to the superpotential coming from the limit of integration at t_* is just an arbitrary constant and is not physically relevant. Thus we write (A.3) instead as the indefinite integral

$$\mathcal{W}(x) = \int (z_i(x) - z_j(x))dx. \tag{A.4}$$

Appendix B. Multi-instanton contributions

In this appendix we demonstrate the computation of multi-instanton corrections to the superpotential using the Polonyi model of section 5 as an example. All the information about these corrections is contained in the deformed superpotential for Φ,

$$\tilde{\mathcal{W}}(x) = \int \left(m(x - a) - \sqrt{m^2(x - a)^2 + mS} \right) dx \tag{B.1}$$

along with the flux superpotential[5]

$$\mathcal{W}_{flux} = \frac{t}{g_s}S + S\left(\log \frac{S}{\Delta^3} - 1 \right). \tag{B.2}$$

where the scale Δ is determined by the one-loop contributions to the matrix model free energy. The models considered in this paper are particularly convenient since the purely quadratic superpotential for the massive adjoint at the transition node guarantees that the flux superpotential will be exact at one-loop order in the associated matrix model [17].

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[5] In the case of the Polonyi model these two terms constitute the entire superpotential. In the more general case, however, there will be more fields with superpotential terms, but it will remain the case that only these two contributions play a role in determining instanton corrections.

Extremizing the flux superpotential and expanding in powers of the instanton action

$$S_{inst} \sim \exp(-t/Ng_s),$$

we can determine multi-instanton contributions to a given superpotential term. Summing up the series contributing to a given Φ^k term in will correspond to computing corrections to a fixed, explicit disc diagram, and so we might expect these series to exhibit some integrality properties.

We first expand the deformed superpotential $\mathcal{W}_1(\Phi)$ as a power series in the glueball superfield S,

$$\tilde{\mathcal{W}}(\Phi) = \int \left(m(x-a) - m(x-a)(1 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}n(2n-2)!}{2^{2n-1}(n!)^2} y^n) \right) dx \qquad (B.3)$$

where the expansion parameter y can also be expanded as a power series in x,

$$y = \frac{S}{m(x-a)^2} = \frac{S}{ma^2} \left(1 + \sum_{n=1}^{\infty} (n+1)(-1)^n \left(\frac{x}{a} \right)^n \right). \qquad (B.4)$$

We can integrate (B.3) term by term to obtain an expansion of the effective superpotential in powers of Φ. However, it will be useful to represent this schematically

$$\mathcal{W}_1(\Phi) = c_1 \mathrm{Tr}\Phi + c_2 \mathrm{Tr}\Phi^2 + \ldots \qquad c_i = \sum_{n=1}^{\infty} c_i^{(n)} S^n$$

where the coefficients c_i are themselves written as power series in S. Extremizing the superpotential with respect to S gives an equation for the values of S

$$\log \frac{S}{\Delta^3} = -\frac{t}{g_s} - \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} n \, c_i^{(n)} S^{n-1} \mathrm{Tr}\Phi^i \qquad (B.5)$$

which can be solved perturbatively in powers of S_{inst}. Re-inserting the resulting values into the original superpotential then allows us to read off the instanton-corrected superpotential of the low energy theory up to any given number of instantons. Below we display the linear and quadratic terms at the three-instanton level.

$$\mathcal{W}_{eff} = \mu \mathrm{Tr}\Phi + m \mathrm{Tr}\Phi^2$$

where

$$
\begin{aligned}
\mu &= \frac{1}{2} \frac{\Delta^3}{a} e^{-\frac{t}{g_s}} - \frac{1}{8} \frac{\Delta^6}{ma^3} e^{-\frac{2t}{g_s}} + \frac{1}{16} \frac{\Delta^9}{m^2 a^5} e^{-\frac{3t}{g_s}} + \ldots \\
m &= -\frac{7}{8} \frac{\Delta^3}{a^2} e^{-\frac{t}{g_s}} + \frac{11}{16} \frac{\Delta^6}{ma^4} e^{-\frac{2t}{g_s}} + \frac{1}{32} \frac{\Delta^9}{m^2 a^6} e^{-\frac{3t}{g_s}} + \ldots
\end{aligned}
\qquad (B.6)
$$

It may be interesting to see if there is some way to relate these to the exact formulae for multicovers derived in the resolution of the singularity in hypermultiplet moduli space when a 2-cycle shrinks in IIB string theory, given (up to mirror symmetry) in [36].

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