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Robert W. Hackenburg

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Physics Department

Brookhaven National Laboratory
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

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R. W. Hackenburg*
Physics Department, Brookhaven National Laboratory, Upton, NY 11973
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This report is a collection of detailed calculations that employ dibaryon propagators and vertex operators to obtain various electromagnetic amplitudes in the low-energy $np/d\gamma$ system.

I. PRELIMINARIES

Consider the low energy reactions depicted in Fig. 1. Amplitudes for these reactions are constructed using vertex operators Y_d and $Y_{d_{M1}}$, off-shell deuteron (dibaryon) propagators $\boldsymbol{D}_d$, and initial and final np and γd non-interacting two-particle wavepacket states $|np_i\rangle$, $|\gamma d_i\rangle$, etc. (d may be either t or s for the spin-triplet 1^+ ground state or spin-singlet 0^+ excited state. Y_d and $Y_{d_{M1}}$ are shorthand for Y_{d^*np} and $Y_{d^*d\gamma_{M1}}$, where the lone subscript refers to the off-shell particle.) The baryonic operator Y_d annihilates a deuteron (1^+ or 0^+) and creates a neutron-proton pair, with no change of spin. There are two types of baryonic–$M1$-electromagnetic operators $Y_{d_{M1}}$. The *isovector* operator $Y_{s_{M1}}$ (or $Y_{t_{M1}}$) annihilates a 0^+ excited (or 1^+ ground state) deuteron and creates a photon and a 1^+ (or 0^+) deuteron, with $\Delta J = 1$. The *isoscalar* operator $Y_{t_{M1(0)}}$ annihilates a deuteron and creates a photon and a deuteron in a different orientation, with $\Delta J = 0$, $\Delta M = \pm 1$. The reactions and their amplitudes are:

$$n + p \to d^* \to n + p \qquad \langle np_f|\, Y_d \boldsymbol{D}_d Y_d^\dagger \,|np_i\rangle \qquad (np \text{ elastic}) \tag{1.1}$$

$$n + p \to d^* \to d + \gamma \qquad \langle \gamma d_f|\, Y_{d_{M1}} \boldsymbol{D}_d Y_d^\dagger \,|np_i\rangle \qquad (\text{radiative capture}) \tag{1.2}$$

$$\gamma + d \to d^* \to n + p \qquad \langle np_f|\, Y_d \boldsymbol{D}_d Y_{d_{M1}}^\dagger \,|\gamma d_i\rangle \qquad (\text{photodisintegration}) \tag{1.3}$$

$$\gamma + d \to d^* \to d + \gamma \qquad \langle \gamma d_f|\, Y_{d_{M1}} \boldsymbol{D}_d Y_{d_{M1}}^\dagger \,|\gamma d_i\rangle \qquad (\gamma d \text{ elastic}) \tag{1.4}$$

These amplitudes do not include the phase space. From Ref. [1], the operators Y_d and $Y_d^\dagger$ are both characterized by the same eigenvalue y_d, which includes the vertex-counting factor of two,

$$C_{d^*np} \equiv C_{\frac{1}{2}\frac{1}{2}}(J_{d^*}, M_{d^*}; M_n, M_p)\ , \quad \langle np|\, Y_d \,|d^*\rangle = C_{d^*np}\, y_d\ , \quad \langle d^*|\, Y_d^\dagger \,|np\rangle = C_{d^*np}\, \overline{y}_d\ , \quad M_{d^*} = M_n + M_p\ , \tag{1.5}$$

$$y_d \equiv i2Y_0^0\sqrt{E} = i\sqrt{E/\pi}\ . \tag{1.6}$$

C_{d^*np} is a shorthand notation for the Clebsch-Gordan coefficient, e.g., as defined in Refs. [2] and [3], and $Y_0^0 \equiv 1/\sqrt{4\pi}$. J_{d^*} and M_{d^*} are the total angular momentum and magnetic quantum numbers for the intermediate, off-shell dibaryon.

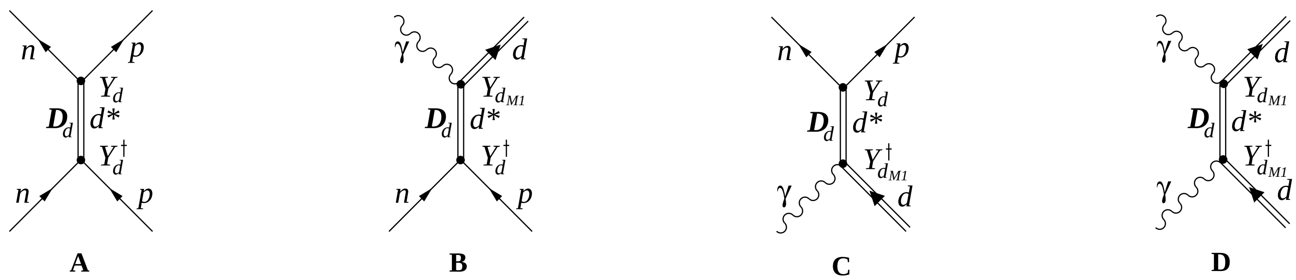

FIG. 1: The uncorrected low-energy s-wave $np/\gamma d$ interactions with an intermediate dibaryon. Each vertex is counted twice, from the number of ways to attach the two initial or two final particles.
A: $np \to d^* \to np$ (elastic) **B**: $np \to d^* \to \gamma d$ (capture) **C**: $\gamma d \to d^* \to np$ (photodisintegration) **D**: $\gamma d \to d^* \to \gamma d$ (elastic)

*hack@bnl.gov

M_n and M_p are the nucleons' magnetic quantum numbers. For a given J_{d^*}, M_{d^*}, the dibaryon propagator is a one-state dyad, thus, from Ref. [1],

$$\boldsymbol{D}_d = |d^*\rangle\, D_d\, \langle d^*| \ , \qquad D_d = i8(-1)^{J-1}/\sqrt{E^2 - m_d^2} \ , \tag{1.7}$$

where m_d is the on-shell deuteron (1^+ ground state or 0^+ excited state) mass. Each npd^* vertex has a correction V_d due to OPE, applied across the np legs, where

$$V_d = (1 - z \pm x)^{-1} \ , \tag{1.8}$$

with the upper sign for the space-symmetric triplet np (V_t) and the lower sign for the space-antisymmetric singlet np (V_s). In Ref. [1], OPE is described in terms of $\pi^\pm$ exchange, through the np exchange operator X and eigenvalue $\pm x$, i.e., $X\,|np\rangle = \pm x\,|np\rangle$; or π^0 exchange, through the np non-exchange operator Z and eigenvalue z, i.e., $Z\,|np\rangle = z\,|np\rangle$. Each d^* propagator has a correction

$$Q_d = [1 + ip(V_d^2 - 1)/\gamma_d]^{-1} \ , \tag{1.9}$$

where p is the np c.m. momentum and γ_d is the scattering wavenumber. Nonrelativistically, $\gamma_d \cong 2m_{np}\epsilon_d$, where m_{np} is the reduced np mass and ϵ_d is the np binding energy of m_d, i.e., $m_d = m_n + m_p - \epsilon_d$. These corrections are to all orders, but ignore non-pionic contributions and the energy dependence of x and z, which is valid at least to a few MeV.

II. ORDINARY $M1$ CAPTURE: DIRECT CONTRIBUTION FROM AN INTERMEDIATE DIBARYON

Consider the magnetic dipole ($M1$) interaction acting directly on dibaryons (Fig. 1). The analog to Eq. (1.5) is

$$C_{d^*\gamma d} \equiv C_{11}(J_{d^*}, M_{d^*}; \gamma, M_d) \ , \ \langle \gamma d|\, Y_{d_{M1}}\, |d^*\rangle = C_{d^*\gamma d}\, y^\gamma_{d_{M1}} \ , \ \langle d^*|\, Y^\dagger_{d_{M1}}\, |\gamma d\rangle = C_{d^*\gamma d}\, \overline{y}^\gamma_{d_{M1}} \ , \ M_{d^*} = M_d + \gamma \ , \tag{2.1}$$

where γ serves double-duty both as the photon-state label in $\langle\gamma d|$ or $C_{d^*\gamma d}$, and as the photon helicity, with $\gamma = \pm 1$ corresponding to right or left circularly-polarized photons. Frequently, γ is used as a sign, i.e., $\gamma = \pm$.

Under the Siegert theorem [4], the baryonic and electromagnetic interactions are separable. If the npd^* vertex operator and the baryonic component of the $d^*d\gamma$ vertex operator are determined by the energy of the off-shell leg, then they are both characterized by the eigenvalue y_d. The $d^*d\gamma$ ($M1$) vertex operator contains an electromagnetic component characterized by the eigenvalue y^γ_{M1}. Following Appendix B in Ref. [2], a plane wave $e^{i\boldsymbol{p}\cdot\boldsymbol{r}}$ describing circularly polarized photons is expanded into a series of spherical vector harmonics $\boldsymbol{X}^M_J(\theta,\phi)$, which are products of a spherical harmonic, a Clebsch-Gordan coefficient ($-\gamma/\sqrt{2}$ – this is not $C_{d^*\gamma d}$), and a spin-vector (polarization) $\boldsymbol{\epsilon}^{(\pm)} = \boldsymbol{\epsilon}^\gamma$. There are only two contributing terms for $M1$, for which $J = 1$, thus,

$$\boldsymbol{X}^\gamma_1 = (-\gamma/\sqrt{2})Y^\gamma_1(\Omega)\boldsymbol{\epsilon}^\gamma = \sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi}\boldsymbol{\epsilon}^\gamma \quad \Rightarrow \quad y^\gamma_{M1} \propto \sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi} \ . \tag{2.2}$$

For $M1$ radiation, the spin of the deuteron defines the Z axis, and the radiation is predominantly in the equatorial plane, i.e., at 90° from the Z axis. Because the intermediate state has no spin, this Z axis has complete 4π freedom-of-choice. By conservation of momentum, the deuteron recoil is exactly opposite to the direction of the photon, in the c.m.

Because the spherical harmonics are contained in y^γ_{M1}, the factor $Y^0_0 = 1/\sqrt{4\pi}$ in y_d must be removed when the interactions described by y_d and y^γ_{M1} are combined, thus,

$$y^\gamma_{d_{M1}} = \sqrt{4\pi}\; y_d\, y^\gamma_{M1} = i2\sqrt{E}\, y^\gamma_{M1} \ . \tag{2.3}$$

The electromagnetic part of the Hamiltonian for a proton in an electromagnetic field is, nonrelativistically [5],

$$H_{\rm em} = (e/2m_p)\boldsymbol{\sigma}\cdot(\boldsymbol{p}_p \times \boldsymbol{A}) + (e/m_p)\boldsymbol{p}_p\cdot\boldsymbol{A} \ ,$$

where $\boldsymbol{\sigma}$ is the Pauli spin operator, e the proton charge, m_p the proton mass, $\boldsymbol{p}_p$ the c.m. momentum of the proton, and $\boldsymbol{A}$ the vector potential. The vector potential is quantized with

$$e\boldsymbol{A} \rightarrow \sqrt{\alpha}\,\boldsymbol{\epsilon}^{(i)} \ , \tag{2.4}$$

FIG. 2: Diagrams for the interaction of a proton with a photon. In both cases $k^\mu + p_p^\mu = p_{p^*}^\mu$.

where $\boldsymbol{\epsilon}^{(i)}$ is the photon polarization vector and α is the electromagnetic coupling[1]. This interaction is represented in Fig. 2, where at least one leg must be off-shell (here, one of the proton legs is off-shell). The on-shell proton momentum is $|\boldsymbol{p}_p| \equiv p_p = \omega$ in the rest frame of the off-shell proton leg, where $\boldsymbol{p}_p = -\boldsymbol{k}$, with $\boldsymbol{k}$ the photon momentum vector, and where the on-shell proton and photon are both incoming or both outgoing. (For one incoming and one outgoing, $\boldsymbol{p}_p = \boldsymbol{k}$.) With $\hat{\boldsymbol{p}} \equiv \boldsymbol{p}_p/\omega$ and Eq. (2.4), the Hamiltonian density is

$$\mathcal{H}_{\rm em} = (1/m_p)\sqrt{\alpha}\,\omega\left[\tfrac{1}{2}\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{p}}\times\boldsymbol{\epsilon}^{(i)}) + \hat{\boldsymbol{p}}\cdot\boldsymbol{\epsilon}^{(i)}\right] .$$

The spin-current term in $\mathcal{H}_{\rm em}$ occurs in the $M1$ interaction. Accounting for the anomalous magnetic moment,

$$\mathcal{H}_{\rm spin} = \mu_p\,\omega\,\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{p}}\times\boldsymbol{\epsilon}^{(i)}) ,$$

where μ_p is the proton magnetic moment. This is a multiple of the nuclear magneton $\mu_{_N} \equiv \sqrt{\alpha}/2m_p$, i.e., $\mu_p = 2\kappa_p\mu_{_N}$. In the s-wave, $\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{p}}\times\boldsymbol{\epsilon}^{(i)})$ can be replaced with 0 or 1, according to the initial and final states considered. With Eq. (2.2),

$$y^\gamma_{M1}(\text{proton}) = \mu_p\,\omega\sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi} . \tag{2.5}$$

The neutron also has a spin-current term, with magnetic moment μ_n. For the neutron,

$$y^\gamma_{M1}(\text{neutron}) = \mu_n\,\omega\sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi} . \tag{2.6}$$

With the isovector magnetic moment $\mu_1 = \mu_p - \mu_n$ for $\Delta J = 1$ (ignoring the d-wave component), the isoscalar magnetic moment $\mu_0 = \mu_p + \mu_n$ for $\Delta J = 0$, and with $\mu_{_{\Delta J}}$ one of μ_1 or μ_0, for an isovector or isoscalar deuteron (or dibaryon),

$$y^\gamma_{M1}(\text{deuteron}) = \mu_{_{\Delta J}}\,\omega\sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi} . \tag{2.7}$$

Then, with Eq. (2.3),

$$y^\gamma_{d_{M1}} = i2\sqrt{E}\,\mu_{_{\Delta J}}\omega\sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi} = i\mu_{_{\Delta J}}\omega\sqrt{3E/4\pi}\,\sin\theta\, e^{i\gamma\phi} . \tag{2.8}$$

In the plane-polarization basis ($\phi = 0°$: "horizontal", $\phi = 90°$: "vertical"),

$$\begin{aligned}
\boldsymbol{X}_{1+} + \boldsymbol{X}_{1-} &= -\sqrt{3/8\pi}\,\sin\theta\left(\sin\phi\;\boldsymbol{\epsilon}^{(\rm h)} + \cos\phi\;\boldsymbol{\epsilon}^{(\rm v)}\right) ,\\
y^{(\rm v/h)}_{M1} &= \mu_{_{\Delta J}}\omega\sqrt{3/16\pi}\,\sin\theta\sqrt{1\pm\cos 2\phi} ,\\
y^{(\rm v/h)}_{d_{M1}} &= i\mu_{_{\Delta J}}\omega\sqrt{3E/4\pi}\,\sin\theta\sqrt{1\pm\cos 2\phi} .
\end{aligned}$$

For ordinary $M1$ capture, the initial state is $|np\rangle = |J,M\rangle = |0,0\rangle$, and the two possible final states are $\langle\gamma t| = \langle\gamma, M_t| = \langle\pm1,\mp1|$, where the magnetic quantum number of the final deuteron is $M_t = -\gamma$. The corrected $\gamma = \pm1$ amplitudes for a direct contribution from an intermediate, off-shell singlet dibaryon are

$$\begin{aligned}
A^\gamma_s &= \langle\gamma t|\,Y_{s_{M1}}\boldsymbol{D}_s Q_s Y_s^\dagger V_s\,|np\rangle = Q_s V_s\,\langle\gamma t|\,Y_{s_{M1}}\,|s^*\rangle\,\boldsymbol{D}_s\,\langle s^*|\,Y_s^\dagger\,|np\rangle = Q_s V_s D_s\,(C_{s^*\gamma t}\,y_{s_{M1}})\,(C_{s^*np}\,\overline{y}_s)\\
&= \mu_1\omega E Q_s V_s D_s \sin\theta e^{i\gamma\phi}/2\pi ,
\end{aligned} \tag{2.9}$$

where $C_{s^*\gamma t} = C_{11}(0,0;\gamma,-\gamma) = 1/\sqrt{3}$, and $C_{s^*np} = 1$ because the np state is expressed in the J,M basis.

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[1] Equation (2.4) often appears with a conventional normalization $1/\sqrt{2\omega}$, where ω is the photon energy [5, 6]. The amplitudes as defined here do not include this normalization, which occurs instead in a covariant phase-space factor [1].

FIG. 3: Capture with off-shell baryons.

III. ORDINARY $M1$ CAPTURE: CONTRIBUTION FROM OFF-SHELL BARYONS

Consider the diagrams in Fig. 3. Let b be either n or p for the neutron or the proton, and let E_{b^*} and p_{b^*} be the off-shell baryon's energy and momentum, which are

$$E_{b^*} = E_b - \omega \ , \qquad p_{b^*} = p - \omega \ , \tag{3.1}$$

with $E_b = \sqrt{m_b^2 + p^2}$, since $p_n = p_p = p$. Then

$$E_{b^*}^2 - p_{b^*}^2 - m_b^2 = -2\omega(E_b - p) \ . \tag{3.2}$$

For a given isospin/spin-state, the baryon propagator is a one-state dyad [1], thus,

$$\boldsymbol{D}_b = |b^*\rangle \, D_b \, \langle b^*| \ , \qquad D_b \cong i2m_b/(E_{b^*}^2 - p_{b^*}^2 - m_b^2) = -im_b/\omega(E_b - p) \ . \tag{3.3}$$

The npd vertex eigenvalues are similar in both Figs. 1 and 3, but with different energies and a different particle off-shell (n^* or p^* instead of d^*), and with a vertex-counting factor of one instead of two. Note that Eq. (1.6) may be written $y_d = i2Y_0^0\sqrt{E_{d^*}}$ because $E_{d^*} = E$. Following Eqs. (1.5) and (1.6), and using Eq. (3.1),

$$C_{dn^*p} \equiv C_{\frac{1}{2}\frac{1}{2}}(J_d, M_d; M_{n^*}, M_p) \ , \quad \langle n^*p|\, Y_{n^*} \,|d\rangle = C_{dn^*p}\, y_{n^*} \ , \quad \langle d|\, Y_{n^*}^\dagger \,|n^*p\rangle = C_{dn^*p}\, \overline{y}_{n^*} \ , \quad M_d = M_{n^*} + M_p \ ,$$
$$C_{dnp^*} \equiv C_{\frac{1}{2}\frac{1}{2}}(J_d, M_d; M_n, M_{p^*}) \ , \quad \langle np^*|\, Y_{p^*} \,|d\rangle = C_{dnp^*}\, y_{p^*} \ , \quad \langle d|\, Y_{p^*}^\dagger \,|np^*\rangle = C_{dnp^*}\, \overline{y}_{p^*} \ , \quad M_d = M_n + M_{p^*} \ ,$$
$$y_b = iY_0^0\sqrt{E_{b^*}} = i\sqrt{(E_b - \omega)/4\pi} \ . \tag{3.4}$$

As with Y_d, etc., Y_n is shorthand for Y_{dn^*p}, etc., where the lone subscript refers to the off-shell particle. The EM vertex eigenvalues are similar to Eq. (2.1), thus,

$$C_{b\gamma b^*} \equiv C_{1\frac{1}{2}}(\tfrac{1}{2}, M_b; \gamma, M_{b^*}) = \gamma\sqrt{2/3} \ , \qquad \langle \gamma b^*|\, Y_{b_{M1}}^\dagger \,|b\rangle = C_{b\gamma b^*}\, \overline{y}_{b_{M1}}^\gamma = \gamma\sqrt{2/3}\, \overline{y}_{b_{M1}}^\gamma \ , \qquad M_b = M_{b^*} + \gamma \ . \tag{3.5}$$

In analogy with Eq. (2.3), with Eqs. (2.5) and (2.6), and μ_b one of μ_p or μ_n,

$$y_{b_{M1}}^\gamma = i\sqrt{(E_b - \omega)/4\pi}\,\sqrt{4\pi}\,\mu_b\omega\sqrt{3/16\pi}\,\sin\theta\, e^{i\gamma\phi} = i\mu_b\omega\sqrt{3(E_b - \omega)/16\pi}\,\sin\theta\, e^{i\gamma\phi} \ . \tag{3.6}$$

The vertex correction V_t is applied to the triplet n^*pt and np^*t vertices, where X and Z operate between the np legs, and the vertex correction V_s is applied to the initial singlet np. (The presence of an internal n^* or p^* leg does not affect the operators X and Z.) For a given $\gamma = \pm 1$, let np be the neutron-proton for the case of the virtual neutron, and let $n'p'$ be the neutron-proton for the case of the virtual proton. (These are the *same* neutron-proton pair, in their two possible singlet spin-states. For quantities other than the spin the prime is omitted, because $E_{n'} = E_n, D_{n'} = D_n$, etc., but note: $\boldsymbol{D}_{n'} \neq \boldsymbol{D}_n$, etc.) For the n^*np contribution, where the neutron undergoes the $M1$ transition,

$$\gamma + M_{n^*} = M_n \ , \ M_t = M_{n^*} + M_p = -\gamma \qquad \Rightarrow \qquad M_p = M_{n^*} = -M_n \ , \ M_n = \tfrac{1}{2}\gamma \ . \tag{3.7}$$

For the $p'^*n'p'$ contribution, where the proton undergoes the $M1$ transition,

$$\gamma + M_{p'^*} = M_{p'} \ , \ M_t = M_{p'^*} + M_{n'} = -\gamma \qquad \Rightarrow \qquad M_{n'} = M_{p'^*} = -M_{p'} \ , \ M_{p'} = \tfrac{1}{2}\gamma \ . \tag{3.8}$$

The initial state may then be written as

$$\tfrac{1}{\sqrt{2}}\left(\left|+\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle - \left|-\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle\right) = \gamma\tfrac{1}{\sqrt{2}}\,|np\rangle - \gamma\tfrac{1}{\sqrt{2}}\,|n'p'\rangle \ . \tag{3.9}$$

FIG. 4: The capture reaction (Fig. 1B) with the $\Delta J = 1$ $M1$ $s^*\gamma t$ vertex expanded to show the virtual baryon contributions. Because the counting factor at the upper s^*np vertex is one, and because the operator Y_s is defined in elastic scattering where the counting factor is two, the operator at that vertex is $\frac{1}{2}Y_s$. The lower nps^* vertex is the same from elastic scattering, and has a counting factor of two. Diagrams with a $t^*\gamma s$ vertex ($\Delta J = 1$) or a $t^*\gamma t$ vertex ($\Delta J = 0$) are similar, with $s^* \to t^*$, $t \to s$, or $s^* \to t^*$, $t \to t$.

By definition of the $np/n'p'$ notation-scheme, $M_{n'} = -M_n$ and $M_{p'} = -M_p$, and only the np part contributes to the n^* transition, and only the $n'p'$ part contributes to the p'^* transition. Note that $C_{s^*np} = \gamma\frac{1}{\sqrt{2}}$ and $C_{s^*n'p'} = -\gamma\frac{1}{\sqrt{2}}$. Then, with Eq. (3.7) and the np part of Eq. (3.9), the n^* amplitude is

$$A_n^\gamma = \langle\gamma t|\, Y_n V_t \boldsymbol{D}_n Y^\dagger_{n_{M1}} V_s \left(\gamma\tfrac{1}{\sqrt{2}}\right)|np\rangle = \gamma\tfrac{1}{\sqrt{2}} V_s V_t \,\langle\gamma|\,\langle t|\, Y_n\, |n^*\rangle\, D_n\, \langle n^*|\, Y^\dagger_{n_{M1}}\, |n\rangle\, |p\rangle$$
$$= \gamma\tfrac{1}{\sqrt{2}} V_s V_t\, \langle t|\, Y_n\, |n^* p\rangle\, D_n\, \langle\gamma n^*|\, Y^\dagger_{n_{M1}}\, |n\rangle = \gamma\tfrac{1}{\sqrt{2}}\, V_s V_t\, y_n D_n \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^\gamma_{n_{M1}} = \tfrac{1}{\sqrt{3}}\, V_s V_t\, y_n D_n\, \overline{y}^\gamma_{n_{M1}}\ , \tag{3.10}$$

where $C_{tn^*p} = C_{tn'p'^*} = 1$ because $M_t = \pm 1$. With Eq. (3.8) and the $n'p'$ part of Eq. (3.9), the p^* amplitude is

$$A_p^\gamma = \langle\gamma t|\, Y_p V_t \boldsymbol{D}_p Y^\dagger_{p_{M1}} V_s \left(-\gamma\tfrac{1}{\sqrt{2}}\right)|n'p'\rangle = -\tfrac{1}{\sqrt{3}}\, V_s V_t\, y_p D_p\, \overline{y}^\gamma_{p_{M1}}\ . \tag{3.11}$$

The combined amplitude is

$$A_{np}^\gamma \equiv A_n^\gamma + A_p^\gamma = \frac{V_s V_t}{\sqrt{3}} \left(y_n D_n\, \overline{y}^\gamma_{n_{M1}} - y_p D_p\, \overline{y}^\gamma_{p_{M1}}\right) = \frac{iV_s V_t}{8\pi}\left(m_n \mu_n \frac{E_n - \omega}{E_n - p} - m_p \mu_p \frac{E_p - \omega}{E_p - p}\right) \sin\theta\, e^{i\gamma\phi}\ . \tag{3.12}$$

At low energies, $m_n \cong E_n \cong E/2$, $m_p \cong E_p \cong E/2$, and $E/2 \gg p, \omega$. Then, with $\mu_1 \cong \mu_p - \mu_n$ and $m_N \equiv (m_n + m_p)/2 \cong E/2$,

$$A_{np}^\gamma \cong \frac{-i\mu_1 V_s V_t m_N}{8\pi} \sin\theta\, e^{i\gamma\phi}\ , \tag{3.13}$$

which reveals the dependence on the isovector magnetic moment μ_1.

IV. ORDINARY $M1$ CAPTURE: EXPANDED $s^*\gamma t$ VERTEX

The $s^*\gamma t$ vertex from Fig. 1B is expanded in Fig. 4 to reveal the virtual baryons. While all three baryons in the loop may be off-shell, the two from the s^* decay are not so-labeled; this is mostly for convenience, but also because these two may be on-shell. The n and p in the expanded vertex are taken to have the same spins as the n and p in the initial state. Then, the n^* (or p^*) has its spin opposite to the initial n (or p). There is an overall sign change when the n and p in the expanded vertex have changed their spins from the initial state, as happens when they are acted on by an odd number of X operators (i.e., $\pi^\pm$ exchange – see Ref. [1]). This is accounted for when the corrections are inserted. The s^*np opening vertex is counted only once, because swapping the n and p gives rise to the n^* and p^* contributions, which are separately accounted for. Therefore, the operator at this s^*np vertex occurs as $\frac{1}{2}Y_s$. The correction V_t is applied to the triplet n^*pt and np^*t vertices, and the correction V_s is applied to both nps^* vertices. The correction Q_s is applied to the singlet dibaryon propagator D_s. There are no corrections applied to the baryon propagators, nor is there a radiative correction applied to the $n^*\gamma n$ or $p^*\gamma p$ vertex. With the $n^*np/p'^*n'p'$ notation of Eqs. (3.8)-(3.9), the amplitudes with an intermediate singlet dibaryon are

$$A_s^\gamma = \langle\gamma t|\, V_t Y_n \boldsymbol{D}_{n^*} Y^\dagger_{n_{M1}} \boldsymbol{D}_n \boldsymbol{D}_p V_s \tfrac{1}{2} Y_s Q_s \boldsymbol{D}_s Y_s^\dagger V_s \left(\gamma\tfrac{1}{\sqrt{2}}\right)|np\rangle + \langle\gamma t|\, V_t Y_p \boldsymbol{D}_{p'^*} Y^\dagger_{p_{M1}} \boldsymbol{D}_{n'} \boldsymbol{D}_{p'} V_s \tfrac{1}{2} Y_s Q_s \boldsymbol{D}_s Y_s^\dagger V_s \left(-\gamma\tfrac{1}{\sqrt{2}}\right)|n'p'\rangle$$

For a particular $\gamma = \pm 1$, the propagators are one-state dyads, thus,

$$\boldsymbol{D}_{n^*} = |n^*\rangle D_{n^*} \langle n^*| \ , \ \boldsymbol{D}_{p'^*} = |p'^*\rangle D_{p^*} \langle p'^*| \ , \ \boldsymbol{D}_n \boldsymbol{D}_p = |n\rangle |p\rangle D_n D_p \langle n| \langle p| \ , \ \boldsymbol{D}_{n'} \boldsymbol{D}_{p'} = |n'\rangle |p'\rangle D_n D_p \langle n'| \langle p'| \ .$$

The asterisk on the subscript of D_{n^*} and D_{p^*} serves to distinguish between the n and n^*, etc., which have different energies. The baryon propagators are all given by Eq. (3.3). Then

$$\begin{aligned}
A_s^\gamma &= \gamma \tfrac{1}{2\sqrt{2}} Q_s V_t V_s^2 \Big(\langle\gamma| \langle t| Y_n |n^*\rangle D_{n^*} \langle n^*| Y^\dagger_{n_{M1}} |n\rangle |p\rangle D_n D_p \langle n| \langle p| Y_s |s^*\rangle D_s \langle s^*| Y_s^\dagger |np\rangle + \\
&\quad - \langle\gamma| \langle t| Y_p |p'^*\rangle D_{p^*} \langle p'^*| Y^\dagger_{p_{M1}} |n'\rangle |p'\rangle D_n D_p \langle n'| \langle p'| Y_s |s^*\rangle D_s \langle s^*| Y_s^\dagger |n'p'\rangle \Big) \\
&= \gamma \tfrac{1}{2\sqrt{2}} Q_s V_t V_s^2 D_s D_n D_p \Big[\langle t| Y_n |n^* p\rangle D_{n^*} \langle \gamma n^*| Y^\dagger_{n_{M1}} |n\rangle \langle np| Y_s |s^*\rangle \left(\gamma \tfrac{1}{\sqrt{2}}\right) \overline{y}_s + \\
&\quad - \langle t| Y_p |n'p'^*\rangle D_{p^*} \langle \gamma p^*| Y^\dagger_{p_{M1}} |p'\rangle \langle n'p'| Y_s |s^*\rangle \left(-\gamma \tfrac{1}{\sqrt{2}}\right) \overline{y}_s \Big] \\
&= \tfrac{1}{4} Q_s V_t V_s^2 D_s D_n D_p \, \overline{y}_s \left[y_n D_{n^*} \left(\gamma \sqrt{\tfrac{2}{3}}\right) \overline{y}^\gamma_{n_{M1}} \left(\gamma \tfrac{1}{\sqrt{2}}\right) y_s + y_p D_{p^*} \left(\gamma \sqrt{\tfrac{2}{3}}\right) \overline{y}^\gamma_{p_{M1}} \left(-\gamma \tfrac{1}{\sqrt{2}}\right) y_s \right] \\
&= \tfrac{1}{4\sqrt{3}} Q_s V_t V_s^2 D_s |y_s|^2 D_n D_p \big(y_n D_{n^*} \overline{y}^\gamma_{n_{M1}} - y_p D_{p^*} \overline{y}^\gamma_{p_{M1}} \big) = \tfrac{1}{4} Q_s V_s D_s |y_s|^2 A^\gamma_{np} D_n D_p \ ,
\end{aligned} \tag{4.1}$$

where $\langle \gamma b^*| Y^\dagger_{b_{M1}} |b\rangle = C_{1\frac{1}{2}}(\tfrac{1}{2}, M_b; \gamma, M_{b^*}) \overline{y}^\gamma_{b_{M1}} = \gamma \sqrt{\tfrac{2}{3}} \, \overline{y}^\gamma_{b_{M1}}$, for $b^* b = n^* n$ or $p'^* p'$, and A^γ_{np} is given by Eq. (3.12), where D_n and D_p correspond to D_{n^*} and D_{p^*} in Eq. (4.1).

V. ISOSCALAR CAPTURE

An initial np in the triplet spin-state may be captured through the $\Delta J = 0$ (isoscalar) operator $Y_{t_{M1(0)}}$. The photon energy is the same as in ordinary capture, but the symmetry of the initial state is different. Consider first the contribution from Fig. 3. For the $n^* np$ contribution, where the neutron undergoes the $M1$ transition,

$$\gamma + M_{n^*} = M_n \qquad \Rightarrow \qquad M_{n^*} = -M_n \ , \ M_n = \tfrac{1}{2}\gamma \ , \ M_t = M_{n^*} + M_p = -\tfrac{1}{2}\gamma + M_p \ . \tag{5.1}$$

For the $p'^* n' p'$ contribution, where the proton undergoes the $M1$ transition,

$$\gamma + M_{p'^*} = M_{p'} \qquad \Rightarrow \qquad M_{p'^*} = -M_{p'} \ , \ M_{p'} = \tfrac{1}{2}\gamma \ , \ M_t = M_{p'^*} + M_{n'} = -\tfrac{1}{2}\gamma + M_{n'} \ , \tag{5.2}$$

From these, it can be seen that the final deuteron must either have $M_t = -\gamma$ for $M = 0$ ($M_n = -M_p$, $M_{n'} = -M_{p'}$) or $M_t = 0$ for $M = \pm 1$ ($M_n = M_p$ – there is no distinction between np and $n'p'$ for $M = \pm 1$). There are three contributing initial states:

$$|1, 0\rangle = \tfrac{1}{\sqrt{2}} \left|+\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle + \tfrac{1}{\sqrt{2}} \left|-\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{2}} |np\rangle + \tfrac{1}{\sqrt{2}} |n'p'\rangle \ , \qquad |1, \pm 1\rangle = \left|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}\right\rangle = |np\rangle \ . \tag{5.3}$$

The amplitudes $A^{\gamma, M_t}_{b(0)} = A^{\gamma, -\gamma}_{b(0)}$ for the $M = 0$ initial states are

$$\begin{aligned}
A^{\gamma,-\gamma}_{n(0)} &= \langle \gamma t| Y_n V_t \boldsymbol{D}_n Y^\dagger_{n_{M1}} V_t \tfrac{1}{\sqrt{2}} |np\rangle = \tfrac{1}{\sqrt{2}} V_t^2 \langle\gamma| \langle t| Y_n |n^*\rangle D_n \langle n^*| Y^\dagger_{n_{M1}} |n\rangle |p\rangle \\
&= \tfrac{1}{\sqrt{2}} V_t^2 \langle t| Y_n |n^* p\rangle D_n \langle \gamma n^*| Y^\dagger_{n_{M1}} |n\rangle = \tfrac{1}{\sqrt{2}} V_t^2 y_n D_n \left(\gamma \sqrt{\tfrac{2}{3}}\right) \overline{y}^\gamma_{n_{M1}} = \gamma \tfrac{1}{\sqrt{3}} V_t^2 y_n D_n \overline{y}^\gamma_{n_{M1}} \ ,
\end{aligned} \tag{5.4}$$

$$\begin{aligned}
A^{\gamma,-\gamma}_{p(0)} &= \langle \gamma t| Y_p V_t \boldsymbol{D}_{p'} Y^\dagger_{p_{M1}} V_t \tfrac{1}{\sqrt{2}} |n'p'\rangle = \tfrac{1}{\sqrt{2}} V_t^2 \langle\gamma| \langle t| Y_p |p'^*\rangle D_p \langle p'^*| Y^\dagger_{p_{M1}} |n'\rangle |p'\rangle \\
&= \tfrac{1}{\sqrt{2}} V_t^2 \langle t| Y_p |n'p'^*\rangle D_p \langle \gamma p'^*| Y^\dagger_{p_{M1}} |p'\rangle = \tfrac{1}{\sqrt{2}} V_t^2 y_p D_p \left(\gamma \sqrt{\tfrac{2}{3}}\right) \overline{y}^\gamma_{p_{M1}} = \gamma \tfrac{1}{\sqrt{3}} V_t^2 y_p D_p \overline{y}^\gamma_{p_{M1}} \ ,
\end{aligned} \tag{5.5}$$

The amplitudes $A^{\gamma, M_t}_{b(0)} = A^{\gamma, 0}_{b(0)}$ for the $M = \pm 1$ initial states are

$$\begin{aligned}
A^{\gamma,0}_{n(0)} &= \langle \gamma t| Y_n V_t \boldsymbol{D}_n Y^\dagger_{n_{M1}} V_t |np\rangle = V_t^2 \langle\gamma| \langle t| Y_n |n^*\rangle D_n \langle n^*| Y^\dagger_{n_{M1}} |n\rangle |p\rangle \\
&= V_t^2 \langle t| Y_n |n^* p\rangle D_n \langle \gamma n^*| Y^\dagger_{n_{M1}} |n\rangle = V_t^2 \left(\tfrac{1}{\sqrt{2}}\right) y_n D_n \left(\gamma \sqrt{\tfrac{2}{3}}\right) \overline{y}^\gamma_{n_{M1}} = \gamma \tfrac{1}{\sqrt{3}} V_t^2 y_n D_n \overline{y}^\gamma_{n_{M1}} \ ,
\end{aligned} \tag{5.6}$$

$$A^{\gamma,0}_{p(0)} = \langle \gamma t| Y_p V_t \boldsymbol{D}_p Y^\dagger_{p_{M1}} V_t |np\rangle = \gamma \tfrac{1}{\sqrt{3}} V_t^2 y_p D_p \overline{y}^\gamma_{p_{M1}} \ . \tag{5.7}$$

Note that there is no change in sign between the n^* and p^* contributions, as for ordinary capture. Define

$$A^{\gamma}_{np(0)} \equiv A^{\gamma,0}_{n(0)} + A^{\gamma,0}_{p(0)} = A^{\gamma,-\gamma}_{n(0)} + A^{\gamma,-\gamma}_{p(0)} = \gamma \tfrac{1}{\sqrt{3}}\; V_t^2 (y_n D_n\, \overline{y}^{\gamma}_{n_{M1}} + y_p D_p\, \overline{y}^{\gamma}_{p_{M1}})\ . \tag{5.8}$$

Like the $s^*\gamma t$ vertex, the $t^*\gamma t$ vertex from Fig. 1B may also be expanded as in Fig. 4 to reveal the virtual baryons. The correction V_t is applied to the triplet n^*pt and np^*t vertices, and to both npt^* vertices. The correction Q_t is applied to the triplet dibaryon propagator D_t. With the $t^*\gamma t$ vertex expanded as shown in Fig. 4, the $\Delta J = 0$ intermediate dibaryon amplitudes for the $M = 0$ ($M_t = -\gamma$) initial states are

$$\begin{aligned}
A^{\gamma,-\gamma}_{t(0)} &= \langle\gamma t|\, V_t Y_n \boldsymbol{D}_{n^*} Y^{\dagger}_{n_{M1}} \boldsymbol{D}_n \boldsymbol{D}_p V_t \tfrac{1}{2} Y_t Q_t \boldsymbol{D}_t Y_t^{\dagger} V_t \tfrac{1}{\sqrt{2}} |np\rangle + \langle\gamma t|\, V_t Y_p \boldsymbol{D}_{p'^*} Y^{\dagger}_{p_{M1}} \boldsymbol{D}_{n'} \boldsymbol{D}_{p'} V_t \tfrac{1}{2} Y_t Q_t \boldsymbol{D}_t Y_t^{\dagger} V_t \tfrac{1}{\sqrt{2}} |n'p'\rangle \\
&= \tfrac{1}{2\sqrt{2}} Q_t V_t^3 \Big(\langle\gamma|\, \langle t|\, Y_n\, |n^*\rangle\, D_{n^*}\, \langle n^*|\, Y^{\dagger}_{n_{M1}}\, |n\rangle\, |p\rangle\, D_n D_p\, \langle n|\, \langle p|\, Y_t\, |t^*\rangle\, D_t\, \langle t^*|\, Y_t^{\dagger}\, |np\rangle + \\
&\quad + \langle\gamma|\, \langle t|\, Y_p\, |p'^*\rangle\, D_{p^*}\, \langle p'^*|\, Y^{\dagger}_{p_{M1}}\, |n'\rangle\, |p'\rangle\, D_n D_p\, \langle n'|\, \langle p'|\, Y_t\, |t^*\rangle\, D_t\, \langle t^*|\, Y_t^{\dagger}\, |np\rangle \Big) \\
&= \tfrac{1}{2\sqrt{2}} Q_t V_t^3 D_t D_n D_p \Big[\langle t|\, Y_n\, |n^* p\rangle\, D_{n^*}\, \langle\gamma n^*|\, Y^{\dagger}_{n_{M1}}\, |n\rangle\, \langle np|\, Y_t\, |t^*\rangle \left(\tfrac{1}{\sqrt{2}}\right) \overline{y}_t + \\
&\quad + \langle t|\, Y_p\, |n'p'^*\rangle\, D_{p^*}\, \langle\gamma p'^*|\, Y^{\dagger}_{p_{M1}}\, |p'\rangle\, \langle n'p'|\, Y_t\, |t^*\rangle \left(\tfrac{1}{\sqrt{2}}\right) \overline{y}_t \Big] \\
&= \tfrac{1}{4} Q_t V_t^3 D_t \overline{y}_t D_n D_p \left[y_n D_{n^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^{\gamma}_{n_{M1}} \left(\tfrac{1}{\sqrt{2}}\right) y_t + y_p D_{p^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^{\gamma}_{p_{M1}} \left(\tfrac{1}{\sqrt{2}}\right) y_t \right] \\
&= \gamma \tfrac{1}{4\sqrt{3}} Q_t V_t^3 D_t |y_t|^2 D_n D_p \left(y_n D_{n^*}\, \overline{y}^{\gamma}_{n_{M1}} + y_p D_{p^*}\, \overline{y}^{\gamma}_{p_{M1}} \right) = \tfrac{1}{4} Q_t V_t D_t |y_t|^2 D_n D_p A^{\gamma}_{np(0)}\ .
\end{aligned} \tag{5.9}$$

The $\Delta J = 0$ intermediate dibaryon amplitudes for the $M = \pm 1$ ($M_t = 0$) initial states are

$$\begin{aligned}
A^{\gamma,0}_{t(0)} &= \langle\gamma t|\, V_t Y_n \boldsymbol{D}_{n^*} Y^{\dagger}_{n_{M1}} \boldsymbol{D}_n \boldsymbol{D}_p V_t \tfrac{1}{2} Y_t Q_t \boldsymbol{D}_t Y_t^{\dagger} V_t\, |np\rangle + \langle\gamma t|\, V_t Y_p \boldsymbol{D}_{p^*} Y^{\dagger}_{p_{M1}} \boldsymbol{D}_n \boldsymbol{D}_p V_t \tfrac{1}{2} Y_t Q_t \boldsymbol{D}_t Y_t^{\dagger} V_t\, |np\rangle \\
&= \tfrac{1}{2} Q_t V_t^3 \Big(\langle\gamma|\, \langle t|\, Y_n\, |n^*\rangle\, D_{n^*}\, \langle n^*|\, Y^{\dagger}_{n_{M1}}\, |n\rangle\, |p\rangle\, D_n D_p\, \langle n|\, \langle p|\, Y_t\, |t^*\rangle\, D_t\, \langle t^*|\, Y_t^{\dagger}\, |np\rangle + \\
&\quad + \langle\gamma|\, \langle t|\, Y_p\, |p^*\rangle\, D_{p^*}\, \langle p^*|\, Y^{\dagger}_{p_{M1}}\, |n\rangle\, |p\rangle\, D_n D_p\, \langle n|\, \langle p|\, Y_t\, |t^*\rangle\, D_t\, \langle t^*|\, Y_t^{\dagger}\, |np\rangle \Big) \\
&= \tfrac{1}{2} Q_t V_t^3 D_t D_n D_p \Big(\langle t|\, Y_n\, |n^* p\rangle\, D_{n^*}\, \langle\gamma n^*|\, Y^{\dagger}_{n_{M1}} |n\rangle\, \langle np|\, Y_t\, |t^*\rangle\, \overline{y}_t + \langle t|\, Y_p\, |np^*\rangle\, D_{p^*}\, \langle\gamma p^*|\, Y^{\dagger}_{p_{M1}} |p\rangle\, \langle np|\, Y_t\, |t^*\rangle\, \overline{y}_t \Big) \\
&= \tfrac{1}{2} Q_t V_t^3 D_t \overline{y}_t D_n D_p \left[\left(\tfrac{1}{\sqrt{2}}\right) y_n D_{n^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^{\gamma}_{n_{M1}} y_t + \left(\tfrac{1}{\sqrt{2}}\right) y_p D_{p^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^{\gamma}_{p_{M1}} y_t \right] \\
&= \gamma \tfrac{1}{2\sqrt{3}}\, Q_t V_t^3 D_t |y_t|^2 D_n D_p \left(y_n D_{n^*}\, \overline{y}^{\gamma}_{n_{M1}} + y_p D_{p^*}\, \overline{y}^{\gamma}_{p_{M1}} \right) = 2 A^{\gamma,-\gamma}_{t(0)}\ .
\end{aligned} \tag{5.10}$$

Because of the occurrence of additional Clebsch-Gordan coefficients in the expanded vertex of Fig. 4, $A^{\gamma,0}_{t(0)} \neq A^{\gamma,-\gamma}_{t(0)}$.

VI. 0^+ LEVEL DECAY

Only the eigenvalues $y^{\gamma}_{s_{M1}}$ of the operator $Y_{s_{M1}}$ are needed, as expanded in Fig. 4. The initial state is the 0^+ level, $|s\rangle = |0,0\rangle$, and the final state is a deuteron and photon $\langle\gamma t|$ in one of two polarization states, with $M_t = -\gamma$. The correction V_t is applied to the triplet n^*pt and np^*t vertices, and the correction V_s is applied to the nps decay vertex. With Eqs. (3.7)-(3.9), the eigenvalues are

$$\begin{aligned}
y^{\gamma}_{s_{M1}} &= \langle\gamma t|\, V_t Y_n \boldsymbol{D}_{n^*} Y^{\dagger}_{n_{M1}} \boldsymbol{D}_n \boldsymbol{D}_p\, \tfrac{1}{2} Y_s V_s\, |s\rangle + \langle\gamma t|\, V_t Y_p \boldsymbol{D}_{p'^*} Y^{\dagger}_{p_{M1}} \boldsymbol{D}_{n'} \boldsymbol{D}_{p'}\, \tfrac{1}{2} Y_s V_s\, |s\rangle \\
&= \tfrac{1}{2} V_t V_s \Big(\langle\gamma|\, \langle t|\, Y_n\, |n^*\rangle\, D_{n^*}\, \langle n^*|\, Y^{\dagger}_{n_{M1}}\, |n\rangle\, |p\rangle\, D_n D_p\, \langle n|\, \langle p|\, Y_s\, |s\rangle \\
&\quad + \langle\gamma|\, \langle t|\, Y_p\, |p'^*\rangle\, D_{p^*}\, \langle p'^*|\, Y^{\dagger}_{p_{M1}}\, |n'\rangle\, |p'\rangle\, D_n D_p\, \langle n'|\, \langle p'|\, Y_s\, |s\rangle \Big) \\
&= \tfrac{1}{2} V_t V_s D_n D_p \Big(\langle t|\, Y_n\, |n^* p\rangle\, D_{n^*}\, \langle\gamma n^*|\, Y^{\dagger}_{n_{M1}}\, |n\rangle\, \langle np|\, Y_s\, |s\rangle + \langle t|\, Y_p\, |n'p'^*\rangle\, D_{p^*}\, \langle\gamma p'^*|\, Y^{\dagger}_{p_{M1}}\, |p'\rangle\, \langle n'p'|\, Y_s\, |s\rangle \Big) \\
&= \tfrac{1}{2} V_t V_s D_n D_p \left[y_n D_{n^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^{\gamma}_{n_{M1}} \left(\gamma\tfrac{1}{\sqrt{2}}\right) y_s + y_p D_{p^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}^{\gamma}_{p_{M1}} \left(-\gamma\tfrac{1}{\sqrt{2}}\right) y_s \right] \\
&= \tfrac{1}{2\sqrt{3}} V_t V_s\, y_s\, D_n D_p \left(y_n D_{n^*}\, \overline{y}^{\gamma}_{n_{M1}} - y_p D_{p^*}\, \overline{y}^{\gamma}_{p_{M1}} \right) = \tfrac{1}{2} y_s D_n D_p A^{\gamma}_{np}\ ,
\end{aligned} \tag{6.1}$$

where A^{γ}_{np} is from Eq. (3.12).

VII. $M1$ PHOTON-DEUTERON ELASTIC SCATTERING

For the amplitude with an intermediate, off-shell singlet dibaryon, the initial deuteron and photon have opposite spins, as do the final deuteron and photon. With γ_i, γ_f the polarization states of the initial and final photon, there are four contributing amplitudes, each with initial and final deuteron spins $M_{ti} = -\gamma_i$ and $M_{tf} = -\gamma_f$. In terms of the expanded $s^*\gamma t$ vertex operator and its eigenvalue (6.1), the elastic γt amplitudes with an intermediate, off-shell singlet dibaryon are

$$A_s^{\gamma_i,\gamma_f} = \langle \gamma_f t_f | \, Y_{s_{M1}} Q_s \boldsymbol{D}_s Y_{s_{M1}}^\dagger \, |\gamma_i, t_i\rangle = Q_s D_s y_{s_{M1}}^{\gamma_f} \, \overline{y}_{s_{M1}}^{\gamma_i} \ . \tag{7.1}$$

VIII. TWO-PHOTON CAPTURE

Consider $\Delta J = 1$ $M1$ radiative capture from an initial triplet np, resulting in a deuteron in the excited 0^+ state and a photon. Unlike ordinary capture, there can be no contribution from $M \equiv M_n + M_p = 0$. For the contribution from off-shell baryons, where there is no intermediate dibaryon, the correction V_s is applied to the singlet n^*ps and np^*s vertices, and the correction V_t is applied to the initial singlet np. Because the intial state has $M = \pm 1$, there is no need to distinguish np between the case of the virtual neutron and the case of the virtual proton, as was done in Eqs. (3.7)-(3.9), For the n^*np contribution, where the neutron undergoes the $M1$ transition,

$$\gamma + M_{n^*} = M_n \ , \ M_s = M_{n^*} + M_p = \gamma + M_t = 0 \qquad \Rightarrow \qquad M_p = -M_{n^*} = M_n = \tfrac{1}{2}\gamma \ . \tag{8.1}$$

For the p^*np contribution, where the proton undergoes the $M1$ transition,

$$\gamma + M_{p^*} = M_p \ , \ M_s = M_{p^*} + M_n = \gamma + M_t = 0 \qquad \Rightarrow \qquad M_n = -M_{p^*} = M_p = \tfrac{1}{2}\gamma \ . \tag{8.2}$$

The initial state is

$$|1, \pm 1\rangle = \left|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}\right\rangle = \left|\gamma\tfrac{1}{2}, \gamma\tfrac{1}{2}\right\rangle = |np\rangle \ . \tag{8.3}$$

Note that $C_{sn^*p} = -\gamma\frac{1}{\sqrt{2}}$ and $C_{snp^*} = \gamma\frac{1}{\sqrt{2}}$. The $M = \pm 1$ amplitudes from Fig. 3 are

$$\begin{aligned}
A_n^\gamma &= \langle \gamma s | \, Y_n V_s \boldsymbol{D}_n Y_{n_{M1}}^\dagger V_t \, |np\rangle = V_s V_t \, \langle \gamma | \, \langle s | \, Y_n \, |n^*\rangle \, D_n \, \langle n^* | \, Y_{n_{M1}}^\dagger \, |n\rangle \, |p\rangle = V_s V_t \, \langle s | \, Y_n \, |n^* p\rangle \, D_n \, \langle \gamma n^* | \, Y_{n_{M1}}^\dagger \, |n\rangle \\
&= V_s V_t D_n \left(-\gamma\tfrac{1}{\sqrt{2}}\right) y_n \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}_{n_{M1}}^\gamma = -\tfrac{1}{\sqrt{3}} \, V_s V_t \, y_n D_n \, \overline{y}_{n_{M1}}^\gamma \ , &(8.4)\\
A_p^\gamma &= \langle \gamma s | \, Y_p V_s \boldsymbol{D}_p Y_{p_{M1}}^\dagger V_t \, |np\rangle = V_s V_t \, \langle \gamma | \, \langle s | \, Y_p \, |p^*\rangle \, D_p \, \langle p^* | \, Y_{p_{M1}}^\dagger \, |p\rangle \, |n\rangle = V_s V_t \, \langle s | \, Y_p \, |np^*\rangle \, D_p \, \langle \gamma p^* | \, Y_{p_{M1}}^\dagger \, |p\rangle \\
&= V_s V_t D_p \left(\gamma\tfrac{1}{\sqrt{2}}\right) y_p \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}_{p_{M1}}^\gamma = \tfrac{1}{\sqrt{3}} \, V_s V_t \, y_p D_p \, \overline{y}_{p_{M1}}^\gamma \ , &(8.5)\\
A_{np}^\gamma &\equiv A_n^\gamma + A_p^\gamma \ . &(8.6)
\end{aligned}$$

But for an overall sign change and substantially different photon energy, these are the same as Eqs. (3.10)-(3.12) for ordinary capture. Like ordinary capture, this is very nearly proportional to $\mu_1 = \mu_p - \mu_n$, i.e., it can also be classified as an isovector reaction. With the $t^*\gamma s$ vertex expanded as shown in Fig. 4, the amplitudes are

$$\begin{aligned}
A_t^\gamma &= \langle \gamma s | \, V_s Y_n \boldsymbol{D}_{n^*} Y_{n_{M1}}^\dagger \boldsymbol{D}_n \boldsymbol{D}_p V_t \tfrac{1}{2} Y_t Q_t \boldsymbol{D}_t Y_t^\dagger V_t \, |np\rangle + \langle \gamma s | \, V_s Y_p \boldsymbol{D}_{p^*} Y_{p_{M1}}^\dagger \boldsymbol{D}_n \boldsymbol{D}_p V_t \tfrac{1}{2} Y_t Q_t \boldsymbol{D}_t Y_t^\dagger V_t \, |np\rangle \\
&= \tfrac{1}{2} Q_t V_t^2 V_s \Big(\, \langle \gamma | \, \langle s | \, Y_n \, |n^*\rangle \, D_{n^*} \, \langle n^* | \, Y_{n_{M1}}^\dagger \, |n\rangle \, |p\rangle \, D_n D_p \, \langle n | \, \langle p | \, Y_t \, |t^*\rangle \, D_t \, \langle t^* | \, Y_t^\dagger \, |np\rangle \, + \\
&\quad + \langle \gamma | \, \langle s | \, Y_p \, |p^*\rangle \, D_{p^*} \, \langle p^* | \, Y_{p_{M1}}^\dagger \, |n\rangle \, |p\rangle \, D_n D_p \, \langle n | \, \langle p | \, Y_t \, |t^*\rangle \, D_t \, \langle t^* | \, Y_t^\dagger \, |np\rangle \Big) \\
&= \tfrac{1}{2} Q_t V_t^2 V_s D_t \overline{y}_t D_n D_p \left(\langle s | \, Y_n \, |n^* p\rangle \, D_{n^*} \, \langle \gamma n^* | \, Y_{n_{M1}}^\dagger \, |n\rangle \, \langle np | \, Y_t \, |t^*\rangle + \langle s | \, Y_p \, |np^*\rangle \, D_{p^*} \, \langle \gamma p^* | \, Y_{p_{M1}}^\dagger \, |p\rangle \, \langle np | \, Y_t \, |t^*\rangle \right) \\
&= \tfrac{1}{2} Q_t V_t^2 V_s D_t \overline{y}_t D_n D_p \left[\left(-\gamma\tfrac{1}{\sqrt{2}}\right) y_n D_{n^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}_{n_{M1}}^\gamma \, y_t + \left(\gamma\tfrac{1}{\sqrt{2}}\right) y_p D_{p^*} \left(\gamma\sqrt{\tfrac{2}{3}}\right) \overline{y}_{p_{M1}}^\gamma \, y_t \right] \\
&= \tfrac{1}{2\sqrt{3}} \, Q_t V_t^2 V_s D_t |y_t|^2 D_n D_p \left(- y_n D_{n^*} \, \overline{y}_{n_{M1}}^\gamma + y_p D_{p^*} \, \overline{y}_{p_{M1}}^\gamma \right) = \tfrac{1}{2} \, Q_t V_t D_t |y_t|^2 D_n D_p A_{np}^\gamma \ . &(8.7)
\end{aligned}$$

A_{np}^γ is given by Eqs. (8.4)-(8.6), where D_n and D_p correspond to D_{n^*} and D_{p^*} in Eq. (8.7).

IX. $E1$ CAPTURE

For $E1$ capture, the initial np is in a triplet spin state, with total spin $S = 1$ and $M_S = \pm 1, 0$, and in a relative p-wave, with orbital angular momentum $L = 1$ and $M_L = \pm 1, 0$. The final deuteron t is in the 1^+ ground state, with spin M_t, and the final photon has helicity $\gamma = \pm 1$. The intermediate baryon contribution, with no intermediate dibaryon, is similar to the $M1$ case (Fig. 3), but there is no contribution from the neutron. $Y_{p_{M1}}$ is replaced with $Y_{p_{E1}}$, and the eigenvalue $y^\gamma_{p_{M1}} = \sqrt{4\pi}\, y_p\, y^\gamma_{M1}$ is replaced with $y^\gamma_{p_{E1}} = \sqrt{4\pi}\, y_p\, y^\gamma_{E1}$, with $y^\gamma_{E1} = (-i\gamma)(i\omega/m_p)\sqrt{3/16\pi}\, \sin\theta\, e^{i\gamma\phi}$. Note that y^γ_{E1} differs from y^γ_{M1} by a factor $-i\gamma$ (among other things), because the electric and magnetic fields in electric multipole radiation are "exchanged", relative to those in magnetic multipole radiation, and it is $\hat{\boldsymbol{p}} \cdot \boldsymbol{\epsilon}^{(i)}$ that is replaced with unity instead of $\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{p}} \times \boldsymbol{\epsilon}^{(i)})$. The spins satisfy

$$\gamma + M_{p^*} = M_p + M_L \ , \ M_t = M_{p^*} + M_n \ , \ M_p = M_{p^*} \ , \ M_S \equiv M_p + M_n \qquad \Rightarrow \qquad M_L = \gamma \ , \ M_S = M_t \ , \quad (9.1)$$

from which it is seen that the final deuteron has the same spin and orientation of the initial np, i.e., $M_t = M_S$. Because $\gamma = M_L$, only $M_L = \pm 1$ contributes. The initial np are in a relative p-wave, so the triplet np wavefunction is spatially antisymmetric. Adopting the $np/n'p'$ notation from Eqs. (3.7)-(3.9) for the $M_t = 0$ case (and modifying it somewhat), the initial state is (By definition, $M_{n'} = -M_n$ and $M_{p'} = -M_p$.)

$$\begin{aligned} &M_t = 0 \qquad \tfrac{1}{\sqrt{2}}\left|+\tfrac12, -\tfrac12\right\rangle - \tfrac{1}{\sqrt{2}}\left|-\tfrac12, +\tfrac12\right\rangle = \tfrac{1}{\sqrt{2}}\left|np\right\rangle - \tfrac{1}{\sqrt{2}}\left|n'p'\right\rangle \ , \qquad M_n = -M_p = \tfrac12 \ , M_{n'} = -M_{p'} = -\tfrac12 \ , \\ &M_t = \pm 1 \qquad \left|\pm\tfrac12, \pm\tfrac12\right\rangle = \left|np\right\rangle \ , \qquad M_n = M_p = \tfrac12 M_t = \pm\tfrac12 \ . \end{aligned} \quad (9.2)$$

With this notation (employed only for $M_t = 0$), there is a contribution from npp^* and another from $n'p'p'^*$. Because p and p' are the same proton in its two possible orientations, the prime is not included in the subscripts of such variables as E_p, D_{p^*}, etc. Note, however, that the Clebsch-Gordan coefficients are not the same, i.e., $C_{tnp^*} = \frac{1}{\sqrt{2}} = -C_{tn'p'^*}$. Let J_{np} specify the total angular momentum of the initial np, with projection $M_{np} = M_S + M_L = M_t + \gamma$. J_{np} is 0, 1, or 2, corresponding to ${}^{2S+1}L_J = {}^3P_0$, ${}^3P_1$, or ${}^3P_2$; amplitudes with different J_{np}, M_{np} do not interfere when integrated over 4π solid angle. In analogy with Eq. (3.5),

$$C_{(np)\gamma(np^*)} \equiv C_{11}(J_{np}, M_{np}; \gamma, M_{np^*}) = C_{11}(J_{np}, M_{np}; \gamma, M_t) \ , \qquad \langle \gamma p^* | Y^\dagger_{p_{E1}} |p\rangle = C_{(np)\gamma(np^*)} \overline{y}^\gamma_{p_{E1}} \ , \quad (9.3)$$

$$y^\gamma_{p_{E1}} = i\sqrt{(E_p - \omega)/4\pi}\, \sqrt{4\pi}\, y^\gamma_{E1} = \gamma(\omega/m_p)\sqrt{\alpha(E_p - \omega)}\sqrt{3/16\pi}\, \sin\theta\, e^{i\gamma\phi} \quad \text{(circular polarization)} \ , \quad (9.4)$$

$$y^{(\mathrm{v/h})}_{p_{E1}} = (\omega/m_p)\sqrt{\alpha(E_p - \omega)}\sqrt{3/16\pi}\, \sin\theta\, \sqrt{1 \mp \cos 2\phi} \quad \text{(plane polarization)} \ . \quad (9.5)$$

The Clebsch-Gordan coefficients are

$$\begin{aligned} &{}^3P_0\ (\gamma = \pm 1; M_t = -\gamma) && C_{11}(0,0;+1,-1) = C_{11}(0,0;-1,+1) = +\tfrac{1}{\sqrt{3}} \ , \\ &{}^3P_1\ (\gamma = \pm 1; M_t = 0, -\gamma) && \left.\begin{aligned} C_{11}(1,+1;+1,0) = C_{11}(1,0;+1,-1) = +\tfrac{1}{\sqrt{2}} \\ C_{11}(1,-1;-1,0) = C_{11}(1,0;-1,+1) = -\tfrac{1}{\sqrt{2}} \end{aligned}\right\} = \gamma\tfrac{1}{\sqrt{2}} \ , \\ &{}^3P_2\ (\gamma = \pm 1; M_t = 0, \pm\gamma) && \begin{aligned} C_{11}(2,+2;+1,+1) = C_{11}(2,-2;-1,-1) = 1 \\ C_{11}(2,+1;+1,0) = C_{11}(2,-1;-1,0) = \tfrac{1}{\sqrt{2}} \\ C_{11}(2,0;+1,-1) = C_{11}(2,0;-1,+1) = \tfrac{1}{\sqrt{6}} \end{aligned} \ . \end{aligned}$$

Since the np^* is in a relative s-wave, in the triplet spin state, the vertex correction V_t is included at the np^*t vertex, where the final-state deuteron t is created and the np^* are annihilated. The p-wave vertex correction V_P is applied to the initial p-wave np (see Ref. [1]). The amplitudes with an intermediate, off-shell baryon (Fig. 3) are, where $JM \equiv J_{np}, M_{np}$,

$$\begin{aligned} A^{\gamma,M_t}_{p,JM} &= \langle \gamma t | V_t Y_p \boldsymbol{D}_p Y^\dagger_{p_{E1}} V_P |np\rangle \ , \qquad A^\gamma_p \equiv V_t V_P\, y_p D_p\, \overline{y}^\gamma_{p_{E1}} \ , \qquad (9.6) \\ A^{\gamma,0}_{p,JM} &= V_t V_P \left[\langle \gamma t | Y_p |p^*\rangle D_p \langle p^* | Y^\dagger_{p_{E1}} \left(\tfrac{1}{\sqrt{2}}\right) |np\rangle + \langle \gamma t | Y_p |p'^*\rangle D_p \langle p'^* | Y^\dagger_{p_{E1}} \left(-\tfrac{1}{\sqrt{2}}\right) |n'p'\rangle \right] \\ &= \tfrac{1}{\sqrt{2}} V_t V_P D_p \left(\langle t | Y_p |np^*\rangle \langle \gamma p^* | Y^\dagger_{p_{E1}} |p\rangle - \langle t | Y_p |n'p'^*\rangle \langle \gamma p'^* | Y^\dagger_{p_{E1}} |p'\rangle \right) \\ &= \tfrac{1}{\sqrt{2}} V_t V_P D_p \left[\left(\tfrac{1}{\sqrt{2}}\right) y_p\, C_{11}(J,M;\gamma,0)\, \overline{y}^\gamma_{p_{E1}} - \left(-\tfrac{1}{\sqrt{2}}\right) y_p\, C_{11}(J,M;\gamma,0)\, \overline{y}^\gamma_{p_{E1}} \right] = C_{11}(J,M;\gamma,0) A^\gamma_p \ , \\ A^{\gamma,\pm1}_{p,JM} &= V_t V_P \langle \gamma t | Y_p |p^*\rangle D_p \langle p^* | Y^\dagger_{p_{E1}} |np\rangle = V_t V_P \langle t | Y_p |np^*\rangle D_p \langle \gamma p^* | Y^\dagger_{p_{E1}} |p\rangle = C_{11}(J,M;\gamma,\pm1) A^\gamma_p \ , \\ \Rightarrow \quad A^{\gamma,M_t}_{p,JM} &= C_{11}(J,M;\gamma,M_t) A^\gamma_p \ . \qquad (9.7) \end{aligned}$$

Contributions from intermediate dibaryons require that the dibaryon posses orbital angular momentum. The p-wave dibaryon propagator is take to be the same as the s-wave dibaryon propagator, but with a p-wave propagator correction Q_P instead of Q_t (see Ref. [1], where it is assumed that a p-wave triplet dibaryon has the same on-shell mass as an s-wave triplet dibaryon). The $t^*t\gamma$ vertex is expanded as shown in the right half of Fig. 4. There is a p-wave vertex correction V_P at either end of the intermediate dibaryon, applied between the np legs, and an s-wave vertex correction V_t at the vertex where the np^* join to form the final triplet dibaryon in a relative s-wave. A_p^γ and $A_{p,JM}^{\gamma,M_t}$ are given by Eqs. (9.6)-(9.7), where D_p corresponds to D_{p^*} in the amplitudes $A_{t,JM}^{\gamma,M_t}$, which are

$$\begin{aligned}
A_{t,JM}^{\gamma,0} &= Q_P V_t V_P^2 \left[\langle \gamma t | \, Y_p \boldsymbol{D}_{p^*} Y_{p_{E1}}^\dagger \boldsymbol{D}_n \boldsymbol{D}_p \tfrac{1}{2} Y_t \boldsymbol{D}_t Y_t^\dagger \left(\tfrac{1}{\sqrt{2}}\right) |np\rangle + \langle \gamma t | \, Y_p \boldsymbol{D}_{p'^*} Y_{p_{E1}}^\dagger \boldsymbol{D}_{n'} \boldsymbol{D}_{p'} \tfrac{1}{2} Y_t \boldsymbol{D}_t Y_t^\dagger \left(-\tfrac{1}{\sqrt{2}}\right) |n'p'\rangle \right] \\
&= \tfrac{1}{2\sqrt{2}} Q_P V_t V_P^2 \Big(\langle \gamma t | \, Y_p \, |p^*\rangle \, D_{p^*} \, \langle p^* | \, Y_{p_{E1}}^\dagger \, |n\rangle \, |p\rangle \, D_n D_p \, \langle n | \, \langle p | \, Y_t \, |t^*\rangle \, D_t \, \langle t^* | \, Y_t^\dagger \, |np\rangle + \\
&\qquad - \langle \gamma t | \, Y_p \, |p'^*\rangle \, D_{p^*} \, \langle p'^* | \, Y_{p_{E1}}^\dagger \, |n'\rangle \, |p'\rangle \, D_n D_p \, \langle n' | \, \langle p' | \, Y_t \, |t^*\rangle \, D_t \, \langle t^* | \, Y_t^\dagger \, |n'p'\rangle \Big) \\
&= \tfrac{1}{2\sqrt{2}} Q_P V_t V_P^2 D_{p^*} D_n D_p D_t \Big[\langle t | \, Y_p \, |np^*\rangle \, \langle \gamma p^* | \, Y_{p_{E1}}^\dagger \, |p\rangle \left(\tfrac{1}{\sqrt{2}}\right) y_t \left(\tfrac{1}{\sqrt{2}}\right) \overline{y}_t + \\
&\qquad - \langle t | \, Y_p \, |n'p'^*\rangle \, \langle \gamma p'^* | \, Y_{p_{E1}}^\dagger \, |p'\rangle \left(-\tfrac{1}{\sqrt{2}}\right) y_t \left(-\tfrac{1}{\sqrt{2}}\right) \overline{y}_t \Big] \\
&= \tfrac{1}{4\sqrt{2}} Q_P V_t V_P^2 D_{p^*} D_n D_p D_t |y_t|^2 \left[\left(\tfrac{1}{\sqrt{2}}\right) y_p \; C_{11}(J,M;\gamma,0) \overline{y}_{p_{E1}}^\gamma - \left(-\tfrac{1}{\sqrt{2}}\right) y_p \; C_{11}(J,M;\gamma,0) \overline{y}_{p_{E1}}^\gamma \right] \\
&= \tfrac{1}{4} C_{11}(J,M;\gamma,0) Q_P V_P D_n D_p D_t |y_t|^2 A_p^\gamma \; , \qquad (9.8) \\
A_{t,JM}^{\gamma,\pm1} &= Q_P V_t V_P^2 \, \langle \gamma t | \, Y_p \boldsymbol{D}_{p^*} Y_{p_{E1}}^\dagger \boldsymbol{D}_n \boldsymbol{D}_p \tfrac{1}{2} Y_t \boldsymbol{D}_t Y_t^\dagger \, |np\rangle \\
&= \tfrac{1}{2} Q_P V_t V_P^2 \, \langle t | \, \langle \gamma | \, Y_p \, |p^*\rangle \, D_{p^*} \, \langle p^* | \, Y_{p_{E1}}^\dagger \, |n\rangle \, |p\rangle \, D_n D_p \, \langle n | \, \langle p | \, Y_t \, |t^*\rangle \, D_t \, \langle t^* | \, Y_t^\dagger \, |np\rangle \\
&= \tfrac{1}{2} Q_P V_t V_P^2 D_{p^*} D_n D_p D_t \, \langle t | \, Y_p \, |np^*\rangle \, \langle \gamma p^* | \, Y_{p_{E1}}^\dagger \, |p\rangle \; y_t \, \overline{y}_t \\
&= \tfrac{1}{2} Q_P V_t V_P^2 D_{p^*} D_n D_p D_t |y_t|^2 \, y_p \, C_{11}(J,M;\gamma,\pm1) \, \overline{y}_{p_{E1}}^\gamma \\
&= \tfrac{1}{2} C_{11}(J,M;\gamma,\pm1) Q_P V_P D_n D_p D_t |y_t|^2 A_p^\gamma \; . \qquad (9.9)
\end{aligned}$$

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