Microscopic Primordial Black Holes and Extra Dimensions

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Abstract

We examine the production and evolution of microscopic black holes in the early universe in the large extra dimensions scenario. We demonstrate that, unlike in the standard four-dimensional cosmology, in large extra dimensions absorption of matter from the primordial plasma by the black holes is significant and can lead to rapid growth of the black hole mass density. This effect can be used to constrain the conditions present in the very early universe. We demonstrate that this constraint is applicable in regions of parameter space not excluded by existing bounds.

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1 Introduction

The possible formation of black holes in the early universe has long been discussed. The idea was first proposed by Carr and Hawking [1], who considered the formation of such primordial black holes (PBHs) by the gravitational collapse of density perturbations and their subsequent evolution. They found that the PBH mass distribution is determined by the initial spectrum of density perturbations and the expansion of the universe, with accretion playing a negligible role. Soon after, Hawking discovered that black holes can emit particles [2], so that microscopic black holes decay very rapidly. PBHs smaller than about 10^{15} g would have evaporated by today, while larger ones could have survived. It has been proposed that these relics could make up the cosmic dark matter [3], while on the other hand their non-observation constrains the initial spectrum of density fluctuations [4]. It has also been suggested that the endpoint of black hole evaporation could be a stable Planck-sized remnant [5], leading to additional observational consequences [6].

Primordial black holes have also been considered in the context of extra dimensional theories [7]. In these theories, the fundamental scale of quantum gravity, called M_*, can be as low as 1 TeV. It is well known that with extra dimensions, the properties of microscopic black holes (those smaller than the size of the extra dimensions) are significantly altered. A black hole in extra dimensions will be colder, larger, and longer lived than one of the same mass in four dimensions [8], with significant cosmological consequences. In particular, production of PBHs by the collapse of primordial density perturbations in large extra dimensions has been studied [7]. The authors show that the unique properties of extra dimensional black holes lead to a relaxation of the bound on the spectral index.

In this paper we discus a different class of PBHs–microscopic black holes produced by high-energy particle collisions in the early universe. The consequences of these tiny black holes have generally been neglected in the literature, under the premise that they are too hot and short lived to have any observational effects. We argue that in the presence of extra dimensions, absorbtion of matter from the surrounding plasma cannot be neglected, and in fact can lead to rapid growth. Consequently, the production and evolution of these black holes must be analyzed. As we will demonstrate, the mass density of PBHs is determined by the initial temperature of the universe T_I, the number of extra dimensions n, and M_*. We find that for different values of n, large regions of T_I–M_* parameter space can be excluded by observational constraints. It is worth noting that the effects of accretion in extra dimensional scenarios have previously been considered [9, 10, 11, 12, 13]. These authors analyzed rapidly growing black holes formed by other mechanisms in Randall-Sundrum cosmologies.

There already exist constraints on theories of large extra dimensions [14, 15], the most stringent of which come from astrophysical considerations. For $n = 1$ a natural value of M_* requires an extra dimension whose size is comparable to that of the solar system. This would lead to obvious conflicts with observation. For $n = 2$ the overheating of neutron stars by captured Kaluza-Klein (KK) gravitons constrains $M_* \gtrsim 700$ TeV. Larger values of n are less tightly constrained.

Independent bounds have already been placed by previous authors on the initial temperature of the universe in these theories [16, 17]. These are obtained by considering the production of KK gravitons in the early universe. The KK bounds can always be evaded by considering high enough values of M_* and n, in which case the gravitons decay too early to have observational consequences. We demonstrate that in many cases these regions of parameter space are excluded by the PBH bounds. In addition, the graviton constraints can be evaded by, for example, rapid graviton decay onto another brane or a heavier graviton spectrum arising from a complicated bulk geometry. The PBH constraints cannot be avoided so easily.

For definiteness, we consider the model of large extra dimensions proposed by Arkani-Hamed, Dimopoulos, and Dvali (ADD) first presented in [18]. In a subsequent paper [14] these authors noted that bounds can be placed on the so-called "normalcy" temperature of the universe–the temperature below which the extra-dimensional bulk must be stable in size and empty. In contrast, we constrain the properties of the universe prior to attaining normalcy. We do not specialize to any specific cosmological model. Instead we consider two possible thermal states for the extradimensional bulk without specifying the dynamics which lead to these states. We examine scenarios where the bulk is cold and empty, and where it is in thermal equilibrium with the brane.

The paper is organized as follows. In Section 2 we analyze the evolution of a single black hole in the hot primordial plasma. In Section 3 we present a simple model for black hole production in the early universe, and use it to derive bounds on T_I and M_*. In Section 4 we show that the results we obtain with this simple model apply also to much less restrictive and more realistic scenarios. Our conclusions are given in Section 5.

2   Evolution of an extra-dimensional black hole

In the ADD model, Standard Model particles are bound to a three-dimensional brane in a $3+n$-dimensional bulk space. The black holes relevant to this analysis are created on the brane and remain there. The brane is populated by a thermal distribution of relativistic Standard Model particles. We claim that, in this scenario, a black hole

upon its creation will instantaneously (compared to the timescale of cosmological evolution) attain a maximum mass. In the case that the bulk is in thermal equilibrium with the brane, this mass will simply be the mass of a black hole the size of the extra dimension,

$$M_{max} = M_L = \left[\left(\frac{M_{pl}}{M_*} \right)^{\frac{2}{n}} \frac{1}{a_n} \right]^{n+1} M_*, \tag{1}$$

where a_n is the dimensionless constant defined in Equation 6. If the bulk is empty of energy density, the mass attained by the black hole depends on the temperature T_0 of the universe when the black hole is created and is given by

$$M_{max} = \left(\gamma_n \frac{M_{pl}}{M_*^3} T_0^2 \right)^{\frac{n+1}{n-1}} M_*, \tag{2}$$

where

$$\gamma_n \equiv \sqrt{\frac{180}{\pi g_*}} \frac{n-1}{2(n+1)} \sigma_4 r_n^2 . \tag{3}$$

The remainder of this section is devoted to demonstrating this claim.

The properties of black holes in infinitely large extra dimensions were first derived in [8]. In ADD, the extra dimensions are of finite size; the four dimensional Planck scale M_{pl} and M_* are related by

$$M_{pl}^2 \simeq L^n M_*^{n+2}, \tag{4}$$

where L is the size of the extra dimensions. We now review the properties of ADD black holes as discussed in [7]. We note that they are only valid for black holes much smaller than L. Larger black holes behave effectively four dimensionally.

The Schwartzschild radius of an ADD black hole of mass M is given by

$$r_s = a_n \frac{1}{M_*} \left(\frac{M}{M_*} \right)^{\frac{1}{n+1}}, \tag{5}$$

where

$$a_n = \left(\frac{8\,\Gamma\left(\frac{n+3}{2}\right)}{(n+2)\pi^{\frac{n+1}{2}}} \right)^{\frac{1}{n+1}} . \tag{6}$$

The temperature of the black hole is given by

$$T_{BH} = \frac{n+1}{4\pi r_s}. \tag{7}$$

We consider a black hole with $r_s \ll L$ submerged in a thermal plasma at temperature T. If we ignore gravitational attraction, the absorption and emission can both

3

be characterized by the Stefann-Boltzmann law. The rate of change of the black hole mass M is then

$$\frac{dM}{dt} = \sigma_4 A_4 (T^4 - T_{BH}^4) + \sigma_{n+4} A_{n+4} (T^{n+4} - T_{BH}^{n+4}).$$ (8)

Here,

$$\sigma_4 = \frac{g_* \pi^2}{120} \quad \text{and}$$

$$\sigma_{n+4} = \frac{g_b \Omega_{n+1} \Gamma(n+4) \zeta(n+4)}{2\pi^{n+3}(n+2)}$$

are the 4 and $4+n$-dimensional Stefann-Boltzmann constants, and A_4 and A_{4+n} are the black hole surface areas on the brane and in the bulk. The number of effective degrees of freedom on the brane is g_* and $g_b = (n+1)(n+4)/2$ is the number of polarization states of a bulk graviton.

In the early universe, the relationship between time, t, and T is determined by the Friedmann equations. We will show in the next section that the phase of black hole growth takes place long before matter-radiation equality and that the fraction of the universe's energy density in black holes is small. Radiation domination can therefore be assumed, and

$$t = \sqrt{\frac{45}{16\pi^3 g_*}} \frac{M_{pl}}{T^2}.$$ (9)

For a given temperature there is a threshold mass above which a black hole will absorb more than it emits. This mass, which we call M_{thresh}, is plotted in Figure 1. It has almost the same value whether the bulk is empty or thermalized. For $M \gg M_{thresh}$, we can neglect Hawking radiation entirely, and Equation 8 becomes

$$\frac{dM}{dt} = \sigma_4 A_4 T^4 + \sigma_{n+4} A_{n+4} T^{n+4}.$$ (10)

If the bulk is empty, the second term on the RHS can be dropped. The resulting equation can be trivially solved to obtain

$$M(T) = M_* \left[\left(\frac{M_0}{M_*} \right)^{\frac{n-1}{n+1}} + \gamma_n \frac{M_{pl}}{M_*^3} \left(T_0^2 - T^2 \right) \right]^{\frac{n+1}{n-1}},$$ (11)

where γ_n is given in Equation 3.

This equation takes into account the competition between the growth of the black hole by the accretion of plasma and the cooling of the plasma by the expansion of
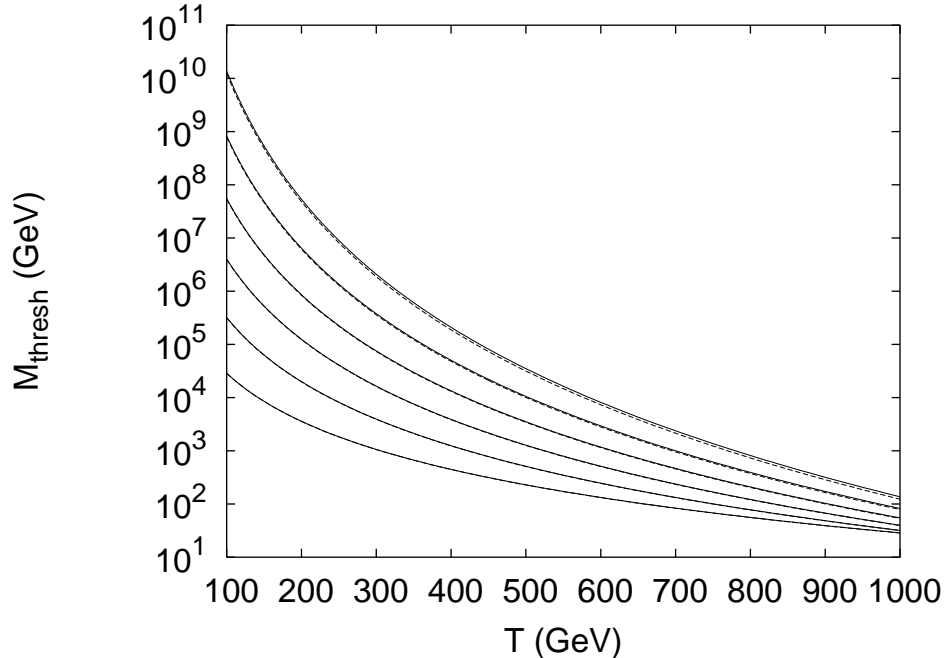
Figure 1: M_{thresh} (see text) as a function of temperature, for a thermalized (solid) and empty (dashed) bulk. From bottom to top, $n = 2, 3, 4, 5, 6, 7$.

the universe. But there is no competition. Because $M_{pl} \gg M_*$, the second term dominates almost immediately and as the universe cools the mass rapidly approaches the value of M_{max} given in Equation 2. This value is plotted as a function of time for different values of n in Figure 2. Numerical integration confirms that, in the regime considered, Hawking radiation is indeed negligible.

In Figure 3, the evolution of a black hole mass with an empty bulk is depicted. The almost instantaneous growth to M_{max} is evident. In this plot, we take a specific value for the initial mass. We find, however, that the rapid growth depicted occurs even for initial masses extremely close to M_{thresh}. Similarly, initial masses even slightly less than M_{thresh} lead to rapid decay of the black hole.

Now consider the opposite case in which the bulk is in thermal equilibrium with the brane. For simplicity, consider integrating Equation 10 with the second term only. Once again the equation can be solved to obtain

$$M(T) = M_* \left[\gamma_n^{bulk} \frac{M_{pl}}{M_*^{n+3}} (T^{n+2} - T_0^{n+2}) + \left(\frac{M_*}{M_0} \right)^{\frac{1}{n+1}} \right]^{-(n+1)}, \qquad (12)$$
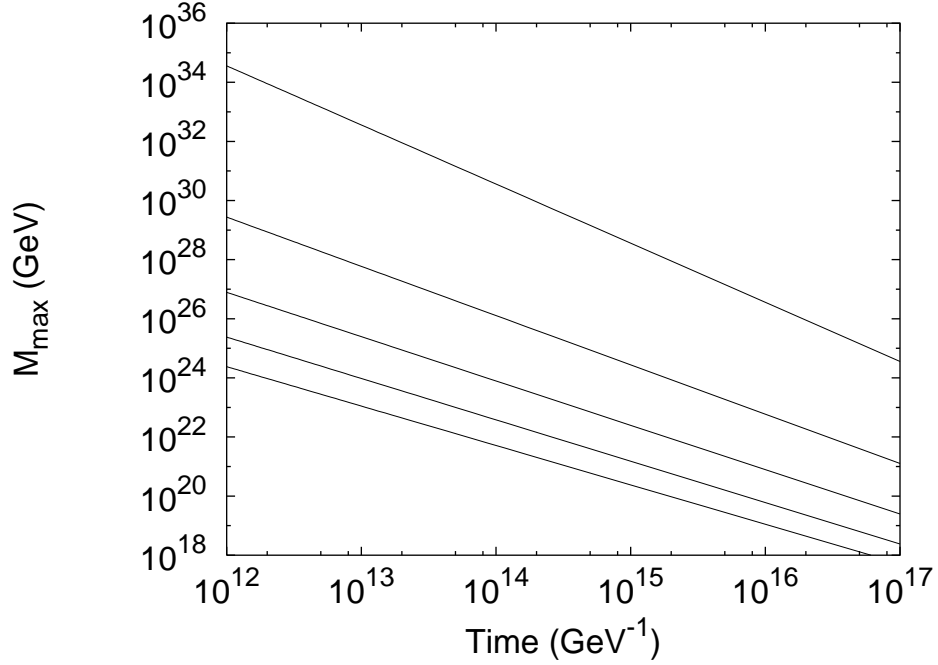
Figure 2: The maximum mass M_{max} attained by a black hole in the empty bulk scenario shown as a function of time after the big bang. From top to bottom, $n = 3, 4, 5, 6, 7$.

where

$$\gamma_n^{bulk} \equiv \sqrt{\frac{45}{4\pi^3 g_*} \frac{\sigma_{n+4}\Omega_{n+2}a_n^{n+2}}{(n+1)(n+2)}} \, .$$

This solution will formally diverge after a finite time if

$$M_0 > M_* \left(\frac{M_*^{n+3}}{\gamma_n^{bulk} M_{pl} T_0^{n+2}} \right)^{n+1} \equiv M_{div}. \tag{13}$$

It should be noted that M_{div} is typically much smaller than M_{thresh}. Therefore the actual threshold for rapid growth of a black hole in the presence of a thermalized bulk is always given by M_{thresh}.

We can also ask how quickly the mass of the black hole grows. The temperature of the universe corresponding to the time when the black hole mass diverges is

$$T_{div} = T_0 \left[1 - \left(\frac{M_{div}}{M_0} \right)^{\frac{1}{(n+1)}} \right]^{\frac{1}{n+2}} . \tag{14}$$
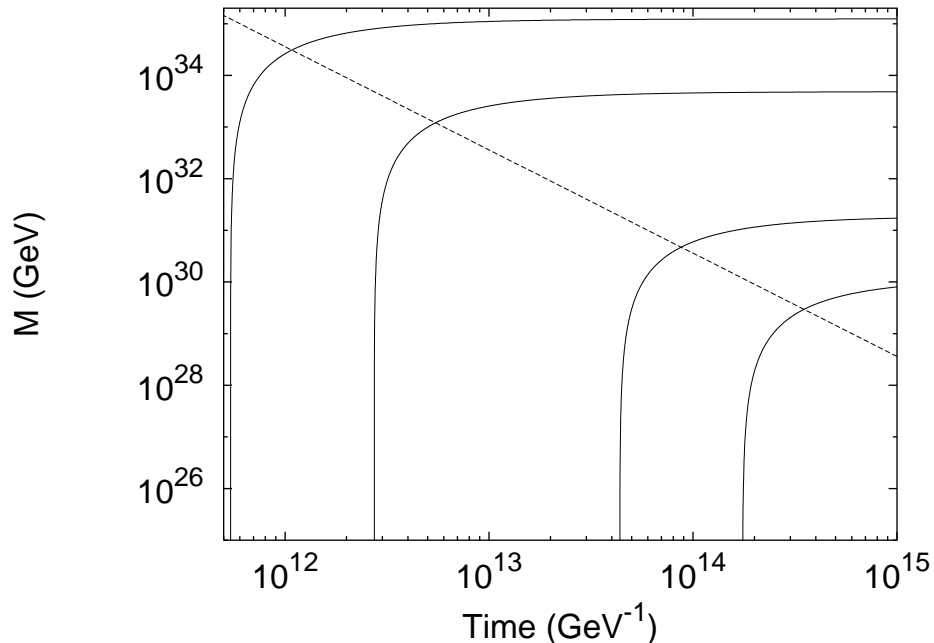
6

Figure 3: The solid lines show the evolution of the mass of a black hole with $M_0 = M_* = 1$ TeV for $n = 3$ assuming an empty bulk. From top to bottom, $T_0 = 1000, 900, 400, 50$ GeV. The dashed line is M_{max}.

From an inspection of Equation 13 it is clear that for $M_0 \gtrsim M_*$, $M_{div} \ll M_0$. Thus the second term in Equation 14 is much smaller than one and the black hole mass diverges almost immediately.

Of course, the mass of the black hole does not actually diverge. In fact, once its size reaches that of the extra dimension the black hole begins behaving four-dimensionally and, as we will see below, the absorption effectively shuts off, so the black hole remains at M_L as claimed.

We have seen that for both an empty and a thermalized bulk the evolution of a microscopic black hole in the early universe can be characterized by a threshold initial mass above which the PBH will rapidly grow and below which it will decay away. In each case any rapidly growing black hole reaches a uniform maximum mass M_{max} which is shown in Table 1 for different values of n for both the empty and thermalized bulk.

For four-dimensional black holes, Carr and Hawking showed [1] that one can neglect absorption. It is useful to review their argument and see why it does not apply for black holes in ADD. In four dimensions the change in the black hole mass

n	3	4	5	6	7
Empty	1.9×10^{35}	1.1×10^{30}	2.7×10^{27}	7.6×10^{25}	7.2×10^{24}
Thermalized	1.4×10^{46}	3.9×10^{43}	1.1×10^{42}	1.0×10^{41}	1.7×10^{40}

Table 1: The maximum mass of a rapidly growing black hole for $M_* = 1\,\mathrm{TeV}$. The two rows correspond to an empty and thermalized bulk. For the empty bulk T_0 is taken to be 1 TeV. For the thermalized bulk the values are independent of temperature.

due to absorption is given by

$$\frac{dM}{dt} = \sigma_4 (4\pi r_{s4}^2) T^4, \tag{15}$$

where

$$r_{s4} = \frac{2M}{M_{pl}^2}. \tag{16}$$

This has the solution

$$M(t \rightarrow \infty) = M_0 \left[1 - \left(\frac{720\sigma_4^2}{\pi g_*} \right)^{1/2} \frac{T_0^2 M_0}{M_{pl}^3} \right]^{-1}, \tag{17}$$

which for black holes formed with sizes smaller than the horizon, never gets very much larger than M_0. This is due to the large M_{pl} suppression in the denominator. For this reason Carr and Hawking rightfully claimed that in four dimensions the absorption of particles from the surrounding plasma can be safely neglected. It is the introduction of the low mass scale, M_*, in theories with large extra dimensions which drastically alters the mass evolution of the PBHs.

3   Black hole production

We showed in the previous section that a black hole formed in the early universe with mass above M_{thresh} will immediately grow to mass M_{max}, while one with mass less than M_{thresh} will decay away. Combining this fact with the rate per unit volume for black hole production by particle collisions on the brane, we can easily evaluate the black hole mass density produced in the early universe. In this section we calculate this density and compare it to the critical density today and to the radiation energy density during BBN to obtain bounds in the T_I–M_* parameter space. These bounds are shown in Figure 5 and consitute the main result of our paper.

8

Define $\Gamma(t)$ to be the total rate per unit volume of black hole production. Then the black hole mass density at time t obeys the equation

$$\frac{dY_{BH}(t)}{dt} = \frac{1}{s(t)} M_{max}(t)\Gamma(t),$$ (18)

where s is the entropy density of the universe, and the black hole mass density $\rho_{BH} = sY_{BH}$. For a thermalized bulk, M_{max} is constant in time.

Here we have made two simplifications. First, during production, we have taken the black hole mass density to be a small fraction of the total energy density of the universe. Our bounds require that, at the very most, the black hole mass density ρ_{BH} equals the radiation density ρ_r at the time of big-bang nucleosynthesis (BBN). As the universe cools, $\rho_{BH} \propto T^3$ and $\rho_r \propto T^4$. Thus, during black hole production, which occurs at $T \gtrsim 100$ GeV $\gg T_{BBN}$, the mass density in black holes is indeed negligible. Second, we ignore Hawking radiation during the production phase. This is a good approximation as the lifetimes of the black holes considered are much longer than the duration of the production phase.

In a particle collision, the cross section for producing a black hole is [19, 20] $\sigma(M) = f\pi r_s(M)^2$, where M is the invariant mass of the two-particle system, $r_s(M)$ is given by Equation 5 and f is an order-one constant. For now we take $f = 1$. If the brane fields are thermalized, the rate per unit volume $d\Gamma$ for creating black holes in the mass range $[M, M + dM]$ by particle collisions on the brane is

$$d\Gamma = dM g_*^2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} f(\vec{k_1}) f(\vec{k_2}) \sigma(M) |\vec{v_1} - \vec{v_2}| \delta\left(\sqrt{(k_1^\mu + k_2^\mu)^2} - M\right),$$ (19)

where $f(\vec{k})$ is the thermal distribution function. Here we make the approximation that all species present in the universe are relativistic, and that for fermions and bosons alike we can use the Boltzmann distribution $f(\vec{k}) = e^{-\frac{k}{T}}$. At the temperatures we are considering, $T \sim 100 - 1000$ GeV, this is valid.

We can do all but one of the integrals analytically, and we are left with

$$\frac{d\Gamma}{dM} = \frac{g_*^2 a_n^2}{16\pi^3} T \left(\frac{M}{M_*}\right)^{\frac{4+2n}{1+n}} \int dk e^{-\frac{k}{T}} \left\{ Me^{-\frac{M^2}{4kT}} - \sqrt{\pi kT} \left[\mathrm{Erf}\left(\frac{M}{2\sqrt{kT}}\right) - 1\right]\right\}.$$ (20)

To obtain the total rate of black hole production at a given temperature, we integrate over k and M numerically. The differential production rate $d\Gamma/dM$ and total production rate Γ are plotted in Figure 4. We see that the early universe is characterized by a period of intense black hole production, which falls off sharply with decreasing temperature. It is also clear from this figure that the period of black hole production ends well before matter-radiation equality, as we assumed in the previous section.
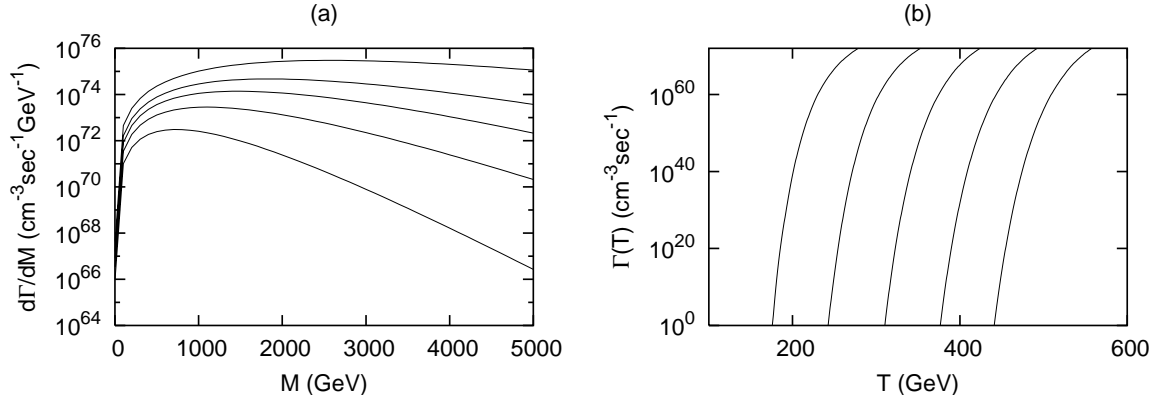
Figure 4: (a) Differential production rate of microscopic black holes in the early universe for $n = 3$. From top to bottom, $T = 700, 500, 400, 300, 200$ GeV. (b) Total production rate. From left to right, $n = 3, 4, 5, 6, 7$.

We solve Equation 18 to obtain ρ_{BH} as a function of time. We now include the effects of Hawking radiation by taking $\rho_{BH} = 0$ at times later than the lifetime of the black holes produced. For a thermal bulk, this is just the lifetime of a black hole of mass M_L. For an empty bulk, the vast majority of black holes are produced at temperatures very close to T_I. In that case we set $\rho_{BH} = 0$ at times later than the lifetime of a black hole of mass $M_{max}(T_I)$. We can now impose the two previously mentioned constraints. First, the black holes must not overclose the universe today. Second, they must not form a significant fraction of the energy density of the universe during big-bang nucleosynthesis. If either constraint were not satisfied, the expansion rate of the universe would be altered in a way that leads to measurable discrepancies with observation. Quantitatively, we require

$$\left.\frac{\rho_{BH}}{\rho_c}\right|_{today} < 1$$

and

$$\left.\frac{\rho_{BH}}{\rho_r}\right|_{BBN} < 1,$$

where ρ_c is the critical density. We use these conditions to constrain the values of T_I and M_*. In Figure 5 (a) and (b) the region above the curves is ruled out for an empty and full bulk, respectively. For an empty bulk the constraints are much weaker due to the lower value of M_{max}. In fact, in this case, for all values of n and M_* considered, no black holes survive until today. All bounds come from the BBN constraint, but for $n > 5$ all black holes decay before BBN and no bound can be obtained.

The M_* dependence of the black hole lifetime accounts for the sharp cutoff on the

10

bound for $n = 5$ with an empty bulk, above which the black holes decay before BBN. It also accounts for the kinks in the bounds for $n = 5$, 6, and 7 with a thermalized bulk, above which the black holes decay before today and the overclosure bound is replaced by the BBN bound. These kinks are barely visible; the two bounds are quantitatively almost identical. This is because the black hole production rate is so incredibly sensitive to T_I, as can be seen in Figure 4(b). As a result, a huge difference in black hole mass density can be acheived with a minute adjustment of T_I.

It is useful to compare these bounds to those obtained by Hannestad [15] and Hall and Smith [17] from gamma ray emission by the decay of KK gravitons. For natural values of M_* and low values of n, these bounds are much stronger than the PBH bounds. Because [15] and [17] only consider $n = 2, 3$, we rederive their bounds for higher n using Equation (12) in [17] and comparing to the COMPTEL data at $E = 4$ MeV as discussed in [17].

We find that the PBH bounds can be stronger in cases where the KK gravitons produced in the early universe decay before today and are not observed. This occurs for higher values of M_* and n. As can be seen in Figure 6, this results in a sharp cutoff of the KK bounds at a certain value of M_* which rapidly decreases for increasing n.

A comparison of Figure 6 to Figure 5(b) shows that, for the full bulk case, for $n > 5$ there are interesting regions of parameter space for which the PBH bounds dominate. We note that a hot bulk in the very early universe may result in additional KK constraints, but a full analysis is beyond the scope of this paper. Comparing to Figure 5(a) shows that, for the empty bulk case, the KK bounds always dominate due to the decay of the black holes. As we will see, however, if we allow for black hole remnants, the cutoffs in the empty bulk PBH bounds go away. In this case, for $n > 5$ and M_* above the KK cutoff these become the dominant constraints.

4   Additional Considerations

In this section, we consider some possible extensions of our simple analysis. We first account for the fact that the black holes, which we took to be Schwartzschild, are embedded in an expanding universe. We then consider in more detail the uncertainties involved with realistic extradimensional black holes at the classical-quantum threshold. We show our conclusions to be robust and at most weakly dependent on the above subtleties. Finally, the consideration of black hole remnants leads to a strengthening of our bounds.
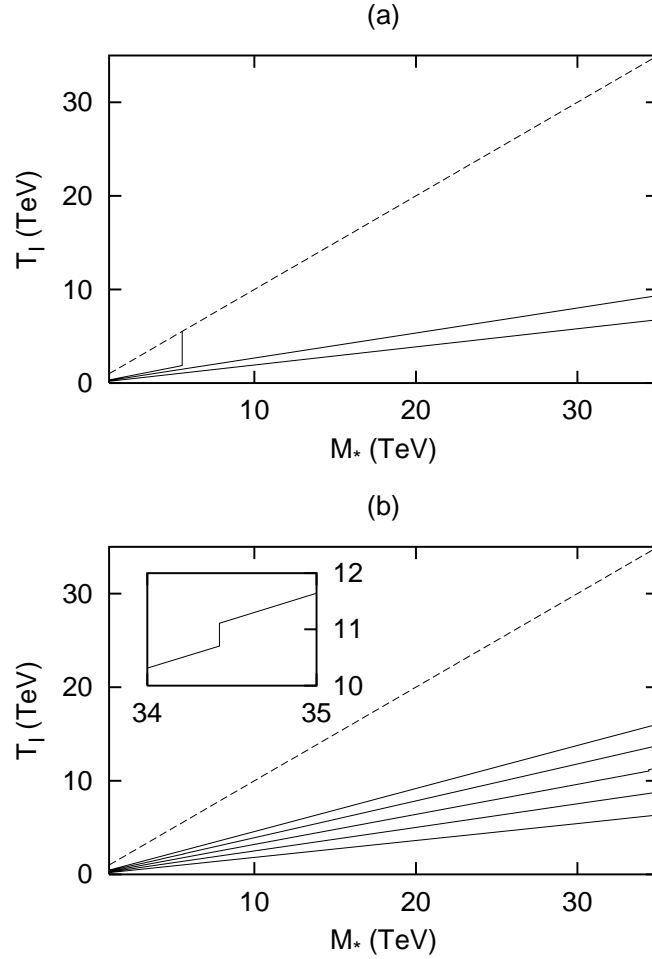
Figure 5: Bounds on T_I and M_* from BBN and overclosure of the universe for an empty bulk. The dashed lines represent $T = M_*$ above which the semi-classical description fails. The regions above the solid lines are ruled out. (a) For an empty bulk, from bottom to top, $n = 3, 4, 5$. (b) For a thermalized bulk, from bottom to top $n = 3, 4, 5, 6, 7$. The inset is a magnification of the $n = 3$ curve showing the kink arising from a switch from the overclosure bound to the BBN bound.
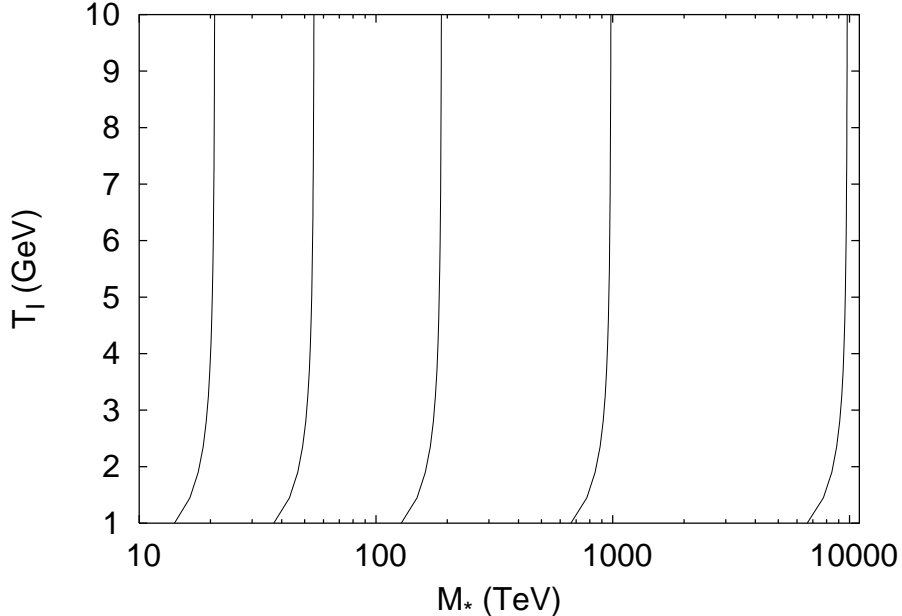
12

Figure 6: Diffuse gamma ray bounds from KK graviton decay. From left to right, $n = 7, 6, 5, 4, 3$. The region to the left of the curves is excluded.

4.1 Black holes in an FRW background

We have been somewhat simplistic in our analysis of black holes in the early universe. In an expanding universe the usual Schwartzschild solution must be replaced by one which, asymptotically, is not flat but FRW. Luckily, as noted by Carr and Hawking [1], these corrections are only important for a black hole whose size approaches that of the horizon. Since we are only considering temperatures lower than M_* we have a lower bound on the horizon size R_h during the relevant epochs. We can compare this bound to the maximum size the black holes attain, as derived in Section 2. With a thermalized bulk, the black holes reach the size of the extra dimensions L. For an empty bulk, the maximal black hole size is smaller. For $M_* = 1$ TeV, $L/R_h(T = M_*)$ is shown in Table 2. We see that for all $n > 2$ the black holes never approach the size

n	2	3	4	5	6	7
L/R_h	17.15	7.45×10^{-5}	1.55×10^{-7}	3.82×10^{-9}	3.23×10^{-10}	5.54×10^{-11}

Table 2: The ratio of the size of the extra dimensions to the horizon size, for $T = M_* = 1$ TeV.

13

of the horizon and we are thus well justified in using the Schwartzschild solution. For $n = 2$, the black holes can grow larger than R_h. In this case our analysis breaks down and a more careful study must be performed. But, because of the existing stringent bounds on extra dimensional theories with $n = 2$, we do not lose much by neglecting this case.

4.2 Properties of extra dimensional black holes

As was noted in the previous sections, the exact properties of extra dimensional black holes of mass close to the fundamental Planck scale are rather poorly understood. Specifically, there is no consensus regarding the production cross section or the exact spectrum of Hawking radiation. So far, we have used the canonical choice of $\sigma(M) = \pi r_s(M)^2$ and a purely thermal spectrum. In doing so, we have neglected the effects of angular momentum and the dissipation of energy through gravitational waves in particle collisions. We have also ignored radiative gray-body factors.

As previously mentioned, the black hole production cross section is given by

$$\sigma(M) = f\pi r_s(M)^2.$$

The factor f depends on the center of mass energy, the orbital angular momentum and the spin of the interacting particles (see [22] and references therein). Typical values for f can reach as low as ~ 0.5. In addition, the total energy actually trapped behind the black hole event horizon is not necessarily $\sqrt{s}$, as we have assumed. Studies have concluded that only approximately $40\% - 90\%$ of the collision center of mass energy actually forms the black hole, with the rest escaping in the form of gravitational radiation.

As we have seen, however, the black hole mass density is exquisitely sensitive to the initial temperature. An order one modification of the cross section will thus have a negligible effect on the bounds on T_I. The percentage of energy converted to black hole mass will affect the number of such black holes which are created above the threshold for rapid growth, and thus the final PBH mass density. But, once again, a tiny modification of the intial temperature would compensate for this effect, leaving the final bounds essentially unchanged. The correction to the spectrum of Hawking radiation due to gray-body factors, because it could modify the black hole lifetime, could change the location of the kinks and cutoffs in Figure 5, but would not qualitatively alter our conclusions.

4.3 Black hole remnants

Many authors have explored the possibility that black holes do not evaporate away completely but leave behind a microscopic remnant. This remnant is typically of the fundamental mass, which in large extra dimensions is M_*. Black hole remnants have been proposed in various contexts. Adler, Chen and Santiago [23] argued that the quantum mechanical uncertainty principal must be generalized in the presence of a curved space-time, causing Hawking radiation to shut off once the black hole reaches the fundamental scale. Rizzo [24] investigated the thermodynamics of black holes in the presence of higher curvature gravity and found that the specific heat of these black holes can become positive. The black hole would then cool as it evaporates, asymptoting to a finite size.

We have incorporated the possibility of black hole remnants by extending the bounds of integration of Equation 19. Originally we integrated over mass values between M_{thresh} and ∞ while assuming that, below M_{thresh}, the black holes would decay away. With remnants, we should extend the region of integration down to the quantum gravity limit, taken to be several times the fundamental scale. To each black hole created below M_{thresh} we assign a final mass M_*. The resulting bounds on T_I are approximately $1-2$ orders of magnitude lower than those displayed in Figure 5 and cover the entire range in M_* for all values of n for both a full and empty bulk. The bounds are sensitive to the value of the remnant mass. Lowering this mass below M_* can substantially strengthen the bounds on T_I.

5 Conclusions

In this paper we have presented a new constraint on cosmological models in theories of large extra dimensions, stemming from the production of rapidly growing microscopic black holes in the very early universe. We found that generic upper bounds can be placed on the temperature of the universe in any post inflationary epoch. As an example we analyzed two simple cases, that of a completely empty bulk and a bulk that is fully thermalized with the standard model brane. We found that in both cases significant regions of $T_I - M_*$ parameter space can be excluded. Notably, these bounds are not sensitive to many of the details relating to black holes at the classical-quantum threshold. This eliminates one of the main sources of uncertainty which plague typical studies of black holes in extra dimensions.

When compared to existing bounds, we showed that the PBH constraints are stronger for high values of n and M_*. Moreover, in the previously mentioned scenarios where the bulk is depopulated, the graviton bounds are weakened even further. On the

other hand, at low values of these parameters, our bounds are generally weaker. This is because while the cross section for black hole production is large, only sufficiently massive ones are long-lived. At low temperatures, these represent a small fraction of the black holes produced. Even if one includes the possibility of remnants, the black hole mass is bounded from below by the quantum gravity threshold. For temperatures much lower than M_* very few will be produced above this limit.

Although we focused on specific examples, it is important to note that the phenomenon of primordial black hole production and subsequent growth is generic. In any theory of extra dimensions where the fundamental Planck scale is low, a large enough energy density will lead to this effect. Generalizations of our simple scenario could include for example non-thermal brane particle distributions, a range of bulk thermal states, and a non-static bulk radius. Rapidly growing microscopic black holes in the early universe represent a conceptually new phenomenon which should be considered in detail.

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