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A Simple Explanation for the $X(3872)$ Mass Shift Observed for Decay to $D^{*0}\bar{D}^0$

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We propose a simple explanation for the increase of approximately 3 MeV/c^2 in the mass value of the $X(3872)$ obtained from $D^{*0}\bar{D}^0$ decay relative to that obtained from decay to $J/\psi\pi^+\pi^-$. If the total width of the $X(3872)$ is 2-3 MeV, the peak position in the $D^{*0}\bar{D}^0$ invariant mass distribution is sensitive to the final state orbital angular momentum because of the proximity of the $X(3872)$ to $D^{*0}\bar{D}^0$ threshold. We show that for total width 3 MeV and one unit of orbital angular momentum, a mass shift $\sim$3 MeV/c^2 is obtained; experimental mass resolution should slightly increase this value. A consequence is that spin-parity 2^- is favored for the $X(3872)$.

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The $X(3872)$ has been seen primarily in its $J/\psi\pi^+\pi^-$ decay mode [1–5], from which a measured mass value of 3871.4 ± 0.6 MeV/c^2 is obtained [6]; an upper limit on the total width of 2-3 MeV has been estimated [1]. Observation of decay to the $J/\psi\gamma$ final state [7, 8] has established positive C-parity, and analysis of the decay angular distributions [9] has narrowed the spin-parity (J^P) possibilities to 1^+ or 2^- for the $X(3872)$. The invariant mass distribution for the $D^0\bar{D}^0\pi^0$ system resulting from B-decay to the $D^0\bar{D}^0\pi^0K$ final state shows a peak near threshold yielding a mass value $3875.2\pm0.7+0.9-1.8$ MeV/c^2 [10], and this has been interpreted as evidence for the decay process $X(3872)\rightarrow D^{*0}\bar{D}^0$. A recent BaBar analysis [11] has confirmed this result, and has obtained the corresponding mass value $3875.1\pm1.1\pm0.5$ MeV/c^2. In each case, the first error quoted is statistical, and the second is systematic. These results are in excellent agreement, and yield a weighted average value 3875.1 ± 1.0 MeV/c^2, to be compared to 3871.4 ± 0.6 MeV/c^2 for the $J/\psi\pi^+\pi^-$ decay mode. The difference in these values is 3.7 ± 1.2 MeV/c^2. Its significance is thus at the three standard deviation level, and given the consistency of the BELLE and BaBar results for $D^{*0}\bar{D}^0$, it would seem to be a real effect. We take the point of view that this is indeed the case, and suggest a possible explanation, which, although very simple, carries some significant physical implications.

Consider the decay process $B\rightarrow KX$, with X is a resonance decaying to a final state F. The invariant mass distribution for the system F takes the form

$$\frac{\mathrm{d}N}{\mathrm{d}m} = C_1 m\frac{|T_B(m)|^2(p/m_B)\int|T_F(m)|^2\mathrm{d}\phi_F(m)}{(m_0^2-m^2)^2+m_0^2\Gamma_{tot}(m)^2}, \quad (1)$$

where C_1 is a constant, m is the invariant mass of system F, $T_B(m)$ is the invariant amplitude describing the B to KX coupling, $T_F(m)$ is the invariant amplitude describing the X to F coupling, $\mathrm{d}\phi_F(m)$ is the element of F decay phase space, and the denominator is the square of the relativistic Breit-Wigner propagator describing the resonance X. The factor m is present because the Lorentz-invariant phase space volume element is proportional to $\mathrm{d}m^2$, and (p/m_B) is the two-body phase space factor for $B\rightarrow KX$ decay, with

$$p = \frac{\sqrt{[m_B^2-(m_K+m)^2]\,[m_B^2-(m_K-m)^2]}}{2m_B}. \quad (2)$$

Equation (1) can be written in terms of the partial width for X decay to F, $\Gamma_F(m)$, as

$$\frac{\mathrm{d}N}{\mathrm{d}m} = \frac{C_2\cdot m\cdot p\cdot|T_B(m)|^2\Gamma_F(m)}{(m_0^2-m^2)^2+m_0^2\Gamma_{tot}(m)^2}. \quad (3)$$

In general, $\Gamma_{tot}(m)$, which is the mass-dependent total width of X, takes the form

$$\Gamma_{tot}(m) = \sum_{i=1}^{M}\Gamma_i(m), \quad (4)$$

where M is the number of decay modes of X, and the Γ_i are the individual partial widths, of which Γ_F is one.

The amplitude T_B is not known. However, if X has spin J, angular momentum conservation requires that there be J units of orbital angular momentum associated with the KX system resulting from B decay, and so we express T_B in terms of the corresponding centrifugal barrier factor as follows:

$$T_B(m) \sim \frac{p^J}{\sqrt{D_J(p,R)}}, \quad (5)$$

where $D_J(p,R)$ is the Blatt-Weisskopf Damping Factor [12], and R is the associated radius parameter, for which we choose the value 5 GeV^{-1} (i.e. 1 Fermi). The D_J functions for $J=0-3$ are summarized in Table I. For the $X(3872)$, the mass range is limited (3.87-3.91 GeV/c^2) and the Q-value for $B\rightarrow KX$ decay is large (0.9 GeV/c^2), so that the m-dependence introduced by this description of the B decay vertex is small.

For the $X(3872)$ only three decay modes have been observed to date [1–5, 7, 8, 10, 11], and of these the

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J	$D_J(p,R)$
0	1
1	$1+(pR)^2$
2	$9+3(pR)^2+(pR)^4$
3	$225+45(pR)^2+6(pR)^4+(pR)^6$

TABLE I: The Blatt-Weisskopf Damping Factors.

$J/\psi\pi^+\pi^-$ and $J/\psi\gamma$ modes are well above their respective mass thresholds. It follows that the m-dependence for each of these contributions to Eq.(4) is very weak. In contrast, for the $D^{*0}\bar{D}^0$ mode, the dependence on m is potentially quite strong because of the proximity of the invariant mass threshold to the $X(3872)$ mass; this is, in fact, the main point of this Letter. If the state F of Eq.(3) is chosen to be the $D^{*0}\bar{D}^0$ final state, the numerator is very sensitive to its dependence on m, as will be discussed in detail below. However, in the denominator, this mass dependence is subsumed into that of the total width by way of Eq.(4). Since the other known modes have very little mass-dependence, and since other as yet unobserved decay modes may contribute to Γ_{tot}, we choose to treat Γ_{tot} as a constant in the computations to follow.

From Eqs.(1) and (3),

$$\Gamma_F(m) \sim \int |T_F(m)|^2 \mathrm{d}\phi_F(m), \tag{6}$$

and with F the $D^{*0}\bar{D}^0$ final state,

$$\mathrm{d}\phi_F(m) \sim (q/m)\mathrm{d}\Omega \tag{7}$$

where

$$q = \frac{\sqrt{[m^2-(m_D+m_D^*)^2]\,[m^2-(m_D-m_D^*)^2]}}{2m} \tag{8}$$

is the momentum in the $D^{*0}\bar{D}^0$ rest frame; we ignore the width of the D^{*0} (< 1 MeV [6]). Integrating over Ω, we obtain

$$\Gamma_F(m) \sim (q/m)|T_F(m)|^2, \tag{9}$$

and following Eq.(5) (with the same value of R) we express $T_F(m)$ as

$$T_F(m) \sim \frac{q^L}{\sqrt{D_L(q,R)}}, \tag{10}$$

where L is the number of units of orbital angular momentum associated with the $D^{*0}\bar{D}^0$ system resulting from decay of the $X(3872)$.

The full expression for the $D^{*0}\bar{D}^0$ invariant mass projection is then

$$\frac{\mathrm{d}N}{\mathrm{d}m} = C_3 \frac{\left(p^{2J+1}/D_J(p,R)\right)\left(q^{2L+1}/D_L(q,R)\right)}{\left(m_0^2-m^2\right)^2+m_0^2\Gamma_{tot}^2} \tag{11}$$

where C_3 is a constant.

If the $X(3872)$ has $J^P = 1^+$ L can be 0 or 2, while for $J^P = 2^-$ L can be 1 or 3, since parity should be conserved in the decay process. Consequently we consider that only L values in the range 0-3 are of relevance to the $X(3872)$. Also we use the following central mass values [6] in our calculations:

$$m(D^0) = 1864.84\,[\pm 0.18]\ \mathrm{MeV/c^2}$$
$$m(D^{*0}) = 2006.96\,[\pm 0.19]\ \mathrm{MeV/c^2}.$$

These yield the threshold mass value 3871.80 ± 0.37 MeV/c^2 for decay to $D^{*0}\bar{D}^0$ (again ignoring the width of the D^{*0}), where the error is obtained by combining the error on twice the D^0 mass and that on the $D^{*0}-D^0$ mass difference in quadrature.

We choose $X(3872)$ mass values 3870.8, 3871.4 and 3872.0 MeV/c^2 (i.e. the PDG 2007 average and plus or minus one sigma), and for each use Γ_{tot} values 2, 3 and 4 MeV. For each of these nine combinations we use Eq.(11) to compute the lineshape for the choices $L = 0$ and 2 with $J = 1$ (i.e. $X(3872)$ $J^P = 1^+$), and $L = 1$ and 3 with $J = 2$ (i.e. $X(3872)$ $J^P = 2^-$). For $L = 2$, all of the lineshapes obtained are very broad, reaching a maximum close to mass 3.90 GeV/c^2, while for $L = 3$ the distribution obtained increases monotonically through 3.91 GeV/c^2, the upper limit of the region investigated. It follows that for $L = 2$ and $L = 3$ Eq.(11) yields behavior which is totally unlike that observed for data [10, 11], and so the possibility that such contributions play a significant role in $X(3872)$ decay to $D^{*0}\bar{D}^0$ is discarded.

For the remaining ($J^P = 1^+ : L = 0, J = 1$) and ($J^P = 2^- : L = 1, J = 2$) possibilities, Eq.(11) does lead to lineshapes which peak at mass values a few MeV/c^2 larger than the input $X(3872)$ values. The mass shift is defined as the difference between the observed peak mass position and the input $X(3872)$ mass value, m_0, and the mass shifts obtained are summarized in Tables II and III respectively. In both Tables, the mass shift increases with decreasing m_0 and increasing Γ_{tot}. However each value in Table II is smaller than the corresponding one in Table III, the difference averaged over the Tables being ∼1.85 MeV/c^2. Furthermore, it seems from Table II to be unlikely that a mass shift of ∼3 MeV/c^2 can be obtained for a reasonable choice of $X(3872)$ mass and width, especially since the $D^{*0}\bar{D}^0$ mass distribution which results becomes considerably broader as the $X(3872)$ mass is decreased and its width increased.

Using $X(3872)$ mass 3871.4 MeV/c^2 and width 3 MeV, we illustrate the lineshape behavior obtained from Eq.(11) for $J^P = 1^+$ and $J^P = 2^-$ in Fig.1 and Fig.2 respectively. In each figure, the curve is obtained using Eq.(11), and is then used to generate the 3000 events shown in the histogram below, which uses the mass intervals from the BaBar analysis [11]. The peak

	Γ_{tot} [MeV]		
$X(3872)$ mass [MeV/c^2]	2	3	4
3870.8	1.54	1.75	1.99
3871.4	0.90	1.17	1.45
3872.0	0.45	0.75	1.02

TABLE II: Dependence of the peak mass shift (in MeV/c^2) on $X(3872)$ mass and total width for $L = 0$ and $J = 1$ (i.e. $J^P = 1^+$).

	Γ_{tot} [MeV]		
$X(3872)$ mass [MeV/c^2]	2	3	4
3870.8	3.79	4.25	4.75
3871.4	2.37	2.99	3.61
3872.0	1.28	1.98	2.66

TABLE III: Dependence of the peak mass shift (in MeV/c^2) on $X(3872)$ mass and total width for $L = 1$ and $J = 2$ (i.e. $J^P = 2^-$).

shift in Fig.2(a) agrees well with the result from experiment [10, 11], and the observed signal shapes seem better represented by that of Fig.2(b) than by that of Fig.1(b). However, it must be acknowledged that the experimental uncertainties are significant, and that even the uncertainty in the location of the $D^{*0}\bar{D}^0$ threshold (above) is relevant on the scale of the effect under discussion.

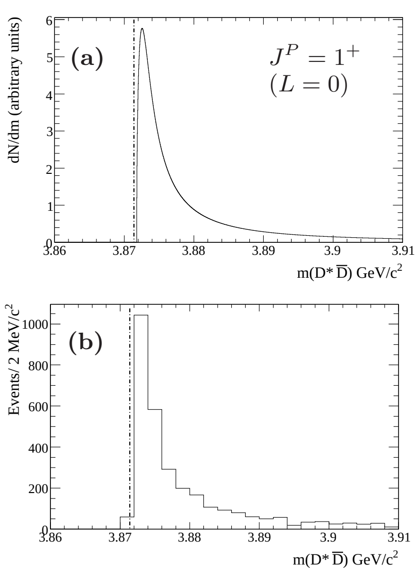

FIG. 1: (a) The $m(D^{*0}\bar{D}^0)$ lineshape obtained from Eq.(11) for $J^P = 1^+$ using $X(3872)$ mass 3871.4 MeV/c^2 (indicated by the dot-dashed line) and width 3 MeV. (b) The histogram for 3000 events generated using the curve shown in (a).

We have made no attempt to study the effect of detector resolution on the mass shifts calculated above.

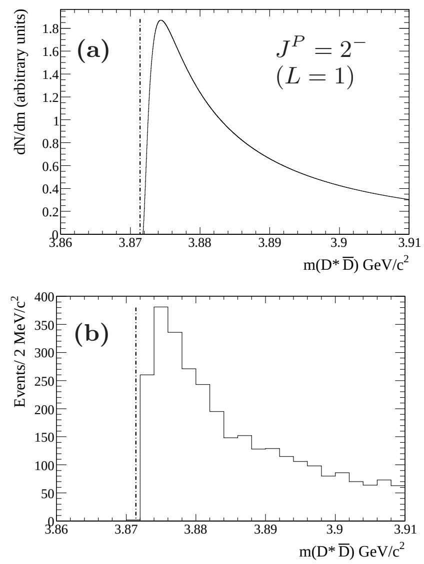

FIG. 2: (a) The $m(D^{*0}\bar{D}^0)$ lineshape obtained from Eq.(11) for $J^P = 2^-$ using $X(3872)$ mass 3871.4 MeV/c^2 (indicated by the dot-dashed line) and width 3 MeV. (b) The histogram for 3000 events generated using the curve shown in (a).

Near threshold, such effects should not be represented by Gaussian smearing in mass, as is done usually, since this will yield contributions below threshold. It is three-momentum resolution which is the source of the smearing, and this must be investigated by full detector simulation for the experiment in question. Since such simulation obviously cannot yield events below threshold, it seems probable that the peak mass shifts calculated above will be increased as a result of experimental resolution. We suspect that such effects will be small ($<$ 1 MeV/c^2), but a thorough investigation making use of detector simulation is necessary.

In summary, we have shown that a simple treatment of the orbital angular momentum involved in $X(3872)$ decay to $D^{*0}\bar{D}^0$ can account for the difference between the mass measured in this mode and that obtained from $J/\psi\pi^+\pi^-$ decay. The results favor $J^P = 2^-$ over $J^P = 1^+$ for the $X(3872)$, but the uncertainty in the measured mass difference (3.7 ± 1.2 MeV/c^2), and the absence of simulated detector resolution effects, prevent a definite conclusion. If our interpretation is correct, a corollary is that the width of the X(3872) cannot be much smaller than $\sim$2 MeV, since otherwise significant displacement of the invariant mass peak for $D^{*0}\bar{D}^0$ would not occur.

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