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Modern Statistical Methods for GLAST Event Analysis

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Abstract. We describe a statistical reconstruction methodology for the GLAST LAT. The methodology incorporates in detail the statistics of the interactions of photons and charged particles with the tungsten layers in the LAT, and uses the scattering distributions to compute the full probability distribution over the energy and direction of the incident photons. It uses model selection methods to estimate the probabilities of the possible geometrical configurations of the particles produced in the detector, and numerical marginalization over the energy loss and scattering angles at each layer. Preliminary results show that it can improve on the tracker-only energy estimates for muons and electrons incident on the LAT.

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INTRODUCTION

The Large Area Telescope (LAT) [1] is the primary instrument on GLAST, and so it is of utmost importance to extract as much information as possible from the response of the LAT to incident photons and particles. While the quantities of primary interest for each event are few (namely the azimuth, elevation and energy of the incident photon/particle), the rich physics of the interactions of the particles/photons with the LAT makes a principled reconstruction algorithm complex. Whilst the objects of primary interest are photons, the interaction of a photon with the LAT is essentially that of an electron-positron pair. Therefore we concentrate on the basic building blocks of event reconstruction, the analysis of the interaction of charged particles with the LAT.

Figure 1 (left) shows the schematic of an interaction between a charged particle and the LAT. Visible in the figure are 1) multiple Coulomb scattering in the tungsten foils; 2) the production of secondary photons; and 3) the production of secondary charged particles. The GEANT4 toolkit [2] is designed to simulate these physics processes in the *forward* direction. The task in event reconstruction is the *inverse* problem – estimating, from the data of the microstrip responses, which physics processes actually occurred in a particular event. The result is an estimate not only for the original particle and its properties, but also of all secondary particles and photons. To accurately estimate the primary particle, it is necessary to estimate accurately all secondaries.

Figure 1 (right) shows the tree of hypotheses for the physical processes at the first two layers of interaction of a charged particle with the LAT. The final leaves describe the *structure* of the hypothesized event reconstructions. Clearly as we descend the layers the number of branches in this tree explodes. But only a small number of their leaves will be consistent with the instrument data - with the pattern of microstrips that fired - and, further, this consideration can be used to exclude branches as the tree is descended, further limiting the number of leaves that must be considered. A full event reconstruction consists of two stages:

1. The enumeration of the possible event structures consistent with the microstrip data.
2. The computation of the parameters of each event structure and their relative probabilities.

Computation of the relative probabilities allows the final event reconstruction to be a weighted average of event structures, weighted according to the probability that the microstrip data actually came from that structure.

METHODOLOGY

We concentrate first on the simplest type of event. If the only physical process that actually occurred was multiple Coulomb scattering, then there is only a single (x-y) pair of microstrips at each layer (excluding noisy strips). This is the case for muons of moderate energy (up to a few hundred MeV), where the probability of producing secondary

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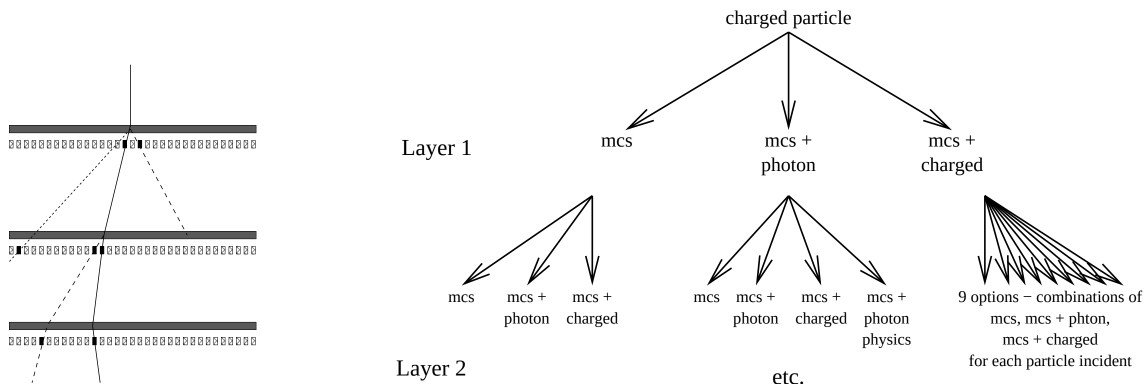

FIGURE 1. LEFT: A schematic of a charged particle interacting with the LAT. The solid line is the incident charged particle. Long-dashed lines indicate secondary charged particles. The short-dashed line is a secondary photon. The solid blobs indicate microstrips that fired. RIGHT: The tree of possible event structures

electrons or photons is extremely small, and hence can be neglected. We parameterize the trajectory of the particle by 1) its origin (a point outside the LAT); 2) the position at which it traverses each conversion layer; and 3) its endpoint (also outside the LAT); and from these we derive the incident directions (θ,ϕ) and the scattering angles, θ_i, at each layer. Finally, we add the incident energy, E, and the energy deposited in each conversion layer δE_i. Denoting by s_i the microstrips at each layer, we can write

$$p(\theta,\phi,E,\theta_1,\delta E_1,\ldots\theta_n,\delta E_n|s_1\ldots s_n) \propto p(s_1\ldots s_n|\theta,\phi,E,\theta_1,\delta E_1,\ldots\theta_n,\delta E_n)\,p(\theta,\phi,E,\theta_1,\delta E_1,\ldots\theta_n,\delta E_n) \quad (1)$$

The first term on the right hand side is the likelihood. It takes one of two values – one if the trajectory described by $\theta,\phi,\theta_1,\ldots\theta_n$ intersects all the microstrips that fired, and zero otherwise. It serves to limit the region of the state space that is of interest. The second term contains all the physics of the interactions of the particle with the LAT. We use conditional independence and decompose it as follows.

$$\begin{array}{rl} p(\theta,\phi,E) & \text{Priors on azimuth, elevation and energy.} \\ \times p(\theta_1,\delta E_1|E) & \text{Distribution of scattering angle and energy loss for a particle of energy } E. \\ \times p(\theta_2,\delta E_2|E,\delta E_1) & \text{Same for the particle at layer 2, which has energy } E-\delta E_1. \\ \times\ldots & \\ \times p(\theta_n,\delta E_n|E,\delta E_1,\ldots,\delta E_n) & \text{At layer } n \text{ the particle has energy } E-\delta E_1-\ldots-\delta E_{n-1}. \end{array} \quad (2)$$

These scattering distributions are the known distributions for particles of a specified energy incident on a LAT foil [3]. The non-Gaussian tails of the scattering angle distributions were modeled by a second Gaussian component. For muons, the energy loss was parameterized as a Landau distribution, with the distribution's parameters being functions of energy.

The parameters of primary interest, however, are the azimuth, elevation and energy, and so the distribution of primary interest is $p(\theta,\phi,E)$. This is obtained from (1) by marginalization. This is performed numerically using Markov chain Monte Carlo (MCMC). Originally developed in physics [4], it has been extensively developed in statistics in the past 20 years, and is now a standard tool for use in the analysis of complex, high-dimensional probability distributions [5]. It works by simulating a Markov chain whose equilibrium distribution is constructed to be the distribution of interest (in this case, the distribution in (1)). Averages over the distribution can then be made by forming averages over the states of the simulated Markov chain. For example, the mean energy is estimated by forming the mean of the energy variables over a length of the simulated chain, while ignoring all the other variables. Collecting all the variables into $\boldsymbol{x}$, and initializing $\boldsymbol{x} \leftarrow \boldsymbol{x}_0$ the MCMC algorithm is iteration of

1. propose a change, $\boldsymbol{x} \leftarrow \boldsymbol{x}'$ with some proposal distribution $\pi(\boldsymbol{x}';\boldsymbol{x})$
2. accept the change with $p_a = \frac{p(\boldsymbol{x}')\pi(\boldsymbol{x};\boldsymbol{x}')}{p(\boldsymbol{x})\pi(\boldsymbol{x}';\boldsymbol{x})}$, and set $\boldsymbol{x} \leftarrow \boldsymbol{x}'$, else retain $\boldsymbol{x}$

The proposal distribution at each stage may be chosen to only change some of the elements of $\boldsymbol{x}$. For this work, we use a cycle of proposals that successively proposes changes to $\theta,\phi,E,\theta_1,\delta E_1,\ldots,\theta_n,\delta E_n$.

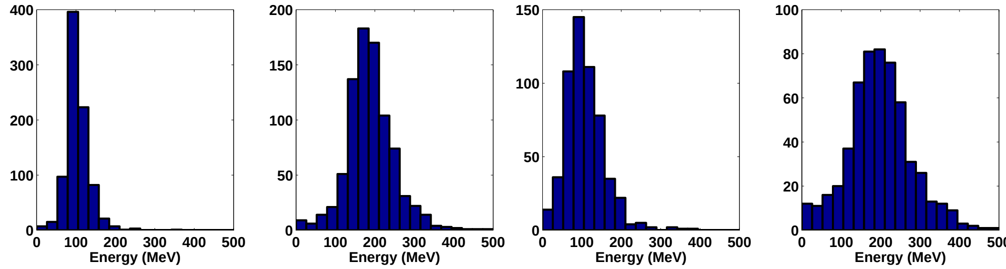

FIGURE 2. Energy estimates for muons (2 left panes) and electrons (2 right panes)

For electrons incident on the LAT foils, as well as multiple Coulomb scattering, there is appreciable probability of producing a secondary photon, and a small probability of producing a secondary electron. (At 100MeV these probabilities are $\simeq 0.25$ and $\simeq 0.01$ respectively.) We restrict the discussion here to events which contain at most secondary photons. Typically, for electrons of a few hundred MeV the secondary photons are not detected. They carry energy away from the electron which is "lost" to the tracker.

In the forward direction this is modeled by a mixture distribution. With probability $p_{\mathrm{ns}}(E)$ no secondary is produced, and the energy loss follows a Landau distribution. With probability $p_{\mathrm{s}}(E)$, a photon is produced and the energy loss has two components, a Landau distributed component from multiple Coulomb scattering, plus a component distributed as $1/E$ representing the energy carried away by the photon.

The samples generated by the MCMC algorithm represent the distribution over the trajectories' parameters. To compute the probability for an event structure it is necessary to compute the normalizing factor that was omitted from equation (1). This can be done by using the MCMC output to construct an importance sampling distribution, and using samples from that distribution to compute the normalizing factor. This will be discussed elsewhere.

RESULTS

One thousand events were simulated [6] for each of four sources – muons and electrons of 100 and 200 MeV. The incidence directions were chosen randomly within a 45 degree cone. Figure 2 shows the energy estimates of the reconstruction. We do not show direction estimates for reasons of space and also because, for charged particles, the accuracy of the direction estimate is determined almost entirely by multiple Coulomb scattering in the top foil. For electron events, those events where two charged tracks were detected by the reconstruction algorithm were not analyzed. In all four cases the estimates are unbiased; the histograms are centered accurately on either 100MeV or 200MeV. The histograms for electrons show more dispersion than those for muons, due to the effect of energy being transferred into photons which are not detected. Note that these estimates were made using only the first 12 regular GLAST layers of the tracker and did not use any information from the calorimeter. In an upcoming paper we will present detailed results and comparisons with current methodology.

ACKNOWLEDGMENTS

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