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Spin Echo and Interference in Synchrotrons*

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Abstract

Spin dynamics in crossing a single depolarization resonance is a well-studied subject. One well-known example is that of Froissart and Stora in 1960 [1]. More is needed to complete the understanding, particularly of the transient effects, when crossing a single resonance [2]-[4], but question arises what happens if we cross two resonances or cross a single resonance twice. When a resonance is crossed twice, the particle's spin dynamics encounters two additional phenomena. First, the two crossings will interfere with each other, leading to an interference effect. Second, there will be a spin echo effect. We discuss these two effects in this report. Two proposals to test these effects experimentally are made at the end.

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Spin Echo and Interference in Synchrotrons

Alex Chao

1 JUMP CROSSING A DEPOLARIZATION RESONANCE

Consider a single particle near a depolarization resonance when its spin tune $G\gamma \approx \kappa$, where κ specifies the resonance location, and

$$G\gamma = \kappa + \alpha(\theta) \tag{1}$$

where $\alpha(\theta)$ is a function of time $\theta =$ (number of turns) $\times\, 2\pi$. Let the resonance strength be ϵ, a complex quantity related to the Fourier component of perturbing depolarizing magnetic fields.

In spinor notation, the spin dynamics is described by [1], [5]-[10]

$$\frac{d\psi}{d\theta} = -\frac{i}{2}\begin{bmatrix} -G\gamma & \epsilon e^{i\kappa\theta} \\ \epsilon^* e^{-i\kappa\theta} & G\gamma \end{bmatrix}\psi \tag{2}$$

where ψ is the two-component spinor. For a planar synchrotron, we will primarily be interested in the vertical y-component of the polarization,

$$P_y(\theta) = \psi^\dagger \sigma_y \psi \qquad \text{with Pauli matrix} \quad \sigma_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3}$$

In crossing a resonance, the simplest case to treat is when the resonance is crossed by a sudden jump in the spin tune. Consider the case of a jumping pattern in $\alpha(\theta)$ as shown in Fig.1. A resonance of strength ϵ_0 is jump-crossed twice at times θ_1 and θ_2.

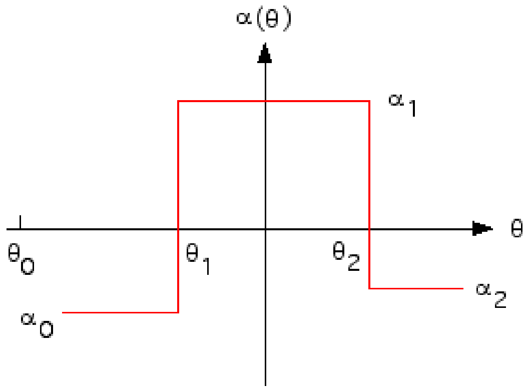

Figure 1: Two crossings of a resonance by sudden jumps of spin tune.

Polarization has been calculated for this case. Details can be found in [11]. We assume that the spin was initially 100% polarized and was adiabatically brought to a launching condition at time $\theta = \theta_0$. The launching y-component of polarization is given by

$$P_y(\theta_0) = \frac{|\alpha_0|}{\sqrt{\alpha_0^2 + |\epsilon_0|^2}} \tag{4}$$

Calculation yields explicit expressions for $P_y(\theta)$ for the three time intervals $\theta_1 > \theta > \theta_0$ (before the first jump), $\theta_2 > \theta > \theta_1$ (between the two jumps), and $\theta > \theta_2$ (after the second jump). Suffice it to say here that the polarization before the first jump is a constant value given by (4), and that the polarization oscillates with frequency $\Omega_1 = \sqrt{\alpha_1^2 + |\epsilon_0|^2}$ between the two jumps and with frequency $\Omega_2 = \sqrt{\alpha_2^2 + |\epsilon_0|^2}$ after the second jump.

A special case occurs when

$$\alpha_0 = -A, \quad \alpha_1 = A, \quad \alpha_2 = -A \tag{5}$$

In this case, our expressions become algebraically simpler, and the oscillation frequencies read $\Omega_1 = \Omega_2 = \Omega = \sqrt{A^2 + |\epsilon_0|^2}$.

2 INTERFERENCE

For the special case (5) it can be shown that there is a complete destructive interference between the two resonance jumps if

$$\Omega(\theta_2 - \theta_1) = 2k\pi \tag{6}$$

where k is an integer. In this case, the two crossings destructively annihilate each other and the final polarization is equal to the launching polarization $|\alpha_0|/\Omega_0$.

There is also a constructive interference that occurs when

$$\Omega(\theta_2 - \theta_1) = (2k+1)\pi \tag{7}$$

In this case, the final polarization reads

$$P_y(\theta > \theta_2) = \frac{|A|}{\Omega^5}\Big[(A^4 - 6A^2|\epsilon_0|^2 + |\epsilon_0|^4) + 4|\epsilon_0|^2(|\epsilon_0|^2 - A^2)\cos\Omega(\theta - \theta_1)\Big] \tag{8}$$

Examples of interferences are shown in Figs.2 (destructive interference) and 3 (constructive interference).

It should be emphasized that, after crossing a resonance, the memory of crossing lasts indefinitely. Resonance crossings should not be generally considered to be separate events. However, this interference effect has conventionally not been taken seriously; in what follows, we will explore the conditions when this is justified.

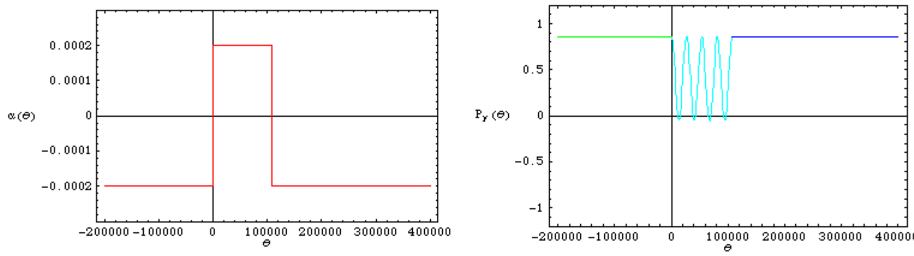

Figure 2: Left figure shows the resonance crossing pattern $\alpha(\theta)$. Right figure shows the polarization $P_y(\theta)$. Parameters used are $A = 2 \times 10^{-4}, |\epsilon_0| = 1.2 \times 10^{-4}, \theta_0 = -2 \times 10^5, \theta_1 = 0$. The two jumps destructively interfere as the polarization makes 4 complete oscillations during the time between the two jumps.

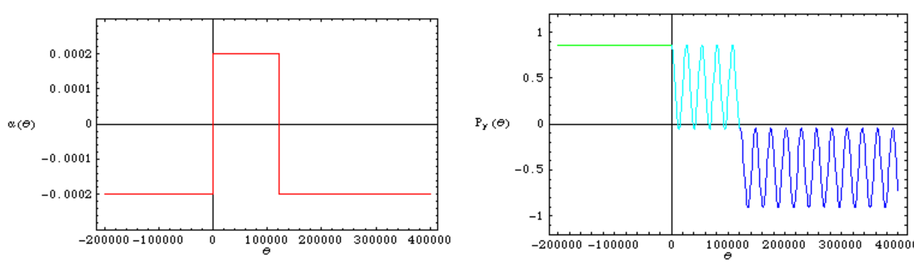

Figure 3: Same as Fig.2, except that the two jumps constructively interfere as the polarization makes $4\frac{1}{2}$ complete oscillations during the time between the two jumps.

3 A BEAM OF PARTICLES WITH ENERGY SPREAD

Consider a case when the on-momentum particle follows the prescription (5). For an off-momentum particle with energy deviation $\delta = \Delta\gamma/\gamma_0$, its spin tune is

$$G\gamma(\theta) = \kappa + \begin{cases} -A + \kappa\delta, & \text{if } \theta < \theta_1 \\ A + \kappa\delta, & \text{if } \theta_1 < \theta < \theta_2 \\ -A + \kappa\delta, & \text{if } \theta_2 < \theta \end{cases} \tag{9}$$

We assume that $|\delta| \ll 1, |\kappa\delta| \ll 1$ and $|\kappa\delta| \ll A$.

We then apply the explicit polarization results to the off-momentum particle, and note that the momentum deviation makes important contributions only through the phases in the sinusoidal terms. The result obtained applies to the case of a single particle. For a beam with energy spread, an averaging over the beam's energy distribution will have to be performed. Assuming the energy distribution is Gaussian with rms σ_δ, the result is

$$\begin{aligned}
P_y(\theta<\theta_1) &\approx \frac{A}{\Omega} \\
P_y(\theta_2>\theta>\theta_1) &\approx \frac{A}{\Omega^3}\left\{A^2 - |\epsilon_0|^2 + 2|\epsilon_0|^2 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta-\theta_1)^2}\cos\Omega(\theta-\theta_1)\right\} \\
P_y(\theta>\theta_2) &\approx \frac{A}{\Omega^5}\left\{(A^2-|\epsilon_0|^2)^2 + 2|\epsilon_0|^4 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(2\theta_2-\theta-\theta_1)^2}\cos\Omega(\theta-\theta_1)\right. \\
&- 2A^2|\epsilon_0|^2 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta-\theta_1)^2}\cos\Omega(\theta+\theta_1-2\theta_2) \\
&+ 2|\epsilon_0|^2(A^2-|\epsilon_0|^2) e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta-\theta_2)^2}\cos\Omega(\theta-\theta_2) \\
&+ \left. 4A^2|\epsilon_0|^2 e^{-\frac{A^2\kappa^2\sigma_\delta^2}{2\Omega^2}(\theta_2-\theta_1)^2}\cos\Omega(\theta_2-\theta_1)\right\}
\end{aligned} \tag{10}$$

In $P_y(\theta_2>\theta>\theta_1)$, the sinusoidal oscillating term with oscillation frequency Ω is the shock response of the beam polarization to the first resonance jump.

In $P_y(\theta>\theta_2)$, there are four oscillating terms, all with oscillation frequency Ω, and each with its own physical meaning. The third oscillating term gives the shock response to the second resonance crossing. The fourth term describes an interference between the two crossings. (This term is independent of time θ, so strictly speaking, it is not an "oscillating" term.) The remaining two terms give rise to spin echo, while the first term will dominate over the second term.

Each of the oscillating terms in (10) contains an exponential factor corresponding to the effect of phase smearing due to the finite beam energy spread. Each oscillating term is damped in $N_{\rm smear}$ turns, where $N_{\rm smear} \approx \frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta}$. All the oscillating terms will be significant only within a time span of the order of $\Delta\theta \sim 2\pi N_{\rm smear}$ centered around specific values of time θ. The shock terms will center around $\theta = \theta_{1,2}$, while the echo term will center around $\theta = 2\theta_2 - \theta_1$.

The interference effect is pronounced when

$$\frac{1}{\kappa} \ll N_{\rm jump} \ll \left\{ \begin{array}{c} \frac{1}{\Omega} \\ \\ \\ \frac{1}{2\pi}(\theta_2 - \theta_1) \end{array} \right\} \ll \frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta} \tag{11}$$

The spin echo effect is pronounced when

$$\frac{1}{\kappa} \ll N_{\rm jump} \ll \left\{ \begin{array}{c} \frac{1}{\Omega} \\ \\ \\ \frac{\sqrt{2}\,\Omega}{2\pi|A|\kappa\sigma_\delta} \end{array} \right\} \ll \frac{1}{2\pi}(\theta_2 - \theta_1) \tag{12}$$

In this regime, the interference term does not contribute, and can be dropped.

4 SPIN ECHO

We are now ready to calculate the echo effect for a beam with energy spread. One example is shown in Fig.4. Upper-left figure reproduces the case of Fig.3 when $\sigma_\delta = 0$. Upper-right and lower figures are cases in the regime (12), and with increasing σ_δ. Each of these two figures contains three separated, peaked responses, centered around $\theta = \theta_1$ (shock response to first crossing), $\theta = \theta_1 + \tau$

(shock response to second crossing), and $\theta = \theta_1 + 2\tau$ (echo response), where $\tau = \theta_2 - \theta_1$ is the time separation between the two jumps.

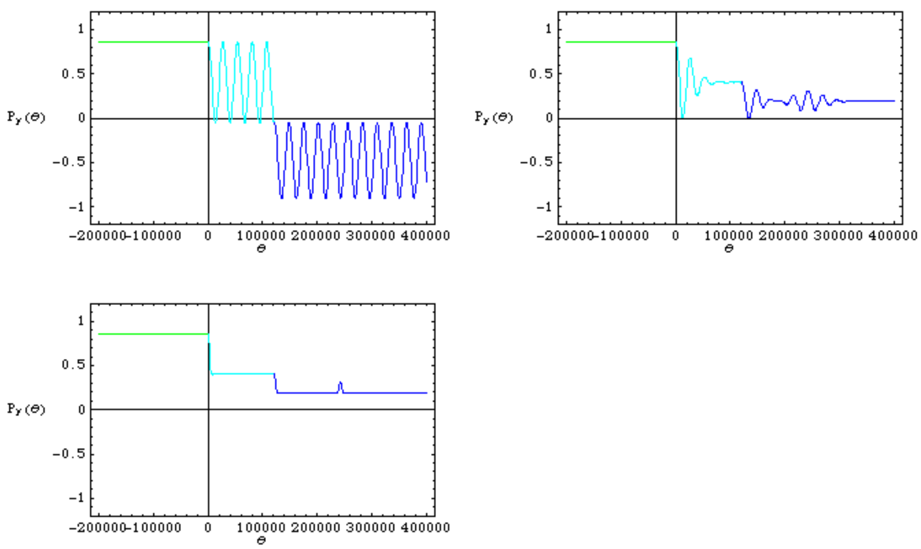

Figure 4: Conditions are the same as in Fig.3, except for an energy spread; $\sigma_\delta = 0$ (upper-left), $\sigma_\delta = 10^{-5}$ (upper-right), $\sigma_\delta = 10^{-4}$ (lower).

The magnitude of the echo signal, relative to its background value, is $P_{y,\text{echo}} = 2A|\epsilon_0|^4/\Omega^5$. It follows that this echo signal is maximum when $|A|_{\text{max. echo}} = \frac{1}{2}|\epsilon_0|$, while $P_{y,\text{max.echo}} = (4/5)^{5/2} = 57\%$, a perhaps surprisingly large value.

5 TWO EXPERIMENTS

We propose two possible experiments, one for detecting echo and one for detecting interference, possibly using a 2.1 GeV/c proton beam of COSY [3]. In these experiments, resonances are introduced using a RF dipole [2, 3]. The strength of the resonance is controlled by the dipole strength. The resonance tune is determined by its RF frequency. The speed of resonance crossing is determined by the speed at which its RF frequency is varied. We will suggest to cross the resonances rapidly to assimilate sudden jumps. The beam energy spread proposed will require electron cooling.

The beam is assumed to be 100% polarized initially away from the resonance. With the resonance strength turned on to the value ϵ_0, the beam is adiabatically brought to a launching position where the spin tune of the beam's on-momentum particle to a distance $-A$ from the resonance $G\gamma = \kappa$. Starting from this launching position, a resonance jump is made (in N_{jump} turns). The on-momentum spin tune is made to be equal to $+A$ after the jump. The beam is then parked there for a period of time τ (or $\tau/2\pi$ turns). At time τ after the first jump, a second resonance jump is performed, bringing the on-momentum spin tune from $+A$ back to $-A$. The beam is then parked at this new position, while beam polarization P_y is measured using a polarimeter. Throughout the procedure, resonance strength is kept at ϵ_0.

5.1 Echo experiment

For the echo experiment, we propose [12]: $\kappa = 4.4, \sigma_\delta = 10^{-4}, |\epsilon_0| = 10^{-3}, A = 0.5 \times 10^{-3}, N_{\text{jump}} < 100$ turns. Figure 5 shows the expected polarization behavior of this experiment when $\tau = \theta_2 - \theta_1 = 2\pi \times 8000$, or 8000 turns. To dramatize the echo effect, one may increase τ by a large factor, e.g. a factor of

1000.

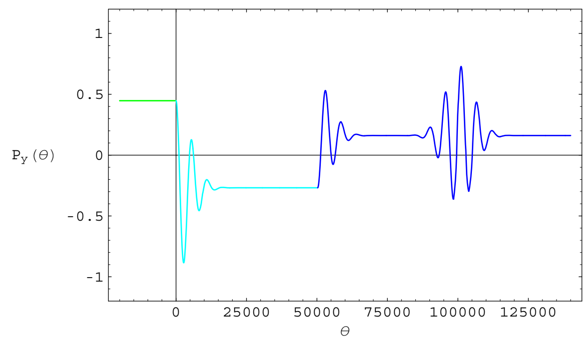

Figure 5: Expected polarization in an echo experiment.

In the experiment, if we assume the polarization measurement accuracy of $\pm 1\%$ when gated at a 200 ms time window (assuming 30 spin-up and 30 spin-down cycles), the expected accuracy of 0.5 ms window would be $\pm 10\%$ assuming 120 spin-up and 120 spin-down cycles [12]. This $\pm 10\%$ statistics is to be compared with the expected echo polarization signal of 57%.

5.2 Interference experiment

For an interference experiment, we suggest [12] $\kappa = 4.4, \sigma_\delta = 10^{-4}, |\epsilon_0| = 3 \times 10^{-4}, A = 6 \times 10^{-4}, \tau = \frac{2\pi}{\Omega} = 9.4 \times 10^3 = 1500$ turns for destructive interference, and $\frac{\pi}{\Omega} = 4.7 \times 10^3 = 750$ turns for constructive interference, and $N_{\rm jump} < 100$ turns. The expected results are shown in Fig.6. The final polarization depends sensitively on the time between the two resonance crossings.

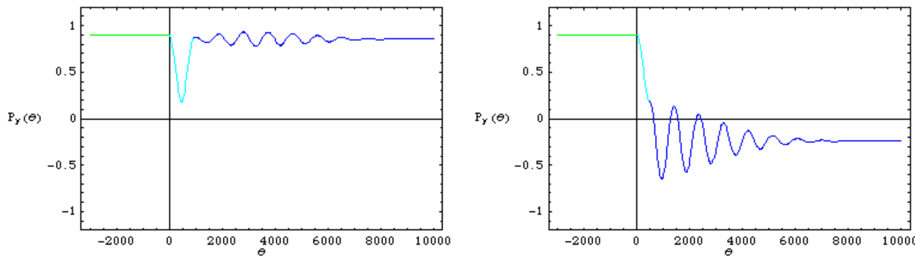

Figure 6: Expected polarization in an interference experiment. The left (right) figures are when the two resonance crossings interfere destructively (constructively).

6 Acknowledgements

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