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# Current and potential distribution in a divertor with toroidally-asymmetric biasing of the divertor plate

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## Abstract

Toroidally-asymmetric biasing of the divertor plate may increase convective cross-field transport in SOL and thereby reduce the divertor heat load. Experiments performed with the MAST spherical tokamak generally agree with a simple theory of non-axisymmetric biasing. However, some of the experimental results have not yet received a theoretical explanation. In particular, existing theory seems to overestimate the asymmetry between the positive and the negative biasing. Also lacking a theoretical explanation is experimentally observed increase of the average floating potential in the main SOL in the presence of biasing. In this paper we attempt to solve these problems by accounting for the closing of the currents (driven by the biasing) in a strong-shear region near the X-point. We come up with the picture which, at least qualitatively, agrees with these experimental results.

Toroidally-asymmetric biasing of the divertor plate may increase convective cross-field transport in SOL and thereby reduce the divertor heat load [1-4]. This phenomenon has been studied in the lower "rib-like" divertor of the MAST spherical tokamak [5 - 7]. The plasma in this divertor was collected by 12 vertical ribs all having the same height over the underlying flat structure (see, e.g., Fig. 3 in Ref. [3]). Every second rib could be biased to a positive or negative voltage with respect to the ground; the rest of the ribs were held at the ground potential. It is the potential difference between the neighboring flux tubes (leaning on the neighboring ribs) that drives strong E×B drifts and thereby induces the plasma convection. A comprehensive theory of this complex phenomenon is still unavailable, although a semi-quantitative analysis presented in Ref. [3, 4] agrees well with most of experimental observations [5 - 7]. However, some of the experimental results have not yet received a theoretical explanation. In particular, existing theory seems to overestimate the asymmetry between the positive and the negative biasing [5]. Also lacking a theoretical explanation is experimentally observed increase of the average floating potential in the main SOL in the presence of positive biasing [5].

The reason for these discrepancies lies probably in an over-simplified model of the current flow between the biased and un-biased divertor ribs used in Refs. [3, 4]. This model was as follows: we considered two adjacent flux tubes, one leaning on the biased rib, and the other leaning on the un-biased rib (the whole structure was a periodic continuation of this system). The opposite end of the flux-tubes was identified with the upper divertor which was un-biased. The plasma potential in the biased flux tube was then determined from the sheath current-voltage characteristics; the plasma potential in the un-biased flux tube remained equal to the floating potential. An expression for the potential difference $\Delta\varphi$ between the biased and unbiased flux-tube obtained from this model was:

$$\Delta\varphi = \frac{T_e}{e}\ln\frac{\exp(e\varphi_b/T_e)+1}{2} \qquad (1)$$

One sees that there would be a strong asymmetry of the attainable potential difference for the positive and the negative biasing: in the first case, at large $\varphi_b >> T_e/e$, one would have $\Delta\varphi \approx \varphi_b$, whereas in the second case the potential difference would not exceed (in absolute value) $(T_e/e)\ln 2$. As in this paper we will be using several parameters of the dimension of potential, and in order to avoid possible mix-up, we define all of them in Table I.

**Table 1** Definitions of parameters of the dimension of electric potential used throughout this paper

| Parameter | Notation | Equation number where first used |
|---|---|---|
| Potential of biased ribs with respect to the ground | $\varphi_b$ | (1) |
| Plasma potential in a flux-tube leaning on the biased rib | $\varphi_{pb}$ | (5) |
| Plasma potential in a flux-tube leaning on the grounded rib | $\varphi_{p0}$ | (6) |
| Difference between the two | $\Delta\varphi \equiv \varphi_{pb} - \varphi_{p0}$ | (1) |
| Floating potential | $\varphi_f$ | (10) |
| Potential of the SOL well above the X-point | $\varphi_{symm}$ | (11) |
| Difference between $\varphi_{symm}$ and the floating potential | $\Delta\varphi_{symm} \equiv \varphi_{symm} - \varphi_f$ | (14) |

The potential difference between the adjacent flux-tubes is the main drive for convection. Accordingly, the simple model based on Eq. (1), would predict a much stronger broadening of the wetted zone for the positive bias (as in this case $\Delta\varphi \sim \varphi_b$) compared to the negative bias (as in this case, according to Eq. (1), $\Delta\varphi << \varphi_b$). On the other hand, the experiment [5] did not reveal a strong asymmetry. The other point that could not be easily explained within the framework of a simple model, was a spatially uniform change of a potential of the upper SOL in the course of biasing (small-scale structures were absent, in agreement with theory [1-4]).

In the present paper, we attempt to solve these problems by accounting for the closing of the currents (driven by the biasing) in a strong-shear region near the X-point. As was pointed out in Ref. [8], a very strong shearing of flux-tubes in the vicinity of the X-point leads to that, on their way to the main SOL, they soon become thinner than the ion gyro-radius. For the typical parameter of the MAST tokamak this happens well before the flux tubes reach the tokamak equatorial plane. In Ref. [9], based on this observation, it was noted that squeezing of flux-tubes to the width much less than the ion gyro-radius, should lead to a significant cross-field current leaks between two adjacent flux-tubes having different potentials in the divertor leg. The closure of the current occurs in the zone extending from just below to just above the X-point (a “transition layer”). In other words, the current would not reach the upper divertor but would rather close near the lower X-point.

The closure of the current through this layer can be described in terms of a boundary condition relating the cross-field current with the potential difference between flux-tubes in the divertor. For a sinusoidally varying potential, with the cross-field wave-number $k$, the boundary condition reads as [9]:

$$j = |k|\sigma_h \Delta\varphi , \tag{2}$$

where $\sigma_h$ is a "heuristic electrical conductivity,"

$$\sigma_h = G \cdot \frac{\omega_{pe}^2}{4\pi\omega_{ce}} , \tag{3}$$

and $G$ is a numerical "adjustment factor" of order 1 accounting for the qualitative nature of the heuristic model of Ref. [9].

For the toroidal wave number of the biasing potential $k_T$, one has $k=(B_T/B_P)k_T$, where $B_T$ is the toroidal field and $B_P$ is the poloidal field halfway between the divertor plate and the X-point. If the major radius of the wetted zone is $R$, then the principal harmonic of the toroidal wave number would be $k_T=N/2R$, where $N$ is the number of ribs. The factor "2" here accounts for the fact that only every other rib is biased. As the potential in the case of the MAST facility is applied to discrete ribs [5], there will also be higher harmonics present in the Fourier decomposition of the applied potential. To account for that fact, we will increase $k_T$ by a factor of "2", ending up with the following expression for $k$:

$$k \approx \frac{NB_T}{RB_P} \tag{4}$$

The $B_P$ here has to be taken somewhere halfway between the divertor plate and the X-point. For typical MAST conditions Ref. [5], $N$=12, $R$=110 cm, $B_T$=0.2 T, $B_P$~0.02 T (the poloidal field at the divertor plate is 0.04 T), one has $k$~1 cm$^{-1}$.

In our model, the current pattern is as follows: it starts on the biased rib, reaches the "control surface" just beneath the X-point, then flows across the magnetic field in a strongly sheared layer in the vicinity of the X-point, and closes to the un-biased rib. The equivalent circuit is represented by Fig. 1. The plasma resistance between the lower and the upper divertor involves a long connection length and is comparable to or higher than the cross-field resistance. For this reason we neglect any current leak to the upper divertor.

Using the standard sheath boundary condition [10], one obtains the following expression for the current flowing from the biased rib:

$$j = -enu + en\frac{\mathrm{v}_{Te}}{2\sqrt{\pi}}\exp\left(\frac{e\varphi_b - e\varphi_{pb}}{T_e}\right) \tag{5}$$

where $n$ is the plasma density, $u$ is the ion velocity at the plasma side of the sheath, $\mathrm{v}_{Te} = \sqrt{2T_e/m_e}$ is the electron thermal velocity, and $\varphi_{pb}$ is the plasma potential over the biased rib. We use the standard convention regarding the sign of the current (it is positive when current flows from the positive to the grounded electrode). The unbiased rib has a zero potential. The plasma potential over the unbiased rib will be denoted by $\varphi_{p0}$. Then, analogously to Eq. (5),

$$j = enu - en\frac{v_{Te}}{2\sqrt{\pi}}\exp\left(\frac{-e\varphi_{p0}}{T_e}\right). \qquad (6)$$

The current closes through the resistive layer near the X-point. Then, according to Eqs. (2)-(4),

$$j = \left(\varphi_{pb} - \varphi_{p0}\right)\Sigma, \qquad (7)$$

where

$$\Sigma = G\cdot\frac{\omega_{pe}^2}{4\pi\omega_{ce}}\cdot\frac{B_T N}{B_P R}. \qquad (8)$$

We neglect the (small) plasma resistance in the divertor legs (Cf. Ref. [3]).

The ion flow velocity to the divertor plate will be described as an average velocity for the half Maxwellian,

$$u = \frac{v_{Ti}}{\sqrt{\pi}}, \qquad (9)$$

where $v_{Ti} = \sqrt{2T_i/m_i}$. This assumption stems from the fact that the electron temperature is substantially less than the ion temperature, so that the ions do not experience any significant ambipolar acceleration on their way through the divertor leg (see, e.g., Ref. [11]).

When there is no biasing, and the plasma does not carry any current, the plasma assumes a "floating potential" [10]:

$$\varphi_f = \frac{T_e}{e}\ln\frac{v_{Te}}{2\sqrt{\pi}u} \qquad (10)$$

The set of equations (5)-(7) describes an equivalent circuit shown in Fig 1. For a given bias potential $\varphi_b$, there are 3 equations for 3 unknowns: $j$, $\varphi_{pb}$, and $\varphi_{b0}$. The induced convection is determined by the potential difference between the biased and unbiased fluxtubes [3,4], i.e., by the difference $\Delta\varphi = \varphi_{pb} - \varphi_{p0}$. The other interesting parameter is the plasma potential well beyond the X point, which was measured experimentally. As was mentioned at the beginning of the paper, the X-point shear leads to an exponential decrease of the toroidally-asymmetric part of the potential perturbations into the main SOL, whereas the toroidally-symmetric part is not affected by the shear and does not change significantly through the transition layer. For the equal width of the positively and negatively charged flux tubes, the toroidally-symmetric part is just

$$\varphi_{symm} = \left(\varphi_{pb} + \varphi_{p0}\right)/2 \qquad (11)$$

(whereas the toroidally-asymmetric part, which is on average zero, is proportional to $\Delta\varphi$). In the unperturbed case (no biasing), $\varphi_{symm}$ is, obviously, just equal to the floating potential $\varphi_f$. Eliminating the current density from Eqs. (5)-(7), normalizing all the potentials to $T_e/e$, and introducing a dimensionless parameter

$$\Xi \equiv \frac{T_e\Sigma}{une^2}, \qquad (12)$$

one obtains the following equations for the (normalized) values of $\Delta\varphi$ and $\Delta\varphi_{symm}\equiv\varphi_{symm}-\varphi_f$:

$$\Delta\varphi + \ln\frac{1+\Delta\varphi\Xi}{1-\Delta\varphi\Xi} = \varphi_b; \qquad (13)$$

$$\frac{\Delta\varphi}{2}+\ln\frac{1}{1-\Delta\varphi\Xi}=\Delta\varphi_{symm}. \quad (14)$$

All the external characteristics of the problem are encapsulated now in a single parameter Ξ. Using Eqs. (8), (9), and (12), one can represent it as

$$\Xi=\frac{\sqrt{\pi}}{2}GN\frac{B_T}{B_P}\frac{\rho_i}{R}\frac{T_e}{T_i}, \quad (15)$$

where $\rho_i$=$v_{Ti}/\omega_{Ci}$ is the ion gyroradius. Large values of Ξ would correspond to a high cross-field electrical conductivity of the transition layer and a substantial reduction of $\Delta\varphi$ compared to $\varphi_b$. This case is illustrated by the curves with Ξ = 2 in Fig. 2. Conversely, small values of Ξ would result in a very small leakage between the neighboring flux-tubes and, accordingly, in larger values of $\Delta\varphi$. This regime is more favorable for enhancement of the cross-field transport by induced convection. This regime corresponds to the curves with Ξ = 0.3 in Fig. 2. For the parameters of MAST ($N$=12, $R$=100 cm, $B_T$=0.2 T, $B_P$~0.02 T, $T_i$=60 eV, $T_e$=10 eV, deuterium), and assuming that the adjustment factor $G$ is equal to 2 (see justification below), one obtains Ξ ~ 0.3, i.e., the MAST better corresponds to a small Ξ case.

By inspecting Eq. (13), one sees that the function $\Delta\varphi(\varphi_b)$ is an odd function, i.e., the absolute value of the cross-field potential difference in the divertor leg does not depend on the sign of the biasing potential $\varphi_b$. This result is in a better agreement with the MAST observations than the earlier model [3, 4] which predicted much weaker effect for a negative biasing than for the positive one.

Eqs. (13), (14) provide also information about the change in average potential of the upper SOL, $\varphi_{symm}$, during the biasing experiment. According to Fig. 2, for the bias potential of +40 V, and $T_e$=10 eV, one has $\varphi_{symm}$-$\varphi_f$= 20 eV, in a reasonable agreement with the experimental measurements [5]. Note that the a toroidally-symmetric potential increase of the SOL will drive some toroidally-symmetric current to the upper divertor. A more complete model must take this current into account. Here we neglect it motivating this by a large connection length between the lower X-point and the upper divertor and the correspondingly large plasma resistance.

After the dependence of $\Delta\varphi$ vs. $\varphi_b$ is found, one can find also the dependence of the current density vs $\varphi_b$ by virtue of Eq. (7), which can be rewritten in the following form:

$$j/j_{sat}=\Xi\Delta\varphi(\varphi_b) \quad (16)$$

where $\Delta\varphi$, as before, is normalized to $T_e/e$. The corresponding set of curves is shown in Fig. 3. By comparing these curves with the experimental dependence (dots, obtained from Fig. 3 of Ref. [5]) one finds that the best fit corresponds to Ξ=0.3 (red curve in Fig. 3). This requires the adjustment parameter $G$ to be equal to 2 – a value that we have chosen for it in the earlier discussion. For comparison, we present also the curve (green) corresponding to a simple model used in Ref. [3], Eq. (32) of Ref. [3]. One sees that the model developed in the present paper fits experimental values much better and yields a reasonable value for the parameter Ξ.

Of course, one should not overestimate the accuracy of our predictions. Our model contains some obvious uncertainties (reflected in part in the presence of an adjustment factor $G$) and is semi-quantitative, at best. However, it is is good enough to produce a qualitatively correct picture. In addition, the value of the unknown parameter *G* derived from the best fit to experimental data is quite reasonable, *G*=2.

Now we evaluate the change of the heat flux to the divertor plates associated with the biasing. In the model where the ions approach the wall as a "half-Maxwellian," the energy flux *Q* to the wall can be easily evaluated as (Cf. Ref. [9] and Eqs. (36), (37) in Ref. [3]):

$$Q_b = nu(2T_i + 2T_e + e\varphi_{pb} - e\varphi_b) \tag{17}$$

$$Q_0 = nu(2T_i + 2T_e + e\varphi_{p0}) \tag{18}$$

where subscripts "*b*" and "*0*" designate the biased and grounded ribs. For the positive biasing, the energy flux to the biased rib is higher than to the grounded rib (and vice versa for the negative biasing). However, as we show below, this trend is weak (and can be easily hidden in the experiment by various other effects). As a measure of the asymmetry, we introduce the ratio $\xi_{asymm}$ of the flux difference, $(Q_0\text{-}Q_b)$, Fig. 4, to the average flux, $(Q_0+Q_b)/2$. After some algebra based on Eqs. (5)-(7), we obtain:

$$\xi_{asymm} = \frac{\ln\dfrac{1+\Xi\Delta\varphi}{1-\Xi\Delta\varphi}}{2\dfrac{T_i}{T_e} + 2 + \ln\dfrac{\mathrm{v}_{Te}}{2\sqrt{\pi}u} + \dfrac{1}{2}\ln\dfrac{1}{1-\Xi^2\Delta\varphi^2}} \tag{19}$$

where $\Delta\varphi$ is normalized to $T_e/e$. Together with Eq. (13), this equation determines the dependence of the asymmetry parameter $\xi_{asymm}$ *vs.* the bias potential. This dependence is illustrated in Fig. 5. One sees that, indeed, the asymmetry in the heat flux remains small. This is not to say that the total power on the biased and un-biased ribs must remain the same: the total power is determined by the wetted surface, which is hard to determine from the simple theory.

The biasing leads to additional energy release in the SOL. According to the equivalent circuit of Fig. 1, part of it is released in the region near the X-point where the cross-field current flows. The fraction $\xi_{X\text{-}point}$ of the total heating power that is deposited in this region is, obviously, $\xi_{X\text{-}point} = (j\Delta\varphi)/j\varphi_b = \Delta\varphi/\varphi_b$. The plot of this fraction *vs.* the biasing voltage is presented in Fig. 6 . We see that, for the typical conditions, about a half of the total power is dissipated near the X-point. This would cause some additional heating of the plasma in this zone, increasing the plasma flow velocity towards the divertor plate, in a qualitative agreement with experimental observations [5].

In summary: We have shown that the model based on the description of the X-point zone in terms of a resistive boundary condition, with the "heuristic" cross-field resistivity of Ref. [9], leads to a reasonable qualitative agreement with observations [5]. It shows that the positive and negative biasing cause effect of roughly the same magnitude, that the potential of the SOL above the X-point changes in the course of biasing, and that substantial fraction of the total Ohmic heating associated with the biasing occurs in the transition zone near and somewhat above the X-point.

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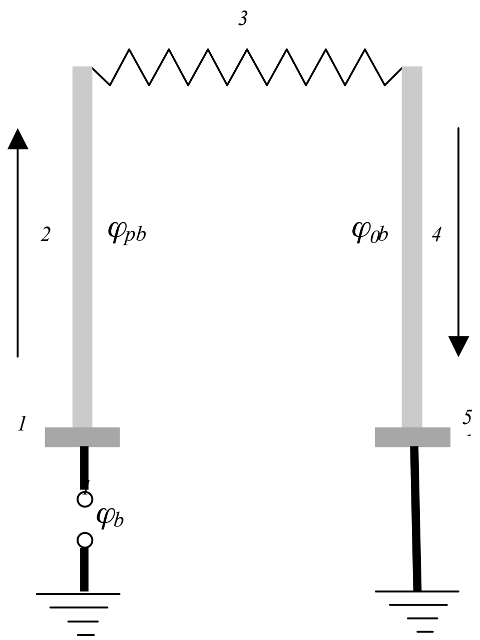


Fig. 1 An equivalent circuit: 1 sheath over the biased rib; 2 plasma flux-tube leaning on the biased rib; 3 a resistor representing the effective plasma resistance near and beyond the X point; 4 plasma flux-tube leaning on the un-biased rib; 5 sheath over the un-biased rib. The direction of the current is shown for the positive bias, $\varphi_b>0$.

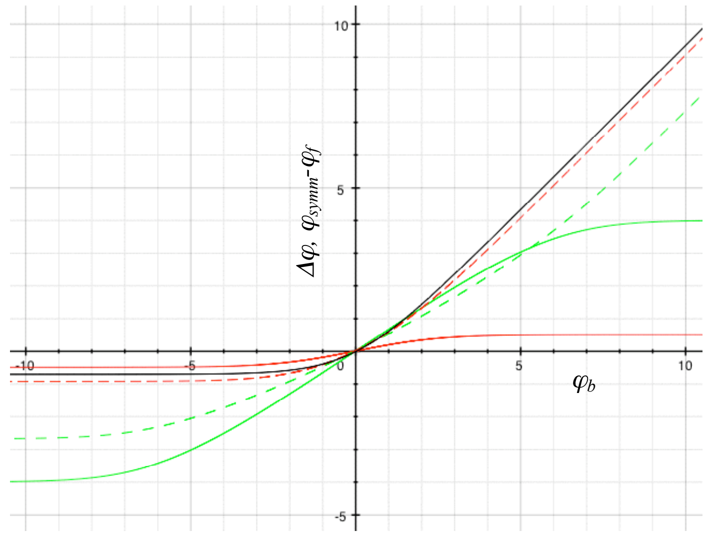


Fig. 2. The result of biasing for various values of the parameter Ξ. Solid lines correspond to $\Delta\varphi_{pb}$, dashed lines correspond to $\Delta\varphi_{symm}$. All the potentials are normalized to $T_e/e$. Red curves correspond to Ξ =2; green curves correspond to Ξ =0.3. The black curve corresponds to Eq. (1).

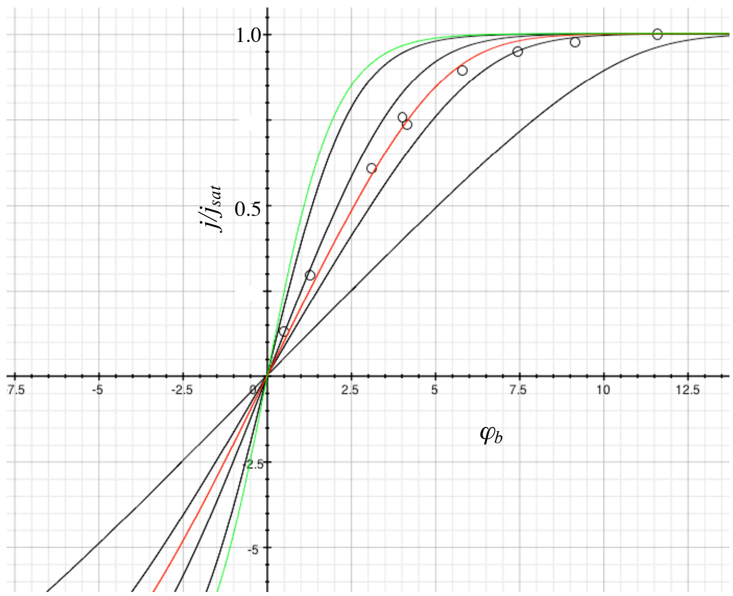


Fig. 3. The current-voltage characteristic (normalized to the saturation current) for various values of the parameter Ξ, Ξ =0.125, 0.25, 0.3, 0.5, and 2 (from below). The upper-most (green) curve corresponds to a different dependence of the current vs. potential, that of Ref. [3]. One sees that the model of our present paper allows one to fit the experimental data much better, leading to an estimate of the parameter Ξ=0.3.

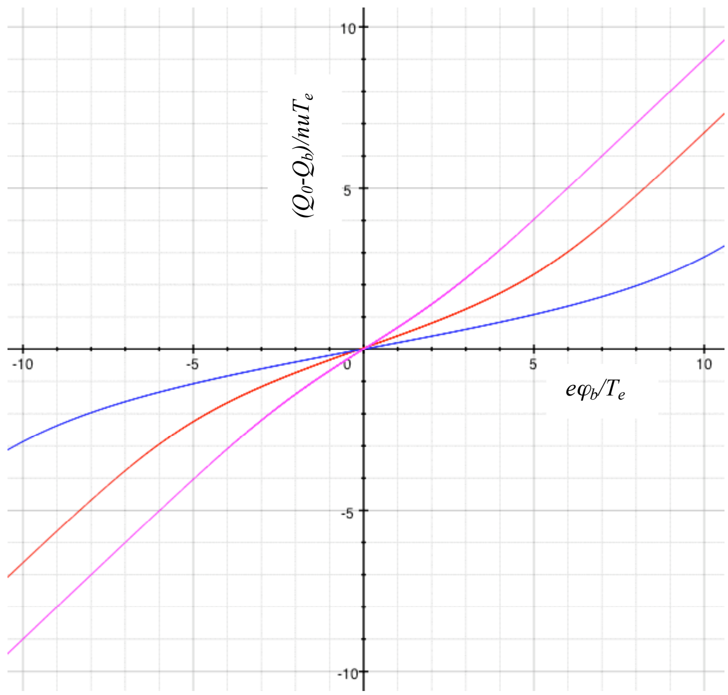


Fig. 4. The difference between heat fluxes to the grounded and biased ribs in the units of $nuT_e$. The parameter X is equal to 0.3 for the green curve, 1.0 for the red curve, and 2.0 for the purple curve.

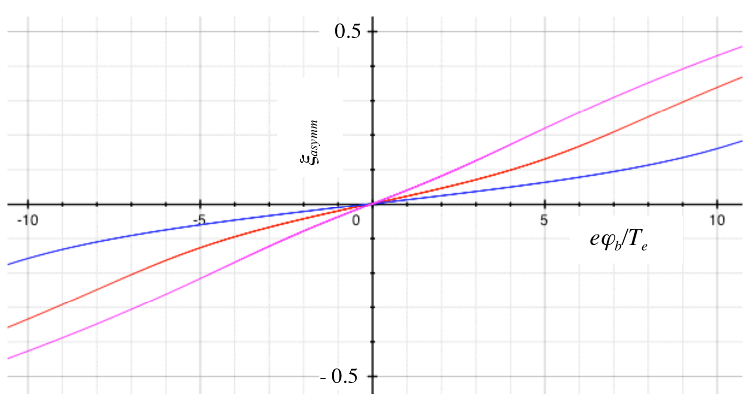


Fig. 5 The difference of the heat fluxes, $(Q_0\text{-}Q_b)$, normalized to their average value, $(Q_0+Q_b)/2$: $\xi_{asymm} = (Q_0\text{-}Q_b)/[(Q_0+Q_b)/2]$, *vs*. the applied bias. The parameter $\Xi$ is equal to 0.3, 1.0, and 2.0 (from the lower to the upper curve). For $\Xi$=0.3, even at the highest value of the applied voltage, the difference does not exceed 15%.

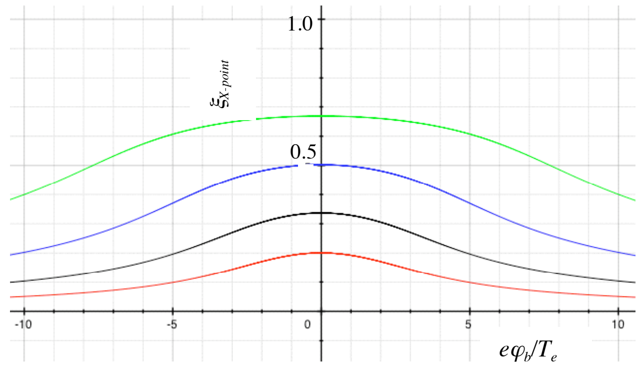


Fig. 6 A fraction of the heating power delivered by the biasing circuit to the plasma in the X-point zone. The parameter $\Xi$ is equal to 2, 1, 0.5, 0.25 from the bottom curve to the top curve.