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NEUTRINO PHYSICS AT VERY HIGH ENERGIES

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Abstract

NAL presents the opportunity to expand our knowledge of neutrino interactions from energies of less than 10 GeV up to more than 300 GeV. We propose an exploratory experiment which is designed to emphasize the physics of very high energy interactions (~ 300 GeV).

Our experiment has the following primary physics capabilities.

- I.) We will make a sensitive search for the Intermediate Boson (W), which is capable of detecting W-bosons of mass up to ~ 15 GeV. The experiment is sensitive to the W independent of its decay mode. It also has the very nice feature that the W mass can be reconstructed to an accuracy $\sim 10\%$ for every event associated with $W \rightarrow \mu + \nu$ decays.
- II.) We will simultaneously explore the behavior of the deep inelastic scattering processes. From these studies we will look for structure in the weak interaction, such as a possible W-meson propagator, and test many hypotheses about the basic hadronic structure. The range of momentum and energy transfers accessible is vastly expanded over that of all previous experiments.
- III.) By taking data at several incident energies, we will obtain information bearing on the fundamental question of whether the total cross section continues to rise linearly to 300 GeV as predicted from a point interaction.

Our experiment relies on a very simple scheme to define the momentum of the incident neutrinos to $\sim \pm 6\%$ and angle to $\sim \pm 0.1$ mrad. These added constraints greatly ease the experimental problems. Also, the monitoring of the neutrino flux becomes very direct. We are therefore proposing a rather modest experiment capable of carrying out the above measurements. The apparatus consists of a target, a calorimeter to determine hadronic energies, and a combination of spark chambers and an iron core magnet to detect the angle and momenta of muons. The rates for the experiment at 250 GeV are 1000 events/day in the inelastic scattering and 15 events/day for a hypothetical 8 GeV W-boson decaying into $\mu\nu$. We request 500 hours of running time and 250 hours of test time for the experiment. ($\sim 2 \times 10^{18}$ p at 400 GeV).

Experimenters

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I. Physics Justification

A. Introduction

One of the most exciting prospects opened up by the high energies available at NAL is the study of the weak interaction. At present, neutrino physics has only been studied for $E_\nu < 10$ GeV. Our experiment is designed as an initial probe of the very high energy neutrino processes, extending to the highest energies accessible ($E_\nu \simeq 300$ GeV).

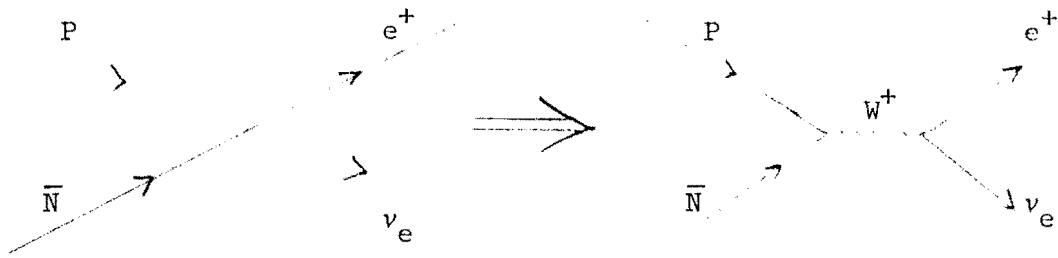
We propose at a very early stage to run in a high energy neutrino beam with a very modest apparatus. The experimental setup described in the next section will do a W-search and measure the deep inelastic scattering simultaneously. We will also obtain information on the A-dependence of the cross section and make a measurement of the energy dependence of the total cross section on heavy nuclei. This will provide an important early look at high energy neutrino physics.

We will also be capable of more refined measurements of the total neutrino-nucleon cross section as a function of energy; antineutrino comparisons; and further measurements of the inelastic cross sections.

The physics which we propose to explore in this extreme high energy region is focused on the most fundamental initial questions. These include:

(a) W-Meson Search

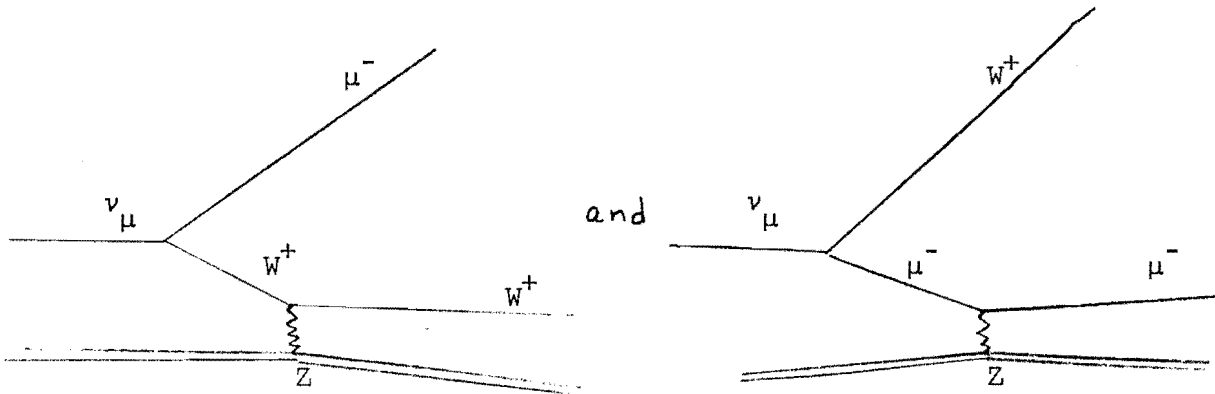
Is there a boson which acts as the intermediary for the weak interactions, in analogy to the strong and electromagnetic interactions? If so, one can replace the four-fermion vertex by connected three particle vertices, as illustrated below for β -decay.



The range of the interaction $R = \frac{h}{M_w c}$ is very small, which implies that the mass M_w is large.

W-mesons, if they exist, should be produced in the neutrino reactions

$$\nu_\mu + Z \rightarrow \mu^- + W^+ + Z'$$



and the decay mode

$$W^+ \rightarrow \mu^+ + \nu_\mu$$

provides a signature for identification of the W by observing two oppositely charged muons in the final state. For W-bosons of mass $> 2 \text{ GeV}/c^2$ many decay modes are possible and it is difficult to calculate the decay rates for pure hadronic decays (e.g., $W^+ \rightarrow \pi^+ + \pi^0$).

However, Yamaguchi⁽¹⁾ has argued that the branching ratio $W \rightarrow \ell + \nu$ is $\sim \frac{1}{6}$ to $\frac{1}{10}$ as $M_W \rightarrow \infty$.

Theoretical cross sections for W-production have been calculated by Wu.⁽²⁾ We have calculated, using Wu's formulae, cross sections per proton for an Fe target extended to the neutrino energies attainable at NAL. These cross sections are shown in Figure 1. Since the coherent production cross section varies as Z^2 it is experimentally advantageous to use a heavy nucleus in regions where the coherent cross section is dominant. Thus far, neutrino experiments at Brookhaven⁽³⁾ and CERN⁽⁴⁾ have set a lower limit on the W mass of $M_W \geq 2 \text{ GeV}/c^2$. From Figure 1 it is clearly desirable to use very high energy neutrinos. In this proposed experiment using neutrinos up to 300 GeV we would be able to detect W-bosons of mass $M_W \leq 15 \text{ GeV}/c^2$.

It should be noted that if the W exists and can be produced at these energies, since only one weak vertex is involved in the production reaction, this process will dominate over second order dimuon production.

(b) Deep Inelastic Scattering

The process $\nu + N \rightarrow \mu + (\text{anything})$ is analogous to the very interesting SLAC inelastic ep scattering.⁽⁵⁾ $\frac{d^2\sigma}{dq^2 d\nu}$, where ν is the energy loss by the leptons and q^2 is the four momentum transfer squared, involves three structure functions for neutrinos and can be written

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{E-\nu}{E} \frac{G^2}{2\pi} \cos^2 \frac{\theta}{2} [W_2(q^2, \nu) + 2 \tan^2 \frac{\theta}{2} (W_1(q^2, \nu) + \frac{2E-\nu}{2M} W_3(q^2, \nu))],$$

$$\text{where } \begin{cases} \nu = E - E' \\ q^2 = 4 EE' \sin^2 \frac{\theta}{2} \end{cases}$$

and E' is the scattered μ energy

E is the incident ν energy

θ is the lab scattering angle,

assuming a current-current form for the weak interaction. Equation (1) looks identical to the ep scattering formula, except that W_3 , which is associated with the parity violating part, is absent in ep scattering, and the factor $G^2/2$ replaces $(2\pi\alpha)^2/q^4$. This latter difference means that the neutrino inelastic scattering cross section does not contain the sharp $1/q^4$ fall-off found in the electron case.

One of the basic questions is whether the neutrino process shows the same behavior as the electron processes. A difference, at the same q^2 and ν , would indicate some fundamental modification to the weak interactions.

Another question that can be investigated is the hypothesis of scaling. Under the assumption that the nucleons consist of fundamental "partons" that themselves have no structure and very little transverse momenta, the functions W_1 , W_2 , and W_3 exhibit scale invariance (i.e., are functions of a single parameter $\omega = \frac{2M\nu}{q^2}$).⁽⁶⁾ The functions are observed to scale in the q^2 region accessible to SLAC ep scattering ($q^2 < 7 \text{ GeV}^2$). It will be of fundamental importance to see whether this scaling extends to the q^2 accessible to 300 GeV neutrinos ($q_{\text{max}}^2 \approx 600 \text{ GeV}^2$). It should be emphasized that only ~5% of the total events at 300 GeV lie in the q^2 and ν region covered at SLAC. This means that most of the data will

be in a new and totally unknown region.

The dependence of the structure functions on ω carries information on the detailed structure of the nucleon. For example,⁽⁶⁾

$$\int \left[[\nu W_2(\omega)] \right] \frac{d\omega}{\omega^2} \propto \frac{\sum Q_i^2}{N} \quad (2)$$

$$\int \left[[\nu W_2(\omega)] \right] \frac{d\omega}{\omega} \propto \sum Q_i^2 \quad (3)$$

where Q_i = charge on the i^{th} parton and N = total number of partons. The asymptotic behavior of $\nu W_2(\omega)$ is not known; that is, whether it approaches some non-zero value. If it does, equation (3) diverges and $N = \infty$. If, on the other hand, $\nu W_2(\omega)$ goes to zero, equations (3) and (2) allow us to determine the number of partons.

There are similar observations concerning W_1 and W_3 . For example, integrals over W_3 are related to a linear combination of baryon number and hypercharge for the constituents.

It is also very important to compare inelastic ν and $\bar{\nu}$ cross sections. Adler⁽⁷⁾ has derived, using current algebra, the following sum rules for the infinite neutrino energy limit

$$\frac{d \sigma(\bar{\nu} p)}{d q^2} - \frac{d \sigma(\nu p)}{d q^2} = \frac{G^2}{\pi} (\cos^2 \theta_c + 2 \sin^2 \theta_c)$$

and

$$\frac{d \sigma(\bar{\nu} n)}{d q^2} - \frac{d \sigma(\nu n)}{d q^2} = \frac{G^2}{\pi} (-\cos^2 \theta_c + \sin^2 \theta_c)$$

θ_c = Cabibbo angle.

(c) σ_T vs E_ν

As we enter a new energy region one of the most basic questions involves the behavior of the total cross sections. For example, will neutrinos scatter like antineutrinos? Will the cross section continue to rise linearly as predicted from simple Fermi theory for scattering from a point particle? Etc.

Experiments at CERN⁽⁴⁾ using a heavy liquid bubble chamber have measured the total cross section for neutrinos up to 12 GeV. The method involved summing the visible energy in the chamber, making an estimated correction for missing energy, and equating the result to the neutrino energy. The results up to $E_\nu \sim 12$ GeV are shown in Figure 2.

A linear relationship

$$\sigma_{\text{tot}} \sim 0.6 \times 10^{-38} E_\nu \text{ (in GeV) cm}^2$$

works at these energies. Fermi scattering from a point particle predicts such a linearly rising cross section. Eventually this rise will violate the unitarity limit ($\sigma < \frac{1}{2} \pi \lambda^2$) but this occurs at $\sim 10^5$ GeV, well beyond NAL energies.

The cross section may, however, turn over at much lower energies.

For example, if the W exists it has a propagator term $\frac{1}{1 + q^2/M_W^2}$.

This term becomes important for $q^2 \sim M_W^2$. At high energies this damping of $\frac{d\sigma}{dq^2}$ at large q^2 will affect the total cross section; for example, we estimate that if $M_W = 25$ GeV, the propagator term produces approximately a 30% damping of σ_T at 300 GeV. A turnover might occur

for other reasons like effects from higher order weak interactions, or because of a form factor if the nucleon does not consist of point constituents. Measurement of the differential spectrum of the outgoing μ might shed light on which effect caused the turnover. High energy neutrino and antineutrino fluxes should be sufficient to measure $\sigma_T(E_\nu)$ and $\sigma_T(E_{\bar{\nu}})$ up to ~ 300 GeV.

B. Basic Experimental Approach

In seeking the best means of studying these physics questions we have given particular attention to the advantages of knowing the incident neutrino energy and angle. We propose to make a "monochromatic" neutrino beam by using the highest energy neutrinos from the decays of the K mesons in a momentum-selected beam. This is done by only looking very near the forward direction, thus obtaining a narrow band of neutrinos from the K decays. In this way we can put neutrino physics more nearly on the same footing as most hadron experiments.

In the W search, the independently-measured neutrino energy and angle permits the mass of the W meson to be reconstructed for each event, with an accuracy of $\sim 10\%$. In addition, the beam is free of antineutrinos (since the charge of the hadron beam is selected unambiguously), which gives an additional handle on the identification of the events. The signature for a W candidate will be the appearance of a μ^+ , μ^- pair, with no hadrons. The W-decay μ^+ has a large transverse momentum, while the μ^- has relatively low energy. The presence of a distinct peak in the mass spectrum for such events will be a great advantage in establishing the existence of the W, as well as providing important information about it. It will also be

possible to investigate the angular distribution of the μ^+ in the W decay.

The decay mode $W \rightarrow e + \nu$, which should be equal in rate to the $W \rightarrow \mu + \nu$ mode, should also be detectable in our apparatus. The high energy electron will produce a short-lived electromagnetic shower, with no hadronic component, and therefore appear anomalous.

The W will be detected even if it does not often decay into leptons. In this case, the processes are

$$\nu + Z \rightarrow W^+ + \mu + Z'$$

followed by

$$W \rightarrow \text{hadrons.}$$

Lee and Yang have pointed out that the μ^- in W^+ production preferentially has low laboratory energy. This allows us to find the W without recourse to the leptonic decay. The cross section for $\nu + Z \rightarrow \mu^- + \text{hadrons}$ will, in this case, show a peak in $d\sigma/dE_\mu$ at low energies relative to the ν energy. The neutrino energy will, of course, be measured independently. This peak will disappear suddenly as the energy of the ν beam is lowered. (See Figure 1.)

The inelastic scattering events will be characterized by a hadron shower plus a μ^- . For such events we shall measure the differential cross section $d^2\sigma/dq^2 d\nu$ and the total cross section as functions of neutrino energy. The monochromatic beam will give a reliable measurement of the neutrino energy with resolution better than 10% and with negligible systematic errors. The energy dependence of $\sigma_{\text{tot}}(E_\nu)$ and $d\sigma/dq^2 d\nu(E_\nu)$ can thus be determined independently of measurements involving the final-state particles, with better accuracy than experiments which rely upon summing the final-state energy. Accurate knowledge of the neutrino angle is important

for the reconstruction of q^2 in inelastic scattering, and in the mass calculation in the W search.

Of particular importance for the total cross section measurement is an accurate determination of the incident neutrino flux. With a narrow-band hadron beam, the neutrino flux can be much more reliably and easily computed and monitored than with a wide-band system, since only hadrons of a given momentum are present. The principal goal of the total cross section measurement is to observe whether there is a deviation from the linear rise with energy at high energies. Such a measurement is extremely sensitive to normalization errors in view of the rapid drop in the neutrino fluxes with increasing energy. Even when the wide-band neutrino spectrum is known exactly, small systematic errors in the energy of individual events produce large errors in normalization. For example, the yield curves for the horn-focused beam show a change in yield of 40% for a 5% error in E near 100 GeV.⁽⁸⁾

An important practical advantage of using a monochromatic neutrino beam is that it is possible to do the experiment with a relatively modest detection apparatus.

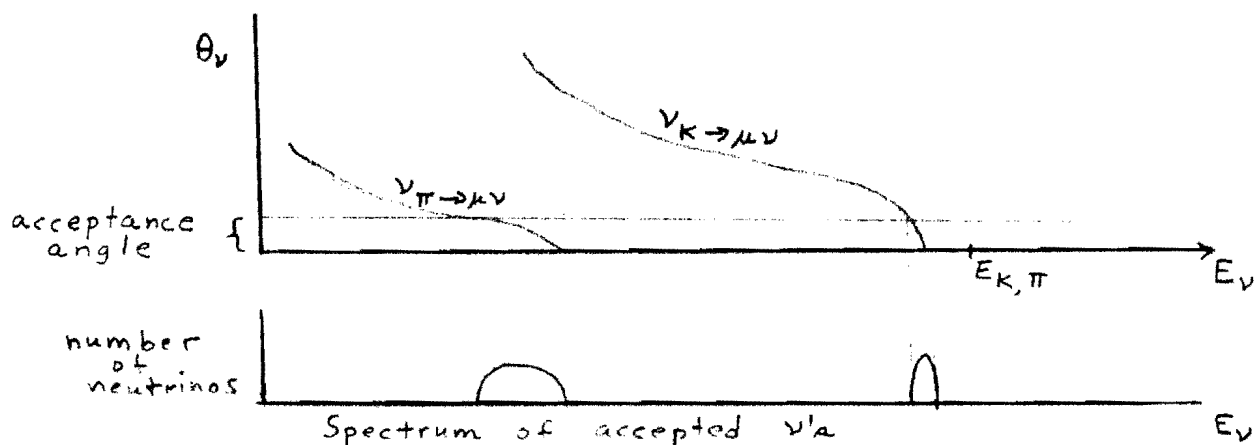
In considering the three problems - W-search, $\frac{d^2\sigma}{dq^2 d\nu}$, and σ_T - we have looked into various choices of target material. In particular, the question of heavy nuclei vs hydrogen as a detector has been considered. Experiments show that the total photon cross section on nuclei is proportional to $\sim A^{0.9}$ at 20 GeV.⁽⁹⁾ Interpretations⁽¹⁰⁾ of this result using a vector dominance model also predict a diminution of the neutrino cross section for small q^2 . In addition, Adler⁽¹¹⁾ has predicted using PCAC, that the differential cross section at zero degrees should be proportional to $A^{2/3}$. There are other versions of PCAC that would give an A-dependence, however.⁽¹²⁾

These considerations do not affect the W-search. While it is highly unlikely that they will affect the deep inelastic cross section, at high q^2 , there may be some effect in the total cross section. We expect to test whether the total cross section is proportional to A, and to check this question at least at one energy with hydrogen. If it becomes desirable, we will be capable of measuring the total cross section on hydrogen over the energy range $100 < E_\nu < 250$ GeV.

II. Experimental Method

A. Introduction

We have tried to devise an experimental setup which is as modest as possible and still sufficient to do the physics. The fact that we have monochromatic neutrinos allows us to relax somewhat the demands on the rest of the apparatus. What we mean by "monochromatic neutrinos" is the following: In the target box we have a simple beam transport system which selects hadrons within a $\pm 5\%$ momentum bite, forms a parallel beam, and directs it down the decay tunnel. The energies of the neutrinos from kaon and pion decay are correlated with their laboratory angles, with the highest energy ν 's going at zero degrees. By making the detection apparatus subtend a small angle we can accept only neutrinos within a small momentum band near the respective end points for kaon and pion decays:



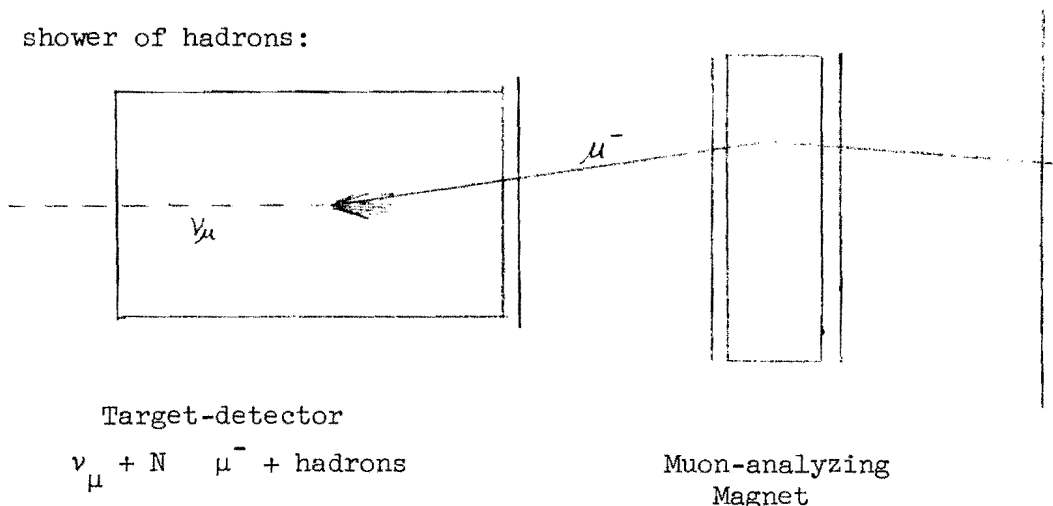
The separation of the two portions of the spectrum is accomplished by very rough measurement of the total final-state energy in the detector. We shall concentrate on the narrow spike of high energy K-decay neutrinos in this proposal.

The overall experiment layout is shown in Figure 3, with a detail of the detection apparatus in Figure 9. The detection apparatus consists of

1 meter² x 10 meters of iron, lead, and aluminum target in sections ~1 interaction length thick, separated by scintillation counters and spark chambers. The scintillators will sample hadron showers to determine roughly their energy. Muon angles will be measured in the spark chambers. Downstream of the target is an iron-core magnet with spark chambers to measure muon momenta.

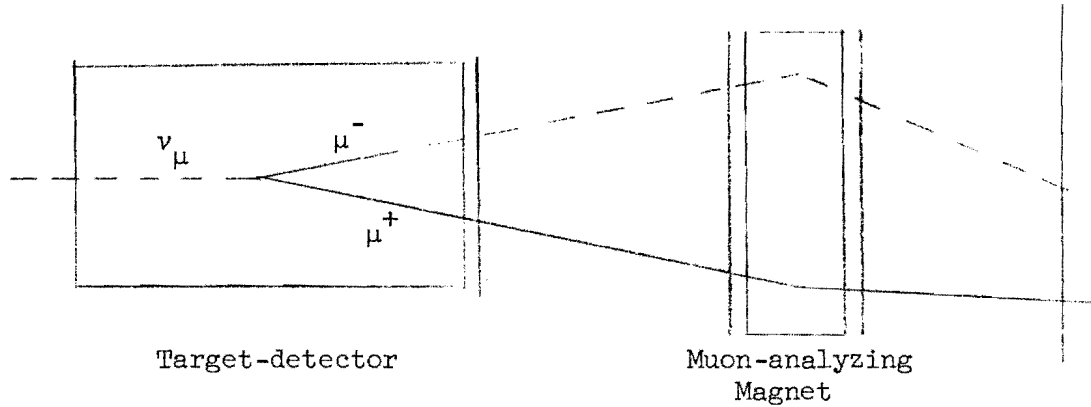
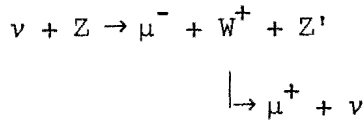
The events of interest will produce the following two main topologies in our apparatus:

- (a) Inelastic scattering. These events are characterized by the appearance of a muon (distinguished by its range) accompanied by a shower of hadrons:



The kinematical variables q^2 and ν are reconstructed from the measured muon energy and angle, and the known neutrino energy and direction. The hadron shower information, besides separating the E_{ν} peaks, further constrains the energy reconstruction.

- (b) W meson production. For these events two muons emerge from the interaction vertex, with no other visible particles.



From the measurement of momentum and angle for both muons, together with the beam energy and direction, the mass of the W can be reconstructed.

W production with decay other than μ, ν will have a topology similar to inelastic scattering, except that the distribution in muon energy will contain an excess of low-energy muons. The size of the effect will depend strongly upon the beam energy. For the particular case $W \rightarrow e\nu$, the shower accompanying the low-energy muon will be much shorter, as there is no hadronic component.

B. Hadron Beam

As the first step in the production of a monochromatic neutrino beam we form a relatively narrow-band, parallel hadron beam at the entrance to the decay tunnel. The requirements on this beam are (1) the momentum should be resolvable within $\Delta p/p = \pm 5\%$; (2) the divergence should be less than ~ 1.2 mrad at the highest energies (this can be relaxed somewhat at lower energies); (3) the beam should be able to go to the highest energies

at which usable fluxes of kaons will be produced by the accelerator at its maximum energy. We take this to be 300 GeV.

A first attempt has been made to design a simple beam meeting the above criteria, which we present here to establish its feasibility. The arrangement is shown in Figure 4. We assume the beam transport elements are to be contained within a 40-inch-wide x 200-foot-long target box. The beam enters the box essentially parallel to its axis and near one side. A quadrupole doublet accepts secondaries produced in the target at zero degrees within as large a solid angle as is practical and forms them into a parallel beam. The beam is dispersed by a pair of C-magnets and passed through a channel in the shielding to select the momentum band. A second identical pair of dipoles removes the angular dispersion in the beam and steers it down the decay tunnel. The proton beam is dumped within the shielding inside the target box.

For definiteness we have taken the magnet dimensions and parameters to be close to those for the "standard" magnets described in the 1969 summer study, SS-37. For running with 400 GeV protons on the target, we use quads of 2-inch-diameter aperture, giving a solid angle acceptance of 4 microsteradians, and a horizontal beam width of 1.5-inches (first quad focuses horizontally). Thus when the beam is displaced by 15 inches, a momentum resolution of $\pm 5\%$ results. The angular divergence in the decay tunnel, due to chromatic aberrations, is about ± 0.1 mrad. With the given parameters, such a beam can run up to about 300 GeV.

A slightly different arrangement is used when 200 GeV protons strike the target. In this case, larger-aperture (4-inch diameter) quads are used to increase the acceptance to 16μ sr, which results in a larger (3-inch-

wide) beam at the momentum slit. In order to maintain the momentum resolution, the bend angles are also increased a factor of two, and the maximum beam momentum is about 150 GeV/c.

In summary, the parameters of this tentative beam are:

400 GeV protons	200 GeV protons
$\Delta\Omega = 4 \mu\text{sr}$	$\Delta\Omega = 16 \mu\text{sr}$
$\Delta p/p = \pm 5\%$	$\Delta p/p = \pm 5\%$
$\Delta\theta_{\text{horiz}} = \pm 0.19 \text{ mr}$	$\Delta\theta_{\text{horiz}} = \pm 0.38 \text{ mr}$
$\Delta\theta_{\text{vert}} = \pm 0.07 \text{ mr}$	$\Delta\theta_{\text{vert}} = \pm 0.14 \text{ mr}$

Using the yield curves from Awshalom⁽¹³⁾ one obtains the K yields shown in Figure 5. It should be noted that it is possible to run the experiment with just the 400 GeV setup.

C. Monitoring System

Because we intend to use a momentum analyzed hadron beam, the monitoring of the neutrino flux becomes simplified and direct. To demonstrate feasibility we mention the following scheme which utilizes a small area downstream of the 10 meters of Fe at the end of the decay region. Figure 6 shows the average number of μ 's per unit area downstream of the Fe as a function of radial position from the center of the hadron beam, for $E_{\text{hadron}} = 300 \text{ GeV}$. Also indicated is the rms radial shift due to multiple scattering in the iron for 150 GeV μ .

The very sharp forward peak from π decay will be a very useful signal to monitor the stability of the hadron beam direction. By equalizing the rates in counters placed at equal positions on each side of the center line, we should be able to keep the center-line of the beam fixed to $< 0.05 \text{ mrad}$.

The relatively broad radial distribution for the K-decay μ 's allows a direct calibration of the K-decay ν 's in our beam. Input to the flux calculation are: the average momentum of the hadron beam, K-meson lifetime, hadron beam length and direction, geometry of the monitoring counters and apparatus, and the number of counts in the $K_{\mu 2}$ monitors. We are very insensitive to the details of the hadron focusing system, and we are completely insensitive to assumptions concerning the hadron flux yields, K/ π ratio, etc.

The monitoring system will be internally checked to determine that the radial distribution of μ 's is as expected. We can also make an absolute calibration, by running at low beam levels and comparing the monitor counts directly to the number of K's in the beam. A very modest counter hodoscope consisting of around ten small counters will accomplish this.

D. Shielding

The full 500 GeV muon shield is probably not required in this experiment. This is because we have (a) dumped the 400 GeV proton beam in the target box, (b) transported only a fraction of the secondary hadrons down the decay channel, (c) utilized a high momentum hadron beam and therefore confined the decay muons to the forward direction, (d) placed our apparatus 600 meters from the end of the decay channel.

A number of possibilities involving pitching the forward muons into the ground using magnetized iron were discussed in the 1969 Summer Study.⁽¹⁴⁾ A very modest version of such a scheme should keep the muons through our apparatus at a tolerable level. As illustration, 100 kg-m of bend at the downstream end of the decay region will displace 300 GeV μ 's by about 20 feet at our apparatus.

E. Muon Beam

In connection with the above suggestion, we note that the narrow-band hadron beam which we propose is also a good source for a muon beam. This opens up the possibility of forming such a beam by using, for example, an iron magnet at the end of the decay tunnel to deflect muons into a muon experimental area. This would double as a sweeping magnet for the neutrino beam. Muon and neutrino experiments could thus run simultaneously in area 1.

A simple extension of this beam would permit bringing muons into the neutrino apparatus. This is very desirable for check out and calibration purposes.

F. Neutrino Beam

The kaons in the hadron beam decay about 64% of the time via $K \rightarrow \mu + \nu$. The kinematics of this reaction is such that, for small neutrino angles,

$$E_\nu = \frac{M_K^2 - m_\mu^2}{P_K \left[\left(\frac{M_K}{P_K} \right)^2 + \theta_\nu^2 \right]} .$$

The relation between E_ν and θ is single-valued, so that the zero mrad neutrinos have the highest energy. An apparatus centered at zero mrad covering finite solid angle will be sensitive to the upper end of the neutrino energy spectrum. Further, if the apparatus is some distance from the decay region, the radial position of interaction allows a measurement of θ_ν , and hence of E_ν . Our apparatus, described in detail below, has a cross sectional area of 1 m^2 , and is placed 600 m downstream of the end of the decay channel.

Figure 7a illustrates the difference between the actual neutrino energy (E_ν) and that calculated (E_{calc}) from the knowledge of the average kaon momentum and some average decay position along the hadron beam. The resolution on the K-neutrino peak is $\text{FWHM} = 36 \text{ GeV}$ out of $\sim 300 \text{ GeV}$.

This figure also illustrates the resolution necessary for the combined measurement of hadrons (E_h) and μ^- (E_μ) energies to make the distinction between K and π neutrinos. For a hadron energy measurement with standard deviation 25% and muon energy measurement 16%, we show in Figure 7b the distribution in $[(E_h + E_\mu) - E_{\text{calc}}]$. This is very close to the distribution that will actually be observed and shows a clear separation of pion neutrinos from kaon neutrinos, even for very large π/K ratios. Once the distinction is made, of course, the resolution on the neutrinos from K-decay is $\pm 6\%$, as shown in Figure 7a.

The yields of neutrinos from $K_{\mu 2}$ decays hitting our apparatus as a function of energy are shown in Figure 8.

G. Target-Calorimeter

A blown up view of the experimental arrangement is shown in Figure 9. We have made the target only 1 meter x 1 meter in cross sectional area. Although it is technically possible to make apparatus of much larger cross sectional area the rates seem sufficient for the smaller size.

The target will consist of repeated modules, one interaction length of material followed by a liquid scintillator. For Pb or Fe this would be 4 inches of target followed by scintillator repeated for a total of 10 meters of target material. Also wire spark chambers will be put with every second scintillator. The liquid scintillator will be used to sample the hadron shower and measure its energy.⁽¹⁵⁾ The wire spark chambers will be used to follow the muon tracks.

The apparatus required in the calorimeter consists of 100 sheets of 4" x 1 meter x 1 meter Fe or Pb, one hundred 1 meter square liquid scintillators, and fifty 1 meter square wire chambers with readout for 4 x and 4 y coords/chamber.

In order to get a feeling for the requirements for the calorimetry in our experiment consider that the very worst case corresponds to a scattered μ^- of zero energy. In this case, the hadron shower contains less than 140 GeV of energy if the neutrino came from π decay; it contains more than 250 GeV if it came from K-decay. Note that the calorimeter does not give the initial neutrino energy, it allows us to distinguish two possibilities which in turn comes from the beam properties. In a more general event, there are three energies of interest: the incident neutrino energy, the final hadron shower energy and the final μ^- energy. Energy conservation allows internal consistency checks among these. The system is self-calibrating.

Calorimeters of the type described for this experiment have been extensively studied, and have been used in cosmic-ray experiments on very high energy p-p collisions.⁽¹⁵⁾ The standard deviation on the energy for real calorimeters has been in the range 20-25%. In real applications, corrections of 20-40% had to be made for unsampled energy. The application of the calorimeter to neutrino interactions is complicated by the fact that the initial hadron system is not a single strongly interacting particle. The initial hadron system, for neutrino interactions, will have unknown multiplicity, charged/neutral ratio, as well as K/ π ratio. Since we measure the neutrino energy independently, our calorimeter can be calibrated directly on neutrino interactions, so that systematic effects should be small.

Considering the experience of previous experimenters and without extensive calculations, we will assume that we can achieve a standard deviation of 25% on the measurement of the energy in the hadronic shower. This is quite adequate for our purposes.

In order to study the dependence of the cross sections upon nuclear mass number, different materials will be used in the target. Good counting rates are obtained with both lead and iron, which gives a variation by a factor of ~ 4 in A. Portions of the target can be replaced with other, less efficient materials for runs at the neutrino energy where the flux is highest. Runs with liquid hydrogen would be made using a slightly different arrangement: in this case the hydrogen would be placed in front of the heavy target, which would still function as a calorimeter.

H. Muon Spectrometer

Rather than trying to stop the high energy muons (it would require 3 meters x 3 meters x 150 meters of Fe at 300 GeV), we propose to measure directly the muon momentum in a magnet. A rather inexpensive solution which we propose is an iron core magnet, toroidal in shape with the axis of the toroid along the ν -beam direction. The magnet, shown in Figure 9, is limited in resolution by the multiple scattering in the iron.

For a configuration with 200" lever arm on each side of the magnet and a 20 kg field for 80" the resolution, including multiple scattering and 1 mm resolution in the spark chambers, is given below.

P (GeV)	$\frac{\Delta P}{P}$ in %	
	<u>Air Core</u>	<u>Iron Core</u>
25	.86	13.0
50	1.64	13.1
100	3.2	13.4
200	6.4	14.5
300	9.7	16.2

I. Resolution

Table I indicates some typical resolutions expected for running at 300 GeV. We first list parameters that are directly measured. The energy errors have already been discussed. The errors on incident neutrino angle are quite small due to the large distance between the decay region and the apparatus. The errors on the outgoing muon angle are dominated by multiple scattering. They are divided into two cases: (1) when there is no large hadron shower, the muon will be measured starting from the interaction point; (2) when there is a large hadron shower, there exists a distance (~1.9 meters) over which the muon is unmeasurable.

From the measured quantities, we have indicated some calculated variables of interest. We have used the constraint on energy conservation, $E_\nu = E_\mu + E_h$ in deriving the resolution on the inelastic scattering variables, v and q^2 .

For the W mass calculation, we take the reaction

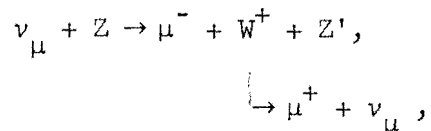


Table I

Resolutions for $E_\nu = 300$ GeV

Parameter	Type	Standard Dev.(SD)	Major Contribution to S.D.
Neutrino Energy (E_ν)	Measured	6%	Momentum Spread in Hadron Beam
Outgoing μ Energy (E_μ)	Measured	13-16%	Multiple Scattering in Iron Core magnet
Hadron Shower Energy (E_H)	Measured	25%	Inherent Statistics in Calorimetry
Incident ν Angle (θ_ν)	Measured	~ 0.1 mrad	Parallax of decay region as viewed from apparatus
Outgoing μ Angle (θ_μ) (hadron shower present)	Measured	15 mrad for $E_\mu = 10$ GeV to 0.5 mrad for $E_\mu = 300$ GeV	Multiple scattering in target detector over the distance of the hadron shower where the μ is not observed
Outgoing μ Angle (θ_μ) (no hadron shower)	Measured	< 0.6 mrad	Multiple scattering of muon in target-detector
$\nu = E_\nu - E_\mu$	Calculated	< 25 GeV $0 < \nu < 300$ GeV	Error in E_h for small ν ; Error in E_ν for large ν .
$q^2 = 2E_\nu E_\mu (1 - \cos \theta_\mu)$	Calculated	$\sim 8-16\%$ over range of E_μ , $q^2 > 1$ BeV ² $\approx .05$ BeV ² for $q^2 \leq 1$ BeV ²	$q^2 > 1$ BeV ² , error in energy determinations $q^2 < 1$ BeV ² , error in angle determination
Angle between μ^+ and W^+ in $W^+ \rightarrow \mu^+ + \nu$ (θ_+)	Calculated	$\delta \theta_+ / \theta_+ < 0.07$	θ_ν and θ_μ for $M_W < 7$ GeV Transverse momentum of recoil nucleon for $M_W > 7$ GeV
W-mass (M_W)	Calculated	$\Delta M_W / M_W < .10$	θ_+ and μ_+ energy

in which Z' predominantly recoils with small momentum transfer. The energy of the μ^- is measured in the detection apparatus and the energy E_0 of the incident neutrino is known, so that E_W is obtained from

$$E_W = E_0 - E_{\mu^-} ,$$

neglecting the energy carried off by the recoiling nucleon. ($E_{\text{transferred}} = q^2/2M_Z \ll E_0$.) The W mass can then be obtained from the relation

$$M_W^2 = 2 E_+ (E_W - P_W \cos \theta_+).$$

Here E_+ is the (measured) energy of the W-decay μ^+ . The angle θ_+ between the μ^+ and W^+ directions is obtained in the reconstruction with an accuracy $\frac{\delta\theta_+}{\theta_+} < 0.07$. This relies upon the fact that the decay μ^+ has large transverse momentum relative to the W-direction, while the (unknown) transverse momentum carried off by the recoil nucleus is small.

J. Rate in the Apparatus

(a) W-Search: Rates are shown in Figure 10. The experiment should be sensitive to W production for W's up to 15 GeV in mass. These rates assume the branching ratio into $\mu\nu \sim 10\%$ (so the total rate for all decay modes is a factor of 10 higher).

(b) $\frac{d^2\sigma}{dq^2 d\nu}(E_\nu)$: Total inelastic rates are shown in Figure 11. The experiment should be capable of measuring the cross sections from 40-300 GeV (rates vary from 100-1000 counts/day over this range in energy).

(c) $\frac{d^2\sigma}{dq^2 d\nu}(E_{\bar{\nu}})$: At 200 GeV, we can make a comparison of the anti-

neutrino cross section (rate of 100/day) with the neutrino cross section.

(d) We can study various materials at 200 GeV for E_ν with reasonable rates. For example. Al gives a rate of 250 events/day and liquid hydrogen $10 \text{ m}^2 \times 10 \text{ m}$ gives ~ 100 events/day.

K. Request for Time

Our rate calculations assumed 400 GeV protons, 10 sec rep rate, and 10^{13} interacting protons/pulse.

We propose to carry out the measurements outlined in this proposal assuming the above parameters with $\sim 2 \times 10^{18}$ interacting protons. This represents 250 hours - test time at low intensity

500 hours - data taking.

It should be noted that our experiment, which could be ready at a very early date, is capable of interesting physics at reduced intensity. For example, for 10^{12} interacting protons/pulse we would produce 10 events/day of hypothetical 8 GeV W-bosons and even an estimated 1 event/day of the type $W \rightarrow \mu + \nu$. Also, we would obtain 100 events/day in the inelastic spectrum.

L. Summary of Experimental Equipment

Below is a summary of the main experimental components needed for this experiment.

- (1) Hadron beam components for the target box.
- (2) μ -dump (either a pitching magnet arrangement or muon shield)
- (3) Target-Calorimeter
 - a) 10 m^3 material (mixture of Fe, Pb, Al)
 - b) 70-1 m^2 liquid scintillators
 - c) 50-1 m^2 wire chambers.

(4) Toroidal Iron-core Magnet

- a) 12 m^3 of Fe (main cost - power supplies and coils are minimal.)
- b) $8\text{-}3 \text{ m}^2$ wire chambers.

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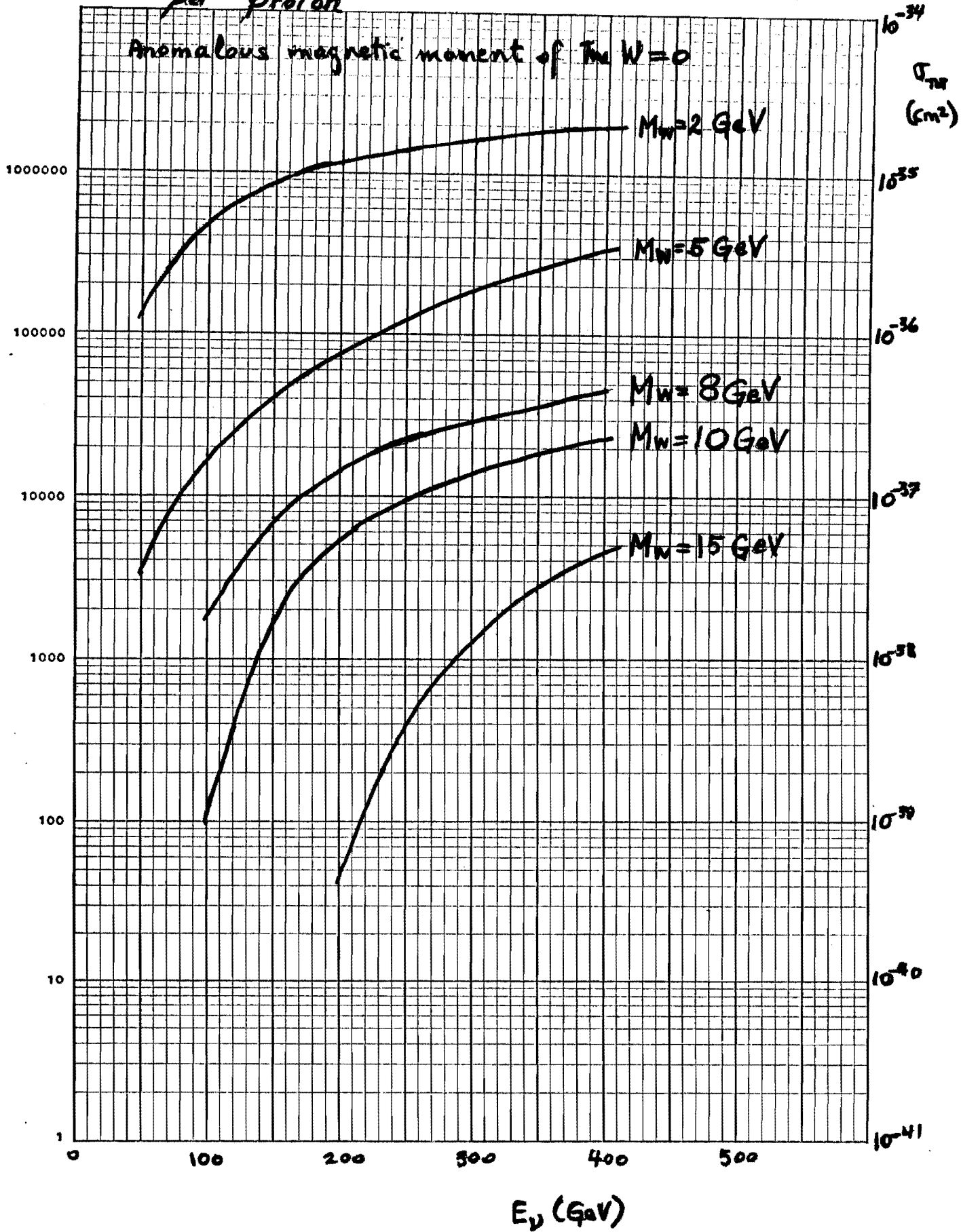
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MODEL

Total cross-section for Fe
per proton

Anomalous magnetic moment of Fe $W=0$



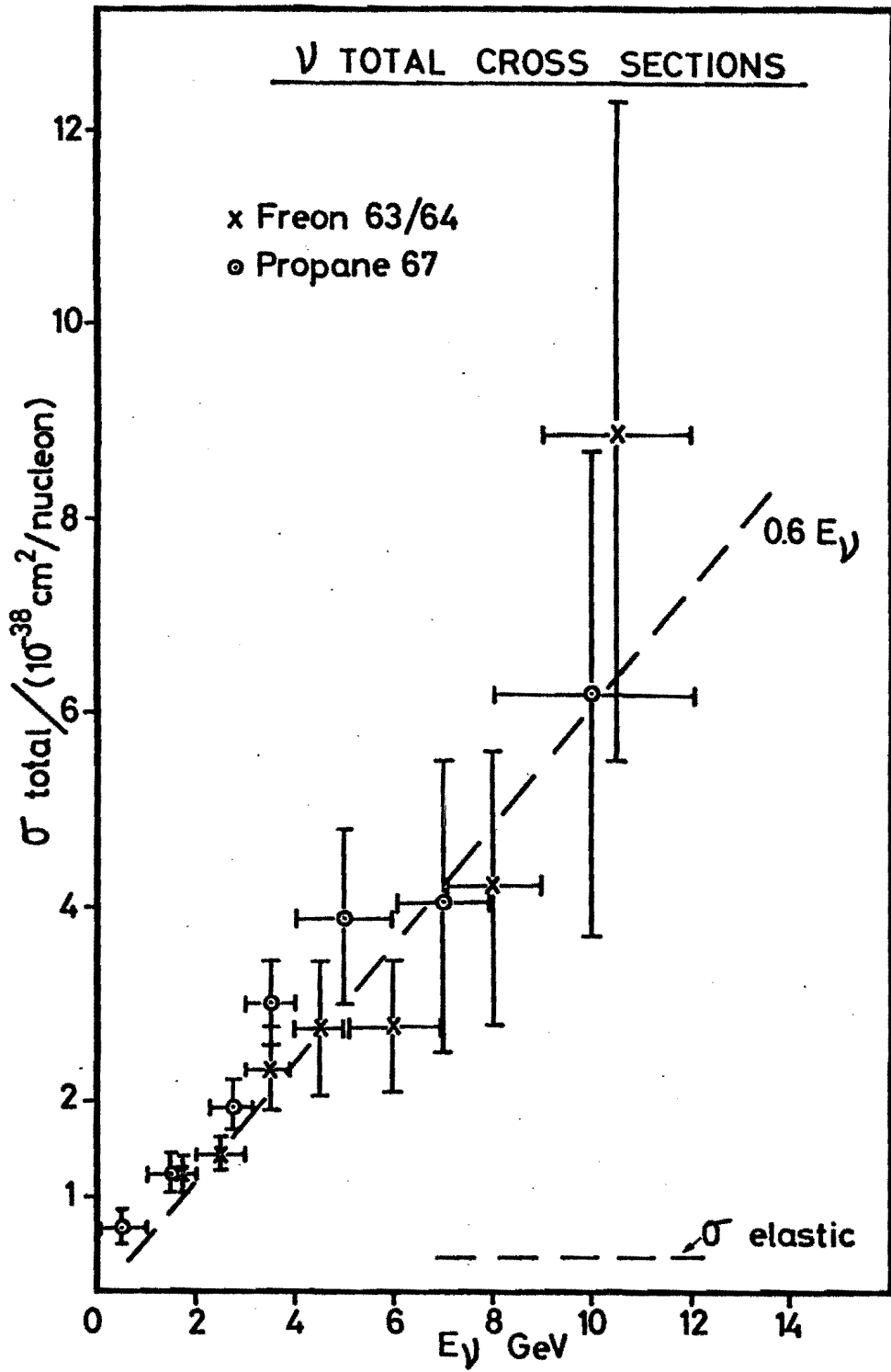
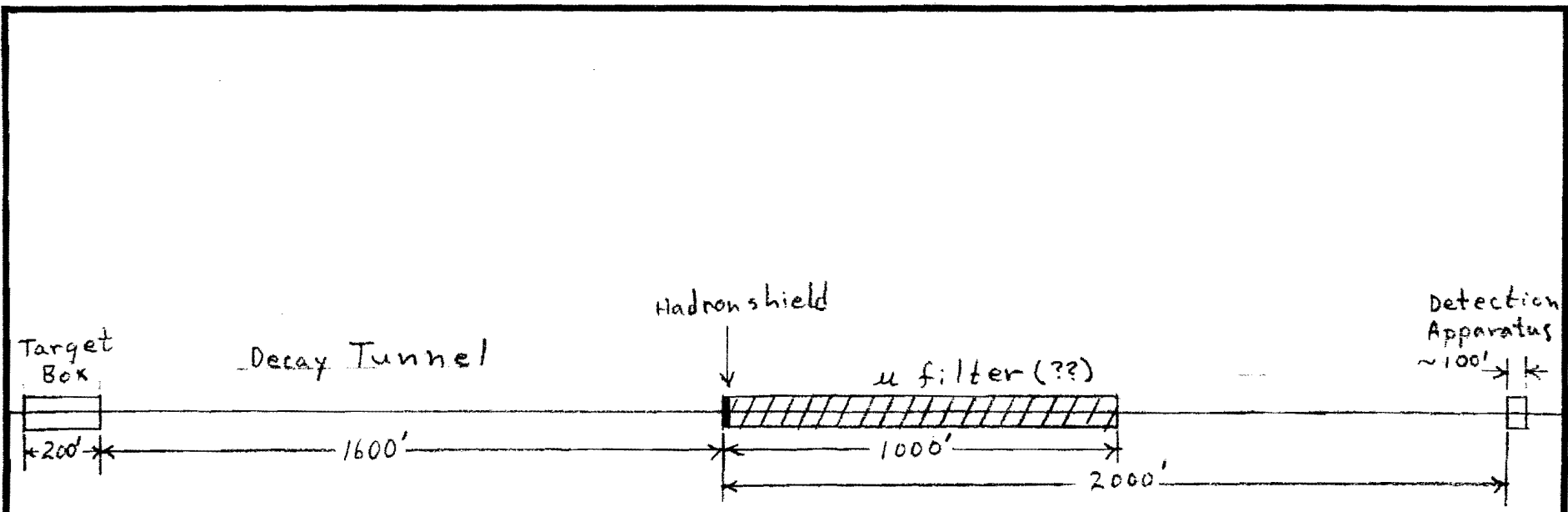
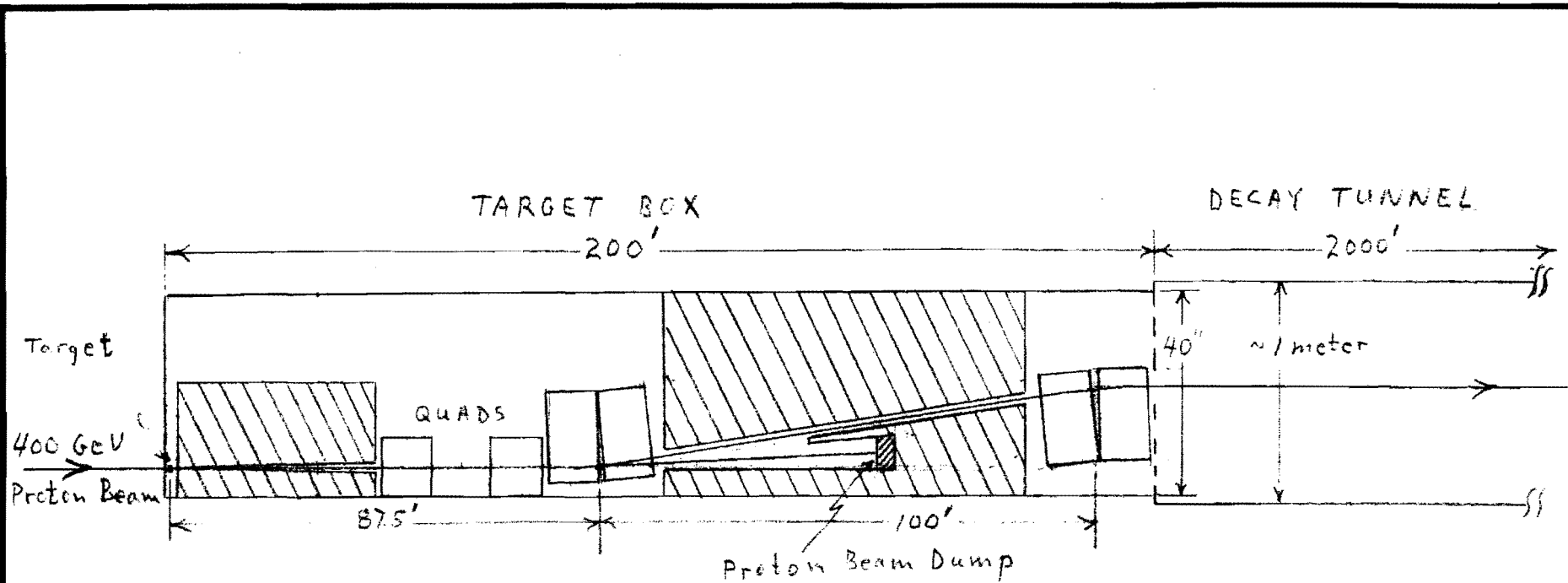


Figure 2



-12a-

CALIFORNIA INSTITUTE OF TECHNOLOGY SYNCHROTRON LABORATORY		
FIGURE 3: BEAM LINE CONFIGURATION		
DRAWN BY	DATE 6/6/70	DRAWING NO.
CHECKED BY	SCALE	
APPROVED BY	W.O.	



-15a-

SCALE:

in

40

30

20

10

Longitudinal: 1" = 32'

Transverse: 1" = 32"

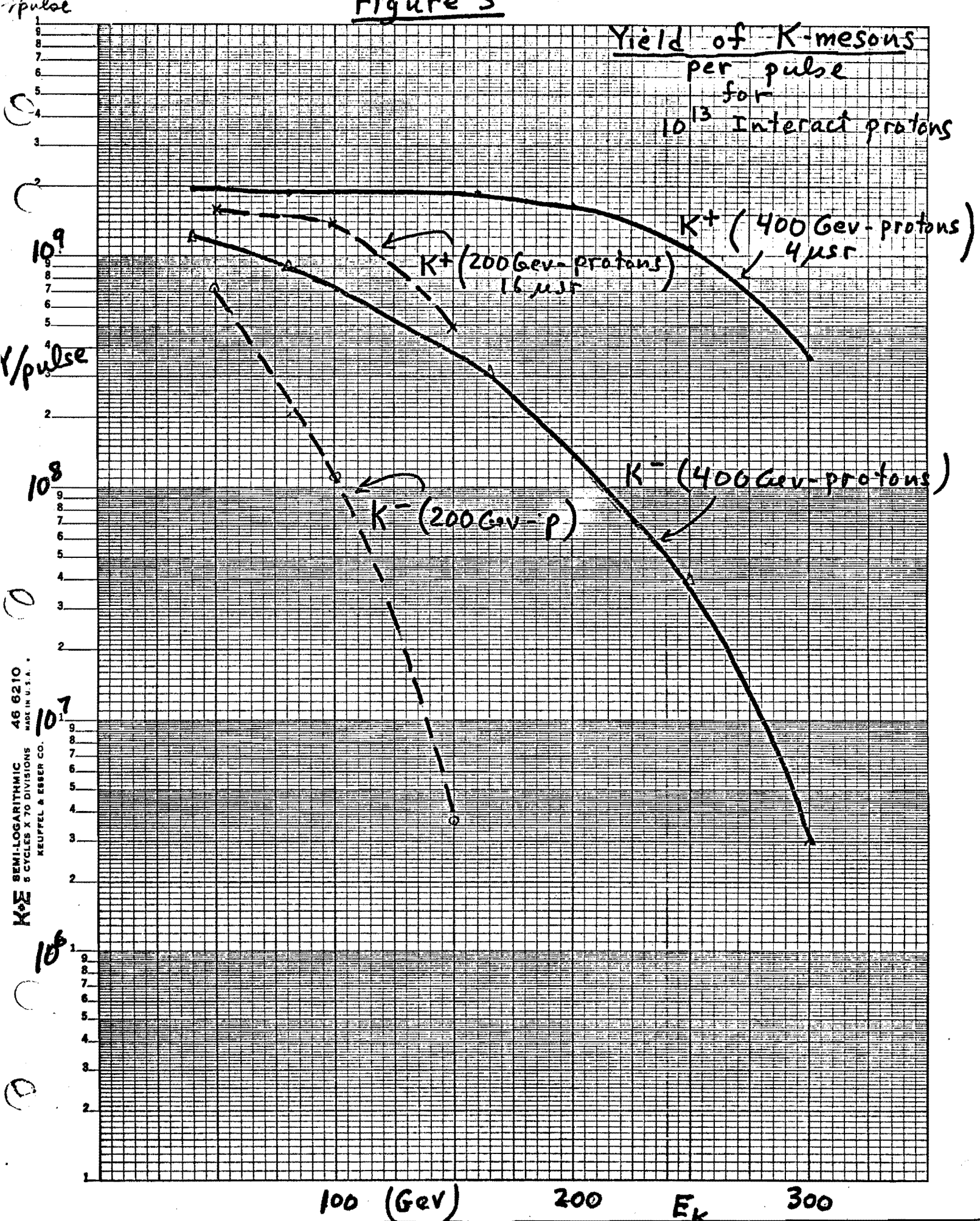
10 20 30 40 ft

FIGURE 4

CALIFORNIA INSTITUTE OF TECHNOLOGY SYNCHROTRON LABORATORY		
HADRON BEAM LAYOUT		
DRAWN BY	DATE 6/4/70	DRAWING NO.
CHECKED BY	SCALE	
APPROVED BY	W.O.	

Figure 3

Yield of K-mesons
per pulse
for
 10^{13} Interact protons



K&E SEMI-LOGARITHMIC
5 CYCLES X 70 DIVISIONS
MADE IN U.S.A.
KEUFFEL & ESSER CO.

10⁶
10⁷
10⁸
10⁹

100 (GeV) 200 E_K 300

FIGURE 6 (Monitoring)

6 June 1970

Average no. of μ 's per unit area downstream of hadron shield at end of 600 m decay region.

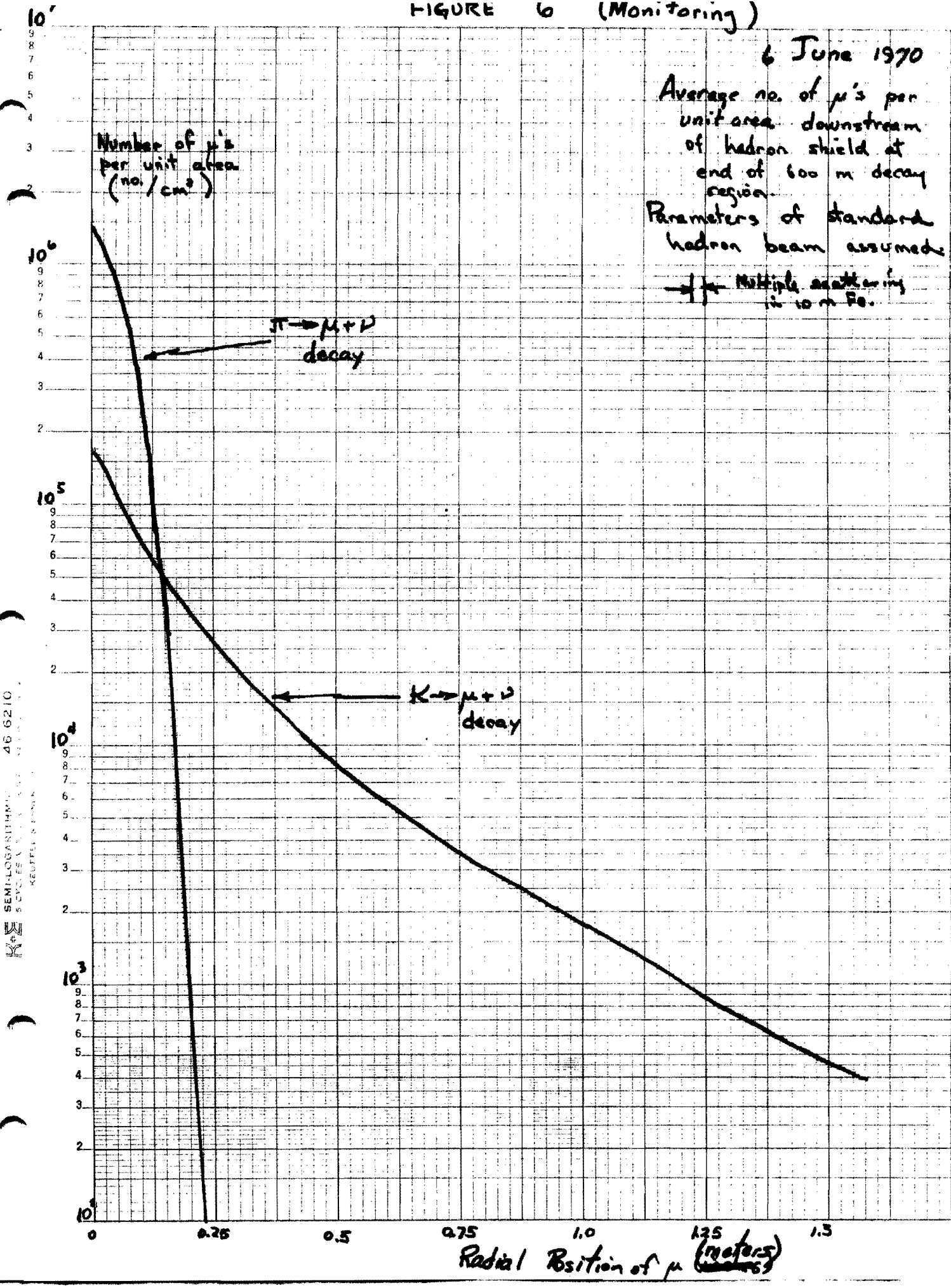
Parameters of standard hadron beam assumed

Multiple scattering in 10 m Fe.

Number of μ 's per unit area (no./cm²)

$\pi \rightarrow \mu + \nu$ decay

$K \rightarrow \mu + \nu$ decay



SEMI-LOGARITHMIC 46 6210
 5 CYCLES
 KEUFEL & LEIN

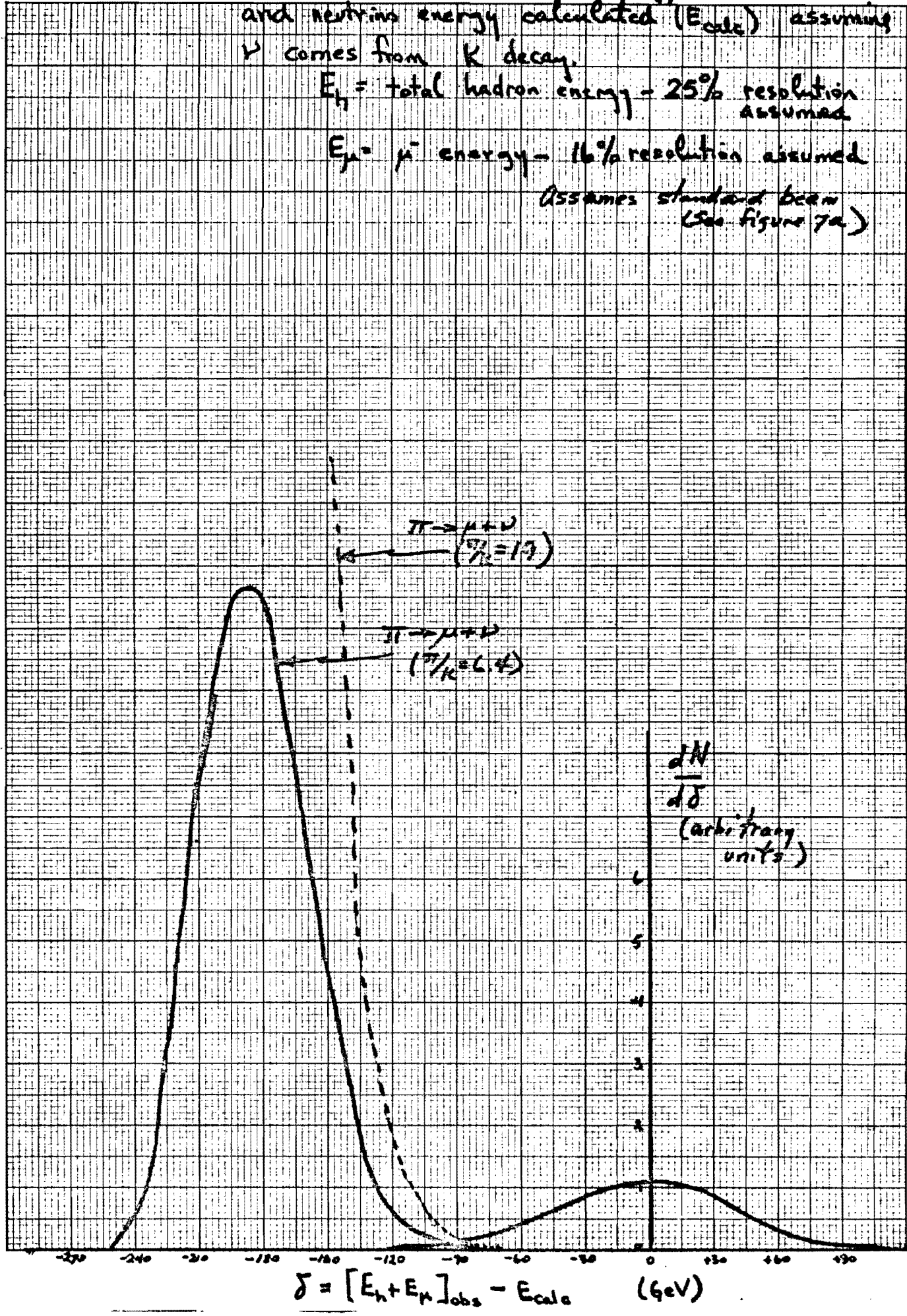
FIGURE 10: Difference between total energy observed

and neutrino energy calculated (E_{calc}) assuming ν comes from K decay.

$E_h = \text{total hadron energy} - 25\% \text{ resolution assumed}$

$E_\mu = \mu^- \text{ energy} - 16\% \text{ resolution assumed}$

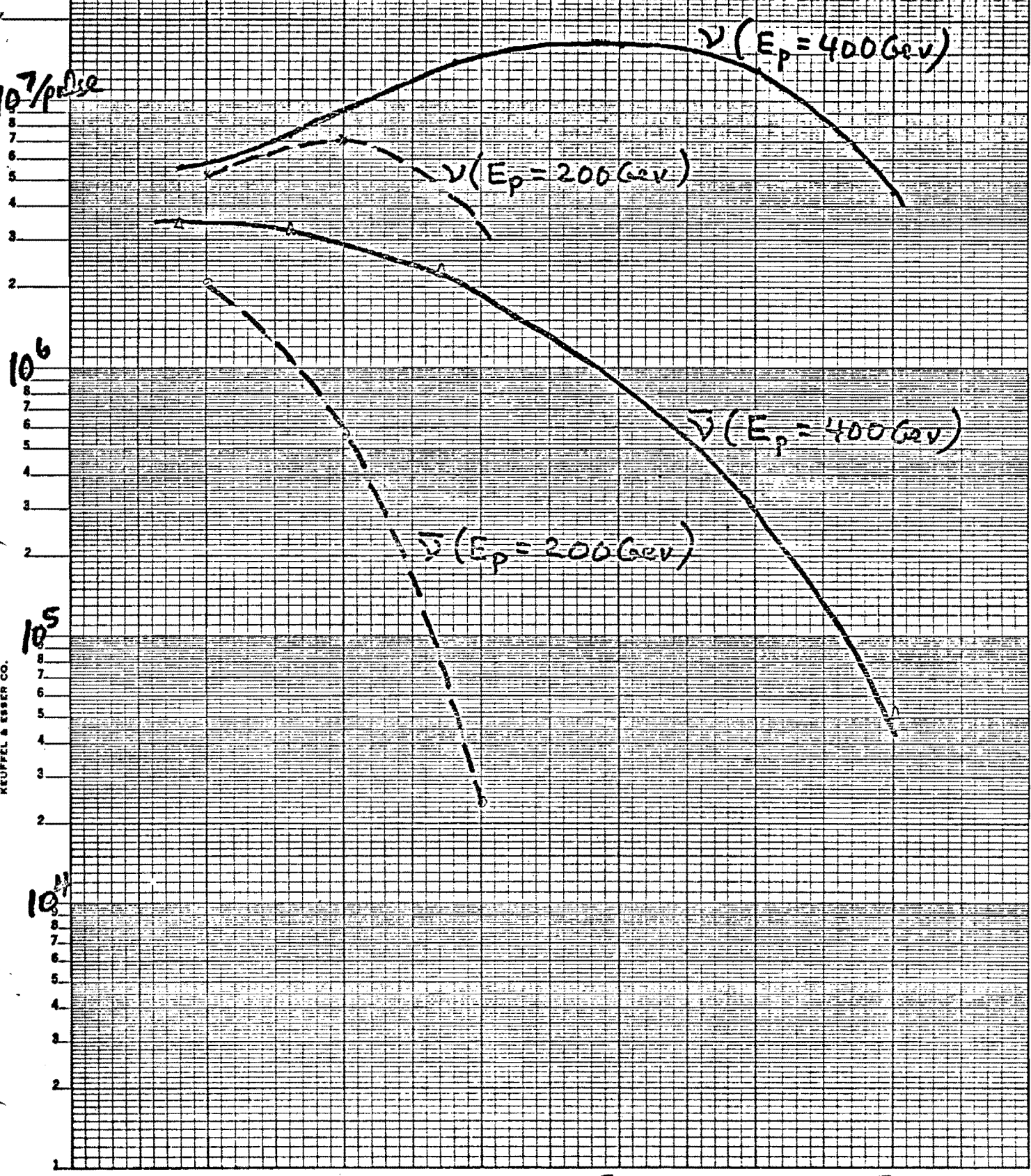
(Assumes standard beam (See figure 7a))



K&S 10X 10 TO THE 1/4 INCH 359-11 KEUFFEL & ESSER CO. MADE IN U.S.A.

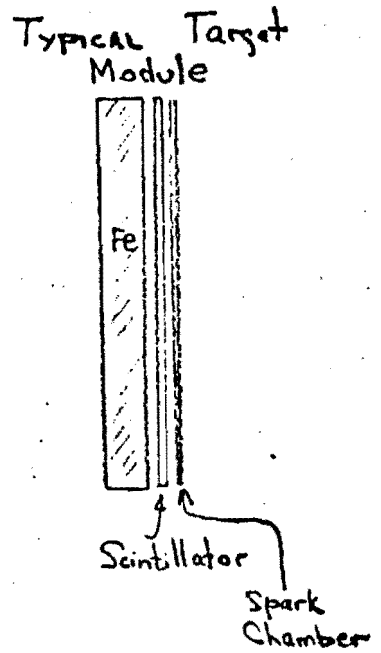
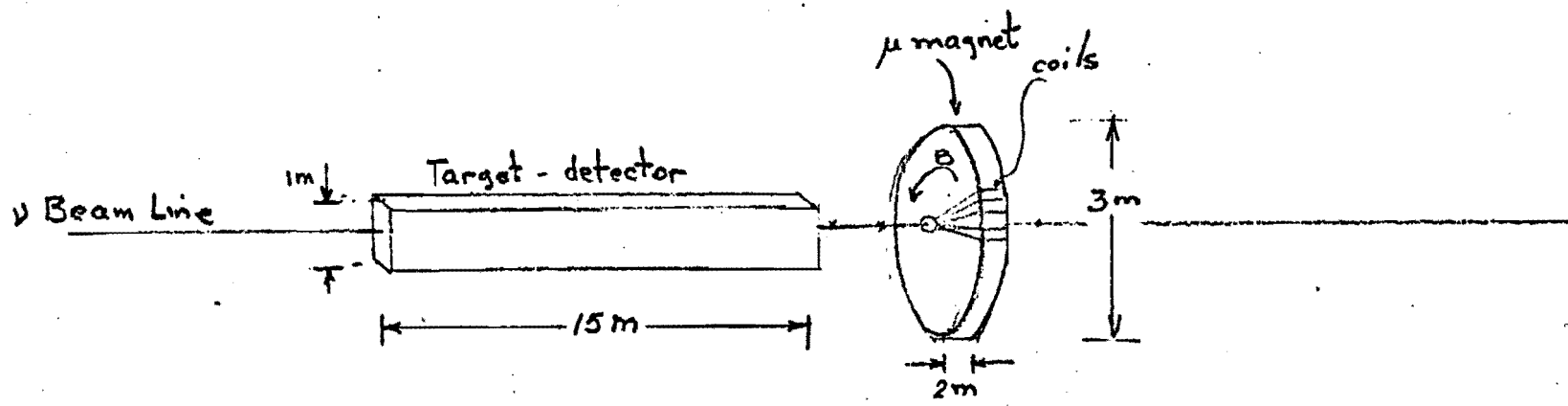
Assumes fluxes Figure 9
 $\nu, \bar{\nu}$ Flux into $(10^{12}$ int protons)
 $1m^2$ apparatus 600m
 from decay region

$10^7/pdise$



100 E_ν (GeV) 200 300

K&E SEMI-LOGARITHMIC 46 6210 MADE IN U.S.A.
 5 CYCLES X 70 DIVISIONS
 KEUFFEL & ESSER CO.

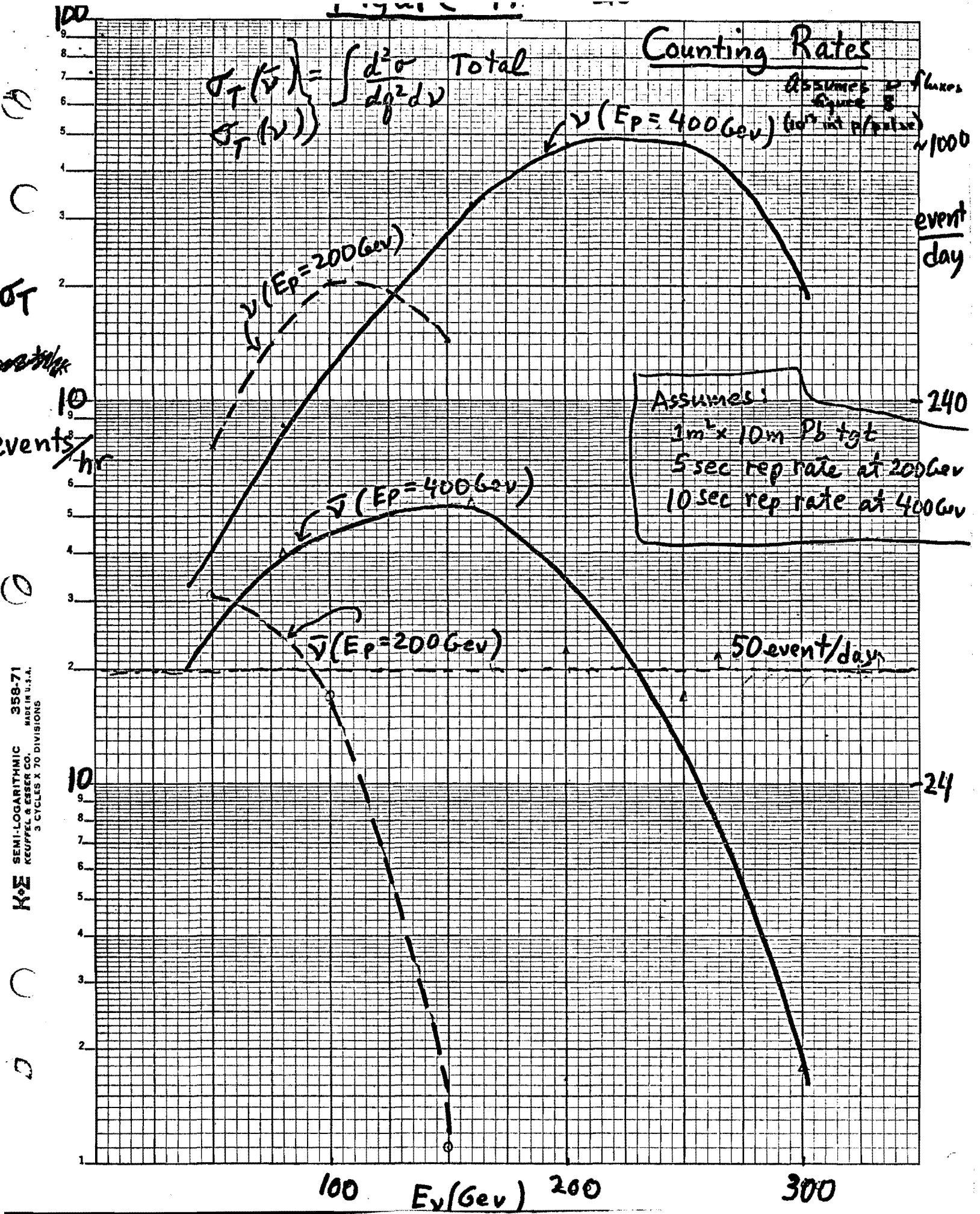


CALIFORNIA INSTITUTE OF TECHNOLOGY SYNCHROTRON LABORATORY		
FIGURE 9		APPARATUS
DRAWN BY	DATE 6 JUNE 70	DRAWING NO.
CHECKED BY	SCALE	
APPROVED BY	W.O.	

Counting Rates

$$\sigma_T(\nu) = \int \frac{d^2\sigma}{d\Omega^2 d\nu} \text{ Total}$$

Assumes ν fluxes
Square σ
(10^8 int ν /pulse) ~ 1000



KE SEMI-LOGARITHMIC 358-71
 KEUFFEL & ESSER CO. MADE IN U.S.A.
 3 CYCLES X 70 DIVISIONS

Observed Events
 $W \rightarrow \mu + \nu$
 • 10 BR. for Assumed
 Assumes ν fluxes of Figure 8
 (10^{12} int protons/pulse)

Events/day
 100

10

$E_p(200 \text{ GeV})$
 $E_\nu(100 \text{ GeV})$
 5 sec rep rate

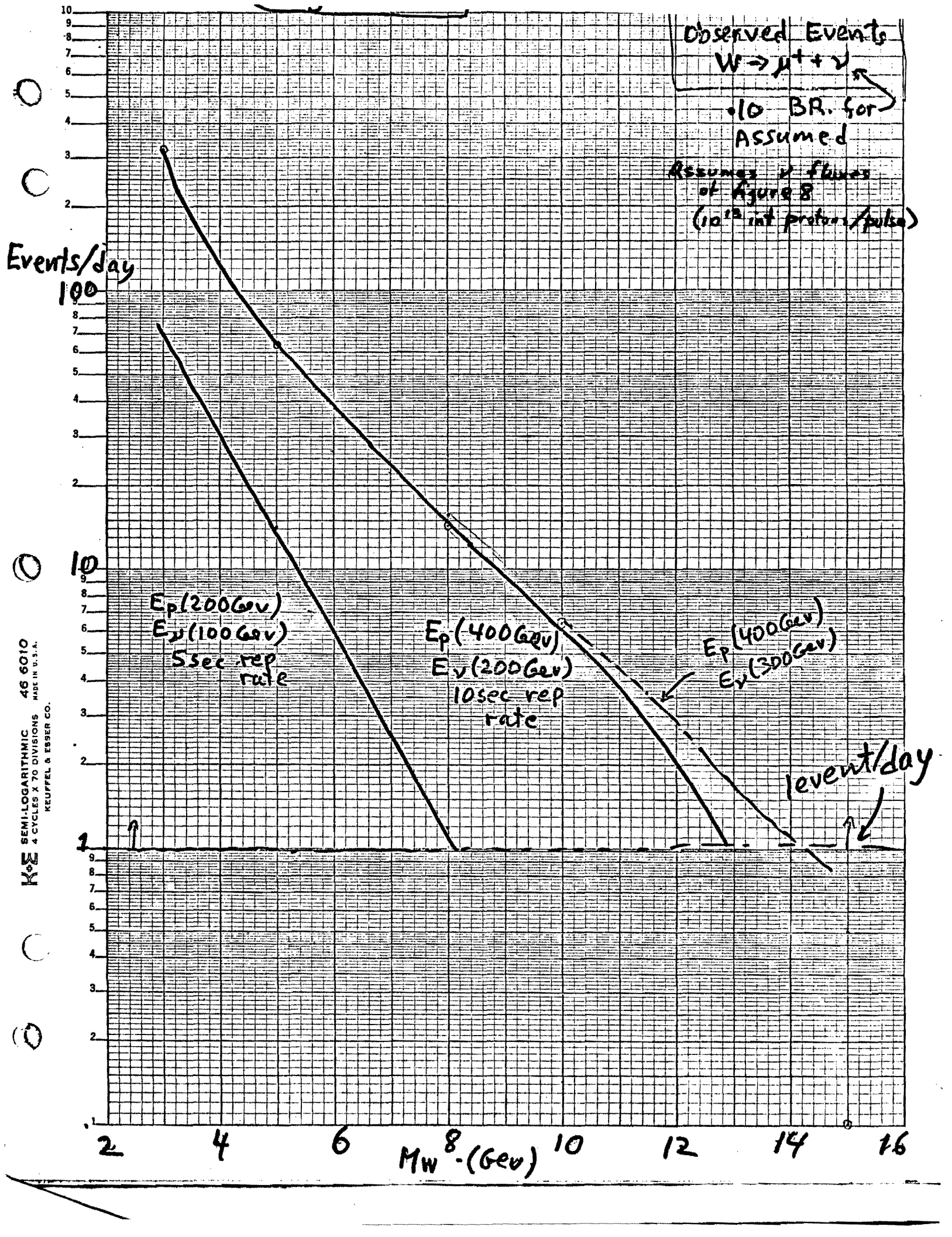
$E_p(400 \text{ GeV})$
 $E_\nu(200 \text{ GeV})$
 10 sec rep rate

$E_p(400 \text{ GeV})$
 $E_\nu(300 \text{ GeV})$

1 event/day

2 4 6 $MW^8 \cdot (\text{GeV})$ 10 12 14 16

K&E SEMI-LOGARITHMIC 46 6010
 4 CYCLES X 70 DIVISIONS
 MADE IN U.S.A.
 KEUFFEL & ESSER CO.



NEUTRINO PHYSICS AT HIGH ENERGIES

July 20, 1970

NAL Proposal #21

Addendum

The material presented in this supplement represents additional information which was not included in the main proposal. This includes discussions of backgrounds, W-boson rates, muon shielding, and the estimated cost of the experiment.

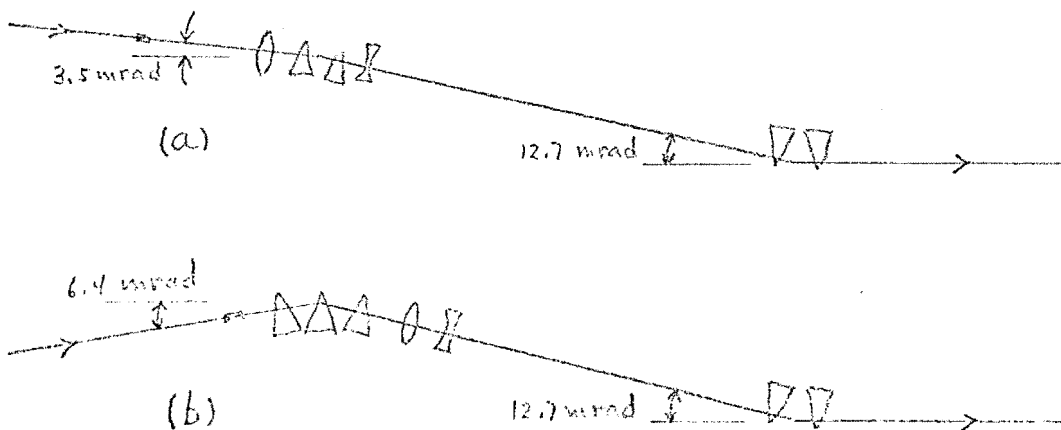
I. Backgrounds

A. Neutrino beam

The neutrino beam was discussed in section IIF of the proposal (p. 18). For the purpose of obtaining a monochromatic neutrino beam, the signal consists of the high energy peak of neutrinos from $K_{\mu 2}$ decay. The background presented by the $\pi_{\mu 2}$ neutrinos was discussed in the proposal. We have calculated the yields for two additional sources of off-energy neutrinos:

(1) $K_{\mu 3}$ and $K_{e 3}$ decays. These decay modes give neutrinos in a broad spectrum extending from zero to $\sim .75 E_K$. Figure A-1 shows the neutrino spectrum with the background effects included, for the case $E_K = 300$ GeV. The $K_{\mu 3}$ decays contribute the shoulder on the high side of the $\pi_{\mu 2}$ peak.

(2) Decays from before the first magnet of the hadron beam. As indicated in Figure 4 of the proposal, the hadrons produced in the target drift for ~ 88 ft. before the charge and momentum selection is made. Although this distance is short compared with the decay tunnel, because of the much wider contributing energy range this background becomes non-negligible at the lower energy settings. A small refinement of the hadron beam will reduce this background significantly. The idea is to arrange that the front-end drift space is not directed parallel to the decay tunnel axis, according to configuration (a) or (b) sketched below:



Neutrino Spectrum

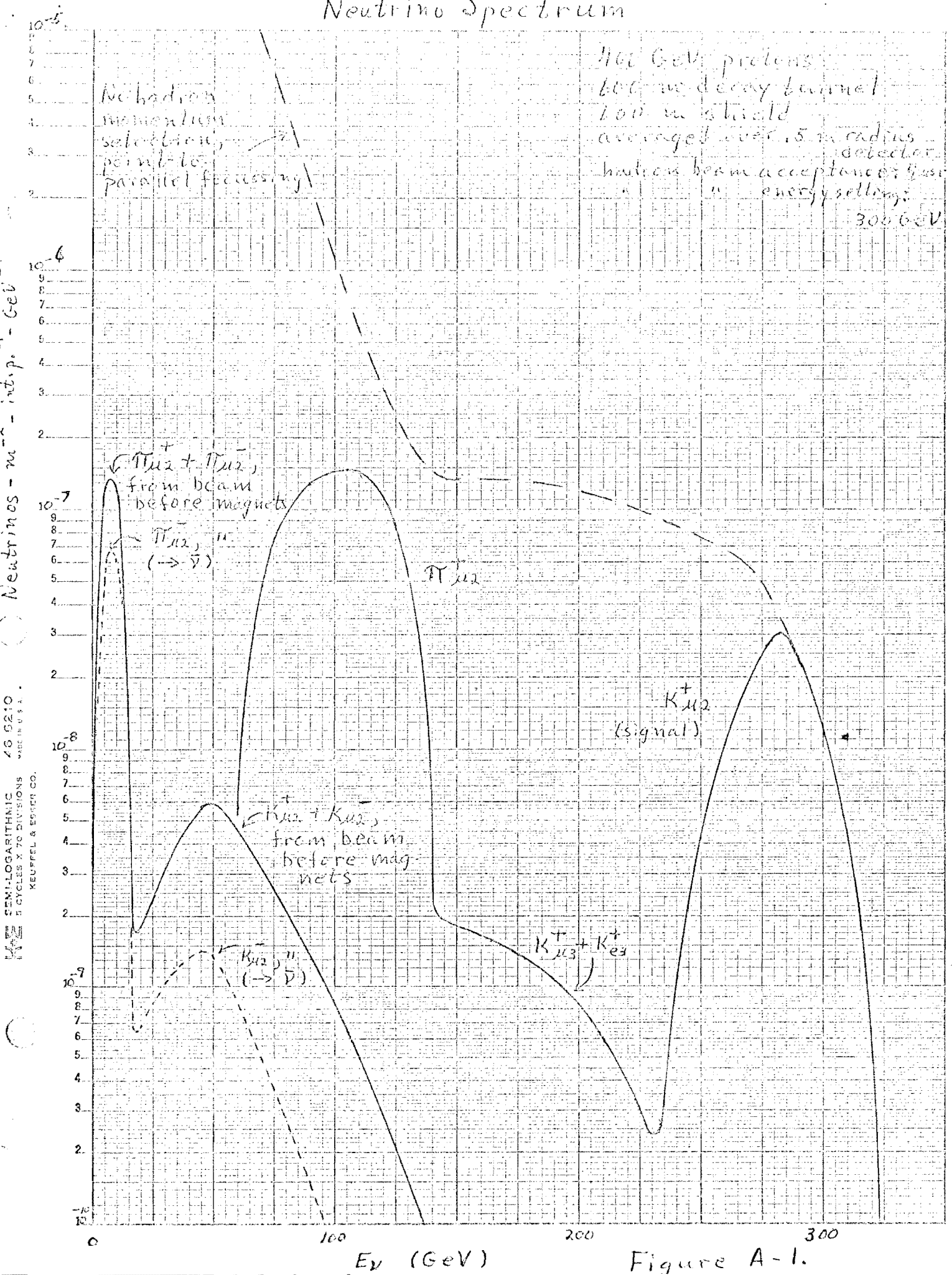


Figure A-1.

The quads magnify or demagnify the first bend as needed to give an achromatic beam at the output. Satisfactory solutions have been calculated for both configurations.

The neutrino background from this source has been calculated and is plotted (for beam configuration (a) above) in Figure A-1. The lower dotted curve gives the antineutrino component.

B. Backgrounds for W-search

In this section we shall discuss briefly the kinds of neutrino interactions that tend to have signatures similar to that of a W meson. If the W is massive it will be produced only by high-energy neutrinos. A feature of the monochromatic beam is that when it is tuned to high energies, the very large low energy part of the neutrino spectrum that would otherwise be present is largely removed (see Figure A-1). The ratio of W-production to total interactions is thereby greatly improved.

For W's detected via the μ, ν decay, the potential backgrounds are:

(1) Single π^+ and multi- π^+ production, with a subsequent $\pi_{\mu 2}^+$ decay, resulting in a dimuon signature with little or no hadronic shower. We estimate the rate of such events by taking a cross section of $.3 \times 10^{-38} \text{ cm}^2$ for single π^+ production, the number of neutrinos (signal and background) striking the detector ($\approx 5 \times 10^7$ per pulse), and a probability for a pion to decay before interacting of $\sim 10^{-3}$, for $E_{\pi} =$ a few GeV. The result for single π^+ background events is $\sim 10^{-3}$ per day, even if the recoiling baryons never give observable showers. Events with observable showers are distinguishable from W's.

Regarding the multipion production, we can establish an upper limit as follows: The total number of neutrino interactions (for signal and background) is ~ 5000 per day. Multiplying by the decay probability of 10^{-3} , we get ~ 5 per day with a π -decay muon. Of this total the number of events with no hadron shower is very

small: conservatively <2%. This gives <.1 background event per day.

(2) A contamination of antineutrinos would be somewhat more serious since the μ^+ from $\bar{\nu} + z \rightarrow \mu^+ + z' + \pi^-$ is more likely to fake a W^+ decay, its energy being more often large. From Figure A-1 it is clear that the $\bar{\nu}$ contamination is very small, and confined to low energy. It can be neglected.

(3) In order to detect the production of W 's without seeing the leptonic decay, one must be able to see a sharp, low energy peak in the spectrum of muons from interactions at a given neutrino energy. The background E_μ spectrum from the inelastic scattering is expected to be quite flat. We can expect to see the low energy peak very clearly since the neutrino energy is well determined for each event. Most of the low energy neutrinos, which could give an excess of low energy background muons in the sample if the energy were misanalyzed, are removed from the narrow-band beam.

C. Separation of Neutrino and Antineutrino Inelastic Processes

The only background for the inelastic processes is the very small contamination from antineutrinos (see Figure A-1), coming from the part of the beam before the first magnet. It is negligible over all of the useful energy range. For running with antineutrinos, the neutrino background is as large as 15% at the worst energy (50 GeV). Because of rate limitations we expect to take antineutrino data at only one energy, which will be chosen to optimize rate consistent with keeping the background at a negligible level. The neutrino background in the antineutrino sample affects only those events in which the muon range is too short to permit determination of its sign.

II. Rates for High Mass W-Bosons ($M_W > 10 \text{ GeV}$)

It is natural to assume that since our beam accepts only a 10% momentum bite, the rates will be lower than those for a beam which does not make this selection. It is the purpose of this section to point out that in the search for very heavy W's the rates for our beam are comparable to those of a wide band system. Rate is not a problem for the rest of the physics in the proposal.

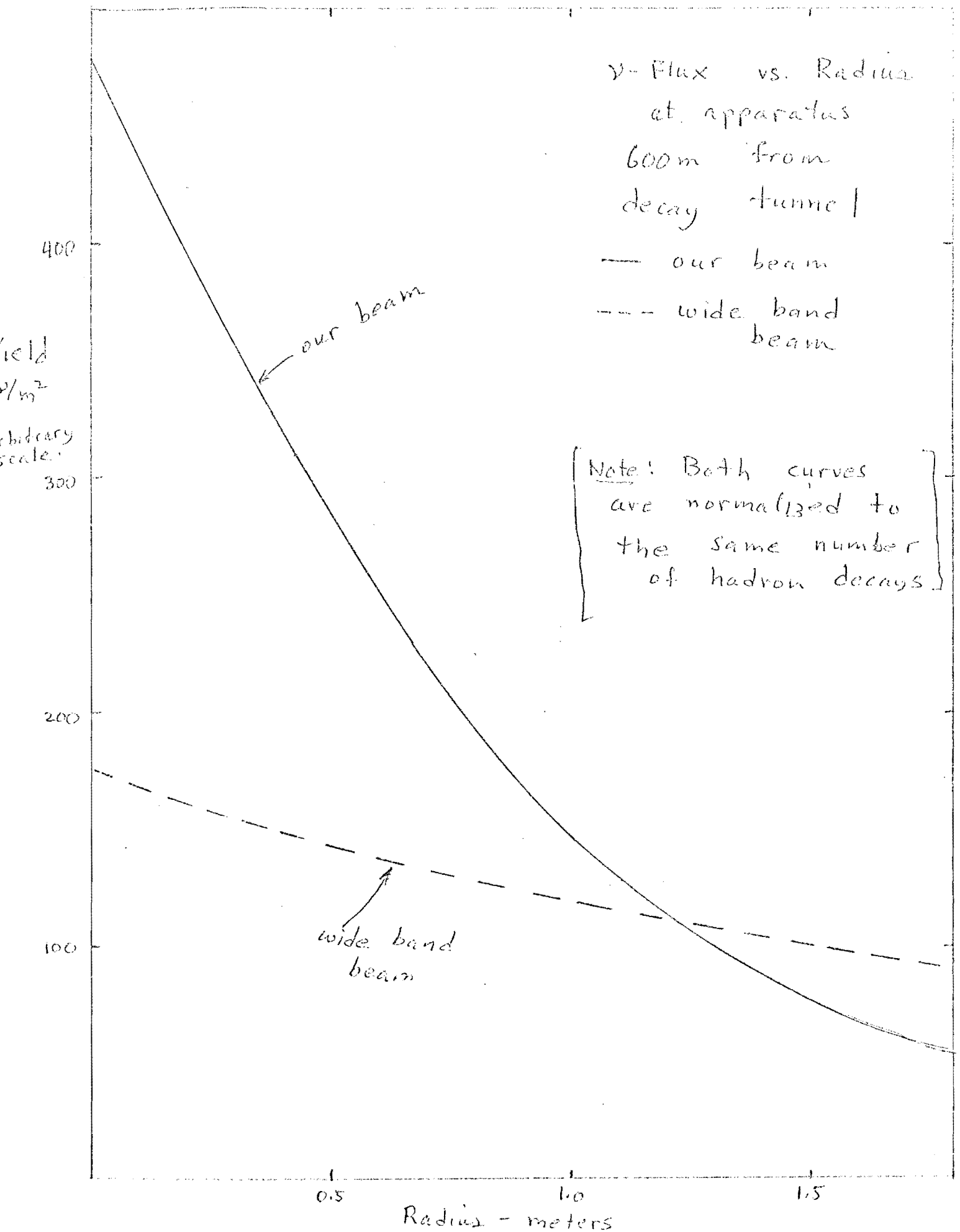
The production of high mass W's is due to the highest energy neutrinos. This can be seen by looking at the W-production curves, Fig. 1 of the proposal. These neutrinos, in turn, come from pions $\geq 300 \text{ GeV}$ and kaons $\geq 150 \text{ GeV}$. Actually, the contribution of 150 GeV K-mesons is less than that of 300 GeV K-mesons, since the cross section rises sharply with energy. As a result, a 30 GeV band of hadrons at $\sim 300 \text{ GeV}$ includes a large fraction of those which contribute to the counting rate.

Another factor makes the rates more favorable for our momentum selected beam. The hadrons are very well collimated ($\sim .2$ meters in diam. at the end of decay tunnel, and $\Delta\theta \sim .1 \text{ mr.}$) for our beam. A wide band system in contrast fills a 1 meter diameter decay tunnel and has much larger angular divergences. As a result, our beam produces a much better collimated beam of neutrinos than does a wide band beam. A comparison of the flux of neutrinos vs. radius at our detector is shown in Fig. A-2 for our beam and for a model of a wide band beam.

In order to match this distribution our apparatus was chosen to be 1 m. x 1 m. in transverse dimensions while any wide band system is usually chosen to be at least 2 m. x 2 m. This means for equal target mass every neutrino in our beam goes through 4 times as much target material.

Calculations have been made comparing in detail a wide band beam with our beam for production of W's $> 10 \text{ GeV}$. Actual rates for these heavy W's are uncertain

Figure A-2



due to the very large differences in K^+/π^+ ratio for different production models. However, we find independent of these assumptions that for equal tonnage, a wide band arrangement with a larger area detector always has less rate than our experiment. The actual factor, which depends on K/π ratio, type of wide band system used, M_W , etc., varies from 1.3 - 3.0.

To summarize, our advantage in rate due to having the neutrinos well collimated more than offsets the losses due to our momentum selection in the hadron beam in the search for the heaviest W's.

III. Muon Shielding

Due to recent questions concerning the muon shield for the bubble chamber, we would like to emphasize the fact that our experiment does not require this shield.

Since our experiment is placed 600 meters downstream of the decay region (in order to obtain our monoenergetic ν -beam), it is possible without compromising the experiment to use an earth shield.

The highest energy muons ever produced in our beam are 300 GeV/c. The energy loss in earth is $\sim .45$ GeV/meter, which means that ~ 670 meters of earth stops our most energetic muons.

The experiment could use either a mixture of earth and iron and remain at 600 meters or pure earth and move to say 800 meters. Moving the experiment gives only $\sim 25\%$ reduction in rate and actually improves the energy and angular resolution on neutrinos.

IV. Estimated Cost of Equipment

We have proposed an apparatus of minimal size and complexity to perform our

experiment. We have been able to relax somewhat the demands on the apparatus, since we independently determine the incoming neutrino angle and energy. The iron core magnet is only 2 meters thick, since 13 - 16% energy resolution on the muons is adequate. Our proposed Calorimeter-target is only ~ 1 m. x 1 m. transverse and is rather coarse in sampling stations. This is possible since the rate is adequate and calorimetry requirements are not severe ($\pm 20 - 25\%$).

Below is an estimate of the cost of our apparatus.

(1) Calorimeter-target

10 m ³ material	\sim \$15K	Fe, Pb, Al mixture
Liquid Scintillator	\sim \$30K	Estimated commercially by Nuclear Enterprises including boxes
Wire Spark Chambers	\sim \$50K	
Electronics	\sim \$55K	(some of this equipment already exists at Caltech)
	<u>\$150K</u>	

(2) Muon Spectrometer

Magnet, coils, power supply, stand. 1.5 m. radius x 2 m. thick	\$125K
Wire Chambers and Electronics	<u>\$25K</u>
	\$150K

(3) Beam Monitor

10 small counters	\sim \$2K
-------------------	-------------

(4) Small Computer for wire chamber readout and data collection

Probably Caltech Σ -2 or Equivalent

(5) Miscellaneous

\$75K

Estimated Grand Total

\$375K