Spin-Dependent Macroscopic Forces from New Particle Exchange

Bogdan A. Dobrescu[1]**, Irina Mocioiu**[2]

[1] *Theoretical Physics Department, Fermilab, Batavia, IL 60510, USA*

bdob@fnal.gov

[2] *Pennsylvania State University, University Park, PA 16802, USA*

irina@phys.psu.edu

May 31, 2006

Abstract

Long-range forces between macroscopic objects are mediated by light particles that interact with the electrons or nucleons, and include spin-dependent static components as well as spin- and velocity-dependent components. We parametrize the long-range potential between two fermions assuming rotational invariance, and find 16 different components. Applying this result to electrically neutral objects, we show that the macroscopic potential depends on 72 measurable parameters. We then derive the potential induced by the exchange of a new gauge boson or spinless particle, and compare the limits set by measurements of macroscopic forces to the astrophysical limits on the couplings of these particles.

Contents

1   Introduction

The electromagnetic and gravitational interactions, mediated by spin-1 and spin-2 particles, are the only macroscopic forces observed so far. However, other macroscopic forces could exist, and more sensitive measurements might reveal them. Searches for long-range spin-independent forces have a long history of substantial improvements achieved by various groups (for recent reviews see Ref. [1, 2]). By contrast, long-range spin-*dependent* forces could lead to a broader variety of observable effects, but so far they have been less intensely investigated. Most searches have been concentrated on two types of spin-dependent long-range forces that could be induced by axion exchange, the so-called dipole-dipole and monopole-dipole interactions [3].

Measurements of forces between macroscopic polarized objects have set limits on new dipole-dipole potentials among electrons [4, 5, 6, 7, 8], and between electrons and nucleons [5, 7]. There are also limits on monopole-dipole forces between polarized electrons and unpolarized objects [5, 9, 10, 11], as well as between polarized nucleons and unpolarized objects [5, 9, 12, 13]. Earlier experiments are reviewed in [5, 7, 14, 15].

Here we study spin-dependent forces between macroscopic objects that could exist given general assumptions within quantum field theory. We focus on rotational-invariant potentials that could be induced by the exchange of new light particles, showing that several new kinds of spin-dependent macroscopic forces may exist and should be searched for in experiments.

The discovery of a new force with a range longer than about a micrometer would have a tremendous impact on our understanding of nature. Furthermore, even if new macroscopic forces will not be discovered, setting limits on the various potentials is important for constraining many extensions of the Standard Model of particle physics. The spontaneous breaking of continuous symmetries leads to the existence of massless or very light (pseudo) Nambu-Goldstone bosons, such as axions, familons, majorons, etc. [16]. It is also possible that new massless gauge bosons associated with unbroken gauge symmetries exist [17]. Such particles have naturally suppressed interactions with ordinary matter, but nevertheless could mediate long-range forces that may be accessible to laboratory experiments. As an application, we derive the limits on the couplings of a new massless spin-1 particle ("paraphoton") from existing measurements of spin-dependent forces.

Massive spin-1 particles with general couplings, or bosons of spin-2 or higher, could also be light enough to mediate macroscopic forces, albeit their low mass and feeble interactions would require very small dimensionless parameters or fine tuning. We will show that the majority of the rotational-invariant spin-dependent potentials are generated by the exchange of a massive spin-1 particle in a Lorentz-invariant theory.

We first construct the most general momentum-space elastic-scattering amplitude for two fermions consistent with rotational invariance (see Section 2). We then Fourier transform to position space in Section 3, and obtain the spin-dependent potential between two fermions. In Section 4 we discuss the potential between macroscopic objects in the case of one-boson exchange in a Lorentz invariant theory (Section 4.1), as well as in more exotic

cases, such as the exchange of a boson obeying a Lorentz-violating dispersion relation [18], or the exchange of two or more particles (see Section 4.2).

We apply this general formalism to the case of spin-1 and spin-0 particle exchange in Sections 5 and 6, respectively. In this context we compare the current experimental limits on spin-dependent forces with the astrophysical limits on very light particles. Our results are summarized in Section 7.

2 Long-range fermion-fermion interactions in momentum space

In order to derive the long-range force between two fermions of masses m and m', mediated by some very light particles, one needs to compute first the nonrelativistic limit of the scattering amplitude represented by the diagram shown in Figure 1. This amplitude can be expressed in terms of scalar invariants formed out of the incoming and outgoing fermion three-momenta, $\vec{p}_1, \vec{p}_1'$ and $\vec{p}_2, \vec{p}_2'$, respectively, and the two fermion spins $\vec{\sigma}$ and $\vec{\sigma}\,'$. In the center-of-mass frame only two momenta are independent, and we choose the following linear combinations:

$$
\begin{aligned}
\vec{q} &\equiv \vec{p}_2 - \vec{p}_1 \\
\vec{P} &\equiv \frac{1}{2}\left(\vec{p}_1 + \vec{p}_2\right) \ .
\end{aligned}
\tag{2.1}
$$

Note that $\vec{q}$ is the momentum transferred to the fermion of mass m, and $\vec{P}$ is the average momentum of that fermion.

With two spins and two momenta, one can construct 16 independent scalars that include all possible spin configurations. Eight of those include an even number of momenta, so they are invariant under a parity transformation:

$$
\begin{aligned}
\mathcal{O}_1 &= 1 \ , \\
\mathcal{O}_2 &= \vec{\sigma} \cdot \vec{\sigma}' \ , \\
\mathcal{O}_3 &= \frac{1}{m^2}\left(\vec{\sigma} \cdot \vec{q}\right)\left(\vec{\sigma}' \cdot \vec{q}\right) \ , \\
\mathcal{O}_{4,5} &= \frac{i}{2m^2}\left(\vec{\sigma} \pm \vec{\sigma}'\right) \cdot \left(\vec{P} \times \vec{q}\right) \ ,
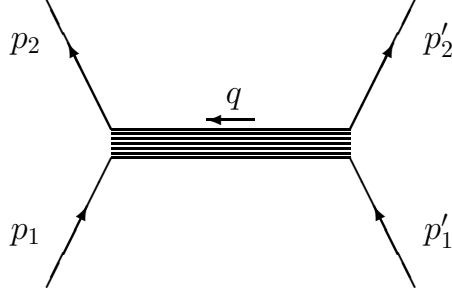\end{aligned}
$$

3

Figure 1: Elastic scattering of two fermions mediated by some very light particles represented generically by the horizontal blob of four-momentum q.

$$\mathcal{O}_{6,7} = \frac{i}{2m^2} \left[\left(\vec{\sigma} \cdot \vec{P} \right) \left(\vec{\sigma}' \cdot \vec{q} \right) \pm \left(\vec{\sigma} \cdot \vec{q} \right) \left(\vec{\sigma}' \cdot \vec{P} \right) \right] ,$$

$$\mathcal{O}_8 = \frac{1}{m^2} \left(\vec{\sigma} \cdot \vec{P} \right) \left(\vec{\sigma}' \cdot \vec{P} \right) . \tag{2.2}$$

We have included powers of the fermion mass m in the denominators such that all these operators are dimensionless (we use the natural unit system: $\hbar = c = 1$). The other eight scalars change sign under a parity transformation:

$$\mathcal{O}_{9,10} = \frac{i}{2m} \left(\vec{\sigma} \pm \vec{\sigma}' \right) \cdot \vec{q} ,$$

$$\mathcal{O}_{11} = \frac{i}{m} \left(\vec{\sigma} \times \vec{\sigma}' \right) \cdot \vec{q} ,$$

$$\mathcal{O}_{12,13} = \frac{1}{2m} \left(\vec{\sigma} \pm \vec{\sigma}' \right) \cdot \vec{P} ,$$

$$\mathcal{O}_{14} = \frac{1}{m} \left(\vec{\sigma} \times \vec{\sigma}' \right) \cdot \vec{P} ,$$

$$\mathcal{O}_{15} = \frac{1}{2m^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q} \right) \right] \left(\vec{\sigma}' \cdot \vec{q} \right) + \left(\vec{\sigma} \cdot \vec{q} \right) \left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q} \right) \right] \right\}$$

$$\mathcal{O}_{16} = \frac{i}{2m^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q} \right) \right] \left(\vec{\sigma}' \cdot \vec{P} \right) + \left(\vec{\sigma} \cdot \vec{P} \right) \left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q} \right) \right] \right\} . \tag{2.3}$$

Any other scalar operator involving at least one of the two spins can be expressed as a linear combination of the operators $\mathcal{O}_i(\vec{q}, \vec{P})$, $i = 1, \ldots, 16$, with coefficients that may depend on the momenta only through the $\vec{q}^2$ or $\vec{P}^2$ scalars. Note that energy-momentum conservation implies $\vec{q} \cdot \vec{P} = 0$. Examples of other operators which can be expressed as linear combinations of $\mathcal{O}_i$, $i = 1, \ldots, 16$, can be found in Appendix A. Although several of

the operators given in Eq. (2.2) have been analyzed in the context of nuclear interactions [20, 21, 22], we believe that the complete set of 16 rotationally invariant operators has not been previously presented in the literature.

The amplitude for elastic scattering of the two fermions depends on the properties of the light particles that mediate it. The long-range nature of the force is due to the propagator of the exchanged particles, which is a function of the square of the four-momentum transferred, q^2. Notice that $q_0 = 0$ due to energy conservation, so that $q^2 = -\vec{q}^2$. We use $\mathcal{P}(\vec{q}^2, m_0)$ to denote the imaginary part of the propagator with the Lorentz structure factored out. The mass dimension of $\mathcal{P}(\vec{q}^2)$ is -2. In the most common case, where the potential is induced by the exchange of one boson within a Lorentz invariant quantum field theory,

$$\mathcal{P}(\vec{q}^2) = -\frac{1}{\vec{q}^2 + m_0^2} \ , \tag{2.4}$$

where m_0 is the mass of the boson. Other forms for the propagator are possible. For example, in the case where *two* massless fermions are exchanged, the effective propagator takes the form [23]

$$\mathcal{P}(\vec{q}^2) = -\frac{1}{12\pi^2 M^2} \ln\left(\frac{\vec{q}^2}{M^2}\right) \ , \tag{2.5}$$

where M is the mass scale that suppresses the four-fermion contact interaction. If Lorentz symmetry is violated, then a boson may have a kinetic term with four or more spatial derivatives, giving a propagator

$$\mathcal{P}(\vec{q}^2) = -\frac{M^{2k-2}}{(\vec{q}^2)^k} \ , \tag{2.6}$$

where $k \geq 2$ is an integer, and M is some mass scale. The case $k = 2$ has been studied in [18]. For the moment we allow a generic form for $\mathcal{P}(\vec{q}^2)$, assuming only that it leads to long-range forces.

Certain generic features of the amplitude can be derived on general grounds. The nonrelativistic amplitude between two fermions may be written in the momentum space as

$$\mathcal{A}\left(\vec{q}, \vec{P}\right) = \mathcal{P}(\vec{q}^2) \sum_{i=1}^{16} \mathcal{O}_i\left(\vec{q}, \vec{P}\right) f_i\left(\vec{q}^2/m^2, \ \vec{P}^2/m^2\right) \ , \tag{2.7}$$

where f_i are dimensionless scalar functions. In the nonrelativistic limit, f_i are polynomials with coefficients that depend on the couplings of the exchanged particles. This is a general

5

result based only on the assumption of rotational invariance (this assumption is not valid in certain Lorentz-violating field theories [18, 19]).

The physical interpretation of the 16 operators is more transparent in the position space, as discussed in the next section.

3 Long-range potentials between fermions

The Fourier transform of the momentum-space amplitude with respect to the momentum transfer $\vec{q}$ gives the position-space potential:

$$V(\vec{r}, \vec{v}) = -\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \mathcal{A}(\vec{q}, m\vec{v}) \ , \tag{3.1}$$

where $\vec{r}$ is the position vector of the fermion of mass m and initial momentum $\vec{p}_1$ with respect to the fermion of mass m' and initial momentum $\vec{p}_1'$. Note that in general the potential depends not only on the position $\vec{r}$, but also on the average velocity of the fermion of mass m in the center-of-mass frame:

$$\vec{v} = \frac{\vec{P}}{m} \ . \tag{3.2}$$

The inverse mass of the boson sets the range of the interaction, so that an experimental setup characterized by a distance scale r_{exp} is sensitive to $1/m_0 \gtrsim r_{\text{exp}}$. We assume that r_{exp} is macroscopic, $r_{\text{exp}} \gtrsim O(1 \text{ mm})$, The most important contributions to the potential come from the momentum-independent terms of the $f_i\left(\vec{q}^2/m^2, \vec{v}^2\right)$ polynomials in Eq. (2.7). Additional powers of $\vec{q}^2/m^2$ lead to terms of order ϵ^2 in the potential, where ϵ is of the order of m_0/m or $1/(r_{\text{exp}}m)$ (see Appendix B). Given that m is the mass of the electron or nucleon, we find $\epsilon < 10^{-10}$, so that it is a good approximation to include only the $\vec{q}^2 = 0$ pieces of the polynomials, $f_i\left(0, \vec{v}^2\right)$. Note that additional powers of $\vec{q}^2/m^2$ also lead to Fourier transforms of the type $\delta(\vec{r})$, or more singular ones, which describe contact interactions rather than long-range potentials.

It is useful to observe that compared to $\mathcal{O}_1$ and $\mathcal{O}_2$, the operators $\mathcal{O}_i$ with $i = 9, 10, 11$ have effects of order ϵ, the operators $\mathcal{O}_i$ with $i = 12, 13, 14$ have effects of order $v = |\vec{v}|$, while the remaining ones have effects suppressed by more powers of ϵ or v. If the $\vec{q}^2$-independent term of f_i vanishes, then the $\vec{q}^2/m^2$ term dominates, and for $i = 1, 2, 9, \ldots, 14$

it might lead to experimentally observable effects. By contrast, the operators $\mathcal{O}_i$ with $i = 3, \ldots, 8, 15, 16$ are already quite suppressed, so that the $\vec{q}^{\,2}$-dependent terms of f_i can be safely neglected in their case. In what follows we will ignore the $\vec{q}^{\,2}$-dependent terms of all f_i, and we only mention that if a physical situation would require the inclusion of some of them, then they could be treated similarly to the $\vec{q}^{\,2}$-independent terms.

The long-range potential between two fermions induced by a Lorentz-invariant, one-boson exchange can be written as

$$V(\vec{r}, \vec{v}) = \sum_{i=1}^{16} \mathcal{V}_i\,(\vec{r}, \vec{v})\, f_i\!\left(0, \vec{v}^{\,2}\right) \,, \tag{3.3}$$

where we defined a complete set of spin-dependent potentials,

$$\mathcal{V}_i(\vec{r}, \vec{v}) = -\int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}}\, \mathcal{P}(\vec{q}^{\,2})\, \mathcal{O}_i(\vec{q}, m\vec{v}) \,, \tag{3.4}$$

with $i = 1, \ldots, 16$. As stated before, $f_i(0, \vec{v}^{\,2})$ are polynomials in $\vec{v}^{\,2}$, with coefficients given by dimensionless parameters that depend on the boson couplings to the fermions.

It is convenient to write the spin-dependent potentials in terms of a dimensionless function of r:

$$
\begin{aligned}
y(r) &\equiv -r \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}}\, \mathcal{P}(\vec{q}^{\,2}) \\
&= -\frac{1}{2\pi^2} \int_0^\infty d|\vec{q}|\, \mathcal{P}(\vec{q}^{\,2})\, |\vec{q}| \sin\left(|\vec{q}| r\right) \,.
\end{aligned} \tag{3.5}
$$

Using the operators $\mathcal{O}_i$ with $i = 1, \ldots, 8$, defined in Eq. (2.2), we obtain the following long-range, parity-invariant potentials:

$$\mathcal{V}_1 = \frac{1}{r}\, y(r) \,,$$

$$\mathcal{V}_2 = \frac{1}{r}\, \vec{\sigma} \cdot \vec{\sigma}'\, y(r) \,,$$

$$\mathcal{V}_3 = \frac{1}{m^2\, r^3}\left[\vec{\sigma} \cdot \vec{\sigma}'\left(1 - r\frac{d}{dr}\right) - 3\left(\vec{\sigma} \cdot \hat{r}\right)\left(\vec{\sigma}' \cdot \hat{r}\right)\left(1 - r\frac{d}{dr} + \frac{1}{3}r^2\frac{d^2}{dr^2}\right)\right] y(r) \,,$$

$$\mathcal{V}_{4,5} = -\frac{1}{2m\, r^2}\left(\vec{\sigma} \pm \vec{\sigma}'\right) \cdot \left(\vec{v} \times \hat{r}\right)\left(1 - r\frac{d}{dr}\right) y(r) \,,$$

7

$$\mathcal{V}_{6,7} = -\frac{1}{2m\,r^2}\left[\left(\vec{\sigma}\cdot\vec{v}\right)\left(\vec{\sigma}'\cdot\hat{\vec{r}}\right)\pm\left(\vec{\sigma}\cdot\hat{\vec{r}}\right)\left(\vec{\sigma}'\cdot\vec{v}\right)\right]\left(1-r\frac{d}{dr}\right)y(r)\ ,$$

$$\mathcal{V}_8 = \frac{1}{r}\left(\vec{\sigma}\cdot\vec{v}\right)\left(\vec{\sigma}'\cdot\vec{v}\right)y(r)\ , \tag{3.6}$$

where r is the length of the $\vec{r}$ vector, and we have defined the unit vector

$$\hat{\vec{r}} \equiv \frac{\vec{r}}{r}\ . \tag{3.7}$$

The operators $\mathcal{O}_i$ with $i = 9,\ldots,16$, defined in Eq. (2.3), give rise to the following long-range, parity-violating potentials:

$$\mathcal{V}_{9,10} = -\frac{1}{2m\,r^2}\left(\vec{\sigma}\pm\vec{\sigma}'\right)\cdot\hat{\vec{r}}\left(1-r\frac{d}{dr}\right)y(r)\ ,$$

$$\mathcal{V}_{11} = -\frac{1}{m\,r^2}\left(\vec{\sigma}\times\vec{\sigma}'\right)\cdot\hat{\vec{r}}\left(1-r\frac{d}{dr}\right)y(r)\ ,$$

$$\mathcal{V}_{12,13} = \frac{1}{2r}\left(\vec{\sigma}\pm\vec{\sigma}'\right)\cdot\vec{v}\,y(r)\ ,$$

$$\mathcal{V}_{14} = \frac{1}{r}\left(\vec{\sigma}\times\vec{\sigma}'\right)\cdot\vec{v}\,y(r)\ ,$$

$$\mathcal{V}_{15} = -\frac{3}{2m^2\,r^3}\left\{\left[\vec{\sigma}\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\left(\vec{\sigma}'\cdot\hat{\vec{r}}\right)+\left(\vec{\sigma}\cdot\hat{\vec{r}}\right)\left[\vec{\sigma}'\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\right\}$$
$$\times\left(1-r\frac{d}{dr}+\frac{1}{3}r^2\frac{d^2}{dr^2}\right)y(r)\ ,$$

$$\mathcal{V}_{16} = -\frac{1}{2m\,r^2}\left\{\left[\vec{\sigma}\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\left(\vec{\sigma}'\cdot\vec{v}\right)+\left(\vec{\sigma}\cdot\vec{v}\right)\left[\vec{\sigma}'\cdot\left(\vec{v}\times\hat{\vec{r}}\right)\right]\right\}\left(1-r\frac{d}{dr}\right)y(r)\ . \tag{3.8}$$

It is interesting that there are both parity-even ($i = 2,3,6,7,8$) and parity-odd ($i = 11,14,15,16$) potentials which induce macroscopic forces between two polarized objects. Among those, $\mathcal{V}_3$ is the so-called dipole-dipole potential. Likewise, there are both parity-even ($i = 4,5$) and parity-odd ($i = 9,10,12,13$) potentials which induce forces between one polarized and one unpolarized object. The so-called monopole-dipole potential is given by $\mathcal{V}_9 + \mathcal{V}_{10}$.

Notice that in the case of identical fermions, only one linear combination of the $\mathcal{V}_4$ and $\mathcal{V}_5$ potentials is relevant. The same is true for the following pairs: $\mathcal{V}_6$ and $\mathcal{V}_7$, $\mathcal{V}_9$ and $\mathcal{V}_{10}$, $\mathcal{V}_{12}$ and $\mathcal{V}_{13}$.

There are several static spin-dependent types of long-range potentials: $\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_9, \mathcal{V}_{10}$ and $\mathcal{V}_{11}$. The other potentials depend on the relative velocity of the two fermions. In general, each of these potentials has an arbitrary coefficient that needs to be measured. Note though, that in simple models only some of the 16 potentials listed above are present. In Sections 5 and 6 we will derive all the spin-dependent potentials that can arise in Lorentz-invariant quantum field theories from exchange of a spin-0 or spin-1 boson.

4 Interactions between macroscopic objects

$\mathcal{V}_i$ with $i = 1, \ldots, 16$, given in Eqs. (3.6) and (3.8), form a complete set of spin-dependent potentials between two fermions, assuming that rotational invariance is an exact symmetry of the Lagrangian. To a good approximation, macroscopic objects are formed of electrons, neutrons and protons, so that a sum over the potential between pairs of fermions belonging to two different objects gives the total potential between those objects. One should keep in mind though that this is just an approximation: some of the mass (a fraction of a percent) of a macroscopic object is due to the nuclear binding energy, which means that if there are long-range forces between electrons and gluons, for example, then their effects would not be fully taken into account by summing over fermion pairs.

In section 2 we have argued that the propagator of the very light particles that mediate macroscopic forces may have various forms. In this section we first discuss the case of standard propagator, given in Eq. (2.4), and later in subsection 4.2 we consider other forms for the propagator, as in Eqs. (2.5) and (2.6).

4.1 Exchange of one boson with standard propagator

In the case of one-boson exchange forces within a Lorentz-invariant quantum field theory, the propagator (2.4) leads to a simple form for the function $y(r)$ defined in Eq. (3.5):

$$y(r) = \frac{1}{4\pi} e^{-m_0 r} \, , \tag{4.1}$$

where m_0 is the mass of the boson exchanged. The ensuing spin-independent potential, $\mathcal{V}_1$, is then of the well-known Yukawa type, such that the static potential between two

point-like, unpolarized objects is given by

$$
\begin{aligned}
V_1(r) &= \Big\{ N_e N_e' \left[f_1^{ee}(0,0) + f_1^{pp}(0,0) + 2 f_1^{ep}(0,0) \right] + N_n N_n' f_1^{nn}(0,0) \\
&\quad + (N_e N_n' + N_n N_e') \left[f_1^{en}(0,0) + f_1^{ep}(0,0) \right] \Big\} \frac{1}{4\pi r} e^{-m_0 r} \quad , \quad (4.2)
\end{aligned}
$$

where N_e, N_n (N_e', N_n') are the number of electrons and neutrons in the first (second) object, respectively, and we assumed that the objects are electrically neutral. The coefficients $f_1^{ee}(0,0)$, $f_1^{\mathcal{N}\mathcal{N}}(0,0)$ and $f_1^{e\mathcal{N}}(0,0)$, with $\mathcal{N} = n$ or p, depend on the couplings of the exchanged boson to the electrons and nucleons, and can be derived as shown in Eq. (2.7) by computing the amplitudes for elastic ee, $e\mathcal{N}$ and $\mathcal{N}\mathcal{N}$ scattering, respectively. The macroscopic forces between unpolarized objects induced by the static potential in Eq. (4.2) have been studied in great detail (see, *e.g.*, Ref. [1, 2]).

Let us study now the spin-dependent forces between a point-like object whose electron spins are polarized on average along a unit vector $\vec{\sigma}$, and a point-like unpolarized object. The average potential between an electron from the polarized object having the spin along $\vec{\sigma}$ and a neutron from the unpolarized object is given by adding the contributions from $\mathcal{V}_i$ with $i = 4, 5, 9, 10, 12, 13$:

$$
V_\sigma^{en}(\vec{r}, \vec{v}) = \frac{1}{8\pi r} \left\{ f_v^{en} \, \vec{\sigma} \cdot \vec{v} + \left[f_r^{en} \, \vec{\sigma} \cdot \hat{\vec{r}} + f_\perp^{en} \, \vec{\sigma} \cdot \left(\vec{v} \times \hat{\vec{r}} \right) \right] \frac{1 + m_0 r}{m_e r} \right\} e^{-m_0 r} \quad , \quad (4.3)
$$

where f_v^{en}, f_r^{en} and $f_\perp^{en}$ are the dimensionless coefficients of the potential when the electron spin is along the center-of-mass velocity $\vec{v}$ of the polarized object with respect to the unpolarized object, along the unit vector $\hat{\vec{r}}$ pointing from the unpolarized object towards the polarized one, or along $\vec{v} \times \hat{\vec{r}}$, respectively. These coefficients are given in terms of the polynomials f_i introduced in Eq.(2.7) by

$$
\begin{aligned}
f_v^{en} &= f_{12}^{en}(0,0) + f_{13}^{en}(0,0) \quad , \\
f_r^{en} &= -f_9^{en}(0,0) - f_{10}^{en}(0,0) \quad , \\
f_\perp^{en} &= -f_4^{en}(0,0) - f_5^{en}(0,0) \quad (4.4)
\end{aligned}
$$

where the upper indices e and n indicate that the fermions of mass m and m' discussed in general in sections 2 and 3 are now specified to be an electron and a neutron, respectively.

10

We have included only the $\vec{q}^{\,2}$- and $\vec{P}^2$- independent terms in f_i because the $\vec{q}^{\,2}$-dependent terms give tiny corrections of order $(m_0/m_e)^2$ while $\vec{P}^2$-dependent terms give relativistic corrections which are also negligible in experiments searching for new macroscopic forces.

The average potential between the electron spin and the protons or electrons in the unpolarized object, V_σ^{ep} and V_σ^{ee}, respectively, may be written analogously to Eq.(4.3). Then the total potential between the object containing the polarized electrons and the unpolarized object is

$$V_{\sigma_e}(\vec{r}, \vec{v}) = N_e \sigma_e \left[N'_p \left(V_\sigma^{ee} + V_\sigma^{ep} \right) + N'_n V_\sigma^{en} \right] \; , \tag{4.5}$$

where N_e is the total number of electrons in the polarized object, σ_e is the polarization (the average projection of the electron spins along $\vec{\sigma}$ in the polarized object), N'_p and N'_n are the numbers of protons and neutrons in the unpolarized object. In writing the above equation we have assumed that the unpolarized object is electrically-neutral. If the polarized object has the neutrons or protons polarized instead of the electrons, then the potentials $V_\sigma^n(\vec{r}, \vec{v})$ or $V_\sigma^p(\vec{r}, \vec{v})$ are given by Eq. (4.5) with the index e replaced appropriately by n or p in Eqs. (4.3)-(4.5). If the boson exchange induces in addition a spin-independent potential, then the total potential is the sum of the terms in Eq. (4.5) and (4.2).

In the case of two polarized objects there are 9 types of spin-spin potentials. Three of those are static,

$$\mathcal{V}_2 \; = \; \frac{1}{4\pi r} \, \vec{\sigma} \cdot \vec{\sigma}' \, e^{-m_0 r} \; ,$$

$$\mathcal{V}_3 \; = \; \frac{1}{4\pi m_e^2 r^3} \left[\vec{\sigma} \cdot \vec{\sigma}'(1 + m_0 r) - \left(\vec{\sigma} \cdot \hat{\vec{r}} \right) \left(\vec{\sigma}' \cdot \hat{\vec{r}} \right) (3 + 3m_0 r + m_0^2 r^2) \right] e^{-m_0 r} \; ,$$

$$\mathcal{V}_{11} \; = \; -\frac{1}{4\pi m_e r^2} \, \left(\vec{\sigma} \times \vec{\sigma}' \right) \cdot \hat{\vec{r}} \, (1 + m_0 r) \, e^{-m_0 r} \; , \tag{4.6}$$

while the other six potentials depend on the relative velocity of the two objects:

$$\mathcal{V}_{6,7} \; = \; -\frac{1}{8\pi m_e r^2} \left[\left(\vec{\sigma} \cdot \vec{v} \right) \left(\vec{\sigma}' \cdot \hat{\vec{r}} \right) \pm \left(\vec{\sigma} \cdot \hat{\vec{r}} \right) \left(\vec{\sigma}' \cdot \vec{v} \right) \right] (1 + m_0 r) \, e^{-m_0 r} \; ,$$

$$\mathcal{V}_8 \; = \; \frac{1}{4\pi r} \left(\vec{\sigma} \cdot \vec{v} \right) \left(\vec{\sigma}' \cdot \vec{v} \right) e^{-m_0 r} \; ,$$

$$\mathcal{V}_{14} \; = \; \frac{1}{4\pi r} \left(\vec{\sigma} \times \vec{\sigma}' \right) \cdot \vec{v} \, e^{-m_0 r} \; ,$$

$$\mathcal{V}_{15} = -\frac{1}{8\pi m_e^2\, r^3} \left\{ \left[\vec{\sigma} \cdot \left(\vec{v} \times \hat{\vec{r}} \right) \right] \left(\vec{\sigma}' \cdot \hat{\vec{r}} \right) + \left(\vec{\sigma} \cdot \hat{\vec{r}} \right) \left[\vec{\sigma}' \cdot \left(\vec{v} \times \hat{\vec{r}} \right) \right] \right\}$$
$$\times \left(3 + 3 m_0 r + m_0^2 r^2 \right) e^{-m_0 r} \ ,$$

$$\mathcal{V}_{16} = -\frac{1}{8\pi m_e\, r^2} \left\{ \left[\vec{\sigma} \cdot \left(\vec{v} \times \hat{\vec{r}} \right) \right] \left(\vec{\sigma}' \cdot \vec{v} \right) + \left(\vec{\sigma} \cdot \vec{v} \right) \left[\vec{\sigma}' \cdot \left(\vec{v} \times \hat{\vec{r}} \right) \right] \right\} \left(1 + m_0 r \right) e^{-m_0 r} \ .$$
$$(4.7)$$

The total spin-spin potential between two macroscopic objects, one of them containing N_e polarized electrons with a polarization σ_e along $\vec{\sigma}$, and the other object containing N_n' polarized neutrons with a polarization σ_n along $\vec{\sigma}'$, is given by

$$V_{\sigma_e \sigma_n'}(\vec{r}, \vec{v}) = N_e \sigma_e N_n' \sigma_n' \sum_i f_i^{en}(0,0) \, \mathcal{V}_i(\vec{r}, \vec{v}) \ , \tag{4.8}$$

where the sum is over the potentials shown in Eqs. (4.6) and (4.7). An analogous potential exists for two objects containing polarized electrons, except that all n indices are replaced by e, and the $f_i^{en}(0,0)$ coefficients may be obtained by computing the $ee \to ee$ amplitude. Similar statements apply to the ep, pp, nn or np spin-spin potentials. Notice that several of the potentials in Eqs. (4.6) and (4.7) include an inverse power of the electron mass, m_e, introduced to keep the f_i functions dimensionless. In the case of the potentials between nucleons, m_e is replaced by m_n (or else the f_i functions need to be rescaled appropriately).

We briefly discuss the experimental limits on the coefficients of the various potentials. Tests of the equivalence principle and of the inverse square law set limits on the Yukawa potential between unpolarized objects. Given that the tests involve macroscopic objects which are electrically neutral, the boson couplings to the electron and proton are not constrained separately. Only their sum is constrained at roughly the same level as the neutron vector coupling. Thus, the limits may be expressed in terms of the three combinations of f_1 coefficients that appear in Eq. (4.2):

$$|f_1^{nn}(0,0)| \ , \ |f_1^{en}(0,0) + f_1^{ep}(0,0)| \ , \ |f_1^{ee}(0,0) + f_1^{pp}(0,0) + 2f_1^{ep}(0,0)| < 10^{-40} - 10^{-48} \ , \tag{4.9}$$

where the weaker limit applies to $1/m_0$ of order 1 cm, while the stronger one applies to $1/m_0 > 10^8$ m, the Earth-Moon distance (see Figure 4 of [1]).

The most stringent limit on the dipole-dipole potential $\mathcal{V}_3$ between electrons is set in

12

Ref. [8] (see also [6, 7], where the potential is explicitly written[1]): $1.2 \pm 2.0 \times 10^{-14}$ times the magnetic interaction of two electrons, for $1/m_0 \gtrsim 10$ cm. At the 1σ confidence level we then find

$$-0.8 \times 10^{-14} < \frac{f_3^{ee}(0,0)}{4\pi^2\alpha} < 3.2 \times 10^{-14} \ . \tag{4.10}$$

Similarly, the limit on the dipole-dipole potential between an electron and a neutron [5] gives

$$\frac{|f_3^{en}(0,0)|}{4\pi^2\alpha|\mu_n/\mu_N|} < 2.3 \times 10^{-11} \ , \tag{4.11}$$

for $1/m_0 \gtrsim 1$ m. Here $|\mu_n/\mu_N| \approx 1.913$ is the ratio of the neutron magnetic moment to the nuclear magneton. The limits on the dipole-dipole potential between nucleons, or between an electron and a proton are weaker [5].

The static spin-spin potential $\mathcal{V}_2$ has not been experimentally searched for. However, the limits on the dipole-dipole potential $\mathcal{V}_3$ provide an indirect constraint on $\mathcal{V}_2$. It is not clear how accurate would be the use of the best limits on $\mathcal{V}_3$, given in Ref. [8] and [5], to constrain $\mathcal{V}_2$, because $\mathcal{V}_3$ includes a $(\sigma \cdot \vec{r})(\sigma' \cdot \vec{r})$ piece which is not present in $\mathcal{V}_2$. By contrast, the limit set in Ref. [4] explicitly applies to the $\sigma \cdot \sigma'$ piece of the dipole-dipole potential between electrons. Given that $\mathcal{V}_2$ falls off as $1/r$ while $\mathcal{V}_3$ falls off as $1/r^3$, we estimate

$$
\begin{aligned}
|f_2^{ee}(0,0)| &\lesssim \frac{4\pi^2\alpha}{m_e^2 r_{\text{exp}}^2} 10^{-11} \\
&\approx 4 \times 10^{-35} \ , \tag{4.12}
\end{aligned}
$$

where $r_{\text{exp}} \approx 10$ cm is the typical distance probed in the experimental setup of Ref. [4].

The only other static spin-spin potential, $\mathcal{V}_{11}$, has also not been directly tested. To the best of our knowledge, the velocity-dependent spin-spin potentials, with coefficients given in Eq. (4.7), have not been experimentally constrained yet.

The monopole-dipole types of interaction given by $\mathcal{V}_9 + \mathcal{V}_{10}$ [see second term in Eq. (4.3)] have also been experimentally searched for [9, 10, 11]. The most stringent limits have been obtained very recently for the interaction between an object with polar-

---

[1]The potential shown in Eq. (1) of Ref. [7] falls off as $1/r$ instead of $1/r^3$. We believe that this is just a typo.

ized electrons and an unpolarized object in Ref. [37]:

$$|f_r^{en}(0,0)| \ , \ |f_r^{ee}(0,0) + f_r^{ep}(0,0)| \ \lesssim \ 10^{-30} - 10^{-36} \ , \tag{4.13}$$

where the weaker limit applies to distances of order 1 m, while the stronger limit is valid for distances above 10^{11} m (the Earth-Sun distance). These limits represent improvements by at least two orders of magnitude over the previous ones given in Refs. [9, 10]. The best limits on the monopole-dipole potential between an object with polarized neutrons and an unpolarized object, set in Ref. [12], give

$$|f_r^{nn}(0,0)| \ , \ |f_r^{ne}(0,0) + f_r^{np}(0,0)| \ \lesssim \ 10^{-27} - 10^{-33} \ , \tag{4.14}$$

for $1/m_0$ in the $1 - 10^6$ m range.

Velocity dependent potentials of the type $\mathcal{V}_4 + \mathcal{V}_5$ and $\mathcal{V}_{12} + \mathcal{V}_{13}$ have been tested for the first time [37] while this paper was being written, and the preliminary limits for distances above 10^{11} m are

$$|f_\perp^{en}| \ , \ |f_\perp^{ee} + f_\perp^{ep}| \ \lesssim \ 10^{-32} \ ,$$

$$f_v^{en} \ , \ f_v^{ee} + f_v^{ep} \ \lesssim \ 10^{-55} \ , \tag{4.15}$$

with $f_\perp$ and f_v defined in Eq. (4.4). We do not show a lower limit for f_v, because the central value obtained in Ref. [37] differs from zero by almost 2σ. Note that the limit on f_v is stronger by many orders of magnitude than the limit on any other f_i coefficient.

4.2 Non-standard dispersion relations

So far we have considered long-range potentials induced by the exchange of a boson whose propagator has the usual pole structure, $1/(q^2 - m_0^2)$, leading to the standard dispersion relation $E^2 = \vec{q}^{\,2} + m_0^2$. This form for the propagator follows from the assumptions that the kinetic term is Lorentz invariant and quadratic in derivatives. If the kinetic term involves higher derivatives, the propagator would include higher inverse powers of q^2, and would lead to new structures for the potentials. However, such kinetic terms lead to instabilities or unitarity violation, so they may not be allowed in well-behaved physical theories.

The propagator (and therefore the dispersion relation) may be modified if Lorentz symmetry is broken, because then the kinetic terms may involve quartic or higher spatial derivatives while the time derivatives are quadratic, as required in a well behaved theory. For example, a dispersion relation of the type $E^2 = \vec{q}^4/M^2$ appears in Ref. [18], where Lorentz symmetry is spontaneously broken. One could imagine a larger class of propagators for a boson which involve higher powers of $1/\vec{q}^2$. In the case of the propagator shown in Eq. (2.6), which is of the $(\vec{q})^{-2k}$ type with $k \geq 2$ integer, the function $y(r)$ defined in Eq. (3.5) may be computed using a Fourier transform given in Eq. (B.2):

$$y(r) = \frac{1}{4\pi} \frac{1}{[2(k-1)]!} r^{2k-2} \ . \tag{4.16}$$

The spin-dependent potentials are given by Eqs. (3.6) and (3.8). Note that the r-dependence is different than in the case of a normal one-boson exchange analyzed in Section 4.1. For example, for $k = 2$ (the case analyzed in Ref. [18]), the static spin-spin potential falls off as $1/r$:

$$V(\vec{r}) \sim -\frac{1}{r} \left[(\vec{\sigma} \cdot \vec{\sigma}') - (\vec{\sigma} \cdot \hat{\vec{r}}) (\vec{\sigma}' \cdot \hat{\vec{r}}) \right] \ . \tag{4.17}$$

Another case of interest is the long-range potential induced by exchange of two or more particles. A well known example is the force due to two-neutrino exchange [23, 24]. The one-loop diagrams involving two neutrinos are equivalent to the tree-level exchange of a single boson with an effective propagator of the type $\sim \ln\left(\vec{q}^2\right)$, as shown in Eq. (2.5). The Fourier transform leads to a potential which falls off as $1/r^5$, and includes a spin-independent term as well as spin-spin terms.

The exchange of two bosons has also been shown to lead to additional types of potentials [24]. In particular, spin-independent potentials falling off as $1/r^3$, $1/r^5$ or $1/r^7$ are induced by the exchange of two spin-0 particles [25]. Unfortunately, the strength of any of the two-particle-exchange macroscopic forces studied so far is many orders of magnitude smaller than the current experimental sensitivity to new particles.

5 Spin-1 exchange forces

The electromagnetic interaction is the only known long-range force induced by a spin-1 particle. Nevertheless, low-mass spin-1 particles other than the photon may exist, and

they would lead to additional long-range forces that could be searched for in experiments.

5.1 New massless gauge boson

A spin-1 particle is naturally kept massless by an unbroken gauge symmetry. In particular, a new $U(1)$ gauge symmetry would require the existence of a massless spin-1 particle, labeled γ' and called paraphoton [26]. If any of the Standard Model fields would be charged under the new $U(1)$ symmetry, then the gauge anomaly cancellation requires the $U(1)$ charge to be proportional to the $B - L$ number, so that the γ' coupling to any electrically-neutral macroscopic object is proportional to the number of neutrons [17]. As a result, tests of the equivalence principle and of the inverse square law (see [27] for a related discussion) set an upper limit on the gauge coupling of γ' orders of magnitude below 10^{-19}, which appears unnatural and also poses theoretical challenges [28].

It is possible, however, that all Standard Model fields have zero charge under the new $U(1)$ symmetry, and yet γ' may interact with the quarks and leptons via dimension-6 operators involving two fermion fields, a paraphoton, and a Higgs doublet [17]. Those operators are gauge invariant and do not depend on the fermion charges. Replacing the Higgs doublet by its vacuum expectation value (VEV), $v_h \simeq 174$ GeV, yields dimension-5 operators in the Lagrangian, representing magnetic- and electric-like dipole moments:

$$\mathcal{L}_{\gamma'} = \frac{v_h}{M^2} P_{\mu\nu} \left[\overline{e}\, \sigma^{\mu\nu} (\mathrm{Re}C_e + i\mathrm{Im}C_e\gamma_5)\, e \,+\, \sum_{\mathcal{N}=n,p} \overline{\mathcal{N}}\sigma^{\mu\nu} (\mathrm{Re}C_{\mathcal{N}} + i\mathrm{Im}C_{\mathcal{N}}\,\gamma_5)\,\mathcal{N} \right] \ . \quad (5.1)$$
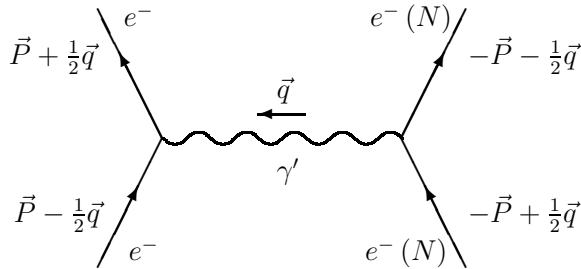

Figure 2: Paraphoton-exchange amplitude for nonrelativistic electron-electron or electron-nucleon scattering. The three-momenta shown here correspond to the center-of-mass frame.

Here $P_{\mu\nu}$ is the field strength of the paraphoton, e is the electron field, $\mathcal{N}$ is the nucleon field, while C_e and $C_\mathcal{N}$ are dimensionless complex parameters (their values are expected to be much less than unity). The γ' coupling to nucleons is an effective low-energy Lagrangian that arises from a similar coupling of γ' to u or d quarks. These couplings may have different strengths, and therefore the values of $C_\mathcal{N}$ when $\mathcal{N}$ is a proton or a neutron may be different. The mass M sets the scale where the dimension-6 operators are generated within an underlying theory which is well-behaved in the ultraviolet (examples of renormalizable models of this type are given in [17]).

One γ' exchange between electrons or nucleons leads to a long-range force between chunks of ordinary matter. In Figure 2 we show the three-momentum flow for the scattering of fermions mediated by γ'. The amplitude for this process is given by

$$\mathcal{A}\left(\vec{q}, \vec{P}\right) = -\frac{1}{\vec{q}^2}\frac{4v_h^2}{M^4}\,S^\nu S'_\nu \; , \tag{5.2}$$

where $\nu = 0, 1, 2, 3$ is a Lorentz index, and we have defined

$$S^\nu = \overline{u}_e(P + q/2)\,q_\mu \sigma^{\mu\nu}(\mathrm{Re}C_e + i\mathrm{Im}C_e\gamma_5)\,u_e(P - q/2) \; . \tag{5.3}$$

The spinor $u_e(p)$ describes the electron field of four-momentum p. In the case of e^-e^- scattering, S'^ν is identical to S^ν except for the spinor $u_e(p')$ which depends on the momentum of the second electron. In the case of $e^-\mathcal{N}$ scattering, S'^ν has the same structure as S^ν but the nucleon spinor $u_\mathcal{N}(p')$ and complex parameter $C_\mathcal{N}$ replace the electron ones.

In what follows we compute the nonrelativistic amplitude for $e^-\mathcal{N}$ scattering, because the result can be immediately adapted to e^-e^- or $\mathcal{N}\mathcal{N}$ scattering. In the nonrelativistic limit, the time-like component of S^ν is given by

$$S^0 = -\mathrm{Im}C_e\,\vec{q}\cdot\vec{\sigma} + \frac{1}{m_e}\mathrm{Re}C_e\left[\left(\vec{P}\times\vec{q}\right)\cdot\vec{\sigma} - \frac{i}{2}\vec{q}^2\right] \quad . \tag{5.4}$$

Relativistic corrections to S^0, of order $\vec{P}^2/m_e^2$ and $\vec{q}^2/m_e^2$, do not introduce new spin-dependent terms. For the nucleon of initial three-momentum $-\vec{P} + \vec{q}/2$,

$$S'^0 = \mathrm{Im}C_\mathcal{N}\,\vec{q}\cdot\vec{\sigma}' + \frac{1}{m_\mathcal{N}}\mathrm{Re}C_\mathcal{N}\left[\left(\vec{P}\times\vec{q}\right)\cdot\vec{\sigma}' - \frac{i}{2}\vec{q}^2\right] \quad . \tag{5.5}$$

In order to compute the space-like components of S^ν and S'^ν, it is useful to recall that energy-momentum conservation implies $\vec{P}\cdot\vec{q} = 0$ and $q^0 = 0$. We find

$$\vec{S} = \mathrm{Re}C_e\left[\vec{q}\times\vec{\sigma} - \frac{i\vec{q}^2}{4m_e^2}\vec{P} + \frac{1}{m_e^2}\left(\vec{P}\cdot\vec{\sigma}\right)\vec{P}\times\vec{q}\right] - \frac{1}{m_e}\mathrm{Im}C_e\,\left(\vec{q}\cdot\vec{\sigma}\right)\vec{P} \; ,$$

$$\vec{S}' = -\mathrm{Re}C_{\mathcal{N}} \left[\vec{q} \times \vec{\sigma}' - \frac{i\vec{q}^2}{4m_{\mathcal{N}}^2} \vec{P} + \frac{1}{m_{\mathcal{N}}^2} \left(\vec{P} \cdot \vec{\sigma}' \right) \vec{P} \times \vec{q} \right] - \frac{1}{m_{\mathcal{N}}} \mathrm{Im}C_{\mathcal{N}} \left(\vec{q} \cdot \vec{\sigma}' \right) \vec{P} \ ,$$

$$(5.6)$$

with relativistic corrections affecting only the above spin-dependent terms.

A lengthy but straightforward computation of the right-hand side of Eq. (5.2) then gives the nonrelativistic amplitude. For the purpose of deriving the long-range potential, we may ignore the terms in $S^\nu S'_\nu$ proportional to $\vec{q}^2$, because upon Fourier transforming to position space they give only contact interactions or contributions additionally suppressed by m_0, as discussed in Section 3.1. The result takes the form of Eq. (2.7), with the functions $f_i\left(0, \vec{v}^2\right)$ being nonzero only for $i = 3, 15$. In the nonrelativistic limit, keeping only the leading order in $\vec{v}^2$, we obtain the following values for these functions:

$$f_3^{e\mathcal{N}}(0,0) \ = \ -\frac{4v_h^2 m_e^2}{M^4} \, \mathrm{Re}\left(C_e^* C_{\mathcal{N}}\right) \ ,$$

$$f_{15}^{e\mathcal{N}}(0,0) \ = \ \frac{4v_h^2 m_e^2}{M^4} \left(1 + \frac{m_e}{m_{\mathcal{N}}}\right) \left[\mathrm{Re}\left(C_e\right) \mathrm{Im}\left(C_{\mathcal{N}}\right) - \mathrm{Im}\left(C_e\right) \mathrm{Re}\left(C_{\mathcal{N}}\right)\right] \ . \quad (5.7)$$

Therefore, the long-range potential between an electron and a nucleon induced by γ' exchange is given by

$$V_{e\mathcal{N}}\left(\vec{r}, \vec{v}\right) = \sum_{i=3,15} \mathcal{V}_i\left(\vec{r}, \vec{v}\right)\bigg|_{m=m_e} f_i^{e\mathcal{N}}(0,0) \qquad (5.8)$$

where the parity-even potential $\mathcal{V}_3$ is given in Eq. (3.6) while the parity-odd potential $\mathcal{V}_{15}$ is given in Eq. (3.8). Notice that the long-range potential induced by paraphoton exchange may be observed only if both objects are polarized.

The long-range potential between electrons due to γ' may be obtained from Eqs. (5.8) and (5.7) by replacing the subscript $\mathcal{N}$ by e (note that $f_{15}^{ee} = 0$, so that the long-range potential is static in this case):

$$V_{ee}\left(\vec{r}, \vec{v}\right) = -\frac{4v_h^2 m_e^2}{M^4} \, |C_e|^2 \, \mathcal{V}_3 \bigg|_{m=m_e} \ . \qquad (5.9)$$

The proton-proton and neutron-neutron long-range potentials have analogous forms with the appropriate replacements of m_e and C_e by the proton and neutron parameters. The proton-neutron potential may include in addition the $\mathcal{V}_{15}$ spin-dependent potential, similarly to Eq. (5.8). Note that the only static potential induced by γ' exchange is $\mathcal{V}_3$, which

18

gives the usual long-range force between two magnetic dipole moments but with an overall strength that depends on the γ' couplings.

Given that the dimension-six operators that give rise to the effective γ' couplings in Eq. (5.1) involve a chirality flip of the fermions, it is expected that its dimensionless coefficients are of the order of or smaller than the corresponding Yukawa coupling to the Higgs doublet. It is therefore useful to factor out the Yukawa coupling from the C_e and $C_{\mathcal{N}}$ parameters:

$$c_e \equiv \frac{v_h}{m_e}|C_e| \; ,$$

$$c_{\mathcal{N}} \equiv \frac{v_h}{m_d}|C_{\mathcal{N}}| \; , \tag{5.10}$$

where m_d is the down-quark mass. The parameters c_e and $c_{\mathcal{N}}$ may be as large as $O(1)$, but could be orders of magnitude smaller than one if the dimension-6 operators are generated at loop level in some renormalizable model with weakly coupled fields. The experimental limits on the paraphoton coupling to the electrons and nucleons can be expressed in terms of $M/\sqrt{c_e}$ and $M/\sqrt{c_{\mathcal{N}}}$, respectively.

The static potential between electrons induced by γ' exchange takes the form

$$V(\vec{r}) = -\frac{c_e^2 m_e^2}{\pi M^4 r^3} \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 - 3 \left(\vec{\sigma}_1 \cdot \hat{\vec{r}} \right) \left(\vec{\sigma}_2 \cdot \hat{\vec{r}} \right) \right] \; , \tag{5.11}$$

so that it is an attractive $\mathcal{V}_3$ potential. Using the 1σ limit shown in Eq. (4.10), we find

$$\frac{c_e^2 m_e^4}{\pi^2 \alpha M^4} < 0.8 \times 10^{-14} \; , \tag{5.12}$$

where α is the fine structure constant. This translates into a limit

$$\frac{M}{\sqrt{c_e}} > 3.3 \text{ GeV} \; . \tag{5.13}$$

Similarly, the limit on the dipole-dipole potential between an electron and a neutron [5], shown in Eq. (4.11), gives

$$\frac{c_e c_{\mathcal{N}} \, m_e^2 m_d m_p}{\pi^2 \alpha M^4 |\mu_n/\mu_N|} \cos \left(\theta_e - \theta_{\mathcal{N}} \right) < 2.3 \times 10^{-11} \; , \tag{5.14}$$

where $\theta_{e,\mathcal{N}}$ are the complex phases of $C_{e,\mathcal{N}}$ and m_p is the proton mass. We find the following constraint on the paraphoton couplings:

$$\frac{M}{\sqrt{c_e c_{\mathcal{N}}}} \cos^{-1/4}(\theta_e - \theta_{\mathcal{N}}) > 4.2 \text{ GeV} \; . \tag{5.15}$$

19

where we used $m_d \sim 4$ MeV.

The bounds from star cooling on $M/\sqrt{c_e}$ and $M/\sqrt{c_\mathcal{N}}$ are three orders of magnitude stronger than Eqs. (5.13) and (5.15), while the limits from primordial nucleosynthesis are also substantially stronger ($M/\sqrt{c_e} \gtrsim 100$ GeV and $M/\sqrt{c_\mathcal{N}} \gtrsim 400$ GeV) [17]. However, there are potential loopholes in these astrophysical and cosmological limits, whereas the limit from searches for new macroscopic forces is robust. A well known loophole in the limit from primordial nucleosynthesis is the possibility of a chemical potential during the early Universe. The limits from star cooling, considered unavoidable in the case of axions [16], could be avoided in the case of the paraphoton if the properties of this massless gauge boson depend on temperature. We contemplate a theory that besides the Standard Model and the new $U(1)$ gauge group includes two or more scalar fields such that there is a mechanism of symmetry non-restoration at high temperatures [29]. Specifically, if a scalar charged under the new $U(1)$ acquires a VEV when in thermal equilibrium in a star, and if this VEV is larger than the star temperature, then the γ' emission from the star is exponentially suppressed. As a result, star cooling via γ' emission could be negligible.

5.2   General spin-1 exchange

So far we have discussed the case of a massless spin-1 field which couples to electrons or nucleons via higher-dimensional operators. Let us turn now to a more general Lorentz-invariant extension of the Standard Model that includes a new spin-1 field, Z', that is electrically neutral. We assume that its mass m_0 is nonzero but smaller than 10^{-3} eV, so that Z' exchange mediates forces with a range longer than a micrometer. We will consider in some cases a mass as small as 10^{-18} eV, which is the inverse Earth-Sun distance.

Without loss of generality, we assume that such a Z' field is the gauge boson associated with a new $U(1)_z$ gauge symmetry that is spontaneously broken by the VEV of a spin-0 field φ, which is a singlet under the Standard Model gauge group. The Z' mass is then related to the gauge coupling g_z and the φ charge z_φ:

$$m_0 = z_\varphi g_z \langle \varphi \rangle . \tag{5.16}$$

The Z' boson couples to the leptons and quarks of the first generation as follows:

$$\mathcal{L}^g_{Z'} = g_z Z'_\mu \left(z_l \, \overline{l}_L \gamma^\mu l_L + z_e \, \overline{e}_R \gamma^\mu e_R + z_q \, \overline{q}_L \gamma^\mu q_L + z_u \, \overline{u}_R \gamma^\mu u_R + z_d \, \overline{d}_R \gamma^\mu d_R \right) , \tag{5.17}$$

where $q_L = (u_L, d_L)$ and $l_L = (\nu_L, e_L)$ are $SU(2)_W$ doublets, while z_l, z_e, z_q, z_u, and z_d are the $U(1)_z$ charges of the leptons and quarks. The ensuing couplings at low energy of the Z' boson to electrons and nucleons are given by

$$\mathcal{L}_{Z'}^g = Z'_\mu \left[\overline{e} \gamma^\mu \left(g_V^e + g_A^e \gamma_5 \right) e + \sum_{\mathcal{N}=n,p} \overline{\mathcal{N}} \gamma^\mu \left(g_V^{\mathcal{N}} + \gamma_5 g_A^{\mathcal{N}} \right) \mathcal{N} \right] , \tag{5.18}$$

where the vector and axial couplings of the electron, proton and neutron are

$$g_{V,A}^e = \frac{g_z}{2} \left(z_e \pm z_l \right) ,$$

$$g_{V,A}^p = \frac{g_z}{2} \left(2z_u + z_d \pm 3z_q \right) ,$$

$$g_{V,A}^n = \frac{g_z}{2} \left(z_u + 2z_d \pm 3z_q \right) . \tag{5.19}$$

In addition to these dimension-4 interactions, there are higher-dimensional interactions as in Eq. (5.1), with $P_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, describing magnetic- and electric-like dipole couplings.

The $U(1)_z$ charges may be treated as arbitrary real parameters. However, there are various requirements that any self-consistent theory that includes the $U(1)_z$ gauge group has to satisfy. The $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_z$ gauge theory must be anomaly free, so that the $U(1)_z$ charges must satisfy several cubic and linear equations. Furthermore, the quark and lepton charges are expected to be commensurate numbers (*i.e.*, their ratios are rational numbers), which makes it much harder to satisfy the cubic equations. It turns out [30], however, that all anomaly cancellation conditions may be satisfied while keeping z_l, z_e, z_q, z_u, and z_d arbitrary, provided there are enough additional fermions charged under $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_z$. Those new fermions charged under the standard model gauge group have not been seen in collider experiments so far, so that they must be heavier than a few hundred GeV. Given that those fermions must be chiral with respect to $U(1)_z$, their masses are less than $4\pi\langle\varphi\rangle$. Hence, the $U(1)_z$ breaking VEV must be of the order of the electroweak scale, or larger, implying that $z_\varphi g_z \lesssim 10^{-14} - 10^{-31}$ for a Z'-induced force of range between a micrometer and the Earth-Sun distance. Notice that this constraint may be satisfied even if g_z is of order one: z_φ may be extremely small, and this situation could arise naturally in theories involving kinetic mixing of several $U(1)$ gauge groups [26], or gauge fields localized in extra dimensions [31].

21

New fermions charged under the standard model gauge group may be avoided in the case of "nonexotic" Z' (see Ref. [32]), where the set of values for z_l, z_e, z_q, z_u, and z_d is restricted such that

$$g_{V,A}^p + g_{V,A}^e = 0 \ ,$$

$$g_{V,A}^n = \frac{g_z}{2} \left[(4 \pm 3) \, z_q - z_u \right] \ . \tag{5.20}$$

Consequently, the long-range forces induced by nonexotic Z' exchange are proportional to the number of neutrons. For $z_q = z_u$ we recover the $U(1)_{B-L}$ gauge group discussed at the beginning of Section 5.1; the associated Z' has no axial couplings, while its vector coupling to neutrons is extremely constrained by tests of the material dependence of the inverse square law, $g_V^n = 3z_q g_z \ll 10^{-19}$. The particular case of $z_q = z_u = 0$ corresponds to the paraphoton.

For $z_q \neq z_u$, even though in the case of nonexotic Z' the $U(1)_z$-breaking VEV is not required to induce a large mass for new fermions, a certain charge times the gauge coupling must still be very small. To see this, note that the quark and charged-lepton mass terms have a $U(1)_z$ charge of $z_q - z_u$. If the Higgs doublet carries charge $z_q - z_u$, then the quark and lepton masses are generated as in the Standard Model, but $z_H g_z$ must be very small such that the Z' is light enough to mediate macroscopic forces. If the Higgs doublet has zero $U(1)_z$ charge, then the masses of the up and down quarks, and of the electron, should be generated by higher-dimensional operators, such as

$$\lambda_d \frac{\varphi}{M} \overline{q}_L d_R H \ , \tag{5.21}$$

where λ_d is a dimensionless parameter smaller than 4π and M is some mass scale larger than $\langle \varphi \rangle$. Therefore, the down-quark mass requires a VEV $\langle \varphi \rangle$ in the MeV range or larger, so that the range for the Z' mass m_0 considered here requires $z_\varphi g_z \lesssim 10^{-9}$.

The only alternative to nonexotic $U(1)_z$ charges that would still avoid the presence of new fermions charged under the Standard Model gauge group involves generation-dependent $U(1)_z$ charges for the quarks and leptons. For example, the electron contributions to the anomalies may be canceled by the muon ones if the charges for the first- and second-generation leptons have opposite signs. In this case $\langle \varphi \rangle$ may be much lower than in the case of nonexotic $U(1)_z$ with Higgs doublet charge different than $z_q - z_u$, but it

still needs to be above 10^{-2} eV in order to accommodate the solar neutrino oscillations. We emphasize though that a low value for $\langle \varphi \rangle$ would in turn lead to the question of what stabilizes the hierarchy between $U(1)_z$ breaking scale and the electroweak scale.

Despite the caveats discussed above, the various couplings of the ultra-light Z' may be treated in general as independent parameters. It is interesting to observe that any of the vector or axial couplings of the electron, proton or neutron, given in Eq. (5.19), may vanish even when the charges of the left- and right-handed quarks and leptons are nonzero. That happens when the charges satisfy certain linear equations (for example, $3z_q = -z_u - 2z_d$ would imply that the neutron has no vector coupling to Z'_μ), which may conceivably be consistent with some grand unified group.

The amplitude for Z' exchange between an electron and a nucleon may be written as

$$\mathcal{A}(\vec{q}, \vec{P}) = -\frac{1}{\vec{q}^2}\left(T^\nu - \frac{2iv_h}{M^2}S^\nu\right)\left(T'_\nu + \frac{2iv_h}{M^2}S'_\nu\right) \tag{5.22}$$

where S^ν, defined in Eq. (5.3), involves the effects of the magnetic- and electric-like dipole couplings of Eq. (5.1), while

$$T^\nu \equiv \overline{u}_e(P + q/2)\,\gamma^\mu\,(g_V + g_A\gamma_5)\,u_e(P - q/2)\ , \tag{5.23}$$

involves the effects of the vector and axial couplings of Eq. (5.18). In the nonrelativistic limit, the time-like component of T^ν is

$$T^0 = g_V^e\left[1 - \frac{i\vec{\sigma}\cdot\left(\vec{P}\times\vec{q}\right)}{4m_e^2}\right] + g_A^e\,\frac{\vec{\sigma}\cdot\vec{P}}{m_e}\ , \tag{5.24}$$

and the space-like component is

$$\vec{T} = \frac{g_V^e}{2m_e}\left(2\vec{P} - i\,\vec{q}\times\vec{\sigma}\right) + g_A^e\left[\vec{\sigma} + \frac{i}{4m_e^2}\left(\vec{P}\times\vec{q} - 2i\vec{P}\,\vec{\sigma}\cdot\vec{P} + \frac{i}{2}\vec{q}\,\vec{\sigma}\cdot\vec{q}\right)\right]\ . \tag{5.25}$$

T'^0 and $\vec{T}'$ have analogous expressions, with the electron couplings replaced by nucleon couplings, $\vec{\sigma}$ replaced by $\vec{\sigma}'$, and a sign change for the terms linear in momenta.

We find that the majority of the long-range potentials listed in Eqs. (4.2), (4.3), (4.6) and (4.7) may be induced by Z' exchange. Their momentum-independent coefficients can be derived by comparing Eqs. (2.7) and (5.22). As expected, there is a Yukawa potential between unpolarized objects like in (4.2) with a coefficient

$$f_1^{e\mathcal{N}}(0, 0) = g_V^e\,g_V^\mathcal{N}\ . \tag{5.26}$$

23

All three potentials between a polarized object and an unpolarized object, shown in Eq. (4.3), have nonzero coefficients:

$$f_r^{e\mathcal{N}} = -4g_V^{\mathcal{N}} \frac{v_h m_e}{M^2} \mathrm{Im} C_e \ , \tag{5.27}$$

for the monopole-dipole potential (this is a linear combination of $\mathcal{V}_9$ and $\mathcal{V}_{10}$), and

$$f_\perp^{e\mathcal{N}} = \left(\frac{1}{2} + \frac{m_e}{m_{\mathcal{N}}}\right) g_V^e\, g_V^{\mathcal{N}} + \frac{m_e^2}{2m_{\mathcal{N}}^2} g_A^e\, g_A^{\mathcal{N}} + 4\left(1 + \frac{m_e}{m_{\mathcal{N}}}\right) g_V^{\mathcal{N}} \frac{v_h m_e}{M^2} \mathrm{Re} C_e \ ,$$

$$f_v^{e\mathcal{N}} = 2\left(1 + \frac{m_e}{m_{\mathcal{N}}}\right) g_A^e\, g_V^{\mathcal{N}} \tag{5.28}$$

for the velocity-dependent potentials (these are linear combinations of $\mathcal{V}_4$ and $\mathcal{V}_5$, and of $\mathcal{V}_{12}$ and $\mathcal{V}_{13}$, respectively).

In the case of two polarized bodies, all three static spin-spin potentials in Eq. (4.6) receive contributions with coefficients given by

$$f_2^{e\mathcal{N}}(0,0) = -g_A^e\, g_A^{\mathcal{N}} \ ,$$

$$f_3^{e\mathcal{N}}(0,0) = \frac{m_e}{4m_{\mathcal{N}}} g_V^e\, g_V^{\mathcal{N}} + \frac{1}{8}\left(1 + \frac{m_e^2}{m_{\mathcal{N}}^2}\right) g_A^e\, g_A^{\mathcal{N}} - \frac{v_h m_e}{M^2}\left(g_V^e \mathrm{Re} C_{\mathcal{N}} - \frac{m_e}{m_{\mathcal{N}}} g_V^{\mathcal{N}} \mathrm{Re} C_e\right)$$

$$\qquad\quad + \ f_3^{e\mathcal{N}}(0,0)\Big|_{\gamma'} \ ,$$

$$f_{11}^{e\mathcal{N}}(0,0) = \frac{1}{2}g_V^e\, g_A^{\mathcal{N}} + \frac{m_e}{2m_{\mathcal{N}}} g_A^e\, g_V^{\mathcal{N}} - 2\frac{v_h m_e}{M^2}\left(g_A^e \mathrm{Re} C_{\mathcal{N}} - g_A^{\mathcal{N}} \mathrm{Re} C_e\right) \ . \tag{5.29}$$

The velocity dependent spin-spin interactions in Eq. (4.7) also receive contributions, with coefficients:

$$f_{6,7}^{e\mathcal{N}}(0,0) = 2\left(1 + \frac{m_e}{M_{\mathcal{N}}}\right) \frac{v_h m_e}{M^2}\left(g_A^e \mathrm{Im} C_{\mathcal{N}} \mp g_A^{\mathcal{N}} \mathrm{Im} C_e\right) \ ,$$

$$f_8^{e\mathcal{N}}(0,0) = -\frac{1}{2}\left(1 + \frac{m_e}{m_{\mathcal{N}}} + \frac{m_e^2}{2m_{\mathcal{N}}^2}\right) g_A^e\, g_A^{\mathcal{N}} \ ,$$

$$f_{15}^{e\mathcal{N}}(0,0) = \frac{v_h m_e}{M^2}\left[\left(\frac{1}{2} + \frac{m_e}{m_{\mathcal{N}}}\right) g_V^e \mathrm{Im} C_{\mathcal{N}} + \frac{m_e}{m_{\mathcal{N}}}\left(1 + \frac{m_e}{2m_{\mathcal{N}}}\right) g_V^{\mathcal{N}} \mathrm{Im} C_e\right] + f_{15}^{e\mathcal{N}}(0,0)\Big|_{\gamma'} \ ,$$

$$f_{16}^{e\mathcal{N}}(0,0) = \frac{m_e}{4m_{\mathcal{N}}}\left(1 + \frac{m_e}{m_{\mathcal{N}}}\right)\left(g_V^e\, g_A^{\mathcal{N}} - g_A^e\, g_V^{\mathcal{N}}\right) + \frac{v_h m_e}{M^2}\left[\left(\frac{1}{2} + \frac{m_e}{m_{\mathcal{N}}} + \frac{m_e^2}{m_{\mathcal{N}}^2}\right) g_A^e \mathrm{Re} C_{\mathcal{N}}\right.$$

$$\qquad\quad \left. + \left(1 + \frac{m_e}{m_{\mathcal{N}}} + \frac{1}{2}\frac{m_e^2}{m_{\mathcal{N}}^2}\right) g_A^{\mathcal{N}} \mathrm{Re} C_e\right] \ . \tag{5.30}$$

The last term in the above formulae for f_3 and f_{15} represents the contribution from the magnetic- and electric-like dipole couplings, given in Eq. (5.7).

The only operator from Eq. (2.7) which does not contribute at this order is $\mathcal{O}_{14}$. Once we include the higher-order corrections proportional to additional powers of $\vec{q}^{\,2}$, $\mathcal{O}_{14}$ is also generated in the vector exchange. As previously mentioned, this contribution is however suppressed by m_0^2/m_e^2, so it is too small to be interesting in practice.

Let us now discuss the limits on the couplings of a low-mass Z'. The limits on the Yukawa potential between unpolarized objects, shown in Eq. (4.9), translate into a limit on the vector coupling of the neutron, and on the sum of the vector couplings of the electron and proton:

$$|g_V^n| \ , \ |g_V^e + g_V^p| \ \lesssim \ 10^{-20} \ , \tag{5.31}$$

for $1/m_0$ of order 1 cm, and almost four orders of magnitude stronger for $1/m_0 > 10^8$ m.

The experimental limits on the dipole-dipole potential $\mathcal{V}_3$ have been used in Section 5.1 to constrain the magnetic- and electric-like dipole couplings. Once nonzero $U(1)_z$ charges are allowed, the coefficient of $\mathcal{V}_3$ receives contributions that also depend on the vector and axial couplings, as displayed in Eq. (5.29). The limit (4.10) becomes:

$$- 0.8 \times 10^{-14} < \frac{g_V^{e\,2} + g_A^{e\,2}}{16\pi^2\alpha} - \frac{c_e^2 m_e^4}{\pi^2\alpha M^4} < 3.2 \times 10^{-14} \ . \tag{5.32}$$

Barring accidental cancellations between the two terms above, we find

$$|g_A^e| \ , \ |g_V^e| \ \lesssim \ 10^{-7} \ . \tag{5.33}$$

The limits from star cooling [34] are stronger by several orders of magnitude, but as discussed at the end of Section 5.1, those limits may be avoided in the case of a spin-1 boson. The indirect limit on the $\mathcal{V}_2$ potential derived in Eq. (4.12) provides the tightest restriction on the Z' axial coupling to the electrons:

$$|g_A^e| \ \lesssim \ 10^{-17} \ . \tag{5.34}$$

The limits on the monopole-dipole potentials shown in Eqs. (4.13) and (4.14) lead to the following constraints on the Z' couplings:

$$4c_e \, \frac{m_e^2}{M^2} \, |\sin\theta_e \, g_V^n| \ \lesssim \ 10^{-30} - 10^{-36} \ , \ \text{ for } 1/m_0 \gtrsim \left(1 - 10^{11}\right) \text{ m} \ ,$$

$$4c_n \frac{m_n^2}{M^2} |\sin\theta_n \, g_V^n| \; \lesssim \; 10^{-27} - 10^{-33} \;\; , \;\; \text{for } 1/m_0 \gtrsim \left(1 - 10^6\right) \text{m} \;\; , \quad (5.35)$$

There are also analogous constraints with g_V^n replaced by $g_V^p + g_V^e$.

The new tests on velocity dependent potentials of the type $\mathcal{V}_4 + \mathcal{V}_5$ and $\mathcal{V}_{12} + \mathcal{V}_{13}$, presented in Ref. [37], set preliminary limits on the combination of couplings of the type g_V^e, g_V^n, and $\text{Re}\, c_e/M^2$ shown in Eq. (5.28). In particular, the constraints on products of axial and vector couplings arising from the second Eq. (4.15) are extremely strong:

$$|g_A^e g_V^n| \;\; , \;\; |g_A^e \left(g_V^e + g_V^p\right)| \; \lesssim \; 10^{-55} \;\; , \quad (5.36)$$

for $1/m_0 > 10^{11}$ m.

6 Spin-0 exchange forces

A very light spin-0 particle, ϕ, can have scalar and pseudoscalar couplings to electrons and nucleons in the low-energy effective Lagrangian:

$$\mathcal{L}_\phi = -\phi \, \overline{e} \left(g_S^e + i\gamma_5 g_P^e\right) e - \phi \, \overline{\mathcal{N}} \left(g_S^{\mathcal{N}} + i\gamma_5 g_P^{\mathcal{N}}\right) \mathcal{N} \;. \quad (6.1)$$

Any higher-dimensional coupling of ϕ to electrons or nucleons can be reduced to the terms in Eq. (6.1) by integrating by parts and using the equations of motion, so that they do not give rise to new types of potentials.

The amplitude for electron-nucleon scattering due to the exchange of ϕ is given by Eq. (2.7) with contributions from the operators $\mathcal{O}_1$ and $\mathcal{O}_{4,5}$ for two scalar couplings, $\mathcal{O}_3$ for two pseudoscalar couplings, and $\mathcal{O}_{9,10}$ and $\mathcal{O}_{15}$ for one scalar and one pseudoscalar coupling. The only $f_i(0,0)$ coefficients that do not vanish are given by:

$$
\begin{aligned}
f_1^{e\mathcal{N}}(0,0) &= -g_S^e g_S^{\mathcal{N}} \\[2mm]
f_3^{e\mathcal{N}}(0,0) &= -\frac{m_e}{4m_{\mathcal{N}}} g_P^e g_P^{\mathcal{N}} \\[2mm]
f_{4,5}^{e\mathcal{N}}(0,0) &= -\frac{1}{4}\left(1 \pm \frac{m_e^2}{m_{\mathcal{N}}^2}\right) g_S^e g_S^{\mathcal{N}} \\[2mm]
f_{9,10}^{e\mathcal{N}}(0,0) &= \frac{1}{2}\left(g_P^e g_S^{\mathcal{N}} \mp g_S^e g_P^{\mathcal{N}} \frac{m_e}{m_{\mathcal{N}}}\right) \\[2mm]
f_{15}^{e\mathcal{N}}(0,0) &= \frac{m_e}{4m_{\mathcal{N}}}\left(g_S^e g_P^{\mathcal{N}} - g_P^e g_S^{\mathcal{N}} \frac{m_e}{m_{\mathcal{N}}}\right) \;.
\end{aligned}
\quad (6.2)
$$

The spin-independent potential between two macroscopic objects induced by ϕ exchange is given in Eq. (4.2), with $f_1^{e\mathcal{N}}(0,0)$ dependent on the scalar couplings as shown in Eq. (6.2), and analogous expressions for the $f_1^{ee}(0,0)$ and $f_1^{\mathcal{N}\mathcal{N}}(0,0)$ coefficients. The limits from tests of the material dependence of the $1/r^2$ force [see Eq. (4.9)] give

$$|g_S^n| \ , \ |g_S^e + g_S^p| \ \lesssim \ 10^{-20} - 10^{-24} \ , \tag{6.3}$$

depending on the range of the interaction, which is set by $1/m_0$ where m_0 is the ϕ mass.

The limit (4.10) on the coefficient of the dipole-dipole potential, $\mathcal{V}_3$, between electrons may be written as

$$\frac{(g_P^e)^2}{16\pi^2\alpha} < 0.8 \times 10^{-14} \ , \tag{6.4}$$

so that the constraint on the pseudoscalar coupling of the electron is

$$|g_P^e| < 0.96 \times 10^{-7} \ . \tag{6.5}$$

The limit (4.11) on the coefficient of the dipole-dipole potential between electrons and neutrons gives $|g_P^e g_P^n| < 0.93 \times 10^{-7}$. Comparing with Eq. (6.5), this places an almost irrelevant bound on the neutron pseudoscalar coupling, $|g_P^n| < 0.97$. Better limits (by three orders of magnitude) on the pseudoscalar couplings to nucleons may be derived by considering two ϕ exchange [33].

The potential between an unpolarized object and an object with polarized electrons is given by Eq. (4.5), with coefficients

$$f_r^{e\mathcal{N}} \ = \ -g_P^e g_S^{\mathcal{N}} \ ,$$

$$f_\perp^{e\mathcal{N}} \ = \ \frac{1}{2} g_S^e g_S^{\mathcal{N}} \ ,$$

$$f_v^{e\mathcal{N}} \ = \ 0 \ , \tag{6.6}$$

and analogous expressions for f_r^{ee}, $f_\perp^{ee}$ and f_v^{ee}. The limits (4.13) and (4.14) on the f_r coefficients of the monopole-dipole potentials, based on the measurements presented in Refs. [37] and [12], respectively, yield constraints on products of scalar and pseudoscalar couplings:

$$|g_P^e g_S^n| \ , \ |g_P^e (g_S^e + g_S^p)| \ \lesssim \ 10^{-30} - 10^{-36} \ , \ \text{for } 1/m_0 \gtrsim \left(1 - 10^{11}\right) \text{m} \ ,$$

$$|g_P^n g_S^n| \ , \ |g_P^n (g_S^e + g_S^p)| \ \lesssim \ 10^{-27} - 10^{-33} \ , \ \text{for } 1/m_0 \gtrsim \left(1 - 10^6\right) \text{m} \ . \tag{6.7}$$

As discussed in Section 4, potentials of the type $\mathcal{V}_{4,5}$ have also been recently constrained in [37]. The limit (4.15) provides a constraint on the scalar couplings different than Eq. (6.3):

$$|g_S^e g_S^n| \ , \ |g_S^e (g_S^e + g_S^p)| \ \lesssim \ 10^{-31} \ , \qquad (6.8)$$

for $1/m_0 > 10^{11}$ m.

The star-cooling limit [16] on the pseudoscalar coupling of the electron to a spin-0 particle, $|g_P^e| < 10^{-12}$, is five orders of magnitude stronger than the one in Eq. (6.5). The scalar coupling to the electron is even more tightly constrained by stellar dynamics, $|g_S^e| < 10^{-14}$, which in conjunction with the constraint from measurements of spin-independent long-range forces given in Eq. (6.3) provides stronger limits than Eqs. (6.7) and (6.8). In the case of the nucleons, the star-cooling limit is $|g_P^{\mathcal{N}}| < 10^{-10}$. Unlike the case of a spin-1 particle, where the astrophysical constraints may be avoided as pointed out at the end of Section 5.1, the star-cooling limits on spin-0 particles are quite robust (some attempts for relaxing the star-cooling constraint on the spin-0 coupling to photons are described in Ref. [35]).

Furthermore, the constraints from searches for new long-range forces may be relaxed in the case of forces mediated by spin-0 exchange if the new particle is self-interacting [36]. By contrast, the constraints on new long-range forces induced by spin-1 exchange are robust: the self-interactions of the paraphoton are forbidden by the $U(1)$ gauge symmetry for operators of dimension 7 or less. Even in the case of a Z', where the gauge symmetry is spontaneously broken, self-interactions may be generated only by higher-dimensional operators which may be adequately suppressed.

7   Conclusions

Assuming energy and momentum conservation, we have shown that rotational invariance restricts the long-range interaction between two fermions to a sum over 16 spin-dependent potentials, given in center-of-mass frame by Eqs. (3.6) and (3.8). The dependence of the potentials on the separation between the fermions, r, is shown in Eqs. (4.2), (4.3), (4.6) and (4.7) for the case of one-boson exchange in a Lorentz invariant theory. If the interaction is induced by two or more particles exchanged (as is the case for neutrino exchange

[23, 24]), or in the more exotic case where the kinetic term of the boson exchanged breaks Lorentz invariance (an example is given in [18]), then different powers of $1/r$ appear in the potentials, as follows from Eqs. (3.6) and (3.8), but the spin dependence remains the same.

Each of the 16 potentials has a dimensionless coefficient which is momentum independent in the non-relativistic limit. The long-range forces between macroscopic objects depend on six different two-particle potentials, e^-e^-, pp, nn, e^-p, e^-n and pn, each of them being described by a different set of 16 dimensionless parameters (or only 12 parameters when the two fermions are identical). Given that searches for new macroscopic forces involve electrically-neutral objects, the following set of parameters needs to be measured: three coefficients of the potential between unpolarized bodies [see Eqs. (4.2)], six coefficients for each of the three potentials between a polarized object and an unpolarized one [see Eqs. (4.3)], six coefficients for each of the nine potentials between two polarized objects given in Eqs. (4.6) and (4.7), with the exception of $\mathcal{V}_7$ where only three coefficients are nonzero. The total number of parameters that need to be measured is 72. Several of those have been already constrained, as discussed in Section 4.1. Many others, both of the static and velocity-dependent types, have not been explored in experiments so far.

In any quantum field theory that extends the Standard Model, these 72 parameters are given in terms of the couplings of some very light particles. Therefore, one expects correlations between the various parameters. We have derived these correlations in the cases of one spin-0 or spin-1 particle exchange, in a general Lorentz-invariant theory. The constraints on the couplings of a spin-0 particle to electrons and nucleons from measurements of spin-dependent macroscopic forces are weaker than the star-cooling constraints. Moreover, they can be further relaxed in the presence of self-interactions. The opposite is true for spin-1 exchange, where the star-cooling constraints may be relaxed, while the searches for macroscopic forces are robust.

If an unbroken $U(1)$ symmetry is added to the Standard Model, theoretical motivation has led to considering only magnetic- and electric-like dipole couplings of the new massless gauge boson to the electrons and nucleons. As seen in section 5.1, this generates only two of the 16 long-range potentials allowed by rotational invariance. The two potentials, one static ($\mathcal{V}_3$) and one velocity dependent ($\mathcal{V}_{15}$), require both objects to be polarized in

order to have an observable effect. If the new $U(1)$ symmetry is spontaneously broken, the vector and axial couplings of the new low-mass gauge boson may also be present (see Section 5.2), and the result is that 15 of the 16 potentials are induced with unsuppressed coefficients, while the remaining potential arises with an extra $(m_0/m_e)^2$ suppression, where m_0 is the mass of the exchanged particle. Remarkably, there are several spin-dependent potentials that fall off as $1/r$: $\mathcal{V}_2$, $\mathcal{V}_8$, $\mathcal{V}_{12} + \mathcal{V}_{13}$ and $\mathcal{V}_{14}$. Measurements of these would be particularly sensitive to new one-boson exchanges in Lorentz invariant theories.

Searching for the macroscopic interactions discussed here could lead to the discovery of new light particles, and at least would provide additional constraints on the properties of any new light particle that couples to electrons or nucleons.

Appendix A: Vector identities

In Section 2 we have stated that any scalar involving two spins and two momenta may be written as a linear combination of spin 16 operators. In this Appendix we give examples of such linear combinations.

In order to find relations between various operators, it is useful to note the following identities:

$$
\begin{aligned}
\left[\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}\right]\left[\left(\vec{a} \times \vec{b}\right) \cdot \vec{d}\right] &= \vec{a}^2 \vec{b}^2 \left(\vec{c} \cdot \vec{d}\right) - \vec{a}^2 \left(\vec{b} \cdot \vec{c}\right)\left(\vec{b} \cdot \vec{d}\right) - \vec{b}^2 \left(\vec{a} \cdot \vec{c}\right)\left(\vec{a} \cdot \vec{d}\right) \\
&\quad + \left(\vec{a} \cdot \vec{b}\right)\left[\left(\vec{a} \cdot \vec{c}\right)\left(\vec{b} \cdot \vec{d}\right) + \left(\vec{a} \cdot \vec{d}\right)\left(\vec{b} \cdot \vec{c}\right) - \left(\vec{a} \cdot \vec{b}\right)\left(\vec{c} \cdot \vec{d}\right)\right],
\end{aligned}
$$

$$
\left[\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}\right]\left(\vec{d} \cdot \vec{e}\right) = \left[\left(\vec{a} \times \vec{b}\right) \cdot \vec{e}\right]\left(\vec{c} \cdot \vec{d}\right) + \left[\left(\vec{b} \times \vec{c}\right) \cdot \vec{e}\right]\left(\vec{a} \cdot \vec{d}\right) - \left[\left(\vec{a} \times \vec{c}\right) \cdot \vec{e}\right]\left(\vec{b} \cdot \vec{d}\right),
$$

$$
\left(\vec{a} \times \vec{b}\right)\left(\vec{c} \times \vec{d}\right) = \left(\vec{a} \cdot \vec{c}\right)\left(\vec{b} \cdot \vec{d}\right) - \left(\vec{a} \cdot \vec{d}\right)\left(\vec{b} \cdot \vec{c}\right), \tag{A.1}
$$

where $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ and $\vec{e}$ are arbitrary 3-vectors.

Given that $\vec{P} \cdot \vec{q} = 0$, we find various nontrivial examples of linear combinations:

$$\left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right]\left[\vec{\sigma}' \cdot \left(\vec{P} \times \vec{q}\right)\right] = \vec{q}^2\vec{P}^2\mathcal{O}_2 - m^2\left(\vec{P}^2\mathcal{O}_3 + \vec{q}^2\mathcal{O}_8\right) ,$$

$$\left[\vec{\sigma} \cdot (\vec{P} \times \vec{q})\right](\vec{\sigma}' \cdot \vec{q}) = m^3\mathcal{O}_{15} - \frac{m}{2}\vec{q}^2\,\mathcal{O}_{14} ,$$

$$i\left[\vec{\sigma} \cdot \left(\vec{P} \times \vec{q}\right)\right]\left(\vec{\sigma}' \cdot \vec{P}\right) = m^3\mathcal{O}_{16} + \frac{m}{2}\vec{P}^2\mathcal{O}_{11} ,$$

$$(\vec{\sigma} \times \vec{q})\,(\vec{\sigma}' \times \vec{q}) = \vec{q}^2\mathcal{O}_2 - m^2\mathcal{O}_3 ,$$

$$\left(\vec{\sigma} \times \vec{P}\right)\left(\vec{\sigma}' \times \vec{P}\right) = \vec{P}^2\mathcal{O}_2 - m^2\mathcal{O}_8 ,$$

$$i\left[\vec{\sigma} \times \left(\vec{P} \times \vec{q}\right)\right] \cdot \vec{\sigma}' = -2m^2\mathcal{O}_7 , \tag{A.2}$$

where $\mathcal{O}_i$ are the operators listed in Eqs. (2.2) and (2.3). Based on these and other similar identities, one can prove by exhaustion that the set of 16 operators $\mathcal{O}_i$ is complete.

Appendix B: Fourier transforms

The Fourier transforms necessary for obtaining the potentials induced by the exchange of one boson with normal propagator (see Section 4.1) are given by

$$\int \frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\frac{1}{\vec{q}^2 + m_0^2}(\vec{q}^2)^l = \frac{1}{4\pi r}e^{-m_0 r}m_0^{2l}(-1)^l ,$$

$$\int \frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\frac{\vec{q}}{\vec{q}^2 + m_0^2}(\vec{q}^2)^l = \frac{i}{4\pi r^2}(1 + m_0 r)e^{-m_0 r}m_0^{2l}(-1)^l \hat{\vec{r}} ,$$

$$\int \frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\frac{(\vec{q}\cdot\vec{n}_1)\,(\vec{q}\cdot\vec{n}_2)}{\vec{q}^2 + m_0^2}(\vec{q}^2)^l = -\frac{1}{4\pi r^3}e^{-m_0 r}m_0^{2l}(-1)^l\left[(1 + m_0 r)\vec{n}_1 \cdot \vec{n}_2\right.$$
$$\left. - \left(3 + 3m_0 r + m_0^2 r^2\right)\left(\hat{\vec{r}} \cdot \vec{n}_1\right)\left(\hat{\vec{r}} \cdot \vec{n}_2\right)\right] , \tag{B.1}$$

where $l \geq 0$ is an integer, while $\vec{n}_1$ and $\vec{n}_2$ are arbitrary 3-vectors. For $l = 1$ we obtain the results used in Section 4.1, with $\vec{n}_1$ and $\vec{n}_2$ replaced by $\vec{\sigma}, \vec{\sigma}', \vec{v}$ or combinations thereof, as needed for the various operators.

We also give here the Fourier transforms relevant for the cases where there are non-standard dispersion relations (see Section 4.2):

$$\int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{(\vec{q}^2)^k} = \frac{1}{2\pi^2}\,r^{2k-3}\sin(k\pi)\Gamma(2 - 2k)$$

$$= -\frac{1}{[2(k-1)]!\,4\pi}\, r^{2k-3} \quad,$$

$$\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{(\vec{q}^2)^k}\vec{q} = -i\vec{\nabla}\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{(\vec{q}^2)^k}$$

$$= \frac{i(2k-3)}{[2(k-1)]!\,4\pi}\, r^{2k-4}\,\hat{\vec{r}} \quad,$$

$$\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{(\vec{q}^2)^k}\left(\vec{q}\cdot\vec{n}_1\right)\left(\vec{q}\cdot\vec{n}_2\right) = \frac{(2k-3)}{[2(k-1)]!\,4\pi}\, r^{2k-5}\left[\hat{\vec{n}}_1\cdot\hat{\vec{n}}_2 + (2k-5)\left(\hat{\vec{r}}\cdot\vec{n}_1\right)\left(\hat{\vec{r}}\cdot\vec{n}_2\right)\right] \quad,$$

$$(B.2)$$

where $k \geq 1$ is an integer.

References

[1] E. G. Adelberger, B. R. Heckel and A. E. Nelson, "Tests of the gravitational inverse-square law," Ann. Rev. Nucl. Part. Sci. **53**, 77 (2003) [arXiv:hep-ph/0307284].

[2] J. C. Long and J. C. Price, "Current short-range tests of the gravitational inverse square law," Comptes Rendus Physique **4**, 337 (2003) [arXiv:hep-ph/0303057].

[3] J. E. Moody and F. Wilczek, "New Macroscopic Forces?," Phys. Rev. D **30**, 130 (1984).

[4] R. C. Ritter, C. E. Goldblum, W.-T. Ni, G. T. Gillies, and C. C. Speake, "Experimental test of equivalence principle with polarized masses", Phys. Rev. D **42**, 977 (1990).

[5] D. J. Wineland, J. J. Bollinger, D. J. Heinzen, W. M. Itano, and M. G. Raizen, "Search for Anomalous Spin-Dependent Forces Using Stored-Ion Spectroscopy", Phys. Rev. Lett. **67**, 1735 (1991).

[6] S.-S. Pan, W.-T. Ni and S.-C. Chen, "Experimental search for anomalous spin spin interactions," Mod. Phys. Lett. A **7**, 1287 (1992).

[7] W.-T. Ni, S.-S. Pan, T.C.P. Chui and B.-Y. Cheng, "Search for anomalous spin spin interactions using a paramagnetic salt with a DC squid," Int. J. Mod. Phys. A **8**, 5153 (1993).

[8] W.-T. Ni, T.C.P. Chui, S.-S. Pan and B.-Y. Cheng, "Search for anomalous spin-spin interactions between electrons using a DC SQUID", Physica B **194-196**, 153 (1994).

[9] A. N. Youdin, D. Krause, K. Jagannathan, L. R. Hunter and S. K. Lamoreaux, "Limits on spin - mass couplings within the axion window," Phys. Rev. Lett. **77**, 2170 (1996).

[10] W. T. Ni, S. S. Pan, H. C. Yeh, L. S. Hou and J. L. Wan, "Search for an axionlike spin coupling using a paramagnetic salt with a dc SQUID," Phys. Rev. Lett. **82**, 2439 (1999).

[11] B. R. Heckel, E. G. Adelberger, J. H. Gundlach, M. G. Harris and H. E. Swanson, "Torsion balance test of spin coupled forces," Prepared for International Conference on Orbis Scientiae 1999: Quantum Gravity, Generalized Theory of Gravitation and Superstring Theory Based Unification, Coral Gables, Florida, 16-19 Dec. 1999.

[12] B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson, Phys. Rev. Lett. **68**, 135 (1992).

[13] J. M. Daniels and W.-T Ni, "Nuclear polarization and the equivalence principle," Mod. Phys. Lett. A **6** (1991) 659.

[14] E. G. Adelberger, B. R. Heckel, C. W. Stubbs and W. F. Rogers, "Searches for new macroscopic forces," Ann. Rev. Nucl. Part. Sci. **41**, 269 (1991).

[15] R. Newman, "Laboratory Gravity: The G Mystery and Intrinsic Spin Experiments", Matters of Gravity **4**, 11 (1994) (http://www.phys.lsu.edu/mog)

[16] S. Eidelman *et al.* [Particle Data Group], "Review of particle physics," Phys. Lett. B **592** (2004) 1.

[17] B. A. Dobrescu, "Massless gauge bosons other than the photon," Phys. Rev. Lett. **94**, 151802 (2005).

[18] N. Arkani-Hamed, H. C. Cheng, M. Luty and J. Thaler, "Universal dynamics of spontaneous Lorentz violation and a new spin-dependent inverse-square law force," JHEP **0507**, 029 (2005) [arXiv:hep-ph/0407034];
H. C. Cheng, M. A. Luty, S. Mukohyama and J. Thaler, "Spontaneous Lorentz breaking at high energies," arXiv:hep-th/0603010.

[19] R. Bluhm and V. A. Kostelecky, "Lorentz and CPT tests with spin-polarized solids," Phys. Rev. Lett. **84**, 1381 (2000) [arXiv:hep-ph/9912542].

[20] T. E. O. Ericson and W. Weise, *Pions And Nuclei*, Oxford University Press (1988), Appendix 10, p. 437.

[21] M. M. Nagels, T. A. Rijken and J. J. de Swart, "Baryon Baryon Scattering In An Obep Approach. 1. Nucleon-Nucleon Scattering," Phys. Rev. D **12**, 744 (1975);
"Baryon Baryon Scattering In A One Boson Exchange Potential Approach. 2. Hyperon - Nucleon Scattering," Phys. Rev. D **15**, 2547 (1977);
"Baryon Baryon Scattering In A One Boson Exchange Potential Approach. 3. A Nucleon-Nucleon And Hyperon - Nucleon Analysis Including Contributions Of A Nonet Of Scalar Mesons," Phys. Rev. D **20**, 1633 (1979).

[22] P. M. M. Maessen, T. A. Rijken and J. J. de Swart, "Soft Core Baryon Baryon One Boson Exchange Models. 2. Hyperon - Nucleon Potential," Phys. Rev. C **40**, 2226 (1989).

[23] S. D. H. Hsu and P. Sikivie, "Long range forces from two neutrino exchange revisited," Phys. Rev. D **49**, 4951 (1994).

[24] G. Feinberg, J. Sucher and C. K. Au, "The Dispersion Theory Of Dispersion Forces," Phys. Rept. **180**, 83 (1989).

[25] J. A. Grifols and S. Tortosa, "Residual long range pseudoscalar forces between unpolarized macroscopic bodies," Phys. Lett. B **328**, 98 (1994) [arXiv:hep-ph/9404249];
F. Ferrer and J. A. Grifols, "Long range forces from pseudoscalar exchange," Phys. Rev. D **58**, 096006 (1998) [arXiv:hep-ph/9805477];

F. Ferrer and M. Nowakowski, "Higgs and Goldstone bosons mediated long range forces," Phys. Rev. D **59**, 075009 (1999) [arXiv:hep-ph/9810550].

[26] B. Holdom, "Two U(1)'S And Epsilon Charge Shifts," Phys. Lett. B **166**, 196 (1986).

[27] S. I. Blinnikov, A. D. Dolgov, L. B. Okun and M. B. Voloshin, "How strong can the coupling of leptonic photons be?," Nucl. Phys. B **458**, 52 (1996) [arXiv:hep-ph/9505444].

[28] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, "The string landscape, black holes and gravity as the weakest force," arXiv:hep-th/0601001.

[29] S. Weinberg, "Gauge And Global Symmetries At High Temperature," Phys. Rev. D **9**, 3357 (1974).

[30] P. Batra, B. A. Dobrescu and D. Spivak, "Anomaly-free sets of fermions," arXiv:hep-ph/0510181.

[31] B. Batell and T. Gherghetta, "Localized U(1) gauge fields, millicharged particles, and holography," Phys. Rev. D **73**, 045016 (2006) [arXiv:hep-ph/0512356].

[32] T. Appelquist, B. A. Dobrescu and A. R. Hopper, "Nonexotic neutral gauge bosons," Phys. Rev. D **68**, 035012 (2003) [arXiv:hep-ph/0212073].

[33] E. Fischbach and D. E. Krause, "New limits on the couplings of light pseudoscalars from equivalence principle experiments," Phys. Rev. Lett. **82**, 4753 (1999) [arXiv:hep-ph/9905408]; "Constraints on light pseudoscalars implied by tests of the gravitational inverse-square law," Phys. Rev. Lett. **83**, 3593 (1999) [arXiv:hep-ph/9906240].

[34] G. G. Raffelt, "Particle physics from stars," Ann. Rev. Nucl. Part. Sci. **49**, 163 (1999) [arXiv:hep-ph/9903472].

[35] E. Masso and J. Redondo, "Evading astrophysical constraints on axion-like particles," JCAP **0509**, 015 (2005) [arXiv:hep-ph/0504202];
P. Jain and S. Mandal, "Evading the astrophysical limits on light pseudoscalars,"

arXiv:astro-ph/0512155;

J. Jaeckel, E. Masso, J. Redondo, A. Ringwald and F. Takahashi, "We need lab experiments to look for axion-like particles," arXiv:hep-ph/0605313.

[36] S. S. Gubser and J. Khoury, "Scalar self-interactions loosen constraints from fifth force searches," Phys. Rev. D **70**, 104001 (2004) [arXiv:hep-ph/0405231];

B. Feldman and A. E. Nelson, "New regions for a chameleon to hide," arXiv:hep-ph/0603057.

[37] B. R. Heckel, C. E. Cramer, T. S. Cook, and E. G. Adelberger, "New CP-violation and preferred-frame tests with polarized electrons", May 2006.