NAL PROPOSAL No. 79

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A MEASUREMENT OF THE REGENERATION PARAMETER IN THE 100 GEV/C RANGE

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ABSTRACT:

We propose a wire chamber experiment to measure the coherent regenerator parameter ρ in hydrogen in the 60 to 120 Gev/c range. In this report we show that, by using the neutral beam at NAL and currently existing apparatus, ρ can be measured in the momentum range from 90 to 110 Gev/c to a precision of $\sim \pm 5 \times 10^{-4}$ and its phase to $\sim \pm 8°$. This results in an uncertainty in $\Delta\sigma = \sigma(\bar{K}^o p) - \sigma(K^o p)$ of $\sim \pm 0.10$mb.

REQUIREMENTS:

1. Beam:
   (a) neutral beam at 15 mrad to primary proton beam
   (b) collimation to 1 in. x 1 in. at 420 feet from target
   (c) 10^{12} protons per pulse in a 1 second spill
   (d) photon filter

   Remarks: proton beam could be reduced and the beam hole widened correspondingly.

2. Apparatus:
   (a) sweeping magnet for electrons from photon filter
   (b) a 20 kilogauss-meter analyzing magnet with a minimum aperture of 24 in. vertically, 30 in. horizontally
   (c) 10 meter long liquid hydrogen target, minimum diameter 10 cm.

   Remarks: the counter-wire spark chamber apparatus is already in existence.

3. Running Time:
   (a) 2 months of setup time in the beam channel
   (b) 500 hours of running time

I. Introduction:

In this report we propose a measurement of the parameter ρ for coherent regeneration of K_S from a K_L beam in the 60 to 120 Gev/c range. Such a measurement leads directly to the determination of the magnitude and phase of $[F(o) - \bar{F}(o)]$, the difference of particle and antiparticle scattering amplitudes on the nuclei of the material used as regenerator. We show that, using (a), currently available estimates for the kaon and neutron fluxes from a target bombarded by 200 Gev/c protons and (b), the Serpukhov total cross-section data, the backgrounds are sufficiently low such that an experiment with a liquid hydrogen target is readily feasible. We also show that if the Serpukhov results should turn out to be in error or the neutron to kaon ratio in the neutral beam less favorable, then a significant experiment testing the validity of the Pomeranchuk Theorem can still be done using plastic scintillator as a regenerator.

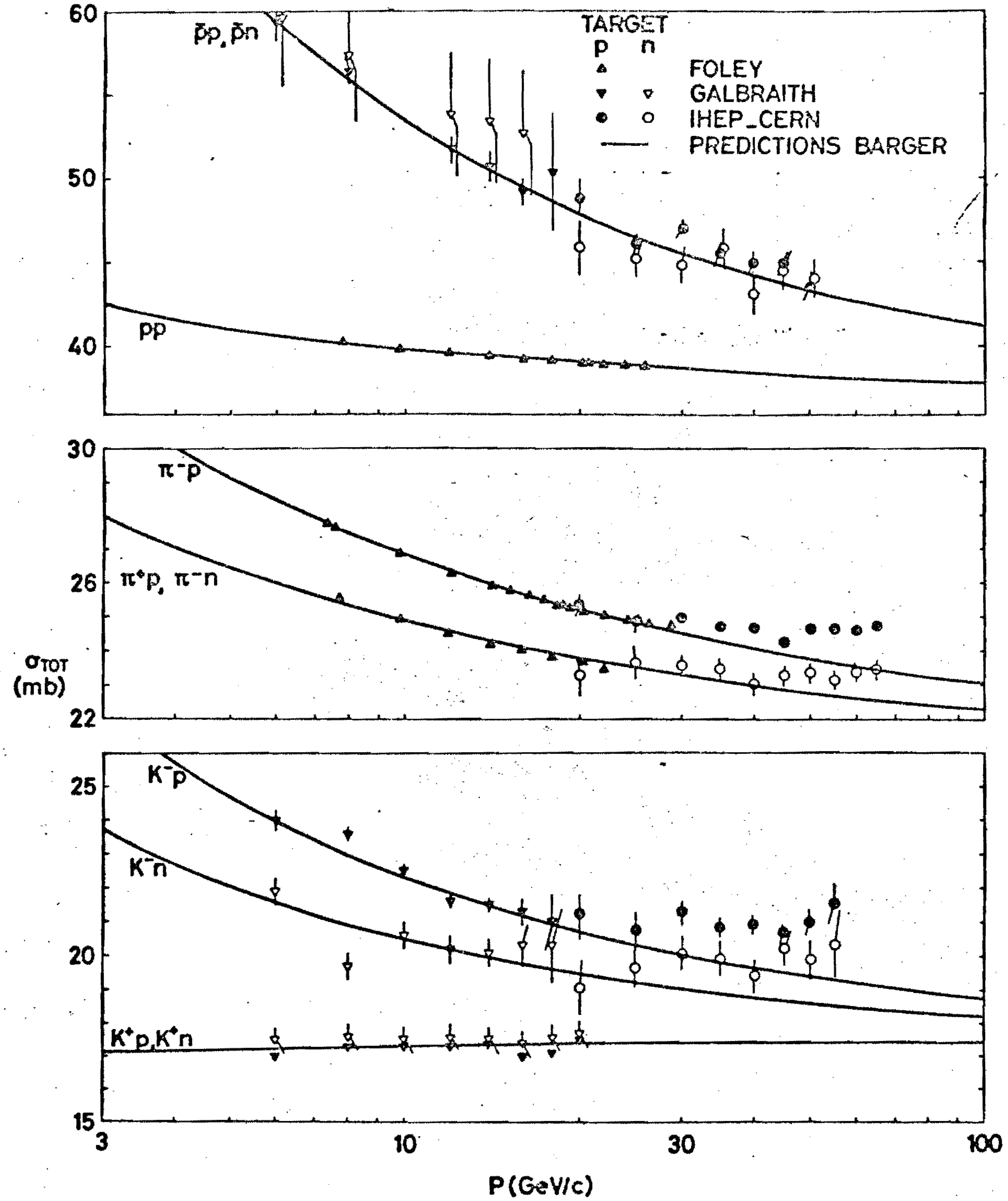
In view of the current budgetary strictures we have tried to select an experiment which does not require a large expenditure or an unwieldy research team. In fact, the proposed experiment can be performed with apparatus which we are currently using for a very similar experiment at SLAC. Due to the fact that in decay processes the transverse momenta does not change, the resolution achieved in the SLAC experiment can be maintained at NAL simply by an increase of the spacing between wire chambers proportional to the increase in kaon momentum.

In Section II we discuss briefly the physics of the experiment, set down the relevant equations and apply them to the situation at hand. The Glauber calculation relevant to the use of carbon as a regenerator is given in Section III. In Section IV we describe our apparatus and give the results of Monte Carlo calculations for trigger rates and resolutions. Finally, Section V contains our estimates of the uncertainties of our final results for various experimental procedures and various possible experimental results.

II. The Physics of the Proposed Experiment:

At high energies a comparison between the forward elastic scattering amplitudes of a particle, $F(o)$, and its antiparticle, $\bar{F}(o)$, on the same hadron is of well-known importance and a detailed justification does not seem worthwhile. The Pomeranchuk Theorem, first reported[1] in 1956, states that as $E \to \infty$, the total cross-sections of particle and antiparticle should become equal to the same constant provided the imaginary parts dominate the separate amplitudes. To test this Theorem, an experimental technique sensitive directly to the quantity $[F(o) - \bar{F}(o)]$ is clearly desirable; it circumvents the normalization and subtraction errors which are unavoidable when particle and antiparticle cross-sections are measured separately.

At this time a careful investigation of this problem seems particularly opportune. Recently Allaby et al[2] reported on the measurements of the total cross-sections of negative particles on protons and neutrons in the momentum range of 20 to 60 Gev/c obtained at Serpukhov. The measurements are reproduced in Figure 1; they suggest that, rather than coalescing at high energies, the cross-sections of bosons and antibosons approach a constant difference. For example, if one assumes that the $\sigma(K^+p)$ and $\sigma(K^+n)$ cross-sections remain constant at 17.5 mb (as extrapolated from measurements from 8 to 20 Gev/c) then the data suggest that the cross-section differences $\Delta\sigma_1 \equiv \sigma(K^-p) - \sigma(K^+p) = \sigma(\bar{K}^o n) - \sigma(K^o n)$ as well as $\Delta\sigma_2 \equiv \sigma(K^-n) - \sigma(K^+n) = \sigma(\bar{K}^o p) - \sigma(K^o p)$ both approach a constant value $\sim$ 3 mb, in clear violation of the Pomeranchuk Theorem. It seems likely that in the not too distant future both K^+ and K^- cross-sections will be measured using an experimental technique which avoids the possibility of normalization errors. Nevertheless, confirmation of this result by means of a technique



60
50
40
$\bar{p}p, \bar{p}n$
TARGET
p n
FOLEY
GALBRAITH
IHEP_CERN
PREDICTIONS BARGER
pp
30
26
22
π^-p
π^+p, π^-n
σ_{TOT} (mb)
25
20
15
K^-p
K^-n
K^+p, K^+n
3
10
30
100
P (GeV/c)

Figure 1

which measures $\Delta\sigma$ directly still seems desirable - especially, if information about the phases of the scattering amplitudes can be secured at the same time. As has been pointed out, for example by Martin[3], if $\Delta\sigma_{1,2} \nrightarrow o$ as $E \to \infty$, then local field theory is in serious difficulties unless the elastic forward scattering amplitudes are dominantly real.

A number of investigators[4] have pointed out that the measurement of the regeneration parameter ρ provides a convenient means to study the quantity $[F(o) - \bar{F}(o)]$ at high energies. When a beam of K_L measons (all in the state $|K_L>$) impinges on a regenerator, the emerging beam is in the quantum state $|K_L> + \rho|K_S>$ and will therefore exhibit K_S decay. The regeneration parameter ρ is given by

$$\rho = \pi N \{i \frac{F(o) - \bar{F}(o)}{P}\} \Lambda_s g(X) \qquad (1)$$

where

N = number of nuclei per unit volume

P = the momentum of the incident beam, given in the same frame in which F and $\bar{F}$ are expressed.

Λ_s = the mean decay distance of the K_S mesons in the laboratory

g = a dimensionless function of the regenerator length L in units of Λ_s, $X = L/\Lambda_s$

$$g(X) = \frac{1 - \exp[(i\delta - 1/2)X]}{-(i\delta - 1/2)}$$

where

$\delta = (m_L - m_S)/\Gamma_s$ is the $K_L - K_S$ mass difference in Γ_s units, Γ_s being the K_S decay probability per unit time.

The magnitude and phase of $g(X)$ are shown in Figure 2; when $X<<1$, $g(X) \approx X$ which somewhat simplifies equation (1) to read

$$\rho \approx \pi NL \{i \frac{F(o) - \bar{F}(o)}{P}\} \qquad (1')$$

Using the optical theorem, equation (1) may also be written in terms of the difference between the antiparticle and particle total cross-sections.

$$\rho = \frac{N}{4} \frac{\Delta\sigma}{\cos\Phi} \Lambda_S \, g(X) \exp(i\Phi) \qquad (2)$$

where $\Delta\sigma = \bar{\sigma} - \sigma$ and $\Phi = \arg\{i[F(o) - \bar{F}(o)]\}$. Equations (1) and (2) demonstrate that, apart from $[F(o) - \bar{F}(o)]$, all quantities which appear in ρ are well-known; thus both $\Delta\sigma$ and Φ can be deduced if ρ is determined.

The parameter ρ can be measured by observing the rate of $\pi^+\pi^-$ decays behind a regenerator. Using $\Gamma_S >> \Gamma_L$, this rate per incident K_L meson is given by

$$\frac{dI}{dx} = \frac{2}{3} e^{-N\sigma L} \{|\rho|^2 e^{-x} + 2|\rho||\eta| e^{-x/2} \cos(\delta x + \phi_\rho - \phi_\eta) + |\eta|^2\} \qquad (3)$$

where

$x = z/\Lambda_S$ with z the distance from the downstream end of the regenerator to the $\pi^+\pi^-$ decay vertex.

σ = the total cross-section for both K_L and K_S in the regenerator

$\rho = |\rho|\exp(i\phi_\rho)$, $\phi_\rho = \Phi + \arg[g(X)]$

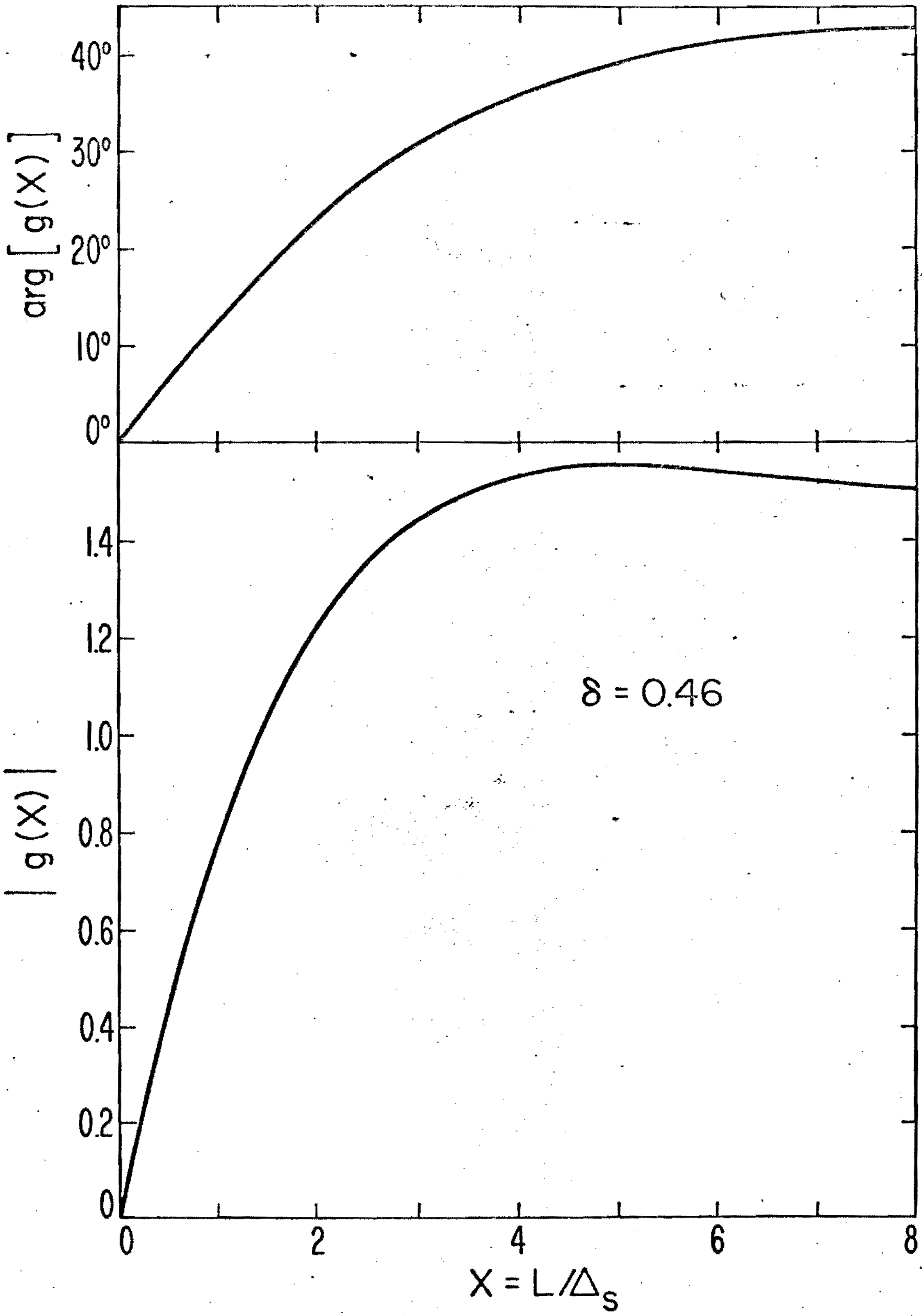


Figure 2

$\eta = |\eta| \exp(i\phi_\eta) = (1.92 \pm 0.05) \times 10^{-3} \exp[i(44 \pm 5)°]$ is the CP violating parameter of $K_L \rightarrow \Pi^+\Pi^-$ decay.

For liquid hydrogen and 100 Gev/c incident kaons, equation (2) (with $\Lambda_s = 5.38\text{m}$) yields

$$\rho = 5.70 \times 10^{-3} \frac{\Delta\sigma}{\cos\Phi} \; g(X) \; e^{i\Phi}$$

with $\Delta\sigma$ in millibarns. Also $\sigma = [\sigma(K^-n) + \sigma(K^+n)]/2 = 18.5$ mb; hence $1/N\sigma = 12.8$ m. Choosing $L = 12.8$ m to make the $\exp(-N\sigma L)$ factor in (3) equal to $1/e$ we find $X = 2.38$; using Figure 2, $g(2.38) = 1.32 \exp(i26°)$. Thus for a 12.8 m long hydrogen target and 100 Gev/c K_L mesons

$$\rho\ (100) = 7.55 \times 10^{-3} \frac{\Delta\sigma}{\cos\Phi} \; e^{i(\Phi + 26°)} \; (\Delta\sigma \text{ in mb}) \qquad (4)$$

For the same target at 50 Gev/c

$$\rho\ (50) = 4.42 \times 10^{-3} \frac{\Delta\sigma}{\cos\Phi} \; e^{i(\Phi + 38°)} \; (\Delta\sigma \text{ in mb}) \qquad (4')$$

If, as suggested by the Serpukhov experiments, $\Delta\sigma_2 \sim 3$mb then at 100 Gev/c, $|\rho|/|\eta| \gtrsim 12$ and at 50 Gev/c, $|\rho|\ |\eta| \gtrsim 7$. Note that the quantity which enters (3) is $|\rho|^2$; thus the sensitivity of the experiment to $\Delta\sigma$ is surely adequate.

A liquid hydrogen regenerator is certainly attractive because then ρ can be related directly to $K^o p$ and $\bar{K}^o p$ scattering. However, it must be kept in mind that the K_S produced by coherent regeneration must be distinguished both from diffraction regenerated K_S and from K_S mesons produced in other processes. The number of diffraction regenerated K_S emitted into a solid angle $\Delta\Omega_{lab}$ in the forward direction per K_S produced by coherent regeneration, Δn is given by

$$\Delta n = \frac{\delta^2 + \frac{1}{4}}{N\,\Lambda_s\,\Lambda^2}\; h(X)\;\Delta\Omega_{lab} \tag{5}$$

where

$\Lambda = h/P$ is the wave length of the incident K_S

$h(X)$ = dimensionless function

$$h(X) = \frac{1-\exp(-X)}{1 - 2\exp(-X/2)\cos(\delta X) + \exp(-X)}$$

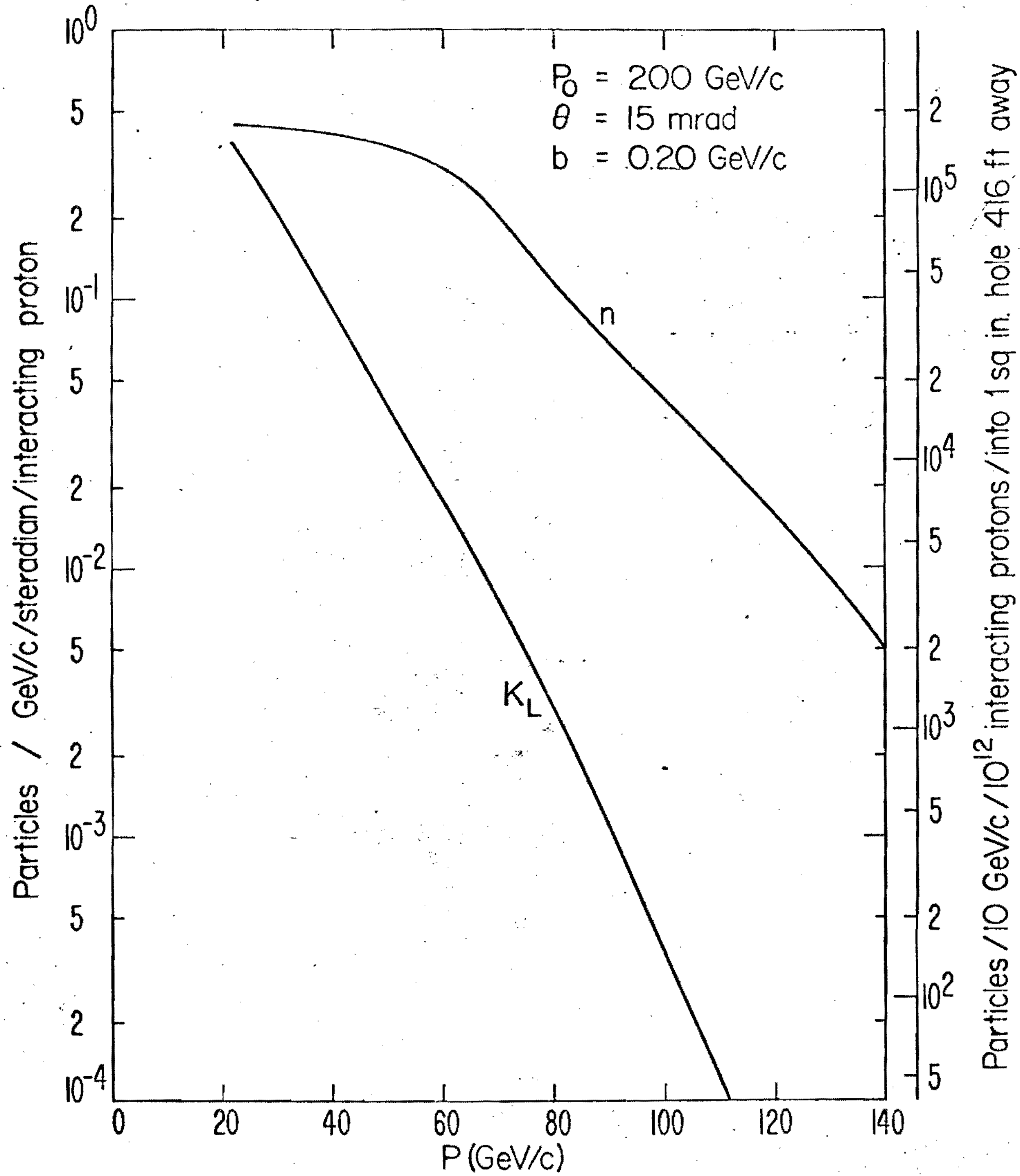
again with $X = L/\Lambda_s$.

For a 12.8 m long liquid hydrogen target at 100 Gev/c, $h(2.38) = 1.11$; hence

$$\Delta n = 1.45 \times 10^4\;\Delta\Omega_{lab} \tag{5'}$$

In a later section we show that the resolution of the proposed apparatus is such that coherently regenerated K_S appear to lie in a forward cone with $\Delta\Omega \leq 10^{-8}$ sterad. The background due to diffraction regenerated K_S can therefore be reduced to a negligible level. Presumably the background due to K_S generated in the forward direction in inelastic processes is even lower.

However, it must be recalled that a neutral beam produced by the impact of the primary proton beam on a target will be very rich in neutrons. Figure 3 shows the best current guess for both the K_L and



P_0 = 200 GeV/c
θ = 15 mrad
b = 0.20 GeV/c
n
K_L
Particles / GeV/c /steradian/interacting proton
Particles /10 GeV/c /10^{12} interacting protons /into 1 sq in. hole 416 ft away
P (GeV/c)
10^0
5
2
10^{-1}
5
2
10^{-2}
5
2
10^{-3}
5
2
10^{-4}
0
20
40
60
80
100
120
140
2
10^5
5
2
10^4
5
2
10^3
5
2
10^2
5

Figure 3

neutron spectra obtained at 15 mrad relative to the primary proton direction for an incident beam momentum of 200 Gev/c. These spectra were obtained from the Cocconi formula[5]

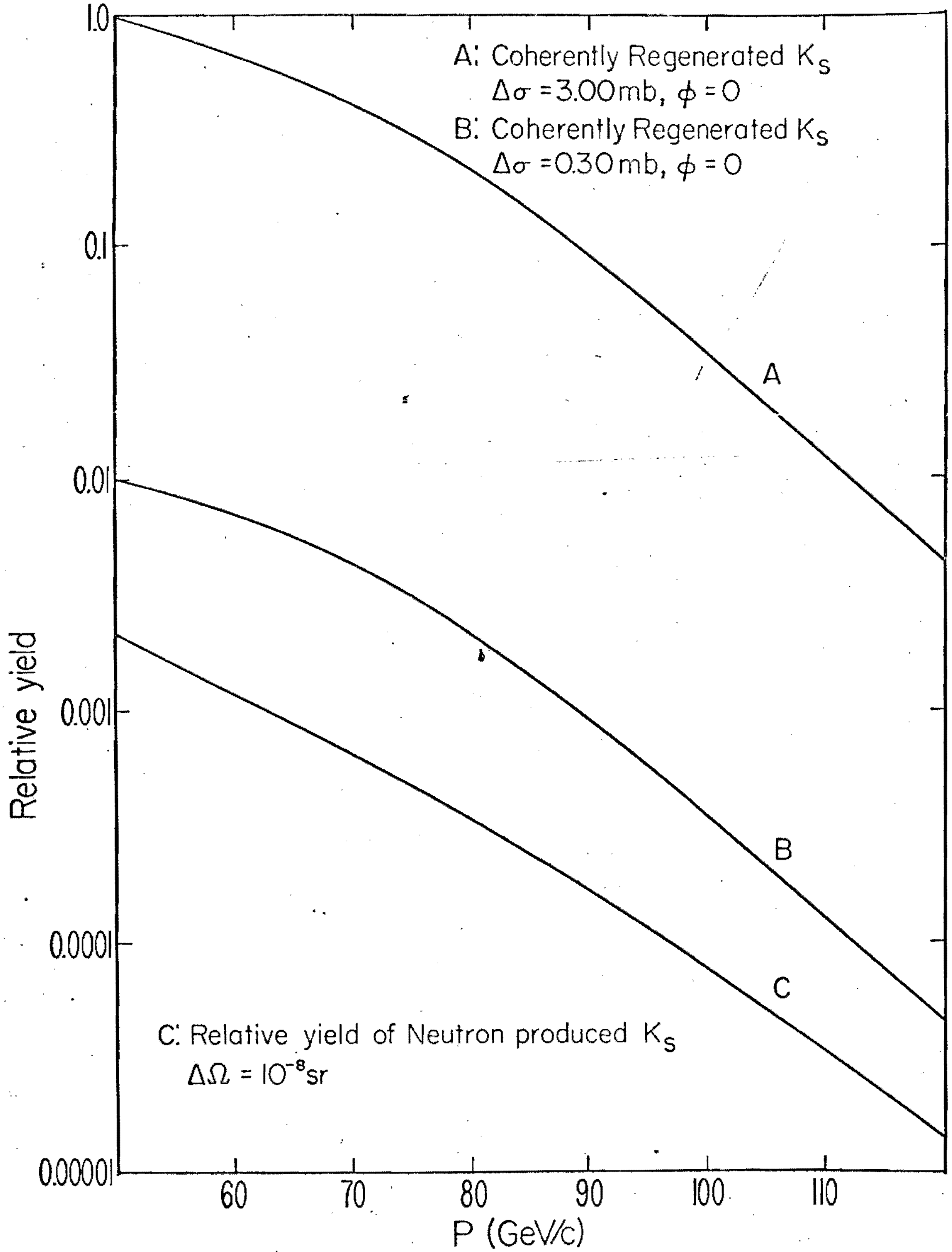
$$dN(200\ \text{Gev/c},p,\Theta) = dN(200\ \text{Gev/c},p,0)\ \exp(-p\Theta/b)$$

with[6] b = 0.20 Gev/c and the forward spectra dN(p,0) estimated by means of the Ranft-Hagedorn model for pion production and the experimental K/Π ratios[7]. Near 100 Gev/c the neutron flux exceeds the kaon flux by more than two orders of magnitudes; thus an estimate of the neutron-generated K_S intensity behind the regenerator is required.

This estimate has been carried out in the following fashion. First of all, curves A and B of Figure 4 show the yields of K_S resulting from coherent regeneration from the K_L beam shown in Figure 3. Both curves are for a 12.8 m long hydrogen target; curve A assumes $\Delta\sigma$ = 3 mb, Φ = o and curve B $\Delta\sigma$ = 0.30 mb, Φ = o. Curve C shows, on the same scale, the yield of K_S due to neutron interactions in the target provided the detection apparatus achieves an angular resolution $\Delta\Omega = 10^{-8}$sr. This curve was obtained by folding the neutron spectrum dN_n(200 Gev/c,P,15 mrad) of Figure 3 with forward K_S production spectra[6,7] due to neutrons $dN_{K_S}(P,p,0)$

$$df(p,0) = \int_P dN_{K_S}(P,p,0)\,dN_n(200\text{Gev/c},P,15\ \text{mrad})$$

A comparison of the curves of Figure 4 reveals that provided $\Delta\sigma \sim 3$ mb and the resolution $\Delta\Omega \sim 10^{-8}$ sterad the neutron produced K_S produce a contamination which is less than 1% of the coherent regeneration signal. Of course, an anticounter at the downstream end of the target would further



A: Coherently Regenerated K_S
$\Delta\sigma = 3.00$ mb, $\phi = 0$
B: Coherently Regenerated K_S
$\Delta\sigma = 0.30$ mb, $\phi = 0$
C: Relative yield of Neutron produced K_S
$\Delta\Omega = 10^{-8}$ sr
A
B
C
Relative yield
1.0
0.1
0.01
0.001
0.0001
0.00001
60
70
80
90
100
110
P (GeV/c)

Figure 4

suppress this contamination because presumably a significant fraction of neutron-produced K_S would be accompanied by other forward-moving charged particles. In addition, this background can be subtracted by extrapolating it from larger production angles under the coherent forward peak. Thus, under the assumptions listed above this background is negligible.

However, in a pessimistic vein, we may assume that "in some way" the Serpukhov K^- cross-sections have a systematic error of the order of 15%. Then $\Delta\sigma$ can be very small, say $\Delta\sigma \sim 0.3$ mb. In addition the kaon to neutron ratio may be smaller than the current best guess which is incorporated in Figure 3. Then the neutron-produced K_S would become a serious background.

We wish to point out that in this eventuality the background due to neutron-produced K_S could be largely eliminated by using a regenerator made of scintillator. When a neutron interacts in the regenerator, charged particles are surely produced in the great majority of the collisions. In a scintillator these give light pulses; thus putting the scintillator itself into anticoincidence would eliminate the majority of neutron-produced events and K_S produced in inelastic K_Lp and K_LC collisions. K_S mesons due to K_LC diffraction scattering produced at angles less than 1 mrad would, of course, not be expected to yield light pulses of sufficient size to be recorded. In the next section we discuss the use of scintillator as a regenerator.

III. Carbon as a Regenerator:

Scintillator is $CH_{1.1}$ and has a density of 1.05 g/cm^3. In what follows we ignore the hydrogen; the density of carbon atoms in scintillator is N = 4.84 x 10^{22} atoms/cm^3.

The forward scattering amplitude F of a complex nucleus with Z protons and A - Z neutrons is given in first approximation by

$$\frac{F^{(1)}}{P} = Z\frac{f_P}{k} + (A - Z)\frac{f_N}{k} \tag{6}$$

where f_p and f_N are the scattering amplitudes on protons and neutrons, respectively in the frame in which the incident momentum is k; P is the incident momentum in the frame in which the total scattering amplitude is F. The more accurate way of dealing with the complex nucleus is by means of the Glauber theory[8] whose validity has recently been confirmed in connection with regeneration phenomena[9]. In the Glauber theory the scattering amplitude of a nucleus with Z protons and A - Z neutrons, $F(\vec{q})$, may be written

$$\frac{F(\vec{q})}{P} = \frac{i}{2\pi}\int d^2\vec{b}\, e^{i\vec{q}\cdot\vec{b}}\Big\{ 1 - \prod^{Z} \Big[1 - \frac{2\pi f_p(o)}{ik}\int\rho_p(\vec{b} + \hat{k}z)dz \Big] \times$$

$$\prod^{A-Z} \Big[1 - \frac{2\pi f_N(o)}{ik}\int\rho_N(\vec{b} + \hat{k}z)dz \Big]\Big\}. \tag{7}$$

where $\vec{q}$ is the momentum transfer and $\rho_P(\vec{r})$ and $\rho_N(\vec{r})$ are the normalized proton and neutron densities, respectively, in the nucleus. We have evaluated equation (7) using a Woods-Saxon density function

$$\rho_p = \rho_N = \frac{C}{1 + \exp[(r-R)/t]} \tag{8}$$

with $R = 1.05\ A^{1/3}$ f, $t = 0.55$ f.

In our calculations we have assumed that the K^o scattering amplitudes on protons and neutrons are equal to each other, and the $\bar{K}^o$ amplitudes also. Thus, with the optical theorem,

$$\begin{aligned} \frac{f_P(o)}{k} &= \frac{f_N(o)}{k} = \frac{\sigma}{4\pi}(i + \alpha) \\ \frac{\bar{f}_P(o)}{k} &= \frac{\bar{f}_N(o)}{k} = \frac{\bar{\sigma}}{4\pi}(i + \bar{\alpha}) \end{aligned} \tag{9}$$

Using (7) with (8) and (9) we have calculated the coherent regeneration parameter ρ for a selection of elements from Li to Pb and have compared the ρ value thus obtained to the $\rho^{(1)}$ values obtained from (6) under simplification (9)

$$\rho^{(1)} = \frac{NA}{4}\{\sigma(i\alpha - 1) - \bar{\sigma}(i\bar{\alpha} - 1)\}\ \Lambda_s\ g\ (X) \tag{10}$$

Figure 5a gives the ratio $\eta = |\rho|/|\rho^{(1)}|$ as a function of $A^{1/3}$. In this calculation we have used $\sigma = 17.5$ mb, $\bar{\sigma} = 20$ mb, as suggested by the Serpukhov results and have set $\alpha = \bar{\alpha} = o$. Figure 5(a) shows that, for carbon, the mutual shielding of the nucleons in the nucleus reduces the effectiveness of each nucleon as a scatterer to 72% of the unshield value given by $\rho^{(1)}$.

We next investigate carbon in more detail. To eliminate the effect of the geometrical factor we write

$$\rho = R\ g(X) \quad \text{and} \quad \rho^{(1)} = R^{(1)}\ g(X) \tag{11}$$

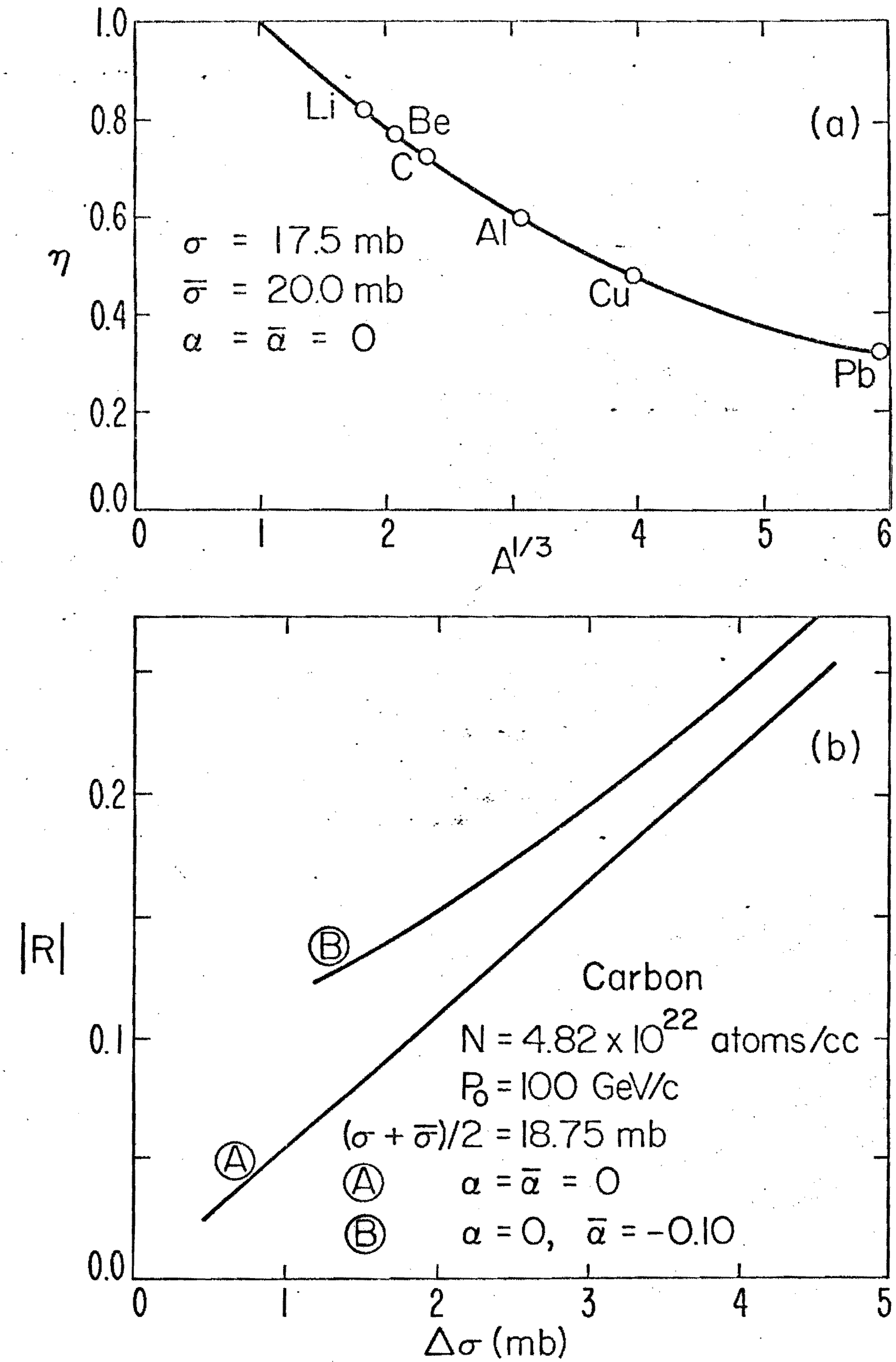

(a)
Li
Be
C
Al
Cu
Pb
σ = 17.5 mb
σ̄ = 20.0 mb
α = ᾱ = 0
η
A^{1/3}
(b)
|R|
Carbon
N = 4.82 x 10^22 atoms/cc
P_0 = 100 GeV/c
(σ + σ̄)/2 = 18.75 mb
A α = ᾱ = 0
B α = 0, ᾱ = -0.10
Δσ (mb)

Figure 5

and list $|R|$ and its phase Φ_R, as well as $|R^{(1)}|$ and $\Phi_R^{(1)}$, for selected σ, $\bar{\sigma}$ and α, $\bar{\alpha}$ pairs at 100 Gev/c in Table 1. It will be noticed that Φ_R is always close to $\Phi_R^{(1)}$ and the ratio $\eta = |R|/|R^{(1)}|$ remains close to 0.72. Figure 5(b) shows $|R|$ as a function of $\Delta\sigma = \bar{\sigma} - \sigma$ for $\alpha = \bar{\alpha} = o$ and for $\alpha = o$, $\bar{\alpha} = -0.10$ at 100 Gev/c. It should be noted that the dispersion claculations of Lusignoli et al[10] and Martin and Poole[11] suggest that α and $\bar{\alpha}$ values much larger than 0.1 are not expected. The main conclusion to be drawn from Table 1 is that carbon faithfully reproduces both the amplitude and the phase of a free mixture of an equal number of protons and neutrons. Its disadvantage is that an average over protons and neutrons is obtained.

To conclude this Section, we consider the number of diffraction regenerated K_S in carbon. The collision mean free path in scintillator is 67 cm and regenerators of that length are likely to be used (see Section IV). Thus equation (5) yields

$$\Delta n = 1.80 \times 10^5 \, \Delta\Omega_{lab} \qquad (5'')$$

at 100 Gev/c. Comparing this with the result for hydrogen (5$'$) we note that for a given resolution, the background due to diffraction is 12.4 times worse in carbon than in hydrogen but with a solid angle resolution $\Delta\Omega < 10^{-8}$ sterad both are negligible.

IV. Set-up, Trigger and Counting Rates:

We assume that the neutral beam used for the experiment is produced at 15 mrad by 200 Gev/c protons. The particle spectrum is then that given in Figure 3. The scale on the right gives "practical units";

Table 1. $R^{(1)}$ and R at 100 Gev/c for selected σ, $\bar{\sigma}$ and α, $\bar{\alpha}$ pairs. Note that a carbon density of 4.84×10^{22} atoms /cc was used.

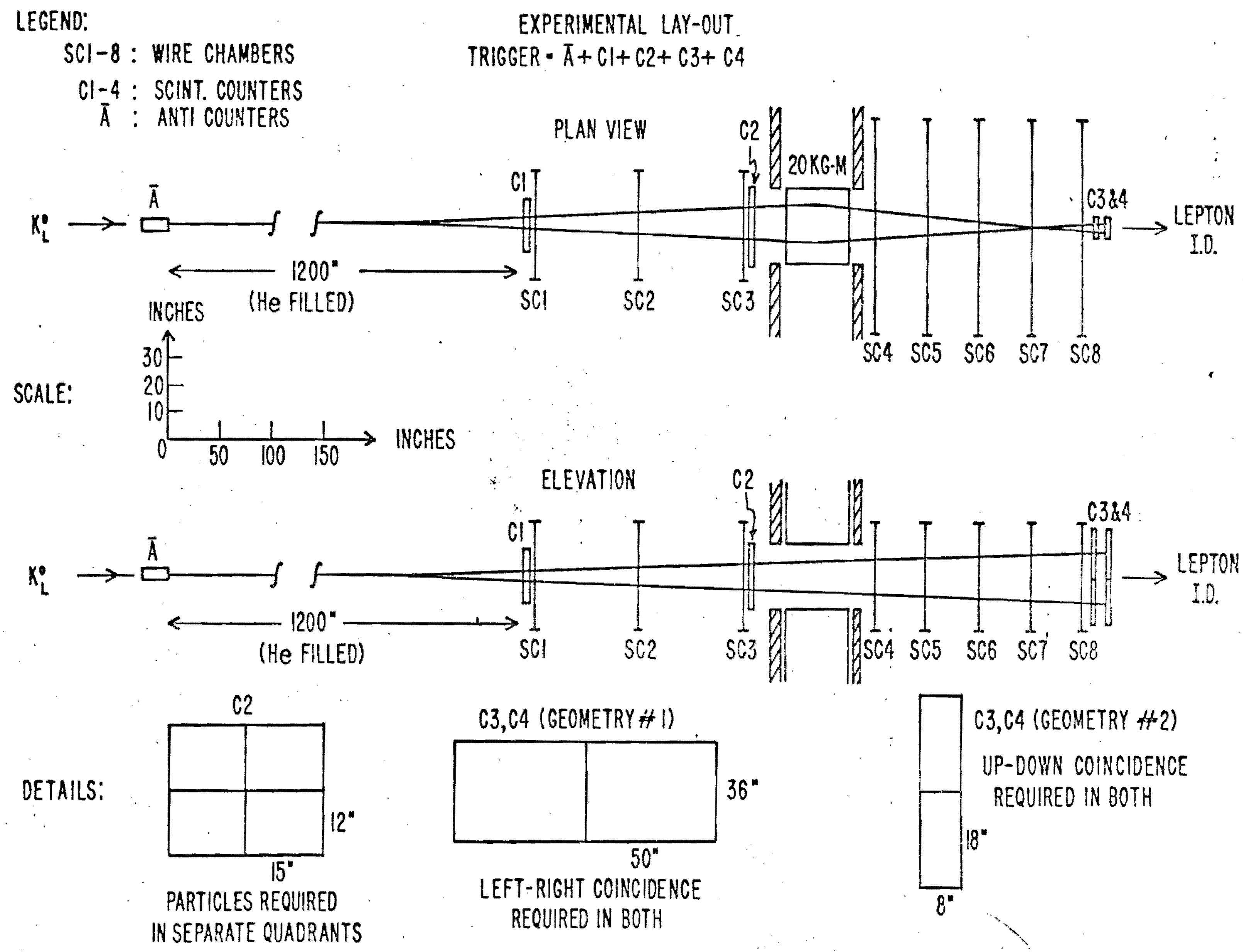
σ (mb)	$\bar{\sigma}$ (mb)	α	$\bar{\alpha}$	$\lvert R^{(1)} \rvert$	$\Phi_R^{(1)}$ (deg)	$\lvert R \rvert$	Φ_R (deg)	η
17.5	20.0	.00	.00	0.188	0	0.136	0	.725
18.0	19.5	.00	.00	0.113	0	0.082	0	.725
16.5	21.0	.00	.00	0.339	0	0.245	0	.724
17.5	20.0	-.05	.00	0.200	-19.3	0.144	-19.7	.723
18.0	19.5	-.05	.00	0.132	-30.9	0.095	-31.4	.723
16.5	21.0	-.05	.00	0.344	-10.4	0.249	-10.8	.724
17.5	20.0	.00	-.10	0.241	38.7	0.174	37.7	.721
18.0	19.5	.00	-.10	0.185	52.4	0.134	51.5	.724
16.5	21.0	.00	-.10	0.374	25.0	0.270	24.0	.724
17.5	20.0	-.10	-.10	0.189	5.7	0.137	3.9	.724
18.0	19.5	-.10	-.10	0.114	5.7	0.082	3.9	.720
16.5	21.0	-.10	-.10	0.340	5.7	0.246	3.9	.723

we assume 10^{12} protons and a 1 sq. in. beam defining hole 416 feet from the target. Under these circumstances the integrated neutron flux is 1.4×10^6 neutrons per pulse.

The apparatus is shown in Figure 6. As indicated in Section I most of its components have been used in recent SLAC experiments of the UCLA group. The beam impinges on a 1 collision mean free path long regenerator (either a 12.8 m long liquid hydrogen target or 67 cm of scintillator). The decay path is 100 ft. long and its downstream end is defined by counter C1. In addition a coincidence of any two of the four C2 as well as both C3 and both C4 are required for a coincidence. In our Monte Carlo trigger analysis we considered two C3-4 configuration. In geometry #1, C3 and 4 are large counters; in geometry #2, which is the one shown in Figure 6, they are "slats" 8 in. wide and 18 in. high. Lepton identifiers consisting of shower counters, e, for electrons, and counters behind a lead wall, μ, for muons, are placed beyond the apparatus and are in anticoincidence to remove the bulk of the leptonic decays which succeed in giving ($\bar{A}$, C1, C2, C3, C4) coincidences. Thus a complete trigger T consists of ($\bar{A}$, C1, C2, C3, C4, $\bar{e}$, $\bar{\mu}$). The coincidence ($\bar{A}$, C1, C2, C3, C4) will be called a t-trigger.

A trigger T fires the wire chambers SC1 - 8 which record the trajectories of the decay particles both upstream and downstream from a 20 kilogauss-m analyzing magnet. This magneti is adjusted to have a 24 in. high, 30 in. wide aperture. The overall length of the apparatus is 150 ft.

Given a net flux of 1.4×10^6 neutrons per pulse and a regenerator whose length represents 1 collision mean free path, $\sim 10^6$ interactions will occur in it per pulse. For a spill length of 1 sec, this gives



LEGEND:
SC1-8 : WIRE CHAMBERS
C1-4 : SCINT. COUNTERS
Ā : ANTI COUNTERS
EXPERIMENTAL LAY-OUT
TRIGGER = Ā + C1 + C2 + C3 + C4
PLAN VIEW
C2
20 KG-M
C1
C3&4
K_L^0
Ā
LEPTON I.D.
1200"
(He FILLED)
SC1
SC2
SC3
SC4
SC5
SC6
SC7
SC8
INCHES
SCALE:
30
20
10
0
50
100
150
INCHES
ELEVATION
C2
C1
C3&4
K_L^0
Ā
LEPTON I.D.
1200"
(He FILLED)
SC1
SC2
SC3
SC4
SC5
SC6
SC7
SC8
DETAILS:
C2
12"
15"
PARTICLES REQUIRED IN SEPARATE QUADRANTS
C3,C4 (GEOMETRY #1)
36"
50"
LEFT-RIGHT COINCIDENCE REQUIRED IN BOTH
C3,C4 (GEOMETRY #2)
UP-DOWN COINCIDENCE REQUIRED IN BOTH
18"
8"

Figure 6

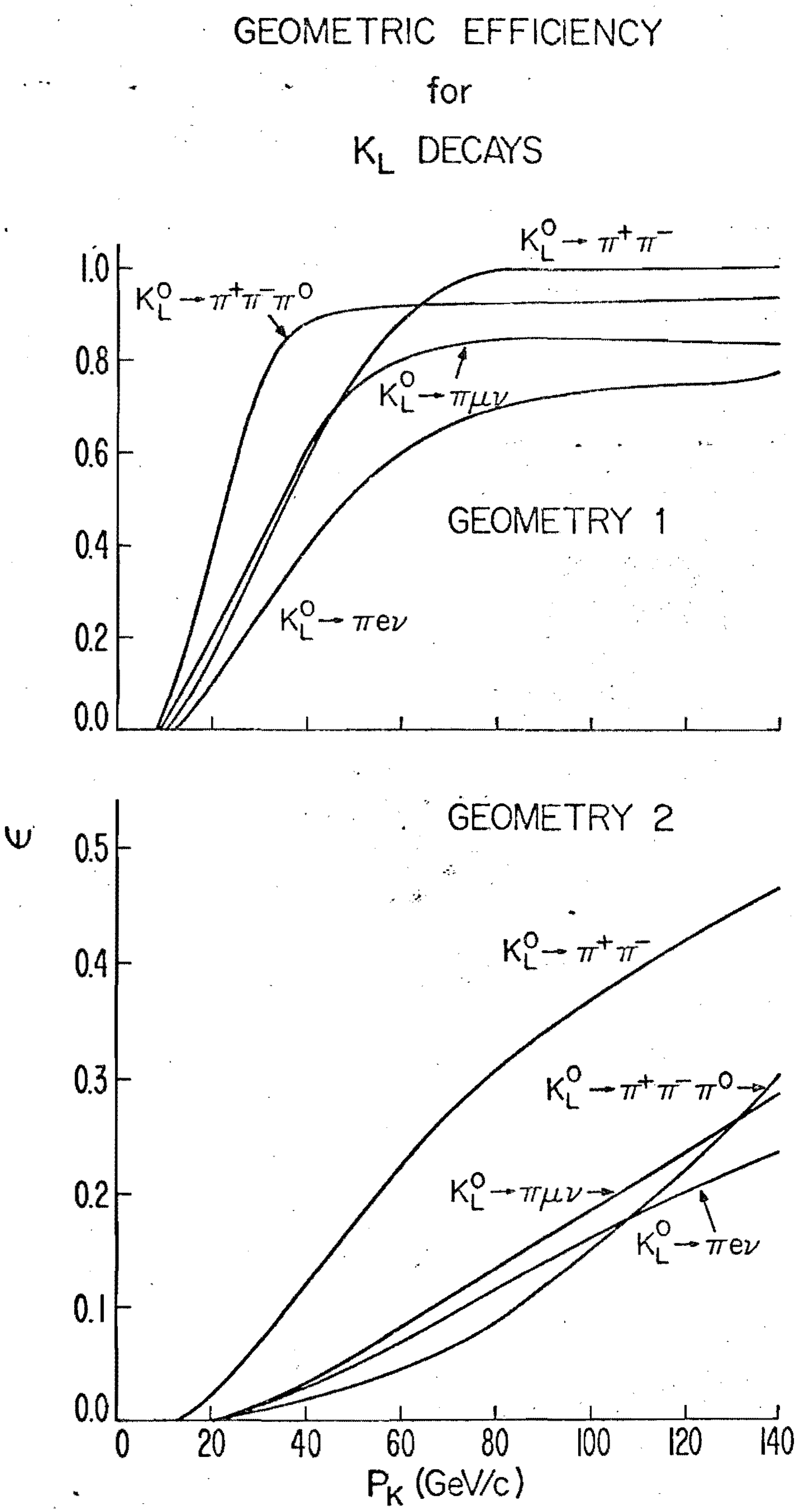
a rate of ~1 per μsec which is readily tolerated by the electronic logic system. However, the flux of secondaries from these interactions may make the operation of SC1 - 3 difficult and they may have to be replaced by proportional chambers.

Before discussing the trigger efficiencies it should be emphasized that no attempt was made at optimization of either the geometry of the apparatus or of the prescribed trigger. We are here more concerned with an "existence proof" - we have drawn an arrangement based on our SLAC experience and proceeded to calculate via using the Monte Carlo program which has been tested using our SLAC data.

The overall t-trigger efficiencies (i.e., assuming the absence of lepton identifiers) obtained by means of Monte Carlo techniques for the two C3C4 counter arrangements are shown in Figure 7 as functions of the K_L momentum. We note that for geometry #1, above 60 Gev/c, all efficiencies are larger than 60%. In geometry #2, three body decays are suppressed relative to $\pi^+\pi^-$ decay by a factor of ~2 but there is also a factor ~3 loss in $\pi^+\pi^-$ decays in the momentum range of 60 - 120 Gev/c.

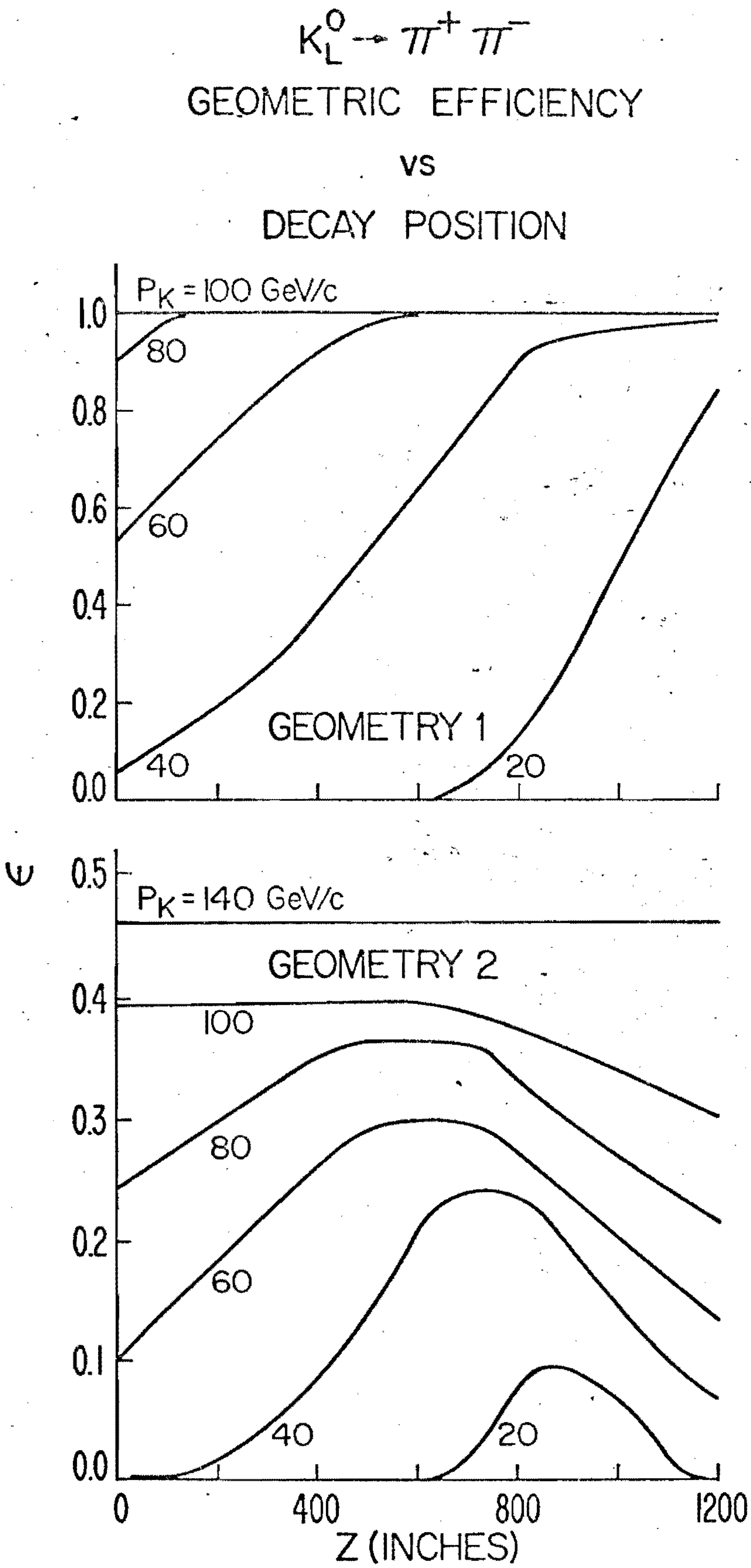
Figure 8 shows the $K \rightarrow \pi^+\pi^-$ decay detection efficiencies as functions of the position of the decay vertex along the decay volume.

In Table 2 we present the results of our calculation of the number of t-trigger per machine pulse based on the spectrum of K_L mesons shown in Figure 3 (right hand scale), the apparatus defined in Figure 6 and the efficiencies given in Figures 7 and 8. The rates with regenerator, given in the last column for each geometry, were calculated using the x-distribution of decays given in equation (3) with $\exp(-N\sigma L) = e^{-1}$, $\rho = 16\eta = 3.04 \times 10^{-2}$ and $\phi_\rho = o$ and the efficiencies as given in Figure 8.



GEOMETRIC EFFICIENCY
for
K_L DECAYS
$K_L^0 \to \pi^+\pi^-$
$K_L^0 \to \pi^+\pi^-\pi^0$
$K_L^0 \to \pi\mu\nu$
$K_L^0 \to \pi e\nu$
GEOMETRY 1
GEOMETRY 2
ϵ
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.4
0.3
0.2
0.1
0.0
0
20
40
60
80
100
120
140
P_K (GeV/c)

Figure 7



$K_L^0 \to \pi^+ \pi^-$
GEOMETRIC EFFICIENCY
VS
DECAY POSITION
P_K = 100 GeV/c
80
60
40
20
GEOMETRY 1
1.0
0.8
0.6
0.4
0.2
0.0
ε
0.5
P_K = 140 GeV/c
GEOMETRY 2
100
80
60
40
20
0.4
0.3
0.2
0.1
0.0
0
400
800
1200
Z (INCHES)

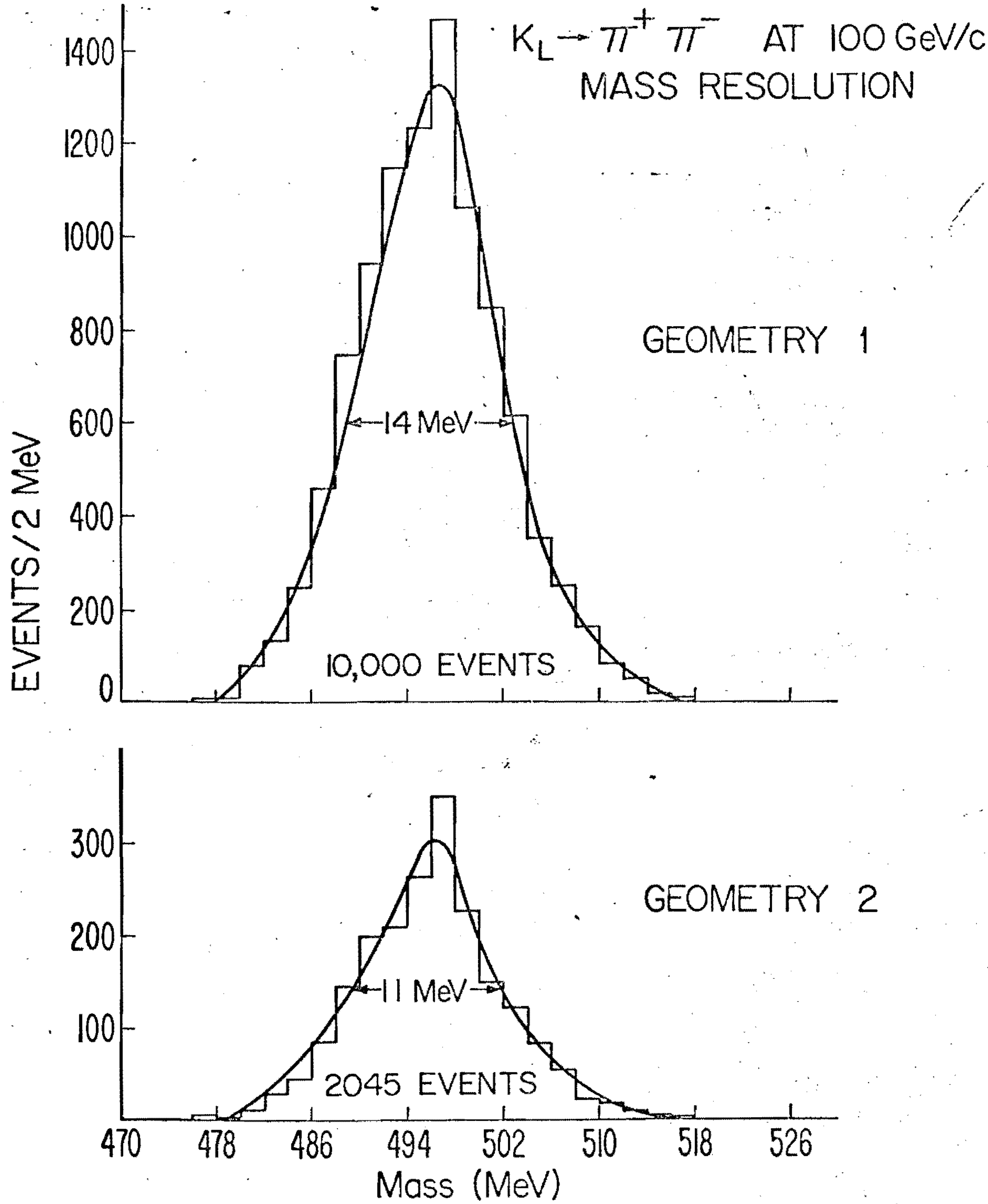
Figure 8

Table 2. t-trigger Rates per Pulse without lepton identifiers based on the Spectrum of Figure 3, setup of Figure 5, and Detection Efficiencies of Figures 7 and 8.

Momentum Range (Gev/c)	Geometry # 1			Geometry # 2		
	3-body decays	$K \to \pi^+\pi^-$		3-body decays	$K \to \pi^+\pi^-$	
		no Reg	with Reg		no Reg	with Reg
10 - 30	2320	3.75	1.38	20.7	0.576	0.212
30 - 50	700	1.47	2.82	36.9	0.309	0.268
50 - 70	121	0.296	2.28	11.9	0.075	0.577
70 - 90	17	0.042	0.68	2.44	0.013	0.199
90 - 110	1.6	0.004	0.09	0.34	0.0013	0.032
110 - 130	0.1	0.0003	0.007	0.03	0.0001	0.003
TOTAL	3159.7	5.55	7.26	72.31	0.975	1.29

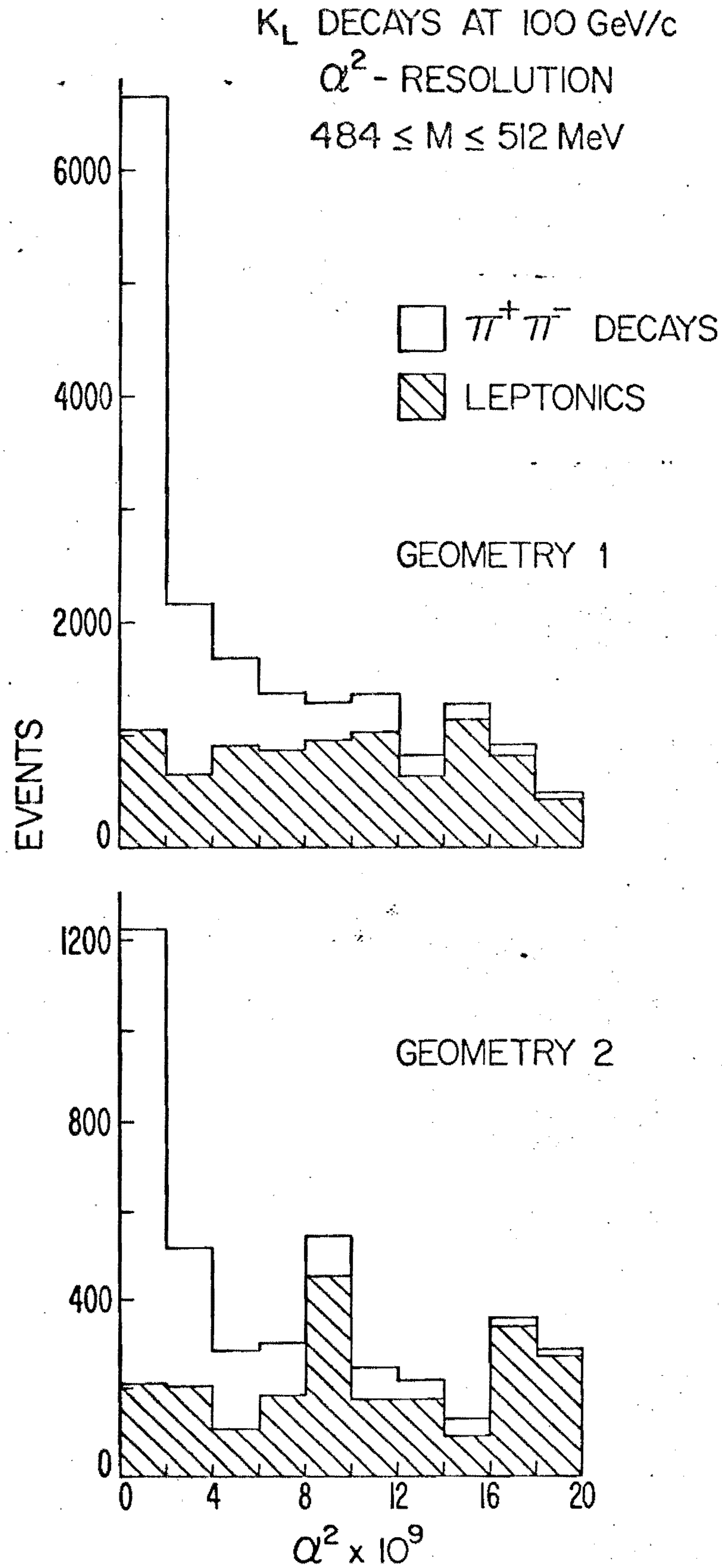
The Table suggests that geometry #1 is unacceptable: it shows that 3200 t-triggers due to three-body decays are expected per pulse. Even if we assume that the lepton identifiers are 97% efficient in suppressing triggers arising from three-body decays (which is probably optimistic) this trigger rate would still be close to 100 per pulse. On the other hand, assuming a 97% efficiency for the rejection of three-body events, the total rates per pulse shown for geometry #2 are quite acceptable. We have also computed the t-trigger rates due to diffraction $K_L \rightarrow K_S$ scattering and K_S-production by neutrons. Combined they give less than 2 triggers per pulse.

In Figure 9 we show the Monte Carlo results for the resolution in the invariant mass of the $\pi^+\pi^-$ pair at 100 Gev/c. The calculation shows that for geometry #2, an 11 Mev wide mass peak can be achieved. The input errors used in the Monte Carlo routines are based on our experience with our "wire chamber package" at SLAC, scaled to take account of the prposed separation between the chambers. In Figure 10 we show the distribution in α^2, the square of the angle in radians between the nominal beam direction and the measured direction of flight of the particle (assuming it suffered $\pi^+\pi^-$ decay). Leptonic decays were treated in the same manner as $\pi^+\pi^-$ decays and were added in the proper proportion. Only events yielding an invariant mass in the range $484 \leq M_{\pi^+\pi^-} \leq 512$ Mev are plotted in Figure 10. Furthermore, Figure 10 shows the "worst" situation: no regenerator (i.e., $\pi^+\pi^-$ decays due to CP violation only) and lepton identifiers inoperative. The resolution of $\pi^+\pi^-$ events from the leptonic background is clearly adequate and confirms the statement made several times before that coherently regenerated events lie within $\Delta\Omega < 10^{-8}$ rad.



$K_L \to \pi^+ \pi^-$ AT 100 GeV/c
MASS RESOLUTION
EVENTS/2 MeV
1400
1200
1000
800
600
400
200
0
GEOMETRY 1
14 MeV
10,000 EVENTS
300
200
100
GEOMETRY 2
11 MeV
2045 EVENTS
470
478
486
494
502
510
518
526
Mass (MeV)

Figure 9



K_L DECAYS AT 100 GeV/c
α^2 - RESOLUTION
$484 \leq M \leq 512$ MeV
$\pi^+\pi^-$ DECAYS
LEPTONICS
GEOMETRY 1
GEOMETRY 2
EVENTS
6000
4000
2000
0
1200
800
400
0
0
4
8
12
16
20
$\alpha^2 \times 10^9$

Figure 10

V. Precision of Final Results:

We have carried out a Monte Carlo study of the precision with which $|\rho|$ and ϕ_ρ can be determined from the $\pi^+\pi^-$ decay data we hope to secure. In this study we restrict ourselves to the 90 - 110 Gev/c momentum interval and consider only geometry #2. Table 2 shows that 1.3×10^{-3} decays are expected in this momentum interval per pulse with the regenerator removed and 3.2×10^{-2} with the regenerator in. In the Monte Carlo calculation we envisage a total running time of one month. Thus for a 1:1 division of the time we expect 420 events with the regenerator removed and 10300 events with the regenerator in situ. One of the first results of the Monte Carlo study was that a run with the regenerator removed was necessary in order to secure a normalization for the decay curve.

In the Monte Carlo study decays were generated along the decay path according to equation (3) with assumed values of ρ and ϕ_ρ, sorted into bins 2.5m long and then analyzed by χ^2 minimization procedures to redetermine ρ and ϕ_ρ and to calculate the expected errors. The results of various trials are shown in Table 3. There columns (a) to (e) list assumed experimental conditions, (f) and (g) give the standard deviations in $|\rho|/|\eta|$ and ϕ_ρ and finally (h) lists the $\Delta\sigma = \bar{\sigma} - \sigma$ values calculated from the assumed values of $|\rho|/|\eta|$ and ϕ_ρ according to equation (2) along with the expected standard deviation based on our statistical analysis. For carbon, column (h) should be divided by 0.72. A study of Table 3, Set A, reveals that in the arrangement which we regard as standard - a 1:1 regenerator in, regenerator out time distribution, a 100 foot decay path and 1 collision mean free path of regenerator, we can measure $\Delta\sigma$ to a precision of 3%

Table 3. Uncertainties $\Delta\left|\frac{\rho}{\eta}\right|$ and $\Delta\phi_\rho$ in the determination of $|\rho|/|\eta|$ and ϕ_ρ and $\Delta\sigma = \bar{\sigma} - \sigma$ for various experimental circumstances, all based on one month total running time.

	(a) $\frac{\lvert\rho\rvert}{\lvert\eta\rvert}$	(b) ϕ_ρ	(c) Reg. in / Reg. out	(d) Decay Path (in)	(e) Reg. Length (coll. mfp's)	(f) $\Delta\frac{\lvert\rho\rvert}{\lvert\eta\rvert}$	(g) $\Delta\phi_\rho$ (°)	(h) $\Delta\sigma$ (mb)
A	16	0	1:1	1200	1	0.44	4.2	3.16 ± .09
	8	0	1:1	1200	1	0.26	5.8	1.58 ± .06
	4	0	1:1	1200	1	0.18	8.4	0.79 ± .03
	1	0	1:1	1200	1	0.22	19	0.20 ± .04
B	4	0	1:1	1200	1/4	0.12	9.0	3.16 ± .10
	2	0	1:1	1200	1/4	0.12	14	1.58 ± .09
	1	0	1:1	1200	1/4	0.14	22	0.79 ± .12
	0.5	0	1:1	1200	1/4	0.16	34	0.41 ± .13
C	16	0	1:1	1200	1	0.44	4.2	3.16 ± .09
	16	60	1:1	1200	1	0.44	3.4	1.58 ± .04
	16	120	1:1	1200	1	0.38	4.4	1.58 ± .04
	16	180	1:1	1200	1	0.36	3.4	3.16 ± .07
	16	240	1:1	1200	1	0.36	5.0	1.58 ± .03
	16	300	1:1	1200	1	0.50	10	1.58 ± .04
D	4	0	1:1	1200	1/4	0.12	9.0	3.16 ± .10
	8	0	1:1	1200	1/2	0.20	4.6	3.16 ± .09
	16	0	1:1	1200	1	0.44	4.2	3.16 ± .09
	32	0	1:1	1200	2	1.36	6.2	3.16 ± .13
E	16	0	1:1	800	1	0.54	7.6	3.16 ± .10
	16	0	1:1	1200	1	0.44	4.2	3.16 ± .09
	16	0	1:1	1600	1	0.38	3.6	3.16 ± .07
F	16	0	2:1	1200	1	0.46	3.8	3.16 ± .09
	16	0	1:1	1200	1	0.44	4.2	3.16 ± .09
	16	0	1:2	1200	1	0.46	5.2	3.16 ± .09

if it is 3 mb, to 20% if it is 0.3 mb. In the same $\Delta\sigma$ range the error in the determination of the phase climbs from 4° to 20°. The remainder of the sets are intended to show that the parameter which we can control are not far from their optimal values.

It is perhaps worth emphasizing that the calculations were carried out for the 90 - 110 Gev/c range only. Going back to Table 2 and recognizing that, according to Figure 8, kaons with momentum larger than 50 Mev/c are detected with adequate sensitivity behind the regenerator to make them useful for regeneration studies, we expect a total of ~30,000 useful $K^+ \rightarrow \pi^+\pi^-$ decays without regenerator and ~250,000 with regenerator. Assuming a lepton rejection with 95% efficiency, these events will come accompanied by 4×10^6 triggers.

VI. Conclusion:

We conclude that the experiment is possible. We would be grateful for the opportunity to carry it out.

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